

An Infrared Topological Approach to Baryon Structure

The 3rd Edition of the Spicy Gluons (胶麻) Workshop for Young Scientists on the Quark-Gluon Matter in Extreme Conditions

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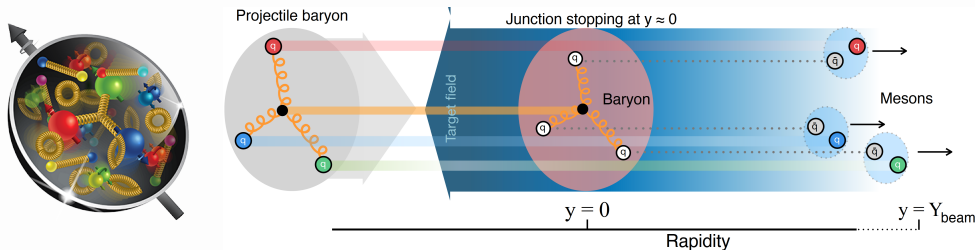
林 凡, 王昕杨 arXiv:2601.20274v3 [hep-th]

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Nucleon Structure: The Mysterious Gluon Structure

Y-Shaped Flux Junction

Both lattice QCD simulations and recent STAR measurements indicate that the 3-quark system inside baryons forms a **Y-shaped flux junction**.

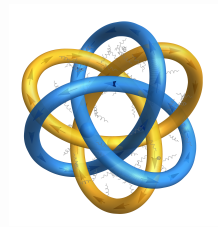


The Central Objective

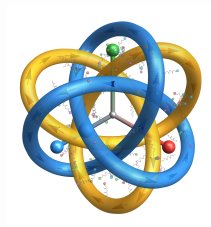
- What non-perturbative gluon field configuration forms the **Y-shaped flux junction**?
- How does it serve as an intrinsic skeleton unifying **confinement**, **mass (gluon condensate and chiral symmetry breaking)**, and **spin (axial anomaly)**?

Central Paradigm: The Topological Gluon Knot

- **Fundamental IR Degree of Freedom:** In the infrared limit of Yang–Mills theory, non-perturbative gluon fields condense into stable topological knots (Hopf solitons) — **Glueballs**.
- **Baryon Frame Architecture:** Physical baryons embed these gluon knots as their central core frame anchoring the three valence quarks.
- **Unified Framework:** A single topological object simultaneously governs chromoelectric flux tube formation, mass generation, and intrinsic angular momentum.



Glueball (Pure Gluon Knot)



Baryon (Knot + 3 Valence Quarks)

From SU(3) Gauge Field to Color Frames: CFN Decomposition

't Hooft Abelian Projection & Monopoles

$$\mathrm{SU}(3) \xrightarrow{\text{Maximal Abelian Gauge (MAG)}} \mathrm{U}(1)^2 \quad \pi_2 \left(\mathrm{SU}(3)/\mathrm{U}(1)^2 \right) \cong \mathbb{Z}^2$$

Singularities of gauge fixing isolate topological chromomagnetic monopoles in the infrared.

Cho–Faddeev–Niemi (CFN) Decomposition

$$A_\mu^a = \underbrace{C_{i,\mu} m_i^a}_{\text{Abelian } \mathrm{U}(1)^2} + \underbrace{H_\mu^a[\mathbf{m}]}_{\text{Magnetic Monopole}} + \underbrace{X_\mu^a[\mathbf{m}]}_{\text{Off-diagonal (Massive)}}$$

where $m_i(x) = g(x)H_i g^\dagger(x) \in \mathrm{SU}(3)/\mathrm{U}(1)^2$ ($i = 1, 2$) defines the local Cartan color frame.

Infrared Abelian / Monopole Dominance

Off-diagonal gluons X_μ^a acquire a dynamical mass gap ($\sim 1\text{--}1.5$ GeV) and decouple in the deep IR, leaving the topological color frame field $\mathbf{m}_i$ and monopole fields H_μ^a as the active long-range degrees of freedom.

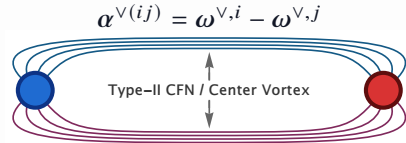
From Monopoles to Center Vortices: CFN Vortex Fractionation

- Monopole charge carries co-root vectors:

$$g_m = \oint_{\Sigma} F^H \cdot d\sigma = 2\pi\alpha^\vee$$

- Type I (co-root) vs Type II (co-weight) fluxes:

$$\Phi_\alpha = 2\pi\alpha^\vee, \quad \Phi_\omega = 2\pi\omega^\vee$$



CFN Co-root Vortex Fractionates into Center-Vortex Pair

Energetic Fractionation into Center Vortices

$$\frac{E_{II}}{E_I} \approx \frac{|\Phi_{\omega^{\vee,i}}|^2 + |\Phi_{\omega^{\vee,j}}|^2}{|\Phi_{\alpha^{ij}}|^2} = \frac{N-1}{N} = \frac{2}{3} < 1$$

An intact CFN vortex tube energetically fractionates into a pair of **center vortices** ($\mathbb{Z}_3$) with fundamental co-weight fluxes.

Center Vortices Network & Topological Stabilization

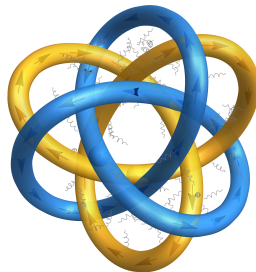
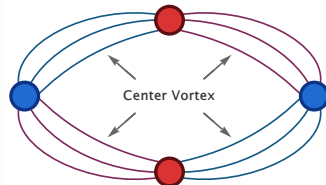
Center Vortices & Area Law Confinement

$$z_k = e^{i\frac{2\pi k}{3}} \in \mathbb{Z}_3 \subset \text{SU}(3), \quad W(C) \longrightarrow z_k W(C)$$

Random piercings of center vortices induce statistical imaginary phase cancellation, generating the Wilson loop **Area Law**:

$$\langle W(C) \rangle \sim \exp [-\sigma \cdot \text{Area}(C)]$$

Topological Stabilization via Knots Non-trivial knot helicity stabilizes the chromomagnetic vortex network against collapse or vacuum decay.



Lattice Center Vortex Percolation & Area Law

J. Greensite, Prog. Part. Nucl. Phys. 51, 1 (2003); M. Ünsal & L. G. Yaffe, PRD 78, 065035 (2008).

Faddeev–Niemi Theory: Knots as Glueball Solitons

Effective Low-Energy Action for Pure Yang–Mills Integrating out massive off-diagonal gluons leads to the non-linear Faddeev–Niemi action:

$$\mathcal{L}_{\text{FN}} = (\partial_\mu m_i^a)^2 + \frac{1}{e^2} (f^{bcd} m_i^b \partial_\nu m_j^c \partial_\mu m_j^d)^2$$

Topological Invariant: The Hopf Charge Target space mapping $\mathbb{S}^3 \rightarrow \text{SU}(3)/\text{U}(1)^2$ possesses non-trivial third homotopy group:

$$\pi_3 \left(\text{SU}(3)/\text{U}(1)^2 \right) \cong \mathbb{Z}, \quad \mathcal{H} = \frac{1}{4\pi^2} \sum_i \int_{\mathbb{S}^3} A_i^H \wedge F_i^H$$

Vakulenko–Kapitanski Bound & Mass Gap

Energy of every knot component is strictly bounded by the Hopf invariant: $E_i \geq c |\mathcal{H}_{(ii)}|^{3/4}$.

Implication: Gluon knots cannot collapse or dissolve. They form **stable solitons (Glueballs)** and provide a non-perturbative foundation for the **Yang–Mills Mass Gap**.

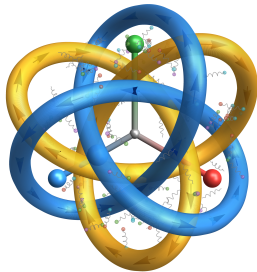
Baryon Structure: Dual Ginzburg–Landau & Y-Junction

Dual Superconductivity Mechanism

$$\mathcal{L}_{\text{GL}} = -\frac{1}{4}M_{\mu\nu}^2 + \sum_{\alpha} \left[|D_{\mu}\chi_{\alpha}|^2 - V(|\chi_{\alpha}|^2) \right]$$

Lattice QCD confirms weak Type-II dual superconductivity: condensation of chromomagnetic monopoles ($\langle\chi\rangle \neq 0$) expels chromoelectric fields via the **Dual Meissner Effect**.

- 3 valence quarks act as chromoelectric charge sources.
- Chromoelectric flux of quarks is squeezed into thin vortex strings.
- The central **gluon knot serves as the pinning junction**, naturally establishing the Y-shaped flux junction observed in Lattice & STAR.



Dual Abrikosov vortex $\rightarrow$ Nucleon 3-string Y-junction

Baryon Mass: Trace Anomaly & Magnetic Catalysis

Gluon Condensate: Trace Anomaly

$$T^\mu{}_\mu = \frac{\beta(g)}{2g} G_{\mu\nu}^a G^{\mu\nu,a} + \sum_f m_f (1 + \gamma_m) \bar{\psi}_f \psi_f$$

- In chiral limit, the entire hadron mass arises from quantum trace anomaly.
- Gluon knot condensate accounts for the dominant $\sim 40\%$ **anomaly mass** in nucleons.

Quark Condensate: Magnetic Catalysis

$$\langle 0 | \bar{\psi} \psi | 0 \rangle_B \sim |g \mathbf{B}|$$

- Highly concentrated chromomagnetic fields inside the knot core act as an effective background $\mathbf{B}^H$.
- **magnetic catalysis of $\chi\mathbf{SB}$** generates large dynamical constituent quark masses.

The gluon knot generates both gluon and quark condensate!

R. J. Crewther, PRL 28, 1421 (1972); Y.-B. Yang et al., PRL 121, 212001 (2018); V. P. Gusynin et al., PRL 73, 3499 (1994).

Topological Current of Gluon Spin

Revisiting the Proton Spin Crisis Total nucleon angular momentum decomposes via Jaffe–Manohar and Ji decompositions:

$$J_{\text{QCD}} = \int d^3x \left[\psi^\dagger \frac{\Sigma}{2} \psi + \psi^\dagger (\mathbf{x} \times -i\nabla) \psi + \mathbf{E}_a \times \mathbf{A}_a + E_a^i (\mathbf{x} \times \nabla) A_a^i \right]$$

Topological Origin of Gluon Spin ΔG

- In the CFN framework, the non-trivial winding of the color frame drives a non-vanishing spatial topological current:

$$\mathbf{H} \propto \mathbf{E}_i^H \times \mathbf{A}_i^H$$

mapping directly into the intrinsic gluon spin operator.

- In Ji's decomposition, non-vanishing $\mathbf{E}^H \times \mathbf{B}^H$ inside the knot yields a finite **gluon Poynting vector angular momentum**:

$$\mathbf{J}_g = \int d^3x \, \mathbf{x} \times (\mathbf{E} \times \mathbf{B})$$

R. L. Jaffe & A. Manohar, NPB 337, 509 (1990); X. Ji, PRL 78, 610 (1997); F. Lin, X. Wang, arXiv:2601.20274.

Vacuum Topology & Axial Anomaly

Homotopy Isomorphism & Hopf-Winding Relation in SU(3)

$$\pi_3 \left(\text{SU}(3)/\text{U}(1)^2 \right) \cong \pi_3(\text{SU}(3)) \cong \mathbb{Z}$$

$$\frac{1}{16\pi^2} \sum_{i=1}^2 \int A_i^H \wedge F_i^H = \frac{1}{4} \sum_{i=1}^2 \mathcal{H}_{(ii)} = \frac{\mathcal{H}}{4} = N(g) = \frac{1}{24\pi^2} \int_{\mathbb{S}^3} \epsilon^{ijk} \text{Tr} \left(g \partial_i g^\dagger g \partial_j g^\dagger g \partial_k g^\dagger \right)$$

Opposite component charges yield $\mathcal{H} = \mathcal{H}_{(11)} + \mathcal{H}_{(22)} = 0$, selecting it as the topologically trivial ground state of nucleons.

Axial Anomaly & Dirac Zero Modes (Banks–Casher Relation)

$$\partial_\mu j^{5,\mu} = 2im\bar{\psi}\gamma^5\psi + \frac{g^2 N_f}{8\pi^2} \text{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) \quad \implies \quad \langle \bar{\psi}\psi \rangle = -\frac{\pi}{V} \text{sign}(m) \overline{\nu(0)}$$

Fluctuations in knot topology drive vacuum tunneling, inducing Dirac zero modes and more χ SB.

S. L. Adler, PR 177 (1969); J. S. Bell & R. Jackiw, Nuovo Cim. A 60 (1969); T. Banks & A. Casher, NPB 169 (1980).

Hadronic Spectra: From Glueballs to Mesons

Scalar Meson $f_0(500)$ (σ)

Identified as the fundamental topological gluon knot excitation, setting the universal IR scale for trace anomaly and χ SB.

Heavy Quarkonia (J/ψ , Υ)

$Q\bar{Q}$ color flux tubes are tightly wound and stabilized by knotted gluon field, matching Cornell potential string tensions.

Glueball Classification

Pure glueball spectrum is systematically classified by knot topological invariants and linkings.

Vector vs Pseudoscalar Mesons

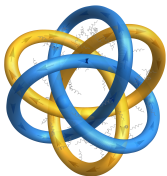
Mass relations $m_\sigma + m_\pi \approx m_\rho$ indicate transient knot components in vector mesons (ρ, ω), while pseudoscalars (K, π) remain unknown as pseudo-Goldstone modes.

G. E. Brown & M. Rho, PRL 66, 2720 (1991); Y.-L. Ma & M. Rho, PPNP 113, 103791 (2020); Y.-L. Li et al., PRD 95, 114011 (2017).

Conclusions: A Unified Topological Picture

A Universal Topological Structure for Hadron Physics

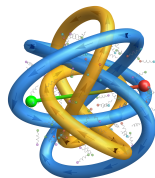
- **Degrees of Freedom:** Stable gluon knots (Hopf solitons) emerge in IR Yang–Mills, naturally contributing the **Yang–Mills mass gap** and glueball spectrum.
- **Baryon Confinement:** Knots act as the central pinning core for Y-shaped chromoelectric flux junctions via dual superconductivity.
- **Mass & Spin:** The knot simultaneously generates the gluon condensate, catalyzes quark chiral masses, and carries angular momentum.



Glueball (Pure Knot)



Baryon (Knot + Y-Junction)



Heavy Meson

Thank You for Your Attention!