The high order chiral Lagrangian

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Outline

- Background & motivations
- Symbols definitions
- A method to get chiral Lagrangian
- Results & summary

Background

- S. Weinberg, Physica 96A, 327 (1979) p^2
- J. Gasser, H. Leutwyler, Ann. Phys.(N.Y.) 158, 142(1984) $p^4, SU(2)$
- J. Gasser, H. Leutwyler, Nucl. Phys. B250, 465(1985) $p^4, SU(3)$
- H.W. Fearing, S. Scherer, Phys. Rev. D53, 315(1996) $p^6, SU(N)$
- J. Bijnens, G. Colangelo, G. Ecker, JHEP 02 (1999) 020 correct $p^6, SU(N)$
- C. Haefeli, M. A. Ivanov, M. Schmid, G. Ecker, arXiv:0705.0576 correct $p^6, SU(2)$
- P. Herrera-Siklódy, J.I. Latorre, P. Pascual, J. Taron, Nucl. Phys. B497, 345(1997) $p^4, U(3)$
- H. Leutwyler, Phys. Lett. B374, 163(1996) $O(1), 1/N_C$
- R. Kaiser, H. Leutwyler, Eur. Phys. J. C17, 623(2000) $O(\delta), 1/N_C$

Background

- J. Wess, B. Zumino, Phys. Lett. B37, 95(1971) $p^4, SU(3)$ anomaly
- H.W. Fearing, S. Scherer, Phys. Rev. D53, 315(1996) $p^6, SU(N)$ anomaly
- T. Ebertshaüser, H. Fearing, S. Scherer, Phys. Rev. D65, 054033 (2002) correct $p^6, SU(N)$ anomaly
- J. Bijnens, L. Girlanda, P. Talavera, Eur. Phys. J. C23,539 (2002) correct $p^6, SU(N)$ anomaly
- O. Catà, V. Mateu, JHEP 09 (2007) 078 to $p^6, SU(N)$ tensor sources
- S.Z. Jiang, Y. Zhang, Q. Wang, Phys. Rev. D87, 094014(2013) correct $p^6, SU(N)$ tensor sources

Background

Numbers in each order

order	p^2			p^4			p^6		
group	n	3	2	n	3	2	n	3	2
SU_n	2	2	2	$11+2$	$10+2$	$7+3$	112+3	90+4	52+4
SU_a	0	0	0	Wezz	Zumino	term	24	23	$5+8$
U	6	6	6		50+7				
SU_t	0	0	0	4	4	4	96+2	90+2	63+2

n:normal parts; a:anormalous parts; t:tensor sources parts

Motivations

- Find a systematic method to get chiral Lagrangian. Do not miss any linear relations

- Check the old results

- Obtain U(N)/U(3)/U(2) in p^6 order

- SU: $a_0^\mu \sim \theta_{\nabla^\mu \hat{\theta} = \partial^\mu \hat{\theta} + 2i\langle a^\mu \rangle}$
  electromagnetic fields$\sim v_0^\mu$

- Can extend to the other effective Lagrangians

Symbols definitions

$$\mathcal{L} = \mathcal{L}^0_{\rm QCD} + \bar{q}(\not{v} + \not{a}\gamma_5 - s + ip\gamma_5 + \sigma_{\mu\nu}\bar{t}^{\mu\nu})q - \frac{\theta}{16\pi^2}\mathrm{tr}_c(G_{\mu\nu}\tilde{G}^{\mu\nu})$$

$$\begin{aligned}
U(x) &= e^{i\phi(x)/F_0} \\
\phi(x) &= \sum_{a=0}^{8} \lambda_a \phi_a(x) \\
&= \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}
\end{aligned}$$

$$U_L(N) \times U_R(N) \to g_L, g_R \qquad U \to g_R U g_L^\dagger$$

LR-basis

$$U$$

$$F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu], \quad r_\mu = v_\mu + a_\mu$$

$$F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu], \quad l_\mu = v_\mu - a_\mu$$

$$\chi = 2B_0(s + ip), \quad \chi^\dagger$$

$$t^{\mu\nu},\ t^{\dagger\,\mu\nu},\ \bar{t}^{\mu\nu} = P_L^{\mu\nu\lambda\rho} t_{\lambda\rho} + P_R^{\mu\nu\lambda\rho} t^\dagger_{\lambda\rho}$$

$$P_R^{\mu\nu\lambda\rho} = \frac{1}{4}(g^{\mu\lambda} g^{\nu\rho} - g^{\nu\lambda} g^{\mu\rho} + i\epsilon^{\mu\nu\lambda\rho}),\ P_L^{\mu\nu\lambda\rho} = (P_R^{\mu\nu\lambda\rho})^\dagger$$

$$\hat{\theta} = i\theta$$

$$X = \langle \ln U \rangle + \hat{\theta}$$

LR-basis

basis	G	C	P
U	$g_R U g_L^\dagger$	U^T	$U^\dagger$
$\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}U$	$g_R\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}U g_L^\dagger$	$(\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}U)^T$	$(\nabla^{\lambda_1}\cdots\nabla^{\lambda_n}U)^\dagger$
χ	$g_R\chi g_L^\dagger$	χ^T	$\chi^\dagger$
$\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}\chi$	$g_R\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}\chi g_L^\dagger$	$(\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}\chi)^T$	$(\nabla^{\lambda_1}\cdots\nabla^{\lambda_n}\chi)^\dagger$
$F^R_{\mu\nu}$	$g_R F^R_{\mu\nu} g_R^\dagger$	$-(F^L_{\mu\nu})^T$	$F_L^{\mu\nu}$
$F^L_{\mu\nu}$	$g_L F^L_{\mu\nu} g_L^\dagger$	$-(F^R_{\mu\nu})^T$	$F_R^{\mu\nu}$
$t^{\mu\nu}$	$g_R t^{\mu\nu} g_L^\dagger$	$t^{\mu\nu,T}$	$t^{\mu\nu,\dagger}$
$\nabla_\mu\hat\theta$	$\nabla_\mu\hat\theta$	$\nabla_\mu\hat\theta$	$-\nabla_\mu\hat\theta$
X	X	X	$-X$

building blocks

$$U = u^2, \quad u(\varphi) \to u(\varphi') = g_R u(\varphi) h^\dagger = h u(\varphi) g_L^\dagger$$

$$\begin{aligned}
u_\mu &= i\{u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i l_\mu)u^\dagger\}, \\
\chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \\
h_{\mu\nu} &= \nabla_\mu u_\nu + \nabla_\nu u_\mu, \qquad \mathcal{O} \to h \mathcal{O} h^{-1} \\
f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u, \\
t_\pm^{\mu\nu} &= u^\dagger t^{\mu\nu} u^\dagger \pm u t^{\mu\nu\dagger} u,
\end{aligned}$$

$$\begin{aligned}
\nabla^\mu \hat{\theta} &= \partial^\mu \hat{\theta} + 2i\langle a^\mu \rangle, \qquad \mathcal{O} \to \mathcal{O} \\
X &= \langle \ln U \rangle + \hat{\theta}
\end{aligned}$$

building blocks

O	P	C	h.c.
u^μ	$-u_\mu$	$(u^\mu)^T$	u^μ
$h^{\mu\nu}$	$-h_{\mu\nu}$	$(h^{\mu\nu})^T$	$h^{\mu\nu}$
$\chi_\pm$	$\pm\chi_\pm$	$(\chi_\pm)^T$	$\pm\chi_\pm$
$f_\pm^{\mu\nu}$	$\pm f_{\pm\mu\nu}$	$\mp(f_\pm^{\mu\nu})^T$	$f_\pm^{\mu\nu}$
$t_\pm^{\mu\nu}$	$\pm t_{\pm\mu\nu}$	$-(t_\pm^{\mu\nu})^T$	$\pm t_\pm^{\mu\nu}$
$\nabla_\mu\hat\theta$	$-\nabla_\mu\hat\theta$	$\nabla_\mu\hat\theta$	$-\nabla_\mu\hat\theta$
X	$-X$	X	X

Power counting

$$U = u^2 \sim O(p^0)$$
$$v_\mu, a_\mu \sim O(p^1)$$
$$\chi \sim O(p^2)$$
$$t_{\mu\nu} \sim O(p^2)$$
$$\hat{\theta} \sim O(p^0)$$
$$X \sim O(p^0)$$
$$\mathcal{L}_n = \sum_{i=1}^{N} C_i(X) O_i$$

$$P(X) = -1 \Rightarrow P(O_i) = \pm 1$$

$p^0 + p^2$ order

$$\begin{aligned}\mathcal{L}_{0+2} &= -W_0(X) + W_1(X)\langle D_\mu U^\dagger D^\mu U\rangle + W_2(X)\langle U^\dagger\chi + \chi^\dagger U\rangle \\ &+iW_3(X)\langle U^\dagger\chi - \chi^\dagger U\rangle + W_4(X)\langle U^\dagger D_\mu U\rangle\langle U^\dagger D^\mu U\rangle \\ &+W_5(X)\langle U^\dagger D_\mu U\rangle\nabla^\mu\hat{\theta} + W_6(X)\nabla_\mu\hat{\theta}\nabla^\mu\hat{\theta}\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{0+2} &= -W_0(X) + W_1(X)\langle u^2\rangle + W_2(X)\langle\chi_+\rangle \\ &+iW_3(X)\langle\chi_-\rangle - W_4(X)\langle u^\mu\rangle\langle u_\mu\rangle \\ &-iW_5(X)\langle u^\mu\rangle\nabla^\mu\hat{\theta} + W_6(X)\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}\end{aligned}$$

Construct chiral Lagrangian

- Pile building blocks (all terms)
- C and Hermitian conjugate is invariant
- Pick out contact terms
- Remove linear depend terms
- By computer

Linear relations

- EOM: remove $h^{\mu}{}_{\mu}$ and $\nabla^{\mu}u_{\mu}$
- Partial integration: only the farthest covariant derivative need be moved

$$\langle \nabla^{\mu}\nabla^{\nu}AB\rangle = -\langle \nabla^{\nu}A\nabla^{\mu}B\rangle$$

- Bianchi identity: only the nearest covariant derivative need be exchanged

$$\langle \nabla^{\mu}\nabla^{\nu}\nabla^{\lambda}AB\rangle \sim \langle \nabla^{\nu}\nabla^{\mu}\nabla^{\lambda}AB\rangle$$
$$\langle \nabla^{\mu}\nabla^{\nu}\nabla^{\lambda}AB\rangle \sim \langle \nabla^{\mu}\nabla^{\lambda}\nabla^{\nu}AB\rangle$$

Linear relations

- Schouten identity: exchange all different indices

$$\epsilon^{\mu\nu\lambda\rho}A^{\sigma}-\epsilon^{\sigma\nu\lambda\rho}A^{\mu}-\epsilon^{\mu\sigma\lambda\rho}A^{\nu}-\epsilon^{\mu\nu\sigma\rho}A^{\lambda}-\epsilon^{\mu\nu\lambda\sigma}A^{\rho}=0$$

- Tensor relations: exchange all possible $t_+ \leftrightarrow t_-$

$$\langle t_+ A t_- B\rangle \to \langle t_- A t_+ B\rangle$$

- Collect all linear relations of O_i':

$$\sum_l R_{il} O'_{lj} = 0, \quad j : \text{trace number}$$

Cayley-Hamilton relations

To all trace number

$$O'_{ij} \to O'_i = [O'_{i1}, O'_{i2}, O'_{i3}, \cdots, O'_{in}]$$

$$\sum_l A'_{ki} O'_{ij} = 0$$

Contact terms

Building blocks:

$$F_L^{\mu\nu} = \frac{1}{2}u^\dagger(f_+^{\mu\nu} + f_-^{\mu\nu})u$$

$$F_R^{\mu\nu} = \frac{1}{2}u(f_+^{\mu\nu} - f_-^{\mu\nu})u^\dagger$$

$$\chi = \frac{1}{2}u(\chi_+ + \chi_-)u \qquad \det\chi$$

$$\chi^\dagger = \frac{1}{2}u^\dagger(\chi_+ - \chi_-)u^\dagger \qquad \det\chi^\dagger$$

$$t^{\mu\nu} = \frac{1}{2}u(t_+^{\mu\nu} + t_-^{\mu\nu})u$$

$$t^{\dagger\mu\nu} = \frac{1}{2}u^\dagger(t_+^{\mu\nu} - t_-^{\mu\nu})u^\dagger$$

$\nabla^\mu\theta$ and their covariant derivative

p^4 order I

Conform to P. Herrera-Siklódy, J.I. Latorre, P. Pascual, J. Taron, Nucl. Phys. B497, 345(1997)

		U(N)			SU(N)$_{\rm I}$ $\langle f_+^{\mu\nu}\rangle \neq 0$ $\langle u^\mu\rangle = 0$			SU(N)$_{\rm II}$ $\langle f_+^{\mu\nu}\rangle = 0$ $\langle u^\mu\rangle \neq 0$			SU(N)$_{\rm III}$ $\langle f_+^{\mu\nu}\rangle = 0$ $\langle u^\mu\rangle = 0$		
m	P	n	3	2	n	3	2	n	3	2	n	3	2
$\langle f_+^{\mu\nu 2}\rangle$	+	1	1	1	1	1	1	1	1	1	1	1	1
$i\langle f_+^{\mu\nu} u^\mu u^\nu\rangle$	+	2	2	2	2	2	2	2	2	2	2	2	2
$\langle f_-^{\mu\nu 2}\rangle$	+	3	3	3	3	3	3	3	3	3	3	3	3
$\langle u^{\mu 2} u^{\nu 2}\rangle$	+	4	4	4	4	4	4	4	4	4	4	4	4
$\langle u^\mu\rangle\langle u^\mu u^{\nu 2}\rangle$	+	5	5	5				5	5	5			
$\langle u^{\mu 2}\rangle\langle u^{\nu 2}\rangle$	+	6	6	6	5	5		6	6	6	5	5	
$\langle u^\mu u^\nu\rangle\langle u^\mu u^\nu\rangle$	+	7	7	7	6			7	7	7	6		
$\langle u^\mu\rangle\langle u^\mu\rangle\langle u^{\nu 2}\rangle$	+	8	8					8	8				
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u^\mu u^\nu\rangle$	+	9	9					9	9				
$\langle u^\mu\rangle\langle u^\mu\rangle\langle u^\nu\rangle\langle u^\nu\rangle$	+	10						10					
$\langle u^{\mu 2}\chi_+\rangle$	+	11	10	8	7	6	5	11	10	8	7	6	5
$\langle u^\mu\rangle\langle u^\mu \chi_+\rangle$	+	12	11	9				12	11	9			
$\langle u^{\mu 2}\rangle\langle \chi_+\rangle$	+	13	12	10	8	7		13	12	10	8	7	
$\langle u^\mu\rangle\langle u^\mu\rangle\langle \chi_+\rangle$	+	14	13					14	13				
$\langle \chi_+^2\rangle$	+	15	14	11	9	8	6	15	14	11	9	8	6
$\langle \chi_+\rangle\langle \chi_+\rangle$	+	16	15	12	10	9	7	16	15	12	10	9	7
$\langle \chi_-\rangle\langle \chi_-\rangle$	+	17	16		11	10		17	16		11	10	
$\langle F_L^{\mu\nu 2}\rangle + \langle F_R^{\mu\nu 2}\rangle$	+	18	17	13	12	11	8	18	17	13	12	11	8
$\langle F_L^{\mu\nu}\rangle\langle F_L^{\mu\nu}\rangle$ + $\langle F_R^{\mu\nu}\rangle\langle F_R^{\mu\nu}\rangle$	+	19	18	14	13	12	9	19	18	14			
$\langle F_L^{\mu\nu}\rangle\langle F_R^{\mu\nu}\rangle$	+	20	19	15									
$\langle \chi\chi^\dagger\rangle$	+	21	20	16	14	13	10	20	19	15	13	12	9
$\det\chi + \det\chi^\dagger$	+			17			11			16			10
$i\langle u^{\mu 2} u^\nu\rangle\nabla\hat\theta^\nu$	+	22	21	18	15	14		21	20	17	14	13	

p^4 order II

$i\langle u^\mu\rangle\langle u^\mu u^\nu\rangle\nabla\hat\theta^\nu$	$+$	23	22	19				22	21	18			
$i\langle u^\mu\rangle\langle u^{\nu 2}\rangle\nabla\hat\theta^\mu$	$+$	24	23	20				23	22	19			
$i\langle u^\mu\rangle\langle u^\mu\rangle\langle u^\nu\rangle\nabla\hat\theta^\nu$	$+$	25	24					24	23				
$i\langle u^\mu\chi_+\rangle\nabla\hat\theta^\mu$	$+$	26	25	21	16	15	12	25	24	20	15	14	11
$i\langle u^\mu\rangle\langle\chi_+\rangle\nabla\hat\theta^\mu$	$+$	27	26	22				26	25	21			
$\langle\chi_-\rangle\nabla\nabla\hat\theta^{\mu\mu}$	$+$	28	27	23	17	16	13	27	26	22	16	15	12
$\langle u^{\mu 2}\rangle\nabla\hat\theta^{\nu 2}$	$+$	29	28	24	18	17	14	28	27	23	17	16	13
$\langle u^\mu u^\nu\rangle\nabla\hat\theta^\mu\nabla\hat\theta^\nu$	$+$	30	29	25	19	18	15	29	28	24	18	17	14
$\langle u^\mu\rangle\langle u^\mu\rangle\nabla\hat\theta^{\nu 2}$	$+$	31	30	26				30	29	25			
$\langle u^\mu\rangle\langle u^\nu\rangle\nabla\hat\theta^\mu\nabla\hat\theta^\nu$	$+$	32	31	27				31	30	26			
$\langle\chi_+\rangle\nabla\hat\theta^{\mu 2}$	$+$	33	32	28	20	19	16	32	31	27	19	18	15
$i\langle u^\mu\rangle\nabla\hat\theta^\mu\nabla\hat\theta^{\nu 2}$	$+$	34	33	29				33	32	28			
$\nabla\nabla\hat\theta^{\mu\mu}\nabla\nabla\hat\theta^{\nu\nu}$	$+$	35	34	30	21	20	17	34	33	29	20	19	16
$\nabla\hat\theta^{\mu 2}\nabla\hat\theta^{\nu 2}$	$+$	36	35	31	22	21	18	35	34	30	21	20	17
$\epsilon^{\mu\nu\lambda\rho}\langle u^\mu\rangle\langle u^\nu f_-^{\lambda\rho}\rangle$	$+$	37	36	32				36	35	31			
$i\epsilon^{\mu\nu\lambda\rho}\langle u^\mu f_-^{\nu\lambda}\rangle\nabla\hat\theta^\rho$	$+$	38	37	33	23	22	19	37	36	32	22	21	18
$i\epsilon^{\mu\nu\lambda\rho}\langle u^\mu\rangle\langle f_-^{\nu\lambda}\rangle\nabla\hat\theta^\rho$	$+$	39	38	34				38	37	33			
$\langle f_+^{\mu\nu} t_+^{\mu\nu}\rangle$	$+$	40	39	35	24	23	20	39	38	34	23	22	19
$\langle f_+^{\mu\nu}\rangle\langle t_+^{\mu\nu}\rangle$	$+$	41	40	36	25	24	21						
$i\langle u^\mu u^\nu t_+^{\mu\nu}\rangle$	$+$	42	41	37	26	25	22	40	39	35	24	23	20
$\langle t_+^{\mu\nu 2}\rangle$	$+$	43	42	38	27	26	23	41	40	36	25	24	21
$\langle t_+^{\mu\nu}\rangle\langle t_+^{\mu\nu}\rangle$	$+$	44	43	39	28	27	24	42	41	37	26	25	22
$\langle u^\mu\rangle\langle u^\nu f_-^{\mu\nu}\rangle$	$-$	45	44	40									
$i\langle u^{\mu 2}\chi_-\rangle$	$-$	46	45	41									
$i\langle u^\mu\rangle\langle u^\mu\chi_-\rangle$	$-$	47	46	42									
$i\langle u^{\mu 2}\rangle\langle\chi_-\rangle$	$-$	48	47	43									
$i\langle u^\mu\rangle\langle u^\mu\rangle\langle\chi_-\rangle$	$-$	49	48										

p^4 order III

$i\langle\chi_+\chi_-\rangle$		$-$	50	49	44									
$i\langle\chi_+\rangle\langle\chi_-\rangle$		$-$	51	50										
$i\det\chi - i\det\chi^\dagger$		$-$			45									
$i\langle u^\mu f_-^{\mu\nu}\rangle\nabla\hat{\theta}^\nu$		$-$	52	51	46									
$i\langle u^\mu h^{\mu\nu}\rangle\nabla\hat{\theta}^\nu$		$-$	53	52	47									
$i\langle u^\mu\rangle\langle f_-^{\mu\nu}\rangle\nabla\hat{\theta}^\nu$		$-$	54	53	48									
$i\langle u^\mu\rangle\langle h^{\mu\nu}\rangle\nabla\hat{\theta}^\nu$		$-$	55	54	49									
$i\langle\chi_+\rangle\nabla\nabla\hat{\theta}^{\mu\mu}$		$-$	56	55	50									
$\langle u^\mu\chi_-\rangle\nabla\hat{\theta}^\mu$		$-$	57	56	51									
$\langle u^\mu\rangle\langle\chi_-\rangle\nabla\hat{\theta}^\mu$		$-$	58	57	52									
$\langle h^{\mu\nu}\rangle\nabla\hat{\theta}^\mu\nabla\hat{\theta}^\nu$		$-$	59	58	53									
$i\langle\chi_-\rangle\nabla\hat{\theta}^{\mu 2}$		$-$	60	59	54									
$i\nabla\hat{\theta}^{\mu 2}\nabla\nabla\hat{\theta}^{\nu\nu}$		$-$	61	60	55									
$\epsilon^{\mu\nu\lambda\rho}\langle f_+^{\mu\nu} f_+^{\lambda\rho}\rangle$		$-$	62	61	56									
$\epsilon^{\mu\nu\lambda\rho}\langle F_L^{\mu\nu} F_L^{\lambda\rho}\rangle$	$+$	$-$	63	62	57									
$\epsilon^{\mu\nu\lambda\rho}\langle F_R^{\mu\nu} F_R^{\lambda\rho}\rangle$														
$i\langle f_+^{\mu\nu} t_-^{\mu\nu}\rangle$		$-$	64	63	58									
$i\langle f_+^{\mu\nu}\rangle\langle t_-^{\mu\nu}\rangle$		$-$	65	64	59									
$\langle u^\mu u^\nu t_-^{\mu\nu}\rangle$		$-$	66	65	60									
$i\langle t_+^{\mu\nu} t_-^{\mu\nu}\rangle$		$-$	67	66	61									
$i\langle t_+^{\mu\nu}\rangle\langle t_-^{\mu\nu}\rangle$		$-$	68	67	62									

p^6 order

Only tensor sources parts can be reduced (SU)

flavor	n	3	2
[1]	120	113	78
[2]	98	92	65
now	97	91	56

[1] O. Cata, V. Mateu, JHEP 09 (2007) 078

[2] S.Z. Jiang, Y. Zhang, Q. Wang, Phys. Rev. D87, 094014(2013)

p^6 order

Numbers of independent terms

Number	$P=+/-$	U(N)			$\mathrm{SU(N)_I}$ $\langle f_+^{\mu\nu}\rangle \neq 0$ $\langle u^\mu\rangle = 0$			$\mathrm{SU(N)_{II}}$ $\langle f_+^{\mu\nu}\rangle = 0$ $\langle u^\mu\rangle \neq 0$			$\mathrm{SU(N)_{III}}$ $\langle f_+^{\mu\nu}\rangle = 0$ $\langle u^\mu\rangle = 0$		
classification	P	n	3	2	n	3	2	n	3	2	n	3	2
$\theta = \bar{t}^{\mu\nu} = 0$	298/225	523	480	305	163	141	82	264	236	143	139	117	61
$\theta \neq 0, \bar{t}^{\mu\nu} = 0$	263/219	482	470	385	126	120	85	252	244	195	117	111	76
$\theta = 0, \bar{t}^{\mu\nu} \neq 0$	163/164	327	316	226	112	106	69	137	131	88	97	91	56
$\theta \neq 0, \bar{t}^{\mu\nu} \neq 0$	66/67	133	133	118	53	53	45	52	52	44	42	42	34
total	790/675	1465	1399	1034	454	420	281	705	663	470	395	361	227

p^6 order I

		U(N)			SU(N)$_{\mathrm{I}}$ $\langle f_+^{\mu\nu}\rangle \neq 0$ $\langle u^\mu\rangle = 0$			SU(N)$_{\mathrm{II}}$ $\langle f_+^{\mu\nu}\rangle = 0$ $\langle u^\mu\rangle \neq 0$			SU(N)$_{\mathrm{III}}$ $\langle f_+^{\mu\nu}\rangle = 0$ $\langle u^\mu\rangle = 0$		
m	P	n	3	2	n	3	2	n	3	2	n	3	2
$i\langle f_+^{\mu\nu} f_+^{\mu\lambda} f_+^{\nu\lambda}\rangle$	+	1	1	1	1	1	1	1	1	1	1	1	1
$\langle \nabla f_+^{\mu\mu\nu} \nabla f_+^{\lambda\nu\lambda}\rangle$	+	2	2	2	2	2	2	2	2	2	2	2	2
$\langle f_+^{\mu\nu 2} u^{\lambda 2}\rangle$	+	3	3	3	3	3	3	3	3	3	3	3	3
$\langle f_+^{\mu\nu} f_+^{\mu\lambda} u^\nu u^\lambda\rangle$	+	4	4	4	4	4	4	4	4	4	4	4	4
$\langle f_+^{\mu\nu} f_+^{\mu\lambda} u^\lambda u^\nu\rangle$	+	5	5	5	5	5	5	5	5	5	5	5	5
$\langle f_+^{\mu\nu} u^\mu f_+^{\nu\lambda} u^\lambda\rangle$ $-$	+	6	6	6	6	6	6	6	6	6	6	6	6
$\langle f_+^{\mu\nu} u^\lambda f_+^{\mu\lambda} u^\nu\rangle$													
$\langle f_+^{\mu\nu}\rangle\langle f_+^{\mu\nu} u^{\lambda 2}\rangle$	+	7	7	7	7	7	7						
$\langle f_+^{\mu\nu 2}\rangle\langle u^{\lambda 2}\rangle$	+	8	8	8	8	8		7	7		7	7	
$\langle f_+^{\mu\nu 2} u^\lambda\rangle\langle u^\lambda\rangle$	+	9	9	9				8	8	7			
$\langle f_+^{\mu\nu} u^\lambda\rangle\langle f_+^{\mu\nu} u^\lambda\rangle$	+	10	10	10	9	9		9	9		8		
$\langle f_+^{\mu\nu}\rangle\langle f_+^{\mu\lambda} u^\nu u^\lambda\rangle$ +	+	11	11	11	10	10	8						
$\langle f_+^{\mu\nu}\rangle\langle f_+^{\mu\lambda} u^\lambda u^\nu\rangle$													
$\langle f_+^{\mu\nu} f_+^{\mu\lambda}\rangle\langle u^\nu u^\lambda\rangle$	+	12	12	12	11	11		10	10		9	8	
$\langle f_+^{\mu\nu} f_+^{\mu\lambda} u^\nu\rangle\langle u^\lambda\rangle$ +	+	13	13	13				11	11	8			
$\langle f_+^{\mu\nu} f_+^{\mu\lambda} u^\lambda\rangle\langle u^\nu\rangle$													
$\langle f_+^{\mu\nu} u^\lambda\rangle\langle f_+^{\mu\lambda} u^\nu\rangle$	+	14	14	14	12	12		12	12		10	9	
$\langle f_+^{\mu\nu} u^\mu\rangle\langle f_+^{\nu\lambda} u^\lambda\rangle$	+	15	15	15	13	13		13	13		11		
$\langle f_+^{\mu\nu}\rangle\langle f_+^{\mu\nu}\rangle\langle u^{\lambda 2}\rangle$	+	16	16		14								
...	...	...	...	...	...	...	...	...	...	...	...	...	...
$i\langle f_+^{\mu\nu} t_-^{\mu\nu}\rangle \nabla\hat{\theta}^{\lambda 2}$	$-$	1445	1379	1015									

p^6 order II

$i\langle f_+^{\mu\nu} t_-^{\mu\lambda}\rangle \nabla\hat\theta^\nu \nabla\hat\theta^\lambda$	–	1446	1380	1016									
$i\langle f_+^{\mu\nu}\rangle\langle t_-^{\mu\nu}\rangle \nabla\hat\theta^{\lambda 2}$	–	1447	1381	1017									
$i\langle f_+^{\mu\nu}\rangle\langle t_-^{\mu\lambda}\rangle \nabla\hat\theta^\nu \nabla\hat\theta^\lambda$	–	1448	1382	1018									
$\langle u^\mu u^\nu t_-^{\mu\nu}\rangle \nabla\hat\theta^{\lambda 2}$	–	1449	1383	1019									
$\langle u^\mu u^\nu t_-^{\mu\lambda}\rangle \nabla\hat\theta^\nu \nabla\hat\theta^\lambda +$ $\langle u^\mu u^\nu t_-^{\nu\lambda}\rangle \nabla\hat\theta^\mu \nabla\hat\theta^\lambda$	–	1450	1384	1020									
$\langle u^\mu t_+^{\mu\nu} t_-^{\nu\lambda}\rangle \nabla\hat\theta^\lambda \quad +$ $\langle u^\mu t_-^{\nu\lambda} t_+^{\mu\nu}\rangle \nabla\hat\theta^\lambda$	–	1451	1385	1021									
$\langle u^\mu\rangle\langle t_+^{\mu\nu} t_-^{\nu\lambda}\rangle \nabla\hat\theta^\lambda$	–	1452	1386	1022									
$\langle u^\mu t_+^{\mu\nu}\rangle\langle t_-^{\nu\lambda}\rangle \nabla\hat\theta^\lambda$	–	1453	1387	1023									
$\langle u^\mu t_-^{\nu\lambda}\rangle\langle t_+^{\mu\nu}\rangle \nabla\hat\theta^\lambda$	–	1454	1388	1024									
$\langle u^\mu t_+^{\nu\lambda}\rangle\langle t_-^{\mu\nu}\rangle \nabla\hat\theta^\lambda$	–	1455	1389	1025									
$\langle u^\mu\rangle\langle t_+^{\mu\nu}\rangle\langle t_-^{\nu\lambda}\rangle \nabla\hat\theta^\lambda$	–	1456	1390										
$i\langle t_+^{\mu\nu} t_-^{\mu\nu}\rangle \nabla\hat\theta^{\lambda 2}$	–	1457	1391	1026									
$i\langle t_+^{\mu\nu}\rangle\langle t_-^{\mu\nu}\rangle \nabla\hat\theta^{\lambda 2}$	–	1458	1392	1027									
$i\langle \nabla t_-^{\mu\mu\nu} t_-^{\nu\lambda}\rangle \nabla\hat\theta^\lambda$	–	1459	1393	1028									
$i\langle \nabla t_-^{\mu\nu\lambda} t_-^{\mu\nu}\rangle \nabla\hat\theta^\lambda$	–	1460	1394	1029									
$i\langle \nabla t_-^{\mu\nu\lambda} t_-^{\nu\lambda}\rangle \nabla\hat\theta^\mu$	–	1461	1395	1030									
$i\langle \nabla t_-^{\mu\mu\nu}\rangle\langle t_-^{\nu\lambda}\rangle \nabla\hat\theta^\lambda$	–	1462	1396	1031									
$i\langle \nabla t_-^{\mu\nu\lambda}\rangle\langle t_-^{\mu\nu}\rangle \nabla\hat\theta^\lambda$	–	1463	1397	1032									
$i\langle \nabla t_-^{\mu\nu\lambda}\rangle\langle t_-^{\nu\lambda}\rangle \nabla\hat\theta^\mu$	–	1464	1398	1033									
$i\langle D t^{\mu\nu\lambda} t^{\dagger\,\mu\nu}\rangle \nabla\hat\theta^\lambda +$ $i\langle t^{\mu\nu} D t^{\dagger\,\mu\nu\lambda}\rangle \nabla\hat\theta^\lambda$	–	1465	1399	1034									

Summary

- Find a method to get chiral Lagrangian
- Give the $U(N) \times U(N)$ chiral Lagrangian to p^6 order
- This method can extend to higher order and other effective Lagrangians.

Thanks!

relations

- Partial integration

$$\langle\nabla^\mu AB\cdots\rangle\langle CD\cdots\rangle\cdots=-\langle A\nabla^\mu B\cdots\rangle\langle CD\rangle\cdots-\cdots$$
$$-\langle AB\cdots\rangle\langle\nabla^\mu CD\rangle\cdots-\langle AB\cdots\rangle\langle C\nabla^\mu D\rangle\cdots-\cdots$$

- Equations of motion

$$\begin{aligned}\nabla_\mu u^\mu &= \frac{i}{2}\frac{W_0'}{W_1}-\frac{i}{2}\frac{W_1'}{W_1}\langle u^\mu u_\mu\rangle+i\frac{W_1'}{W_1}(\langle u_\mu\rangle+i\nabla_\mu\hat{\theta})u^\mu\\ &+\frac{i}{2}\frac{W_2}{W_1}\chi_- -\frac{1}{2}\frac{W_3}{W_1}\chi_+ -\frac{i}{2}\frac{W_2'}{W_1}\langle\chi_+\rangle+\frac{1}{2}\frac{W_3'}{W_1}\langle\chi_-\rangle\\ &+\left(\frac{i}{2}\frac{W_5'}{W_1}-\frac{i}{2}\frac{W_6'}{W_1}\right)(\nabla_\mu\hat{\theta})(\nabla^\mu\hat{\theta})+\frac{i}{2}\frac{W_5}{W_1}\nabla_\mu\nabla^\mu\hat{\theta}\end{aligned}$$

- Bianchi identity

$$\nabla_\mu\Gamma_{\nu\lambda}+\nabla_\nu\Gamma_{\lambda\mu}+\nabla_\lambda\Gamma_{\mu\nu}=0$$

- Schouten identity

$$\epsilon^{\mu\nu\lambda\rho}A^\sigma-\epsilon^{\sigma\nu\lambda\rho}A^\mu-\epsilon^{\mu\sigma\lambda\rho}A^\nu-\epsilon^{\mu\nu\sigma\rho}A^\lambda-\epsilon^{\mu\nu\lambda\sigma}A^\rho=0$$

relations

- Tensor relations

$$\epsilon_{\mu\nu\lambda\rho} t_{\pm}^{\lambda\rho} = 2it_{\mp\mu\nu}$$

- Cayley-Hamilton relations

$$AB + BA - A\langle B\rangle - B\langle A\rangle - \langle AB\rangle + \langle A\rangle\langle B\rangle = 0, n_f = 2$$

$$\begin{aligned}
&ABC + ACB + BAC + BCA + CAB + CBA \\
&-AB\langle C\rangle - AC\langle B\rangle - BA\langle C\rangle - BC\langle A\rangle - CA\langle B\rangle \\
&-CB\langle A\rangle - A\langle BC\rangle - B\langle AC\rangle - C\langle AB\rangle - \langle ABC\rangle \\
&-\langle ACB\rangle + A\langle B\rangle\langle C\rangle + B\langle A\rangle\langle C\rangle + C\langle A\rangle\langle B\rangle + \langle A\rangle\langle BC\rangle \\
&+\langle B\rangle\langle AC\rangle + \langle C\rangle\langle AB\rangle - \langle A\rangle\langle B\rangle\langle C\rangle = 0, n_f = 3
\end{aligned}$$

- Contact terms

Reduced building blocks

$$\Gamma_{\mu\nu} = \frac{1}{4}[u_\mu, u_\nu] - \frac{i}{2} f_{+\mu\nu} \;\to\; f_{+\mu\nu} \sim \Gamma_{\mu\nu}$$

$$\left[\nabla_\mu, \nabla_\nu\right] O = \left[\Gamma_{\mu\nu}, O\right] \;\to\; \left[\nabla_\mu, \nabla_\nu\right] O \sim 0$$

$$\chi_{\pm\mu} = \nabla_\mu \chi_\pm - \frac{i}{2}\{\chi_\mp, u_\mu\} \;\to\; \chi_{\pm\mu} \sim \nabla_\mu \chi_\pm$$

Mathematical relations

- Einstein summation convention

$$A^\mu A_\mu = A^\nu A_\nu$$

- Trace relations

$$\langle AB \rangle = \langle BA \rangle$$

- Symmetry and anti-symmetry indices

$$A^{\mu\nu} = A^{\nu\mu} \qquad B^{\mu\nu} = -B^{\nu\mu}$$

- C-numbers

$$\langle O_1 \rangle \langle O_2 \rangle = \langle O_2 \rangle \langle O_1 \rangle$$

Mathematical relations

- Label the factors and terms

operator	$\langle$	$\rangle$	∇	u	h	χ_+	χ_-	f_-	Γ
number	82	83	1004	1011	1058	1021	1141	1071	1007
index	μ	ν	λ	ρ	σ	δ	α	β	
number	1	2	3	4	5	6	7	8	

$$\langle u^\mu u^\nu h_{\mu\nu}\rangle \to (82, 1011, 1011, 1058, 83, 1, 2, 1, 2)$$

- Use the above relations, adopt the smallest one

Each equal terms have the unique structure

Substitutions and classifications

- Substitutions:$O_i \to O_i'$

$$\begin{aligned}
h_{\mu\nu} &= \nabla_\mu u_\nu + \nabla_\nu u_\mu \\
f_-^{\mu\nu} &= -\nabla^\mu u^\nu + \nabla^\nu u^\mu \\
\langle \Gamma^{\mu\nu} \rangle &= \langle \nabla^\mu \Gamma^\nu \rangle - \langle \nabla^\nu \Gamma^\mu \rangle \\
\nabla^\mu u_\mu &\sim 0 \qquad \text{EOM} \\
O_i &= \sum_j A_{ij} O_j'
\end{aligned}$$

- Classifications: by kinds of sources and traces Linear relations cannot out of these classifications (without CH-relations)
- Get all possible terms

Linear relations

$$O_i = \sum_j A_{ij} O'_j$$

$$P = \sum_j B_j O'_j = \sum_k c_k O_k$$

$$\Rightarrow \sum_j c_i A_{ij} O'_j = \sum_j B_j O'_j$$