

Spin Entanglement in QCD

from Parton Scattering and Topological Fluctuations



Guowei(国维) Yan(严) ¹

¹ School of Physics and Astronomy, SYSU

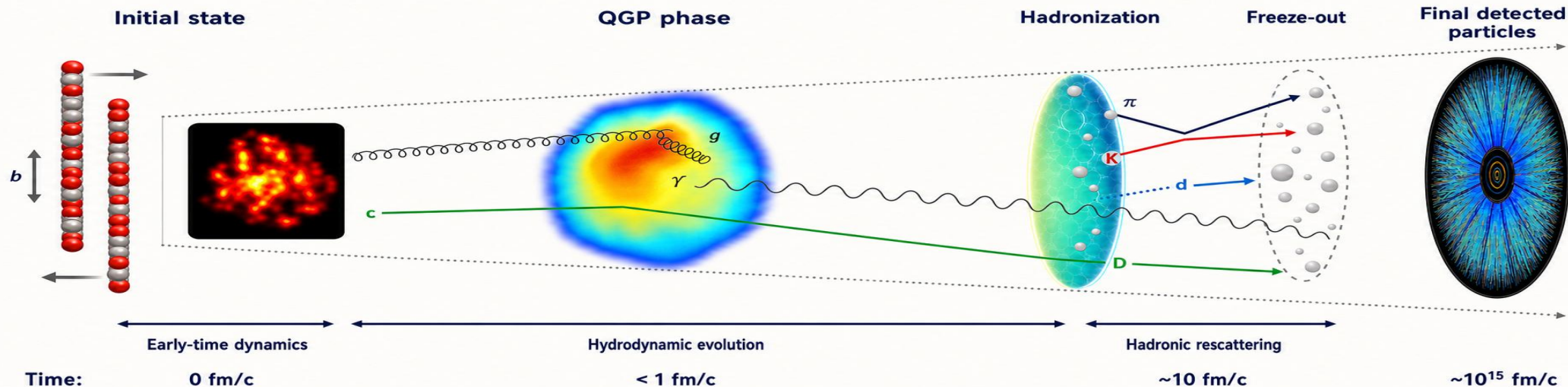
² Institute of Modern Physics, Fudan University



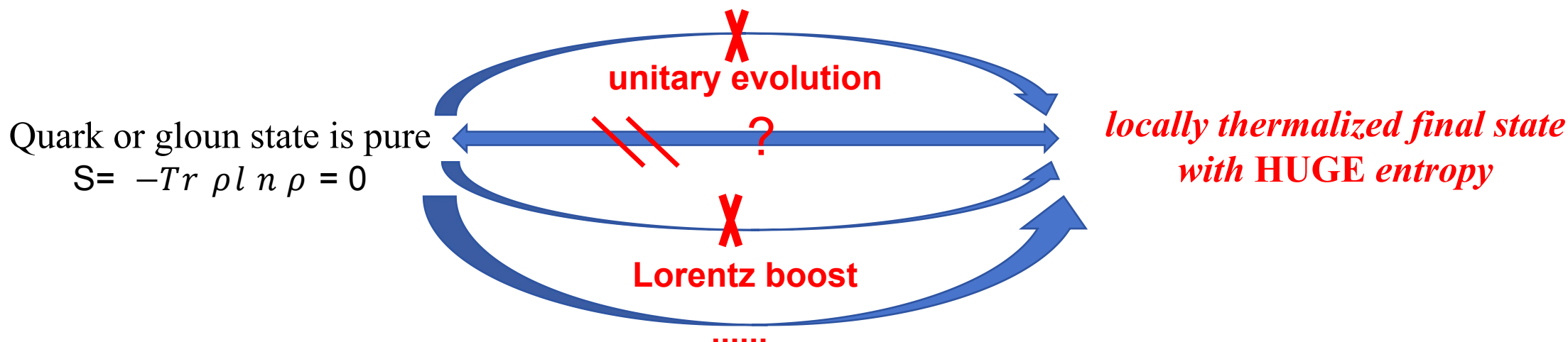
with Chenxi(晨曦) Liang(梁) ², Zhishun(志顺) Chen(陈) ¹, Shu(树) Lin(林) ¹, Li(力) Yan(严) ²

10.19 at Wuhan

Entropy paradox in QCD matter

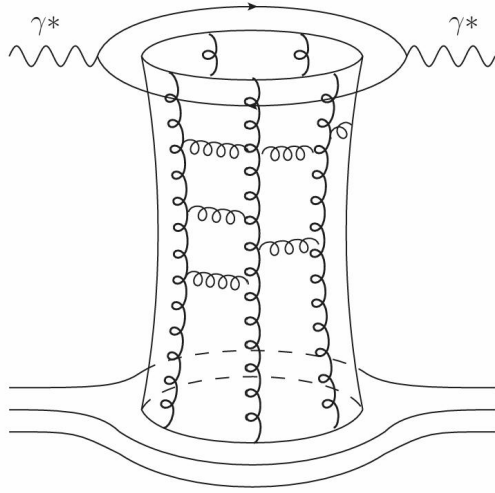


Melting nuclei in relativistic heavy ion collision $\rightarrow$ **new state of QCD matter** $\rightarrow$ **fast equilibrium ~ 1 fm/c**



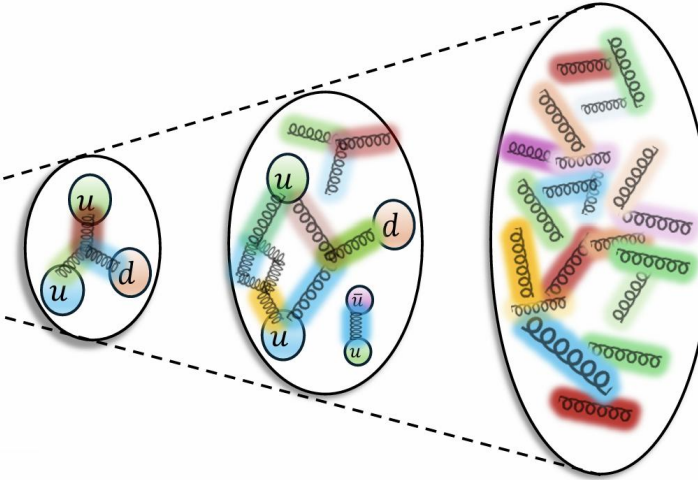
- A *Mismatch* between Quantum and Classical entropy for an isolated matter.

Entropy Paradox in QCD Matter



Hadron

Parton distribution function describes the probability of finding a quark or gluon



Low

Momentum

High

D.Kharzeev e.t. al:2410.22331v2
:1702.03489v3

- Non-linear evolution of QCD gives maximum of entanglement entropy among partons, under limit of high momentum scale and small x .

- Scale-dependent entropy is natural in DIS picture.
- $$S(x) = \ln[xG(x)] \simeq \Delta \ln[1/x] \simeq \Delta \ln \frac{L}{\epsilon}$$

CFT

$$S_E = \frac{c}{3} \ln \frac{L}{\epsilon}$$

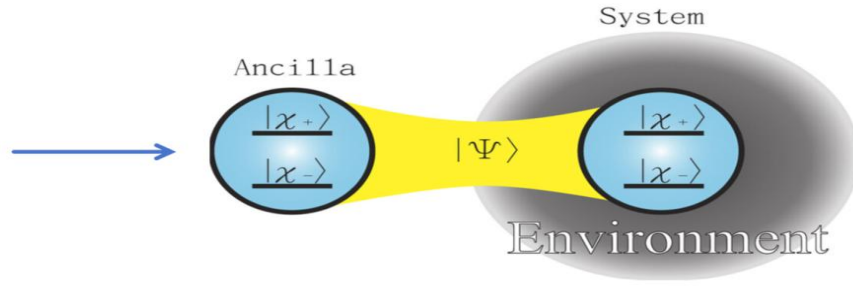
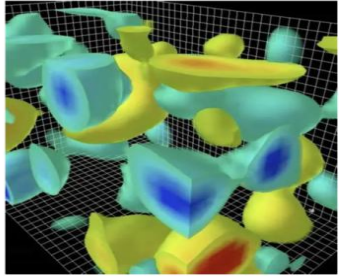
Entangle Entropy in CFT

$$S_E \sim \frac{c}{3} \ln \left(\frac{1}{\pi T \epsilon} \right) \sinh(\pi T L).$$

finite temperature

- Problems Not Solved:** Entanglement entropy $\neq$ thermal entropy
- How to evolve entanglement entropy into thermal entropy?**
- In which process does Entanglement among partons come from?**

Entanglement from Topological Effect



Vacuum topology(instanton ensemble) $\longrightarrow$ *entanglement*

Prompt Quark Production by exploding Sphalerons

Edward Shuryak and Ismail Zahed

Department of Physics and Astronomy, State University of New York, Stony Brook, NY 11794
(November 7, 2018)

hadron and nucleus-nucleus collisions at large $\sqrt{s}$. Simple arguments indicate that while in hadron-hadron scattering production via sphalerons, it is only a small part of the multiplicity, in nucleus-nucleus collisions it is not so and hundreds of sphalerons may be released, contributing substantially to the total entropy produced [16]. In view of that, the production of $2N_F$ chiral quarks per cluster may make the quark-gluon plasma produced in heavy-ion

total number of qq collision, the production of sphaleron-like:

$$N_{coll} = 8\pi\sigma_{qq}n_q^2 \int_0^R dr_t r_t (R^2 - r_t^2)$$

$$= 3^{4/3} 2^{-5/3} \pi\sigma_{qq}n_q^2 \left(\frac{AN_q}{\pi n_q} \right)^{4/3},$$

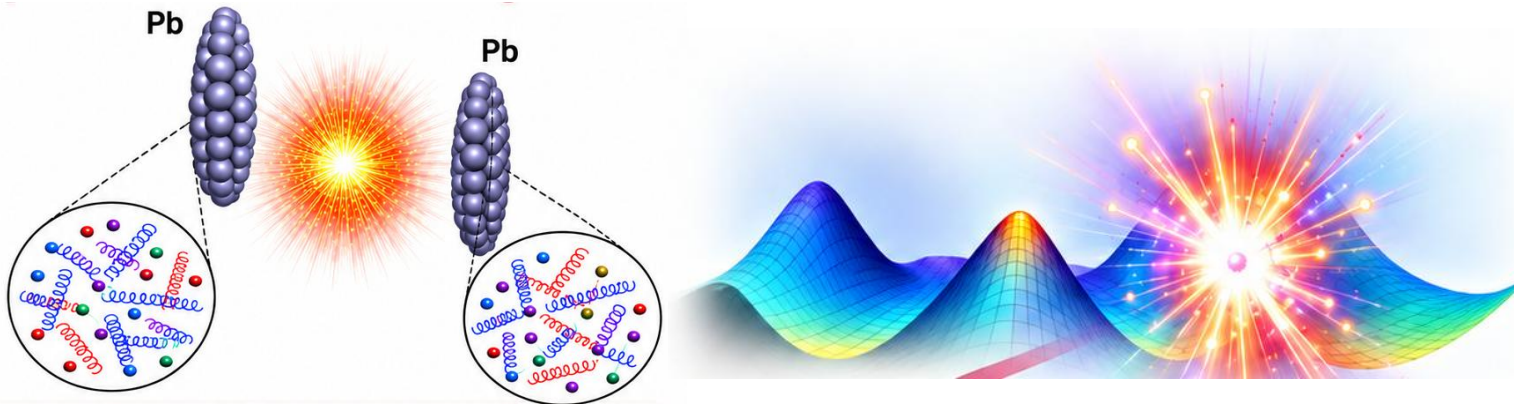
at RHIC energies

$$N_{objects} \approx 400,$$

E. Shuryak. [hep-ph/0205031](#)
[hep-ph/0206022](#)

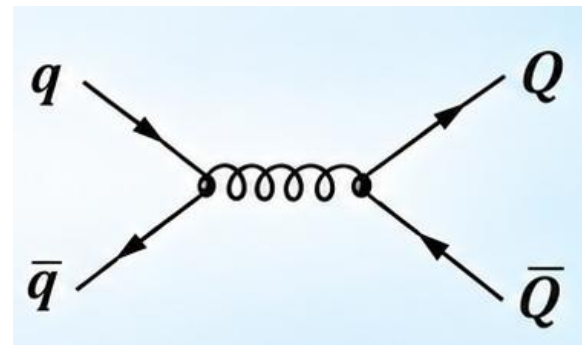
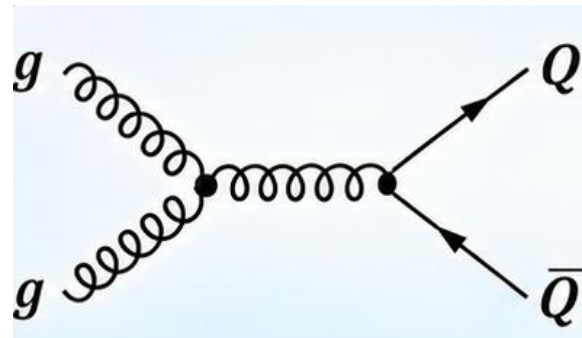
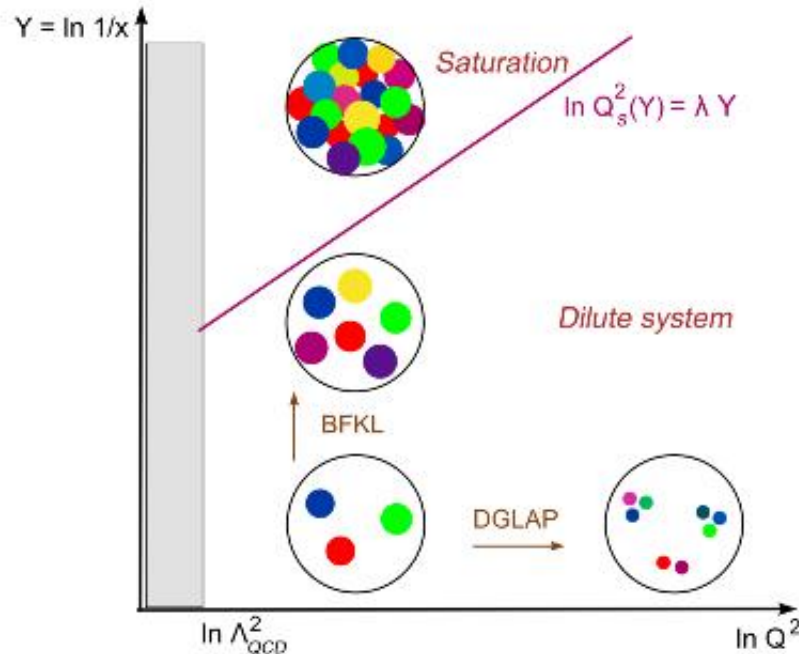
- In evolution of Sphalerons, they explode into many partons, which potentially leads to entropy production
- Entanglement as a novel probe of early QCD matter
 - Not only providing entropy among partons
 - But also reflecting the non-commutative Topo effects

Entanglement From Parton Scattering



- Two sources of Parton:
Nucleus & **Sphaleron**

Color Glass Condensate (CGC)



- Partons scatter to form entanglement in the **strong colour background**.
- ❖ **Scattering induced entanglement** is dominant for heavy flavours due to **low** production rate from Sphaleron.

Amplitude to Concurrence WorkFlow



Initial (4 x 4)

$$\hat{\rho}_{in} = \frac{1}{N_a N_b} \sum_{s_a, s_b} |s_a, s_b\rangle \langle s_a, s_b|$$

evolve by scattering



$$\hat{R} = \mathcal{M} \hat{\rho}_{in} \mathcal{M}^\dagger$$

Final state Density Matrix

$$R^{(\eta_2, \zeta_2)}_{(\eta_1, \zeta_1)} = \frac{1}{N_a N_b} \sum_{s_a, s_b} \mathcal{M}_{\eta_1, \zeta_1}^{s_a, s_b} \mathcal{M}_{s_a s_b}^{\dagger \eta_2 \zeta_2}, \quad \rho_{out} = \frac{R}{\text{Tr} R}$$

Decompose it with Correlation

$$\rho = \frac{1}{4} \left(\mathbb{1} \otimes \mathbb{1} + \tilde{\mathcal{C}}_{ij} \hat{\sigma}_i \otimes \hat{\sigma}_j \right)$$

$$\tilde{\rho} = (\sigma_2 \otimes \sigma_2) \rho (\sigma_2 \otimes \sigma_2)$$



$$\lambda_i \in \sqrt{\text{EigenV}(R_w := \rho \tilde{\rho})}$$

W.K. Wothers, Phys. Rev. Lett. **80**, 2245 (1998)

Concurrence (并发度)

$$\mathcal{C} = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

$$= \frac{1}{2} \max(0, -C_{nn} + |C_{rr} + C_{kk}|)$$

- Concurrence is measurable

$$t\bar{t} \rightarrow (l^+ \nu b) (l^- \bar{\nu} \bar{b})$$

Nature **633**, 542-547 (2024) ATLAS

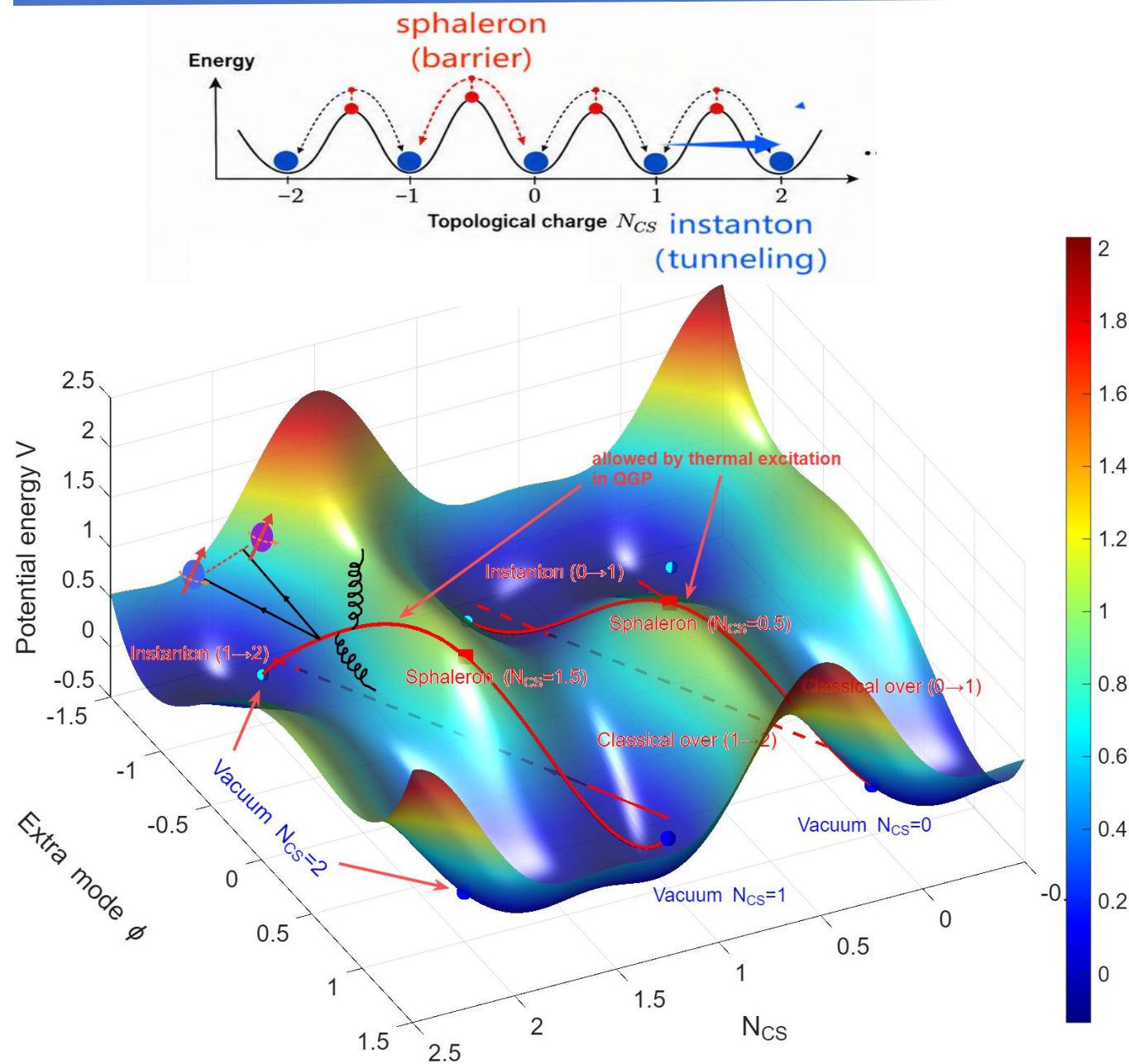
φ : Angles between two leptons

$$D = \langle \cos \varphi \rangle = - \frac{1 + 2\mathcal{C}[\rho]}{3}$$

- Entanglement of Formation (Free From Ambience & Projective to trivial knot)

$$E_F(\mathcal{C}(\rho)) := h\left(\frac{1 + \sqrt{1 - \mathcal{C}^2}}{2}\right), \quad h(x) = -x \log_2 x - (1-x) \log_2 (1-x)$$

QCD Topo-induced Entanglement



- Each field configuration of QCD vacuum is a valley labelled with **Chern-Simon number**:

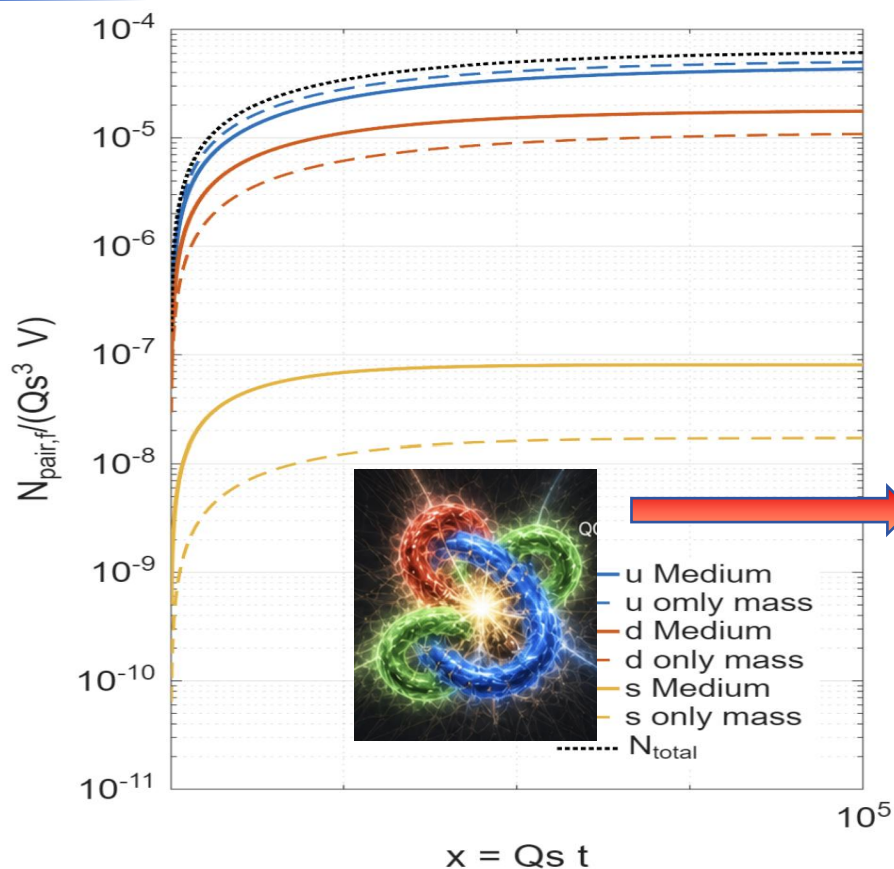
$$n = \frac{1}{24\pi^2} \int d^3x \epsilon^{ijk} \text{Tr}[(U^\dagger \partial_i U)(U^\dagger \partial_j U)(U^\dagger \partial_k U)] \in \mathbb{Z}.$$

- Sphaleron is a static classical solution A_μ^{cl} with non-trivial topo number in Path Integral

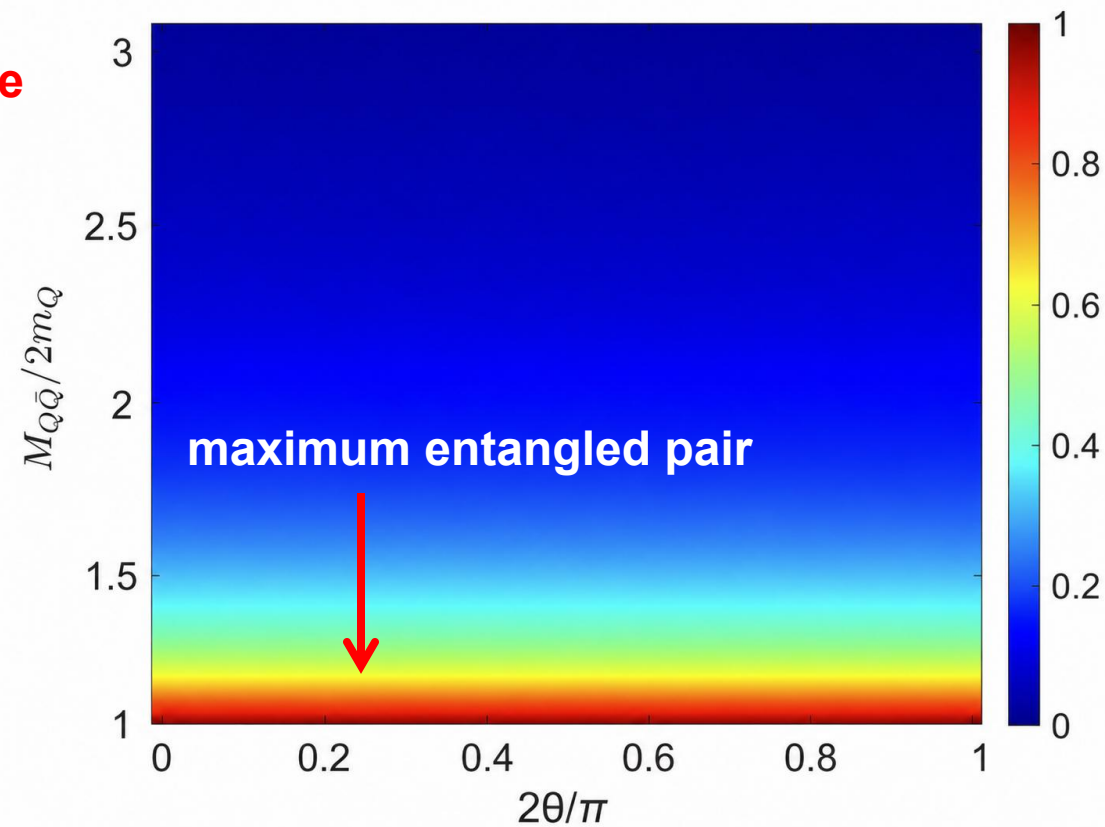
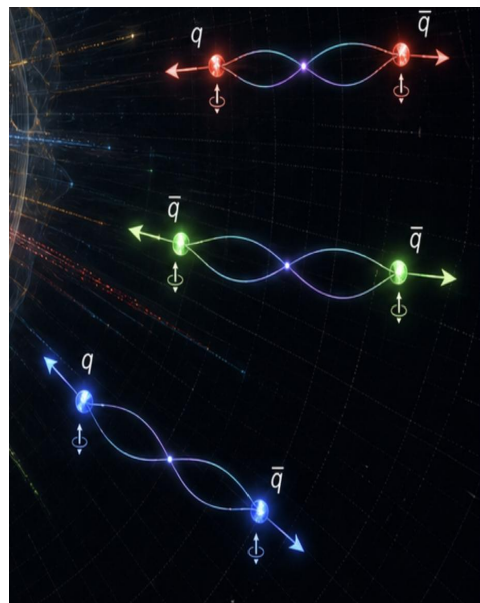
$$Z[A_\mu, \bar{q}_f, q_f] = \int \mathcal{D}A_{inst}^\mu \prod_f \mathcal{D}\bar{q}_f \mathcal{D}q_f \exp\left\{-\int d^4x \left[\frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a + \sum_{f=1}^{N_f} \bar{q}_f (\not{D}) q_f\right]\right\} \\ + \int \mathcal{D}A_{excited}^\mu \prod_f \mathcal{D}\bar{q}_f \mathcal{D}q_f \exp\left\{-\int d^4x \left[\frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a + \sum_{f=1}^{N_f} \bar{q}_f (\not{D}) q_f\right]\right\}$$

- **Fermi zero mode leads to divergence in second part of path integral, absorbed by external quark line, which gives rise to real quarks production.**

$Q\bar{Q}$ Entanglement from Sphaleron



quark pair concurrence



$$\mathcal{N}_f(t) = 2 \int d\rho \delta(\rho - \rho_*(t)) \int d\Phi_2 C_{\text{color}} |S_{\text{spec}}^{(f)}(\rho, t)|^2 \left(\frac{4\pi^2 \rho^3}{N_c} \right)^2 \mathcal{F}_0^2(p\rho/2) \mathcal{F}_0^2(q\rho/2) 2p \cdot q.$$

- Spin entanglement can be extracted from quark pair state, with amplitude $\mathcal{M}$ matched using sphaleron rate.

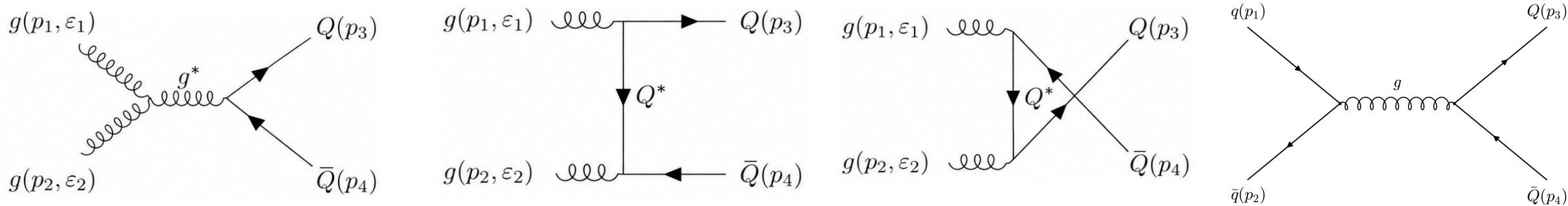
$$dP_f = r_f(t) \Gamma_{\text{sph}}^{\text{neq}}(t) d^3X dt.$$

$$\mathcal{M}_{Q,f;s\bar{s}} = e^{i\delta_Q(t)} \sqrt{\frac{r_f(t) \Gamma_{\text{sph}}^{\text{neq}}(t)}{\mathcal{N}_f(t)}} T_{Q,f;s\bar{s}}.$$

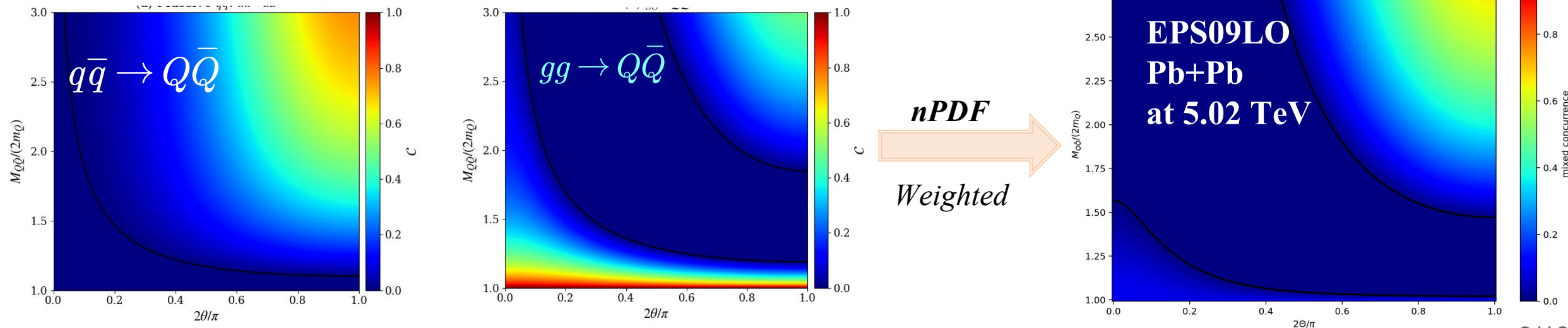
- Spectator flavor and Schwinger–Keldysh propagators are included in T matrix, non-equilibrium medium effect **enhances** the pair production.

Entanglement from Parton Scattering

- Heavy quarks entanglement mainly comes from scattering, four processes to be considered for $Q\bar{Q}$ pair production (vacuum)



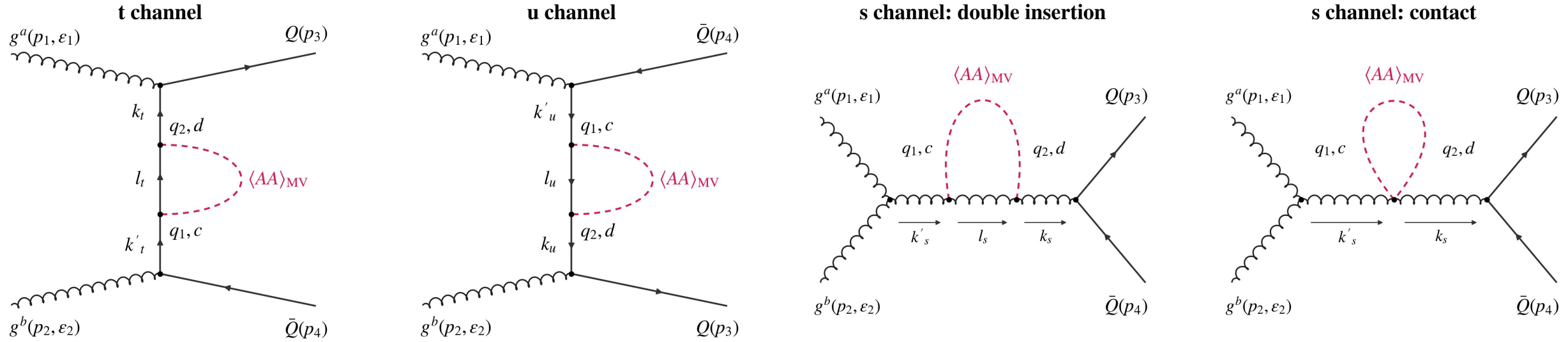
- Amplitude of scattering is used to extract the concurrence as:



Entanglement from Scattering in CGC



- Partons scatter under the **strong colour background**, whose effects are to be considered as **propagator correction** with insertion of two BG averaged fields



- Correction to $q\bar{q}$ is reduced in normalizing the density matrix
- Background is considered with MV model of CGC

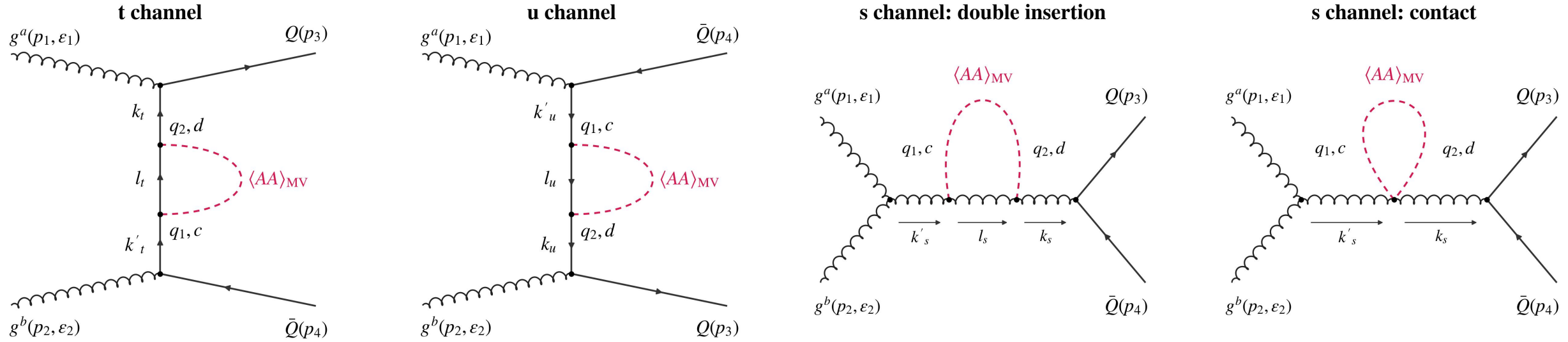
- two point function of colour source $\langle \rho^a(\mathbf{x}_\perp) \rho^b(\mathbf{y}_\perp) \rangle = \delta^{ab} \mu^2 \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$. *BG strength*

- two point function of classical field $\langle A^{+a}(q) A^{+b}(q') \rangle = \frac{\delta(q^-) (\delta q'^-) g^2 \langle \varrho^a(\mathbf{q}_\perp) \varrho^b(\mathbf{q}'_\perp) \rangle}{(q_\perp^2 + m_{\text{IR}}^2) (q'^2_\perp + m_{\text{IR}}^2)}$ *Infrared cutoff*

Entanglement from Scattering in CGC



- Partons scatter under the **strong colour background**, whose effects are to be considered as **propagator correction** with insertion of two BG averaged fields



- As an example, propagator correction to t, u channel can be computed as

$$\begin{aligned} \delta \bar{D}_{t,u}^{(2)}(k, k') &= g^2 \int \frac{d^4 l}{(2\pi)^4} D_0(k) \gamma^- T^a S D_0(l) \gamma^- T^b D_0(k') \langle A^{+,a}(k-l) A^{+,b}(l-k') \rangle \\ &= \boxed{\Delta_3(k-k')} g^4 C_F \mu^2 S_0(k) \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{1}{(q_\perp^2 + m_{\text{IR}}^2)^2} \boxed{\int \frac{dl^+}{2\pi} \gamma^- S_0(l) \gamma^- S_0(k')} \end{aligned}$$

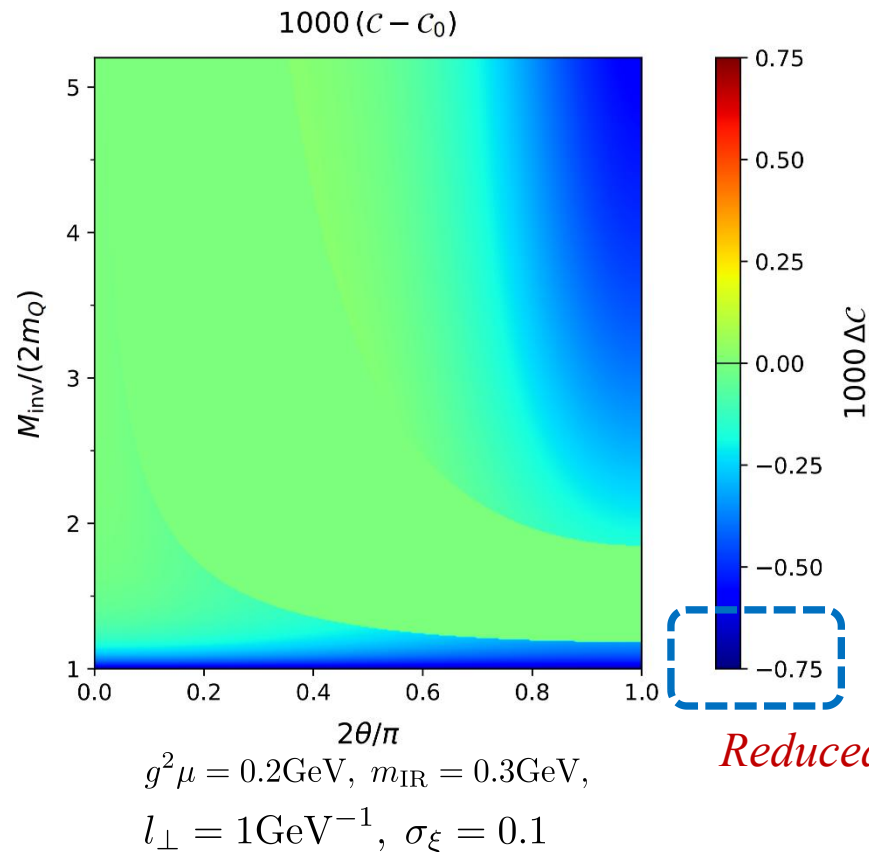
$$\langle \varrho^a(\mathbf{x}) \varrho^b(\mathbf{y}) \rangle_{\ell_\perp} = \delta^{ab} \mu^2 \frac{e^{-|\mathbf{x}-\mathbf{y}|^2/(4\ell_\perp^2)}}{4\pi \ell_\perp^2}$$

- To deal with this delta function, the external gluon line is treated as a wave packet of width σ_ξ

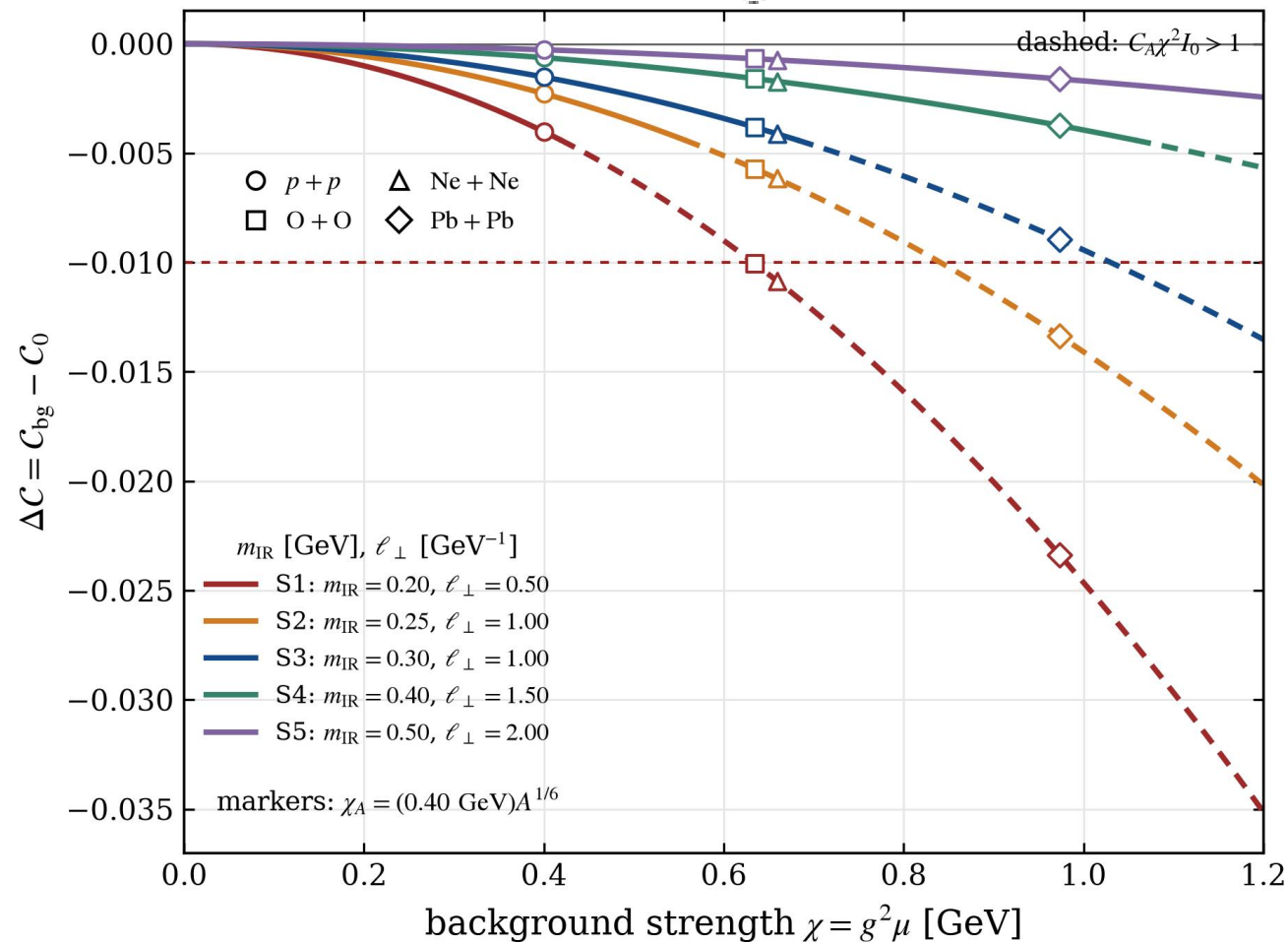
- Divergence in this integral can be regulated assuming finite correlation length for MV model

Entanglement from Scattering in CGC

- After arduous calculations of amplitude, they give the concurrence correction as:

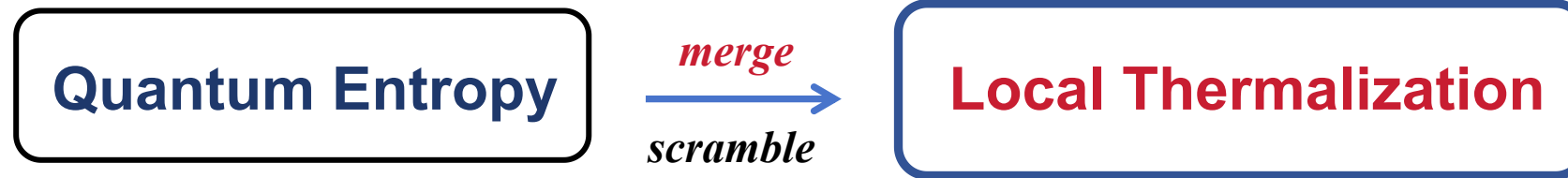


$$m_c = 1.50 \text{ GeV}, E = 1.732 \text{ GeV}, \beta = 0.50, \theta = \pi/2; \sqrt{s_{NN}} = 5.36 \text{ TeV}, x_{\text{hard}} \simeq 6.46 \times 10^{-4}$$



- The size of collided nuclei is associated with BG strength, stronger background reduces more entanglement
- As partons traverse the nuclei background, this reduction accumulates,
 i.e. *the two body entangled entropy is reduced in the evolution of the QCD matter!*

- Where does the generated entanglement go?
- What is the quantitative relation of entropy in thermalization?
- Is the entanglement entropy partial or all of the thermal entropy?
- **Eigenstate Thermalization Hypothesis**



Entropy Relation in Quantum Thermalization



➤ R generating rules for general entanglement

$$E^{irr} = \begin{array}{c} \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \text{[Diagram 4]} + \dots \\ + \text{[Diagram 5]} + \text{[Diagram 6]} + \text{[Diagram 7]} + \text{[Diagram 8]} + \dots \end{array}$$

$$d_K = |\Delta_K(-1)|$$

$$a_J(K) = \|V_K(t) - V_K(t^{-1})\|_1$$

$$r_{\text{Kh}}(K) = \sum_{i,j} \text{rank Kh}^{i,j}(K)$$

$$\tau_{\text{Kh}}(K) = \sum_{i,j} \text{rank Tor Kh}^{i,j}(K)$$

$$\mathcal{R} : \mathfrak{M} \rightarrow \text{Hyp}(P)$$



$$E_k(\rho_P; K) = \sum_{e \in E_K[k]} \omega_{K,e} \mathcal{Q}_k(\rho_e).$$

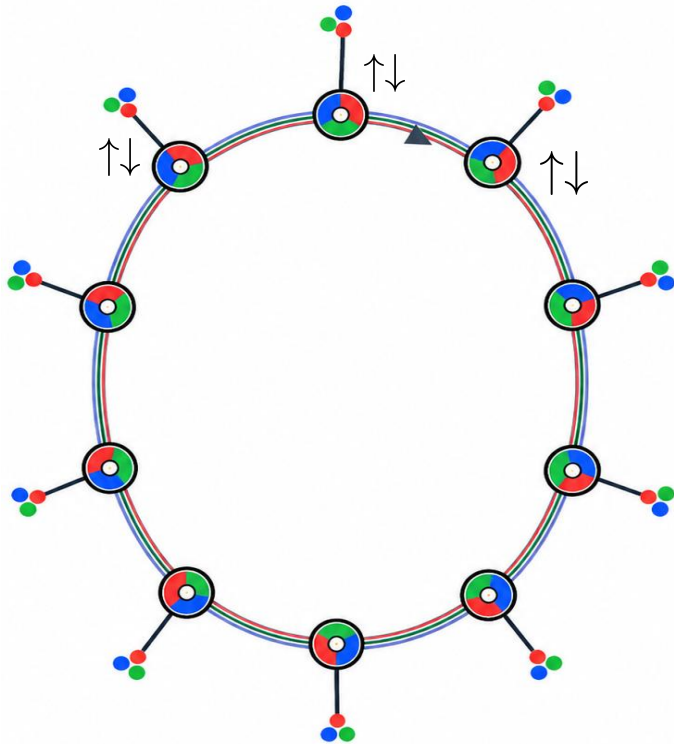
$$\omega_{K,e} = \frac{|\mathcal{Z}_1(K, e)|^2}{|\mathcal{Z}_0(e)|^2 + |\mathcal{Z}_1(K, e)|^2}$$

• Thermalization rigorous condition in terms of entanglement:

$$\rho_A(t) \xrightarrow{\|\cdot\|_1} \tau_A \iff \mathfrak{L}_{R,A}(\rho_A(t) \parallel \tau_A) \longrightarrow 0 \quad |S(\rho_A(t)) - E_R^{\text{irr}}(A; \rho(t))| \leq |\mathcal{C}_R(A; \tau) - \mathcal{I}_R(A; \tau)|$$

$$+ \delta_A(t) \log(d_A - 1) + h_2(\delta_A(t)) + \sum_a g_a(\delta_{e_a}(t)).$$

SU(3) link Quantum Simulation



Notation



MPS writes the wavefunction as

$$|\Psi\rangle = \sum_{s_0, \dots, s_{N-1}} A^{[0]s_0} A^{[1]s_1} \dots A^{[N-1]s_{N-1}} |s_0 s_1 \dots s_{N-1}\rangle$$

second-order Trotter step is

$$e^{-iH\Delta t} = e^{-iH_m\Delta t/2} e^{-iH_{\text{even}}\Delta t} e^{-iH_{\text{odd}}\Delta t} e^{-iH_m\Delta t/2} + O(\Delta t^3).$$

Kogut–Susskind-type Hamiltonian has the structure

$$H_{\text{KS}} = m \sum_i (-1)^i \psi_i^\dagger \psi_i - \kappa \sum_i \left(\psi_i^\dagger U_{i,i+1} \psi_{i+1} \text{h.c.} \right) \\ \frac{g^2}{2} \sum_i E_i^2 \frac{1}{2g^2} \sum_{\square} \left(2N_c - \text{Tr } U_{\square} - \text{Tr } U_{\square}^\dagger \right).$$

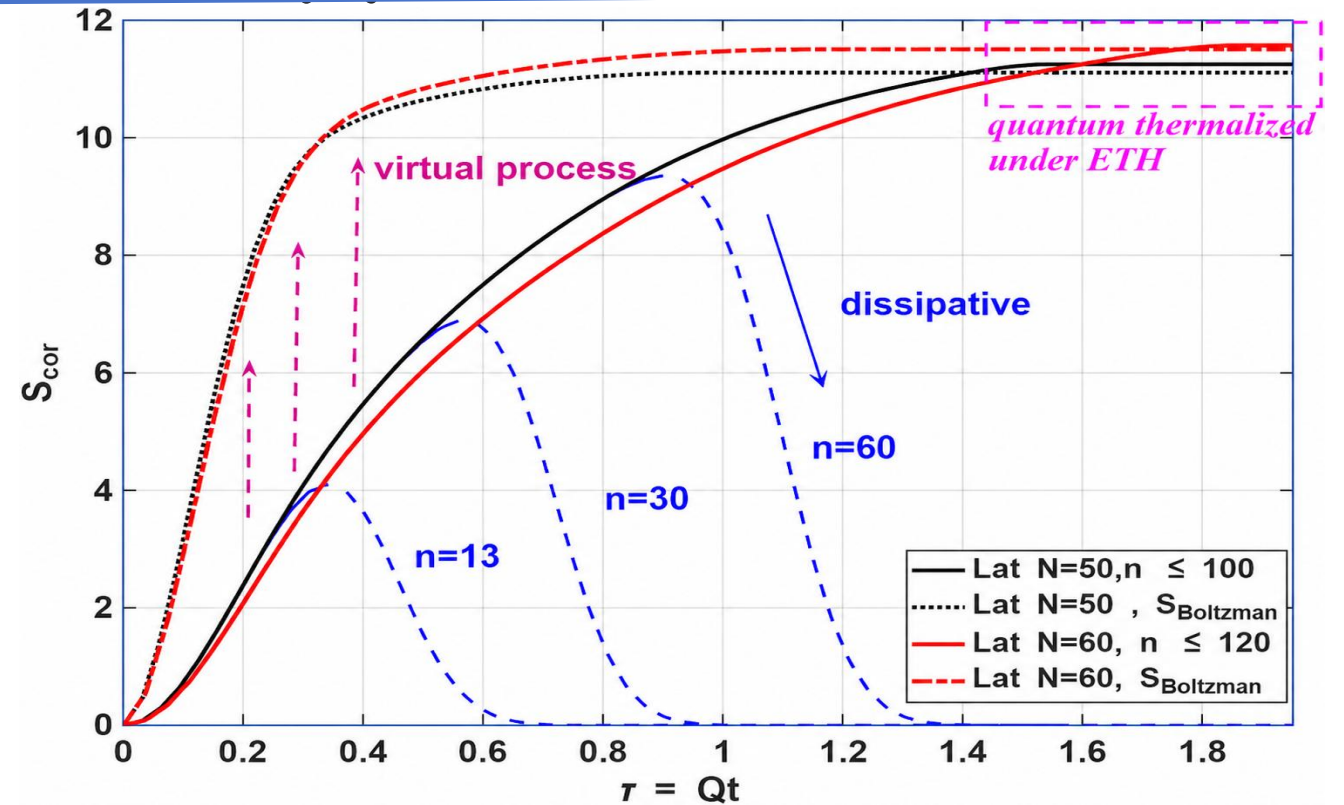
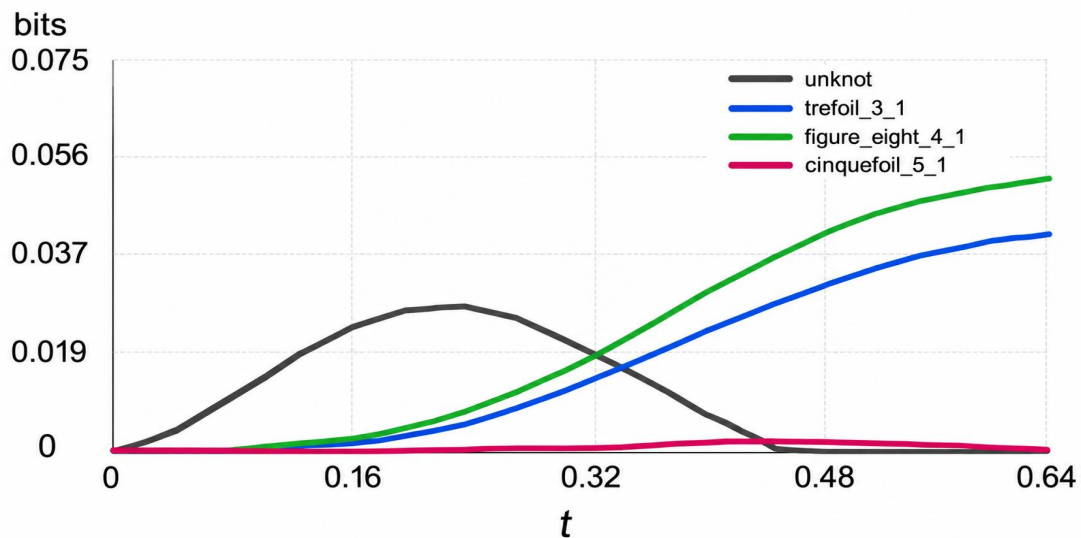
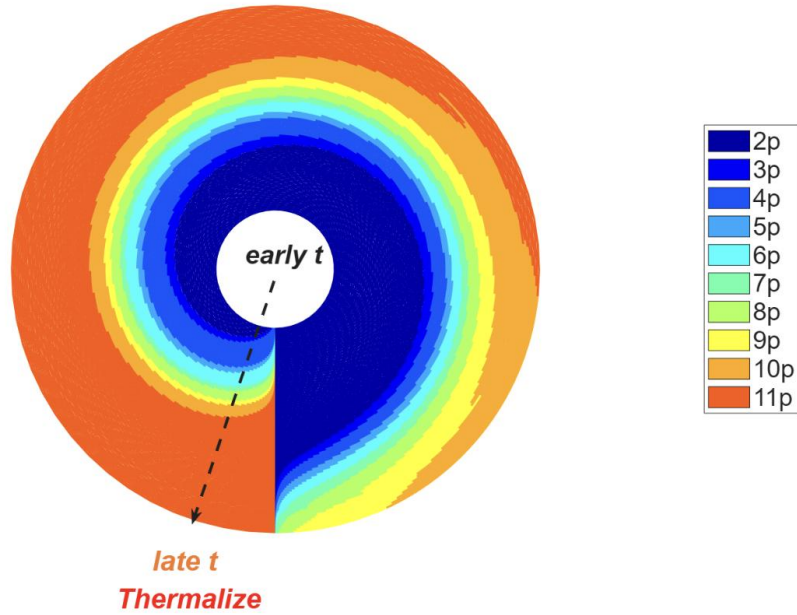
effective model:

$$H_{\text{eff}} = m \sum_{i=0}^{N-1} (-1)^i n_i - w \sum_{i,c} \left(c_{i,c}^\dagger c_{i+1,c} + c_{i+1,c}^\dagger c_{i,c} \right) + J \sum_{i=0}^{N-2} \mathbf{Q}_i \cdot \mathbf{Q}_{i+1}$$

TEBD evolution gate: $\tilde{\Theta}_{\alpha\gamma}^{s'_i s'_{i+1}} = \sum_{s_i, s_{i+1}} G_{s_i s_{i+1}}^{s'_i s'_{i+1}} \Theta_{\alpha\gamma}^{s_i s_{i+1}}$

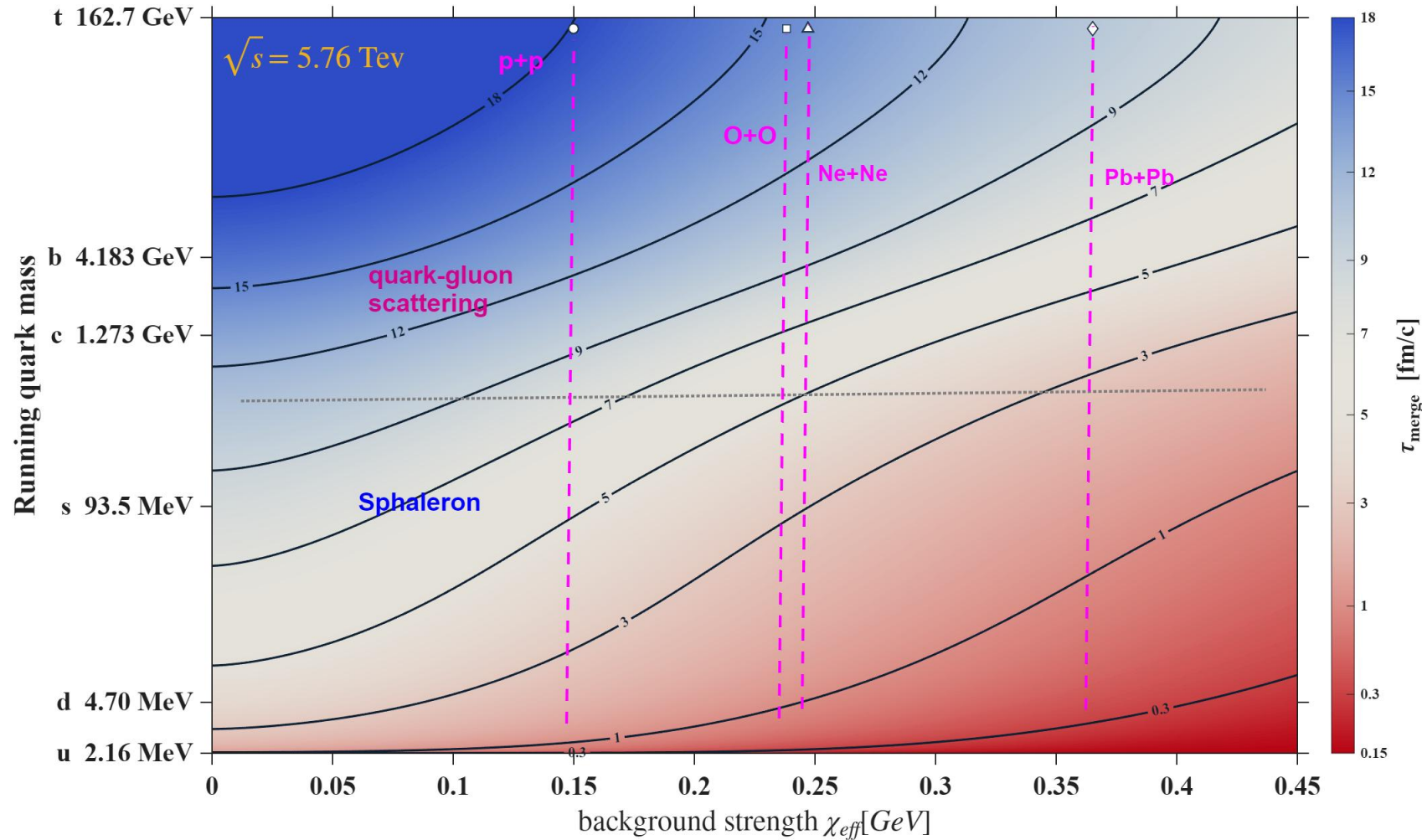
$|\Psi'(+\Delta t)\rangle$

Quantum & Thermal Entropy Evolution



- In entropy evolution, few-body entanglement “*disperse*” into many-body entanglement by quantum scrambling.
- Quantum entropy *merges* and *matches* the thermal entropy in window, **a signal of thermalization**

Quantum & Thermal Entropy Evolution



- We interpret the time of merging of quantum entropy as the thermalization time.
- Background strength facilitates the merging process for different quarks.
- For lighter sector and larger system, by action, quantum and thermal entropy can reach the matching window within shorter time . (τ in unit of $1/Q$)

Summary

- We Propose that *entanglement entropy* can provide a new perspective for the **thermlization time problem in HIC**.
- Entanglement entropy can come from QCD topological sphaleron effects and parton scattering at the early stage of collisions.
- By quantum simulation, we find that quantum entanglement entropy can evolve to thermal entropy by quantum scrambling.
- **The time for thermalization can be estimated by quantum scrambling time that merges the many body entangled entropy.**