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Update for the study of $\gamma\gamma \rightarrow \omega J/\psi$ at Belle and Belle II

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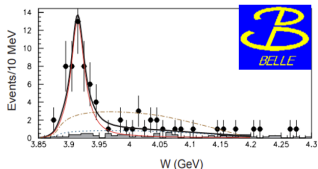
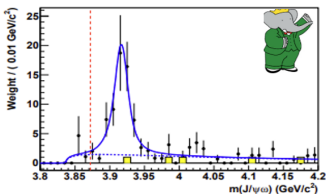
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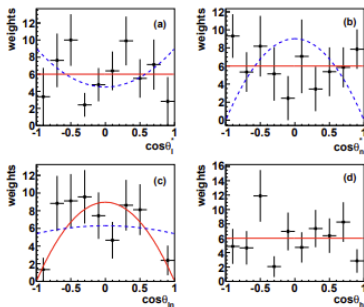
Recall: Motivation

- An excess above the expected background is observed in the $M_{\omega J/\psi} > 4 \text{ GeV}/c^2$ region in both Belle and BaBar data.
- The efficiency in the previous Belle measurement can be improved, this can improve the resonance parameter measurement.



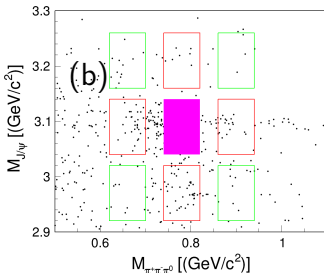
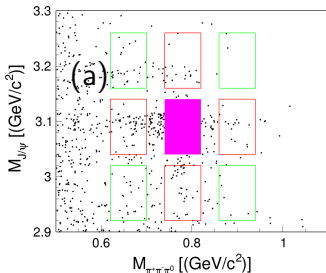
Recall: Motivation

- With the improved statistics, WG readers recommend us to perform an amplitude analysis.
- Previous BaBar's angular analysis is limited to 1D projections.



Recall: sidebands and number of expected events

- 2D sidebands from M_{ω} and $M_{J/\psi}$ are used.
- Event counts (blinded): 287 for Belle, 175 for Belle II Run I.
- Sideband counts : 26 for Belle, 14 for Belle II Run I.
- Event number in Toy MC (pure signal) set as 500.



Fit to the $M_{\omega J/\psi}$ Spectrum (Toy MC)Fit to the $M_{\omega J/\psi}$ Spectrum
(Toy MC)

Signal Modeling: PDF Construction

- Two configurations: Single S -wave BW / Incoherent sum of two S -wave BWs.
- For the two-resonance hypothesis, the relative magnitude of the two states is parameterized by a yield ratio R :

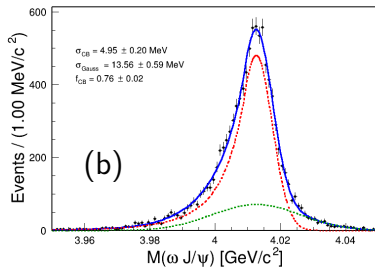
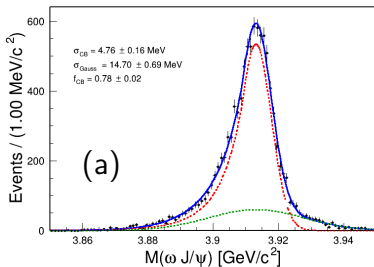
$$\mathcal{P}_{\text{sig}}(M) \propto \mathcal{P}_1(M) + R \cdot \mathcal{P}_2(M) \quad (1)$$

$$\mathcal{P}_k(M) \propto \left[\left(|\text{BW}_k(M)|^2 \cdot p^*(M) \right) \otimes \mathcal{R}_k(M) \right] \quad (2)$$

- $\text{BW}_k(M)$: The S -wave relativistic BW describing resonance line shape.
- $p^*(M)$: The two-body kinematic phase space factor governing the near-threshold truncation.
- $\mathcal{R}_k(M)$: The detector resolution function accounting for experimental mass smearing.

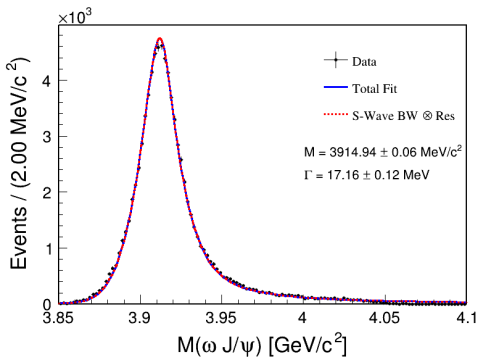
Detector Resolution

- 1M zero-width Belle II MCri generated and reconstructed.
- Using Crystal Ball + Gaussian to describe.



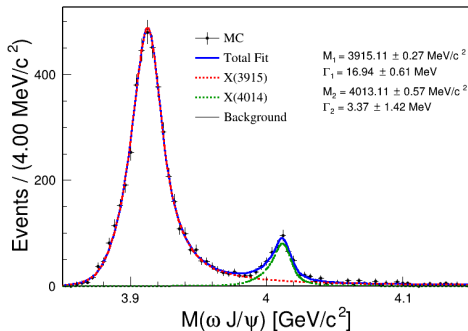
I/O Check - Configuration I

- $M_{X(3915)} = 3915 \text{ MeV}/c^2$, $\Gamma_{X(3915)} = 17 \text{ MeV}$ (from previous Belle work).
- 1M Belle II MCri generated and simulated.



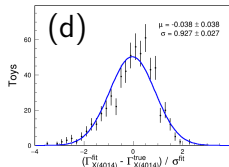
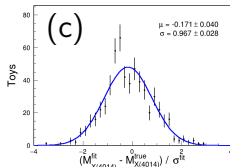
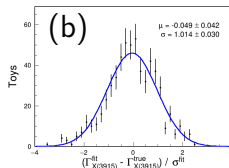
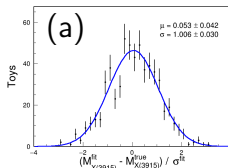
I/O Check - Configuration II

- $M_{X(3915)} = 3915 \text{ MeV}/c^2$, $\Gamma_{X(3915)} = 17 \text{ MeV}$ (from previous Belle work).
- $M_{X(4014)} = 4014 \text{ MeV}/c^2$, $\Gamma_{X(4014)} = 3 \text{ MeV}$ (from theoretical prediction), $R = 0.1$.
- 5000 events ($10N_{\text{expected}}$) used for the unbinned fit.



Pull distribution - Configuration II

- 500 Toy MC samples generated to validate fit stability.
- 500 events with poisson fluctuation in each Toy MC samples.
- Mean values μ are quoted as the systematic uncertainty.

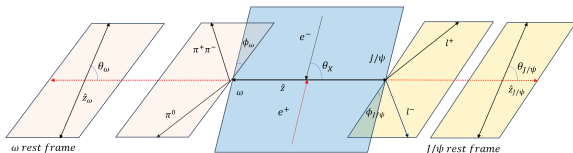


Amplitude Analysis for $X(3915)$ (Toy MC)

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Kinematic Variables

- Five independent angles: $\xi \equiv (\theta_X, \theta_\omega, \phi_\omega, \theta_{J/\psi}, \phi_{J/\psi})$
- θ_X : Polar angle of the J/ψ momentum in the $X(3915)$ rest frame.
- $\theta_\omega, \theta_{J/\psi}$: Polar helicity angles. θ_ω is defined by the normal vector of the 3π decay plane.
- $\phi_\omega, \phi_{J/\psi}$: Dihedral angles between the production plane and the decay planes of the ω and J/ψ .



Maximum Likelihood Fit Framework

- Extended likelihood:

$$\mathcal{L} \propto \prod_{i=1}^N [N_{\text{sig}} \mathcal{P}_{\text{sig}}(\xi_i; \alpha) + N_{\text{bkg}} \mathcal{P}_{\text{bkg}}(\xi_i)]$$

- Signal PDF with MC normalization:

$$\mathcal{P}_{\text{sig}}(\xi; \alpha) = \frac{|\mathcal{M}(\xi; \alpha)|^2}{\frac{1}{N_{\text{MC}}^{\text{gen}}} \sum_{j=1}^{N_{\text{MC}}^{\text{acc}}} |\mathcal{M}(\xi_j^{\text{MC}}; \alpha)|^2}$$

- Normalization via PHSP Monte Carlo ($20 \sim 100 N_{\text{sig}}$).
- Background PDF: modeled using sideband events (will be added in the future).
- Minimization using MINUIT.

Helicity Amplitude Formalism

- The transition amplitude for $X(3915) \rightarrow \omega J/\psi$ in the helicity frame:

$$\mathcal{M}_{M,\lambda_1,\lambda_2}(\theta, \phi) = \mathcal{A}_{\lambda_1,\lambda_2} D_{M,\lambda}^{J*}(\phi, \theta, 0)$$

where $\lambda = \lambda_1 - \lambda_2$.

- **Initial state restrictions** for $\gamma\gamma \rightarrow X(3915)$:
 - $J = 0 \implies M = 0$
 - $J = 2 \implies M = \pm 2$ (and $M = 0$ in general)
- $\lambda_{1(2)}$: Helicity of ω (J/ψ) meson.
- $D_{M,\lambda}^J$: Wigner D-functions representing rotation to the helicity frame.

Full Transition Amplitude

- The full amplitude is constructed by a **coherent sum** over intermediate helicity states:

$$\mathcal{A}(M, \Delta\lambda_\ell; \xi) = \sum_{\lambda_1, \lambda_2} \mathcal{M}_{M, \lambda_1, \lambda_2}(\Omega) \cdot \mathcal{T}_\omega(\Omega_\omega, \lambda_1) \cdot \mathcal{T}_{J/\psi}(\Omega_{J/\psi}, \lambda_2, \Delta\lambda_\ell)$$

- **Sub-decay amplitudes:**

- $J/\psi \rightarrow \ell^+ \ell^-$: Proportional to $D_{\lambda_2, \Delta\lambda_\ell}^{1*}(\Omega_{J/\psi})$.
 - $\omega \rightarrow \pi^+ \pi^- \pi^0$: Described by the decay-plane normal \vec{n} , proportional to $D_{\lambda_1, 0}^{1*}(\Omega_\omega)$.
- $\Delta\lambda_\ell = \lambda_{\ell^+} - \lambda_{\ell^-}$: Helicity difference of final state leptons.

Total Observable Intensity

- To obtain the probability density, an **incoherent sum** is performed over unobserved quantum numbers:

$$|\mathcal{M}(\xi)|^2 \propto \sum_{M \in \{0, \pm 2\}} \rho_{MM} \sum_{\Delta\lambda_\ell \in \{\pm 1\}} |\mathcal{A}(M, \Delta\lambda_\ell; \xi)|^2$$

- ρ_{MM} : Diagonal elements of the spin density matrix for $X(3915)$ produced via two-photon fusion.
- This intensity $|\mathcal{M}(\xi)|^2$ is used directly in the maximum likelihood fit mentioned before.

From Helicity to L - S Coupling

- Helicity amplitudes $\mathcal{A}_{\lambda_1, \lambda_2}$ describe the kinematics but mask the underlying dynamics.
- L - S coupling reveals the Partial Wave composition (S, P, D waves...).

$$\mathcal{A}_{\lambda_1, \lambda_2} = \sum_{L, S} \underbrace{G_{L, S}}_{\text{Dynamics}} \times \underbrace{C(L, S, J; \lambda_1, \lambda_2)}_{\text{Clebsch-Gordan Coefficients}}$$

- For 0^+ hypothesis: S_0, D_2
- For 2^+ hypothesis: S_2, D_0, D_1, D_2, G_2

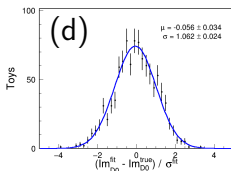
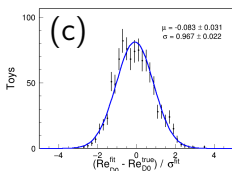
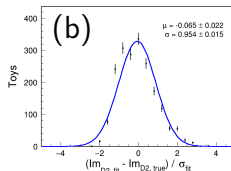
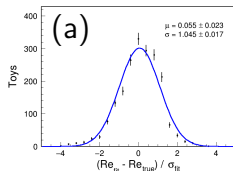
Toy MC Generation Settings

- Non-zero values are arbitrarily chosen for tests.
- G_2 is fixed to 0 as it's physically suppressed.

Hypothesis	Parameter	Input Value
0^+	S_0 (Re, Im)	1.0, 0.0 (Fixed)
	D_2 (Re, Im)	1.5, 1.8
2^+	S_2 (Re, Im)	1.00, 0.00 (Fixed)
	D_0 (Re, Im)	1.2, -1.6
	D_1 (Re, Im)	1.5, -1.9
	D_2 (Re, Im)	1.8, 2.2
	G_2 (Re, Im)	0.0, 0.0 (Fixed)
	ρ_{00}	0.5

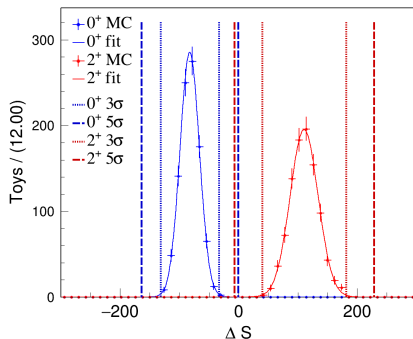
Pull Distributions

- 1000 Toy MC samples generated to validate fit stability
- 500 events with poisson fluctuation in each Toy MC samples.
- (a) and (b) from D_2 in 0^+ MC, (c) and (d) from D_0 in 2^+ MC.



Statistical Significance for J^{PC} Assignment

- **Ensembles:** 1000 Toy MC samples generated for both 0^+ and 2^+ hypotheses.
- **Method:** Each sample is cross-fitted under both assignments to construct the test statistic distributions:
$$\Delta S = -2 \ln(\mathcal{L}_{0^+} / \mathcal{L}_{2^+}).$$



Summary and Outlook

■ Summary:

- Mass fit preliminary validated for both single- and double-resonance hypotheses with signal Toy MC.
- Amplitude analysis framework preliminary established and validated with signal Toy MC.

■ Next steps:

- Validate the framework with Belle MC samples.
- Incorporate background modeling using 2D sideband events.
- Assess systematic uncertainties for the amplitude analysis.

Thanks!