Regge trajectory of the $f_0(500)$ resonance from a dispersive connection to its pole

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Motivation

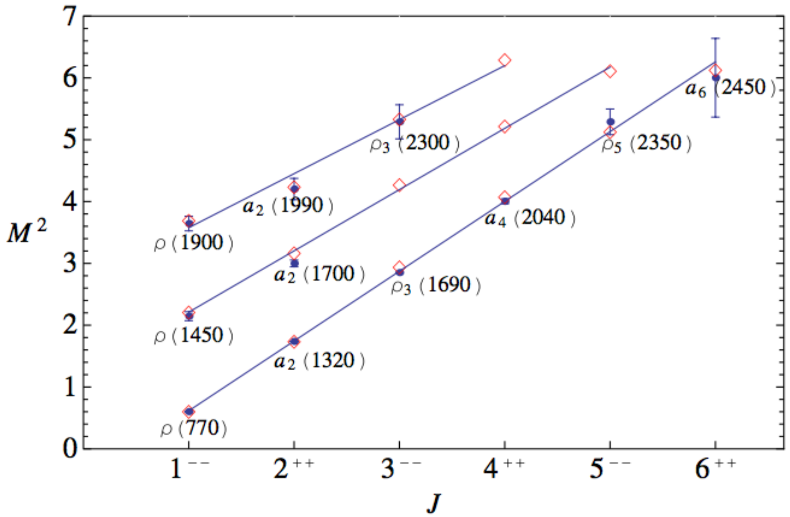
- In S-matrix theory, unitarity in the *s* channel is the key to determine the properties of resonances and bound states
- Singularities in the complex *J*-plane (Regge poles) reflect the important contributions of the crossed channels on the direct channel -> contain in principle the most complete description of resonance parameters

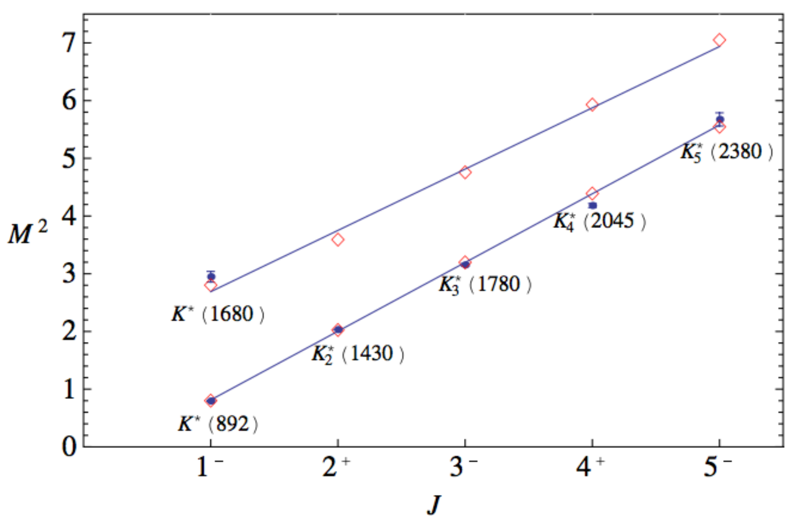
In this work we parametrize the Regge poles corresponding to the ρ and σ resonances and fix the parameters by fitting to the experimental data on the physical poles.

Regge trajectories

- **Experimental observation**

  Take particles with the same quantum numbers and signature ($\tau=(-1)^J$) and plot (spin) vs. (mass)^2





Particles can be classified in **linear trajectories** with a universal slope

Regge Theory

The Regge trajectories can be understood from the analytic extension to the complex angular momentum plane (Regge Theory)

However, light scalars, particularly the $f_0(500)$, do not fit in

> are doubled due to two flavor components, $n\bar{n}$ and $s\bar{s}$. We do not put the enigmatic σ meson [11–14] on the $q\bar{q}$ trajectory supposing σ is alien to this classification. The broad state

Anisovich-Anisovich-Sarantsev-Phys.Rev.D62.051502

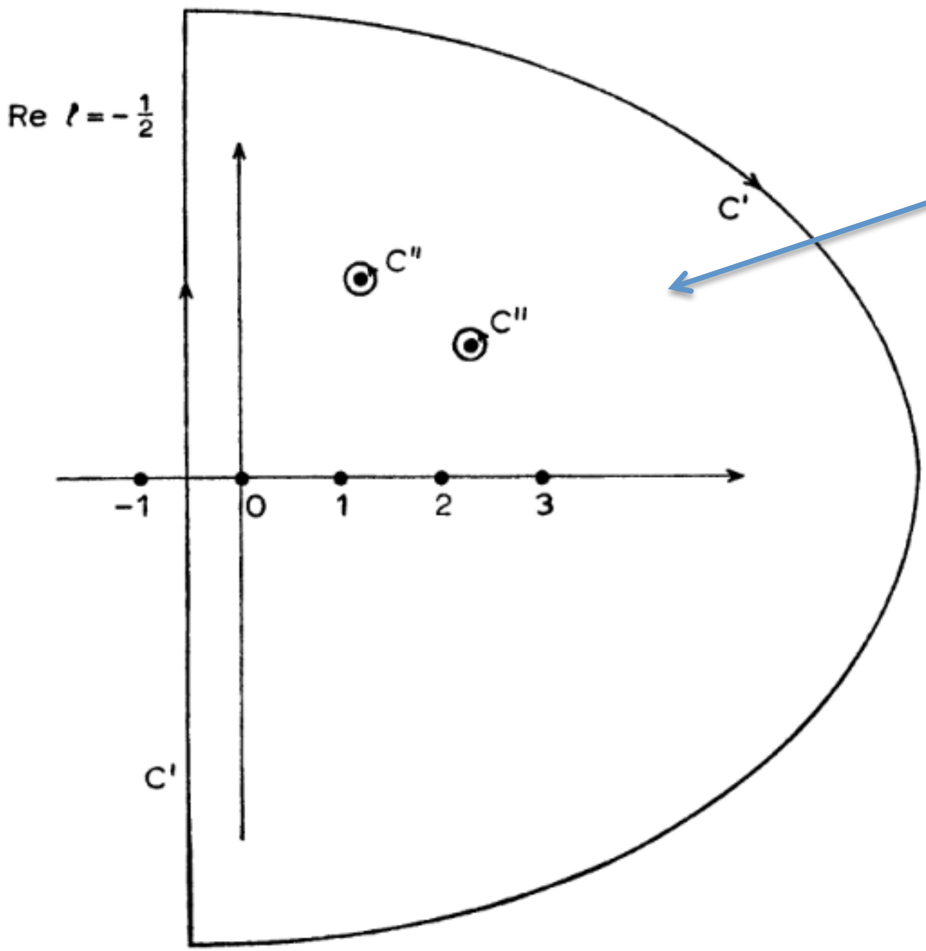
Regge Theory

- **The concept of partial wave can be expanded to complex values of *J***, which will be valid in the entire *t*-plane

  Procedure: Sommerfeld-Watson transform

$$T(s,t) = \sum_{J=0}^{\infty} (2\,J+1) f_J(s)\, P_J(z) \quad \longrightarrow \quad T(s,t) = -\frac{1}{2\,i} \int_C \frac{(2\,J+1) f(J,s)\, P_J(-z)}{\sin \pi\, J} \mathrm{d}J$$

Regge Theory

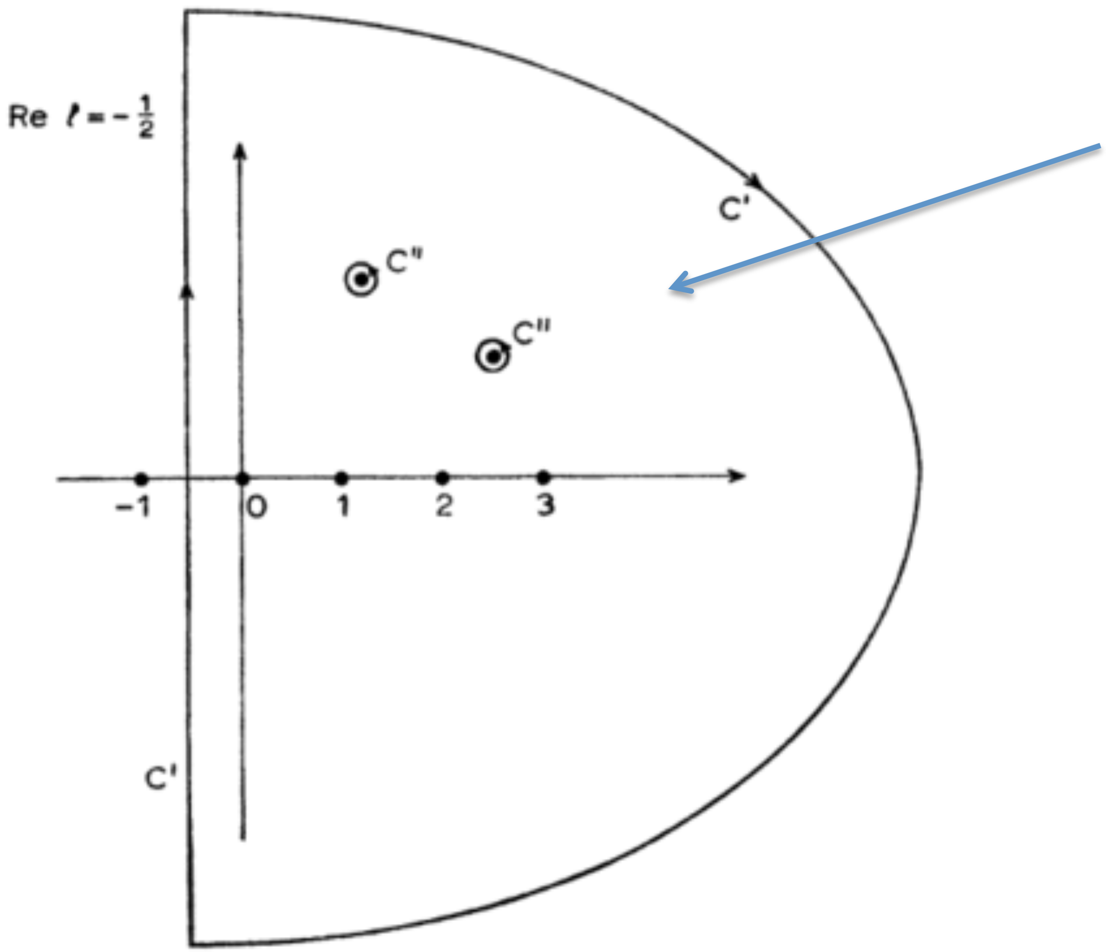


Regge poles

Position $\alpha^{\pm}(s)$

Residue $\beta^{\pm}(s)$

Regge Theory



Regge poles

Position $\alpha^{\pm}(s)$

Residue $\beta^{\pm}(s)$

Regge Trajectory

Regge Theory

- **Relevance of Regge poles in the *s*-channel**

Contribution of a single Regge pole to a physical partial wave amplitude

$$f(J,s) = \hat{f} + \frac{\beta(s)}{J - \alpha(s)}$$

regular function

analytic functions
α: right hand cut $s>4m^2$
β: real

Parametrization of the amplitudes

- Unitarity condition on the real axis implies

$$\text{Im}\ \alpha(s) = \rho(s)\beta(s)$$

- Further properties of β*(s)*

threshold behavior

$$\hat{s} = \frac{s - 4m^2}{\tilde{s}}$$

$$\beta(s) = \frac{\hat{s}^{\alpha(s)}}{\Gamma(\alpha(s) + \frac{3}{2})}\gamma(s)$$

suspress poles
of full amplitude

$$(2\alpha + 1)P_\alpha(z_s) \sim \Gamma(\alpha + \frac{3}{2})$$

analytical function:
β*(s)* real on real axis
⇒ phase of *Υ(s)* known
⇒ Omnès-type disp. relation

S. -Y. Chu, G. Epstein, P. Kaus, R. C. Slansky and F. Zachariasen, Phys. Rev. 175, 2098 (1968).

Parametrization of the amplitudes

- **Twice-subtracted dispersion relations**

$$\alpha(s) = A + B(s - s_0) + \frac{(s - s_0)^2}{\pi} \int_{\mathrm{thr.}}^{\infty} \frac{\mathrm{Im}\alpha(s')ds'}{(s' - s)(s' - s_0)^2}$$

$$\gamma(s) = g^2 \exp\left\{ C(s - s_0) + \frac{(s - s_0)^2}{\pi} \int_{\mathrm{thr.}}^{\infty} \frac{\phi_\gamma(s')}{(s' - s)(s' - s_0)} ds' \right\}$$

with $$\beta(s) = \frac{\hat{s}^{\alpha(s)}}{\Gamma(\alpha(s) + \frac{3}{2})} \gamma(s) = \frac{\mathrm{Im}\alpha(s)}{\rho(s)}$$

Parametrization of the amplitudes

System of integral equations:

$$\mathrm{Re}\alpha(s) = \alpha_0 + \alpha' s + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\alpha(s')}{s'(s'-s)},$$

$$\mathrm{Im}\alpha(s) = \rho(s) b_0 \frac{\hat{s}^{\alpha_0+\alpha' s}}{|\Gamma(\alpha(s)+\frac{3}{2})|} \exp\Big(-\alpha' s[1-\log(\alpha'\tilde{s})] + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\alpha(s')\log\frac{\hat{s}}{\hat{s}'} + \arg\Gamma\left(\alpha(s')+\frac{3}{2}\right)}{s'(s'-s)}\Big)$$

In the scalar case a slight modification is introduced (Adler zero)

Determination of the parameters

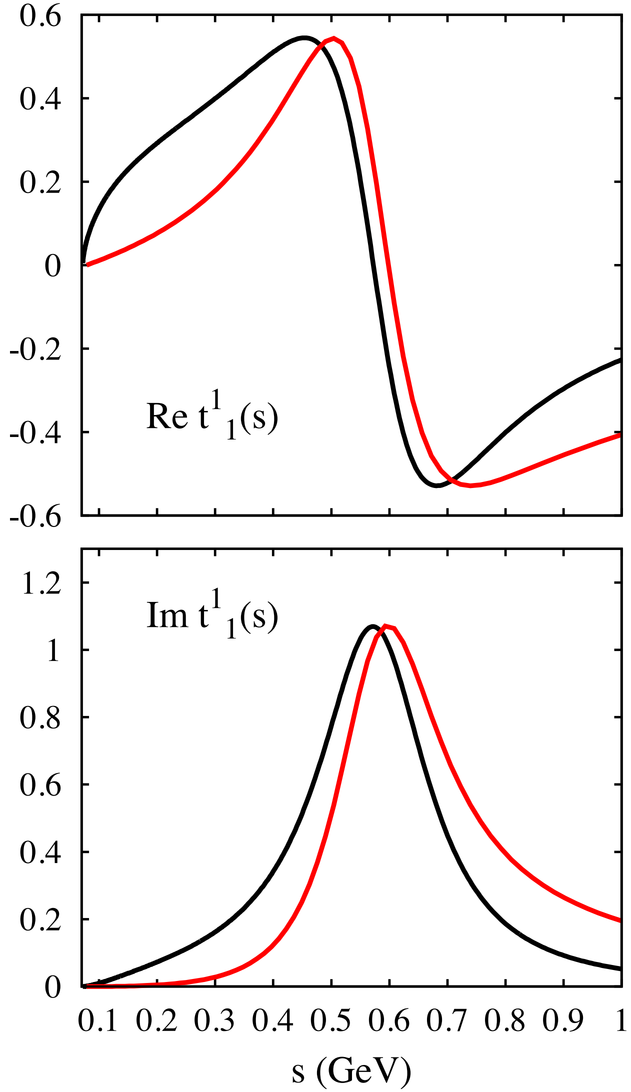
- for a given set of α_0, α' and b_0:
  - solve the coupled equations
  - get α(s) and β(s) in real axis
  - extend to complex *s*-plane
  - obtain pole position and residue

$$f^{II}(J,s) = \hat{f} + \frac{\beta(s)}{J - \alpha^{II}(s)}$$

- fit α_0, α' and b_0 so that **pole position and residue** coincide with those given by a **dispersive analysis of scattering data**

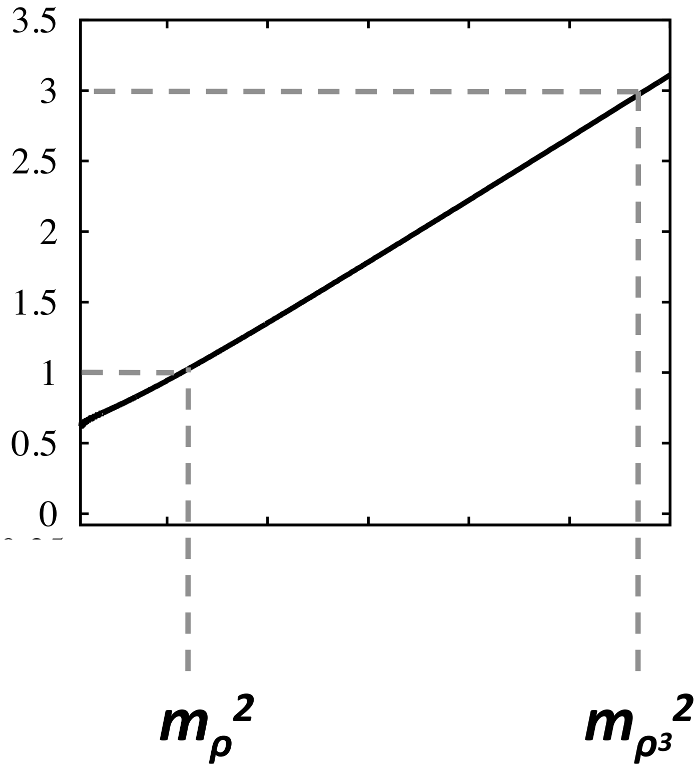
Garcia-Martin, Kaminski, Pelaez and Ruiz de Elvira, Phys. Rev. Lett. **107**, 072001 (2011)

Results: *ρ case (I = 1, J = 1)*



We recover a fair representation of the amplitude, in good agreement with the GKPY amplitude

Results: *ρ case (I = 1, J = 1)*



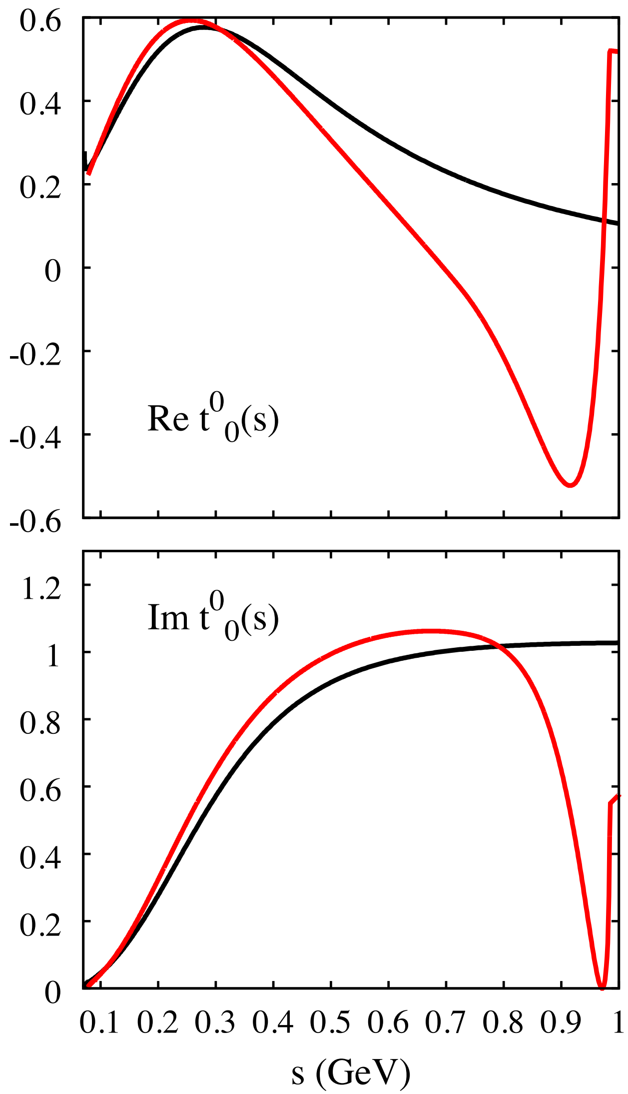
But we also get a prediction for the *ρ* Regge trajectory, which is:

- $\alpha(s)$ almost real
- almost linear $\alpha(s) \sim \alpha_0 + \alpha' s$
- intercept $\alpha_0 = 0.52$
- slope $\alpha' = 0.913\ \text{GeV}^{-2}$

Remarkably consistent with the literature, taking into account our approximations

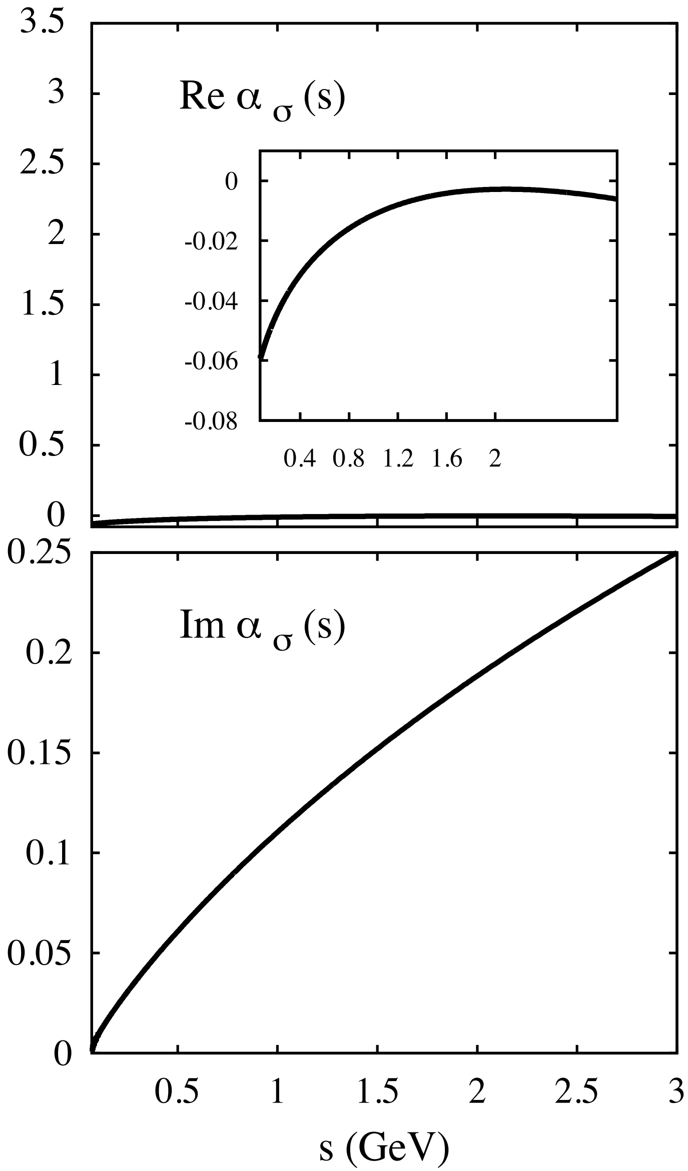
m_ρ -> 0.756 GeV; $m_{\rho 3}$ -> 1.697 GeV, PDG: *ρ_3(1690)*

Results: *σ case (I = 0, J = 0)*



Even better agreement with the parameterized GKPY amplitude

Results: *σ case (I = 0, J = 0)*



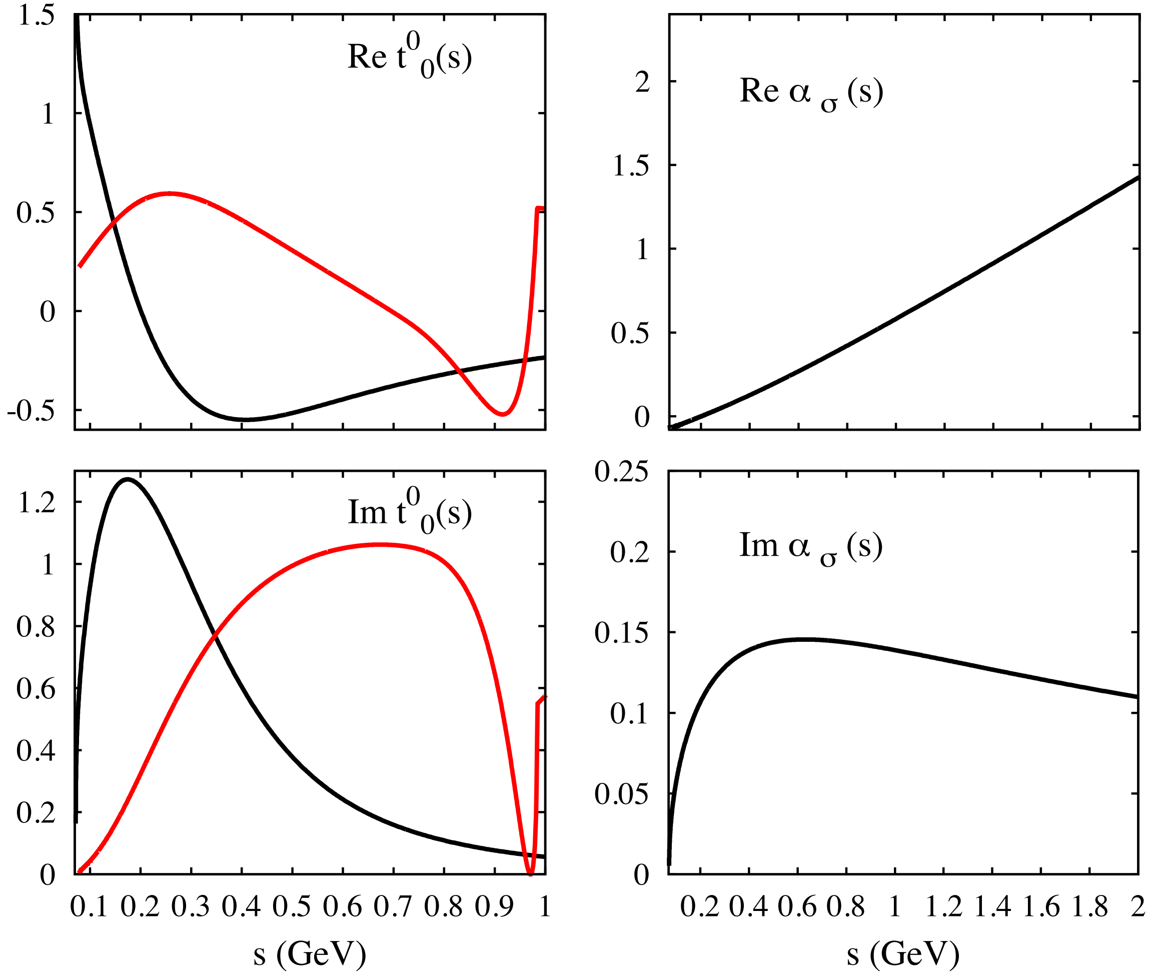
Prediction for the *σ* Regge trajectory, which is:

- NOT real
- NOT linear
- intercept α_0= -0.087
- slope α' = 0.002 GeV^{-2}

Two orders of magnitude flatter than other hadrons
The sigma does NOT fit the usual classification

Results: *σ case (I = 0, J = 0)*

If we fix the *α'* (~ slope in the "normal" Regge trajectories) to a natural value (that of the *ρ* trajectory)

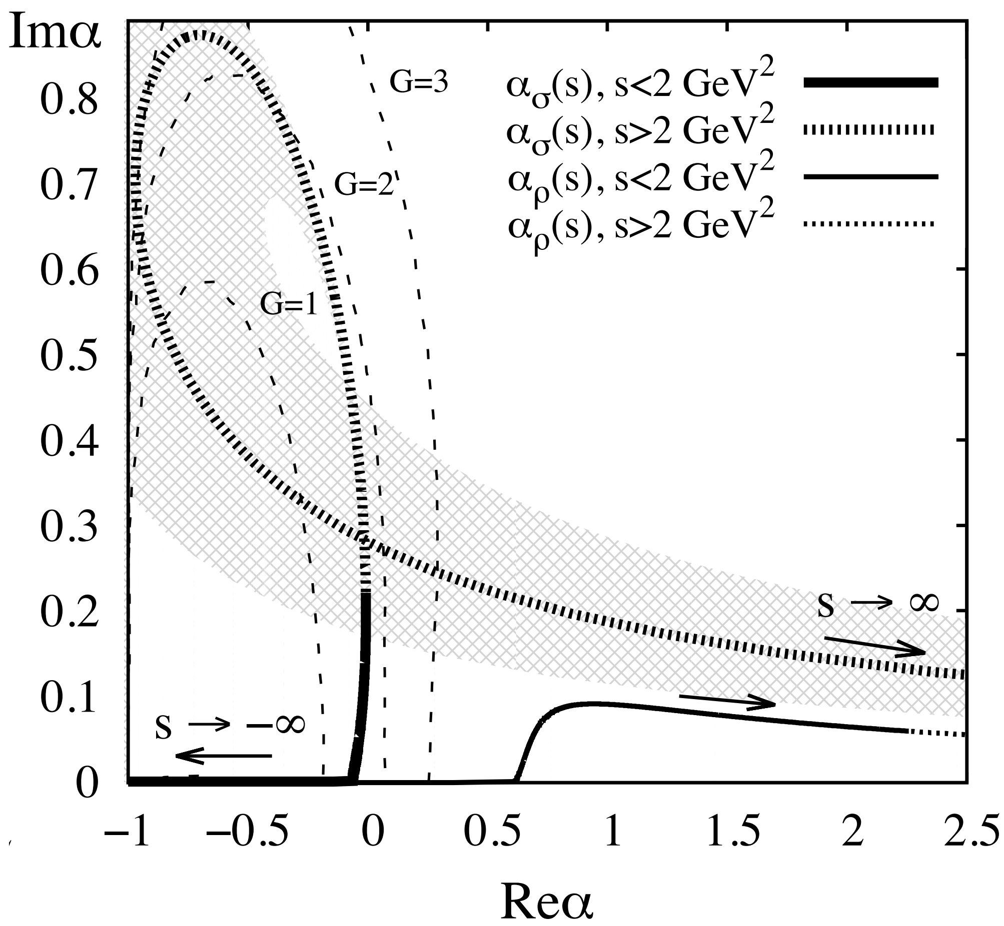


Summary

- We are studying the Regge trajectories that pass through the ρ and σ resonances
- **By fitting to the pole position and residue**, we get the parameters of the Regge parametrization (in particular, the slope of the Regge trajectory)
- ρ trajectory: parameters consistent with literature
- σ trajectory: slope of the trajectory **two orders of magnitude smaller than natural**
- If we force the σ trajectory to have a natural slope, the description of the pole parameters is ruined

Thank you!

Results: comparison to Yukawa potential

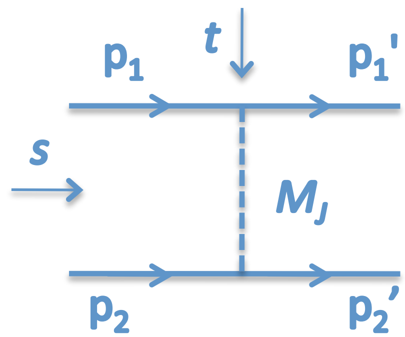


- **Large energy behavior of amplitudes**
  - Froissart bound (amplitude analiticity + unitarity)

$$|T(s,t=0)| \leq c\, s\,(\log s)^2, \qquad s \to \infty$$

  - t-channel exchange of a particle of mass *M* and angular momentum *J*

$$T(s,t\approx M_J^2) \approx \frac{g^2 P_J(\cos\theta_t)}{M_J^2 - t}$$



Since $\cos\theta_t = 1 + \dfrac{2s}{t-4m^2}$, at fixed *t* and large *s*

$$P_J(\cos\theta_t) \sim s^J \qquad \Rightarrow \qquad T(s,t\approx M_J^2) \sim s^J$$

To reconcile both behaviors:

$$\left.\begin{array}{l} |T(s,t=0)| \leq \mathrm{c}\, s\, (\log s)^2 \\ \\ T(s,t\approx M_J^2) \sim s^J \end{array}\right\} \quad T(s,t) \sim \beta(t) s^{\alpha(t)}$$

with

$\alpha(t<0) < 1$ (physical values of t)

$\alpha(t=M_J^2) = J$ the Regge trajectory!

High energy behavior interpreted as an interpolation in *J* between poles with different spin → justifies the continuation of the partial waves to complex values of *J*

Complex angular momentum

- **We want to extend the concept of partial wave to complex values of *J***, which will be valid in the entire *t*-plane

  Procedure: Sommerfeld-Watson transform

$$T(s,t) = \sum_{J=0}^{\infty} (2\,J+1) f_J(s)\, P_J(z) \quad \longrightarrow \quad T(s,t) = -\frac{1}{2\,i} \int_C \frac{(2\,J+1) f(J,s)\, P_J(-z)}{\sin \pi\, J} \mathrm{d}J$$

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$$T^{\pm}(s,t)=-\frac{1}{2i}\int_C\frac{(2J+1)f^{\pm}(J,s)\,P_J(-z)}{\sin\pi J}\mathrm{d}J$$

Next step is to deform the contour.

To be sure that it can be done, *f*(*J*,*s*) must have some analytic properties -> we must redefine it:

Froissart-Gribov projection

For J bigger than the number of subtractions that we need to make the integrals converge

$$f^{\pm}(J,s) = \frac{1}{\pi}\int_{z_0}^{\infty}\{D_t(s,z) \pm D_u(s,z)\}\, Q_J(z)\,\mathrm{d}z$$

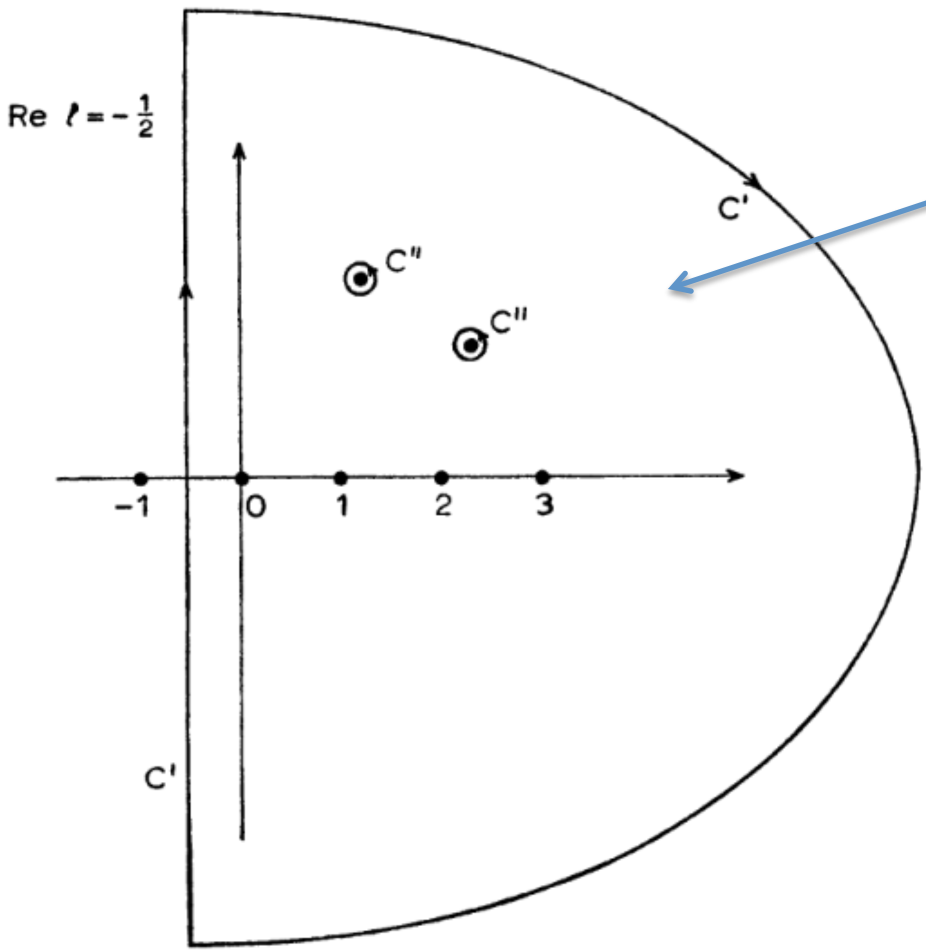
$$f_J(s) = f^{+}(J,s) \quad \text{for even } J$$

$$f_J(s) = f^{-}(J,s) \quad \text{for odd } J$$

Associated $T^{\pm}(s,z)$ such that

$$T(s,z) = \frac{1}{2}\left\{T^{+}(s,z) + T^{+}(s,-z) + T^{-}(s,z) - T^{-}(s,-z)\right\}$$

In non-relativistic scattering, Regge found that the only singularities of $f^{\pm}(J,s)$ in the region $Re\ J > -½$ are poles in the upper half J-plane:



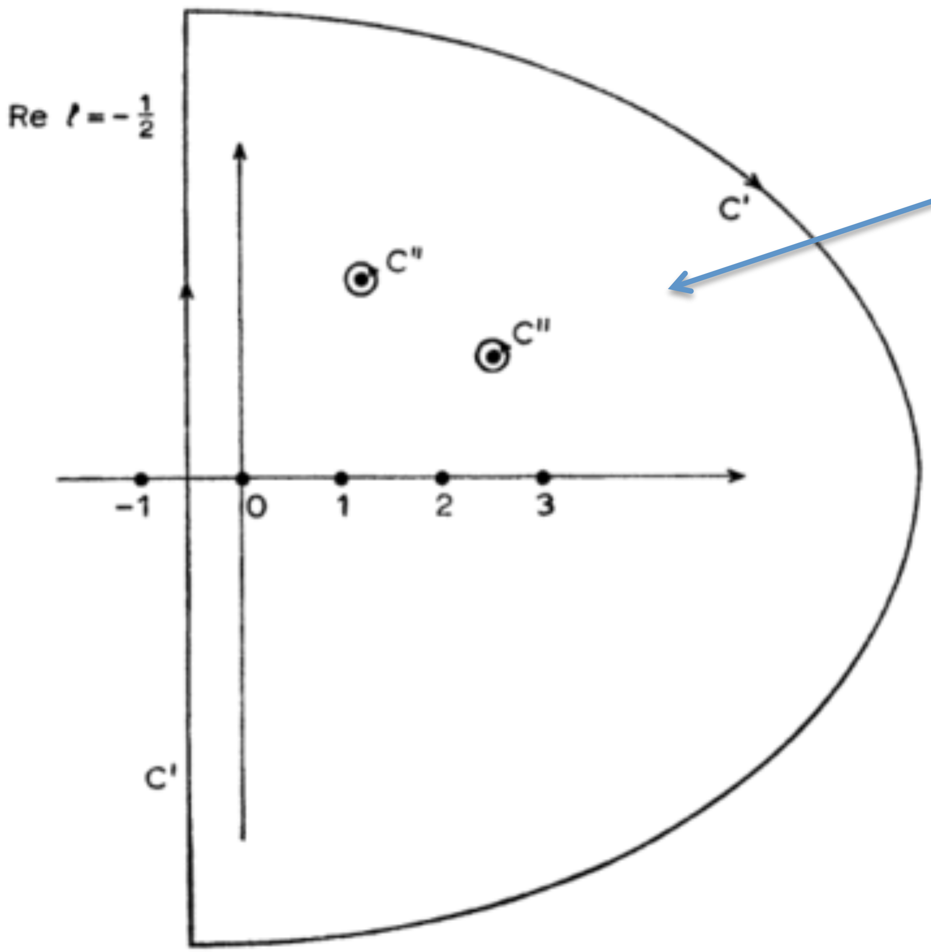
Regge poles

Position $\alpha^{\pm}(s)$

Residue $\beta^{\pm}(s)$

(In relativistic scattering, Mandelstam showed that there could be branch points, but we will ignore them)

In non-relativistic scattering, Regge found that the only singularities of $f^{\pm}(J,s)$ in the region $Re\ l > -½$ are poles in the upper half l-plane:



Regge poles

Position $\alpha^{\pm}(s)$

Residue $\beta^{\pm}(s)$

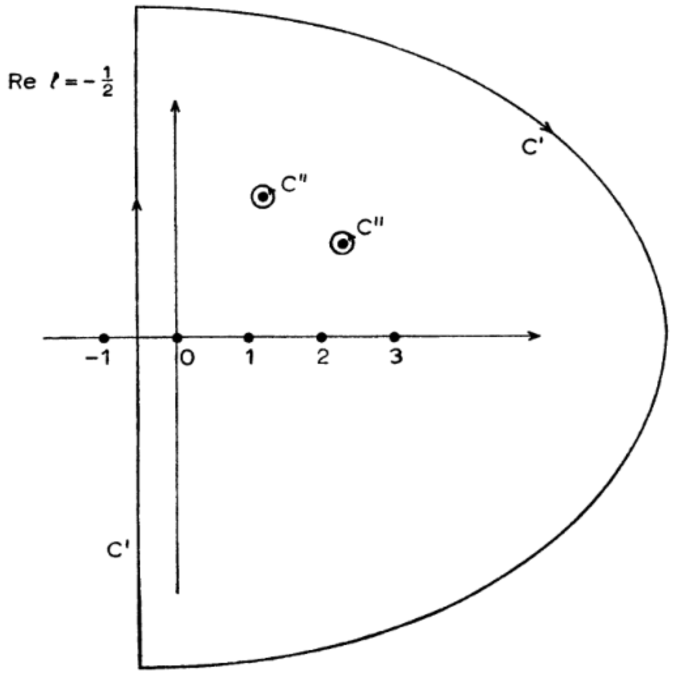
Regge Trajectory

(In relativistic scattering, Mandelstam showed that there could be branch points, but we will ignore them)

Integration on the contour →

$$T^{\pm}(s,z) = -\frac{1}{2i}\int_{C'} \frac{(2J+1)f^{\pm}(J,s)P_J(-z)}{\sin\pi J}\mathrm{d}J - \sum_i \frac{\pi(2\alpha_i^{\pm}(s)+1)\beta_i^{\pm}(s)P_{\alpha_i^{\pm}}(-z)}{\sin\pi\alpha_i^{\pm}(s)}$$

Background term



$$T^{\text{poles}}(s,t) =$$

$$-\sum_i \frac{\pi(2\alpha_i^{\pm}(s)+1)}{2\sin\pi\alpha_i^{\pm}}\beta_i^{\pm}(s)(P_{\alpha_i^{\pm}}(-z)\pm P_{\alpha_i^{\pm}}(z))$$

α⁺(s) only contribute to the amplitude when the trajectory passes through even integer values (and viceversa)

- **Relevance of Regge poles in the *s*-channel**

Contribution of a single Regge pole to a physical partial wave amplitude

$$f_l^{\text{pole}}(s) = \frac{1}{2}\int_{-1}^{1} P_l(z) T^{\text{pole}}(s,t)\,dz = -\frac{1}{2}(1 \pm (-1)^l)\beta(s)\frac{2\alpha(s)+1}{(\alpha(s)-l)(\alpha(s)+l+1)}$$

even signature poles only contribute to *even* pw amplitudes

Near the Regge pole:

$$f_l(s) = \hat{f}_l + \frac{\beta(s)}{l-\alpha(s)}$$

analytic functions

α: right hand cut *s>4m²*

β: real

regular function

- **Relevance of Regge poles in the *t*-channel**

Assymptotic behavior of $P_\alpha(z)$ when $z \to \infty$ $(t \to \infty)$

$$\sqrt{\pi}P_\alpha(z) \sim \frac{(\alpha-\frac{1}{2})!}{\alpha!}(2z)^\alpha + \frac{(-\alpha-\frac{3}{2})!}{(-\alpha-1)!}(2z)^{-\alpha-1}$$

Dominated by leading Regge pole (largest Re α)

$$T(s,t) \sim \phi(s)t^{\alpha(s)}$$

- **Relevance of Regge poles in the *s*-channel (cont.)**

The whole family of resonances in the Regge trajectory (with spins spaced by two units) contributes to the amplitude