

Comparative analysis of hybrid star stability and dynamics

Zhihan Li

Tsung-Dao Lee Institute
Shanghai Jiao Tong University

*Spicy Gluons 2026: Workshop for Young Scientists on the
Quark-Gluon Matter in Extreme Conditions*

In collaboration with

Zhenyu Zhu (RIT), Sophia Han (TDLI), and Tianqi Zhao (UC Berkeley)

October 17–20, 2026, Wuhan

Overview

- 1 Introduction
- 2 Motivation
- 3 Preliminary results
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Hydrostatic Equilibrium of NSs

- Tolman–Oppenheimer–Volkoff (TOV) equations:

$$\frac{dP}{dr} = -\frac{G(\varepsilon + P)(m + 4\pi r^3 P/c^2)}{c^2 r(r - 2Gm/c^2)}, \quad \frac{dm}{dr} = \frac{4\pi r^2 \varepsilon}{c^2} \quad \rightarrow \quad \text{mass-radius relation}$$

$$P = P(\varepsilon) \quad (\text{EOS}), \quad \varepsilon : \text{total energy density}$$

$$\begin{array}{ccc} r = 0 & \xrightarrow{\text{integrate TOV outward}} & r = R \\ m(0) = 0, \quad P(0) = P_c & & P(R) = 0, \quad M = m(R) \end{array}$$

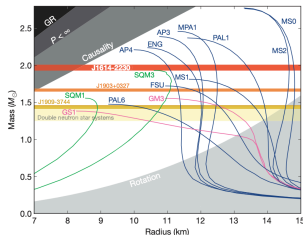


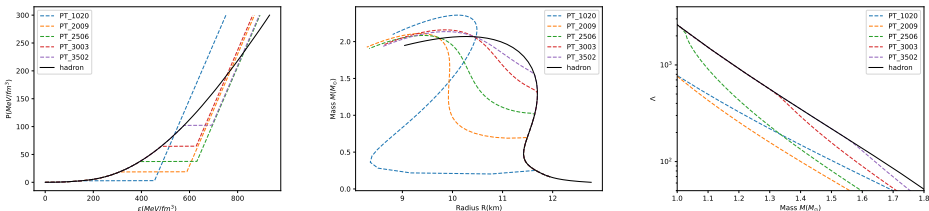
Figure 3: An example of an NS mass-radius diagram (P. B. Demorest et al., Nature, 2010)

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Hybrid EOSs and stellar structure

- Hadronic models describe the lower-density phase. Quark-matter models describe a possible high-density phase.
- A strong first-order PT leads to many significant changes in astronomical observables, such as mass-radius relation or tidal deformability.
- A sufficiently strong transition, introducing a density discontinuity, can produce a disconnected stable branch and twin-star configurations.



An example of hybrid-star EOSs, corresponding mass-radius relations, and tidal deformabilities

EOS using constant speed of sound parameterization

- Hadronic EOS with a constant-speed-of-sound quark phase:

$$P(\varepsilon) = \begin{cases} P_{\text{had}}(\varepsilon), & \varepsilon \leq \varepsilon_{\text{tr}}, \\ P_{\text{tr}}, & \varepsilon_{\text{tr}} < \varepsilon < \varepsilon_{\text{tr}} + \Delta\varepsilon, \\ P_{\text{tr}} + c_{s,Q}^2(\varepsilon - \varepsilon_{\text{tr}} - \Delta\varepsilon), & \varepsilon \geq \varepsilon_{\text{tr}} + \Delta\varepsilon. \end{cases}$$

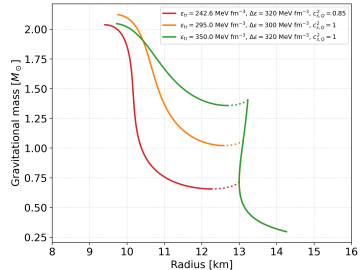
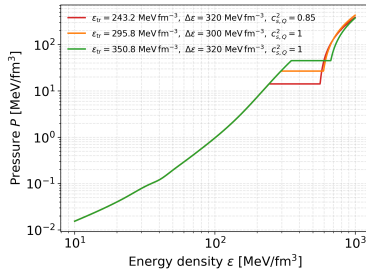


Figure 4: Cold, β -equilibrated hybrid EOSs (left) and the corresponding mass–radius relations (right). (*Unstable branch: $dM/dP_c < 0$, where equilibrium configurations are unstable against radial perturbations.*)

- **What are the final outcomes of hybrid stars initially on the unstable branch?**

We perform general-relativistic (GR) hydrodynamics simulations to investigate which stellar branches they ultimately migrate to and whether their initial quark cores survive or disappear.

- **How does the EOS treatment change this evolution?**

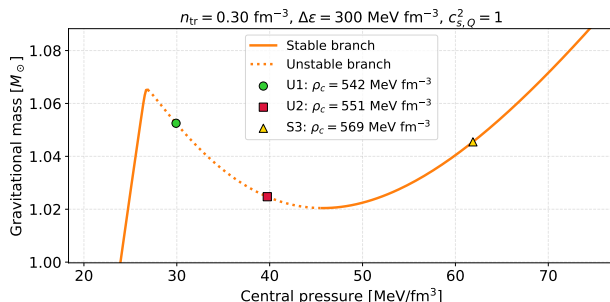
We impose different thermodynamic constraints on the same hybrid EOS to assess how *thermal and compositional degrees* of freedom affect the stellar evolution.

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Initial configurations

$n_{\text{tr}} [\text{fm}^{-3}]$	$\Delta\epsilon [\text{MeV fm}^{-3}]$	$c_{s,Q}^2$
0.25	320	0.85
0.30	300	1
0.35	320	1



U1, U2

unstable hybrid branch

S3

stable hybrid branch

3 EOSs $\times$ 3 configurations = 9 initial configurations

Initial settings from mass-central pressure relations

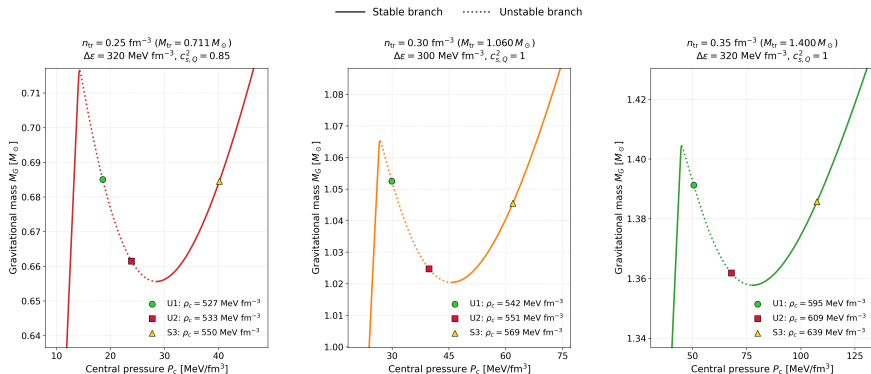


Figure 5: Mass–central pressure relations for the hybrid neutron star models based on previous EOSs. The colored points denote representative stellar configurations chosen for subsequent simulations.

Construction of finite-temperature EOSs

Four prescriptions, with two thermal-index choices for the cold EOS.

Treatment	Pressure response
Cold + $\Gamma_{\text{th}} = 1$	$P_{\text{cold}}(\rho)$, no gamma-law thermal pressure
Cold + $\Gamma_{\text{th}} = 5/3$	$P = P_{\text{cold}}(\rho) + (\Gamma_{\text{th}} - 1)\rho\epsilon_{\text{th}}$ (Gamma-law thermal prescription)
Full-3D	$P(\rho, T, Y_e)$, full temperature and Y_e dependence
Frozen- T	fixed $T = 0.01$ MeV
Frozen- β_{eq}	$Y_e = Y_e^\beta(\rho, T)$, imposed β equilibrium

These prescriptions impose different thermodynamic responses during stellar evolution.

Numerical method and branch selection

WhiskyTHC within the Einstein Toolkit

- GR hydrodynamics with CCZ4 spacetime evolution
- Octant reflection symmetry
- TOV initial data without an imposed perturbation

9 configurations $\times$ 5 settings = 45 simulations, each about 5 ms

Q Persistent quark core with a hadronic envelope

H Purely hadronic configuration without a quark core

Classification criteria rely on the maximum-density history with radial profiles.

Outcomes for the 45 simulations

Q: quark-core retention

H: hadronic outcome

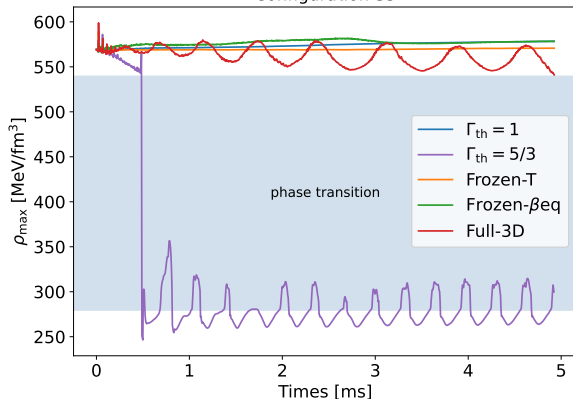
n_{tr}	Model	$\Gamma_{\text{th}} = 1$	$\Gamma_{\text{th}} = 5/3$	Frozen- T	Frozen- β_{eq}	Full-3D
0.25 fm^{-3}	U1	H	H	Q	Q	Q
	U2	H	H	Q	Q	Q
	S3	Q	H	Q	Q	Q
0.30 fm^{-3}	U1	H	H	Q	Q	H
	U2	H	H	Q	Q	Q
	S3	Q	H	Q	Q	Q
0.35 fm^{-3}	U1	H	H	Q	Q	H
	U2	Q	H	Q	Q	Q
	S3	Q	H	Q	Q	Q

29 Q and 16 H

Cold EOSs fail to capture the full thermal and compositional effects. Evolution outcomes depend on both the thermodynamic treatment and the initial configuration.

Significance of thermal pressure contribution

$n_{\text{tr}} = 0.30 \text{ fm}^{-3}$, $\Delta\epsilon = 300 \text{ MeV fm}^{-3}$, $c_{s,Q}^2 = 1$
Configuration S3



$$P_{\text{th}} = (\Gamma_{\text{th}} - 1)\rho\epsilon_{\text{th}}$$

$\Gamma_{\text{th}} = 1$: **Q**

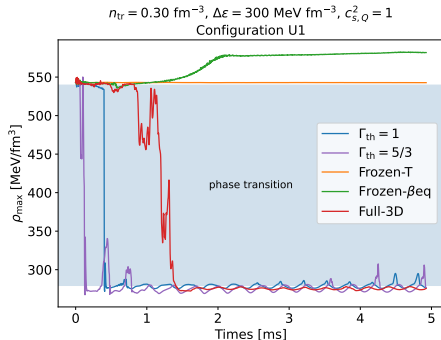
$\Gamma_{\text{th}} = 5/3$: **H**

The same S3 model responds differently to thermal pressure.

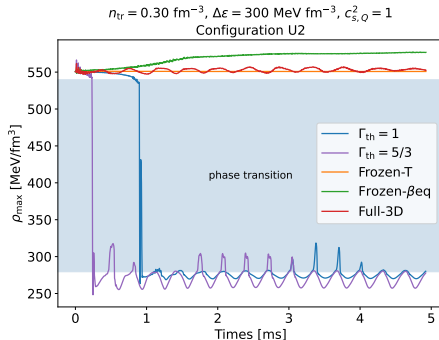
Increasing Γ_{th} from 1 (cold EOS) to 5/3 unexpectedly pushes Q back to H.

Full-3D results depend on the initial configuration

Same EOS: $n_{\text{tr}} = 0.30 \text{ fm}^{-3}$, $\Delta\varepsilon = 300 \text{ MeV fm}^{-3}$, $c_{s,Q}^2 = 1$



U1: Full-3D gives rise to **H**



U2: Full-3D gives rise to **Q**

Different points on the same unstable branch can have different fates.

Current challenges and future directions

Conclusions and limitations

- No simple, universal trend in quark-core survival across the configurations and EOS treatments studied.
- Phase transition may introduce additional heating limited by complicated neutrino scheme.

Next steps

- Test more publicly available 3D EOS tables with strong first-order phase transitions and unstable equilibrium branches.
- Add finite-rate weak interactions and neutrino transport for consistent thermal and compositional evolution.

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Summary

Thermodynamic treatment changes quark-core survival.

- 45 evolutions lead to 29 Q and 16 H outcomes over about 5 ms.
- Frozen- T and Frozen- β_{eq} retain cores in all sampled models.

Thermal-index changes affect the evolution.

- Raising Γ_{th} from 1 to 5/3 changes four Q classifications to H.

Both cold+ Γ_{th} and full-3D evolution depend on the EOS and initial model.

- A common initial branch label does not imply the same fate.

Thank you for your attention!

- ① **FLT EOS**: Zhenyu Zhu, Shuai Zha, and Sophia Han.
Three-dimensional equation of state extension of quark matter in the Fermi-liquid theory.
Phys. Rev. D **112**, 103003 (2025).
- ② **CSS parameterization**: Mark G. Alford, Sophia Han, and Madappa Prakash.
Generic conditions for stable hybrid stars.
Phys. Rev. D **88**, 083013 (2013).
- ③ **WhiskyTHC**: David Radice, Luciano Rezzolla, and Filippo Galeazzi.
Beyond second-order convergence in simulations of binary neutron stars in full general relativity.
Mon. Not. R. Astron. Soc. **437**, L46–L50 (2013).
- ④ **Gamma-law thermal approximation**: H.-Th. Janka, Th. Zwerger, and R. Moenchmeyer.
Does artificial viscosity destroy prompt type-II supernova explosions?
Astron. Astrophys. **268**, 360–368 (1993).

Backup Slides

Backup: thermal-index scan

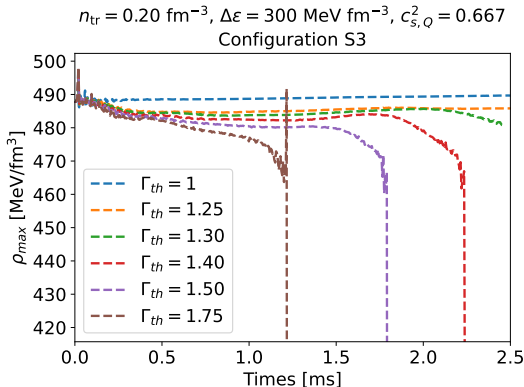


Figure 6: Thermal-index scan for a configuration initially on the stable hybrid branch with the supplementary EOS: $n_{\text{tr}} = 0.20 \text{ fm}^{-3}$, $\Delta\varepsilon = 300 \text{ MeV fm}^{-3}$, and $c_{s,Q}^2 = 2/3$. Larger Γ_{th} gives an earlier departure from the initial high-density state. Greater thermal pressure drives stellar expansion and causes migration to the purely hadronic branch.

Backup: complete density histories

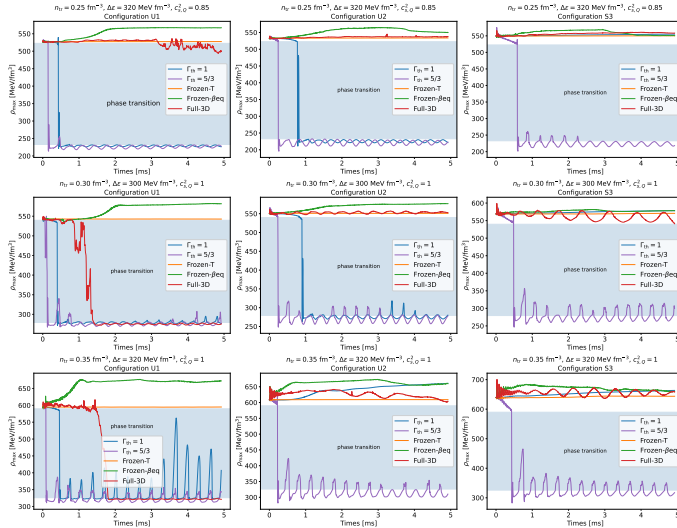


Figure 7: Maximum-density evolution for nine initial configurations under five EOS treatments. Rows correspond to the three EOSs; columns show U1, U2, and S3.

Backup: radial profiles, $n_{\text{tr}} = 0.25 \text{ fm}^{-3}$

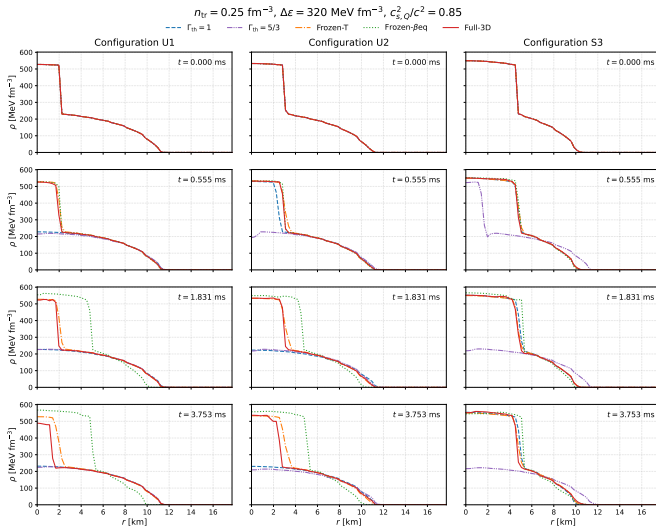


Figure 8: Radial rest-mass density profiles at selected times for $n_{\text{tr}} = 0.25 \text{ fm}^{-3}$. Columns show U1, U2, and S3; rows show different times. Colors distinguish the five EOS treatments.

Backup: radial profiles, $n_{\text{tr}} = 0.30 \text{ fm}^{-3}$

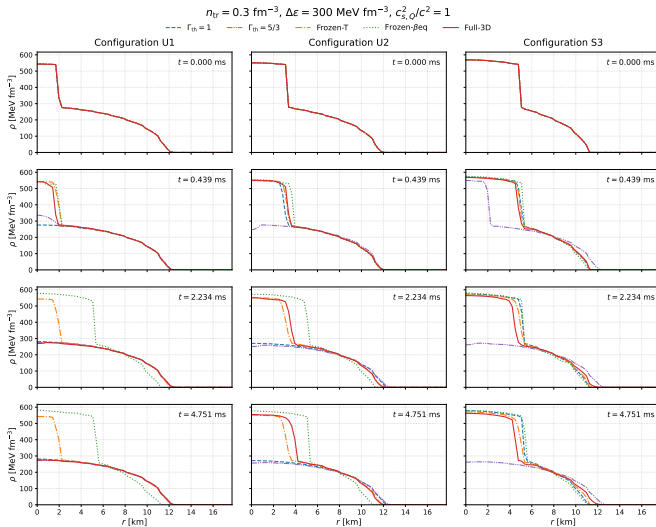


Figure 9: Radial rest-mass density profiles at selected times for $n_{\text{tr}} = 0.30 \text{ fm}^{-3}$. Columns show U1, U2, and S3; rows show different times. Colors distinguish the five EOS treatments.

Backup: radial profiles, $n_{\text{tr}} = 0.35 \text{ fm}^{-3}$

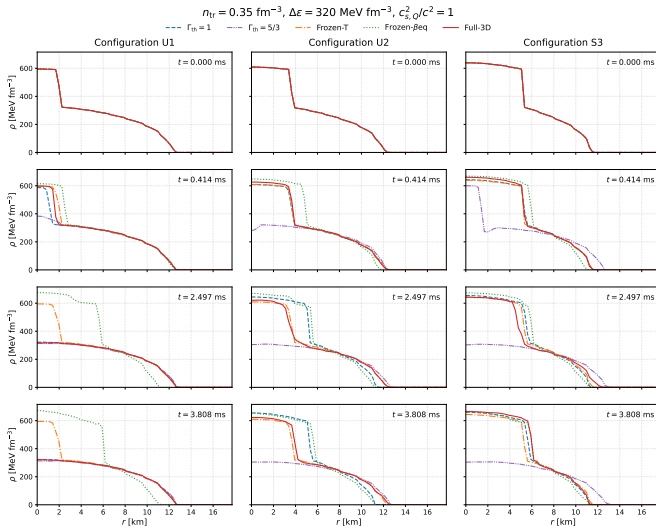


Figure 10: Radial rest-mass density profiles at selected times for $n_{\text{tr}} = 0.35 \text{ fm}^{-3}$. Columns show U1, U2, and S3; rows show different times. Colors distinguish the five EOS treatments.

Backup: cold mass ordering and branch selection

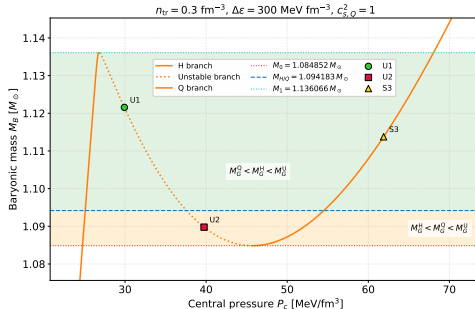


Figure 11: Baryonic mass versus central pressure along the cold equilibrium sequence. Markers identify U1, U2, and S3; horizontal lines correspond to fixed baryonic mass.

Baryon conservation

Compare states on the same horizontal line.

Energy release

A stellar final state must have lower gravitational mass.

Possible endpoints

Both H and Q are energetically allowed here.

Mass ordering alone does not determine the final branch.

Backup: EOS dependence on electron fraction

Full 3D EOS: $P(\rho)$ at fixed T , varying Y_e

— $Y_e = 0.01$ — $Y_e = 0.02$ — $Y_e = 0.03$ — $Y_e = 0.06$ — $Y_e = 0.10$ — $Y_e = 0.20$

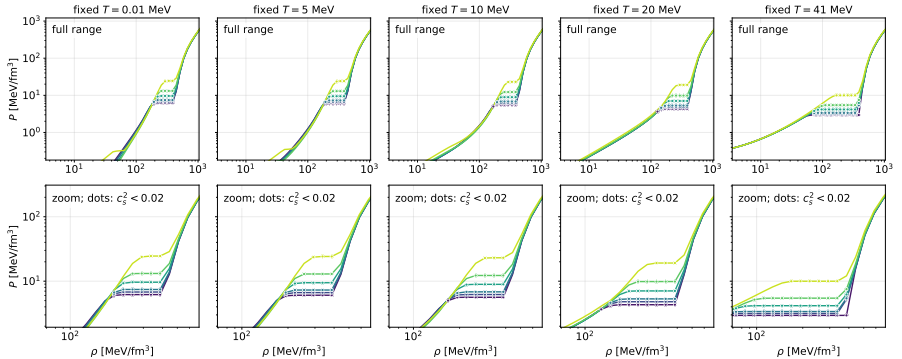


Figure 12: Pressure–density relations for different electron fractions at fixed temperatures. Top panels show the full density range; bottom panels show enlarged views of the transition region. Dots mark points with $c_s^2 < 0.02$ in units of c^2 .

Backup: EOS dependence on temperature

Full 3D EOS: $P(\rho)$ at fixed Y_e , varying temperature

— $T = 0.01$ MeV — $T = 5$ MeV — $T = 10$ MeV — $T = 20$ MeV — $T = 41$ MeV

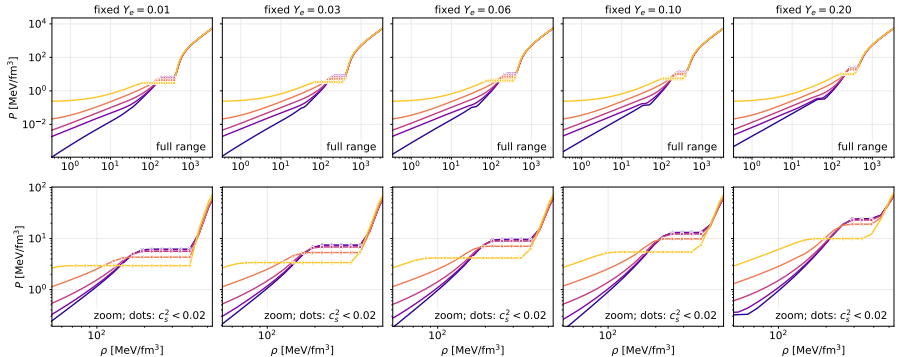


Figure 13: Pressure–density relations for different temperatures at fixed electron fractions. Top panels show the full density range; bottom panels show enlarged views of the transition region. Dots mark points with $c_s^2 < 0.02$ in units of c^2 .

Backup: resolution check for unstable hybrid branch

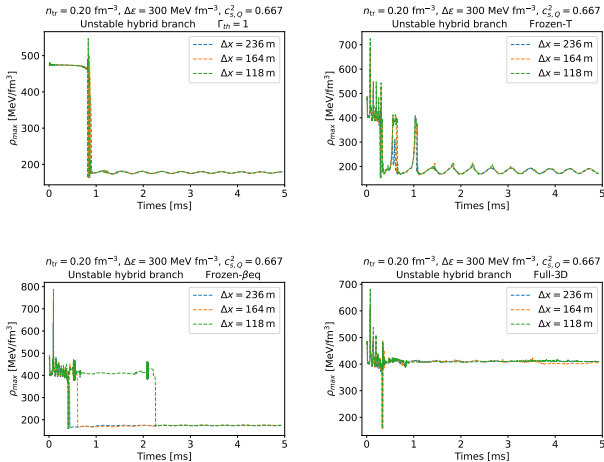


Figure 14: Maximum-density evolution for an initially unstable hybrid star at grid spacings of 236, 164, and 118 m. Some migration times vary, while the late-time outcomes agree within each EOS treatment.

Backup: resolution check for stable hybrid branch

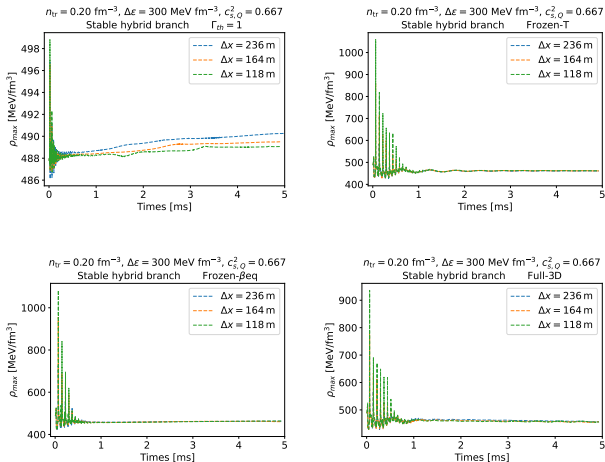


Figure 15: Maximum-density evolution for an initially stable hybrid star at grid spacings of 236, 164, and 118 m. All four EOS treatments retain quark cores over the simulated time.