

Non-hydro simplified

- We propose a simple extended hydro that works beyond small gradients.
- Test it in different theories
- Analytically analyze why it works.

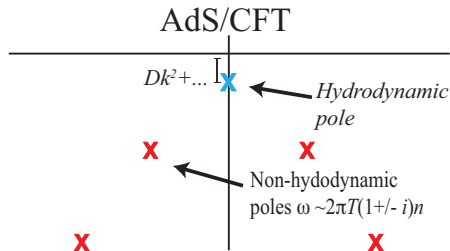
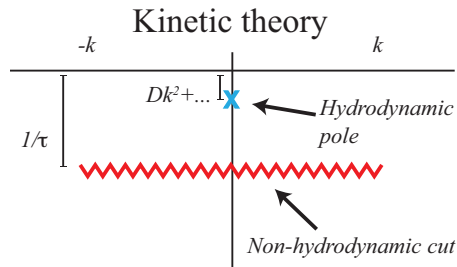
Speakers: **Zhenkang Lu and Mufeng Lei**

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Hydro and non-hydro modes



[Kurkela, Wiedemann & Wu'19, EPJC]

- Hydro is a universal EFT of a few conserved densities, valid at small gradients. Non-hydro modes, however, differ across classes of theories.
- At finite gradients, many non-hydro modes of different types can contribute.
- **Common wisdom:** describing the non-hydro regime can be complicated, and an effective description may not exist at all.

Main results

- We find a simple, systematic description of dominant (least-damped) mode **beyond small gradients** for several classes of microscopic theories.
 - Coupling to only a minimal set of non-hydro modes already gives a good approximation.
 - In kinetic-like theories, we identify an effective small “gradient” even when hydro is no longer applicable.
 - The realization of this minimal coupling may distinguish different classes of theories.
- These results may help explain why hydro modeling works in small collision systems.
[Zhao, Shen & Schenke’22, PRL]

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}})f = C[f] \stackrel{\text{RTA}}{\simeq} -\frac{f - f_{\text{eq}}}{\tau_R}.$$

Static, homogeneous, isotropic equilibrium.

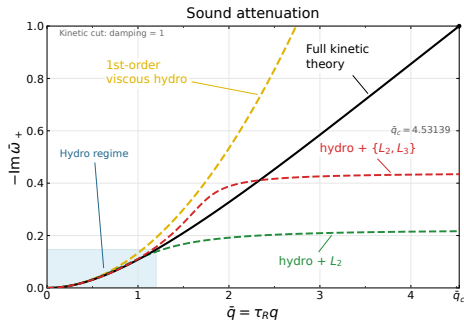
$$\mathcal{L}_n = \int \frac{d^3p}{(2\pi)^3} E_p P_n(\cos(\theta)) \delta f, \quad \mathcal{L}_0 = \delta\varepsilon, \quad \mathcal{L}_1 = \delta T^{0z}.$$

Longitudinal wave moment equations:

$$\begin{aligned} -i\omega \mathcal{L}_0 &= -iq \mathcal{L}_1, \\ -i\omega \mathcal{L}_1 &= -iq \left(\frac{1}{3} \mathcal{L}_0 + \frac{2}{3} \mathcal{L}_2 \right), \\ -i\omega \mathcal{L}_2 &= -iq \left(\frac{2}{5} \mathcal{L}_1 + \frac{3}{5} \mathcal{L}_3 \right) - \tau_R^{-1} \mathcal{L}_2, \\ &\dots \end{aligned}$$

- Higher moments describe non-hydro modes, e.g., $\mathcal{L}_2 = \frac{3}{2} \pi^{zz}$.
- Solving this linear system gives its eigenmodes, including the dominant mode.
- To give an effective description, we truncate the system.

Simple truncation fails



Green dashed: $\mathcal{L}_{n \geq 3} = 0$ Red dashed: $\mathcal{L}_{n \geq 4} = 0$.

Yellow dashed: first-order viscous hydro. Black: full kinetic theory.

- The full kinetic theory has an isolated dominant mode—the *ground-state mode*.
- Discarding higher-order non-hydro information makes the approx fail rapidly beyond the hydro regime.
- We need a practical way to retain higher-moment information!

Our method

- Keep only the first three equations. Instead of setting $\mathcal{L}_3 = 0$, approximate

$$\mathcal{L}_3(\bar{\omega}, \bar{q}) \approx \mathcal{L}_3(\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2).$$

Absorb the non-local $\bar{\omega}, \bar{q}$ dependence into lower moments.

- An intuitive approximation is

$$\mathcal{L}_3(\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2) \equiv \frac{\mathcal{L}_3}{\mathcal{L}_2} \mathcal{L}_2 \approx c_2 \frac{\mathcal{L}_2}{\mathcal{L}_1} \mathcal{L}_2.$$

- For general K , we propose

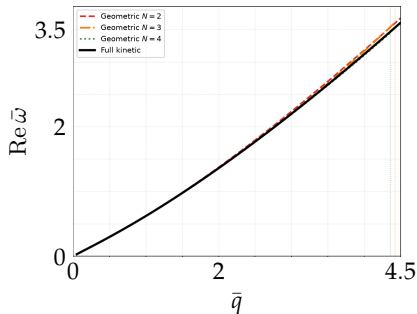
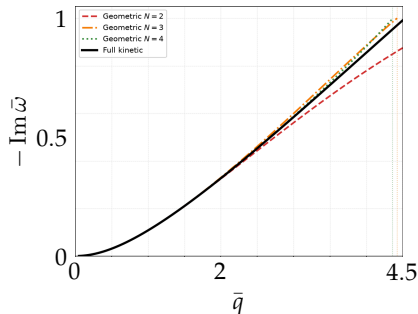
$$r_K \simeq c_K r_{K-1} \quad \text{with} \quad r_n \equiv \frac{\mathcal{L}_{n+1}}{\mathcal{L}_n}.$$

- As $K \rightarrow \infty$, $\mathcal{L}_K \rightarrow 0$ and we expect

$$c_K \rightarrow 1, \quad r_K, r_{K-1} \rightarrow r^*, \quad |r^*| < 1.$$

Does this simple approximation give an accurate result?

Sound mode approx results



$K = 2$ $K = 3$ $K = 4$ Black: full kinetic theory. $\bar{q} = q\tau_R$, $\bar{\omega} = \omega\tau_R$.
 $K + 1$: order of highest retained moment.

- Adding more non-hydro modes improves the ground-state mode prediction, especially $\text{Im } \omega$, even at finite gradient. **How to prove this?**
- This resembles a Taylor expansion: adding higher-order contributions improves the prediction. It implies an effective “gradient” parameter that remains small even at large $\bar{q}$. **How to find this?**

Moments describe modes

Expanding the kinetic equation in moments commonly yields a tridiagonal secular equation:

$$\lambda \mathcal{L}_n = b_n \mathcal{L}_{n-1} + a_n \mathcal{L}_n + c_n \mathcal{L}_{n+1}$$

Gradient $\bar{q}$ is encoded in coefficients a_n, b_n, c_n

Where does error come from?

For $K \times K$ matrix, $\mathcal{L}_0, \mathcal{L}_1, \dots, \mathcal{L}_{K-1}$ are variables

Last row reads:

$$\lambda \mathcal{L}_{K-1} = b_{K-1} \mathcal{L}_{K-2} + a_{K-1} \mathcal{L}_{K-1} + c_{K-1} \mathcal{L}_K$$

First row gives:

$$\delta \lambda \sim \delta r_0$$

We need to relate δr_{K-1} to δr_0

We approximate this



$$\mathcal{L}_K = r_{K-1} \mathcal{L}_{K-1}$$



Error propagation

Rewrite secular equation gives recurrence relation of relative error δr_n :

$$\delta r_{n-1} = \left(\frac{c_n}{b_n} r_n r_{n-1} \right) \delta r_n := q_n \delta r_n$$

We use it to propagate error back to δr_0 .

The diagram shows a matrix structure within large parentheses. A vertical dashed line separates the matrix into two parts. To the left of the line, the matrix is upper triangular with elements a_0 , $c_0 r_0$, $b_0 r_0^{-1}$, and ellipses. To the right of the line, the matrix is lower triangular with elements $b_{K-1} r_{K-2}^{-1}$, a_{K-1} , and $c_{K-1} r_{K-1}$. The element $c_{K-1} r_{K-1}$ is enclosed in a red box. A blue arrow labeled "Error propagation" points from the red box towards the top-left. The text "Error introduced here" is placed next to the vertical dashed line.

Analytical expression for relative error

Ground-state mode relative error can be deduced

$$\delta\lambda = \left|1 - \frac{a_0}{\lambda}\right| \left(\prod_{n=1}^{K-1} q_n\right) \delta r_{K-1}$$

Ground-state modes exist **require**:

$$|q_n| < 1$$

Every order is compressive. Even if the gradient is large!

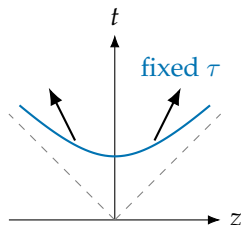
A new small parameter for extended hydro regime has been found!

If there is a typical r_* satisfying $q_n \sim r_*^2$, a rough scaling law show up:

$$\delta\lambda \sim r_*^{2(K-1)} \delta r_{K-1}$$

To improve accuracy of extended hydro, we can increase the moments order and/or look for better approximation for r_{K-1}

Bjorken expansion



Boost invariant; Transversely homogeneous

$$3 + 1d \rightarrow 0 + 1d (\tau)$$

$$\tau = \sqrt{t^2 - z^2}, \quad w = \tau/\tau_R$$

$$\text{Gradient } \bar{q} = \frac{1}{w} = \frac{\tau_R}{\tau}$$

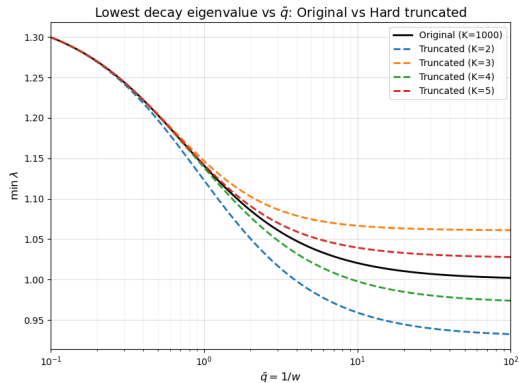
Secular equation can be expand by moments.

$$\lambda \mathcal{L}_n = b_n \mathcal{L}_{n-1} + [a_n + w(1 - \delta_{n0})] \mathcal{L}_n + c_n \mathcal{L}_{n+1}$$

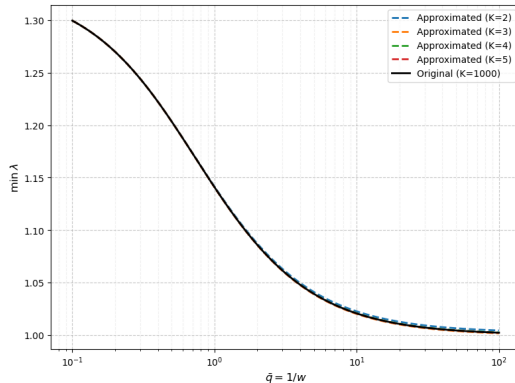
Pick first K moments and close with our simple approximation.

$$r_{K-1} \approx r_{K-2}$$

Approximation improvement



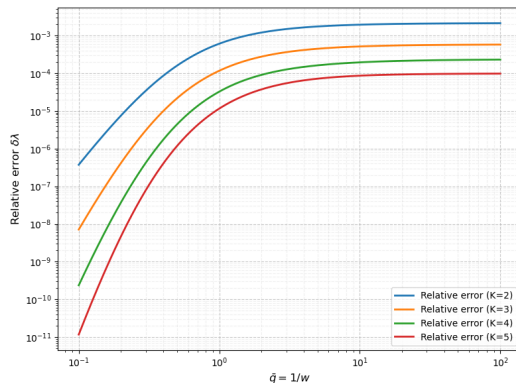
Hard truncation



Approximated

Simple approximation greatly improves the accuracy!

More moments improvement



Relative error of different order approximation

More moments systematically reduce relative errors.

Discrete $U(1)$ gauge field chain

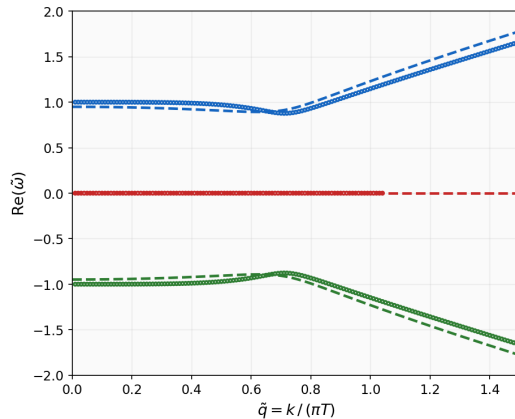
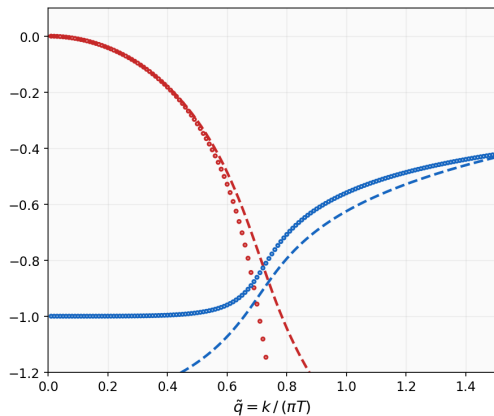
- Holographic Maxwell dynamics in $\mathbf{AdS}_5$ can be represented by a chain of K coupled $U(1)$ sites with horizon dissipation at site 1.
- The radial direction emerges as $K \rightarrow \infty$.

$$\boxed{\sum_{n=1}^K [\beta_n \Delta \mathcal{E}_n^a \Delta \mathcal{E}_n + \epsilon_n \mathcal{E}_n^a \mathcal{E}_n] + \frac{i\sigma_1}{\omega} \mathcal{E}_1^a \mathcal{E}_1} \xrightarrow{K \rightarrow \infty} \boxed{-\frac{1}{2g_5^2} \int_{\mathcal{M}_{\text{ext}}} d^5x \sqrt{-g} F_{MN}^a F^{MN} + I_{\text{horizon}}^{\text{eff}}}$$

[An, Brants, Heller & Yin'25, PRL]

- $\mathcal{E}_n$: longitudinal electric field at site n .
 $\Delta \mathcal{E}_n = \mathcal{E}_{n+1} - \mathcal{E}_n$, σ_1 : horizon dissipation.
- **Equation of motion:** $\beta_{n-1} \Delta \mathcal{E}_{n-1} - \beta_n \Delta \mathcal{E}_n + \epsilon_n \mathcal{E}_n = 0$.
- **Boundary conditions:** $(\beta_1 + \epsilon_1 + i\sigma_1/\omega) \mathcal{E}_1 - \beta_1 \mathcal{E}_2 = 0$, $\mathcal{E}_{K+1} = 0$.

$K = 1$ approximation to holography



- Solid: $K \rightarrow \infty$ (AdS/CFT) vs dashed: $K = 1$ (minimum coupling) approximation.
- Ground-state mode with crossover.

Summary and outlook

Summary

- We develop an extended hydro framework for the ground-state mode beyond small gradients, applicable to weakly coupled kinetic theory and strongly coupled holography.
- In kinetic-like theories, we explicitly identify effective “gradient” parameters that control the approximation even beyond the hydro regime.
- We analytically quantify how the approximation improves as more non-hydro information is included.

Outlook

- Identify the effective “gradient” parameters in more theories and understand how they depend on the structure of non-hydro modes.
- Explore applications of our framework to collective behavior in small collision systems.

Thank you!