Chiral Symmetry Breaking and Restoration with mixing between quarkonium and tetraquark

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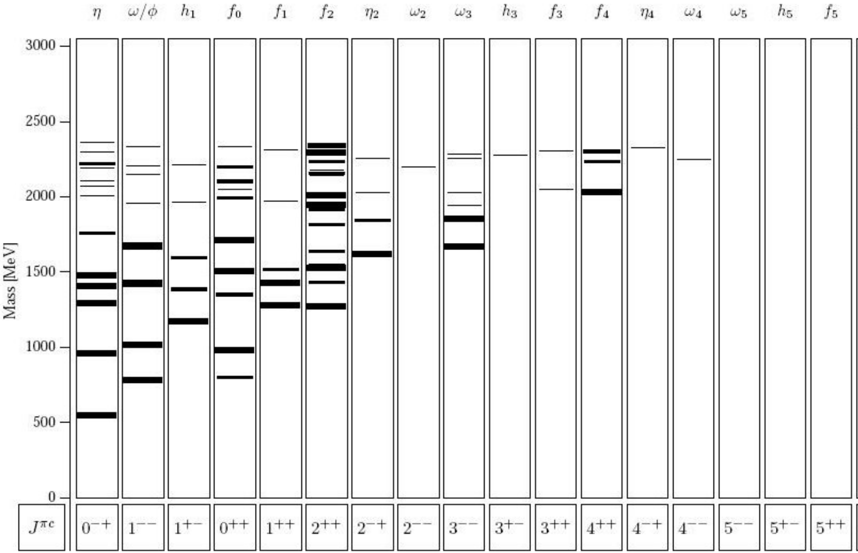
- Motivation
- Model set up
- Result
- Summary & Outlook

Scalar Mesons



Unusual Spectroscopy

Vector Mesons:

I = 1:	m[$\rho(776)$] $\approx 776 MeV$	$n\bar{n}$
I = 0:	m[$\omega(783)$] $\approx 783 MeV$	$n\bar{n}$
I = $\frac{1}{2}$:	m[$K^\star(892)$] $\approx 892 MeV$	$n\bar{s}$
I = 0:	m[$\phi(1020)$] $\approx 1020 MeV$	$s\bar{s}$

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Scalar Mesons:

I = 0:	m[$f_0(600)$] $\approx 500 MeV$	$\sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$
I = $\frac{1}{2}$:	m[κ] $\approx 800 MeV$	$\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d$
I = 0:	m[$f_0(980)$] $\approx 980 MeV$	$\bar{s}s$
I = 1:	m[$f_0(980)$] $\approx 980 MeV$	$\bar{u}d, \bar{d}u, \sqrt{\frac{1}{2}}(\bar{u}u - \bar{d}d)$

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Scalar Mesons:

I = 0: m[$f_0(600)$] $\approx 500 MeV$ $\sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$

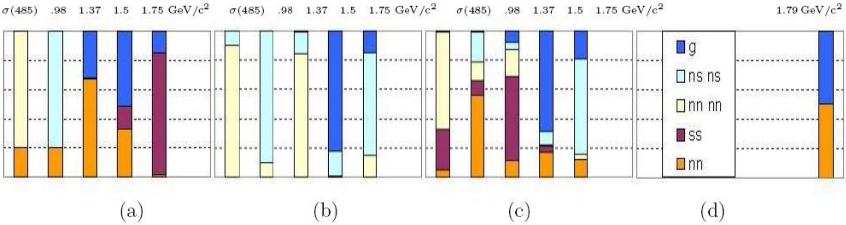
I = $\frac{1}{2}$: m[κ] $\approx 800 MeV$ $\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d$

I = 0: m[$f_0(980)$] $\approx 980 MeV$ $\bar{s}s$

I = 1: m[$f_0(980)$] $\approx 980 MeV$ $\bar{u}d, \bar{d}u, \sqrt{\frac{1}{2}}(\bar{u}u - \bar{d}d)$

Light Scalars are tetraquark state: Jaffe (Phys. Rev. D 15 (1977))
The States above consecutively can be represented as:
$nn\bar{n}\bar{n}, nn\bar{n}\bar{s}, ns\bar{n}\bar{s}, ns\bar{n}\bar{s}$

Model Prediction

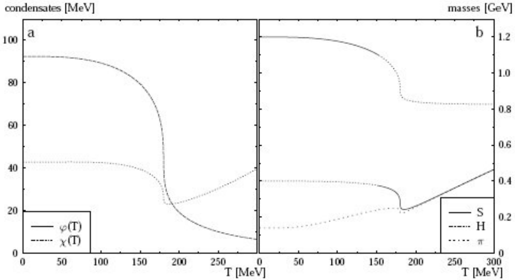


Decomposition of scalar isoscalar states into different components.

see.: E. Klempt, A. Zaitsev, Phys. Rept. 454:1-202,2007.

Effect of Mixing

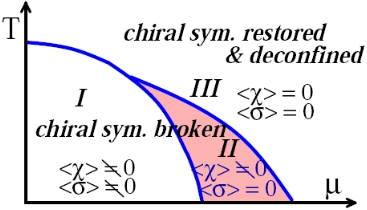
$$V = \frac{\lambda}{4}(\phi^2 + \vec{\pi}^2 - F^2)^2 - \epsilon\,\phi + \frac{1}{2}\,M_\chi^2\,\chi^2 - g\,\chi\,(\phi^2 + \vec{\pi}^2)$$



- $f_0(600)$ is tetraquark dominated meson while $f_0(1370)$ is quarkonium dominated. Near T_c their role interchanges.
- $T > T_c$: ϕ approaches to zero but χ tends to rise. A. Heinz et. al., Phys.Rev. D79 037502

Alternate Symmetry breaking

$$V = A\sigma^2 + B\chi^2 + \sigma^4 + \chi^4 - \sigma^2\chi + D\chi^3$$



- Alternative breaking of chiral symmetry in dense matter. $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V \times (Z_{N_f})_A \to SU(N_f)_V$. M. Harada et al. arXiv:0908.1361

Formalism: Model

- Effective fields: a 2×2 matrix field Φ which denotes the bare quarkonia field and a 2×2 matrix field Φ' which denotes the bare tetraquark field:
  $\Phi = \frac{1}{2}(\sigma_b + \eta_b) + \frac{1}{2}(\vec{\alpha_b} + i\vec{\pi_b}).\vec{\tau} \quad \Phi' = \frac{1}{2}(\sigma'_b + \eta'_b) + \frac{1}{2}(\vec{\alpha'_b} + i\vec{\pi'_b}).\vec{\tau}$
- Transformation properties under $U(2)_L \times U(2)_R$:
  $$\Phi \rightarrow U_L \Phi U^{\dagger}{}_R, \quad \Phi' \rightarrow U_L \Phi' U^{\dagger}{}_R \ ,$$
  where $U_{L,R}$ are group elements of the $U(2)_L \times U(2)_R$ symmetry.
- The thermodynamic potential in our model ($N_f = 2$) has two parts:
  $\Omega = -2TN_f N_c \int \frac{d^3q}{(2\pi)^3}[\log(1 + e^{-(E_q-\mu)/T}) + \log(1 + e^{-(E_q+\mu)/T})]$
  $+U(\sigma, \chi)$; Where, $E_q = \sqrt{q^2 + m^2}$ and the constituent quark mass: $m = g_3\sigma + g_4\chi$. σ and χ being the vacuum expectation values of the quarkonia and tetraquark effective fields srespectively.

Formalism: Model

- Mesonic part of the thermodynamic potential is calculated from the effective Lagrangian:

$$\mathcal{L}_{\mathrm{m}} = \mathrm{Tr}(\partial_\mu \Phi \partial^\mu \Phi^\dagger) + \mathrm{Tr}(\partial_\mu \Phi' \partial^\mu \Phi^{\dagger\prime}) - {m_\Phi}^2 \mathrm{Tr}(\Phi^\dagger \Phi) - {m_{\Phi'}}^2 \mathrm{Tr}(\Phi^{\dagger\prime} \Phi')$$
$$+ \frac{\lambda_1}{2} \mathrm{Tr}(\Phi^\dagger \Phi \Phi^\dagger \Phi) + \frac{\lambda_2}{2} \mathrm{Tr}(\Phi^{\dagger\prime} \Phi' \Phi^{\dagger\prime} \Phi') + g_2 \mathrm{Tr}(\Phi' \Phi' \Phi')$$
$$- g_1 \mathrm{Tr}(\Phi') \mathrm{Tr}(\Phi) \mathrm{Tr}(\Phi) + k[\mathrm{Det}(\Phi) + h.c.] - h[\mathrm{Tr}(\Phi) + h.c.]$$

- Mean field mesonic potential: $U(\sigma,\chi) = -\frac{1}{2}{m_\Phi}^2\sigma^2 - \frac{1}{2}{m_{\Phi'}}^2\chi^2 + \frac{1}{16}\lambda_1\sigma^4 + \frac{1}{16}\lambda_2\chi^4 + \frac{1}{4}g_2\chi^3 - g_1\sigma^2\chi + \frac{1}{2}k\sigma^2 - 2h\sigma$
- The mesonic spectrum consist of sixteen physical mesons: scalar isoscalar $\{f_0(600), f_0(1370)\}$, pseudoscalar isoscalar $\{\eta_p, \eta_p'\}$, scalar isovector $\{\vec{\alpha_p}, \vec{\alpha_p}'\}$ and pseudoscalar isovector $\{\vec{\pi_p}, \vec{\pi_p}'\}$. Here, η_p and η_p' are composed of u and d quarks only.

Bare mass matrix

For f_0 mesons:

$$(M_{f_0}^2) = \begin{bmatrix} \frac{1}{2}\lambda_1 \sigma^2 + 2\frac{h}{\sigma} & -2 g_1 \sigma \\ -2 g_1 \sigma & \frac{1}{2}\lambda_2 \chi^2 + \frac{3}{4} g_2 \chi + g_1 \frac{\sigma^2}{\chi} \end{bmatrix} \quad (1)$$

For pions we have:

$$(M_{\pi}^2) = \begin{bmatrix} 2 g_1 \chi + 2\frac{h}{\sigma} & 0 \\ 0 & g_1 \frac{\sigma^2}{\chi} - \frac{9}{4} g_2 \chi \end{bmatrix} \quad (2)$$

For η we have,

$$(M_{\eta}^2) = \begin{bmatrix} 4 g_1 \chi - 2k + 2\frac{h}{\sigma} & 2 g_1 \sigma \\ 2 g_1 \sigma & -\frac{9}{4} g_2 \chi + g_1 \frac{\sigma^2}{\chi} \end{bmatrix} \quad (3)$$

Parameter fixing & Results

- Vacuum stability condition: $\frac{\partial U(\sigma,\chi)}{\partial \sigma} = 0, \ \frac{\partial U(\sigma,\chi)}{\partial \chi} = 0$
- Physical input mass: $(R^{-1})M^2_{bare}(R) = M^2_{phys}$
- Pion decay constant: $f_\pi = \sigma$
- Input value for constituent quark mass in vacuum.

- Case I ($\lambda_2, g_2, k, h = 0$): **Scenario 1:** $f_0(600)$ is a quarkonium dominated meson, whereas the heavier one $f_0(1370)$ is tetraquark dominated. For **Scenario 2:** $f_0(1370)$ is quarkonia dominated and $f_0(600)$ is tetraquark dominated

Mesons	$m_{f_0(600)}$ (GeV)	$m_{f_0(1370)}$ (GeV)	m_{π_p} (GeV)	$m_{\pi'_p}$ (GeV)
Scenario 1	0.8	1.5	0.14	1.3
Scenario 2	0.8	1.5	0.14	1.1

Figure: Values of physical meson masses used in Case 1.

Results

Parameters	σ (GeV)	χ (GeV)	${m_\Phi}^2$ (GeV^2)	${m_{\Phi'}}^2$ (GeV^2)	λ_1	g_1 (GeV)
Scenario 1	0.0924	0.021	0.426	-1.69	281.1	4.15
Scenario 2	0.0924	0.029	0.595	-1.21	393.55	4.17

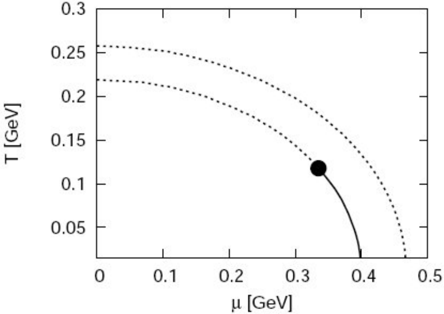


Figure: Phase diagram for case 1. The dotted line represents second order phase transition and the solid line stands for first order phase transition. The **upper phase boundary** line corresponds to **scenario 2** and the **lower one** for **sceanrio 1**

Results Contd..

Fields	$m_{f_0(600)}$	$m_{f_0(1370)}$	m_{π_p}	$m_{\pi'_p}$	m_{η_p}	$m_{\eta'_p}$
Mass (GeV)	0.6	1.35	0.14	1.29	0.55	1.3

Figure: Values of physical meson masses used in Case II, III and IV

- Case II, III and IV correspond to the values of the coupling constant (cubic self interaction term for the tetraquark field) g_2 being $g_2 = 0$, $g_2 > 0$ and $g_2 < 0$ respectively.
- In cases II, III and IV we find the lowest scalar ($f_0(600)$) is quarkonia dominated and the heavier counterpart $f_0(1370)$ is tetraquark dominated.
- behaviour of the order parameters σ and χ with the variation of temperature and chemical potential are qualtiatively same for case II and III.

Results Contd..

Parameters	σ (GeV)	χ (GeV)	$m_{\Phi}{}^2$ (GeV^2)	$m_{\Phi'}{}^2$ (GeV^2)	λ_1	λ_2	g_1 (GeV)	h (GeV^3)	k (GeV^2)
Value	0.0924	0.00523	0.019	-1.6	87.99	9103.07	1.02	0.00042	-0.149

Figure: Output parameters for Case II

Parameters	σ (GeV)	χ (GeV)	$m_{\Phi}{}^2$ (GeV^2)	$m_{\Phi'}{}^2$ (GeV^2)	λ_1	λ_2	g_1 (GeV)	g_2 (GeV)	h (GeV^3)	k (GeV^2)
Value	0.0924	0.00513	0.019	-1.63	87.93	7527.9	1.016	2.25	0.00042	-0.149

Figure: Output parameters for Case III

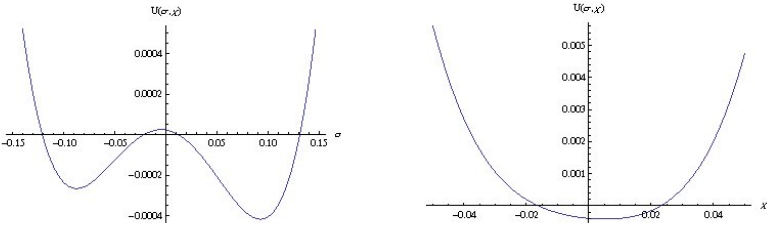


Figure: Nature of the mesonic potential $U(\sigma, \chi)$in vacuum. Parameters are for case II

Results Contd..

Parameters	σ (GeV)	χ (GeV)	$m_\Phi{}^2$ (GeV^2)	$m_{\Phi'}{}^2$ (GeV^2)	λ_1	λ_2	g_1 (GeV)	h (GeV^3)	k (GeV^2)
Value	0.0924	0.00523	0.019	-1.6	87.99	9103.07	1.02	0.00042	-0.149

Parameters	σ (GeV)	χ (GeV)	$m_\Phi{}^2$ (GeV^2)	$m_{\Phi'}{}^2$ (GeV^2)	λ_1	λ_2	g_1 (GeV)	g_2 (GeV)	h (GeV^3)	k (GeV^2)
Value	0.0924	0.00924	0.027	-0.63	89.88	25785.1	1.02	-34.88	0.000038	-0.145

Figure: Output parameters for Case II (first table) & IV (second table)

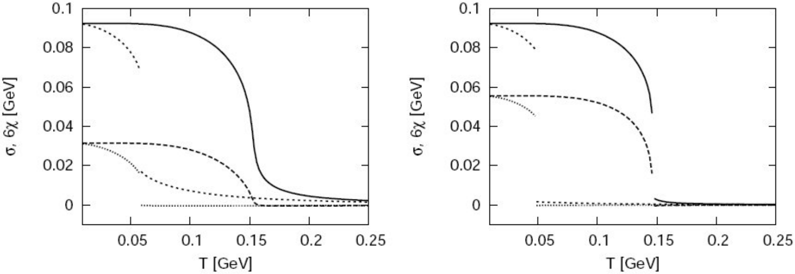


Figure: Left panel for Case II & the right one for Case IV. The solid ($\mu = 0$ GeV) and short dash ($\mu = 0.27$ GeV) lines are for variation of σ. Variation of χ is represented by long dash ($\mu = 0$ GeV) and points ($\mu = 0.27$ GeV) for both figs.

Results Contd..

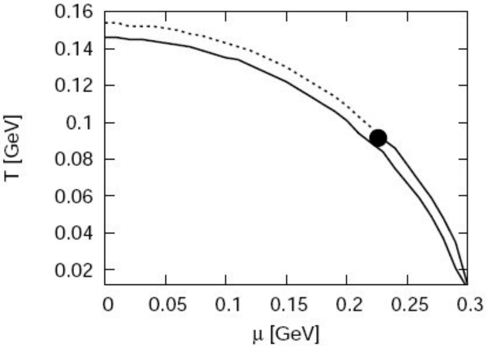


Figure: Phase diagram for case III. Solid line indicates first order phase transition and the dashed line is for crossover transition. The **upper phase boundary** is for $g_2 = 2.25$ and the **lower one** is for $g_2 = -34.88$. The bold **circle** indicates location of the **CEP**

- Two flavoured quark-meson model is used to study the effect of mixing between quarkonium and tetraquark fields on chiral phase transition.
- For cubic self interaction coupling constant (tetraquark) $g_2 = 0$ and positive the phase structure resembles the conventional one.
- with a strong and negative g_2 makes not only the transition of χ 1^{st} order but the transition for σ as well becomes 1^{st} order. No CEP! in the phase diagram.
- The strong and negative g_2 also makes the chiral phase transition temperature lower than that for the case of $g_2 = 0$ or positive.
- For all the various scenarios considered in our study: $T_c(\sigma) = T_c(\chi)$ for all values of the chemical potential.
- Quantum and thermal fluctions of the mesons has been neglected. Ultraviolet divergent vacuum contribution to the thermodnamic potential has also not been considered. Systematic study on these aspects required to have a better understanding.

THANK YOU!