

Anisotropic Hydrodynamics Expands the Domain of Applicability of Hydrodynamics

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S.Zhao, Y. Peng, H. Heinz and H. Song, Phys.Rev.Lett. 137 (2026) 6, 062301

Y. Peng, V. Ambrus, C. Werthmann, S. Schlichting, U. Heinz and H. Song, Phys.Rev.C 113 (2026) 2, 024918

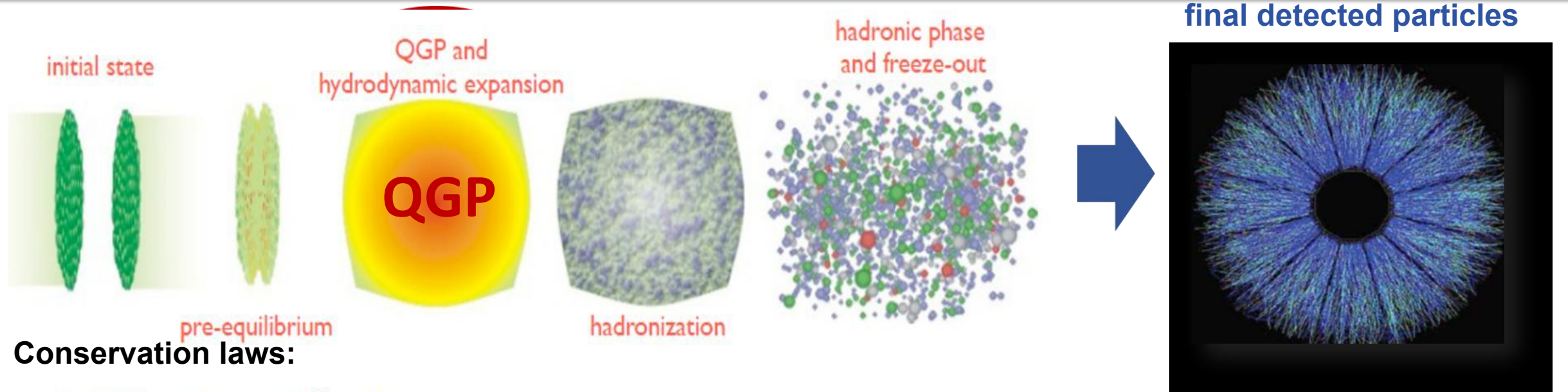
Y. Peng, Y. Wang, S. Zhao and H. Song, paper in preparation

Y. Wang, Y. Peng, W. Zhao and H. Song, paper in preparation

Outline

- **Background of hydrodynamics and its applicability**
- **Brief introduction of viscous anisotropic hydrodynamics (VAH)**
- **Comparison between kinetic theory, VAH, and traditional viscous hydrodynamics**
- **Apply VAH in pp collisions at 13 TeV**
- **Apply VAH in O+O/Ne+Ne collisions at 5.36 TeV**

Quark Gluon Plasma (QGP) and Hydrodynamics



Conservation laws:

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N_i^\mu = 0.$$

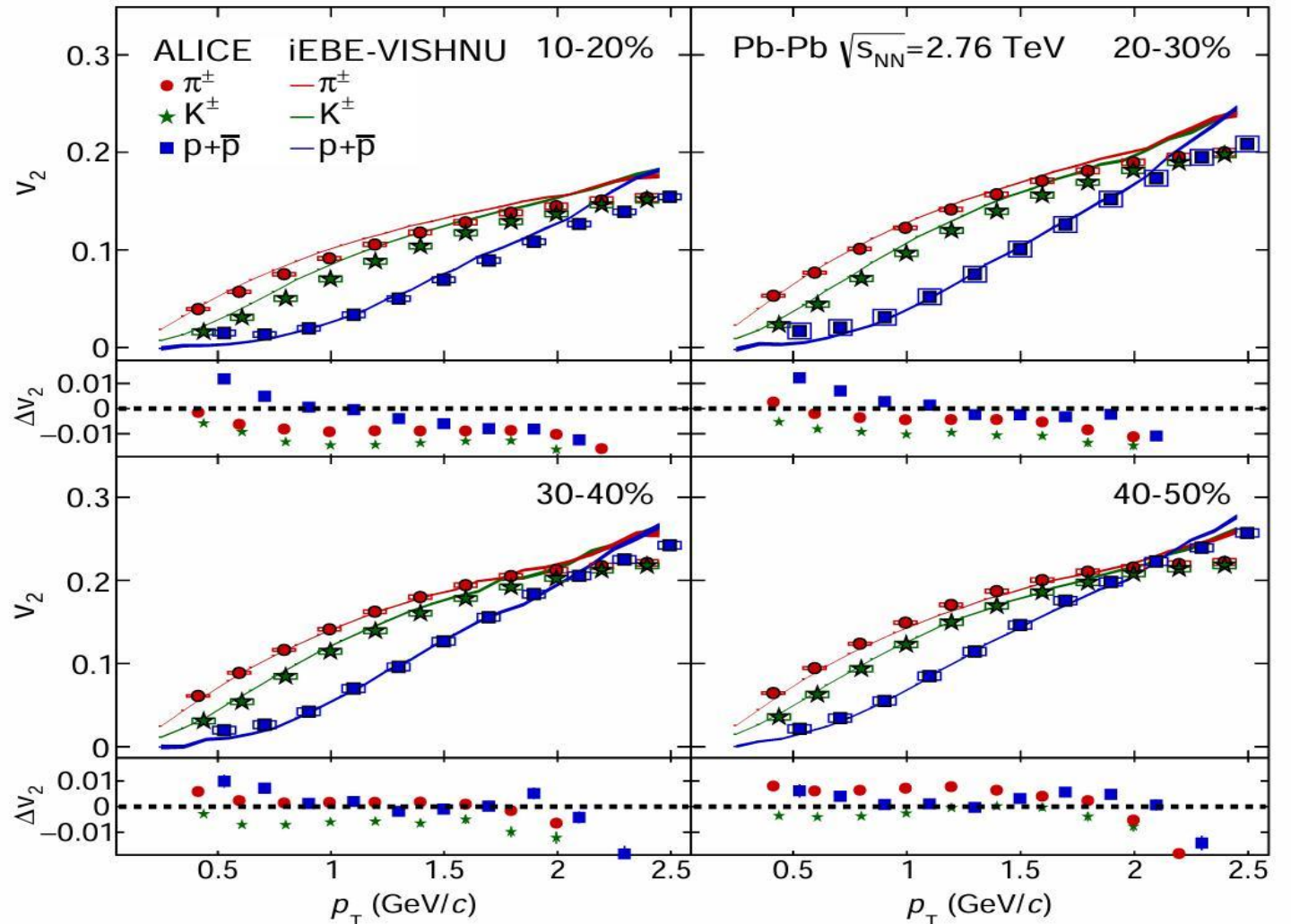
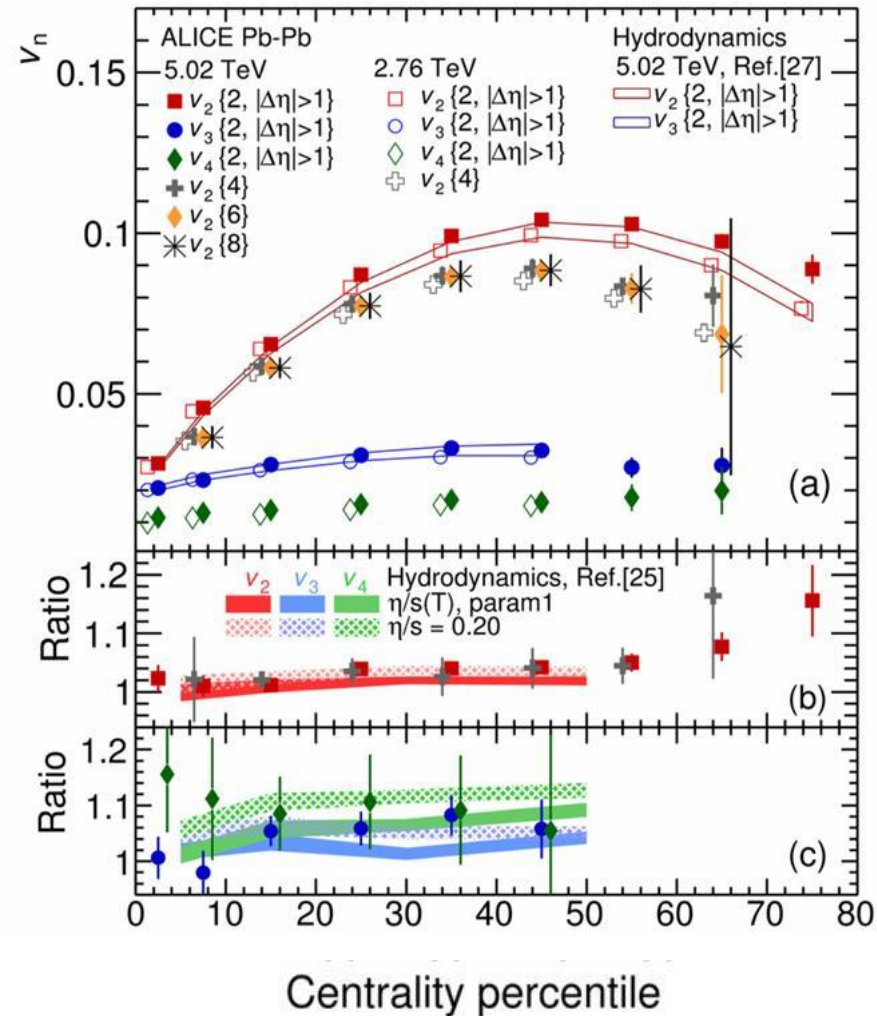
2nd order I-S eq:

$$\begin{aligned} \dot{\Pi} &= -\frac{1}{\tau_\Pi} \left[\Pi + \zeta\theta - l_{\Pi q} \nabla_\mu q^\mu + \Pi \zeta T \partial_\mu \left(\frac{\tau_\Pi u^\mu}{2\zeta T} \right) \right], \\ \dot{q}^{\langle\mu\rangle} &= -\frac{1}{\tau_q} \left[q^\mu + \lambda \frac{nT^2}{e+p} \nabla^\mu \frac{\nu}{T} + l_{q\pi} \nabla_\nu \pi^{\mu\nu} + l_{q\Pi} \nabla^\mu \Pi - \lambda T^2 q^\mu \partial_\nu \left(\frac{\tau_q u^\nu}{2\lambda T^2} \right) \right], \\ \dot{\pi}^{\langle\mu\nu\rangle} &= -\frac{1}{\tau_\pi} \left[\pi^{\mu\nu} - 2\eta \nabla^{\langle\mu} u^{\nu\rangle} - l_{\pi q} \nabla^{\langle\mu} q^{\nu\rangle} + \pi^{\mu\nu} \eta T \partial_\alpha \left(\frac{\tau_\pi u^\alpha}{2\eta T} \right) \right]. \end{aligned}$$

Input: EoS $\mathcal{E} = \mathcal{E}(\mathcal{P})$; Initial condition

Success in Large Systems

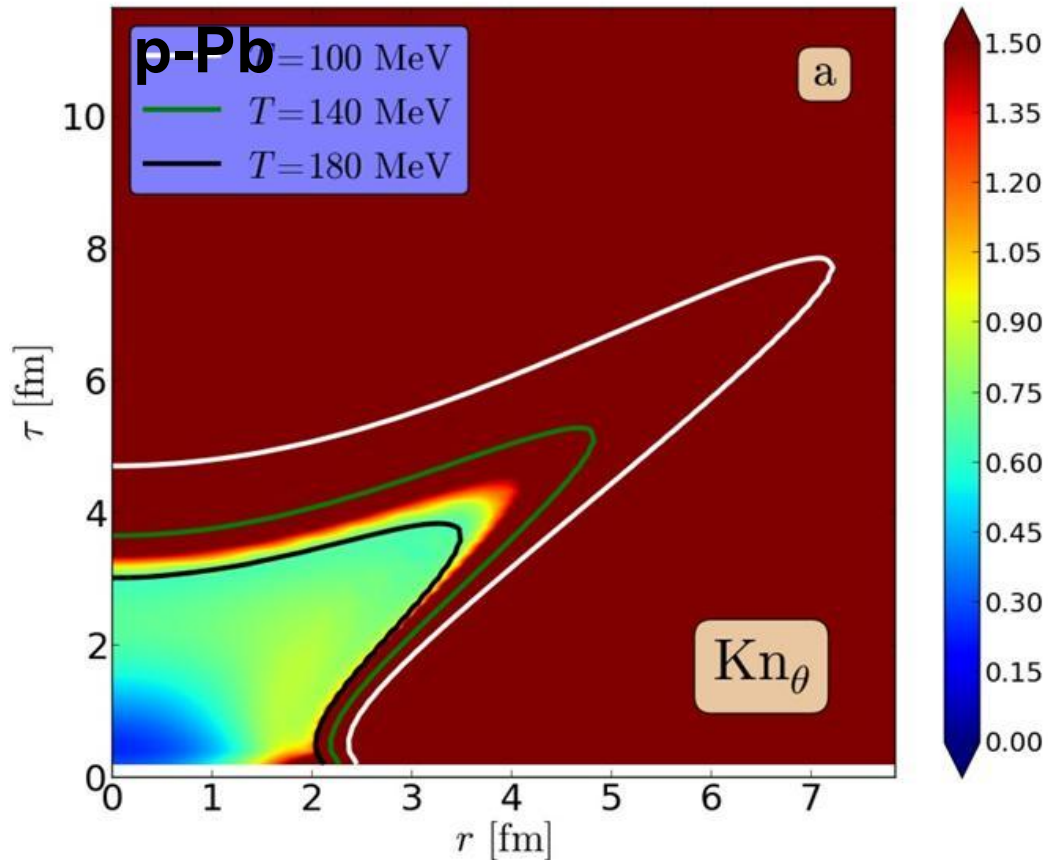
Hydrodynamics nice description of integrated and differential flow of charged and identified hadrons.



Challenge in Small Systems

Applicability of hydro challenged in small systems(p-Pb, p-Au, O-O, p-p, ...):

Large Knudsen number:

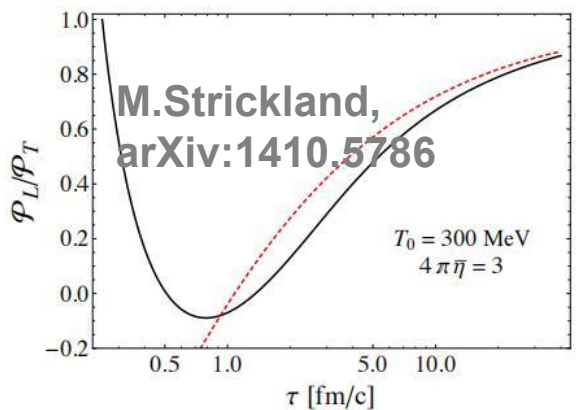
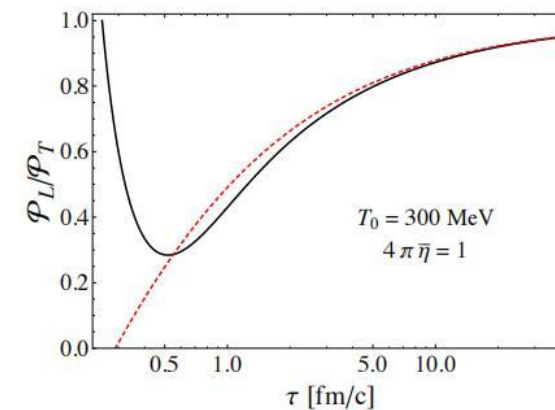
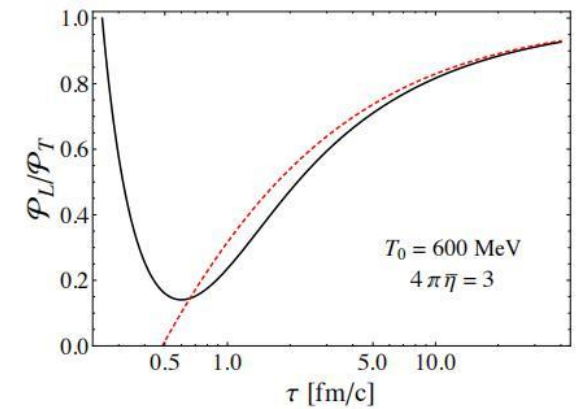
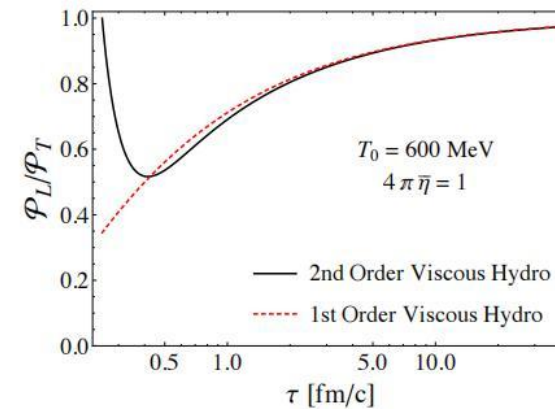


H. Niemi, G.S. Denicol,
arXiv:1404.7327

Early-time pressure anisotropy (large inversed Reynolds number):

Longitudinal pressure: $\mathcal{P}_L = \mathcal{P} + \Pi + \tau^2 \pi^{\eta_s \eta_s}$

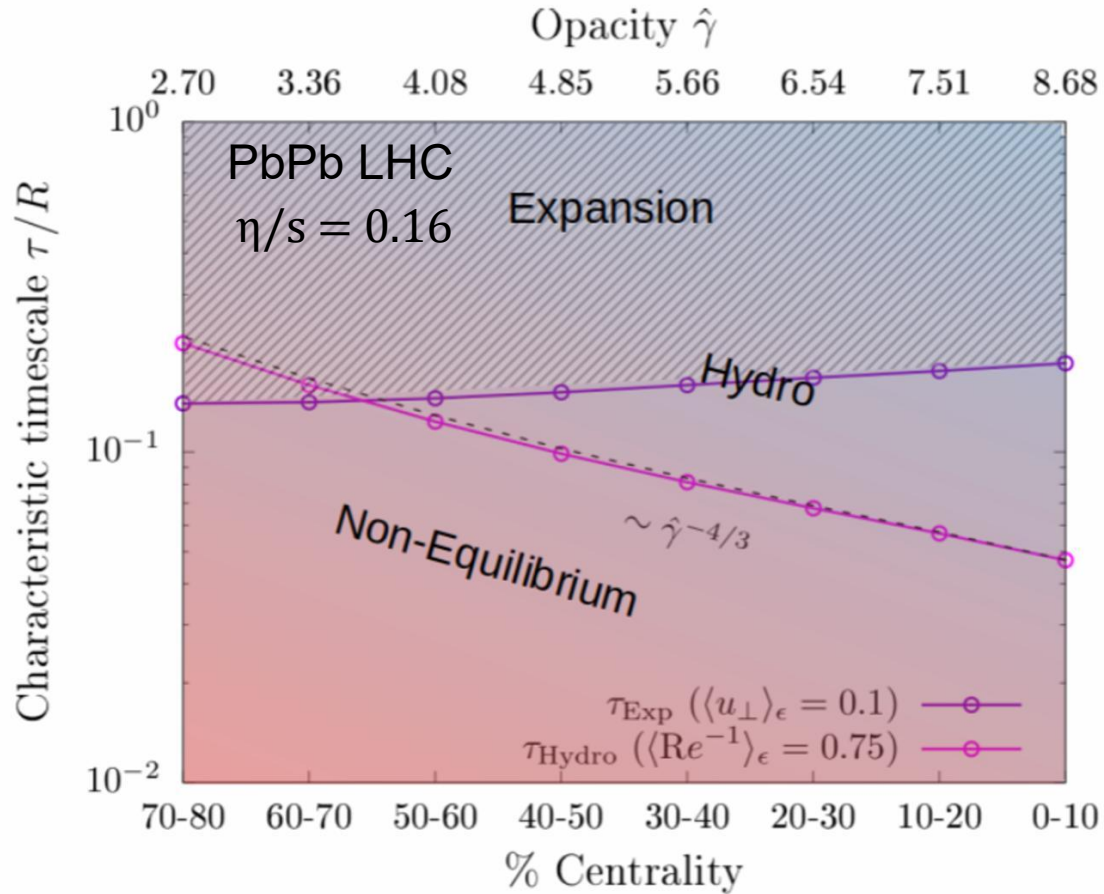
Transverse pressure: $\mathcal{P}_T = \mathcal{P} + \Pi + (\pi^{xx} + \pi^{yy})/2$



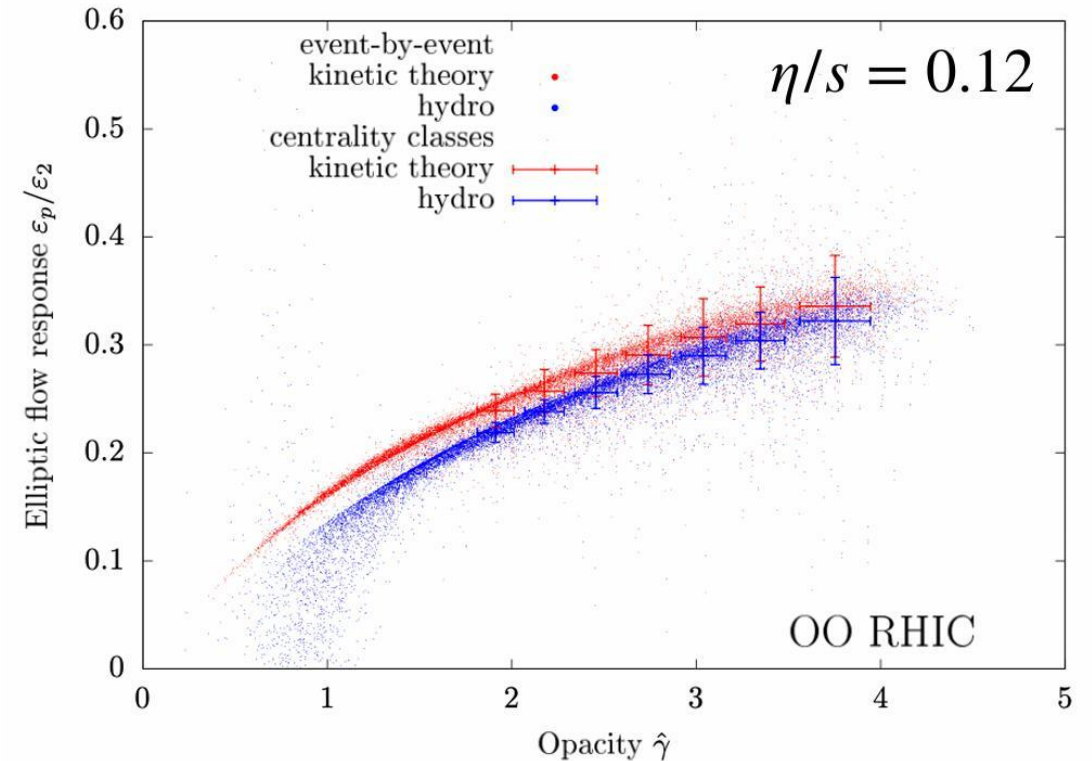
M.Strickland,
arXiv:1410.5786

Non-hydro Effects on Final-state Observables

Transverse expansion before hydrodynamization in dilute systems, leading to overestimated dissipative effects on collective flow in hydrodynamics simulation.



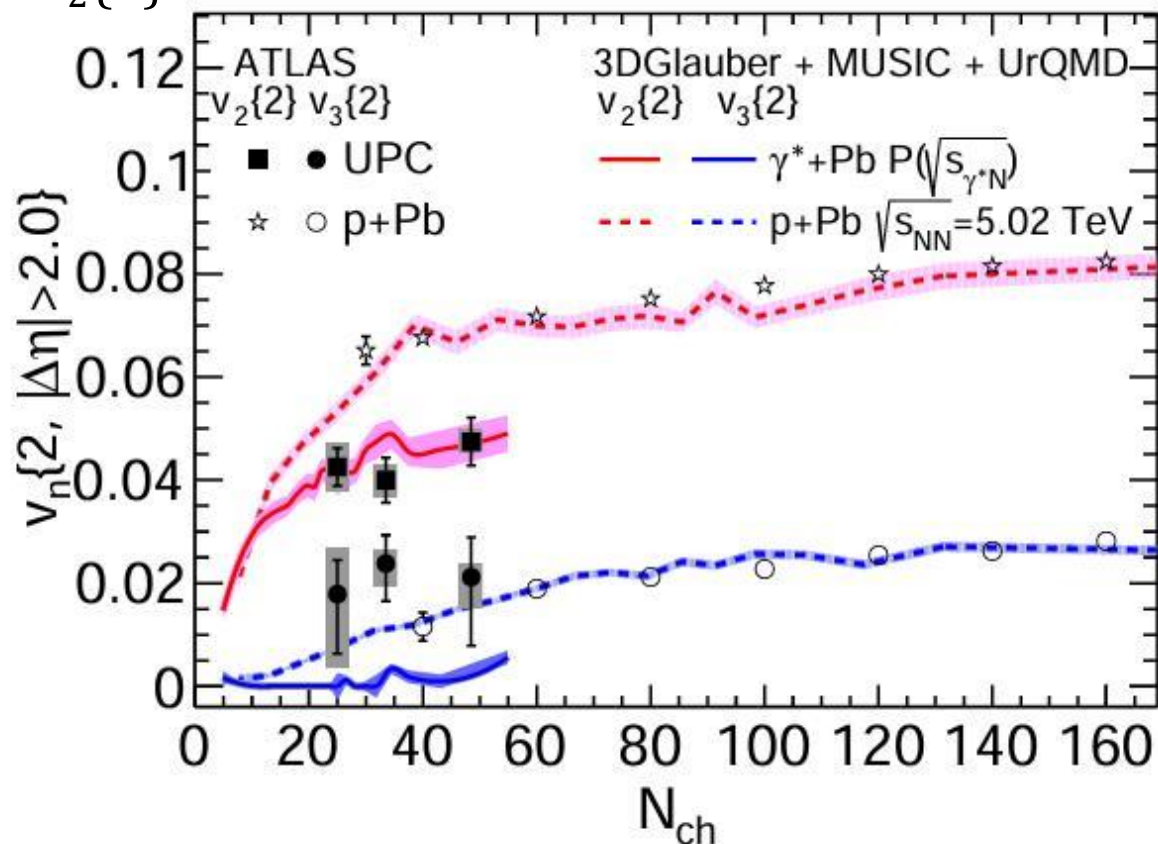
V. Ambrus, S. Schlichting and C. Werthmann,
Phys. Rev. Lett. 130, 152301 (2023)



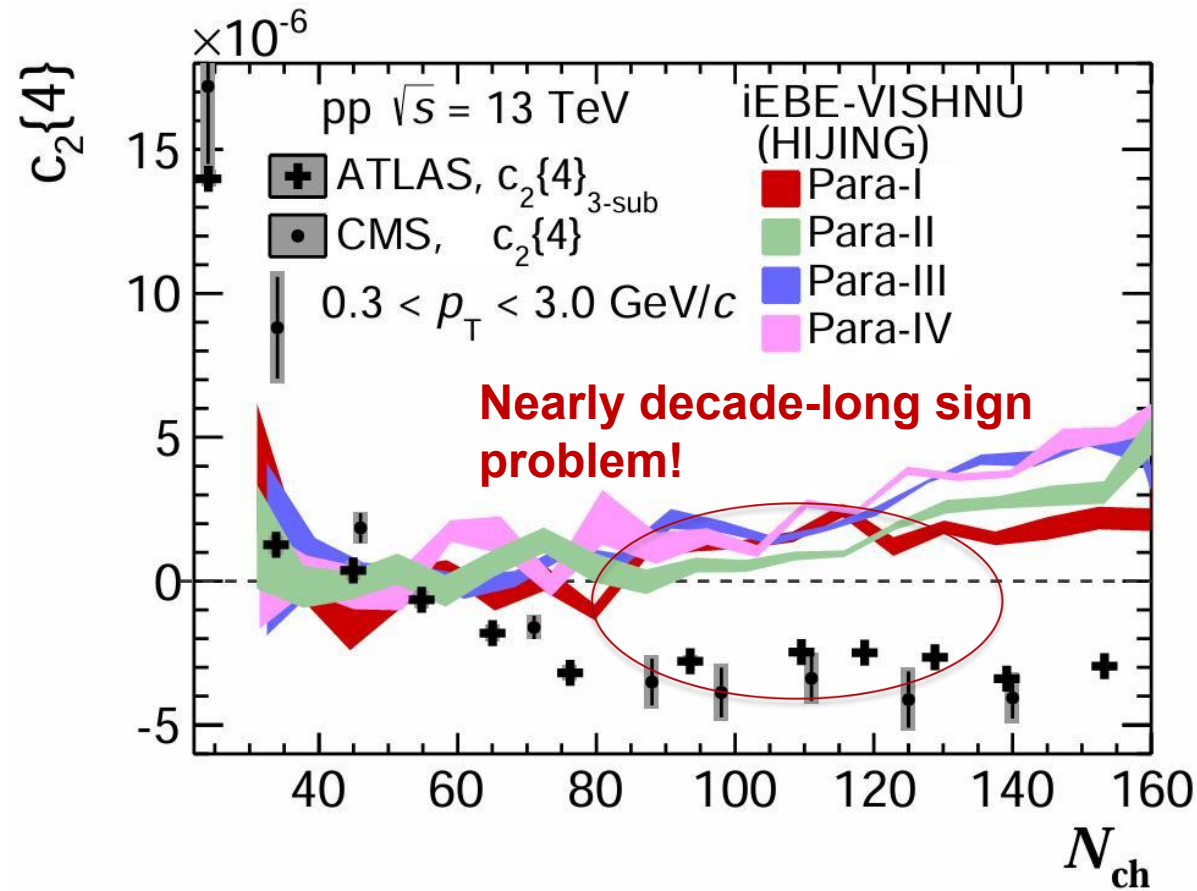
V. Ambrus, S. Schlichting and C. Werthmann,
Phys. Rev. D 111, 054024 (2025)

Hydro Simulations in Small Systems

Imperfect description of 2-particle cumulants ($v_n\{2\}$) and even wrong sign of 4-particle cumulant $c_2\{4\}$:



W.Zhao, C.Shen and B.Shenke,
Phys.Rev.Lett. 129 (2022) 252302



W.Zhao et al., Phys.Lett.B 780 (2018) 495-500

Can we improve viscous hydrodynamics?

Viscous Anisotropic Hydrodynamics (VAH)

- Difference between VAH and traditional hydro:

Traditional hydro:

$$T^{\mu\nu} = \boxed{\mathcal{E} u^\mu u^\nu - (\mathcal{P}_{eq} + \Pi) \Delta^{\mu\nu}} + \boxed{\pi^{\mu\nu}}.$$

dominant terms

small corrections

$$f = f_{eq} + \delta f$$

$$\boxed{f_{eq} = \left[\exp \left(\sqrt{\mathbf{p}^2 + m^2/T} \right) + \Theta \right]^{-1}}$$

$$\boxed{\delta f = f_{eq}(1 - \Theta f_{eq}) \left[\frac{p^i p^j \pi^{ij}}{2E_{\mathbf{p}} \beta_\pi} - \frac{\Pi}{E_{\mathbf{p}} T \beta_\Pi} (m^2 - (1 - 3c_s^2) E_{\mathbf{p}}^2) \right]}$$

VAH:

$$T^{\mu\nu} = \boxed{\mathcal{E} u^\mu u^\nu + \mathcal{P}_L z^\mu z^\nu - \mathcal{P}_\perp \Xi^{\mu\nu}} + \boxed{2 W_{\perp z}^{(\mu} z^{\nu)} + \pi_\perp^{\mu\nu}}$$

dominant terms

small corrections

$$f(x, p) = f_a(x, p) + \delta \tilde{f}(x, p)$$

$$\boxed{f_a = \left[\exp \left(\sqrt{\frac{\mathbf{p}_L^2}{\alpha_L^2} + \frac{\mathbf{p}_\perp^2}{\alpha_\perp^2} + m^2/\Lambda} \right) + \Theta \right]^{-1}}$$

$$\boxed{\delta \tilde{f} = f_a(1 - \Theta f_a) \left(\frac{p_z \mathbf{p}_\perp \cdot \mathbf{W}_{\perp z}}{\mathcal{J}_{4210}} + \frac{p_\perp^i p_\perp^j \pi_\perp^{ij}}{2 \mathcal{J}_{4020}} \right)}$$

- Relation between different decomposition:

$$\pi^{\mu\nu} = \pi_\perp^{\mu\nu} + 2 W_{\perp z}^{(\mu} z^{\nu)} + \frac{1}{3} (\mathcal{P}_L - \mathcal{P}_\perp) (2 z^\mu z^\nu - \Xi^{\mu\nu}), \quad \Pi = \frac{2 \mathcal{P}_\perp + \mathcal{P}_L}{3} - \mathcal{P}_{eq}.$$

- The evolution more reasonable with large $\mathcal{P}_{L,\perp} - \mathcal{P}_{eq}$.

- Return to traditional hydro with small $\mathcal{P}_{L,\perp} - \mathcal{P}_{eq}$.

M. McNelis, D. Bazow, and U. Heinz,
Com. Phys. Comm 267, 108077 (2021)

Viscous Anisotropic Hydrodynamics (VAH)

Relaxation equations of VAH:

$$\begin{aligned}\dot{\mathcal{P}}_L = & -\frac{\bar{\mathcal{P}} - \mathcal{P}_{\text{eq}}}{\tau_{\Pi}} - \frac{\mathcal{P}_L - \mathcal{P}_{\perp}}{3\tau_{\pi}/2} + \bar{\zeta}_z^L z_{\mu} D_z u^{\mu} + \bar{\zeta}_{\perp}^L \theta_{\perp} - 2W_{\perp z}^{\mu} \dot{z}_{\mu} + \bar{\lambda}_{Wu}^L W_{\perp z}^{\mu} D_z u_{\mu} \\ & + \bar{\lambda}_{W\perp} W_{\perp z}^{\mu} z_{\nu} \nabla_{\perp\mu} u^{\nu} - \bar{\lambda}_{\pi}^L \pi_{\perp}^{\mu\nu} \sigma_{\perp,\mu\nu},\end{aligned}$$

Longitudinal Pressure

$$\begin{aligned}\dot{\mathcal{P}}_{\perp} = & -\frac{\bar{\mathcal{P}} - \mathcal{P}_{\text{eq}}}{\tau_{\Pi}} + \frac{\mathcal{P}_L - \mathcal{P}_{\perp}}{3\tau_{\pi}} + \bar{\zeta}_z^{\perp} z_{\mu} D_z u^{\mu} + \bar{\zeta}_{\perp}^{\perp} \theta_{\perp} + W_{\perp z}^{\mu} \dot{z}_{\mu} + \bar{\lambda}_{Wu}^{\perp} W_{\perp z}^{\mu} D_z u_{\mu} \\ & - \bar{\lambda}_{W\perp}^{\perp} W_{\perp z}^{\mu} z_{\nu} \nabla_{\perp\mu} u^{\nu} + \bar{\lambda}_{\pi}^{\perp} \pi_{\perp}^{\mu\nu} \sigma_{\perp,\mu\nu}.\end{aligned}$$

Transverse Pressure

$$\begin{aligned}\dot{W}_{\perp z}^{\{\mu\}} = & -\frac{W_{\perp z}^{\mu}}{\tau_{\pi}} + 2\bar{\eta}_u^W \Xi^{\mu\nu} D_z u_{\nu} - 2\bar{\eta}_{\perp}^W z_{\nu} \nabla_{\perp}^{\mu} u^{\nu} - (\bar{\tau}_z^W \Xi^{\mu\nu} + \pi_{\perp}^{\mu\nu}) \dot{z}_{\nu} + \bar{\delta}_W^W W_{\perp z}^{\mu} \theta_{\perp} \\ & - \bar{\lambda}_{Wu}^W W_{\perp z}^{\mu} z_{\nu} D_z u^{\nu} + \bar{\lambda}_{W\perp}^W \sigma_{\perp}^{\mu\nu} W_{\perp z,\nu} + \omega_{\perp}^{\mu\nu} W_{\perp z,\nu} + \bar{\lambda}_{\pi u}^W \pi_{\perp}^{\mu\nu} D_z u_{\nu} - \bar{\lambda}_{\pi\perp}^W \pi_{\perp}^{\mu\nu} z_{\alpha} \nabla_{\perp\nu} u^{\alpha},\end{aligned}$$

Longitudinal momentum diffusion

$$\begin{aligned}\dot{\pi}_{\perp}^{\{\mu\nu\}} = & -\frac{\pi_{\perp}^{\mu\nu}}{\tau_{\pi}} + 2\bar{\eta}_{\perp} \sigma_{\perp}^{\mu\nu} - 2W_{\perp z}^{\{\mu} \dot{z}^{\nu\}} - \bar{\delta}_{\pi}^{\pi} \pi_{\perp}^{\mu\nu} \theta_{\perp} - \bar{\tau}_{\pi}^{\pi} \pi_{\perp}^{\alpha\{\mu} \sigma_{\perp,\alpha}^{\nu\}} + 2\pi_{\perp}^{\alpha\{\mu} \omega_{\perp,\alpha}^{\nu\}} + \bar{\lambda}_{\pi}^{\pi} \pi_{\perp}^{\mu\nu} z_{\alpha} D_z u^{\alpha} \\ & - \bar{\lambda}_{Wu}^{\pi} W_{\perp z}^{\{\mu} D_z u^{\nu\}} + \bar{\lambda}_{W\perp}^{\pi} W_{\perp z}^{\{\mu} z_{\alpha} \nabla_{\perp}^{\nu\}} u^{\alpha}.\end{aligned}$$

Transverse shear stress tensor

VAH, Hydro and Kinetic Theory Comparison

Assumptions:

- Finite transverse extent with longitudinal boost invariance ((2+1)-D simulation)
- Systems consist of massless particles (conformal symmetry).

$$\mathcal{P}_L + 2\mathcal{P}_\perp = 3\mathcal{P} = \mathcal{E}, \quad \Pi = 0$$

Boltzmann equation in the relaxation time approximation (RTA):

$$p \cdot \partial f(x, p) = -\frac{u \cdot p}{\tau_R(x)} [f(x, p) - f_{eq}(x, p)], \quad \tau_R = 5\eta/(sT) \longrightarrow \text{Benchmark}$$

Traditional viscous hydro:

$$\begin{aligned} \tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_\lambda^{\langle\mu} \omega^{\nu\rangle\lambda} - \delta_{\pi\pi} \pi^{\mu\nu} \theta \\ &\quad - \tau_{\pi\pi} \pi^{\lambda\langle\mu} \sigma_{\lambda}^{\nu\rangle} + \phi_7 \pi_\alpha^{\langle\mu} \pi^{\nu\rangle\alpha}, \end{aligned}$$

Y. Peng, V. Ambrus, C. Werthmann,
S. Schlichting, U. Heinz and H. Song,
Phys.Rev.C 113 (2026) 2, 024918

VAH:

$$\begin{aligned} \dot{P}_L &= -\frac{P_L - P_\perp}{3\tau_\pi/2} + \bar{\zeta}_z^L \theta_L + \bar{\zeta}_\perp^L \theta_\perp - \bar{\lambda}_\pi^L \pi_\perp^{\mu\nu} \sigma_{\perp,\mu\nu}, \\ \dot{\pi}_\perp^{\{\mu\nu\}} &= -\frac{\pi_\perp^{\mu\nu}}{\tau_\pi} + 2\bar{\eta}_\perp \sigma_\perp^{\mu\nu} - \bar{\lambda}_\pi^\pi \pi_\perp^{\mu\nu} \theta_L - \bar{\delta}_\pi^\pi \pi_\perp^{\mu\nu} \theta_\perp \\ &\quad - \bar{\tau}_\pi^\pi \pi_\perp^{\lambda\{\mu} \sigma_{\perp,\lambda}^{\nu\}} + 2\pi^{\lambda\{\mu} \omega_{\perp,\lambda}^{\nu\}}, \end{aligned}$$

Coefficients in consistency
with RTA (e.g., $\tau_\pi = \tau_R$)

VAH, Hydro and Kinetic Theory Comparison

Quantities of interest:

Transverse energy

$$\frac{dE_{\perp}}{d\eta} = \tau \int_{\mathbf{x}_{\perp}} (T^{xx} + T^{yy}) \quad \Rightarrow \quad \text{Cooling behavior}$$

Inversed Reynolds number

$$\langle \text{Re}^{-1} \rangle_{\varepsilon} = \left\langle \left(\frac{\pi^{\mu\nu} \pi_{\mu\nu}}{\varepsilon^2} \right)^{1/2} \right\rangle_{\varepsilon} \quad \Rightarrow \quad \text{Hydrodynamization}$$

Average transverse velocity

$$\langle u_{\perp} \rangle_{\varepsilon} = \left\langle (u_x^2 + u_y^2)^{1/2} \right\rangle_{\varepsilon}$$

Momentum anisotropy

$$\varepsilon_p = \frac{\int_{\mathbf{x}_{\perp}} (T^{xx} - T^{yy})}{\int_{\mathbf{x}_{\perp}} (T^{xx} + T^{yy})}$$

Radial expansion

Response to initial spatial anisotropy (resemble with v_2)

Transverse dynamics
(**comparison never made before!**)

Criterion of diluteness (Opacity):

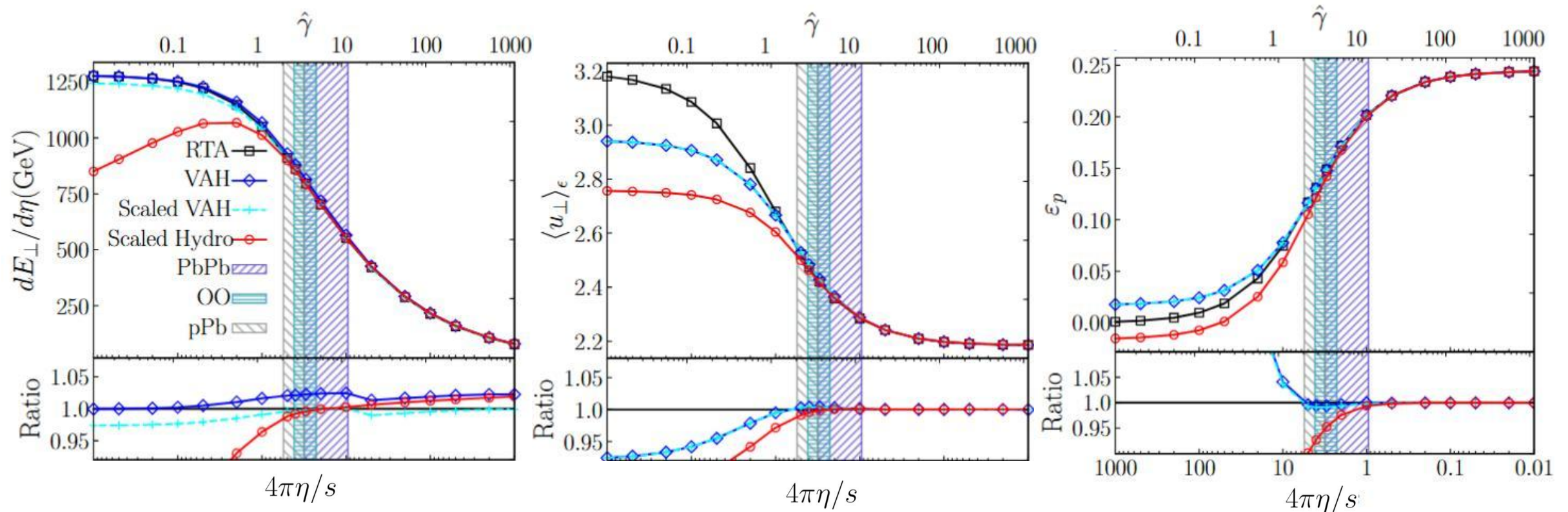
$$\hat{\gamma} = \frac{1}{5\eta/s} \left(\frac{R}{\pi a} \frac{dE_{\perp}^0}{d\eta} \right)^{1/4}$$

- The only parameter that determines the evolution.
- Encoding **system size**, **viscosity** and **energy dependence**.
- Transition from free-streaming (ultra-dilute) to ideal Hydro (ultra-dense) with opacity increasing from 0 to ∞ (shear viscosity from ∞ to 0).

Final-state Results

Final state results ($\tau = 3R$) as functions of opacity:

- Traditional hydro gradually deviates from RTA in O+O and p+Pb collisions, while VAH maintains good agreement.
- VAH fails to capture transverse dynamics at exceptionally large shear viscosity (small opacity).



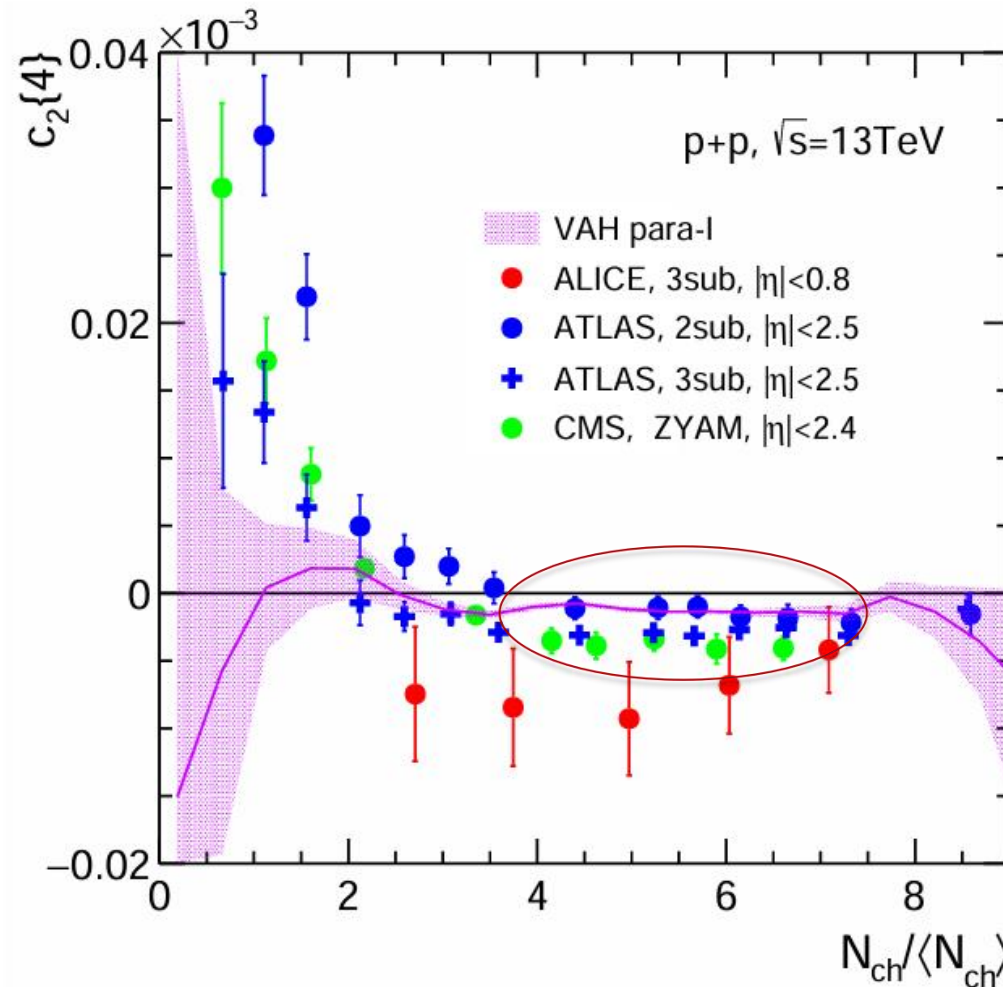
Y. Peng, V. Ambrus, C. Werthmann, S. Schlichting, U. Heinz and H. Song, Phys. Rev. C, accepted [DOI: 10.1103/gbpd-m1mh]

Apply VAH in p+p at 13 TeV

S. Zhao, Y. Peng, U. Heinz and H. Song,

(2+1)-D simulation in the TRENTo3D+VAH+SMASH framework: Phys.Rev.Lett. 137 (2026) 6, 062301

- Produce correct sign of $c_2\{4\}$ (Traditional hydrodynamics produces a wrong sign...)

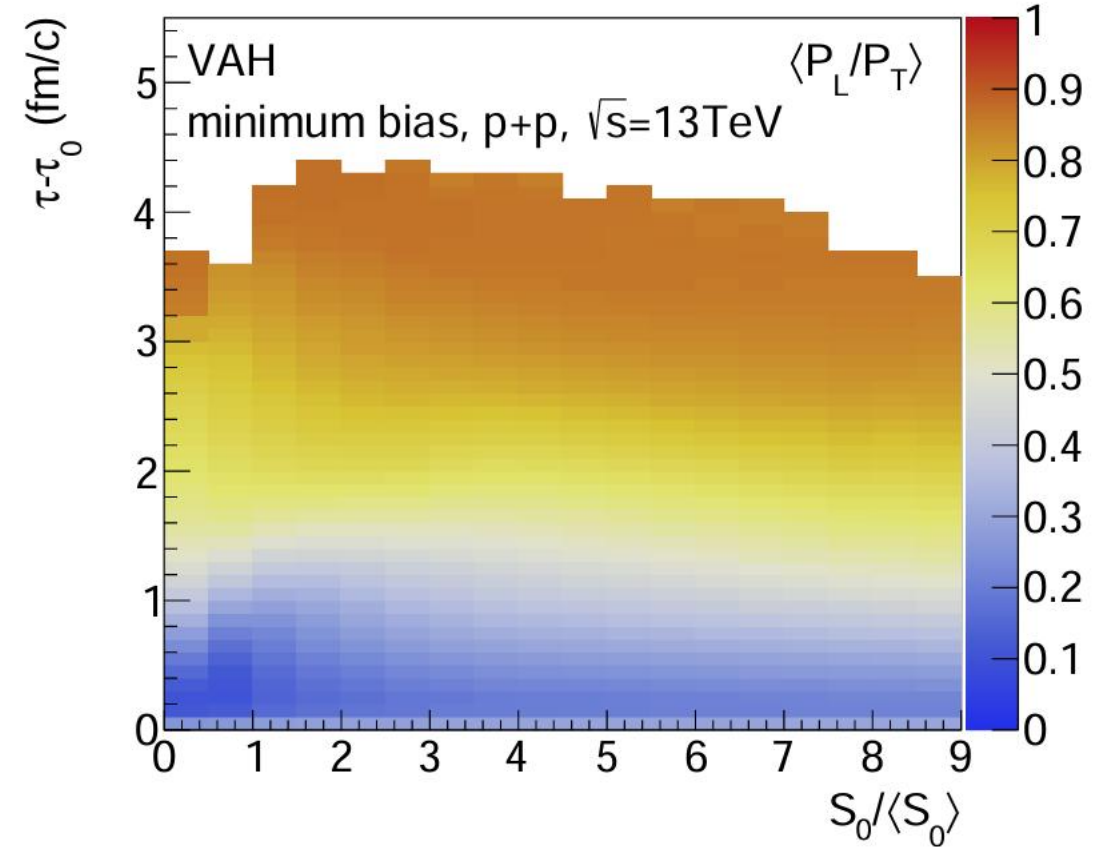
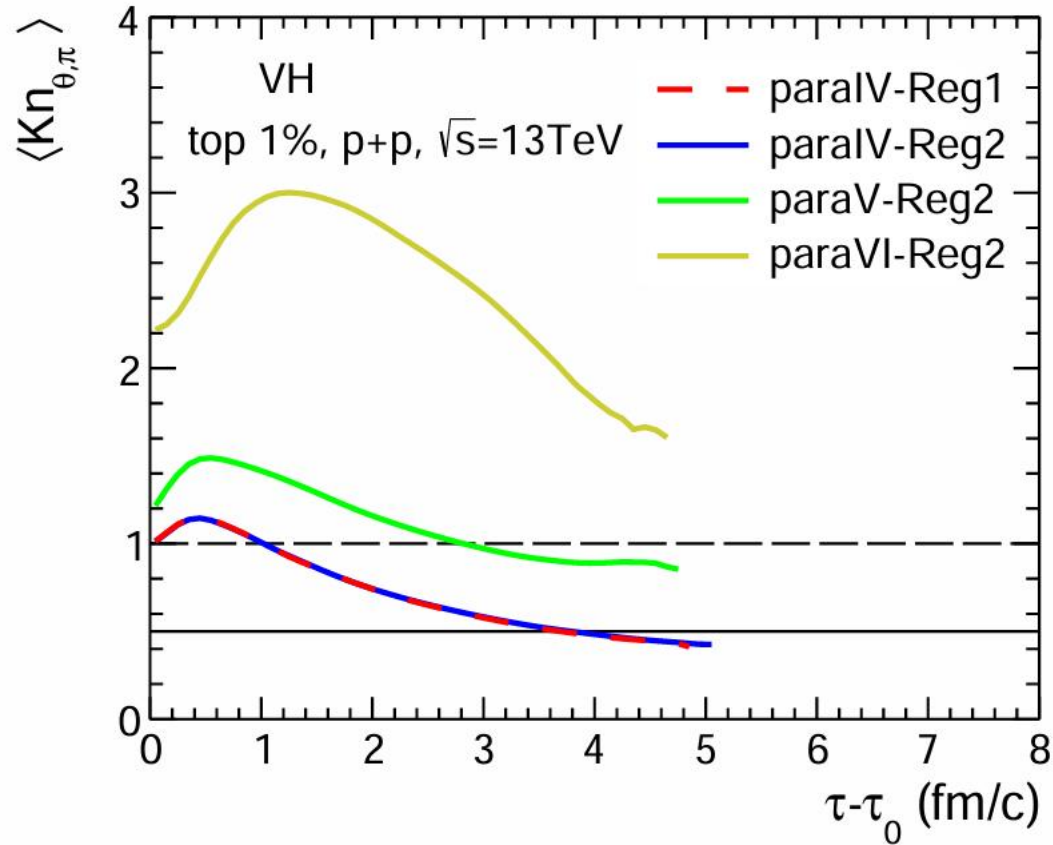


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- Extremely large Knudsen number in traditional hydrodynamics simulation;
- High anisotropy for most of its evolution according to VAH simulation.



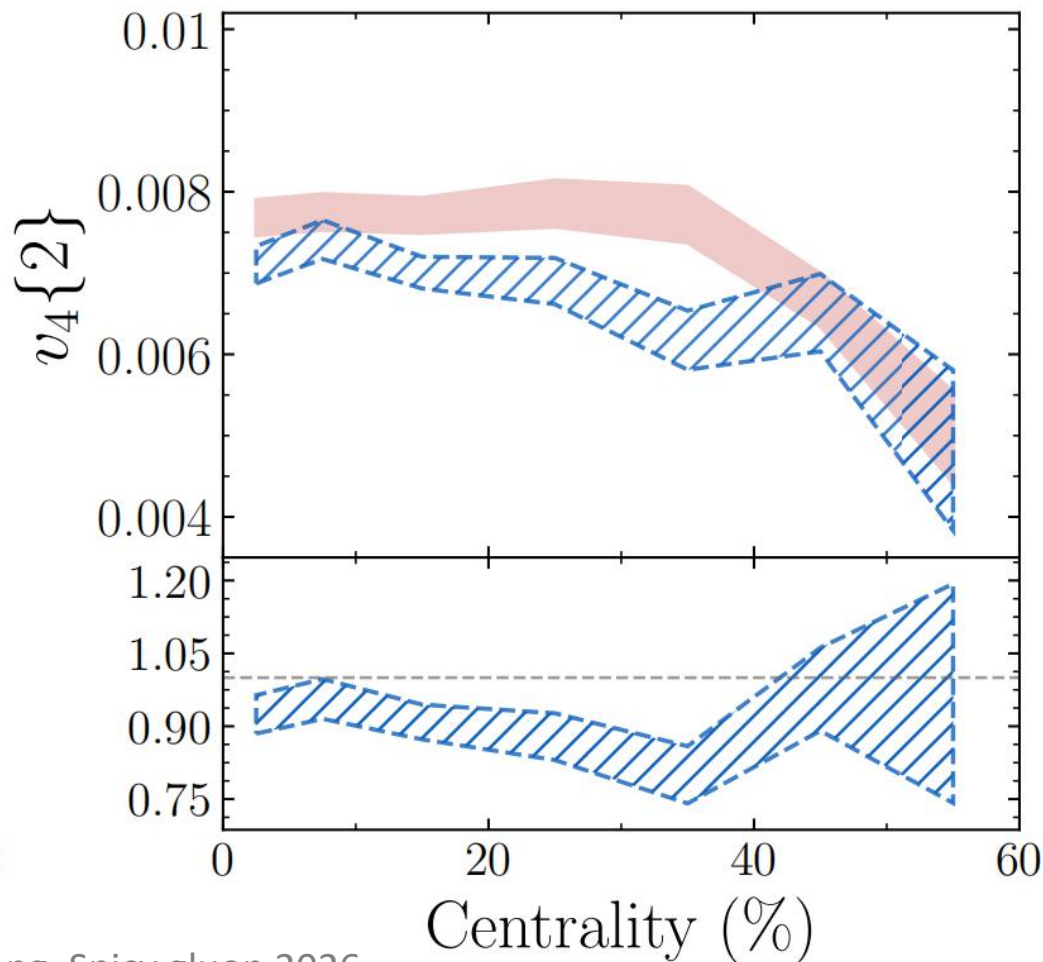
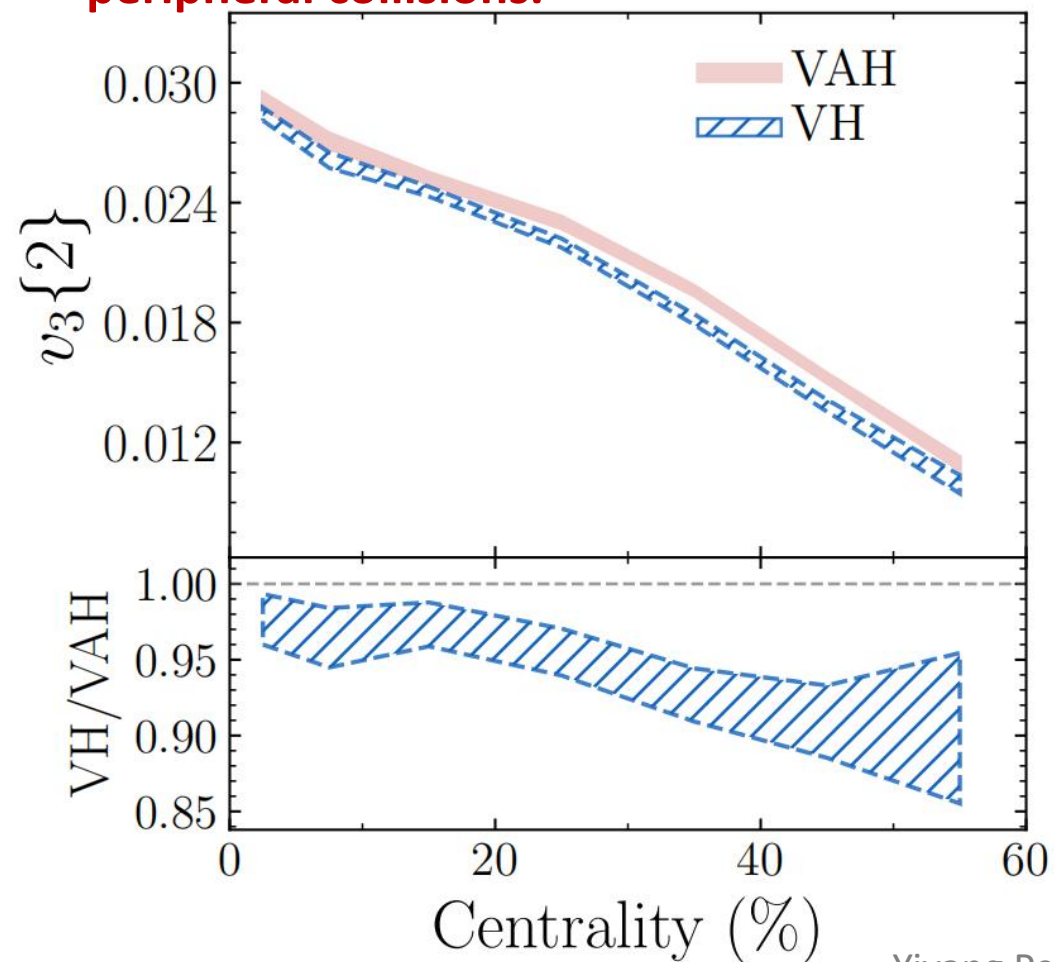
Hydro Applicability in O+O, Ne+Ne at 5.36 TeV

Comparison between VAH and viscous hydro(VH) in collective flow:

- Same pre-equilibrium stages for controlled comparison.
- Higher order flow harmonics sensitive to non-hydro behaviors in peripheral collisions.

Y. Peng, Y. Wang, S. Zhao and H. Song,
paper in preparation

Y. Wang, Y. Peng, W. Zhao and H. Song,
paper in preparation



See Yili Wang's
talk for details

Summary

- **Viscous anisotropic hydrodynamics (VAH) is an extended version of viscous hydrodynamics with self-consistent assumptions.**
- **Extending the applicability of hydrodynamics into dilute systems.**
- **Properly capture the sign of 4-particle cumulant in p+p collisions at 13 TeV.**
- **Collective flow in peripheral O+O/Ne+Ne collisions indicates sensitivity to non-hydro behaviors.**

References

S.Zhao, Y. Peng, H. Heinz and H. Song, Phys.Rev.Lett. 137 (2026) 6, 062301

Y. Peng, V. Ambrus, C. Werthmann, S. Schlichting, U. Heinz and H. Song, Phys.Rev.C 113 (2026) 2, 024918

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