



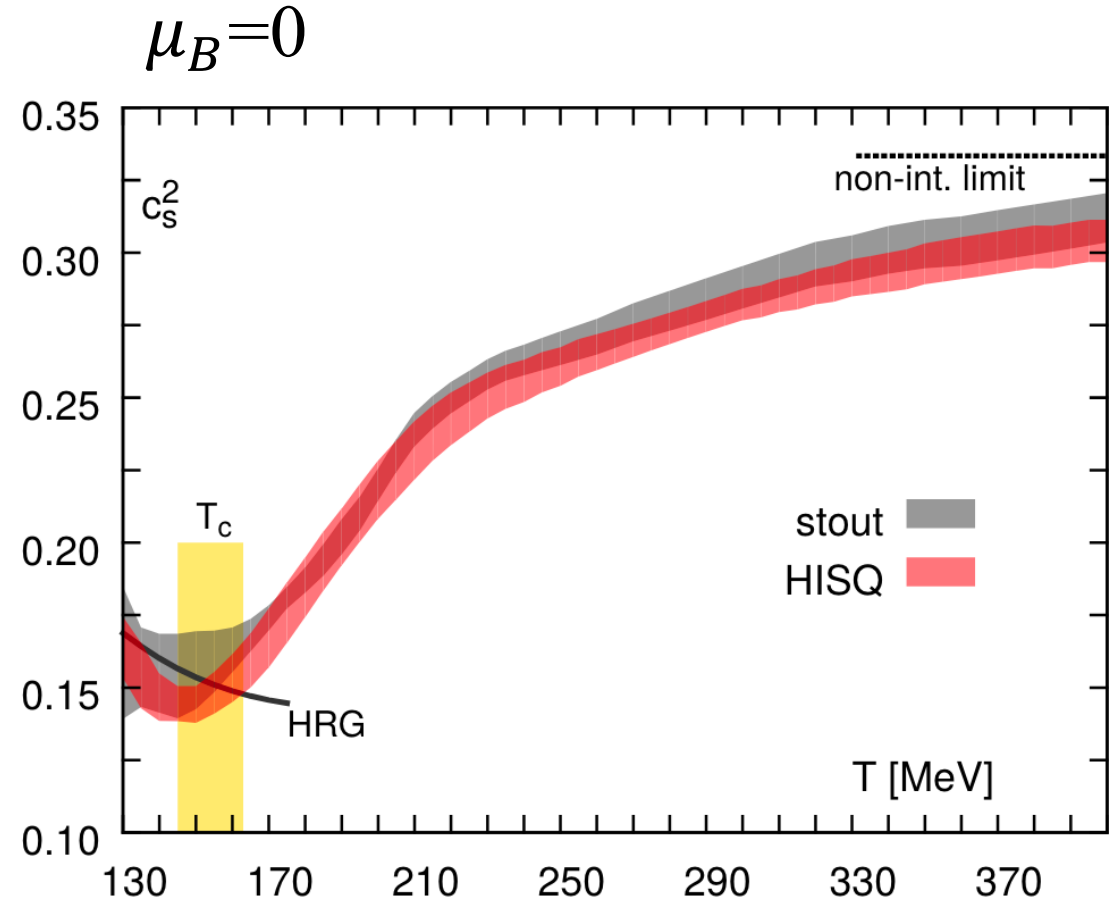
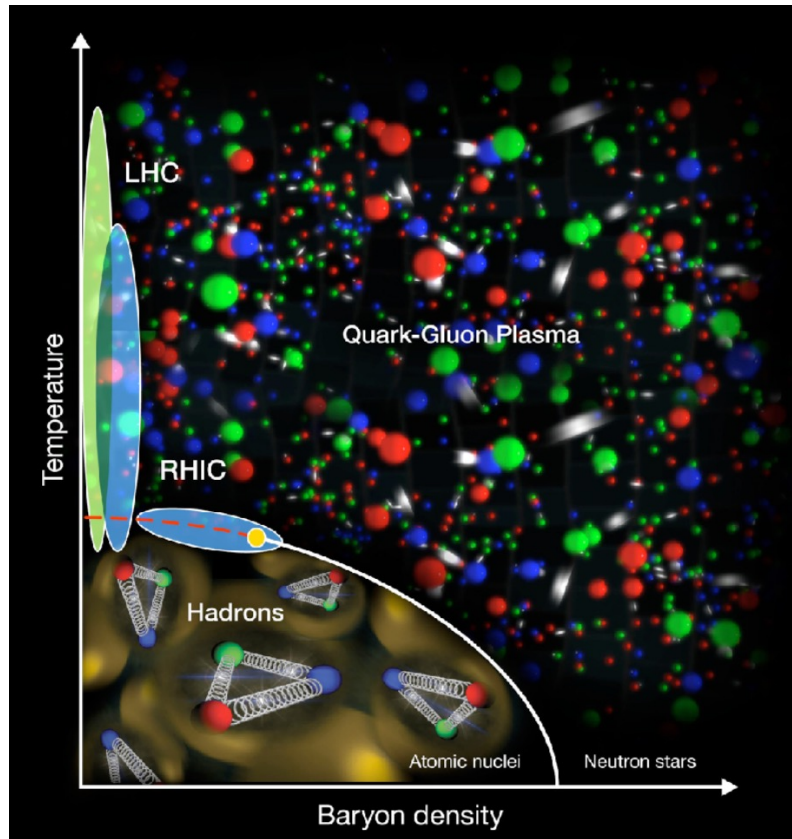
Extracting the QCD speed of sound in relativistic heavy-ion collisions

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Spicy Gluon 2026

Relativistic heavy-ion collisions



A direct measurement of QCD equation of state is challenging in heavy-ion study.

The QCD speed of sound in heavy-ion collisions

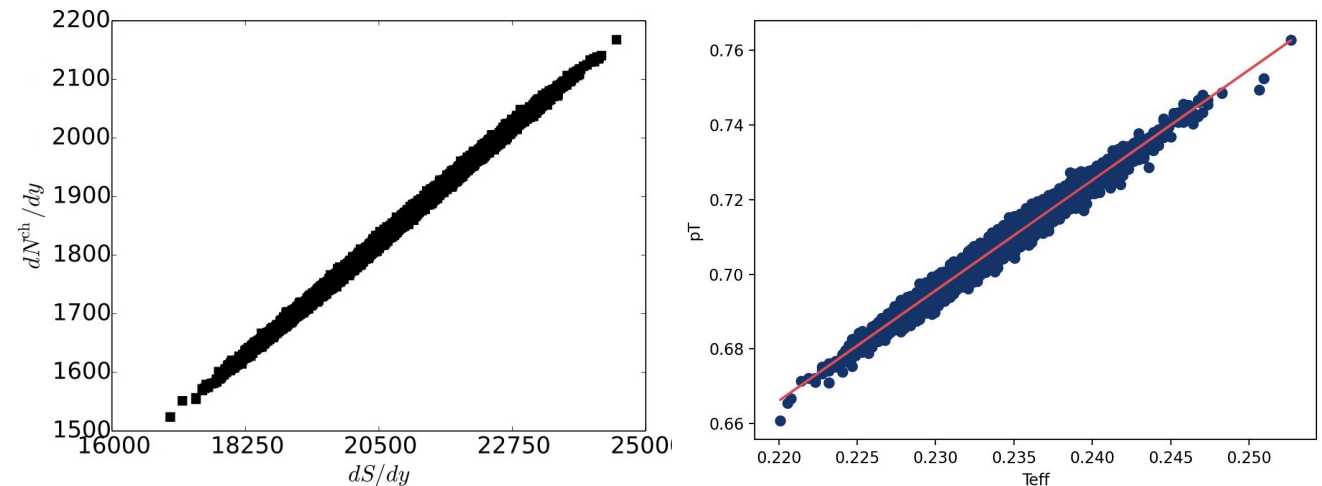
$$\begin{aligned}
 c_s^2 &\equiv \frac{dP}{de} \xrightarrow{\substack{dP = sdT, \\ de = Tds}} = \frac{d \ln T}{d \ln s} \xrightarrow[\substack{\text{constant} \\ \text{volume}}]{s \propto S} = \frac{d \ln T}{d \ln S} \xrightarrow[\substack{S \propto N_{ch} \\ T \propto \langle p_T \rangle}]{\substack{d \ln S = d \ln N_{ch} \\ d \ln T = d \ln \langle p_T \rangle}} = \frac{d \ln \langle p_T \rangle}{d \ln N_{ch}}
 \end{aligned}$$



Thermodynamics of hot strong-interaction matter from ultrarelativistic nuclear collisions

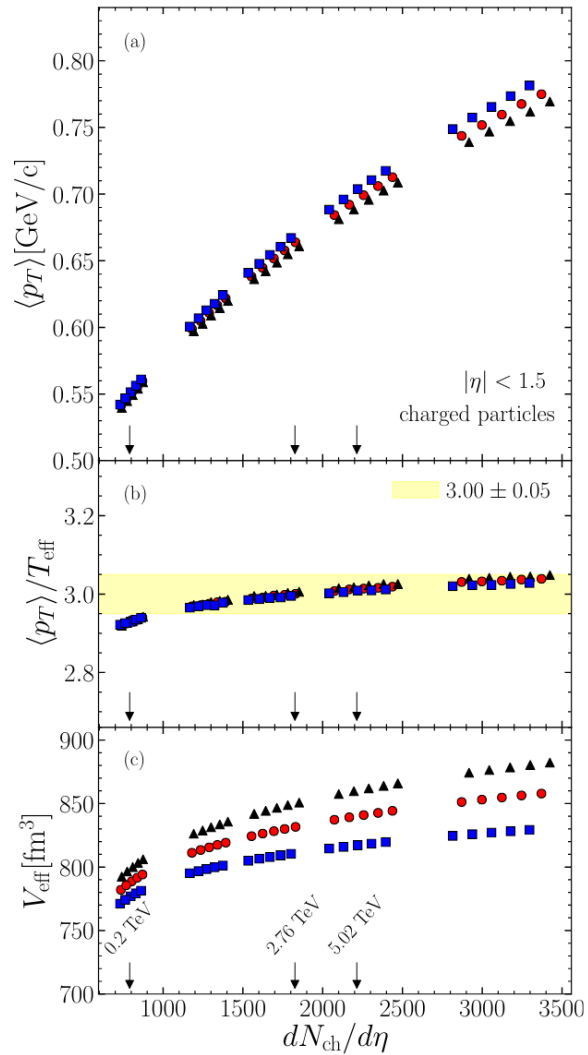
Fernando G. Gardim^{1,2}, Giuliano Giacalone², Matthew Luzum³ and Jean-Yves Ollitrault²✉

[F. Gardim et al, 1908.09728, Nature Physics]



[C. Shen, Comput.Phys.Commun. 199 (2016) 61-85]

Effective temperature and effective volume



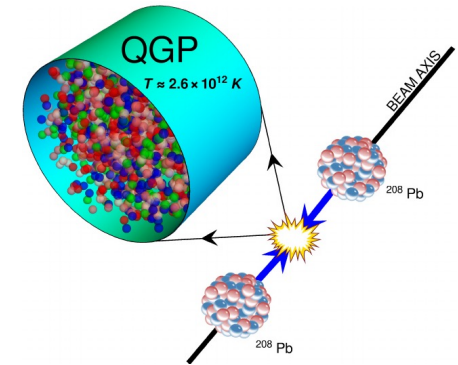
[F. Gardim et al , 2403.06052]

- A uniform fluid at rest with T_{eff} and V_{eff} :

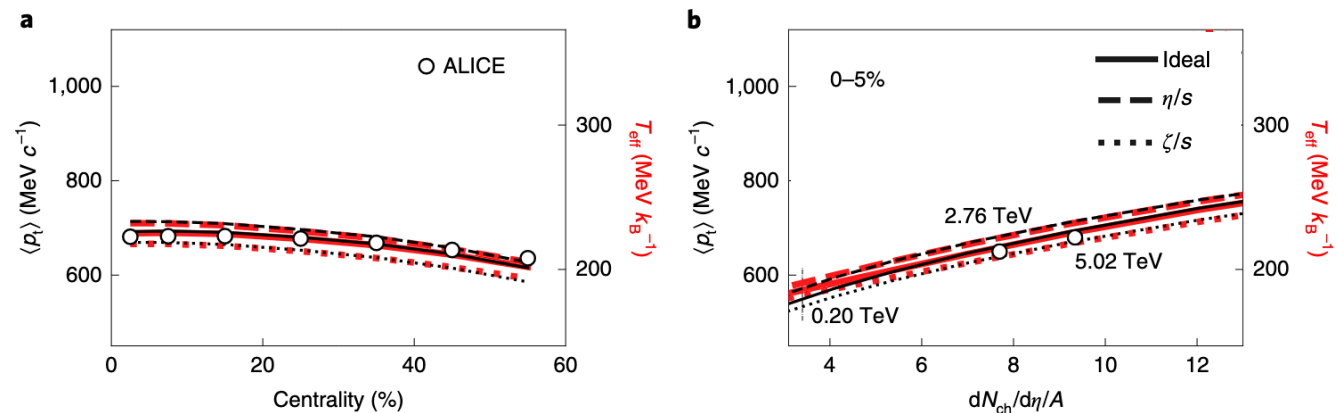
$$E = \int_{f.o.} T^{0\mu} d\sigma_\mu = \epsilon(T_{eff}) V_{eff}$$

$$S = \int_{f.o.} su^\mu d\sigma_\mu = s(T_{eff}) V_{eff}$$

QGP uniform fluid

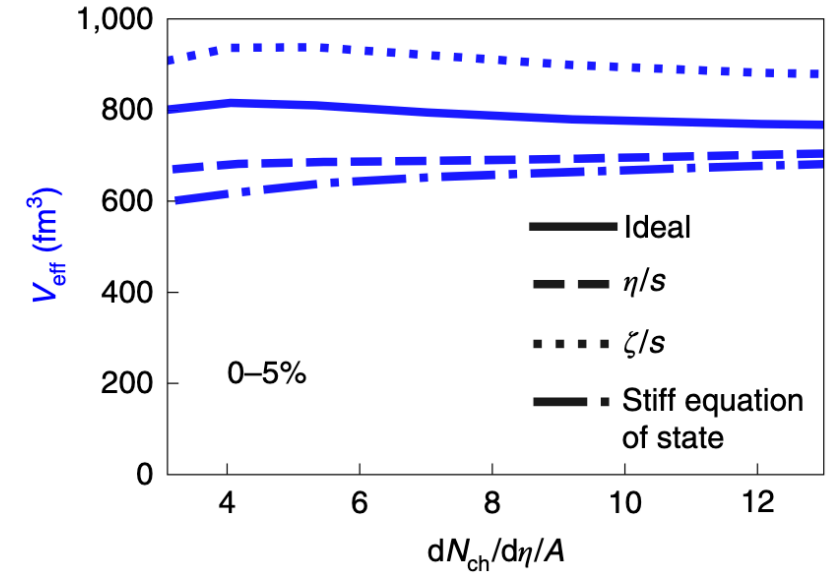
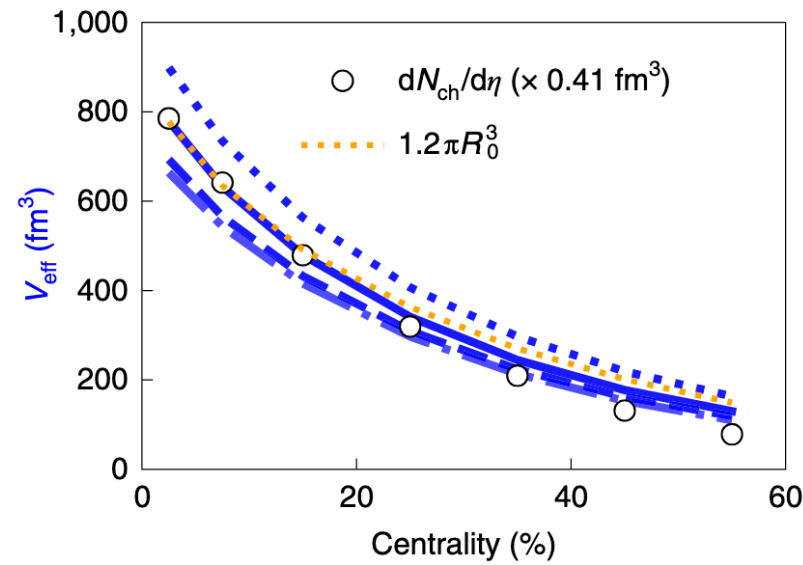
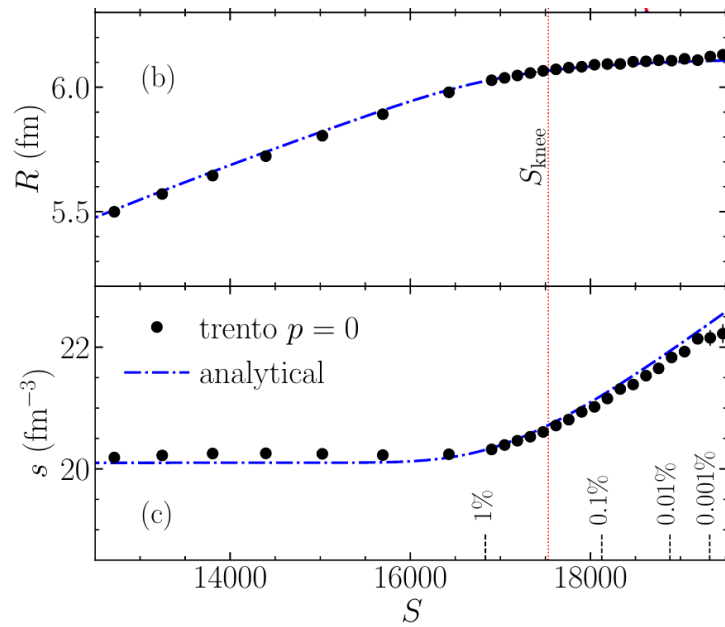


- In boost-invariant fluid, $\langle p_T \rangle \approx 3 T_{eff}$



[F. Gardim et al , 1908.09728, Nature Physics]

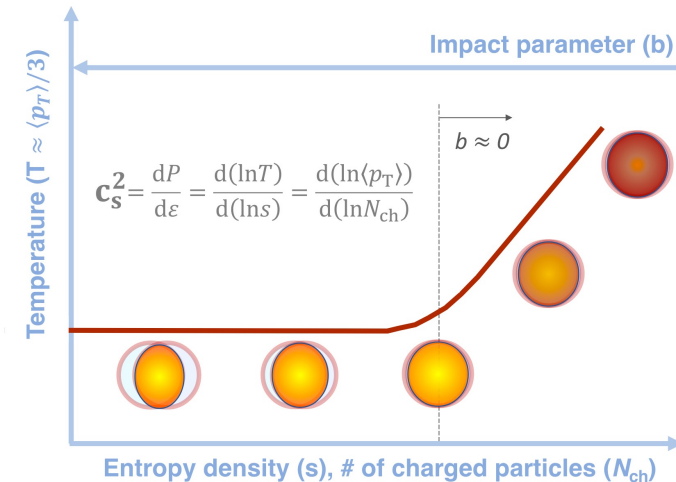
Ultra-central nucleus-nucleus collisions (UCC)



[F. Gardim et al, 1908.09728, Nature Physics]

[F. Gardim et al, 1909.11609]

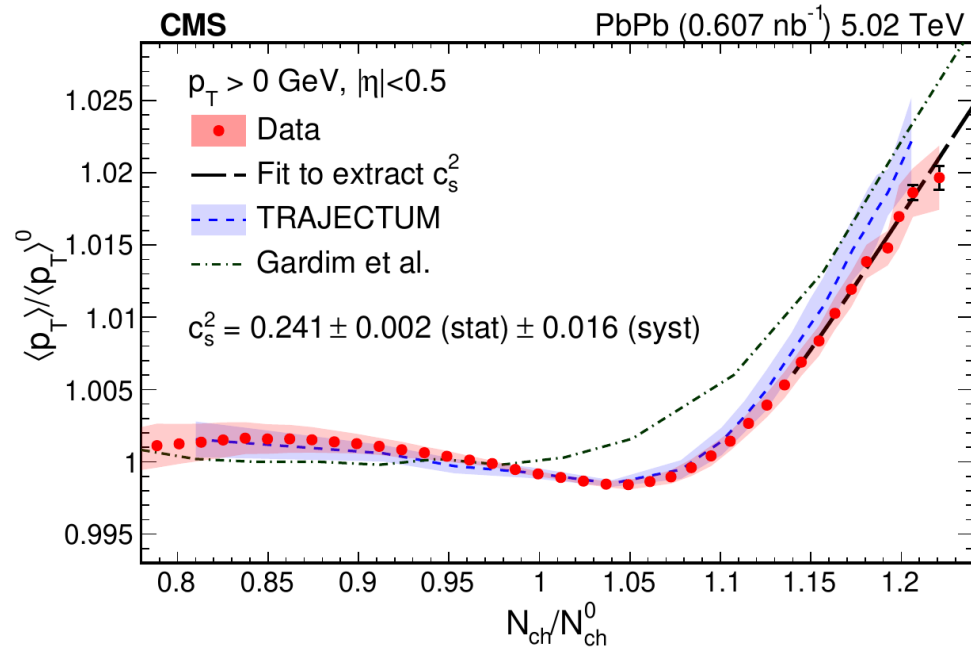
- S_{knee} is the mean value of the entropy at $b = 0$.
- The volume is fixed in UCC events.



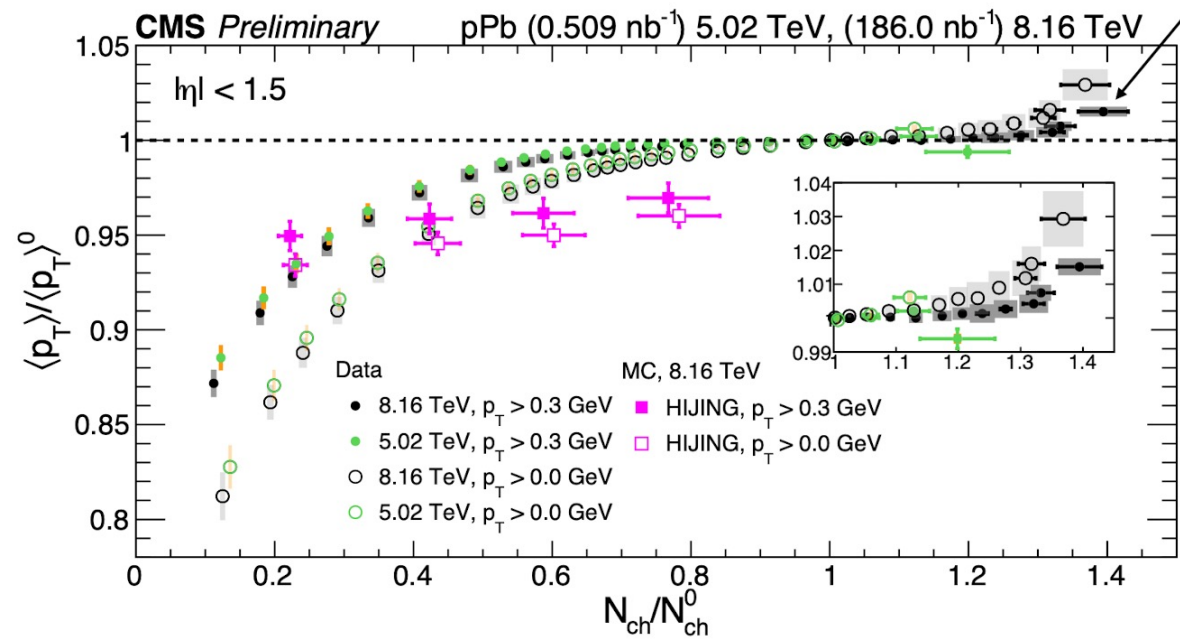
Extract the speed of sound in experiments

$$c_s^2 = \frac{d \ln \langle p_t \rangle}{d \ln N_{ch}} \longrightarrow \frac{\Delta_p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta_N}{N_0} \quad \text{with} \quad \begin{cases} \Delta_p \equiv \langle p_T \rangle - \langle p_T \rangle_0 \\ \Delta_N \equiv N_{ch} - N_0 \end{cases} \quad \begin{array}{l} \text{where } \langle p_T \rangle_0 \text{ and } N_0 \text{ are the} \\ \text{averaged values over the} \\ \text{ultra-central events} \end{array}$$

[CMS collaboration, 2401.06896]



[CMS collaboration, QM2025]

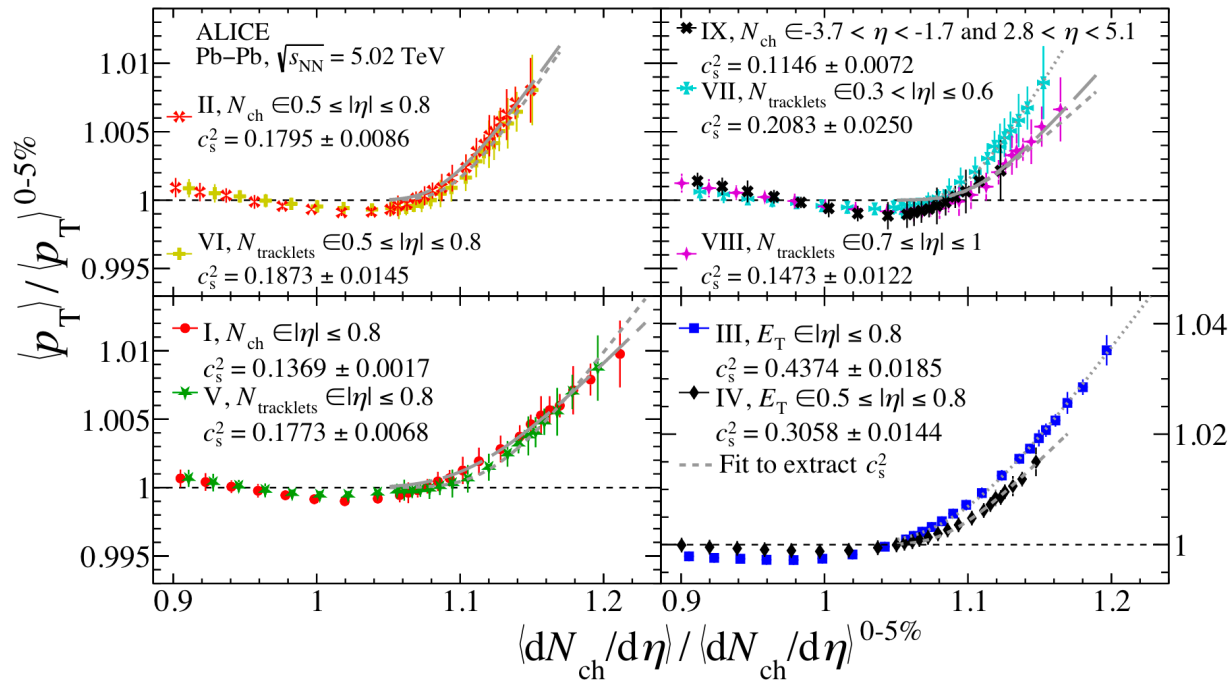


Extract the speed of sound in experiments

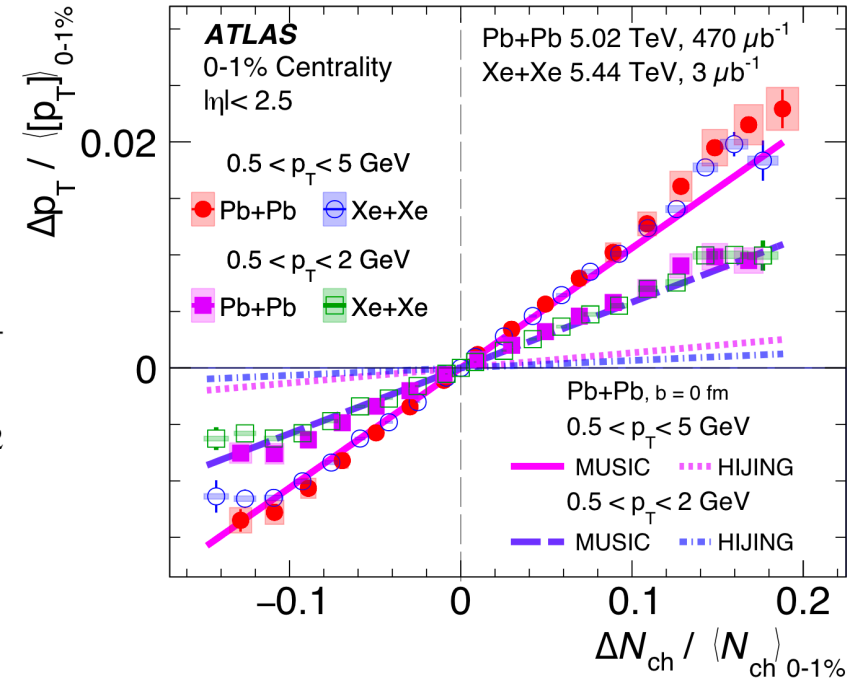
$$c_s^2 = \frac{d \ln \langle p_T \rangle}{d \ln N_{ch}} \longrightarrow \frac{\Delta p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta N}{N_0} \quad \text{with} \quad \begin{cases} \Delta p \equiv \langle p_T \rangle - \langle p_T \rangle_0 \\ \Delta N \equiv N_{ch} - N_0 \end{cases}$$

where $\langle p_T \rangle_0$ and N_0 are the averaged values over the ultra-central events

[ALICE collaboration, 2506.10394]

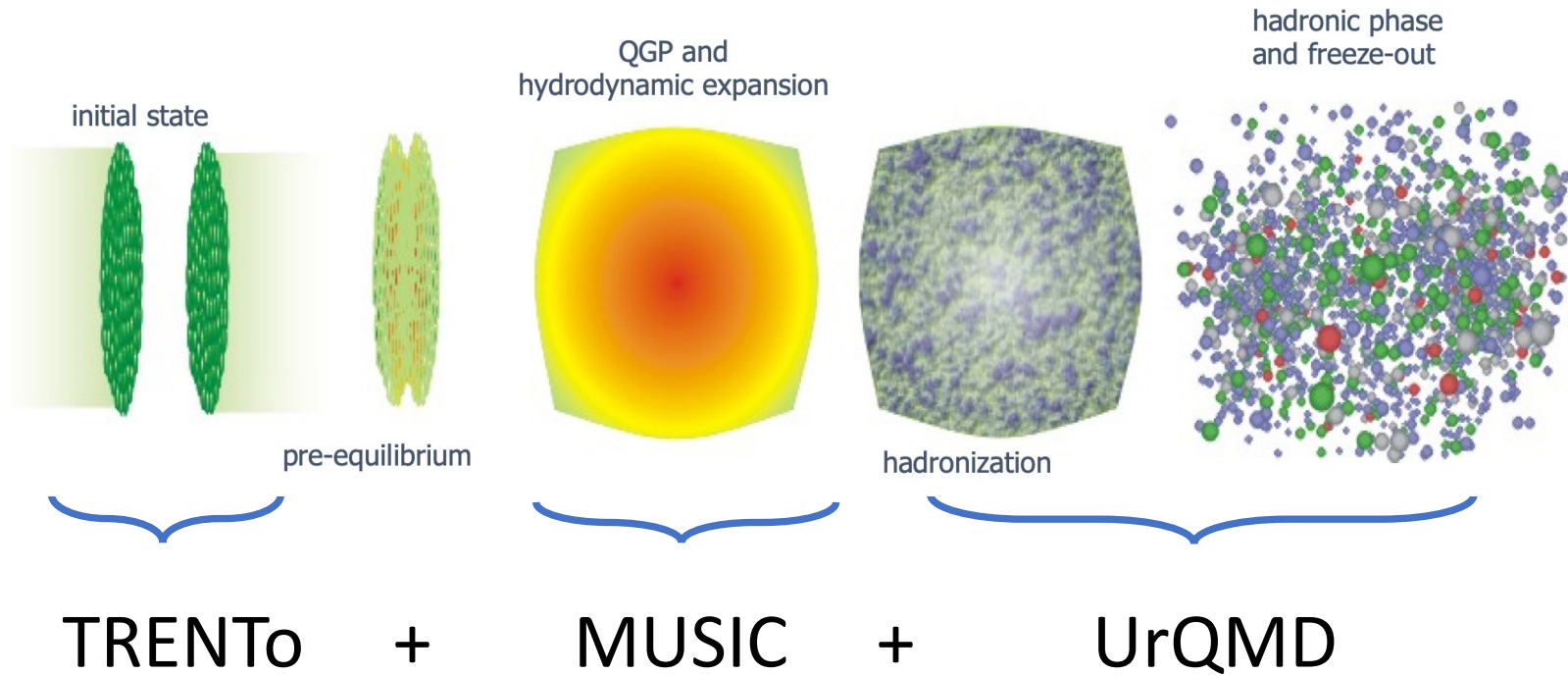


[ATLAS collaboration, 2407.068413]



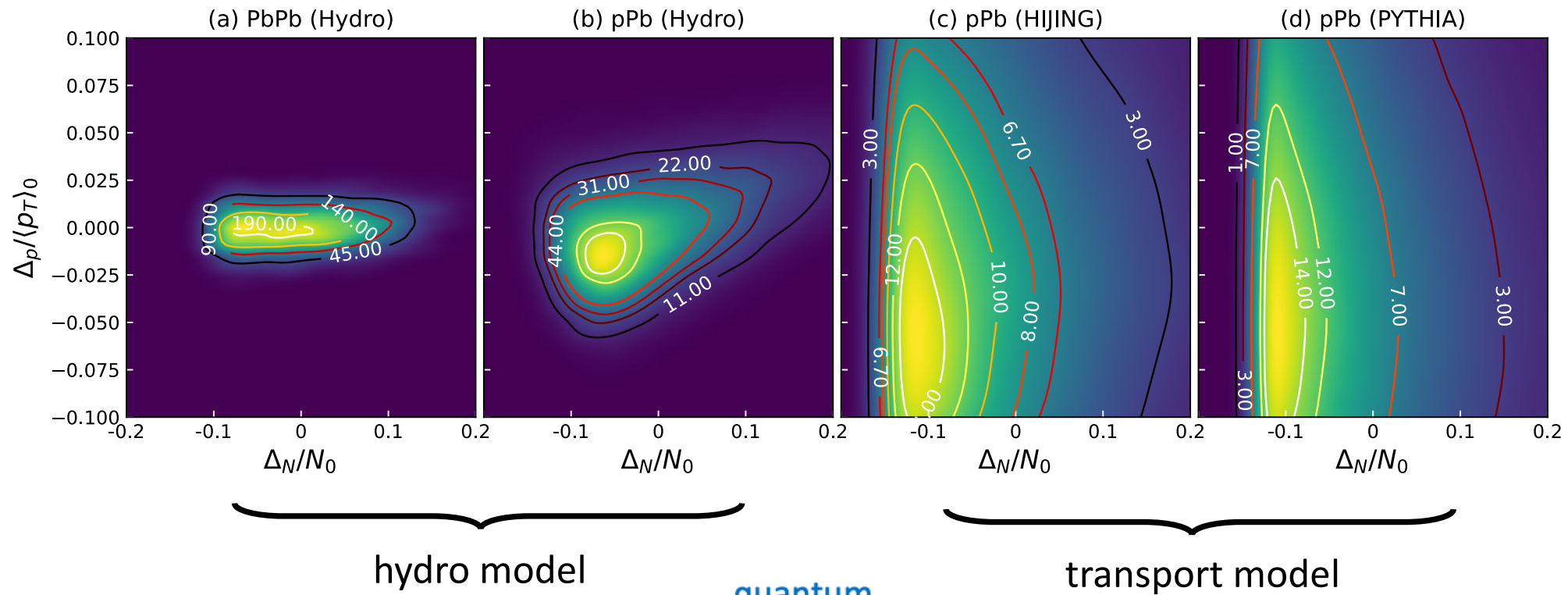
Two sets of model studies

- Thermal model



- Non-thermal model HIJING, Pythia

Quantum fluctuations in realistic collisions



$$\frac{\{\Delta_p\}_I}{\langle p_T \rangle_0} = c_s^2 \frac{\{\Delta_N\}_I}{N_0} \xrightarrow{\text{quantum fluctuation term}} \frac{\{\Delta_p\}_I}{\langle p_T \rangle_0} = c_s^2 \frac{\{\Delta_N\}_I + \{\delta\}_I}{N_0}$$

The presence of quantum fluctuations leads to 2D distribution of Δ_N and Δ_p .

Extracting the speed of sound in the presence of quantum fluc.

- In a thermalized system,

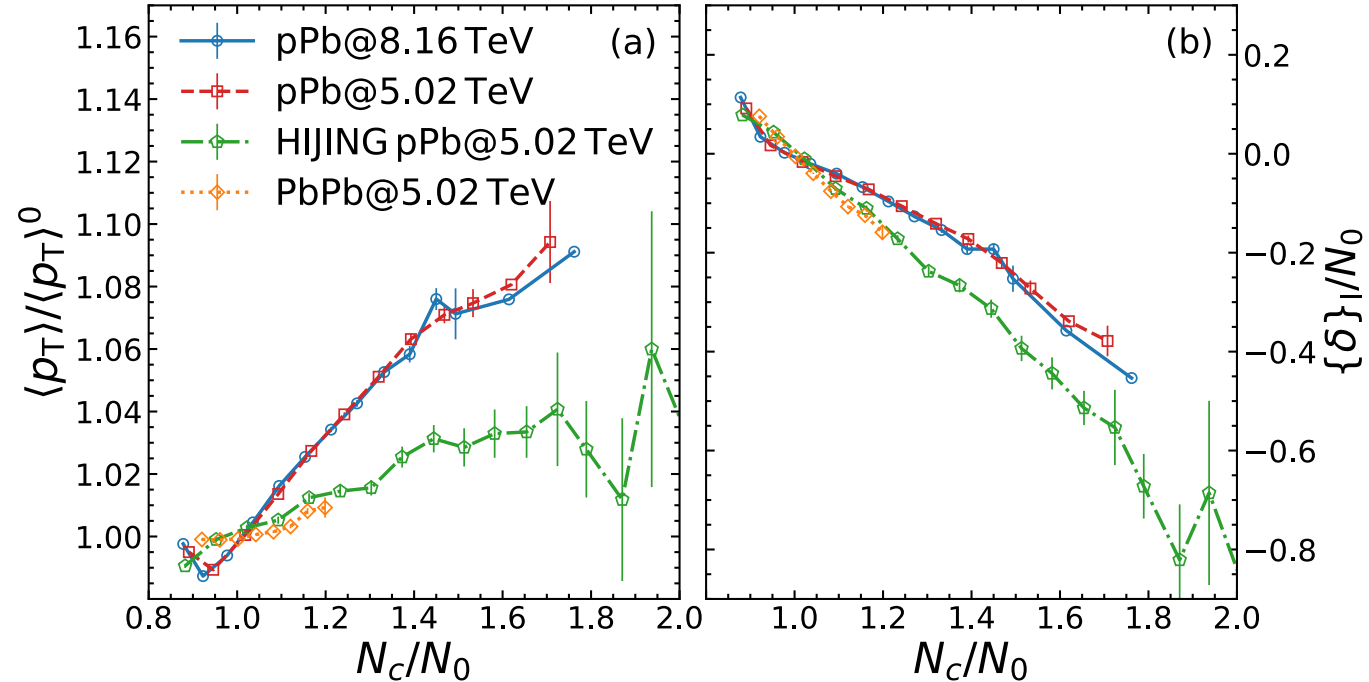
$$\frac{\Delta_p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0} \longrightarrow \delta = \frac{N_0}{c_s^2 \langle p_T \rangle_0} \Delta_p - \Delta_N$$

- δ denotes a quantum contribution.
- The distribution of δ is Gaussian according to Central Limit Theorem (CLT).
- Zero skewness condition: $\{\delta^3\}_c = 0$

$$\underbrace{(c_s^2)^3 \frac{\{\Delta_N^3\}}{N_0^3}}_{\text{mixed skewness of } \langle p_T \rangle \text{ and } N_{ch}} - 3 \underbrace{(c_s^2)^2 \frac{\{\Delta_N^2 \Delta_p\}}{N_0^2 \langle p_T \rangle_0}}_{\text{which can be measured in experiments}} + 3 \underbrace{c_s^2 \frac{\{\Delta_N \Delta_p^2\}}{N_0 \langle p_T \rangle_0^2}}_{\text{which can be measured in experiments}} - \underbrace{\frac{\{\Delta_p^3\}}{\langle p_T \rangle_0^3}}_{\text{which can be measured in experiments}} = 0$$

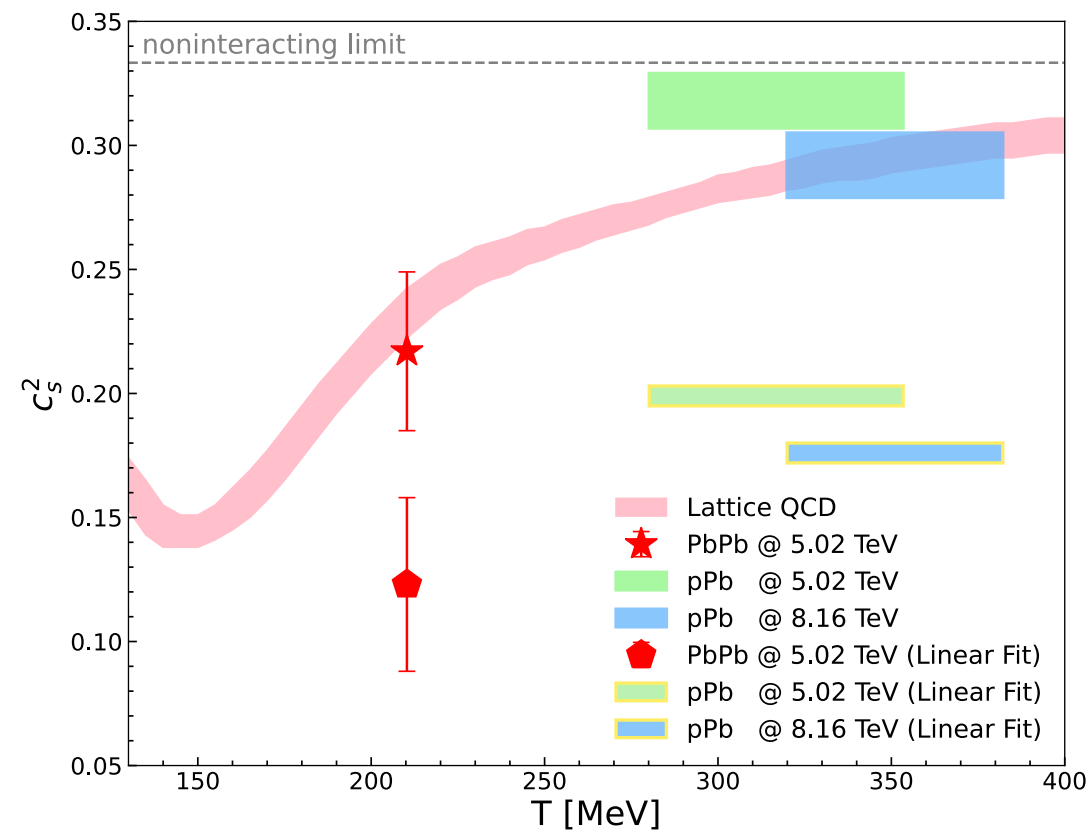
mixed skewness of $\langle p_T \rangle$ and N_{ch}
which can be measured in experiments

Quantum fluctuations are anti-correlated with N_{ch}



- $\frac{\{\Delta_p\}_I}{\langle p_T \rangle_0} = c_s^2 \frac{\{\Delta_N\}_I + \{\delta\}_I}{N_0}$
- $\{\delta\}_I = -\alpha \{\Delta_N\}_I$ which leads to $c_s^2 \rightarrow c_s^2 - \alpha$.
- The slope from simple linear fit can be suppressed by the quantum fluctuations.

Extracted speed of sound from different methods



c_s^2	Sub-bin slope	$\{\delta^3\}_c = 0$	$\{\delta^5\}_c = 0$	LEOS	$T_{\text{eff}}(\text{ MeV})$
PbPb (Hydro, 5.02 TeV)	0.123 ± 0.035	0.217 ± 0.032	0.216 ± 0.041	0.222–0.242	210.3
pPb (Hydro, 8.16 TeV)	0.176 ± 0.004	0.292 ± 0.013	0.287 ± 0.012	0.282–0.309	319.9–382.1
pPb (Hydro, 5.02 TeV)	0.197 ± 0.004	0.318 ± 0.011	0.313 ± 0.008	0.269–0.304	280.2–353.4

Probe of thermalization

- δ behaves differently in thermalized and non-thermalized systems.

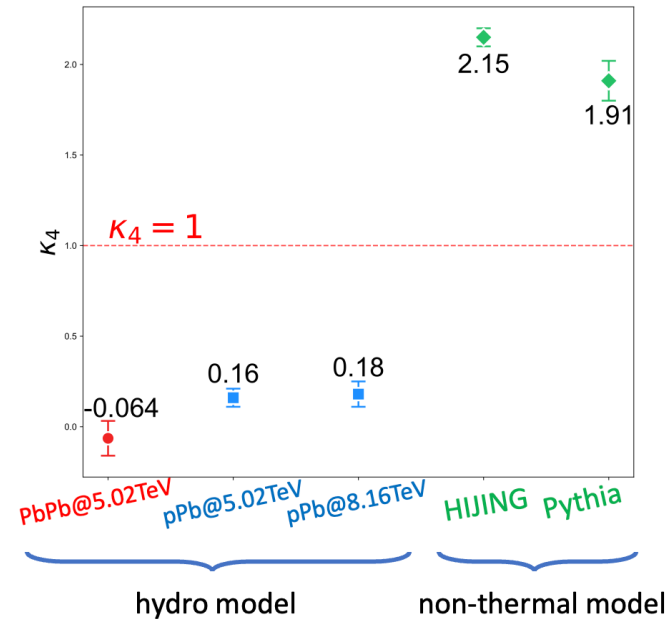
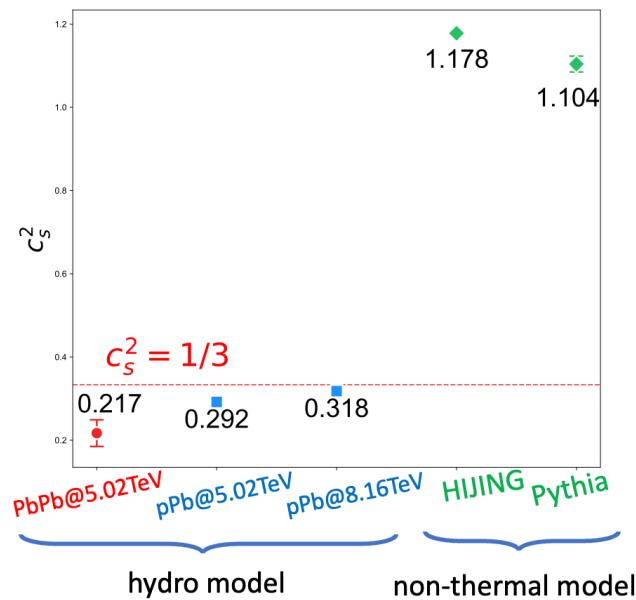
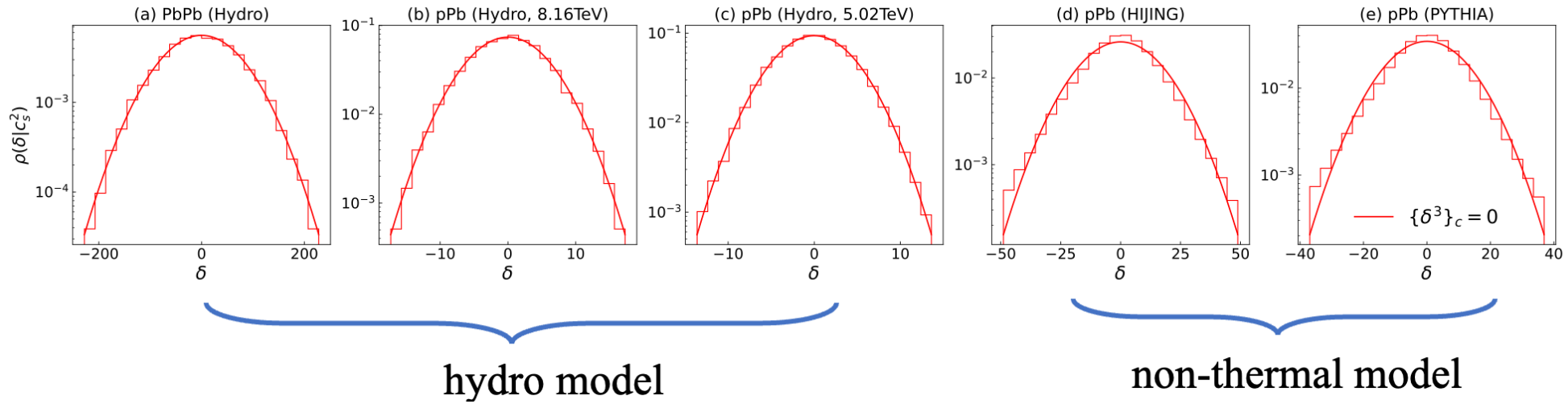
Thermalized system: $\frac{\Delta_p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0} \longrightarrow \text{thermal response} + \text{quantum noise}$

Non-thermalized system: $\frac{\Delta_p}{\langle p_T \rangle_0} = \kappa \frac{\Delta_N + \delta}{N_0} \longrightarrow \text{quantum response} + \text{quantum noise}$

- The extracted value is physical only in thermalized systems.
- δ distribution is Gaussian only in thermalized systems.
- Deviations from thermalization can be quantified by the standard kurtosis.

$$\kappa_4 = \{\delta^4\}/\{\delta^2\}^2 - 3$$

Probe of thermalization



Summary

- The QCD speed of sound can be extracted even in the presence of quantum fluctuations;
- Thermalization can be probed by examining the Gaussianity of quantum fluctuation δ and the physical validity of the extracted speed of sound, which can be quantified by the standardized kurtosis of δ .

Backup

Back up

$$\{\delta^5\}_c = 0$$

$$(c_s^2)^5 \frac{\{\Delta_N^5\}}{N_0^5} - 5(c_s^2)^4 \frac{\{\Delta_N^4 \Delta_p\}}{N_0^4 \langle p_T \rangle_0} + 10(c_s^2)^3 \frac{\{\Delta_N^3 \Delta_p^2\}}{N_0^3 \langle p_T \rangle_0^2} - 10(c_s^2)^2 \frac{\{\Delta_N^2 \Delta_p^3\}}{N_0^2 \langle p_T \rangle_0^3} + 5c_s^2 \frac{\{\Delta_N \Delta_p^4\}}{N_0 \langle p_T \rangle_0^4} - \frac{\{\Delta_p^5\}}{\langle p_T \rangle_0^5} = 0.$$

