

Fluctuations and Correlations of Conserved Charges in a Magnetized QCD Matter

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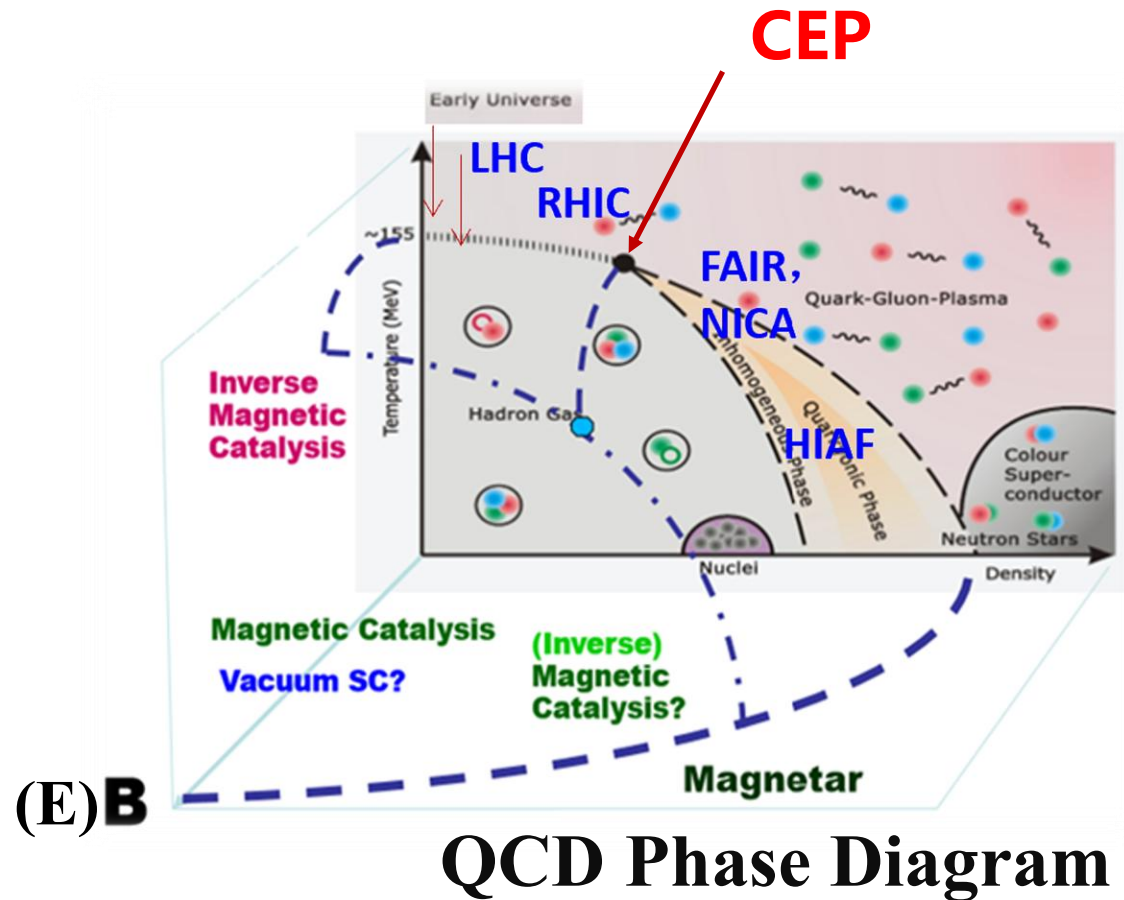
Supervisor: Prof. Shijun Mao

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arXiv:2605.14674 (2026).

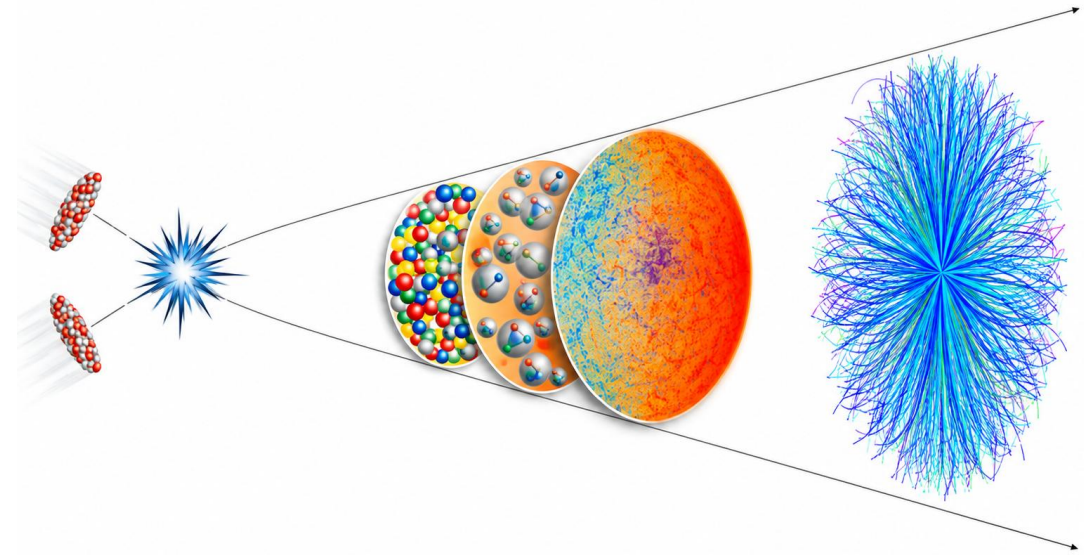
Outline

- QCD Matter
- PNJL model
- Fluctuations and Correlations
- Summary
- Backup

QCD phase transition (chiral and deconfinement)

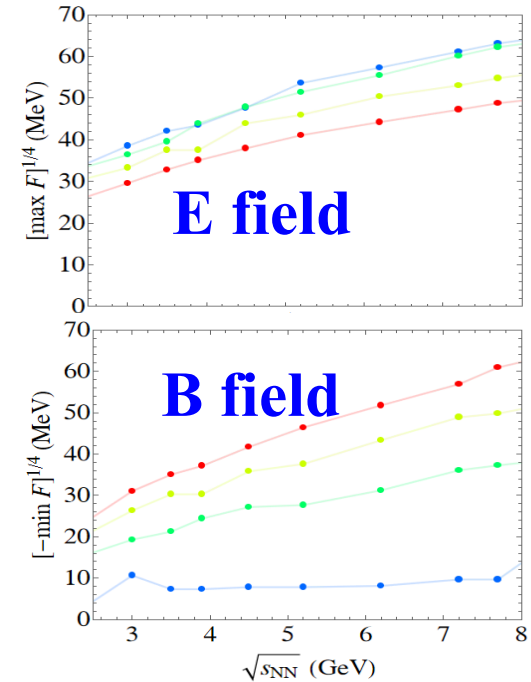
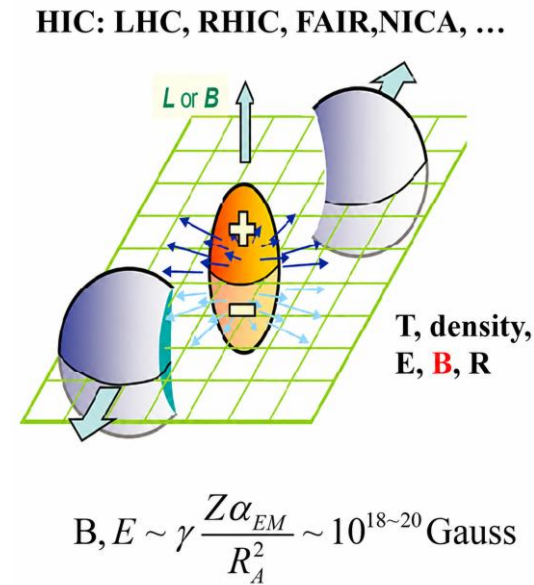
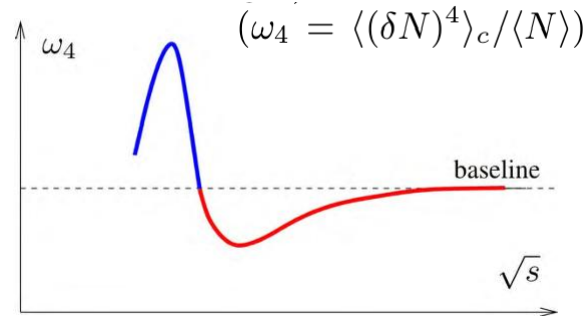
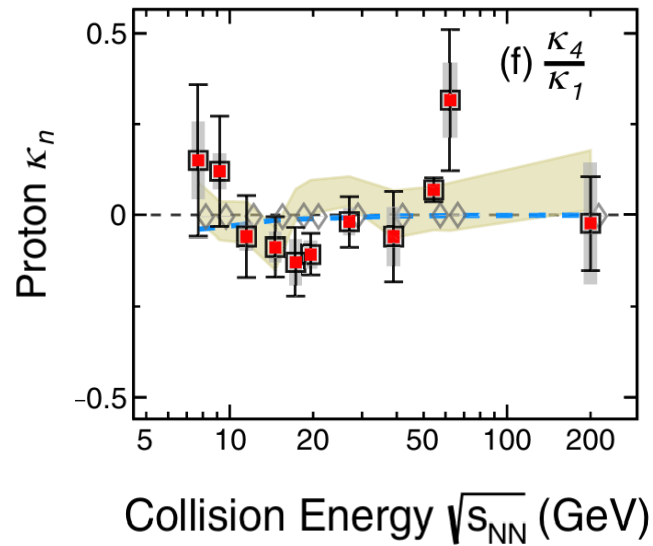


From ALICE Experiment



Heavy-Ion Collisions

QCD phase transition



Baryon number $\delta N = N - \langle N \rangle$, $C_1 = \langle N \rangle$, $C_2 = \langle \delta N^2 \rangle$

$$C_3 = \langle \delta N^3 \rangle, \quad C_4 = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2$$

$$\kappa_1 = C_1, \kappa_2 = -C_1 + C_2, \kappa_3 = 2C_1 - 3C_2 + C_3,$$

$$\kappa_4 = -6C_1 + 11C_2 - 6C_3 + C_4$$

PNJL model:

PNJL:

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu D_\mu - \hat{m}_0) \psi + \mathcal{L}_4 + \mathcal{L}_6 - \mathcal{U}(\Phi, \bar{\Phi}),$$

$$\mathcal{L}_4 = G \sum_{\alpha=0}^8 [(\bar{\psi} \lambda^\alpha \psi)^2 + (\bar{\psi} i\gamma_5 \lambda^\alpha \psi)^2],$$

$$\mathcal{L}_6 = -K [\det \bar{\psi} (1 + \gamma_5) \psi + \det \bar{\psi} (1 - \gamma_5) \psi],$$

$$\mathcal{U}(\Phi, \bar{\Phi}) = T^4 \left[-\frac{b_2(t)}{2} \bar{\Phi} \Phi - \frac{b_3}{6} (\bar{\Phi}^3 + \Phi^3) + \frac{b_4}{4} (\bar{\Phi} \Phi)^2 \right]$$

$$\Omega = 2G(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) - 4K\sigma_u\sigma_d\sigma_s + \mathcal{U}(\Phi, \bar{\Phi}) + \Omega_q,$$

$$\Omega_q = - \sum_{f=u,d,s} \frac{|Q_f B|}{2\pi} \sum_l \alpha_l \int \frac{dp_z}{2\pi} \left[3E_f \right. \\ \left. + T \ln \left(1 + 3\Phi e^{-\beta E_f^+} + 3\bar{\Phi} e^{-2\beta E_f^+} + e^{-3\beta E_f^+} \right) \right. \\ \left. + T \ln \left(1 + 3\bar{\Phi} e^{-\beta E_f^-} + 3\Phi e^{-2\beta E_f^-} + e^{-3\beta E_f^-} \right) \right],$$

Pauli–Villars
(PV):

$$\sum_l \int \frac{dp_z}{2\pi} F(E_f) \longrightarrow \sum_l \int \frac{dp_z}{2\pi} \sum_{i=0}^N c_i F(E_{f,i}),$$

Magnetic-Field-Independent
Regularization
(MFIR):

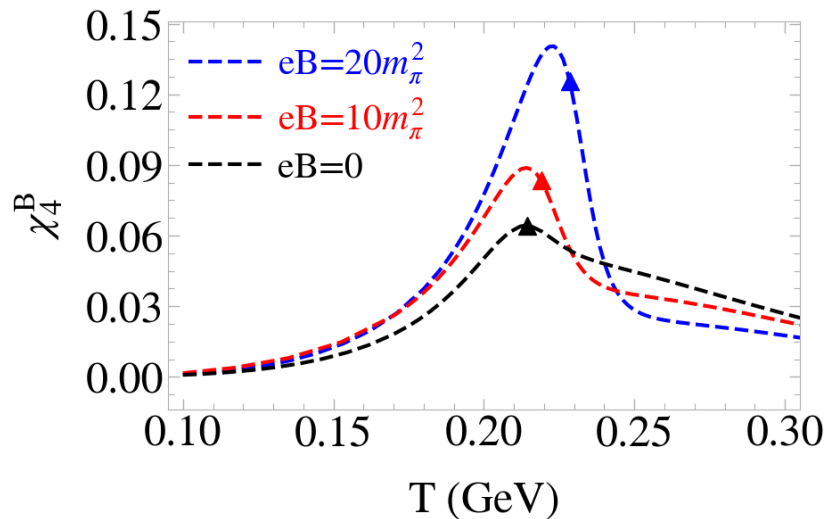
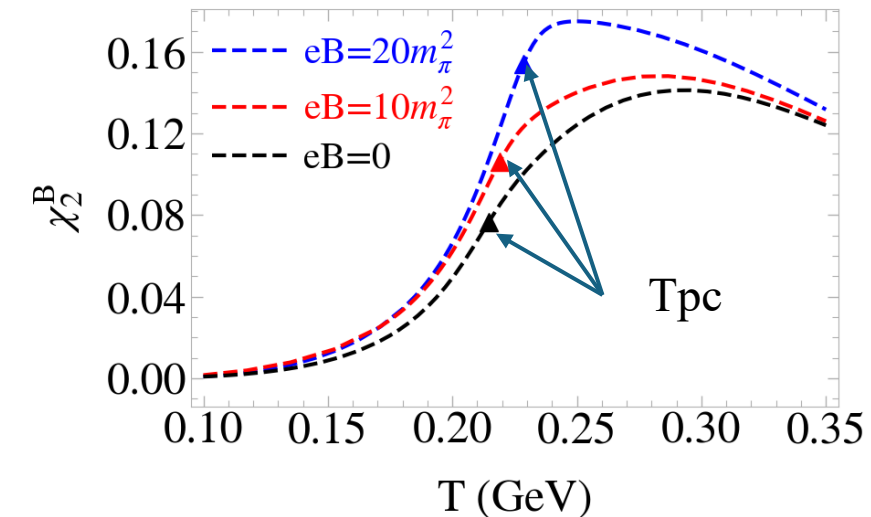
$$\Omega^{vac} = -2N_c \sum_{f=u,d,s} \int \frac{d^3 p}{(2\pi)^3} [p^2 + M^2]^{1/2}$$

$$\Omega^{mag} = -\frac{N_c}{2\pi^2} \sum_{f=u,d,s} (|q_f|eB)^2 \times \left[\zeta'(-1, x) - \frac{1}{2} (x^2 - x) \ln x + \frac{x^2}{4} \right]$$

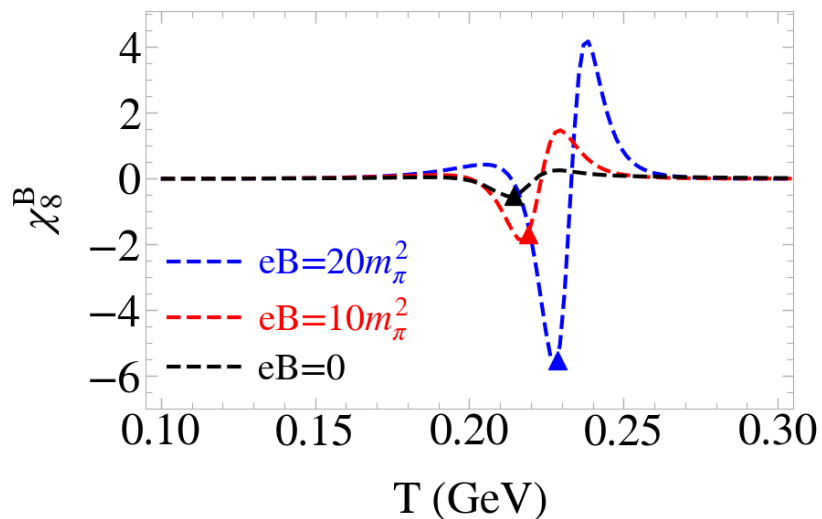
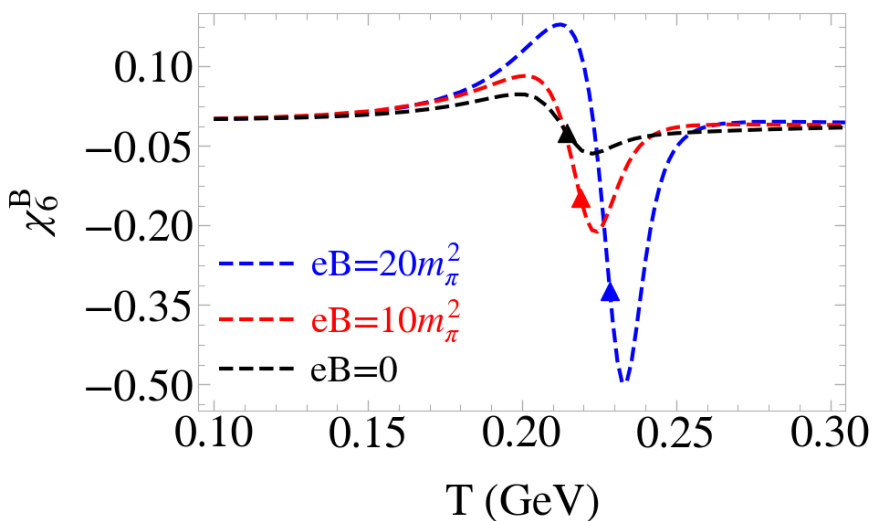
Phys. Rev. C 79, 035807

Fluctuations and Correlations

———— Fluctuations of conserved charges (B)



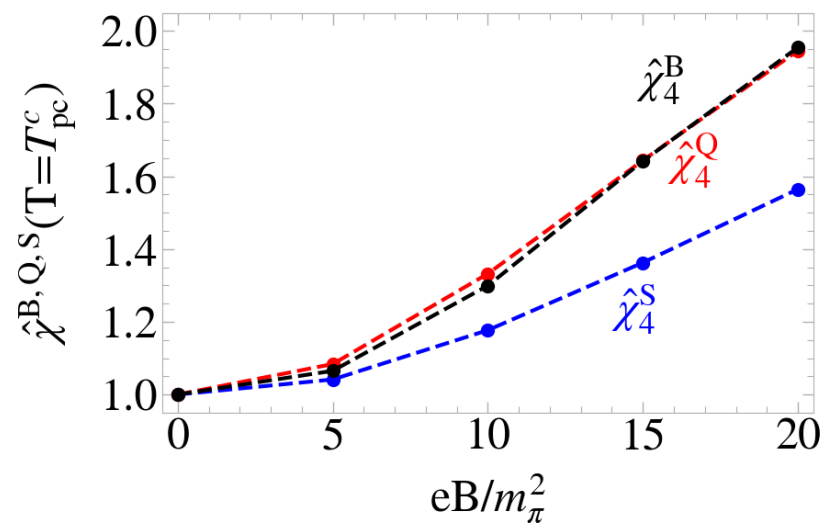
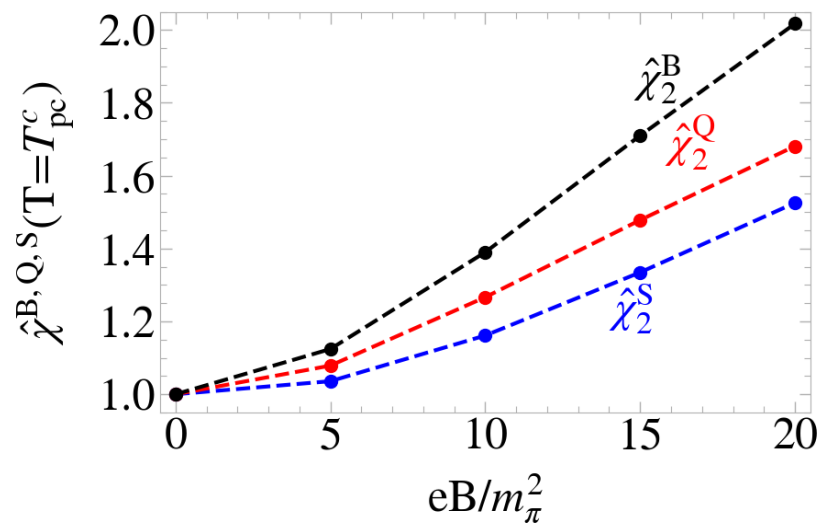
$$\chi_{i,j}^{B,Q,S} = - \left. \frac{\partial^{i+j+k} (\Omega/T^4)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu_X=0}.$$



$$\begin{aligned} \chi_2^X &= \frac{1}{VT^3} \langle \delta N_X^2 \rangle, \\ \chi_3^X &= \frac{1}{VT^3} \langle \delta N_X^3 \rangle, \\ \chi_4^X &= \frac{1}{VT^3} \left(\langle \delta N_X^4 \rangle - 3 \langle \delta N_X^2 \rangle^2 \right), \\ &\dots \\ X &= B, Q, S \end{aligned}$$

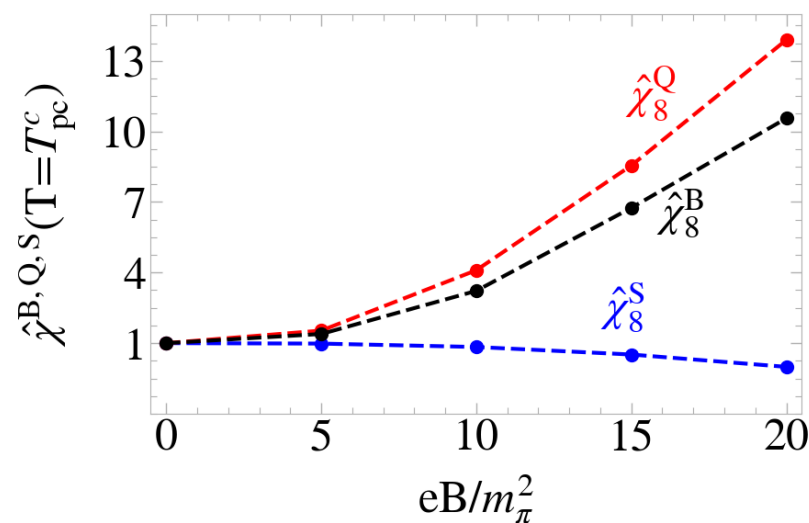
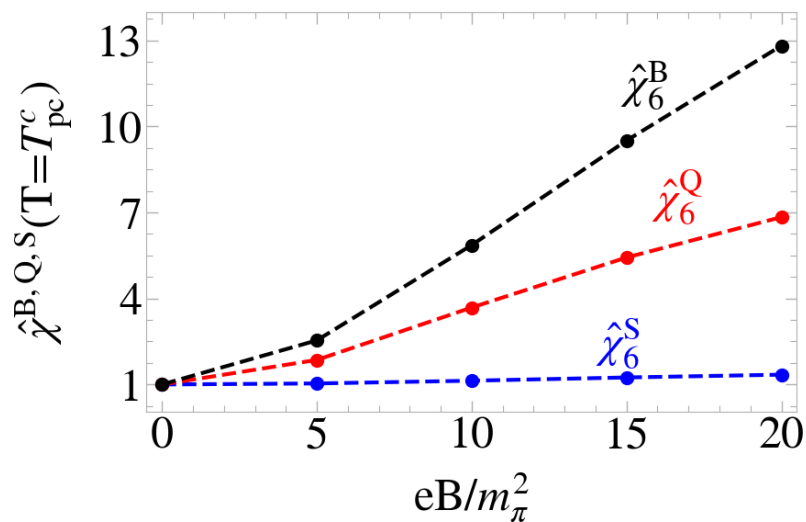
Fluctuations and Correlations

— Fluctuations of conserved charges (B)



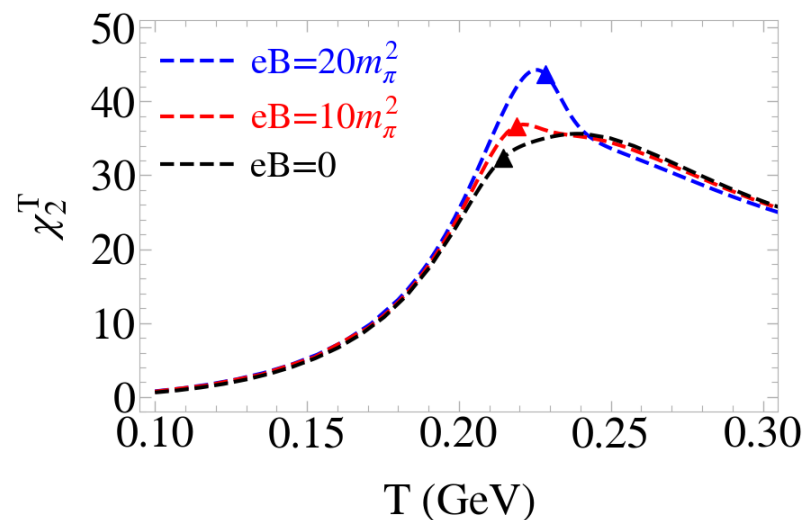
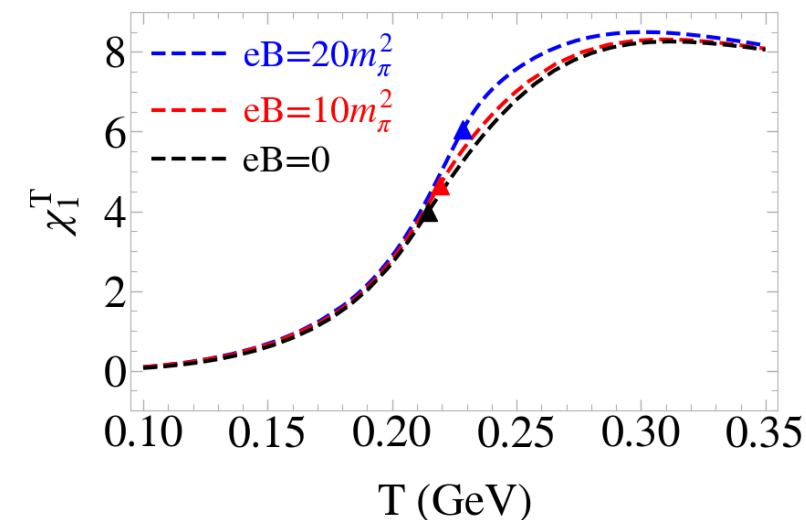
$$\hat{\chi}_n^X = \frac{\chi_n^X(eB, T_{pc}^c(eB))}{\chi_n^X(eB=0, T_{pc}^c(eB=0))}$$

$$X = B, Q, S$$

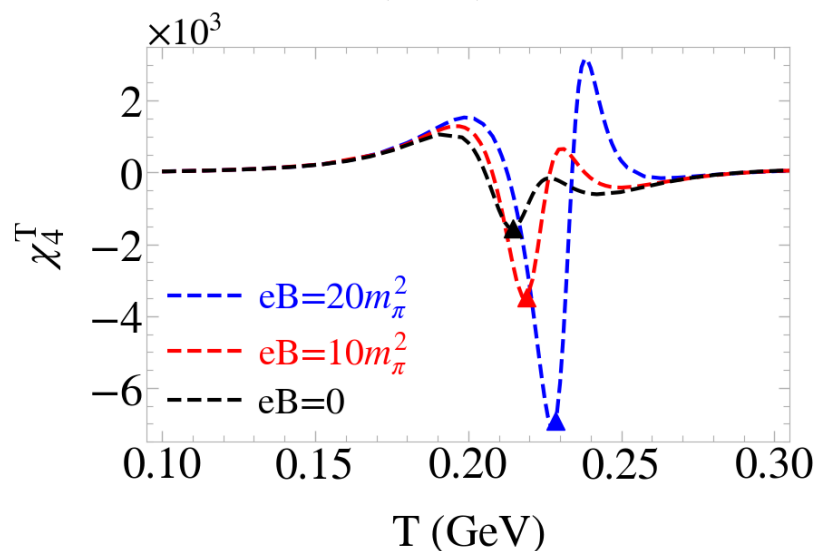
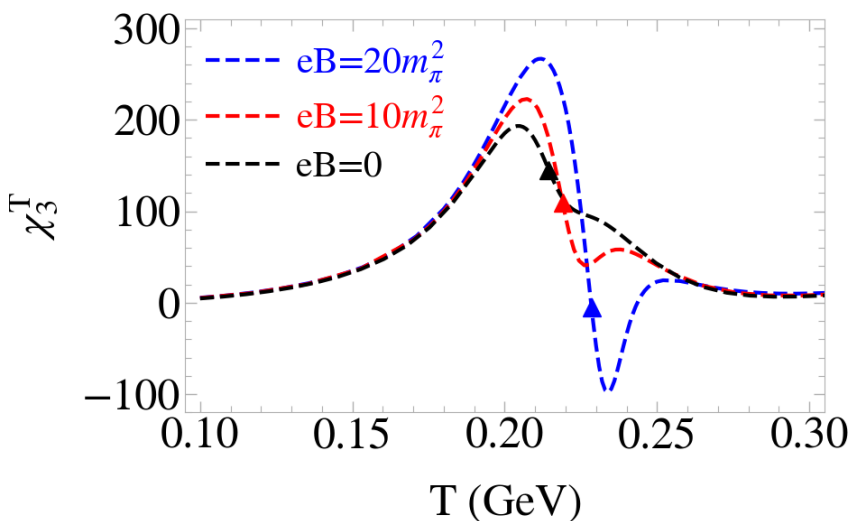


Fluctuations and Correlations

———— Fluctuations of the temperature (T)



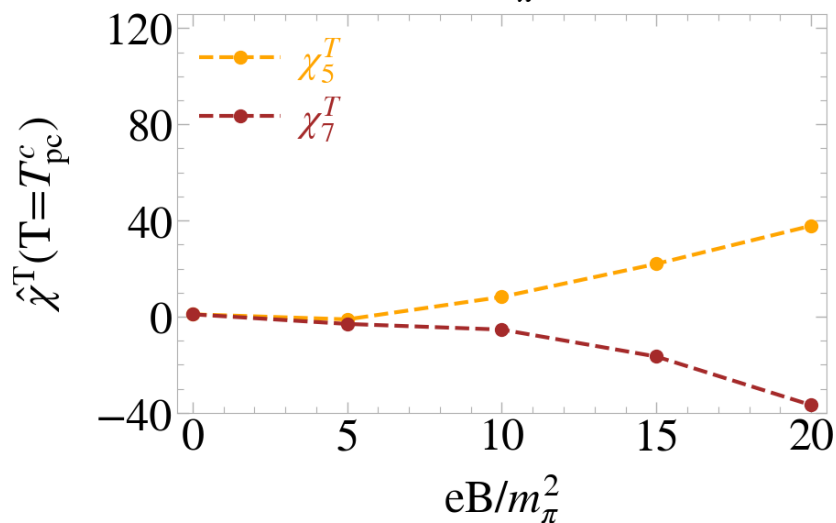
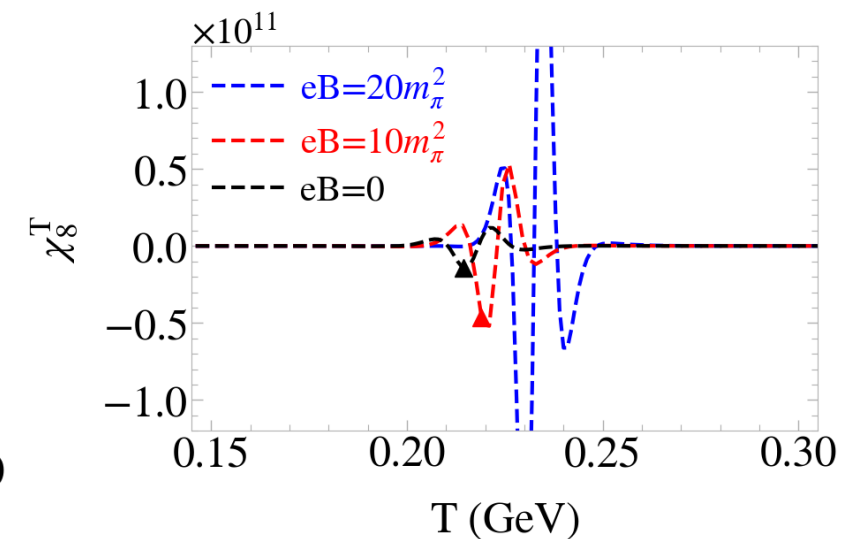
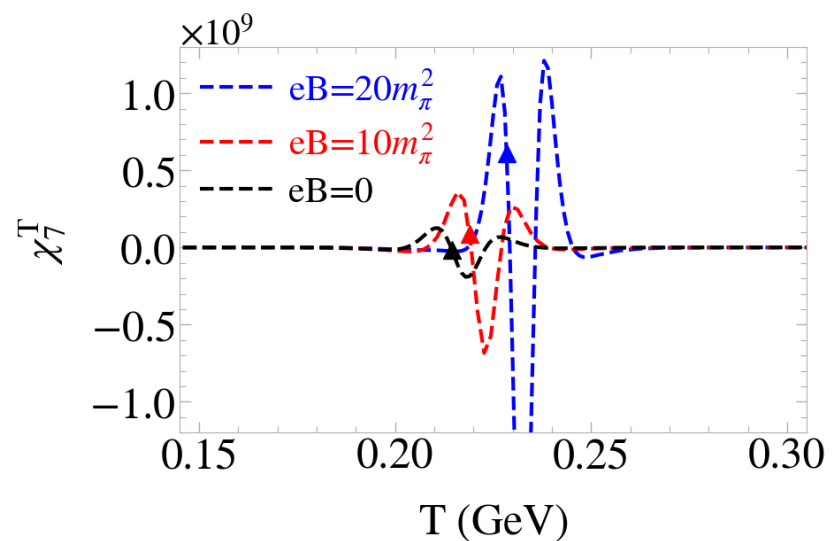
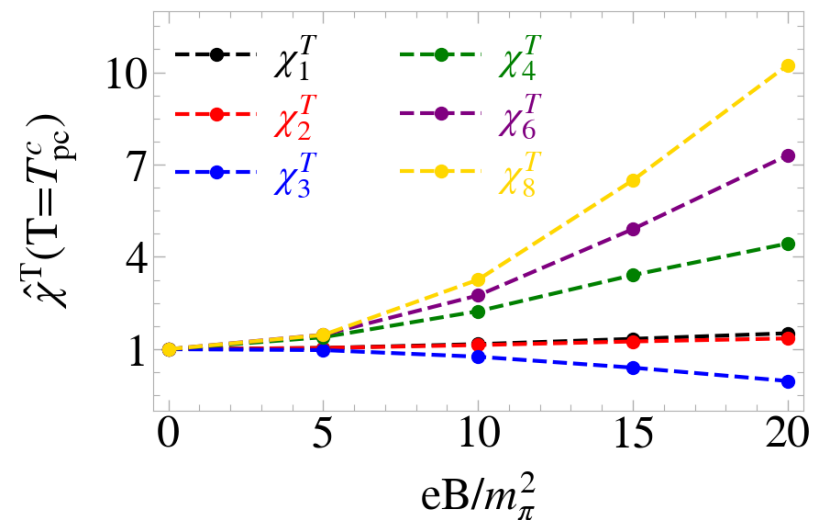
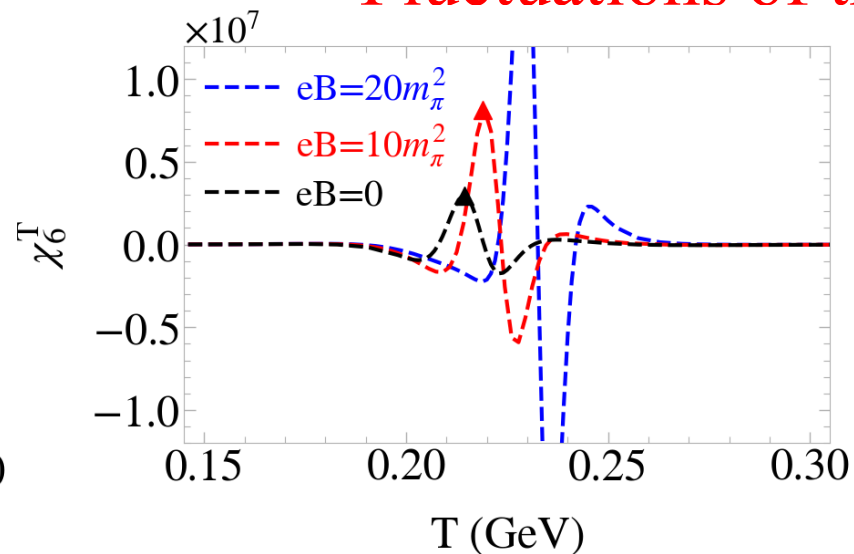
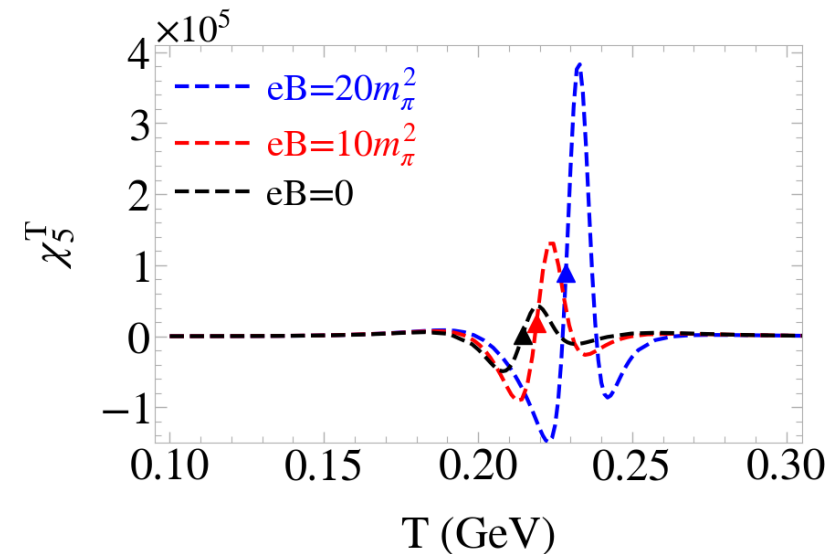
$$\chi_n^T = T^{n-4} \frac{\partial^n P}{\partial T^n}.$$



$$\hat{\chi}_n^T = \frac{\chi_n^T(eB, T_{pc}^c(eB))}{\chi_n^T(eB=0, T_{pc}^c(eB=0))}$$

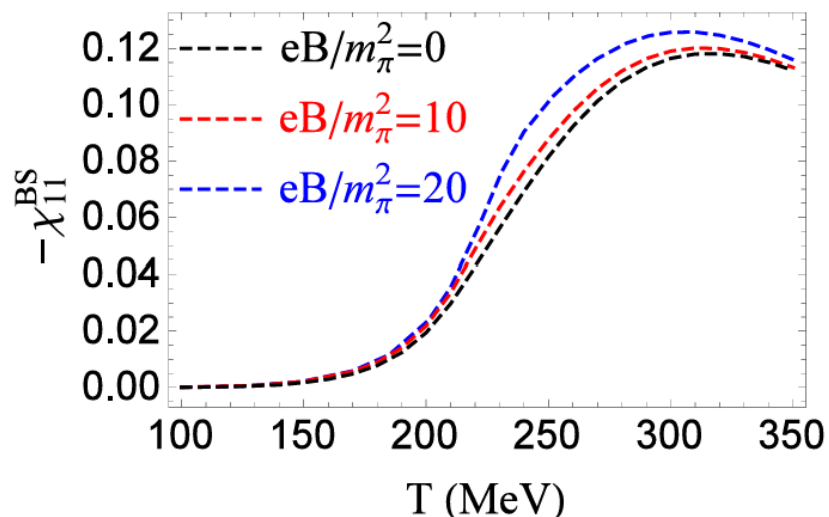
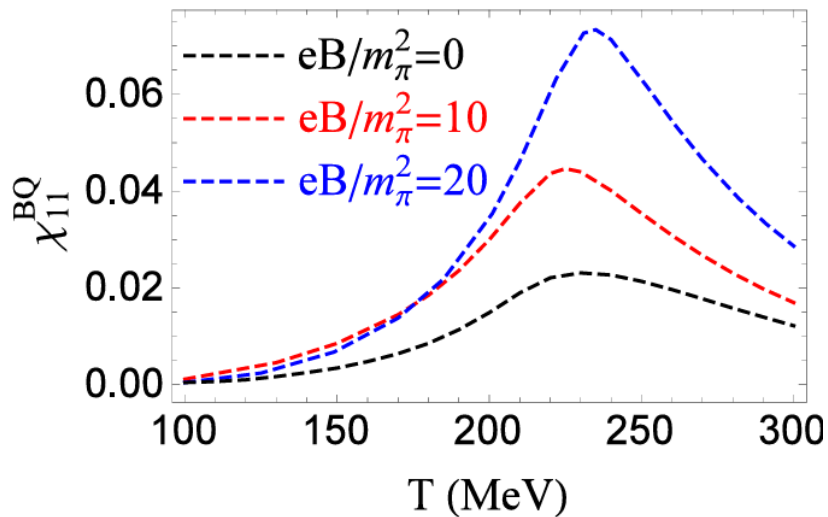
Fluctuations and Correlations

— Fluctuations of the temperature (T)

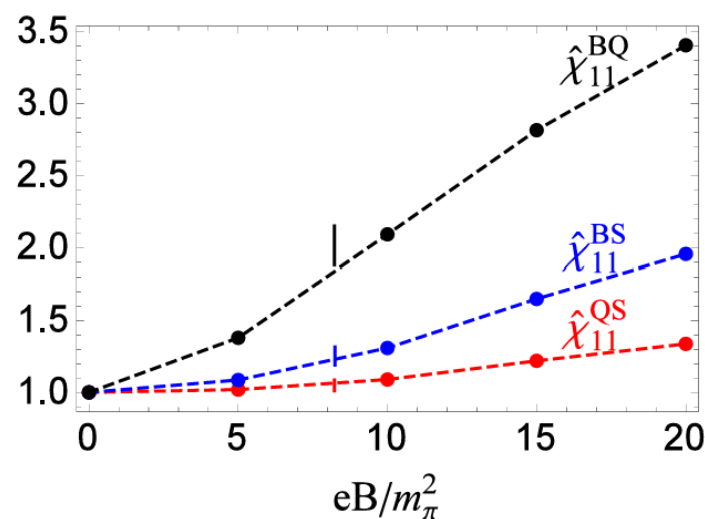
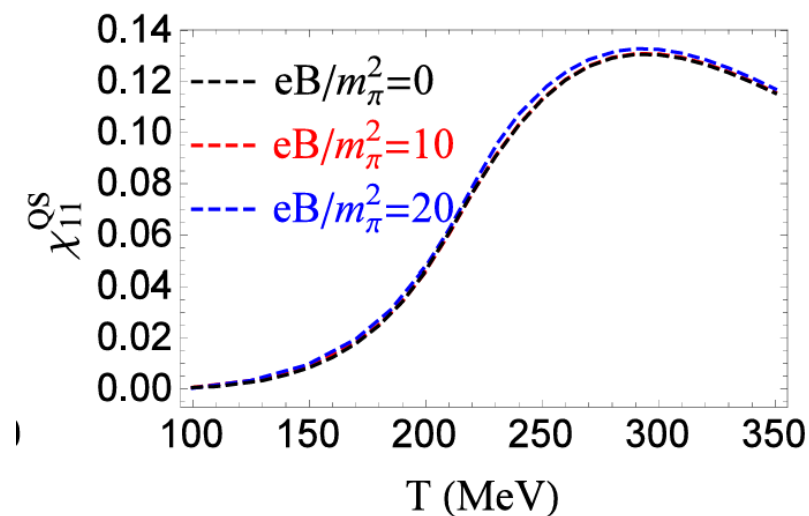


Fluctuations and Correlations

———— Correlations of conserved charges (BQS)



$$\chi_{i,j}^{B,Q,S} = - \frac{\partial^{i+j+k} (\Omega/T^4)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \Big|_{\mu_X=0}.$$

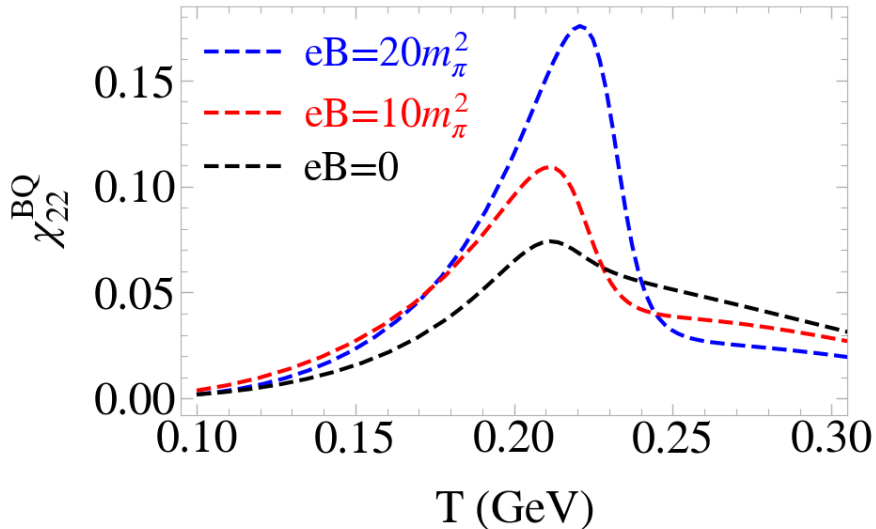
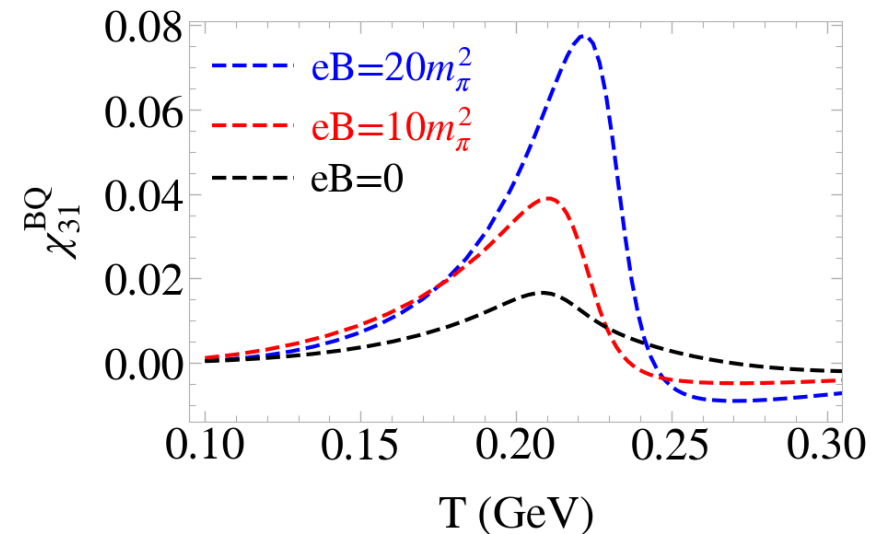


$$\hat{\chi}_{mn}^{XY} = \frac{\chi_{mn}^{XY}(eB, T_{pc}^c(eB))}{\chi_{mn}^{XY}(eB=0, T_{pc}^c(eB=0))}$$

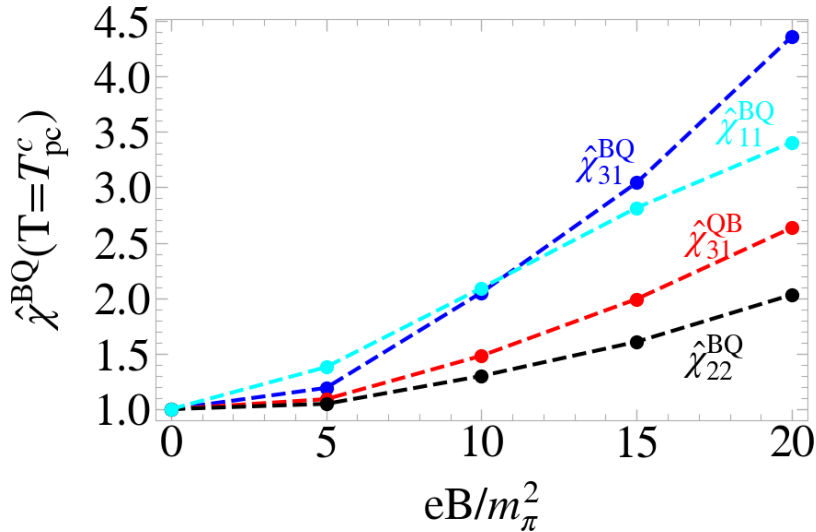
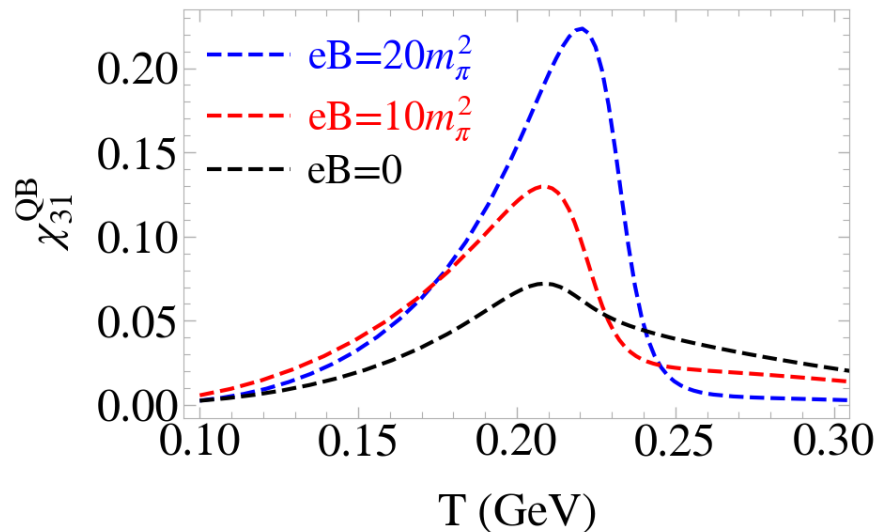
$$X, Y = B, Q, S$$

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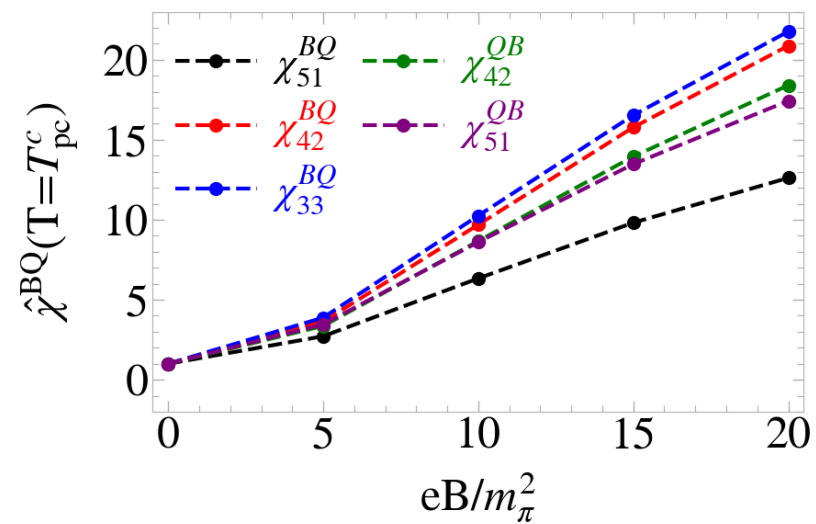
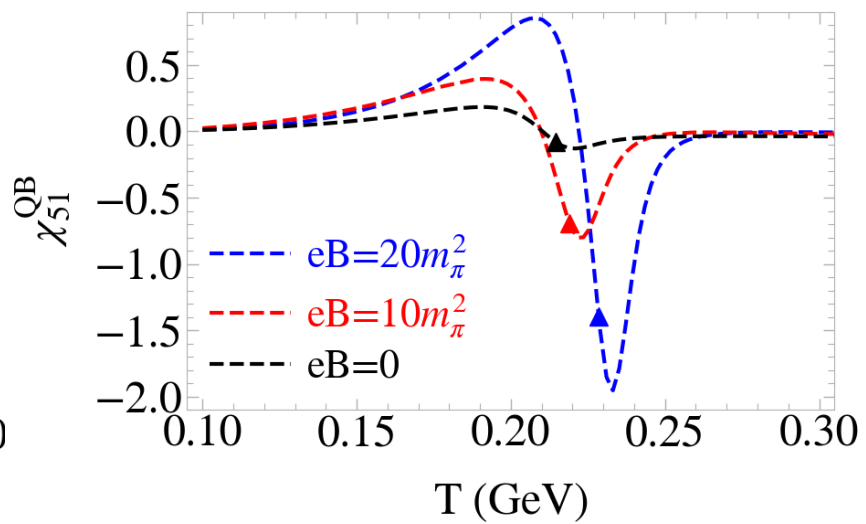
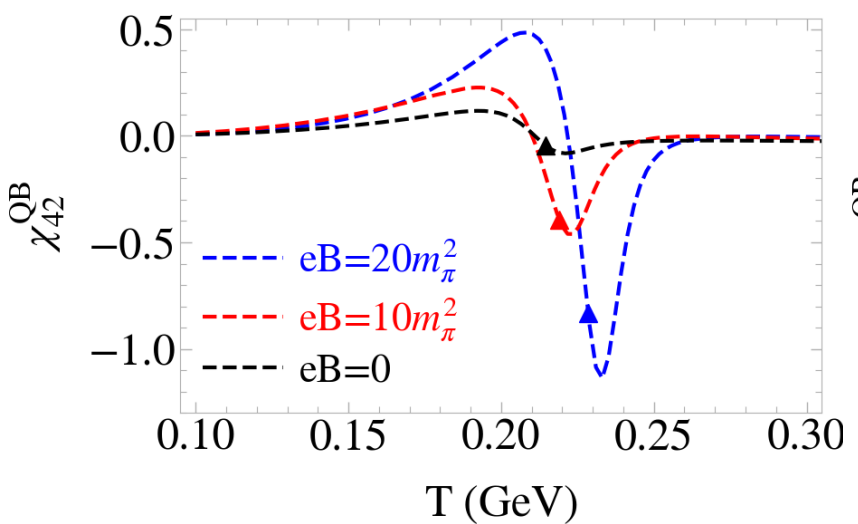
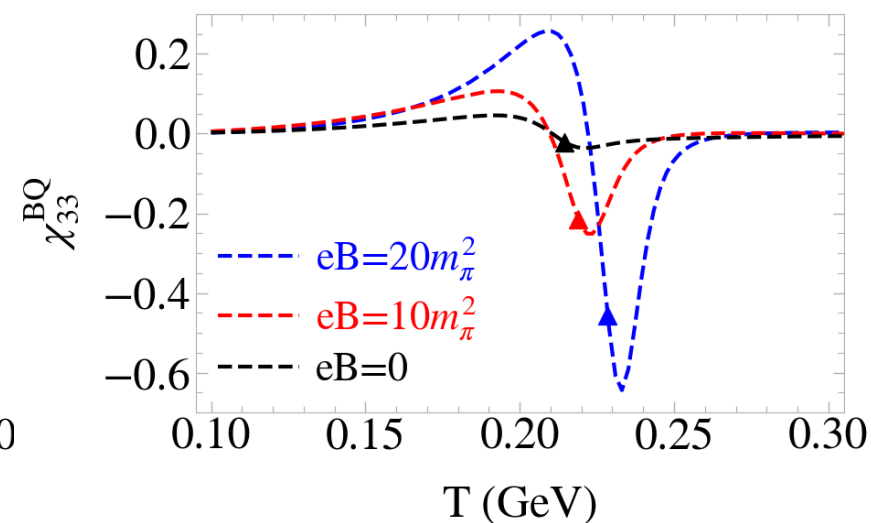
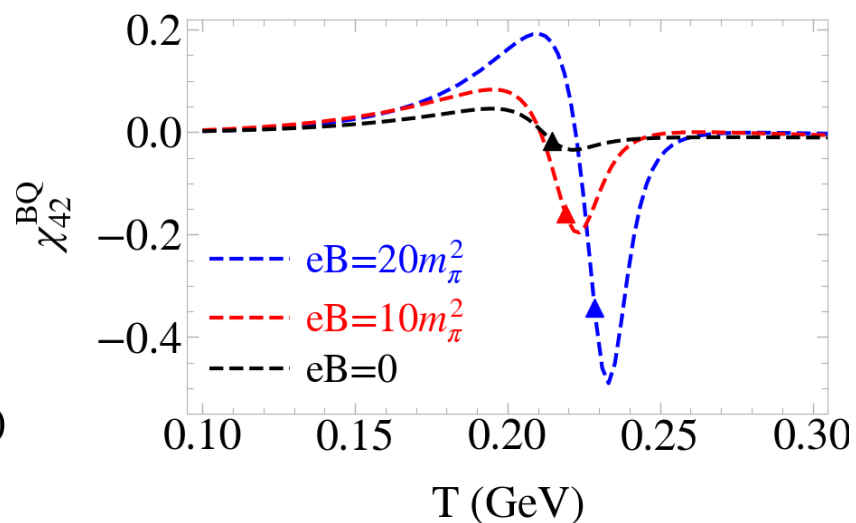
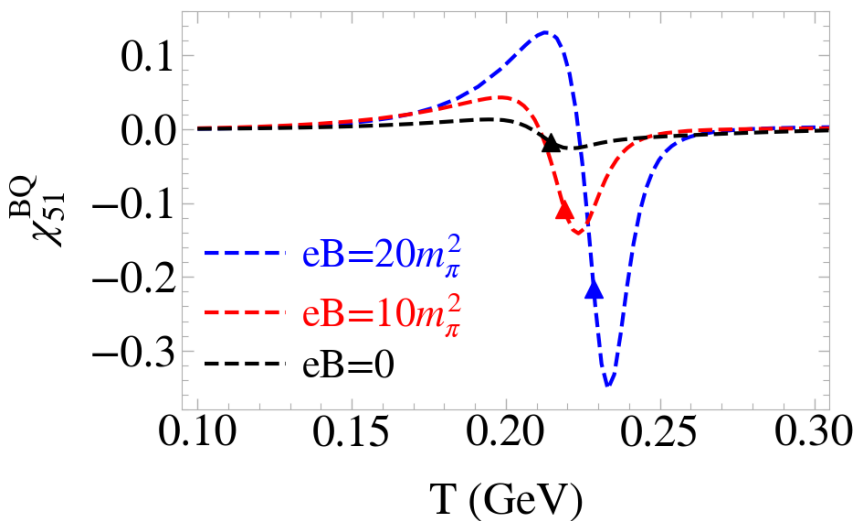


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$$X, Y = B, Q, S$$

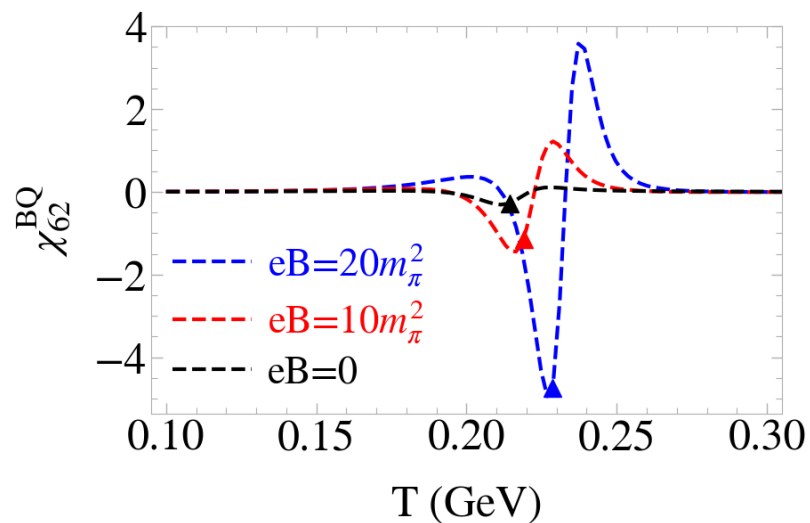
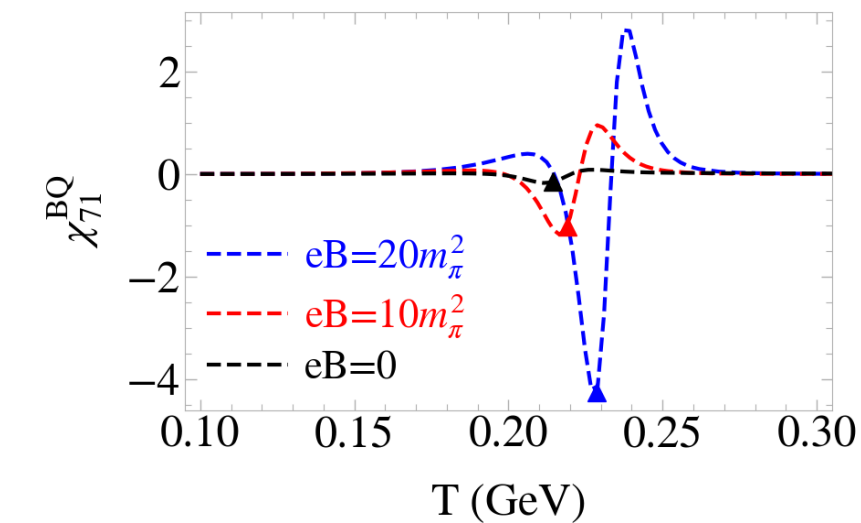
Fluctuations and Correlations

Correlations of conserved charges (BQS)

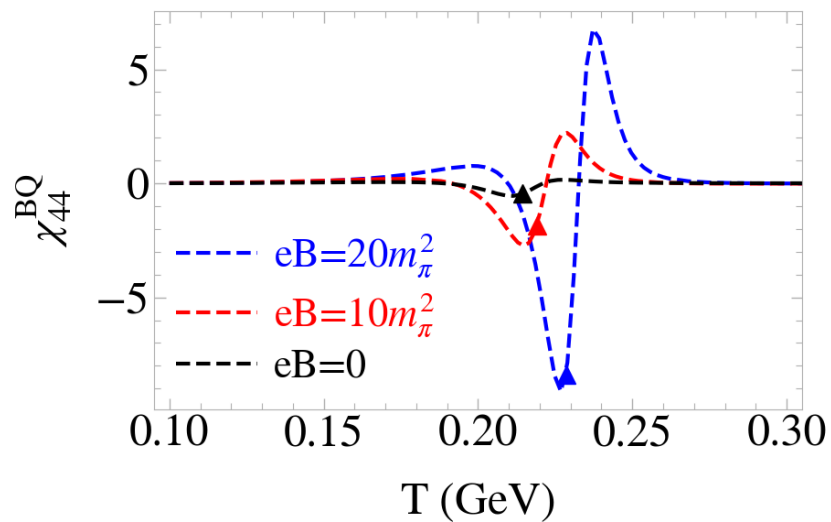
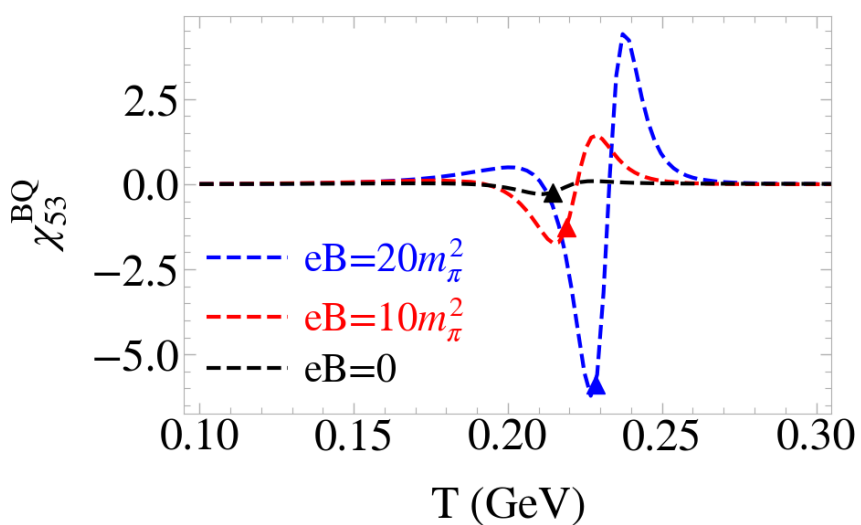


Fluctuations and Correlations

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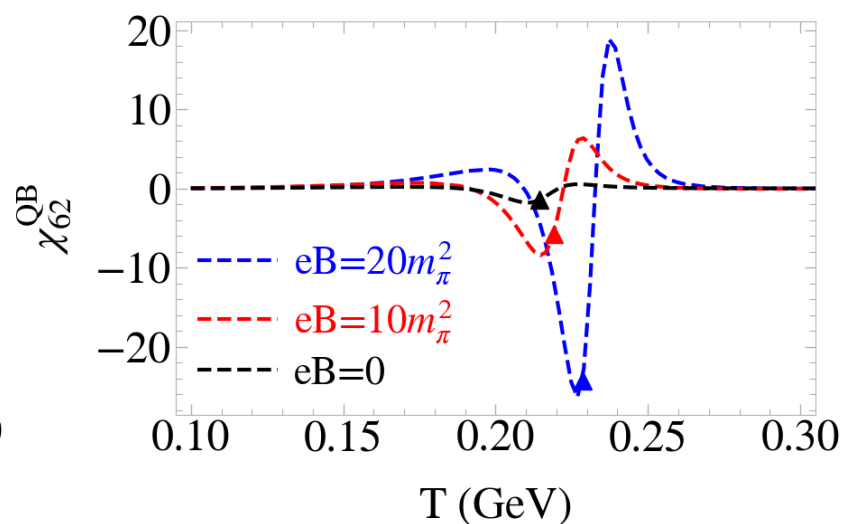
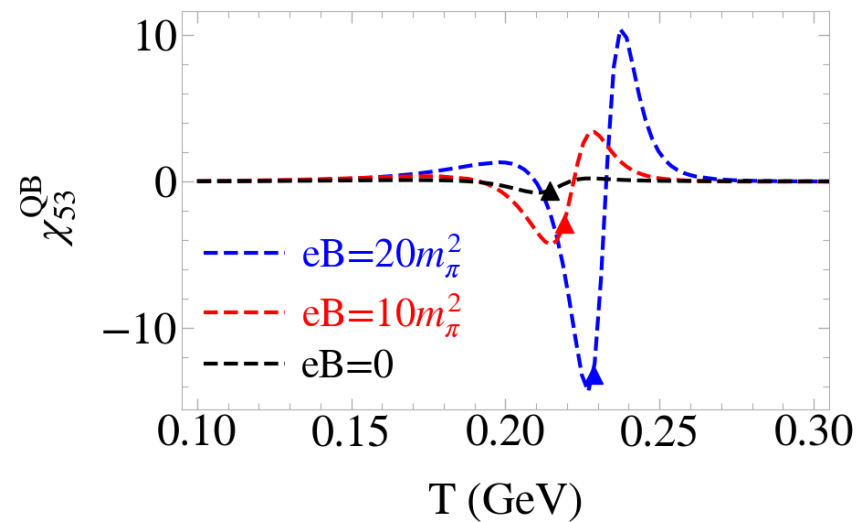


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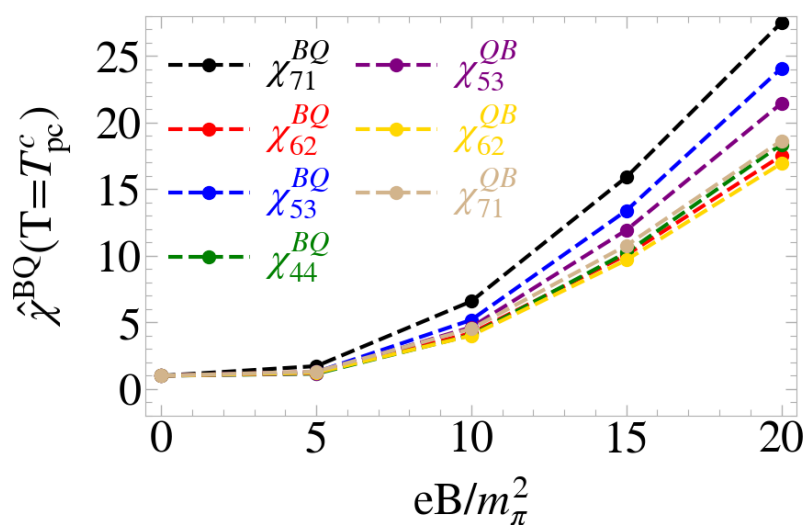
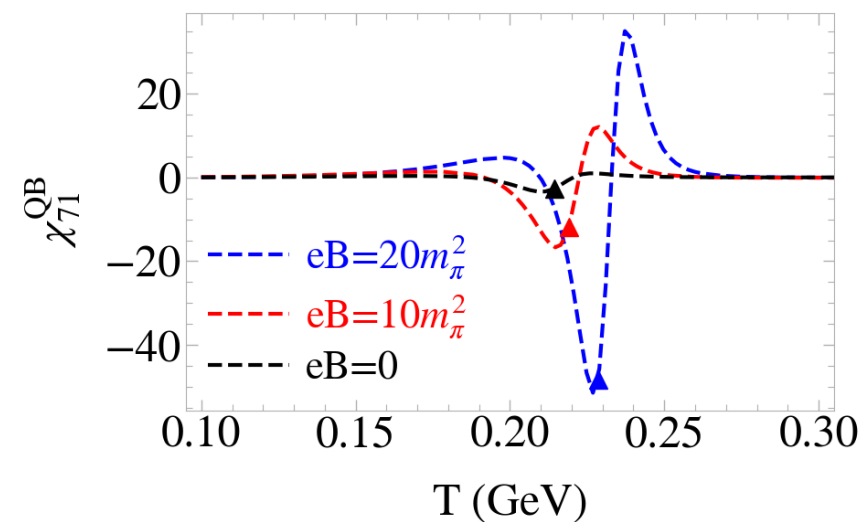


Fluctuations and Correlations

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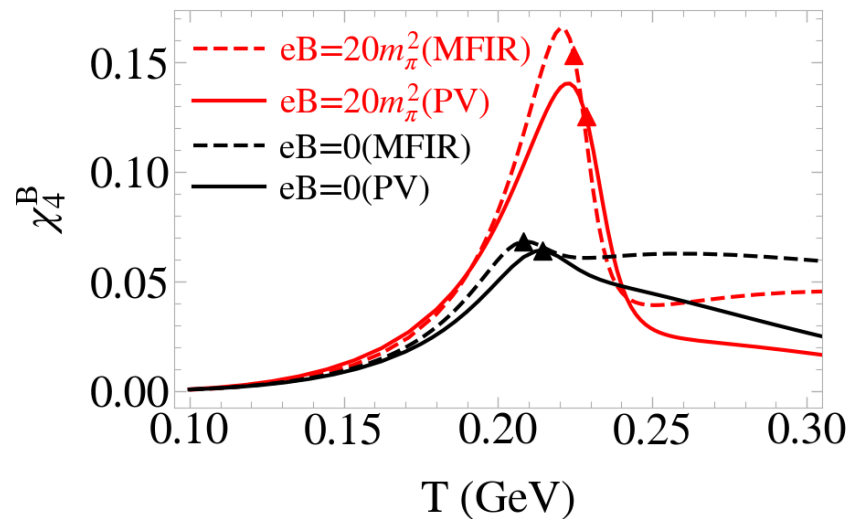
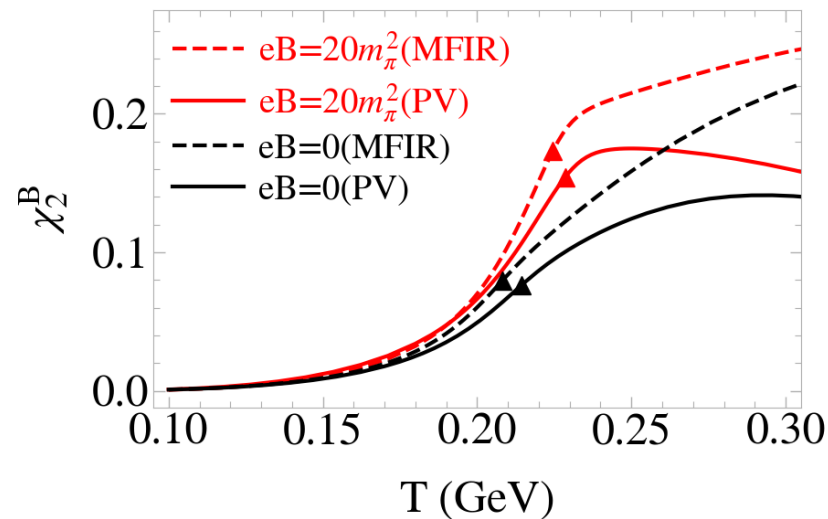


$X, Y = B, Q, S$

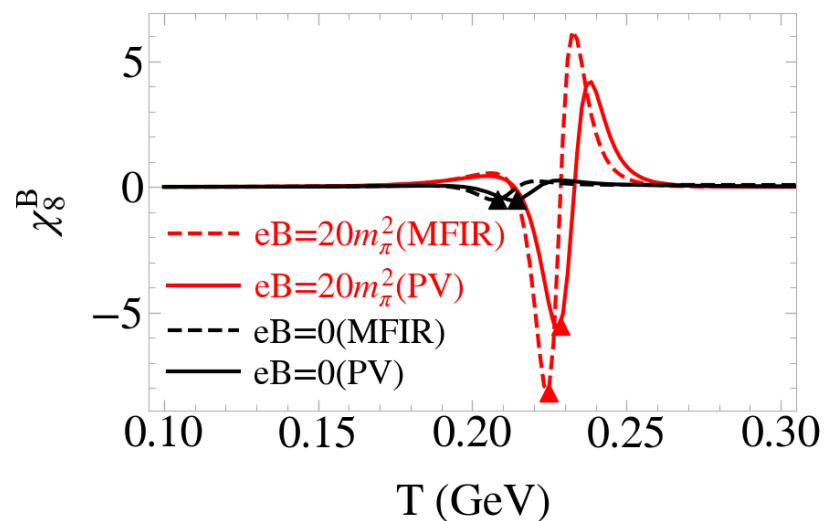
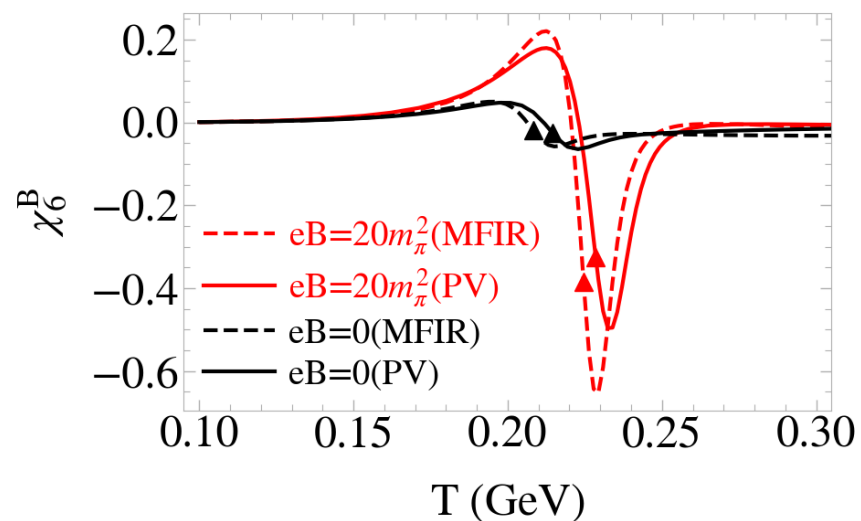
Summary

- Second and fourth order correlations can be served as a **effective magnetometer** of QCD.
- Higher order fluctuations and correlations exhibit more pronounced **nonmonotonic behavior**
- χ_{71}^{BQ} shows **more strong sensitivity** to the magnetic field and may serve as a potential **magnetometer** for QCD matter.

Backup



$$\chi_{i,j}^{B,Q,S} = - \frac{\partial^{i+j+k} (\Omega/T^4)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \Big|_{\mu_X=0}.$$

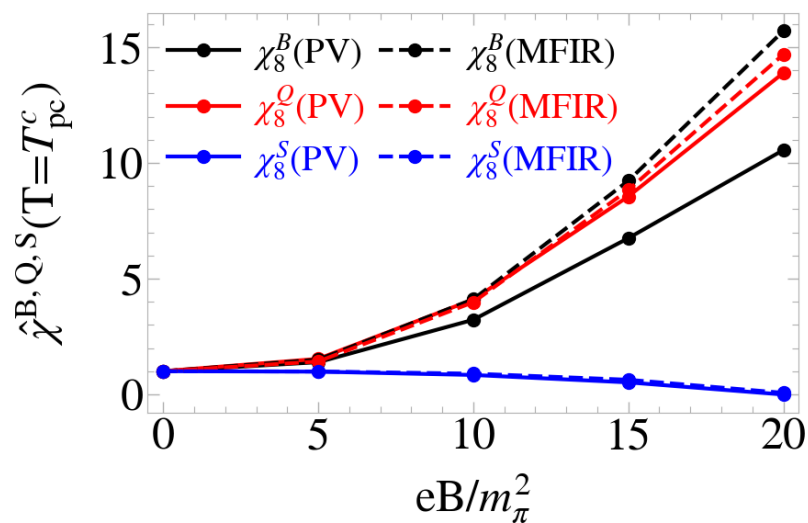
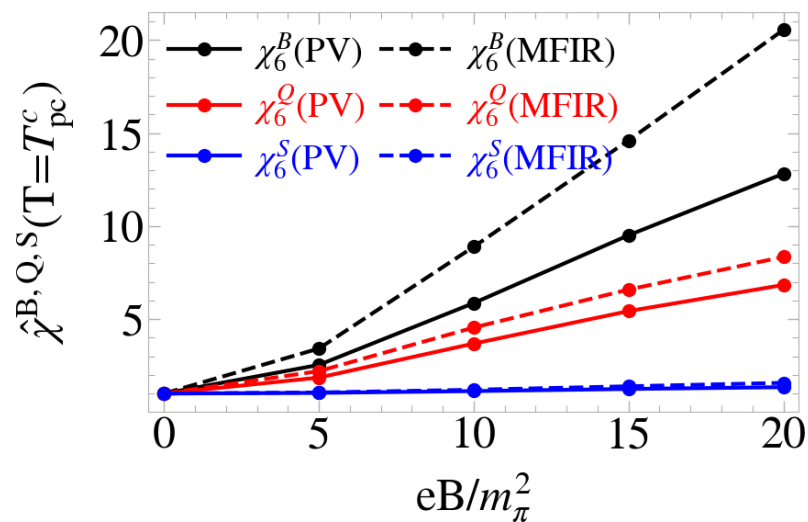
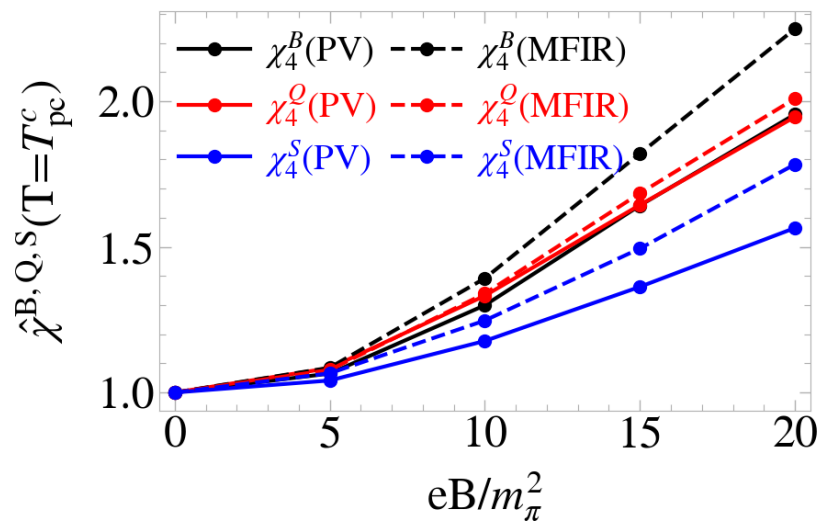
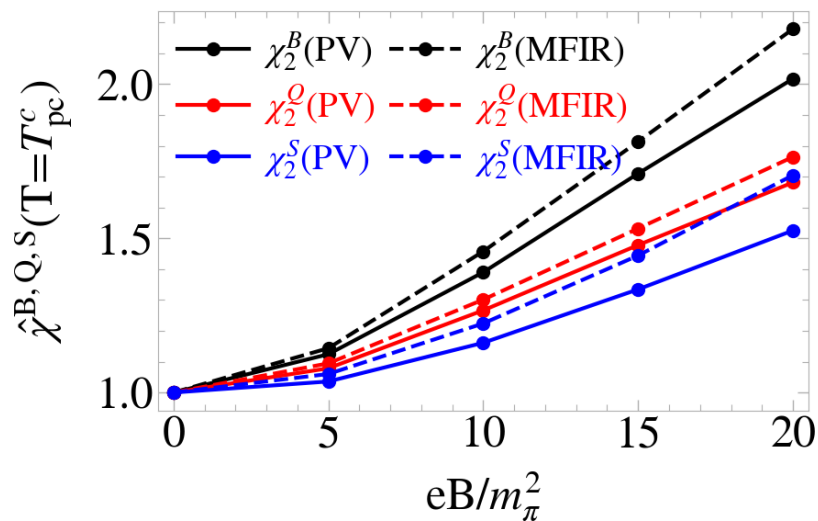


$$\chi_2^X = \frac{1}{VT^3} \langle \delta N_X^2 \rangle,$$

$$\chi_3^X = \frac{1}{VT^3} \langle \delta N_X^3 \rangle,$$

$$\chi_4^X = \frac{1}{VT^3} \left(\langle \delta N_X^4 \rangle - 3 \langle \delta N_X^2 \rangle^2 \right),$$

Backup



$$\hat{\chi}_n^X = \frac{\chi_n^X(eB, T_{pc}^c(eB))}{\chi_n^X(eB=0, T_{pc}^c(eB=0))}$$

$$X = B, Q, S$$