



Isospin-Driven Splitting of Chemical Potentials in Isobar Collisions from Lattice QCD

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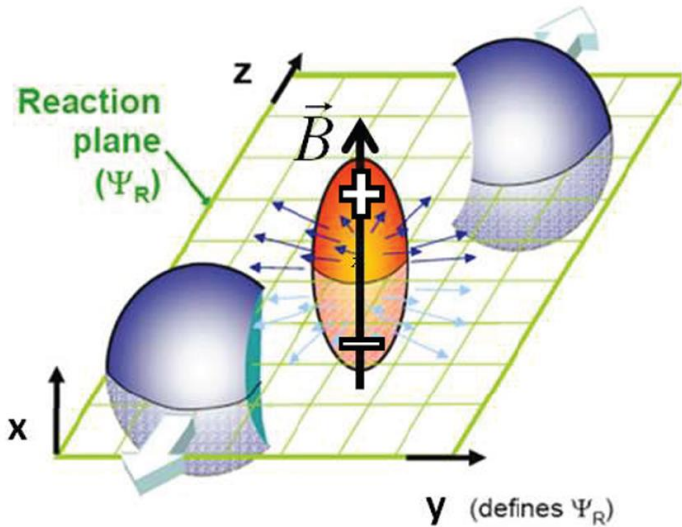
In collaboration with Heng-Tong Ding(丁亨通), Jin-Biao Gu(顾锦彪) and Arpith
Kumar

Based on arXiv:2606.30164 [hep-lat]

Spicy Gluons

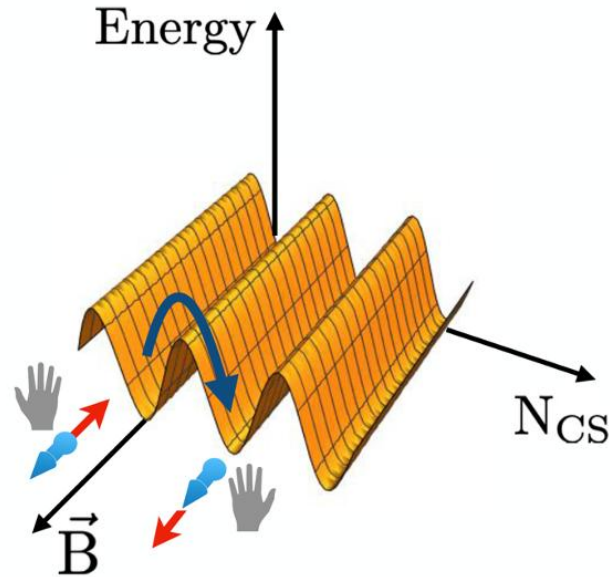
Wuhan · October 2026

Strong magnetic fields and the chiral magnetic effect

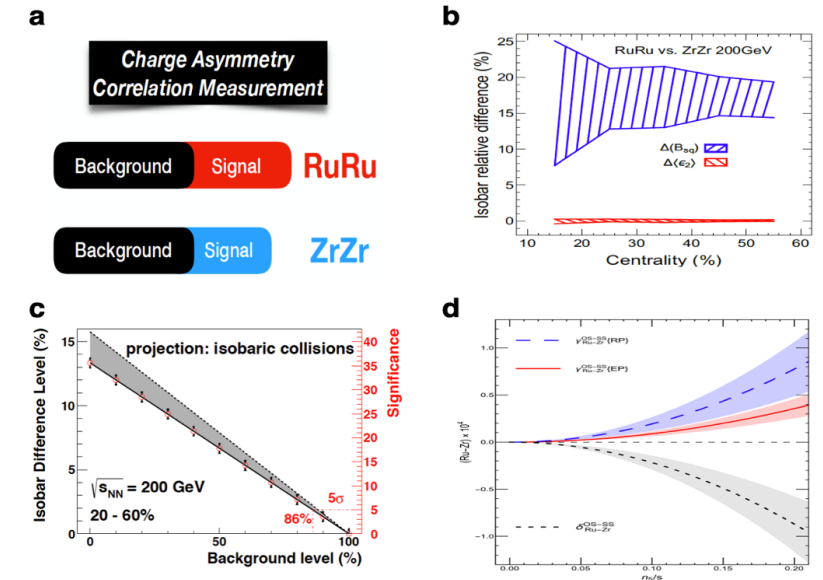


$10^{17} - 10^{18} \text{G}$ orders of Λ_{QCD}^2

V. Skokov, A. Illarionov, V. Toneev,
IJMPA 24 (2009) 592



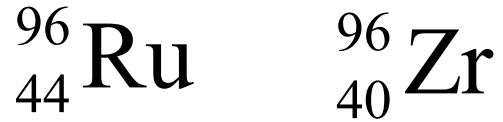
D.E. Kharzeev and J. Liao, Nature Rev.
Phys. 3 (2021) 55 [2102.06623].



D.E. Kharzeev and J. Liao,
Nature Rev. Phys. 3 (2021) 55
[2102.06623].

The chemical-potential splittings in STAR isobar data

Nuclear composition



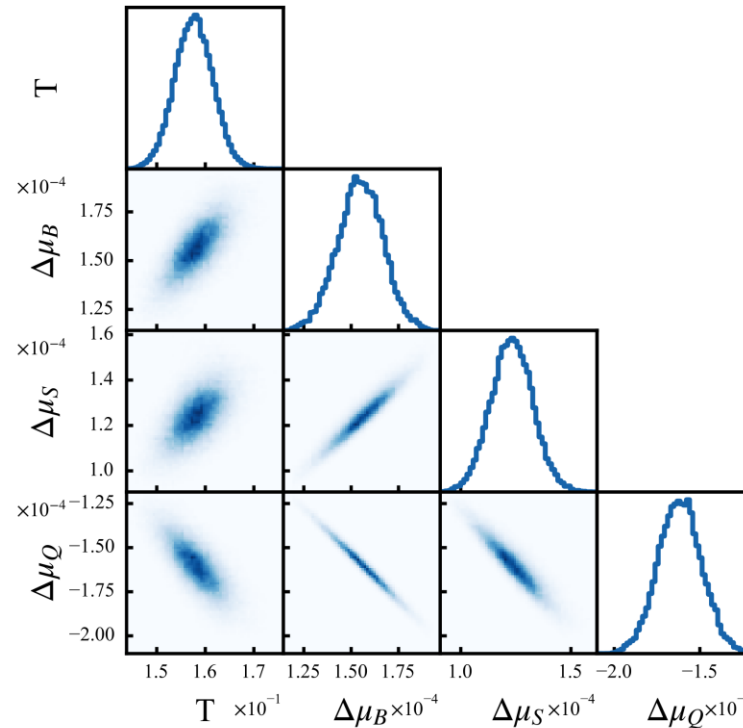
charge-to-baryon ratio

$$r \equiv \frac{n_Q}{n_B} \simeq \frac{Z}{A} \quad r_{\text{Ru}} \simeq 0.458 \quad r_{\text{Zr}} \simeq 0.417$$

chemical potential splittings

$$\Delta\mu_B, \Delta\mu_Q, \Delta\mu_S$$

Relate to Zr-Ru Splitting Ratios in Isobar Experiments



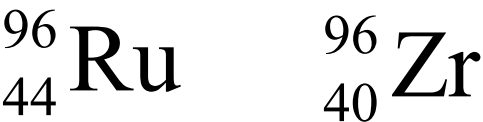
$\Delta\mu_B$	$0.156^{+0.011}_{-0.012}$
$\Delta\mu_S$	$0.124^{+0.01}_{-0.01}$
$\Delta\mu_Q$	$-0.161^{+0.013}_{-0.012}$
$\frac{\Delta\mu_B}{\Delta\mu_Q}$	$-0.968^{+0.011}_{-0.012}$
$\frac{\Delta\mu_S}{\Delta\mu_Q}$	$-0.769^{+0.021}_{-0.026}$
$\frac{\Delta\mu_B}{\Delta\mu_S}$	$1.259^{+0.028}_{-0.032}$

0~10% centrality class
Bayesian analyses posterior

J. Grefa, C.Y. Tsang, R. Kumar, V. Dexheimer, C. Ratti, Z. Xu; arXiv:2601.21232 [nucl-th]

The chemical-potential splittings in STAR isobar data

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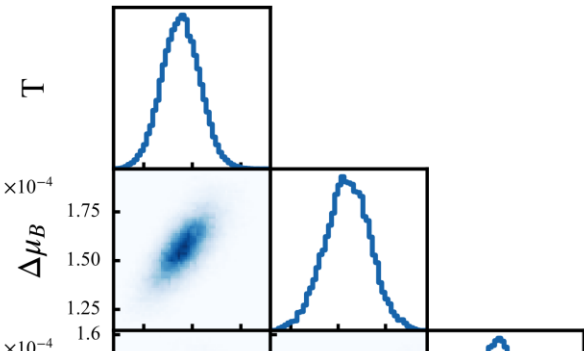
$$r \equiv \frac{n_Q}{n_B} \simeq \frac{Z}{A}$$

$r_{\text{Ru}} \simeq 0.458$
 $r_{\text{Zr}} \simeq 0.417$

chemical potential splittings

$$\Delta\mu_B, \Delta\mu_Q, \Delta\mu_S$$

Relate to Zr-Ru Splitting Ratios in Isobar Experiments



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	$0.59^{+0.028}_{-0.032}$

Can we provide a first-principles QCD benchmark for these isospin-driven splittings ?

quality class

Bayesian analyses posterior

J. Grefa, C.Y. Tsang, R. Kumar, V. Dexheimer,C. Ratti, Z. Xu; arXiv:2601.21232 [nucl-th]

From conserved-charge fluctuations to q_1 and s_1

Heavy-ion constraints

$$n_S = 0$$

$$n_Q = rn_B$$

$$r \simeq Z/A$$

Lattice-QCD input

$$\chi_{ijk}^{\text{BQS}}(T, eB) = \left. \frac{\partial^{i+j+k} (p / T^4)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\hat{\mu}_B = \hat{\mu}_Q = \hat{\mu}_S = 0}$$

Solve the constraints at leading order

$$\hat{\mu}_Q = q_1(r, T, eB) \hat{\mu}_B + q_3(r, T, eB) \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5)$$

$$\hat{\mu}_S = s_1(r, T, eB) \hat{\mu}_B + s_3(r, T, eB) \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5)$$

From q_1 and s_1 to splitting ratios

At common T and eB

Zr–Ru convention

$$\Delta\mu_X \equiv \mu_X(r_{\text{Zr}}) - \mu_X(r_{\text{Ru}})$$

One algebraic step

$$\begin{aligned}\Delta\mu_X &= c_X(r_{\text{Zr}})\Delta\mu_B \\ &+ [c_X(r_{\text{Zr}}) - c_X(r_{\text{Ru}})] \\ c_Q &= q_1, \quad c_S = s_1\end{aligned}$$

The splitting ratios

$$\frac{\Delta\mu_Q}{\Delta\mu_B} = q_1(r_{\text{Zr}}) + \Delta q_1 x$$

$$\frac{\Delta\mu_S}{\Delta\mu_B} = s_1(r_{\text{Zr}}) + \Delta s_1 x$$

$$\Delta q_1 = q_1(r_{\text{Zr}}) - q_1(r_{\text{Ru}})$$

$$\Delta s_1 = s_1(r_{\text{Zr}}) - s_1(r_{\text{Ru}})$$

Nuclear composition:

$$r_{\text{Zr}}, r_{\text{Ru}}$$

Lattice QCD:

$$q_1, s_1, \Delta q_1, \Delta s_1$$

Bayesian input:

$$x = \mu_B(r_{\text{Ru}}) / \Delta\mu_B$$

Lattice Setup

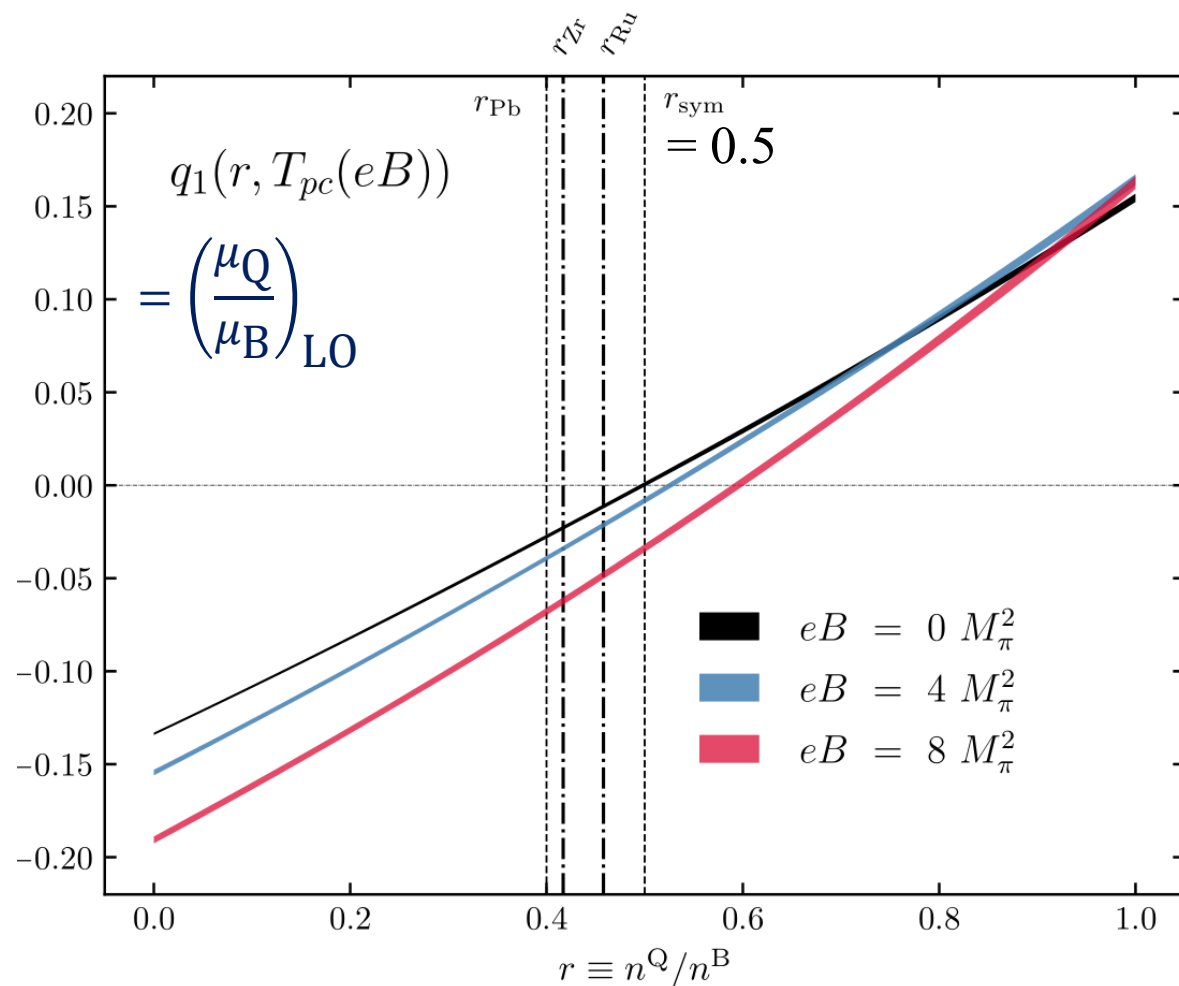
2+1 flavor QCD on Lattice

- Highly improved staggered fermions and a tree-level improved Symanzik gauge action
- physical pion mass : $m_l = m_s / 27$
- Lattice : $32^3 \times 8$ and $48^3 \times 12 \rightarrow \text{cont.est.}$

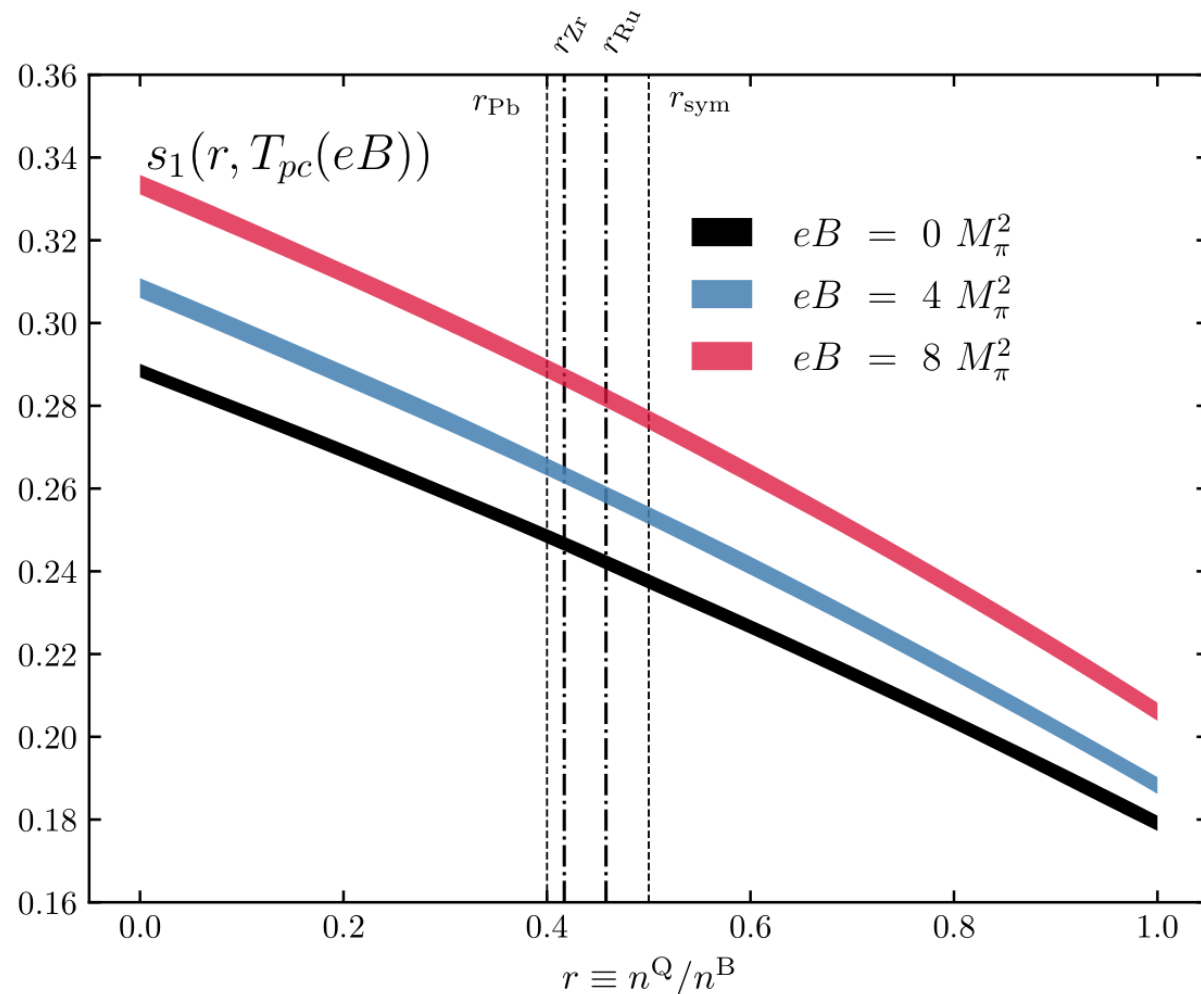
simulated T and eB range

- Temperature: along the $T_{pc}(eB)$ at finite eB
- Magnetic field : $0 \leq eB \lesssim 8M_\pi^2 \sim [0, 0.15] \text{ GeV}^2$

r –dependence of q_1 and s_1 along $T_{pc}(eB)$

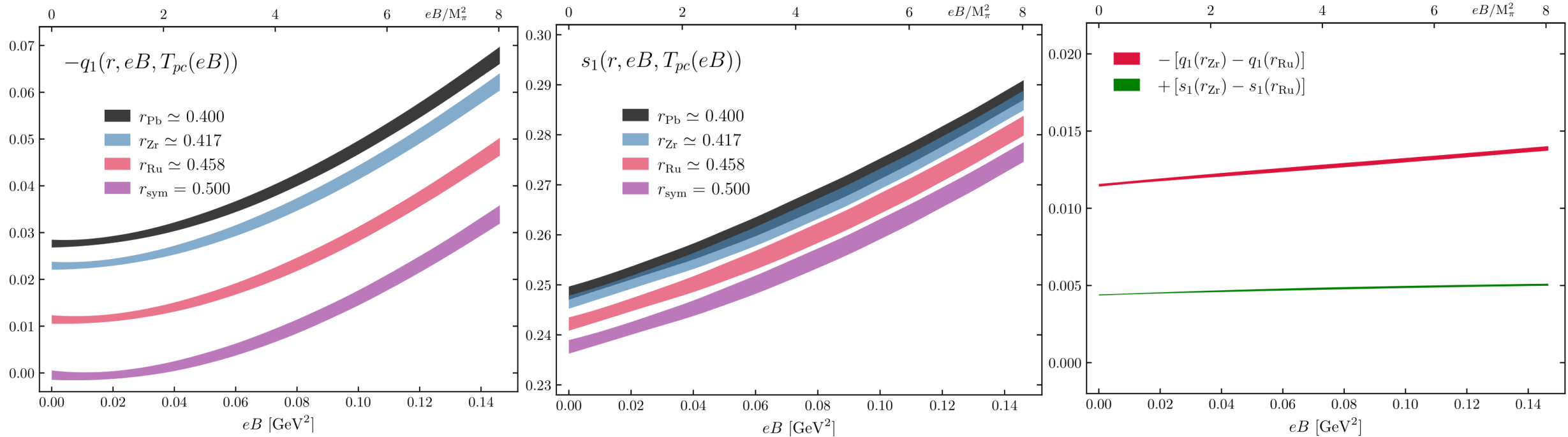


$$|q_1(r_{Pb})| > |q_1(r_{Zr})| > |q_1(r_{Ru})|$$



$$s_1(r_{Pb}) > s_1(r_{Zr}) > s_1(r_{Ru}) > 0$$

eB –dependence at fixed isospin parameter r



$$\frac{\Delta\mu_Q}{\Delta\mu_B} = q_1(r_{\text{Zr}}) + [q_1(r_{\text{Zr}}) - q_1(r_{\text{Ru}})] \frac{\mu_B(r_{\text{Ru}})}{\Delta\mu_B}$$

$$\frac{\Delta\mu_S}{\Delta\mu_B} = s_1(r_{\text{Zr}}) + [s_1(r_{\text{Zr}}) - s_1(r_{\text{Ru}})] \frac{\mu_B(r_{\text{Ru}})}{\Delta\mu_B}$$

Intercepts : $-q_1$ grows strongly, while s_1 grows mildly.

Slopes : $-\Delta q_1$ increases mildly, while Δs_1 is nearly eB -independent.

QMHRG baselines & STAR-cut proxies within PDGHRG

Fluctuations expressed in terms of hadron and resonance species R:

$$\chi_{ijk}^{\text{BQS}}(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S) = \sum_R B_R^i Q_R^j S_R^k \frac{\partial^{i+j+k} p_R / T^4}{\partial \hat{\mu}_R^{i+j+k}} \Rightarrow \begin{matrix} q_1^{\text{QMHRG}} \\ s_1^{\text{QMHRG}} \end{matrix} \Rightarrow \left\{ \frac{\Delta\mu_Q}{\Delta\mu_B}, \frac{\Delta\mu_S}{\Delta\mu_B}, \frac{\Delta\mu_S}{\Delta\mu_Q} \right\}^{\text{QMHRG}}$$

R. Bellwied et al., Phys. Rev. D 101, 034506 (2020)

π, K, p proxy

$$\tilde{B} \equiv \tilde{p}$$

$$\tilde{Q}_{\text{PID}} \equiv \tilde{\pi} + \tilde{K}^+ + \tilde{p}$$

$$\tilde{S} \equiv \tilde{K}^+$$

Λ -extended proxy

$$\tilde{B} \rightarrow \tilde{p} + \tilde{\Lambda}$$

$$\tilde{Q}_{\text{PID}} \equiv \tilde{\pi} + \tilde{K}^+ + \tilde{p}$$

$$\tilde{S} \rightarrow \tilde{K}^+ - \tilde{\Lambda}$$

STAR acceptance

$$|y| < 0.5$$

species-dependent p_T cut:

$$0.2 < p_T < 2.5 \text{ GeV} / c \quad \text{for } \pi, K,$$

$$0.45 < p_T < 2.5 \text{ GeV} / c \quad \text{for } p,$$

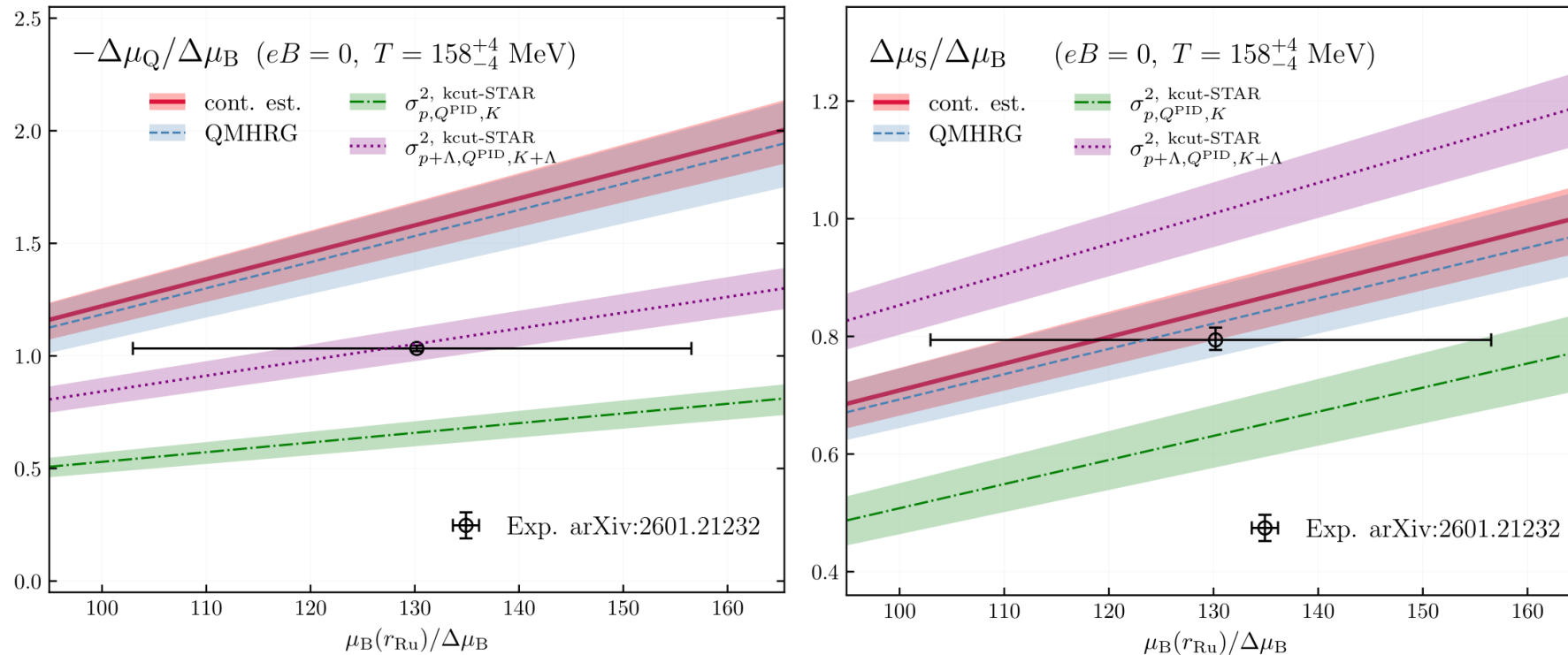
$$0.4 < p_T < 1.6 \text{ GeV} / c \quad \text{for } \Lambda.$$

STAR-cut proxies

$$\{\pi, K, p\} : \sigma_{p, Q^{\text{PID}}, K}^{2, \text{kcute-STAR}}$$

$$\{\pi, K, p, \Lambda\} : \sigma_{p+\Lambda, Q^{\text{PID}}, K+\Lambda}^{2, \text{kcute-STAR}}$$

Isospin-driven splitting ratios at vanishing magnetic field



The electric-charge ratio is displayed as $-\Delta\mu_Q/\Delta\mu_B$

Bayesian thermal-analysis extraction:

$$\mu_B(r_{Ru}) = 20.31^{+3.8}_{-4.0} \text{ MeV}$$

$$\Delta\mu_B = 0.156^{+0.011}_{-0.012} \text{ MeV}$$

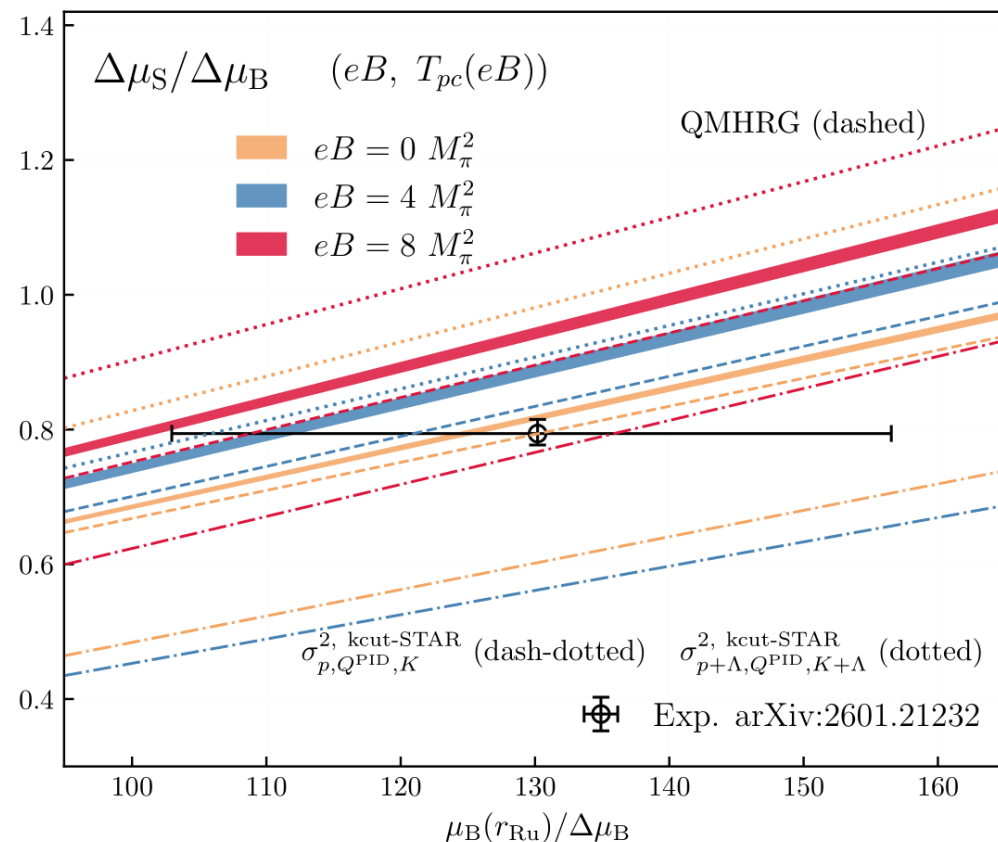
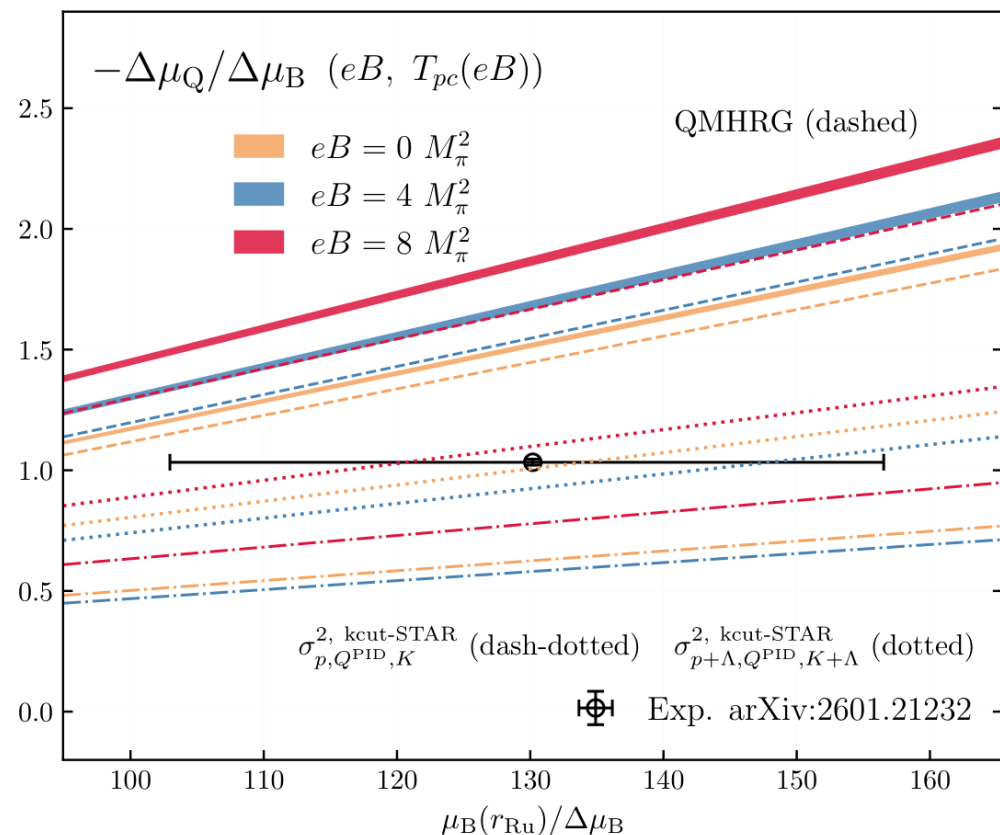
$$\frac{\mu_B(r_{Ru})}{\Delta\mu_B} \simeq 130$$

LQCD $\Delta\mu_Q < 0$, $\Delta\mu_S > 0$; $\Delta\mu_S/\Delta\mu_B$ is closest to the Bayesian agreement.

QMHRG Follows the lattice results trend in the Bayesian window.

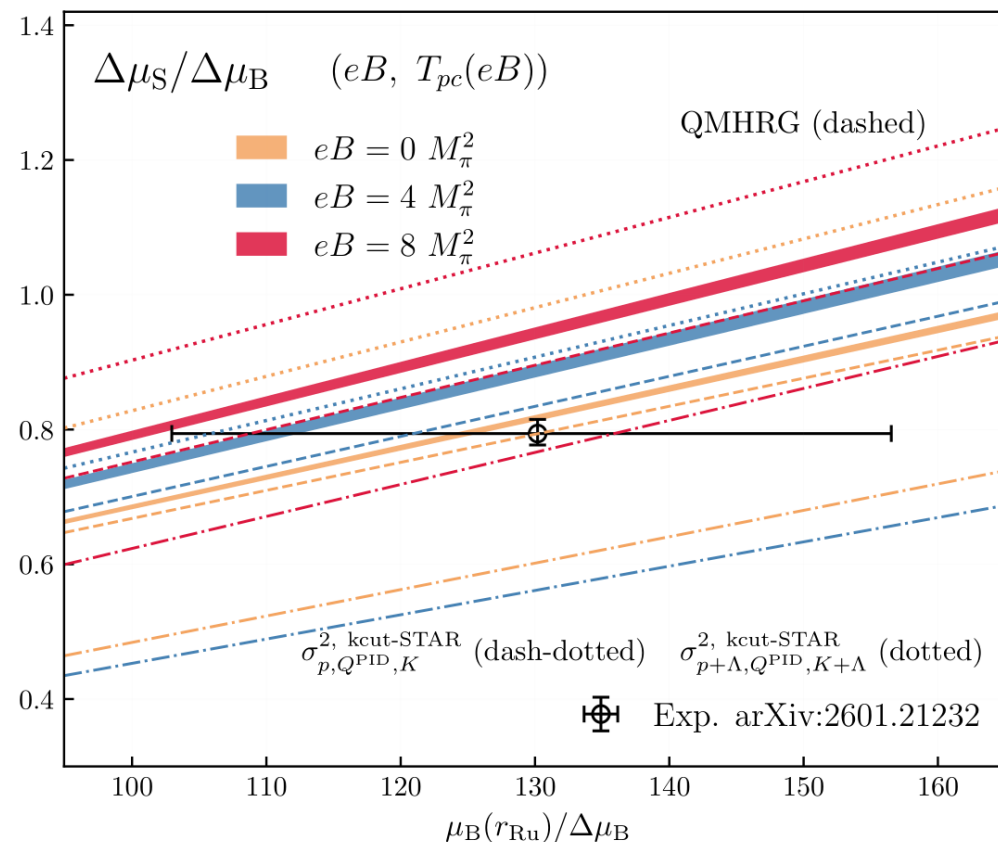
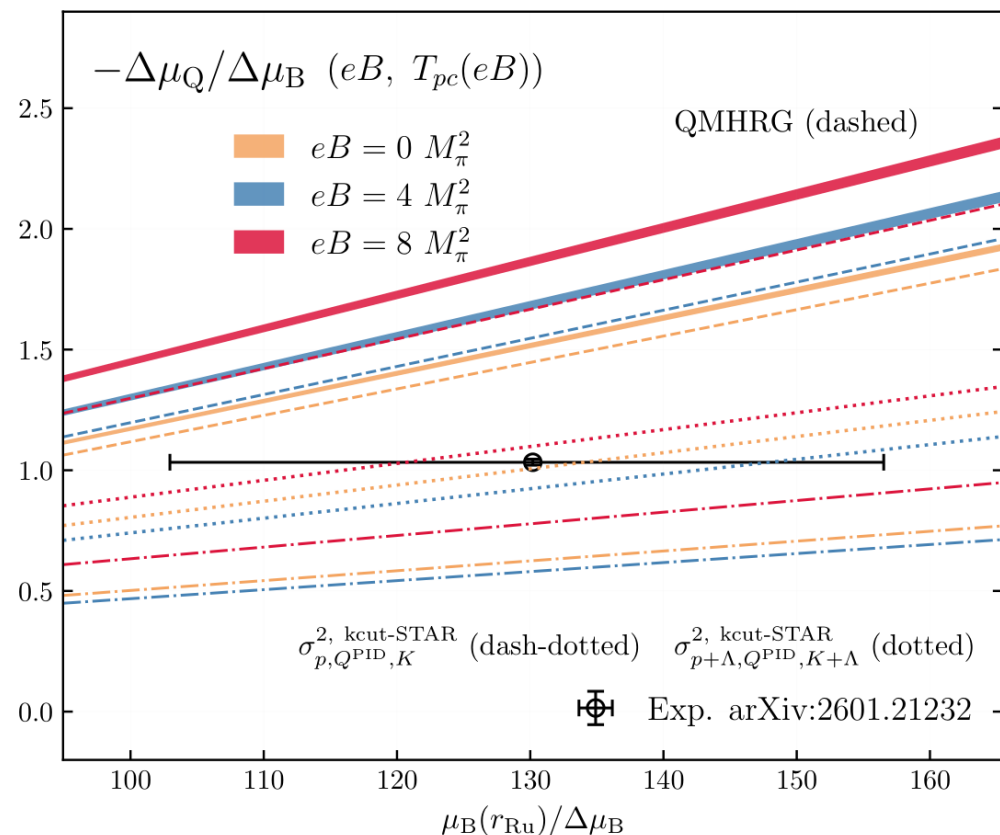
Proxies Adding Λ improves the charge ratio, but overshoots the strangeness ratio.

Magnetic fields modify the splitting ratio



- LQCD** Both ratios increase with eB ; the charge sector responds more strongly.
- QMHRG** Follows the lattice overall trend.
- Proxies** Show sizeable offsets from full conserved-charge results.

Magnetic fields modify the splitting ratio



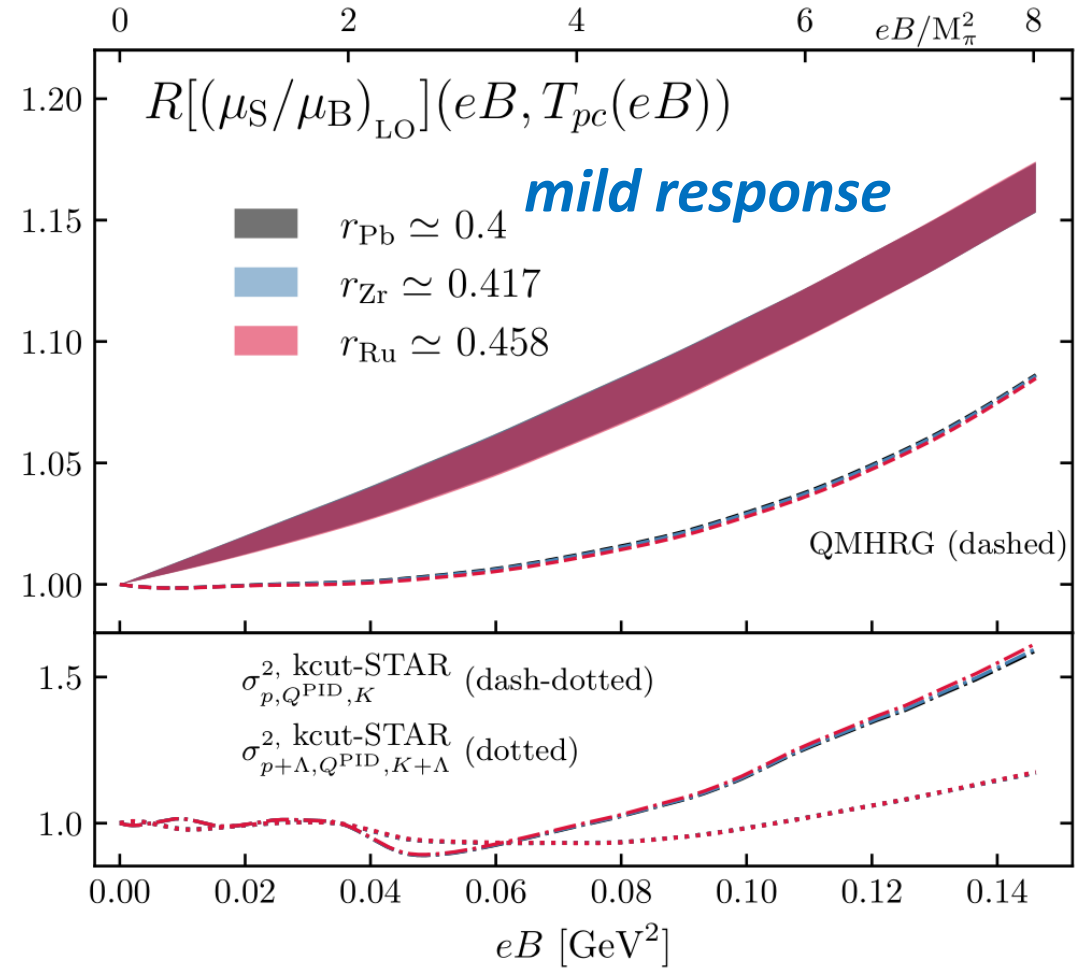
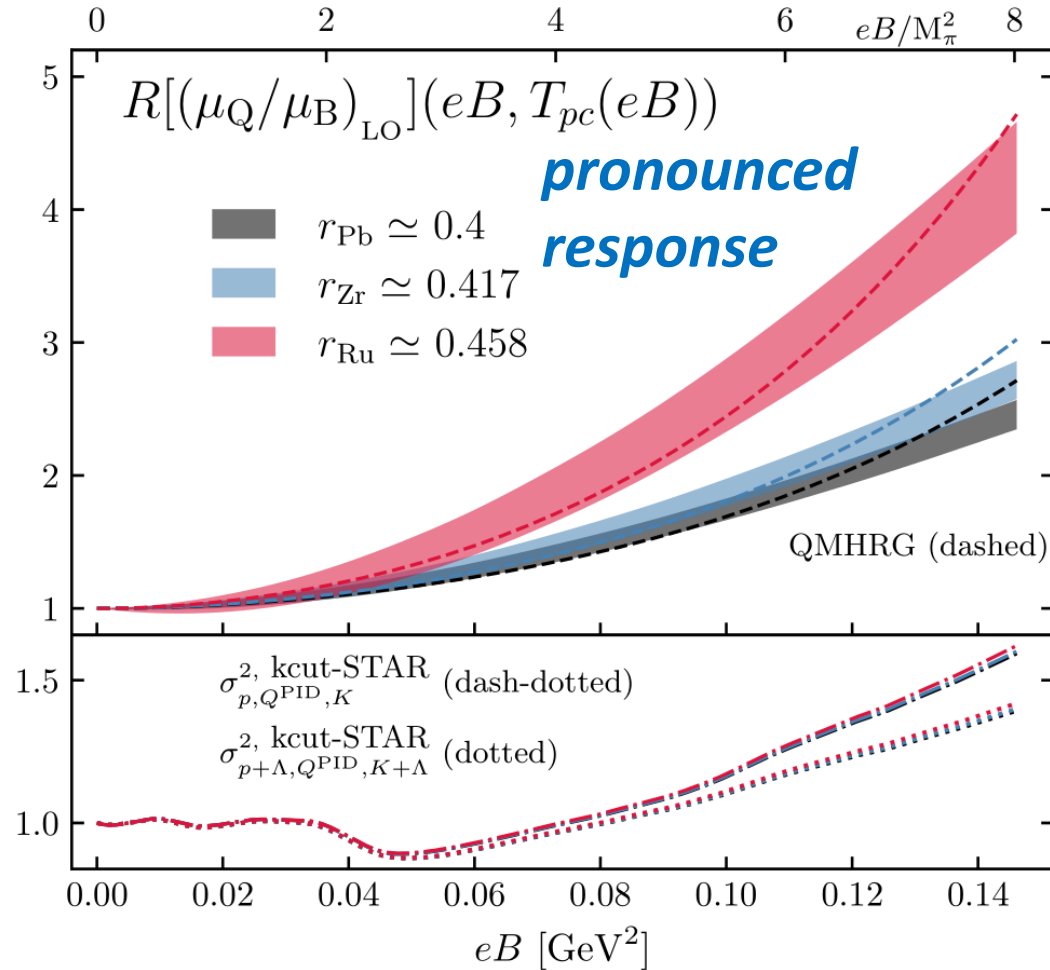
$$\frac{\Delta\mu_Q}{\Delta\mu_B} = q_1(r_{Zr}) + \left[q_1(r_{Zr}) - q_1(r_{Ru}) \right] \frac{\mu_B(r_{Ru})}{\Delta\mu_B}$$

Dominate, but weak eB -dependence

Can we construct a sensitivity observable ?

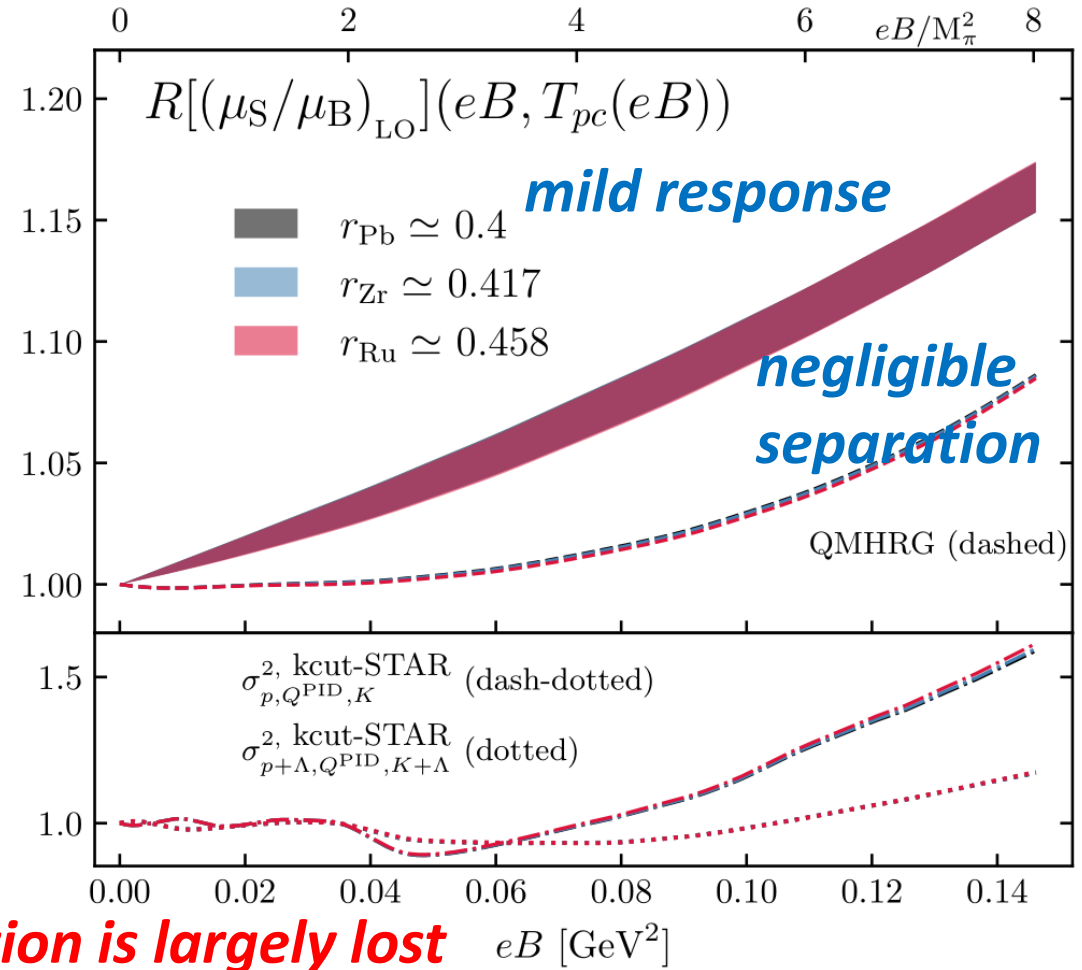
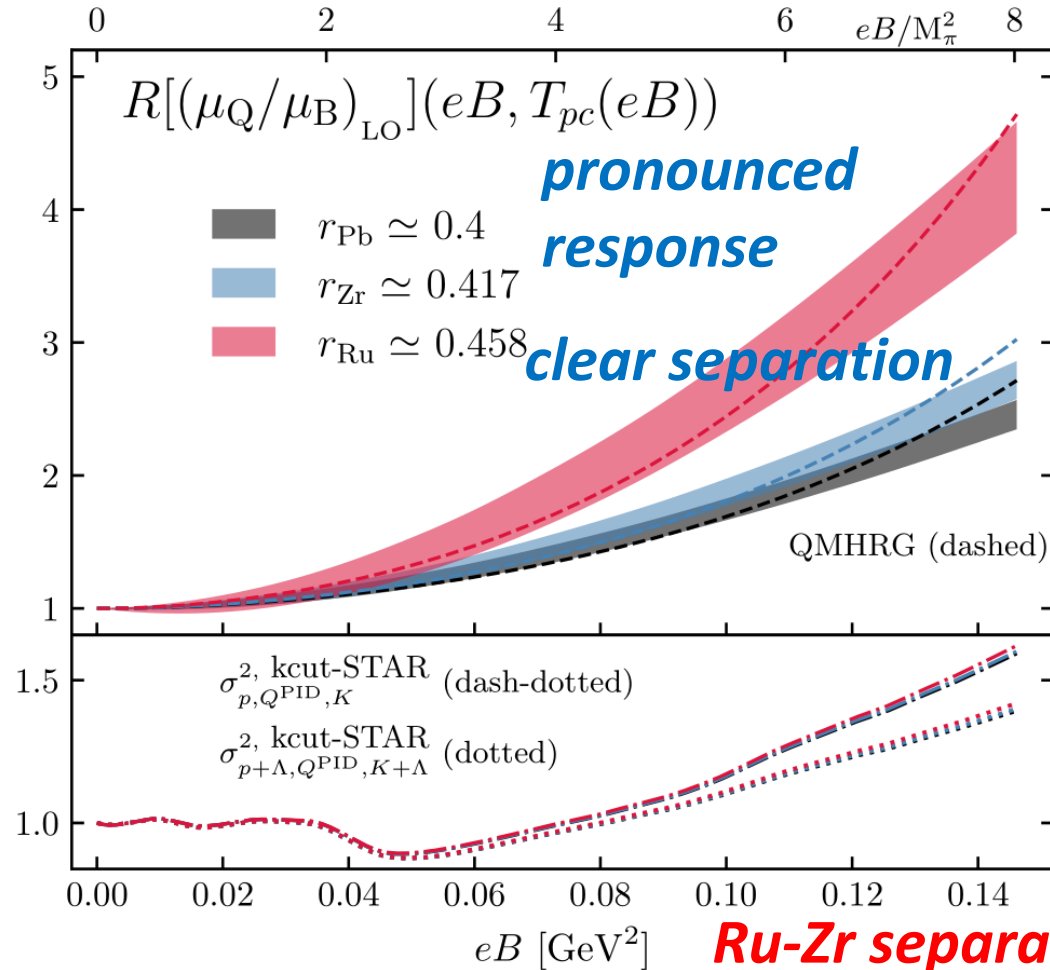
Normalized ratios reveal a pronounced eB response

$$R[O](eB) \equiv \frac{O(r, eB, T_{pc}(eB))}{O(r, eB=0, T_{pc}(0))}, \text{ where } O \in \left\{ \left(\frac{\mu_Q}{\mu_B} \right)_{LO}, \left(\frac{\mu_S}{\mu_B} \right)_{LO} \right\}$$



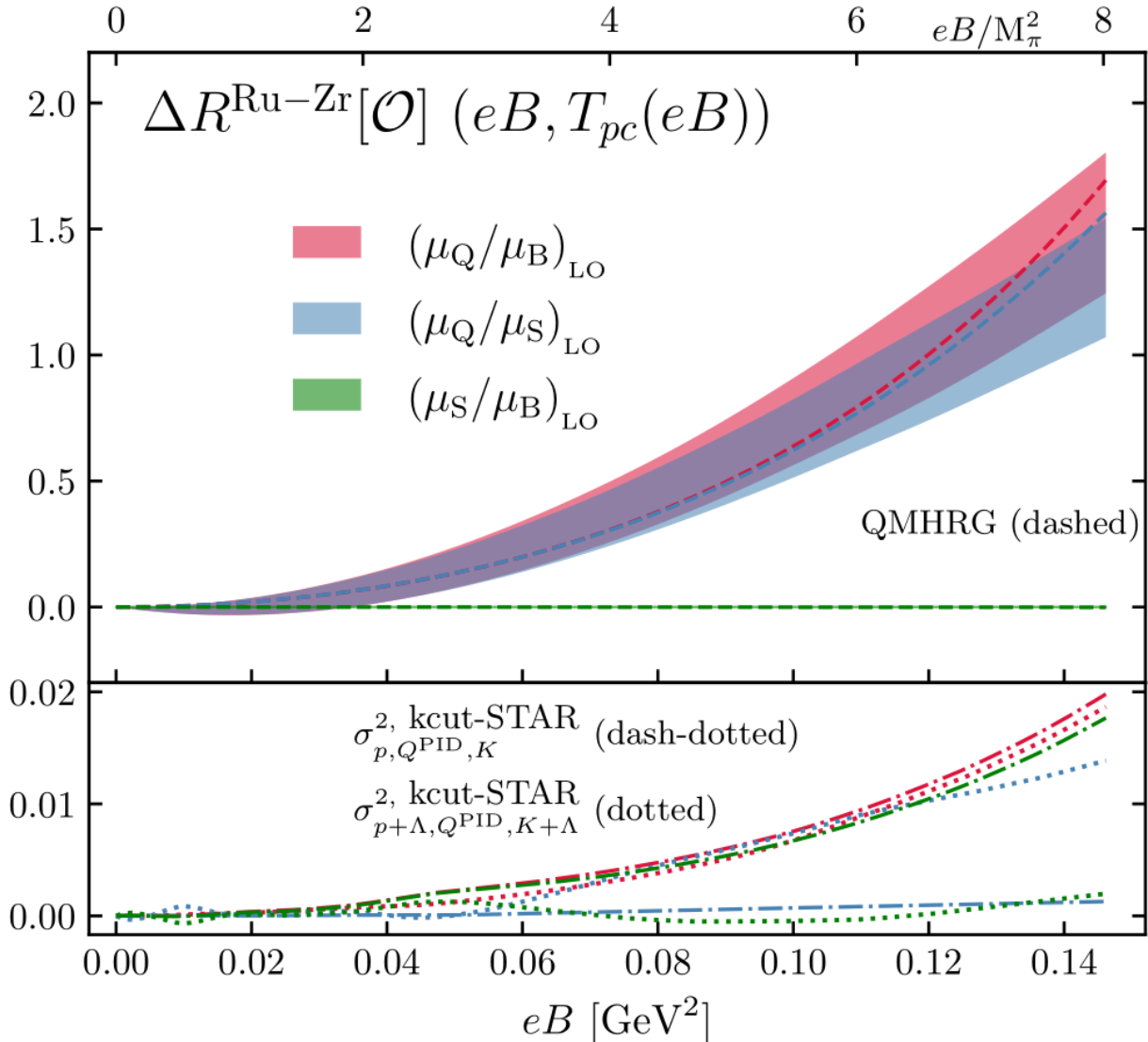
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Ru-Zr separation is largely lost

Ru-Zr difference of normalized magnetic responses



$$\Delta R^{\text{Ru-Zr}}[O] \equiv R_{\text{Ru}}[O] - R_{\text{Zr}}[O]$$

LQCD/QMHRG:

Charge and mixed differences grow to ~ 1.5 .

Strangeness:

Difference remains near zero.

STAR-cut proxies:

Ru-Zr contrast falls to $O(10^{-2})$.

Restricted charged-hadron content, notably missing the n and K^0

Summary

- Framework** q_1 and s_1 map isospin asymmetry to chemical-potential splitting ratios.
- At $eB = 0$** $\Delta\mu_Q < 0, \Delta\mu_S > 0$; $\Delta\mu_S/\Delta\mu_B$ agrees best with the Bayesian extraction.
- At $eB > 0$** – $\Delta\mu_Q/\Delta\mu_B$ and $\Delta\mu_S/\Delta\mu_B$ increase, while their slopes change only mildly.
- Normalized response** Pronounced Ru-Zr contrast in μ_Q -related ratios, but nearly none in μ_S/μ_B .
- Proxies** Overall eB response retained, Ru-Zr separation largely lost.

Thank you!

The NLO splitting ratio reads:

$$\left(\frac{\Delta\mu_Q}{\Delta\mu_B}\right)_{\text{NLO}} \equiv q_1(r_{\text{Zr}}) + (q_1(r_{\text{Zr}}) - q_1(r_{\text{Ru}})) \frac{\mu_B(r_{\text{Ru}})}{\Delta\mu_B} \quad \text{LO}$$
$$+ (q_3(r_{\text{Zr}}) - q_3(r_{\text{Ru}})) \hat{\mu}_B^2(r_{\text{Ru}}) \left(\frac{\mu_B(r_{\text{Ru}})}{\Delta\mu_B}\right) + 3q_3(r_{\text{Zr}}) \hat{\mu}_B^2(r_{\text{Ru}}) \quad \text{NLO}$$

Near the isobar freeze out point:

$$\hat{\mu}_B = \frac{20.31}{158} \simeq 0.129,$$
$$\hat{\mu}_B^2 \simeq 0.017 \simeq 2\%.$$

The lattice estimate:

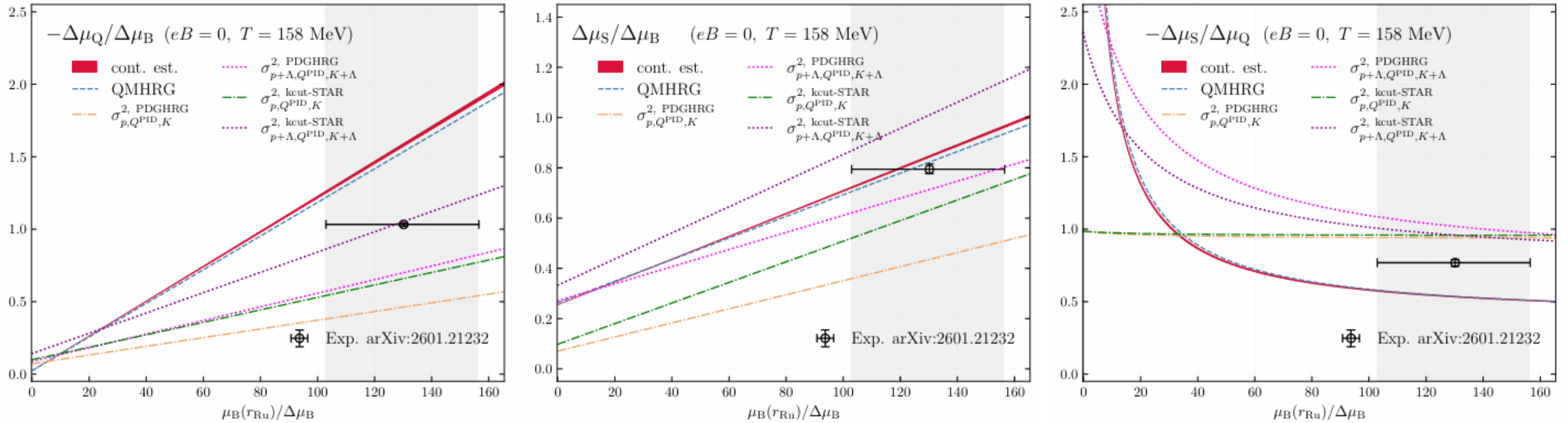
$$\frac{q_3}{q_1} \lesssim 2\%$$

H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li and J.-H. Liu,
Phys. Rev. Lett. 132 (2024) 201903 [2312.08860].

- Suppresses the NLO corrections to $\Delta\mu_Q/\Delta\mu_B$ to $O(10^{-3})$ relative to the LO.
- Similarly for $\Delta\mu_S/\Delta\mu_B$, $\Delta\mu_S/\Delta\mu_Q$

Results in an extended $\mu_B(r_{Ru})/\Delta\mu_B$ window

BACKUP



- The charge and strangeness splitting ratios grow linearly with $\mu_B(r_{Ru})/\Delta\mu_B$, with the isospin-difference terms dominating in the experimental window.
- The mixed ratio begins to saturate near $1/2$ as the common $\mu_B(r_{Ru})/\Delta\mu_B$ dependence cancels.
- The Λ -extended proxy improves the strangeness sector at small $\mu_B(r_{Ru})/\Delta\mu_B$, but current proxies do not reproduce the lattice behavior in the experimental window.

B pointing along the z direction:

$$u_x(n_x, n_y, n_z, n_t) = \begin{cases} \exp[-iqa^2BN_xn_y] & (n_x = N_x - 1) \\ 1 & (\text{otherwise}) \end{cases}$$

No sign problem!

$$u_y(n_x, n_y, n_z, n_t) = \exp[iqa^2Bn_x]$$

$$u_z(n_x, n_y, n_z, n_t) = u_t(n_x, n_y, n_z, n_t) = 1$$

Quantization of the magnetic field

$$qB = \frac{2\pi N_b}{N_x N_y} a^{-2} \quad \longrightarrow \quad eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$$

The periodicity of $U(1)$ links implies the constrain : $0 \leq N_b < N_x N_y / 4$

In this work, we restrict to $N_b \leq 6 \Leftrightarrow eB \simeq 8M_\pi^2$

Landau gauge

G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz, S. Krieg et al., JHEP 02 (2012) 044.

From chemical potential to coefficient q_1 and s_1

Taylor expansion $\hat{\mu}_{\{Q,S\}} \equiv \hat{\mu}_{\{Q,S\}}(T, eB, \hat{\mu}_B)$

$$\hat{\mu}_Q = q_1(r, T, eB) \hat{\mu}_B + q_3(r, T, eB) \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5)$$

$$\hat{\mu}_S = s_1(r, T, eB) \hat{\mu}_B + s_3(r, T, eB) \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5)$$

Initial nuclei
constraint

$$n_Q / n_B = r, n_S = 0$$



$$q_1 = \frac{r(\chi_2^B \chi_2^S - \chi_{11}^{BS} \chi_{11}^{BS}) - (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}{(\chi_2^Q \chi_2^S - \chi_{11}^{QS} \chi_{11}^{QS}) - r(\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}$$

$$s_1 = -(\chi_{11}^{BS} + q_1 \chi_{11}^{QS}) / \chi_2^S$$



The fluctuations of conserved charge

$$\chi_{ijk}^{BQS}(T, eB) = \left. \frac{\partial^{i+j+k} (p / T^4)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\hat{\mu}_B = \hat{\mu}_Q = \hat{\mu}_S = 0}$$

Isospin-driven chemical-potential splitting ratios

The splittings of chemical potential:

$$\Delta\hat{\mu}_X \equiv \hat{\mu}_X(r_{Zr}) - \hat{\mu}_X(r_{Ru}), \quad X \in \{B, Q, S\}$$



$$q_1 = \left(\frac{\mu_Q}{\mu_B}\right)_{LO}, s_1 = \left(\frac{\mu_S}{\mu_B}\right)_{LO}$$

$$\begin{aligned} \Delta\hat{\mu}_Q &\equiv q_1(r_{Zr})\hat{\mu}_B(r_{Zr}) - q_1(r_{Ru})\hat{\mu}_B(r_{Ru}), \\ &= q_1(r_{Zr})(\hat{\mu}_B(r_{Ru}) + \Delta\hat{\mu}_B) - q_1(r_{Ru})\hat{\mu}_B(r_{Ru}) \end{aligned}$$

$$\begin{aligned} \Delta\hat{\mu}_S &\equiv s_1(r_{Zr})\hat{\mu}_B(r_{Zr}) - s_1(r_{Ru})\hat{\mu}_B(r_{Ru}) \\ &= s_1(r_{Zr})(\hat{\mu}_B(r_{Ru}) + \Delta\hat{\mu}_B) - s_1(r_{Ru})\hat{\mu}_B(r_{Ru}) \end{aligned}$$

