



Spin Physics in Nuclear and Collider Experiments

- Spin as a probe of QCD matter and hadron structure -

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Quick self-introduction

Univ. of Tsukuba



- ▶ Name: Takafumi Niida (新井田 貴文)
- ▶ PhD from University of Tsukuba (筑波大学)

* Tsukuba city is ~1h away from Tokyo by train

- RHIC-PHENIX experiment
- Pion and kaon femtoscopy (two-particle momentum correlation, quantum statistical effect)
- ▶ Postdoc and Research assistant professor at Wayne State University
 - Joined RHIC-STAR experiment
 - Anisotropic flow, Chiral Magnetic Effect, **Hyperon polarization in heavy-ion collisions**
- ▶ Faculty at University of Tsukuba
 - RHIC-STAR experiment, CBM experiment at FAIR, and other small projects
- ▶ First time to visit CCNU/Wuhan!

感谢您的邀请

Contents of this lecture

- ▶ Day-1: The birth of spin physics: Stern-Gerlach to Wu
- ▶ Day-2: Spin structure of nucleon and Transverse spin phenomena
- ▶ Day-3: Spin polarization in relativistic heavy-ion collisions (1)
- ▶ Day-4: Spin polarization in relativistic heavy-ion collisions (2)
- ▶ Day-5: Spin correlations and Future directions

Day-1

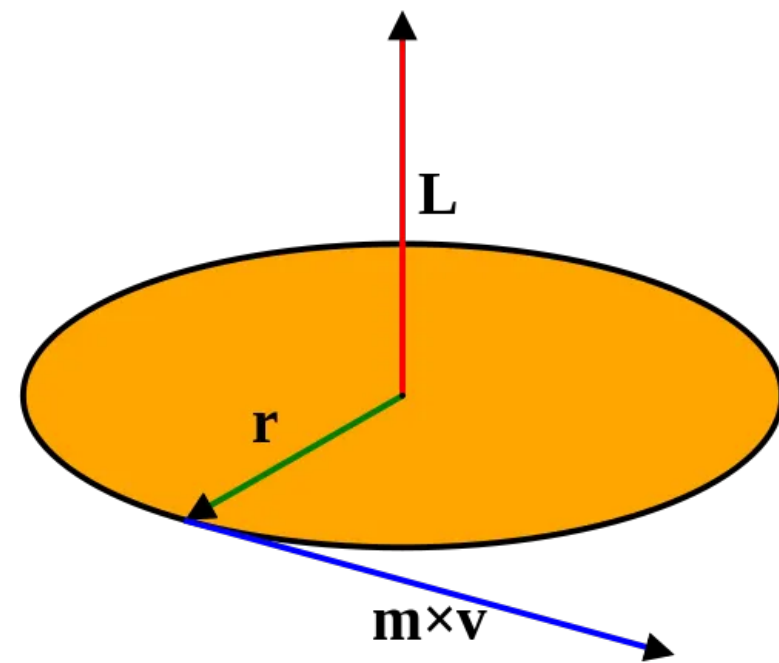
The birth of spin physics: Stern-Gerlach to Wu

What is spin?

Orbital angular momentum

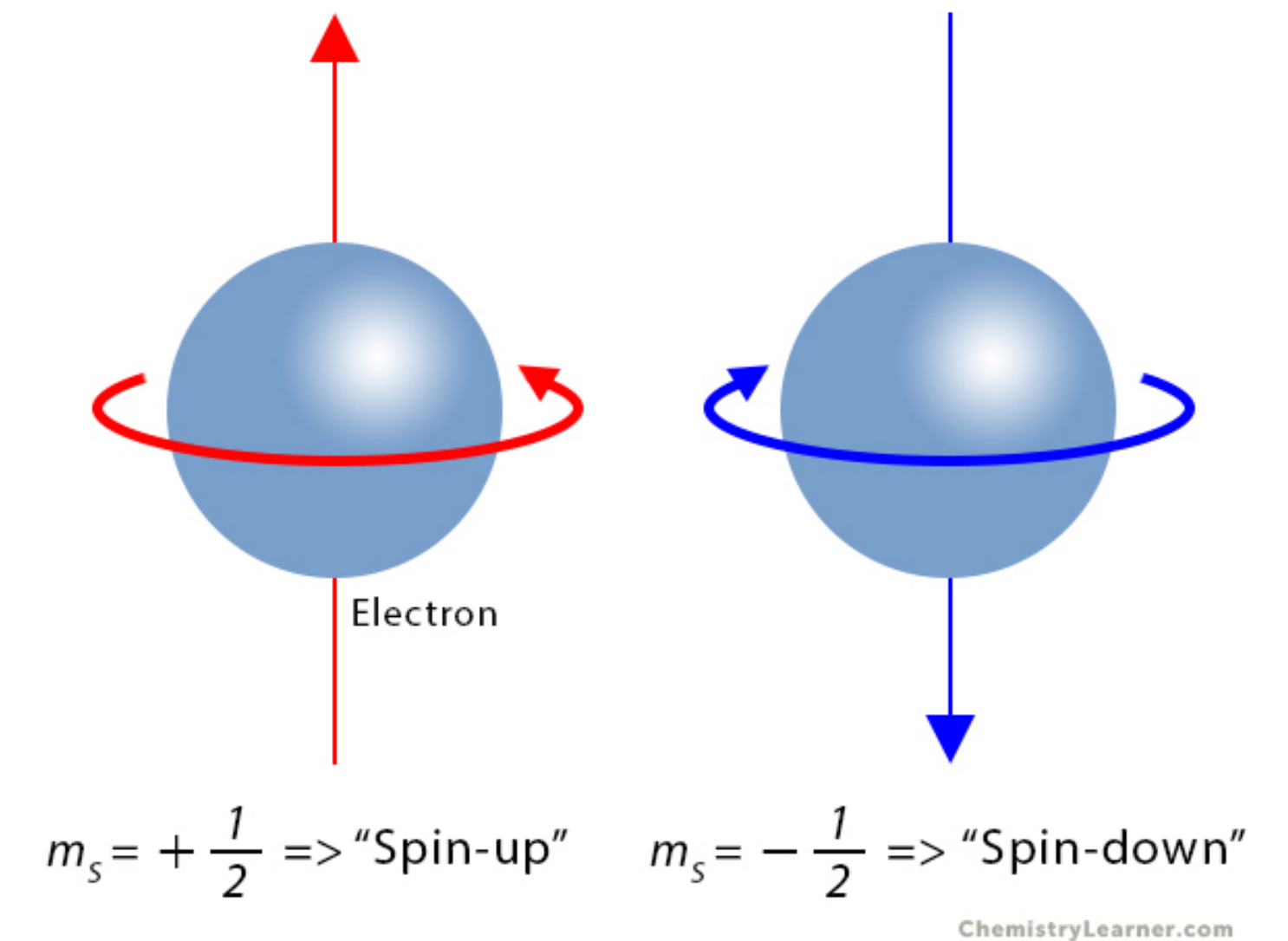
Orbital angular momentum is a measure of the rotational momentum of a body.

$$\vec{L} = \vec{r} \times \vec{p}$$



Spin angular momentum

Spin angular momentum is an intrinsic property of a particle, not due to classical rotation



Particles have a unique value of spin:

Integer spin $\rightarrow$ bosons (photon, gluon, W,Z boson...)

Half-integer $\rightarrow$ fermions (quark, electron, neutrino...)

Angular momentum in quantum mechanics

In quantum mechanics, both orbital and spin angular momenta are quantized

$$L^2 = l(l+1)\hbar^2, \quad L_z = m_l\hbar$$

$$l = 0, 1, 2, \dots$$

$$m_l = -l, -l+1, \dots, +l$$

(2l+1) possible projections

$$S^2 = s(s+1)\hbar^2, \quad S_z = m_s\hbar$$

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(2s+1) possible projections

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(2s+1) possible projections

e.g. for electron, $s=1/2 \rightarrow m_s = +1/2, -1/2$

A spin-1/2 particle has only two possible spin projections along a chosen axis.

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(2s+1) possible projections

e.g. for electron, $s=1/2 \rightarrow m_s = +1/2, -1/2$

A spin-1/2 particle has only two possible spin projections along a chosen axis.

Orbital angular momentum has integer values, while spin can take half-integer, which is a purely quantum-mechanical property

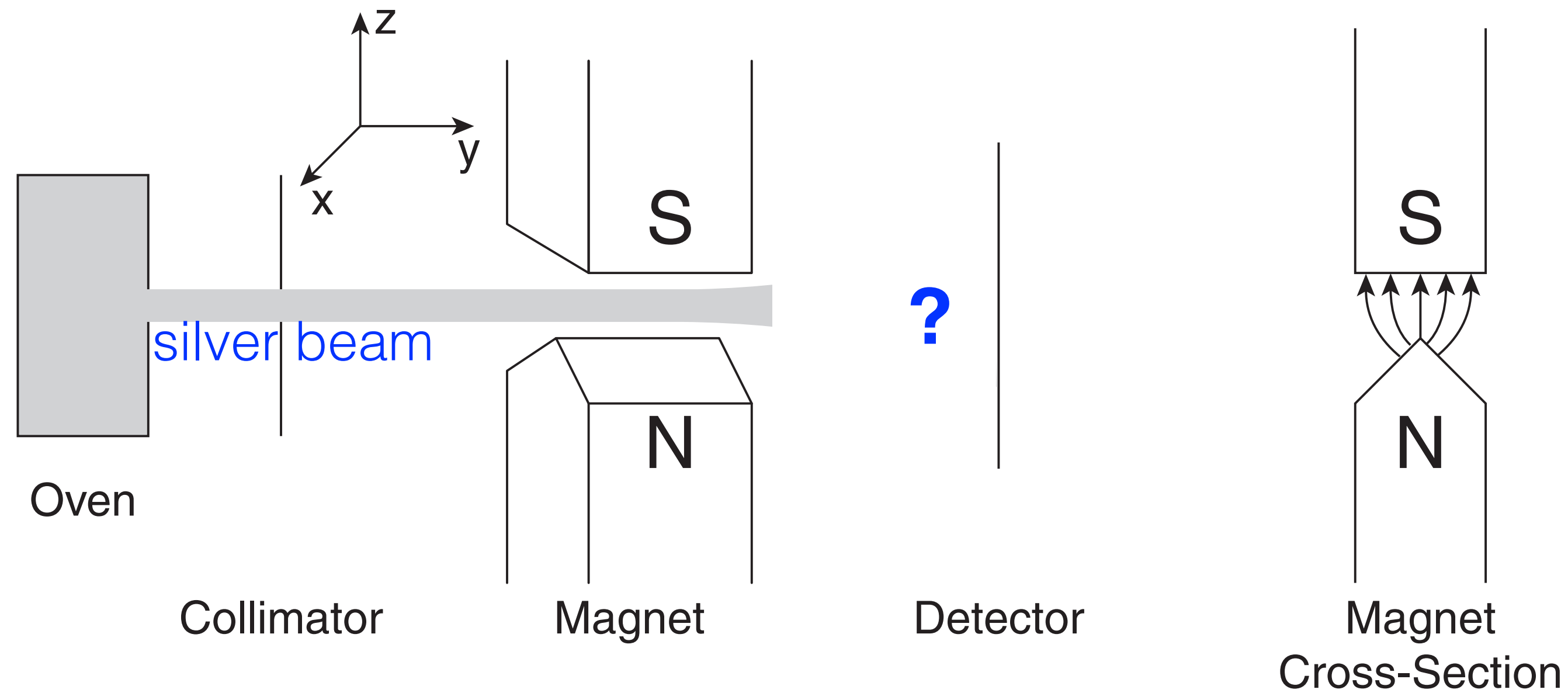
How can we experimentally observe angular momentum quantization?

Discovery of spin



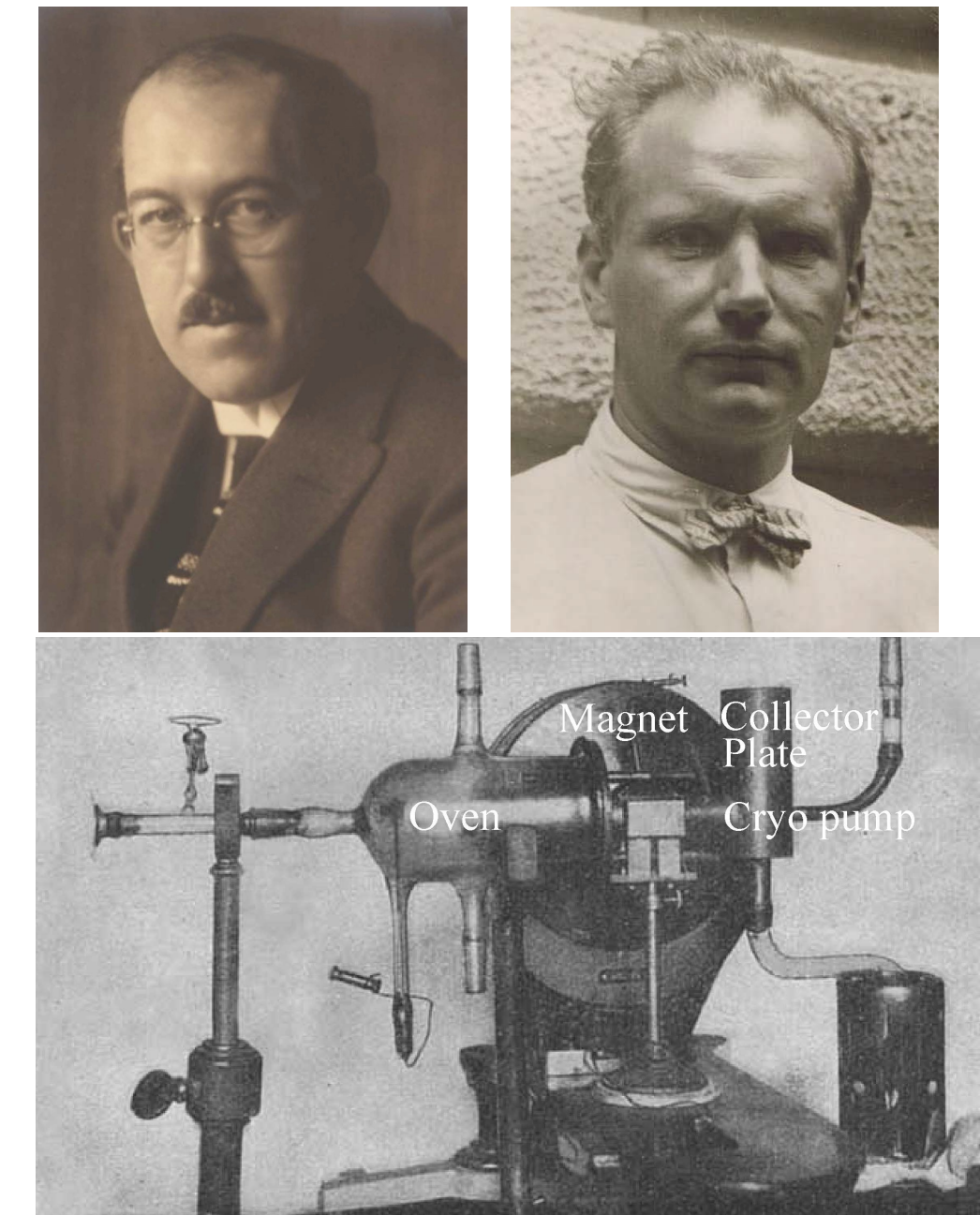
Stern-Gerlach experiment (1922)

W. Gerlach and O. Stern, Z.Phys. 9, 349-352 (1922)



Silver atoms were evaporated from a heated oven and collimated into an atomic beam

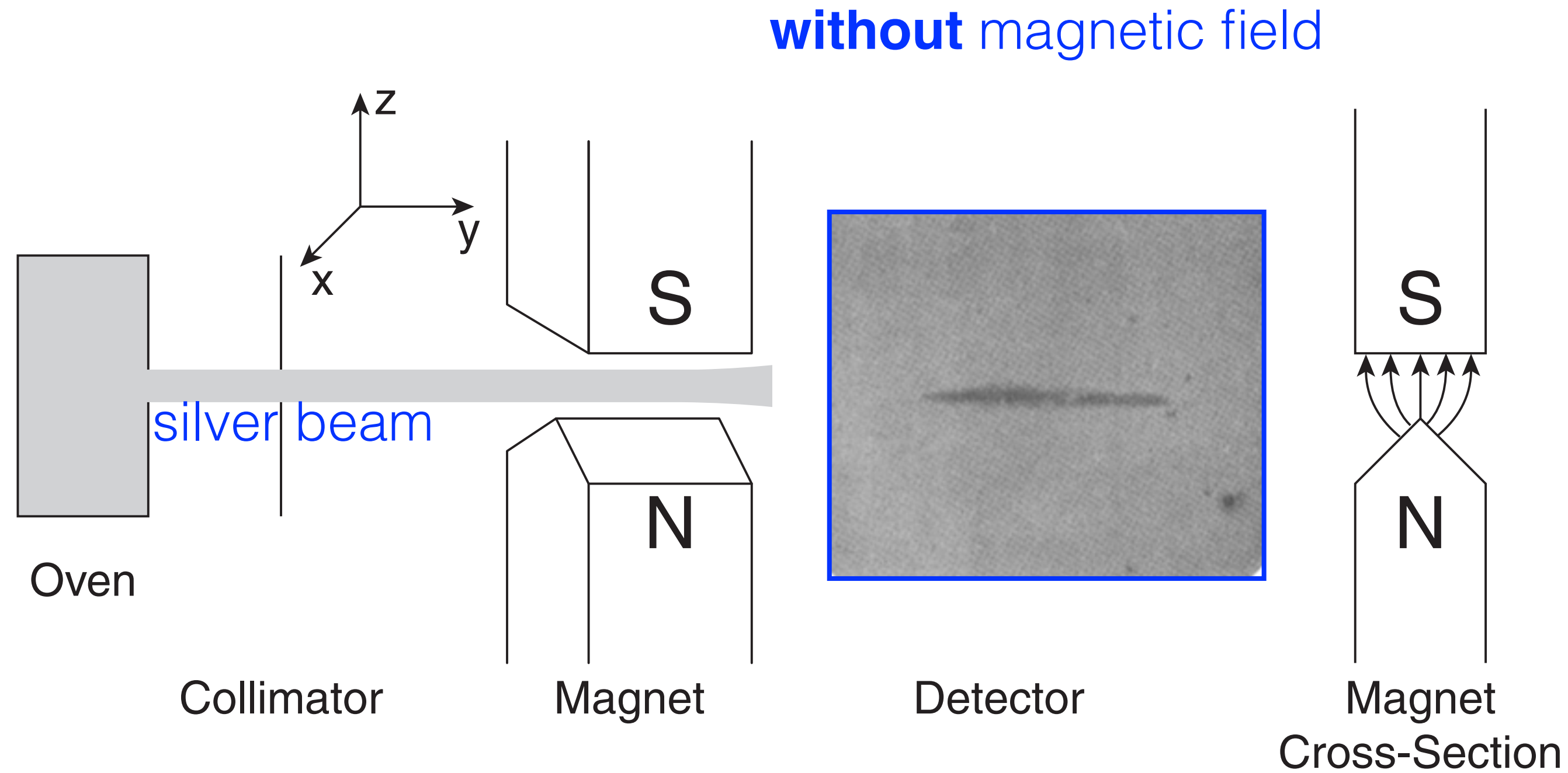
Otto Stern Walther Gerlach



B. Friedrich, *Isr. J. Chem.* 2023, 63, e202300047.

Stern-Gerlach experiment (1922)

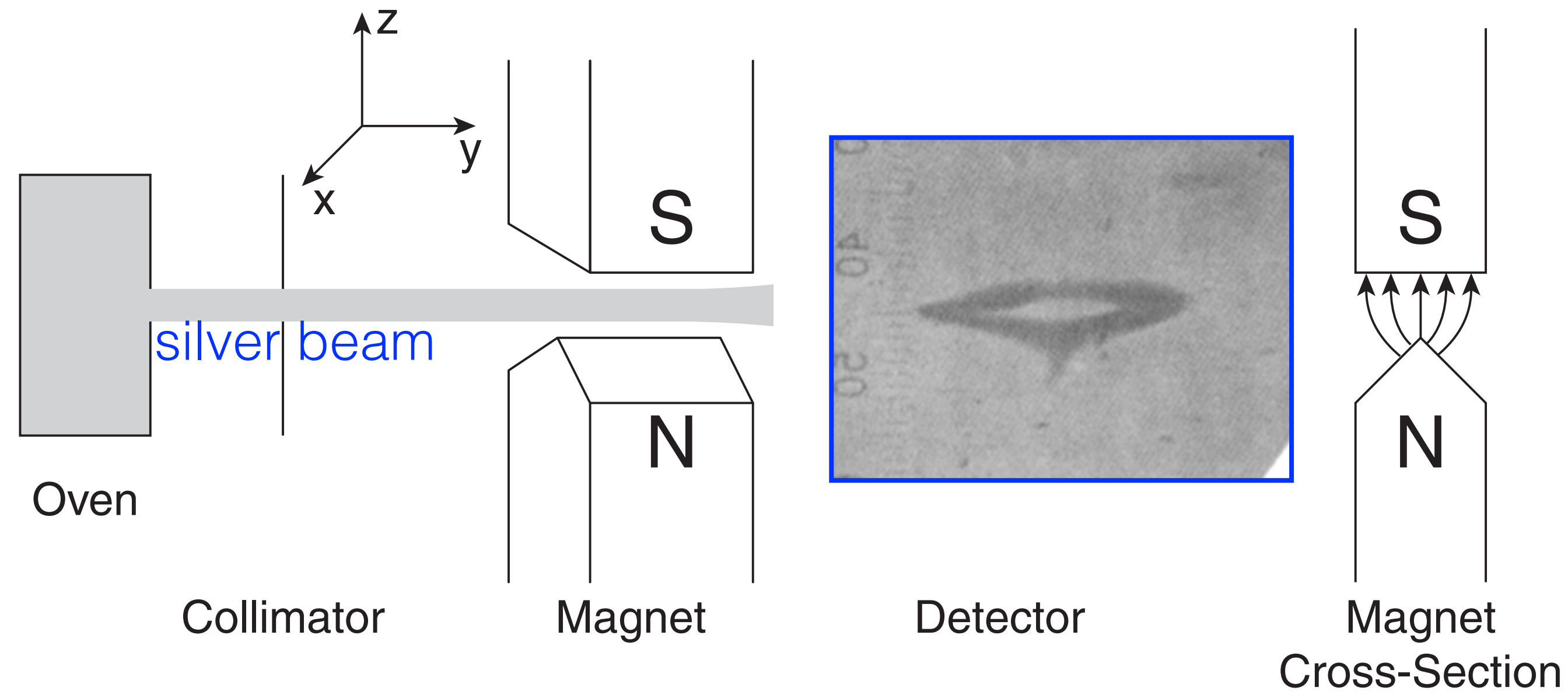
W. Gerlach and O. Stern, Z.Phys. 9, 349-352 (1922)



Stern-Gerlach experiment (1922)

W. Gerlach and O. Stern, Z.Phys. 9, 349-352 (1922)

with (non-uniform) magnetic field



Clear splitting observed
in case of B-field applied.

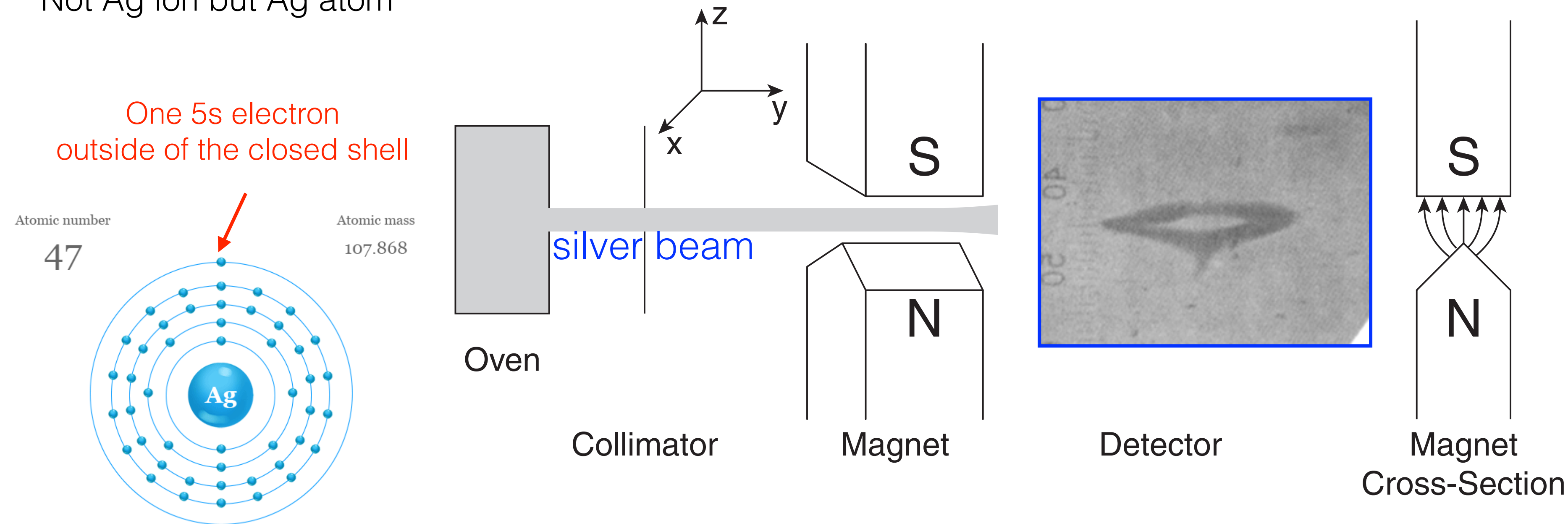
First experimental evidence for quantization of angular momentum!

Stern-Gerlach experiment (1922)

W. Gerlach and O. Stern, Z.Phys. 9, 349-352 (1922)

Not Ag ion but Ag atom

with (non-uniform) magnetic field



$$\begin{aligned}\mu &= \mu_L + \mu_S \\ &= -\frac{e}{2m_e}(\mathbf{L} + g_s \mathbf{S})\end{aligned}$$

μ_{Ag} is determined by one 5s electron μ_{5s-e}
and 5s electron has $L=0$

$$\begin{aligned}F_z &= \frac{\partial}{\partial z}(\mu \cdot \mathbf{B}) \\ &\simeq \mu_z \frac{\partial B_z}{\partial z}\end{aligned}$$

Force along the direction of B-field

What did Stern and Gerlach know in 1922?

► 1922: Stern–Gerlach experiment

- ❑ Observed two discrete components
- ❑ Demonstrated space quantization of angular momentum
- ❑ Electron spin was not yet known

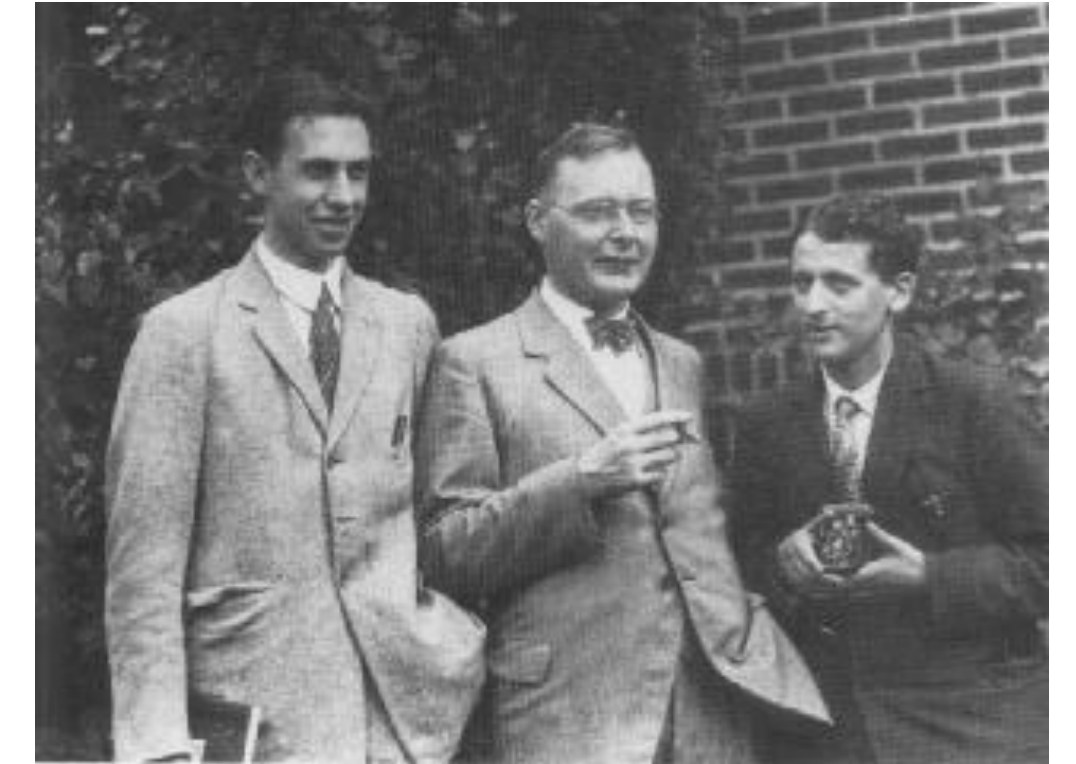
(3 years later)

► 1925: Electron spin was proposed

- ❑ Uhlenbeck & Goudsmit proposed $s=1/2$
- ❑ The two-fold splitting was later understood in terms of the **spin magnetic moment of the unpaired electron**, and cannot be explained by $L=0$ of the 5s electron

Discovery of electron spin

Hendrik Kramers
George Uhlenbeck Samuel Goudsmit



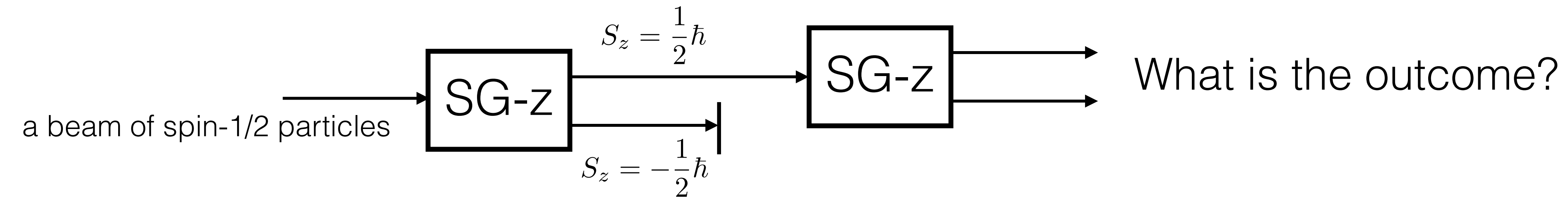
From <https://lorentz.leidenuniv.nl/>

- ▶ 1922: Stern-Gerlach experiment
- ▶ 1925: G. Uhlenbeck & S. Goudsmit proposed the idea of electron spin
 - A few months before: R. Kronig proposed the idea of electron spin, which was strongly criticized by Pauli and Heisenberg (unpublished)
 - One objection to the classical picture of a “spinning electron” was that its surface would have to move faster than light.
 - P. Ehrenfest encouraged Uhlenbeck & Goudsmit (published)
 - “You are both young enough to be able to afford a stupidity!”*
- ▶ 1927: Pauli formulated the quantum theory of electron spin using Pauli matrices
- ▶ 1928: Dirac formulated a relativistic quantum theory of electrons, in which spin naturally emerges and it predicted antiparticle

Stern-Gerlach experiment Quiz.

Setup: A beam of spin-1/2 particles first passes through SG-z device

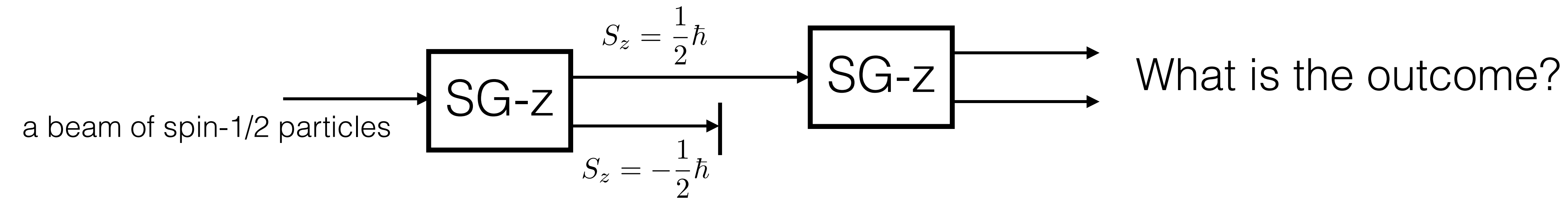
* "SG" box represents Stern-Gerlach device



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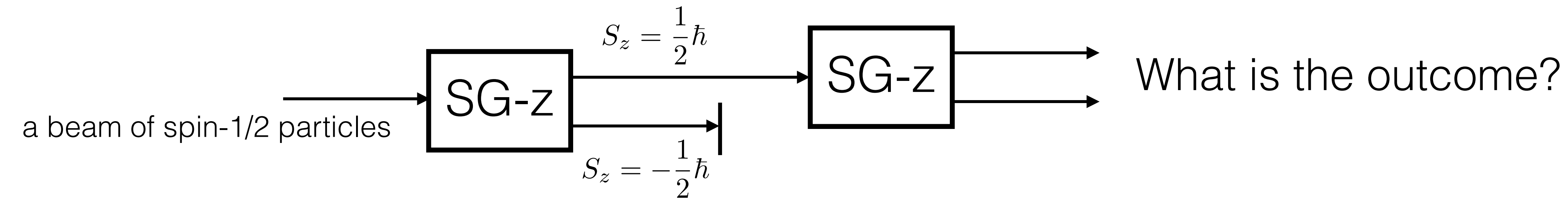


- A. 100% $S_z = +1/2$
- B. 50% $S_z = +1/2$, 50% $S_z = -1/2$
- C. 100% $S_z = -1/2$

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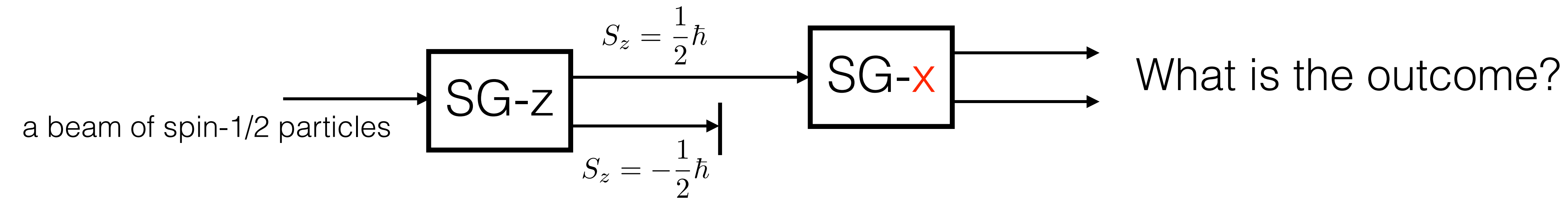
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Answer is A.

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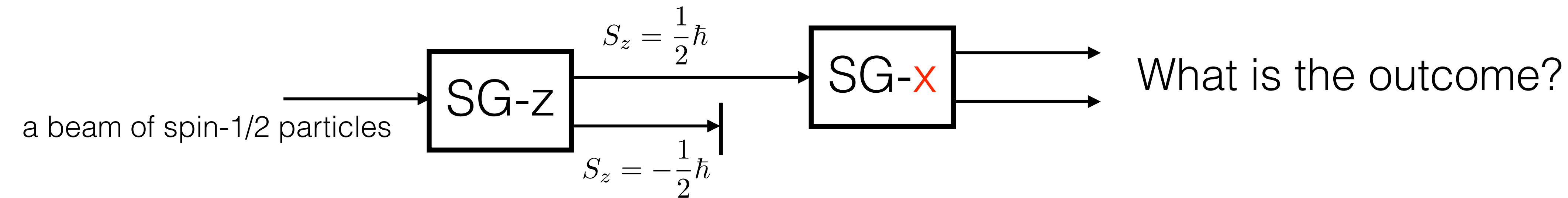


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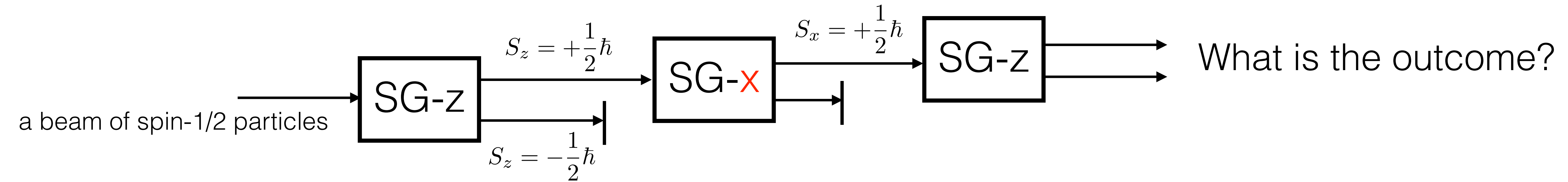
Answer is B.

Because $|+z\rangle = \frac{1}{\sqrt{2}}(|+x\rangle + |-x\rangle)$

Stern-Gerlach experiment Quiz.

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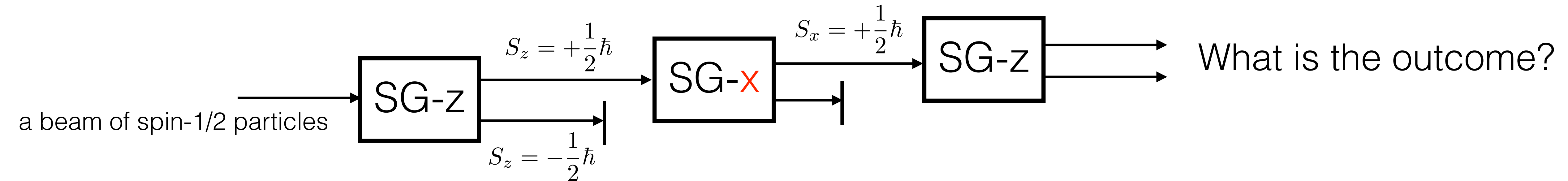
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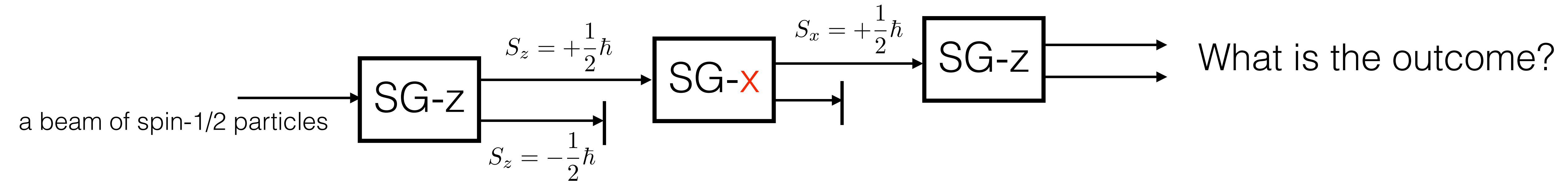


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- D. No splitting

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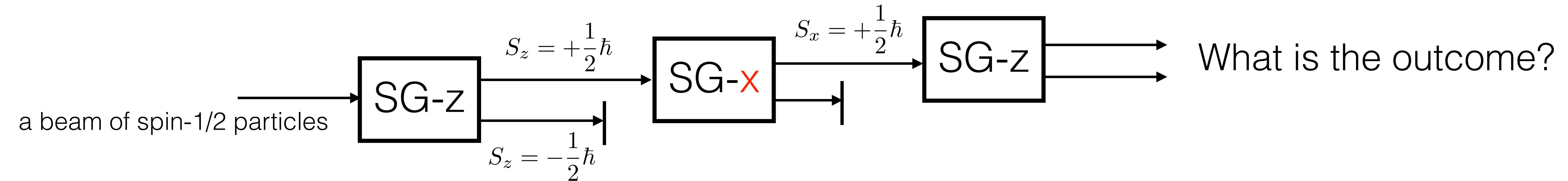
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After the SG-x measurement, the particle no longer has a definite S_z .

$$[\hat{S}_x, \hat{S}_z] \neq 0$$

S_x and S_z operators do not commute.

They are incompatible observables and cannot have definite values simultaneously.

Spin density matrix

A quantum state can be described by density matrix

$$\rho = |\psi\rangle\langle\psi|$$

For a spin-1/2 particle $|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

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Then, a mixture of spin-up and spin-down states can be written as:

$$\rho = p_{\uparrow}|\uparrow\rangle\langle\uparrow| + p_{\downarrow}|\downarrow\rangle\langle\downarrow| = \begin{pmatrix} p_{\uparrow} & 0 \\ 0 & p_{\downarrow} \end{pmatrix}$$

$$p_{\uparrow} + p_{\downarrow} = 1, \quad p_{\uparrow} - p_{\downarrow} = P$$

$p_{\uparrow}, p_{\downarrow}$: population fraction

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The density matrix ρ can be rewritten with polarization P :

$$\rho = \frac{1}{2} \begin{pmatrix} 1 + P & 0 \\ 0 & 1 - P \end{pmatrix}$$

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The density matrix describes the spin composition of an ensemble

$P=0$ (unpolarized): $\rho = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$0 < P < 1$ (partially polarized):

$$\rho = \frac{1}{2} \begin{pmatrix} 1+P & 0 \\ 0 & 1-P \end{pmatrix}$$

$P=1$ (fully polarized):

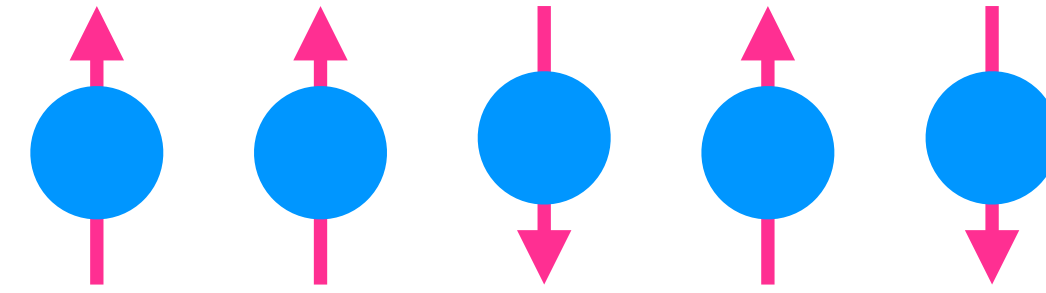
$$\rho = \frac{1}{2} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

Off-diagonal elements: quantum coherence

Statistical mixture

$$\rho = p_{\uparrow} |\uparrow\rangle\langle\uparrow| + p_{\downarrow} |\downarrow\rangle\langle\downarrow| = \begin{pmatrix} p_{\uparrow} & 0 \\ 0 & p_{\downarrow} \end{pmatrix}$$

$p_{\uparrow}, p_{\downarrow}$: statistical probability (population fraction)



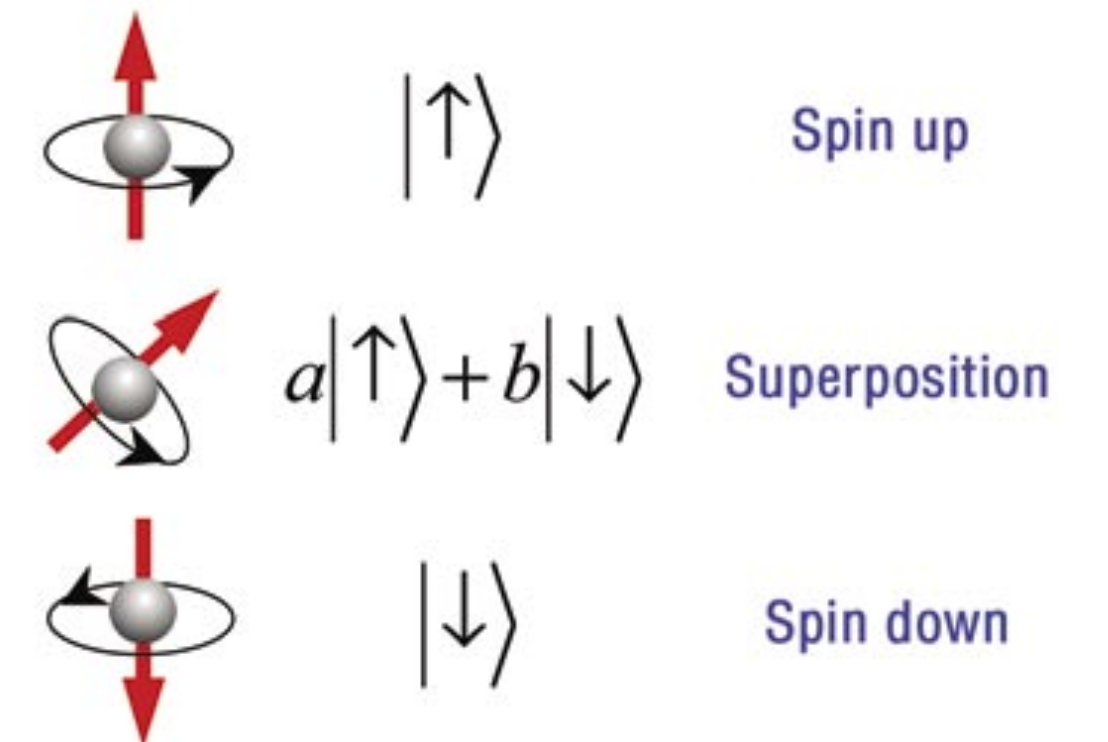
each particle is either up $\uparrow$ or down $\downarrow$

Coherent superposition

A single spin state can be a superposition of spin $\uparrow$ and $\downarrow$ states, with a definite relative phase

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$$

$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix}$$

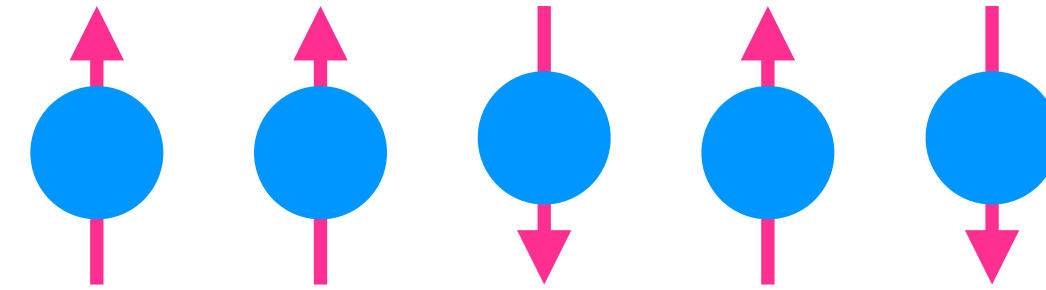


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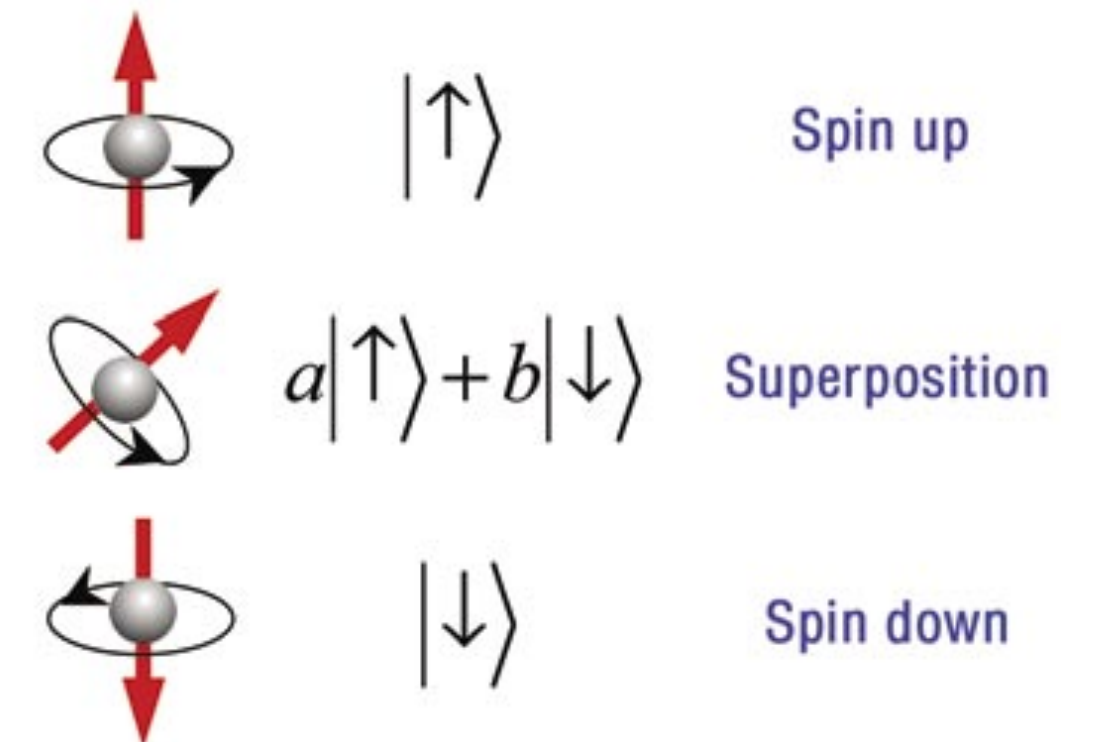
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Diagonal elements:
Probability/populations of spin states along the quantization axis

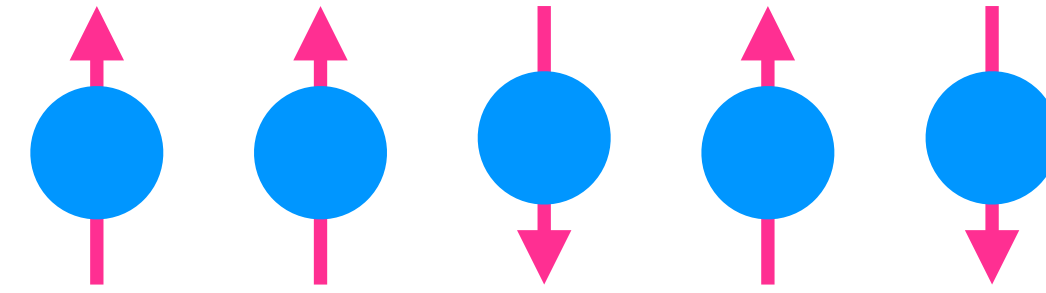


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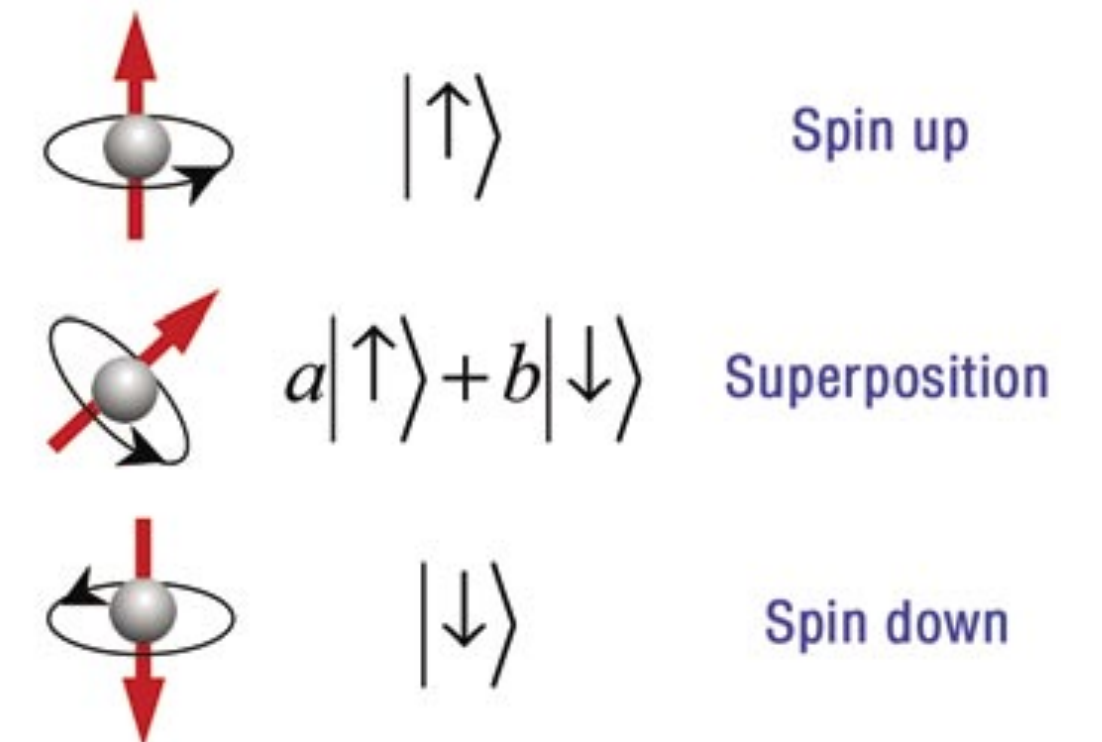
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Diagonal elements:
Probability/populations of spin states along the quantization axis

Off-diagonal elements:
Coherence between $\uparrow$ and $\downarrow$ states



General form of spin density matrix

$$\rho = \sum_i p_i |\psi\rangle \langle \psi|, \quad \sum_i p_i = 1$$

For spin-1/2

$$\begin{aligned} \rho &= \frac{1}{2} (\mathbf{I} + \mathbf{P} \cdot \boldsymbol{\sigma}) \\ &= \frac{1}{2} \begin{pmatrix} 1 + P_z & P_x - iP_y \\ P_x + iP_y & 1 - P_z \end{pmatrix} \end{aligned}$$

$\mathbf{P}$: polarization vector
 $\boldsymbol{\sigma}$: Pauli matrices

*choose z-axis as the quantization axis

For spin-1

$$m = +1, 0, -1$$

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1,-1} \\ \rho_{01} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$

The diagonal elements represent the populations of the three spin-projection states.

$$\text{Tr}(\rho) = \rho_{11} + \rho_{00} + \rho_{-1,-1} = 1$$

In case for equal populations: $\rho_{11} = \rho_{00} = \rho_{-1,-1} = \frac{1}{3}$

→ Vector-meson spin alignment

Polarization vs. spin alignment

*choose z-axis as the quantization axis

For spin-1/2

$$\rho = \begin{pmatrix} p_{\uparrow} & 0 \\ 0 & p_{\downarrow} \end{pmatrix}, \quad P_z = p_{\uparrow} - p_{\downarrow}$$

For spin-1

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1,-1} \\ \rho_{01} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$

If unpolarized, all three states are equally populated

$$\rho_{11} = \rho_{00} = \rho_{-1,-1} = \frac{1}{3}$$

Vector polarization: imbalance between $m=+1$ and $m=-1$

$$P_z \propto (p_{+1} - p_{-1})$$

Spin alignment: imbalance between $m=0$ and $m=\pm 1$ populations

Even for vector polarization $P_z=0$,
“spin alignment” ($\rho_{00} \neq 1/3$) could happen, and vice versa

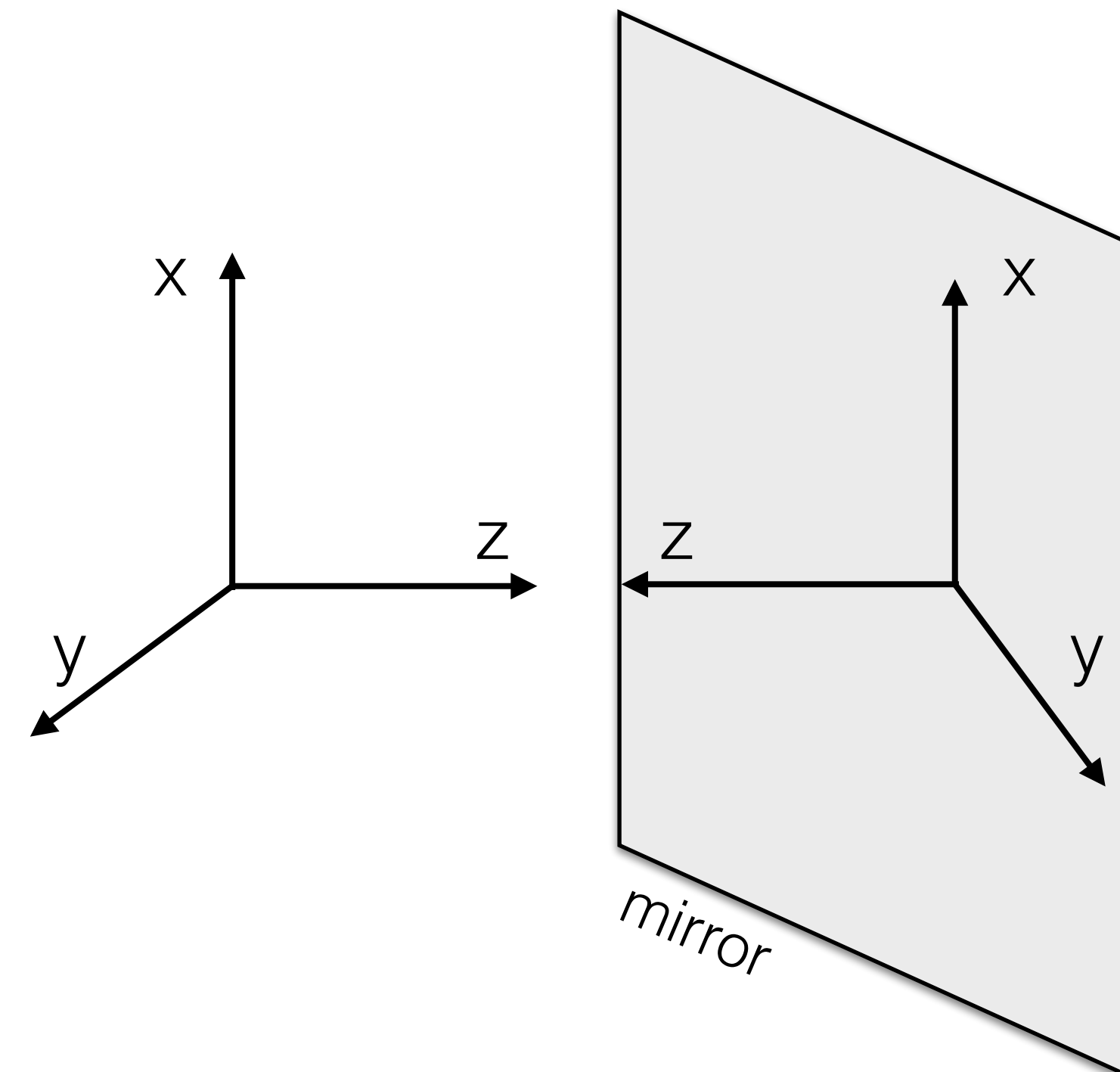
What is parity?

- Parity (P) conservation is a fundamental symmetry: physical laws don't change when flipping the sign of spatial position, $\mathbf{r} \rightarrow -\mathbf{r}$, $\mathbf{p} \rightarrow -\mathbf{p}$

- $\mathbf{r}$ and $\mathbf{p}$ are called polar (true) vector, while $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ is called axial vector (pseudovector)

$$\mathbf{L} \xrightarrow{P} (-\mathbf{r}) \times (-\mathbf{p}) = \mathbf{L}$$

- “Mirror image” is often used to explain the parity transformation but it flips only one of the direction (x, y, z)



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- Parity = Mirror + 2D-rotation

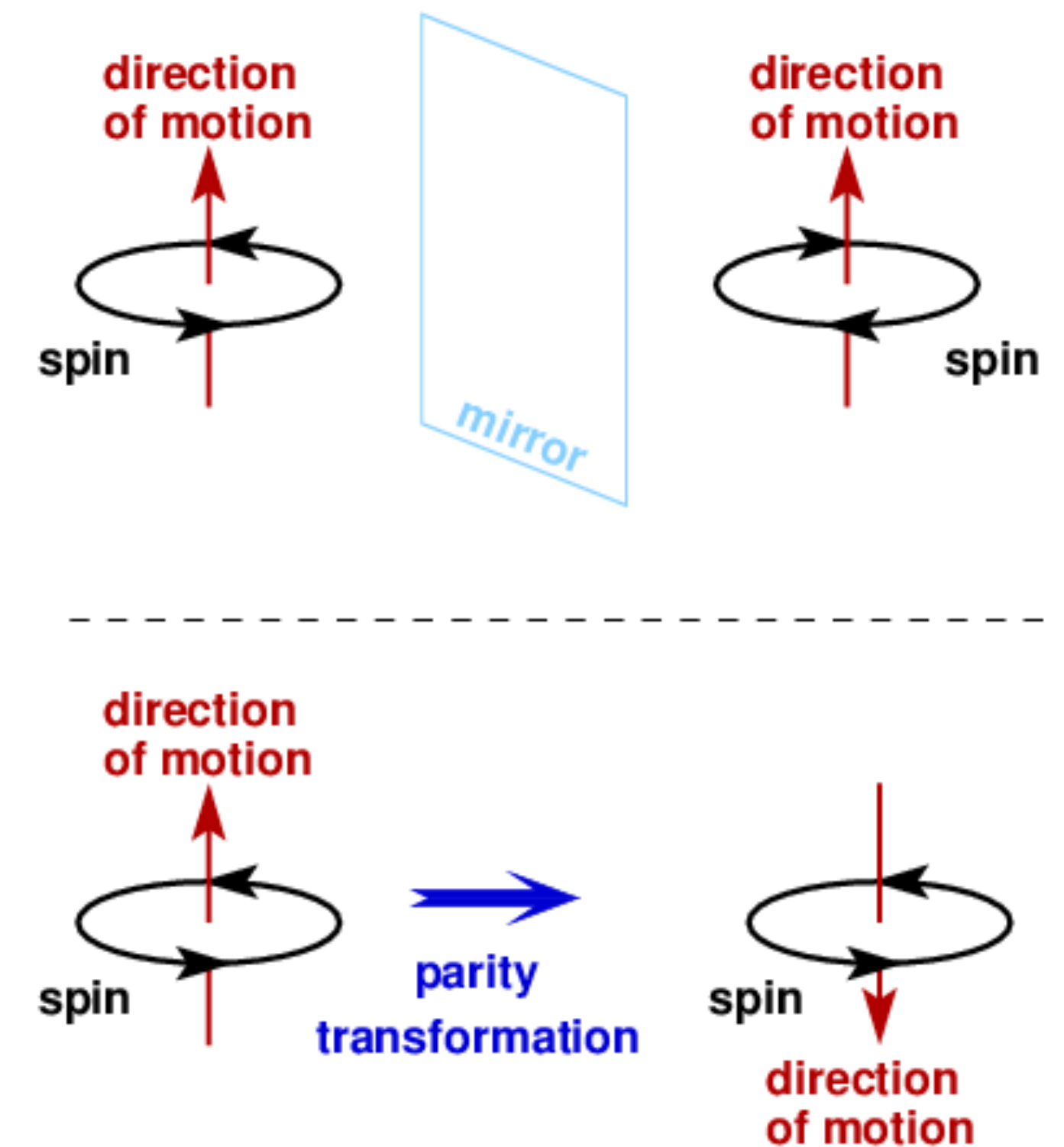
e.g. flip z and rotate in xy by π

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \xrightarrow{P} \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

Parity violation in weak interaction

- In 1950s, experimental studies showed that parity is conserved in strong and electromagnetic interactions. But Lee and Yang noticed that there is no experiment to prove parity conservation in weak interaction.



Parity violation in weak interaction

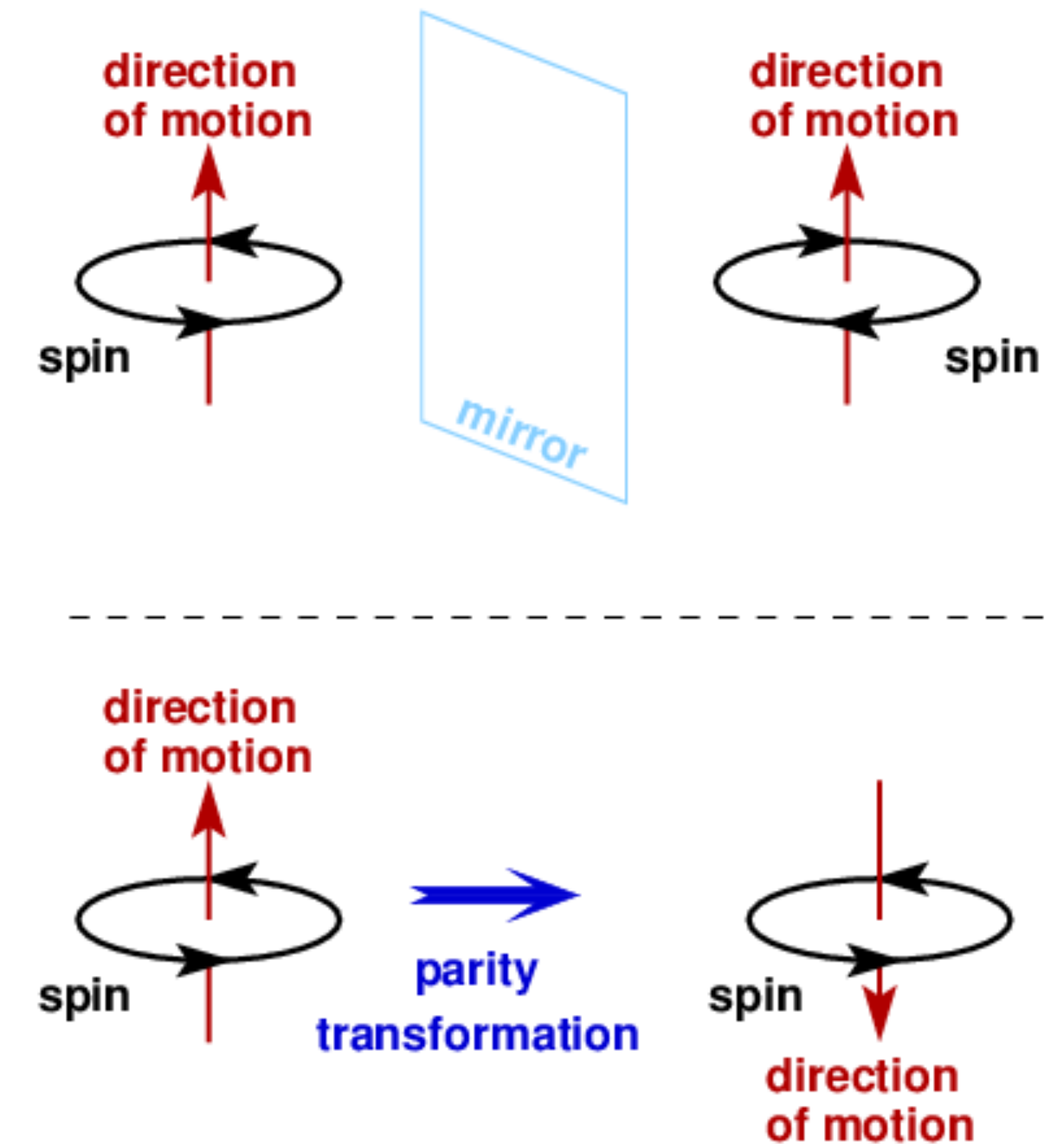
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- τ - Θ puzzle

- “ τ -particle” and “ Θ -particle” named at that time have the same mass, charge and lifetime but decay differently

$$\Theta \rightarrow \pi^+ \pi^0$$

$$\tau \rightarrow \pi^+ \pi^+ \pi^-$$



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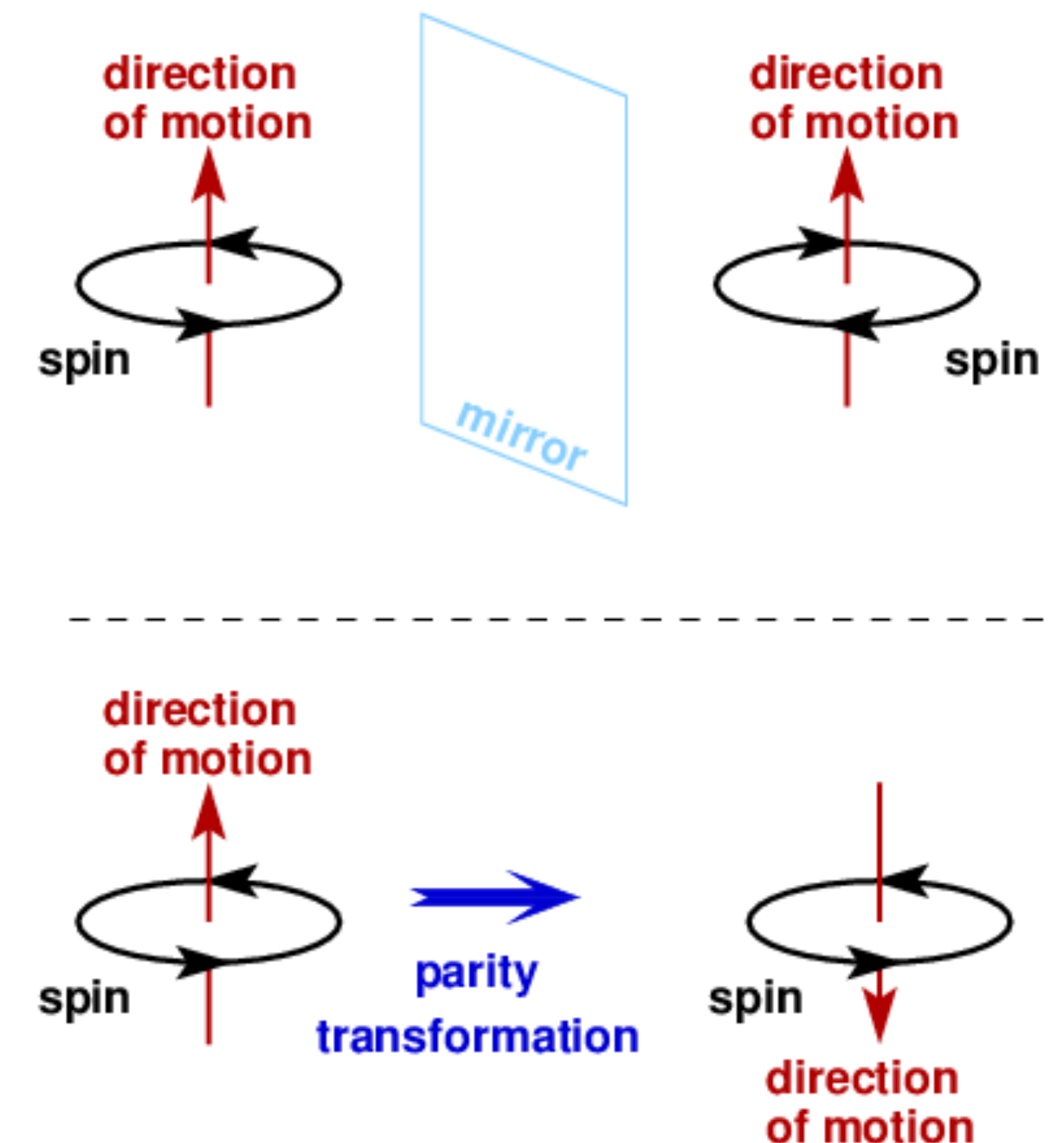
$$\Theta \rightarrow \pi^+ \pi^0$$

$$\tau \rightarrow \pi^+ \pi^+ \pi^-$$

pion has spin 0 and negative parity

$$2\pi \text{ decay: } P = (-1)^2 = 1$$

$$3\pi \text{ decay: } P = (-1)^3 = -1$$



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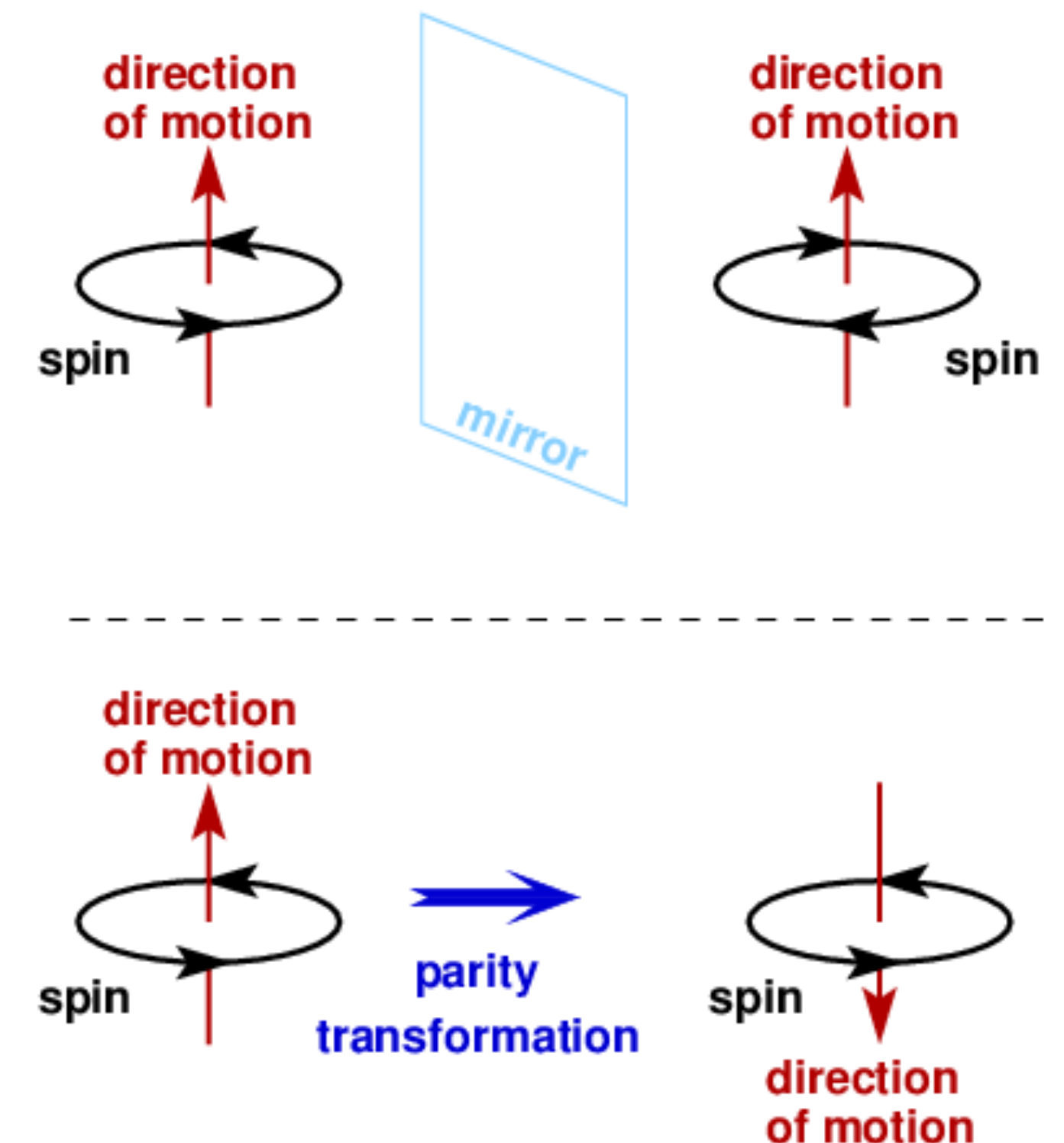
$$\Theta \rightarrow \pi^+ \pi^0$$

$$\tau \rightarrow \pi^+ \pi^+ \pi^-$$

pion has spin 0 and negative parity

$$2\pi \text{ decay: } P = (-1)^2 = 1$$

$$3\pi \text{ decay: } P = (-1)^3 = -1$$



If they are the same particle, the puzzle is solved but the parity is broken in weak interaction

Parity violation in weak interaction

- In 1950s, experimental studies showed that parity is conserved in strong and electromagnetic interactions. But Lee and Yang noticed that there is no experiment to prove parity conservation in weak interaction.

- τ - Θ puzzle

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$$\Theta \rightarrow \pi^+ \pi^0$$

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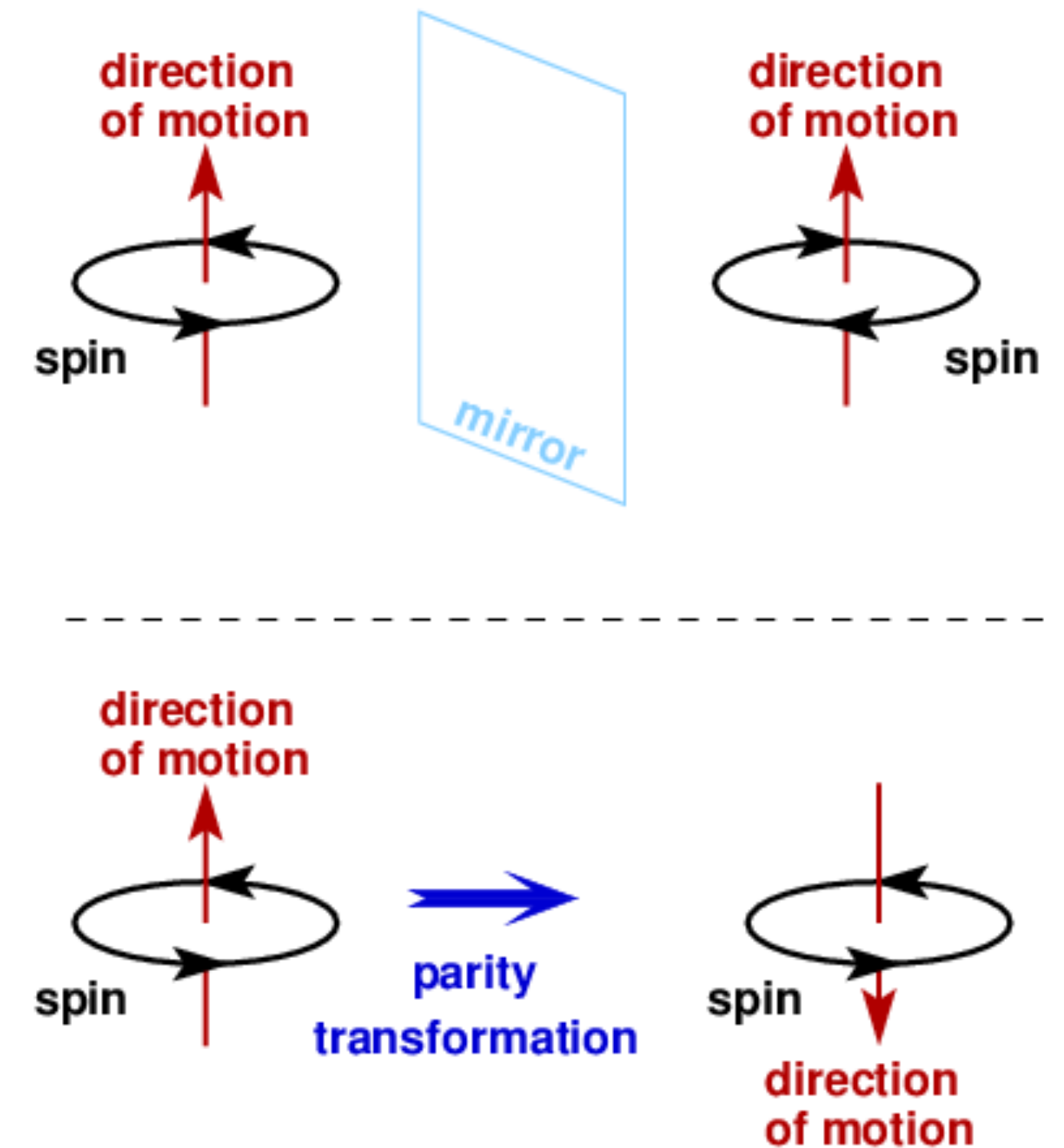
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If they are the same particle, the puzzle is solved but the parity is broken in weak interaction

Now we know that the τ - Θ puzzle is two decay modes of K^+ meson



How we test parity conservation?

- Spin is an axial vector like orbital angular momentum

$$\mathbf{r}, \mathbf{p} \xrightarrow{P} -\mathbf{r}, -\mathbf{p}$$

$$\mathbf{L} \xrightarrow{P} (-\mathbf{r}) \times (-\mathbf{p}) = \mathbf{L} \quad \mathbf{S} \xrightarrow{P} \mathbf{S}$$

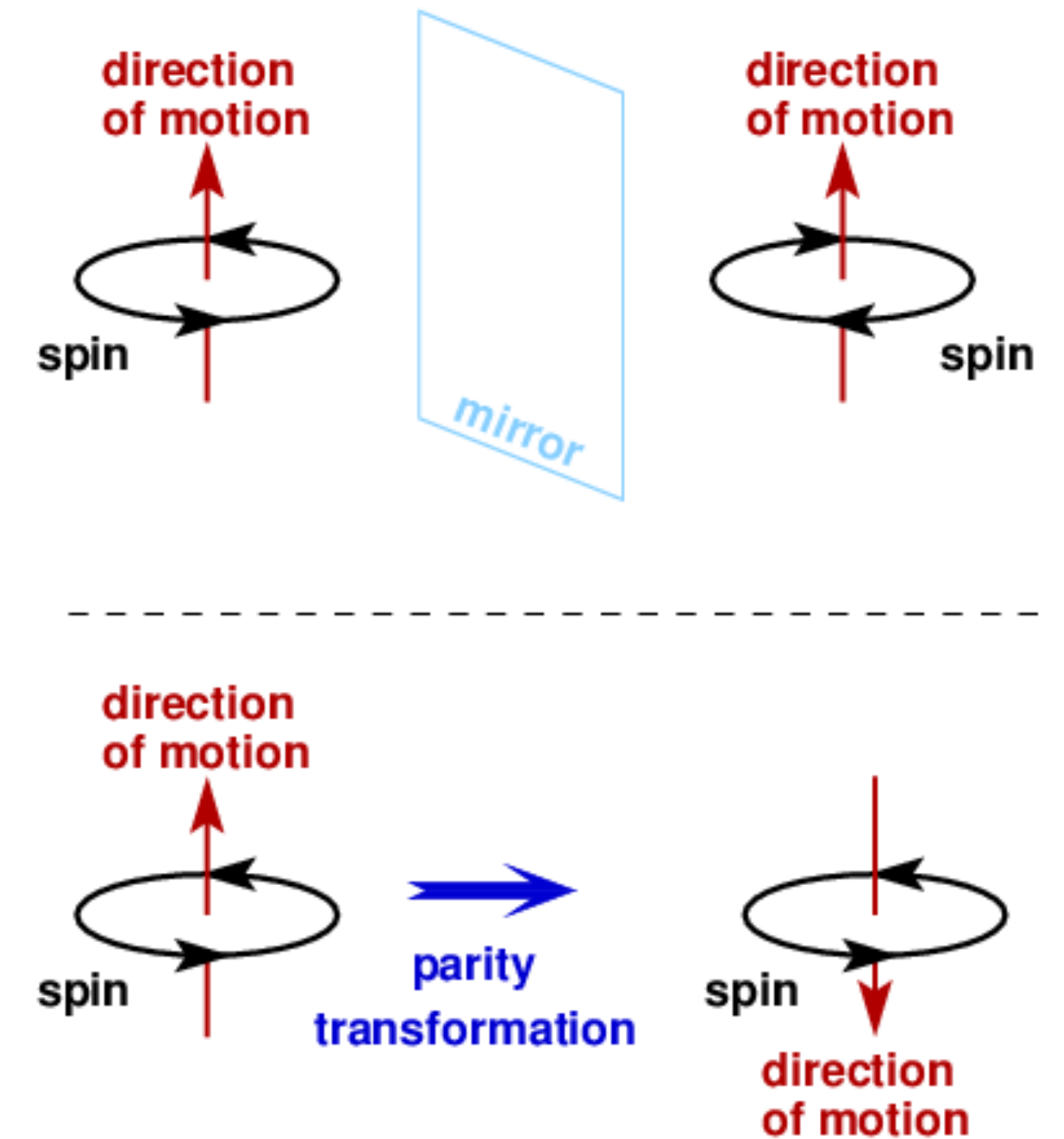
- Consider the following beta decay: $X \rightarrow X' + e^- + \bar{\nu}_e$

- Spin of nucleus X is J, momentum of electron is $\mathbf{p}_e$

$$\mathbf{J} \cdot \mathbf{p}_e \xrightarrow{P} -\mathbf{J} \cdot \mathbf{p}_e$$

If parity is conserved, two configurations above must occur with equal probability.

In other words, angular distribution needs to be symmetric (A=0)



$$\frac{dN}{d\Omega} \propto 1 + A \hat{\mathbf{J}} \cdot \hat{\mathbf{p}}_e$$

Parity violation in weak interaction

Photos from <https://www.aps.org/apsnews>

- 1956: Proposed by Lee and Yang

T.D.Lee and C.N.Yang, Phys.Rev.104, 254(1956)

- 1957: Wu et al. discovered P-violation in β -decay of ^{60}Co

C.S. Wu et al., Phys.Rev.105, 10413(1957)

PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

T. D. LEE, *Columbia University, New York, New York*

AND

C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

Experimental Test of Parity Conservation in Beta Decay*

C. S. WU, *Columbia University, New York, New York*

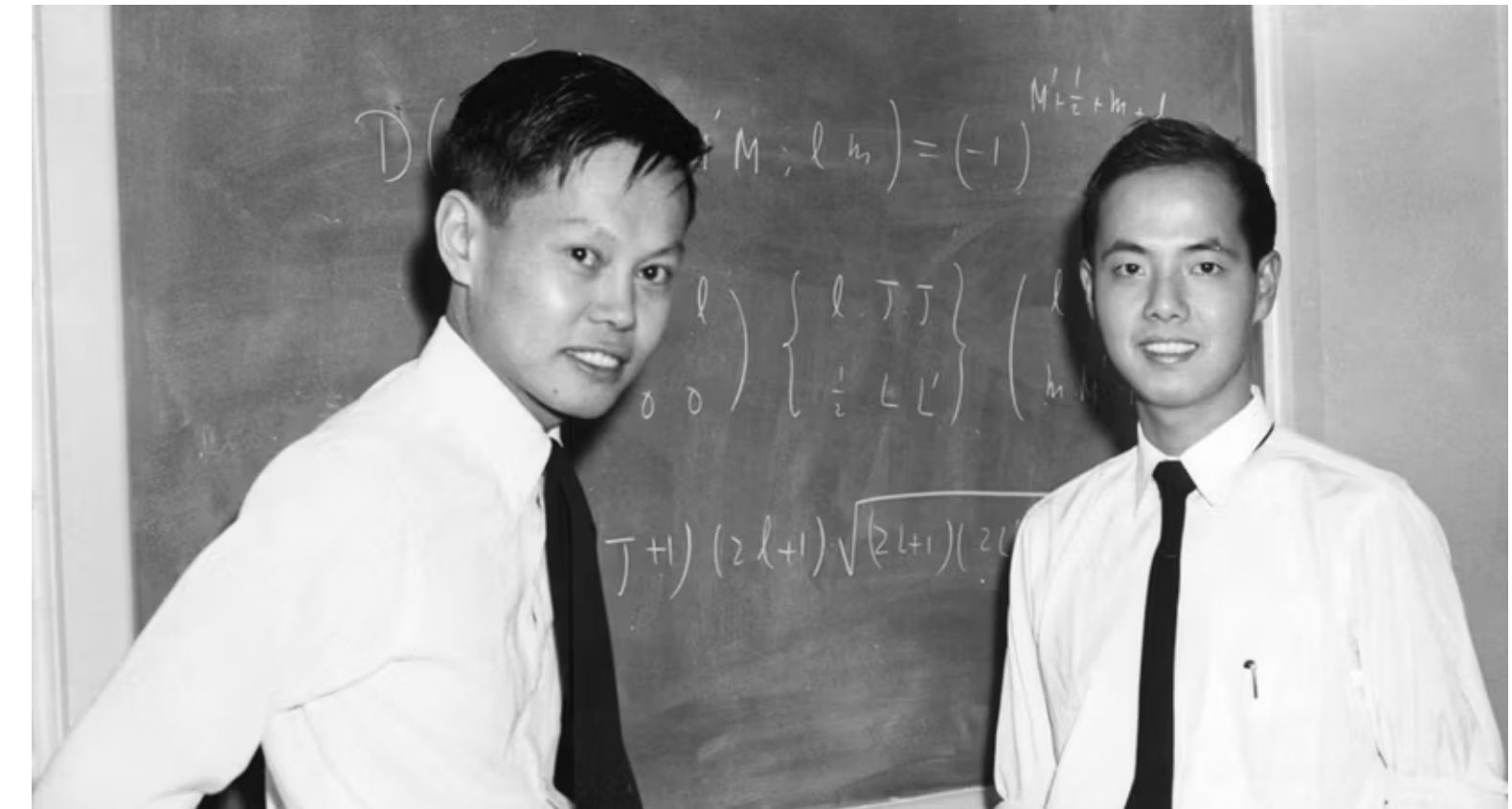
AND

E. AMBLER, R. W. HAYWARD, D. D. HOPPES, AND R. P. HUDSON,
National Bureau of Standards, Washington, D. C.

(Received January 15, 1957)

[Article on PhysicsWorld \(Feb. 2026\)](#)

*“Twenty-three nominations, yet no Nobel prize:
how Chien-Shiung Wu missed out on the top award in physics”*



Chen-Ning Yang

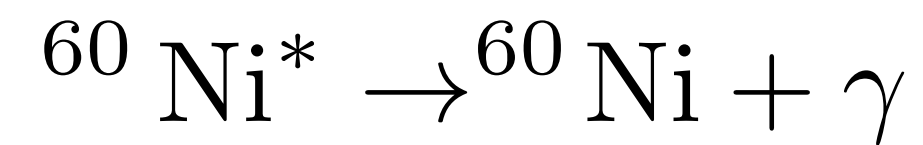
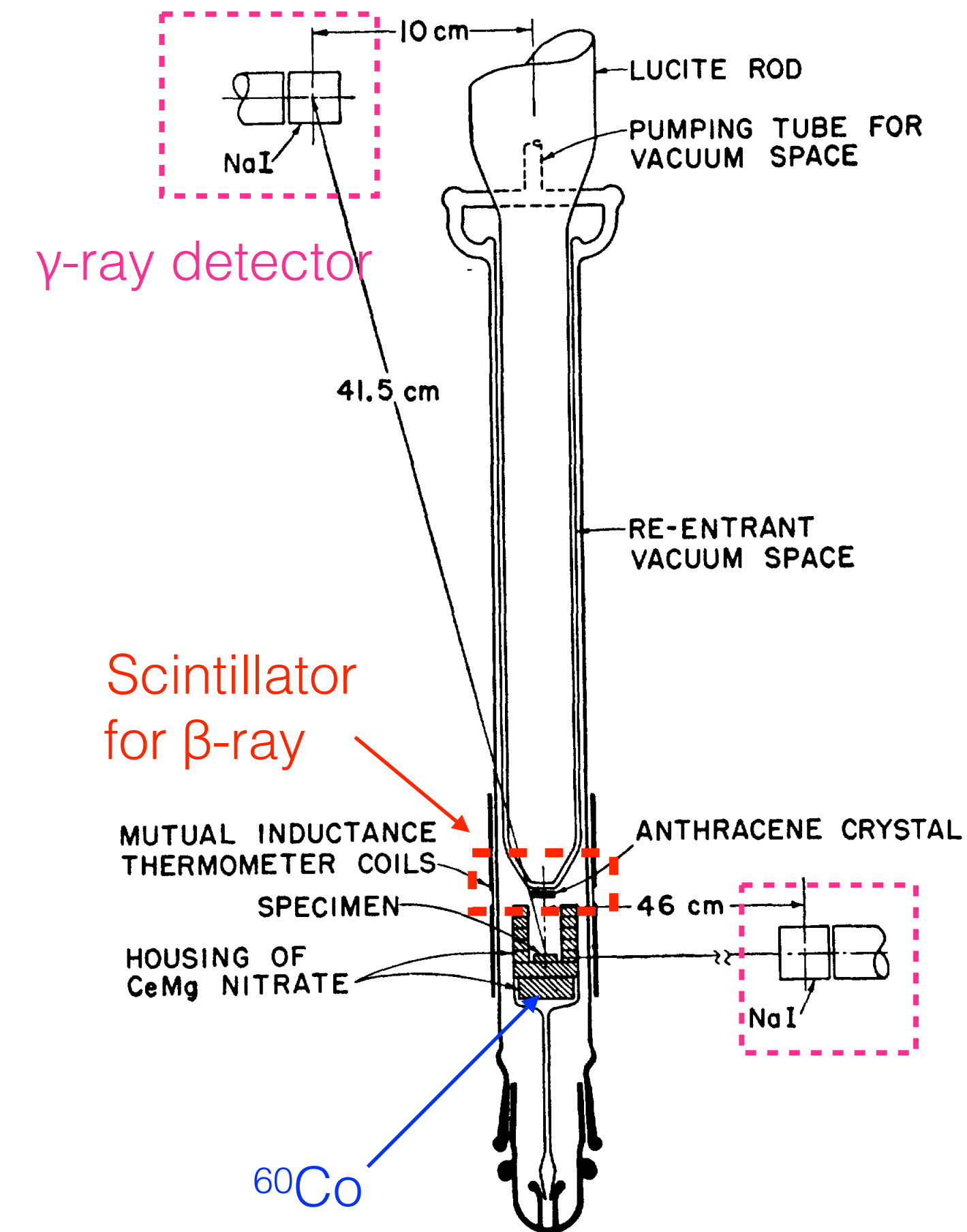
Tsung-Dao Lee

Chien-Shiung Wu

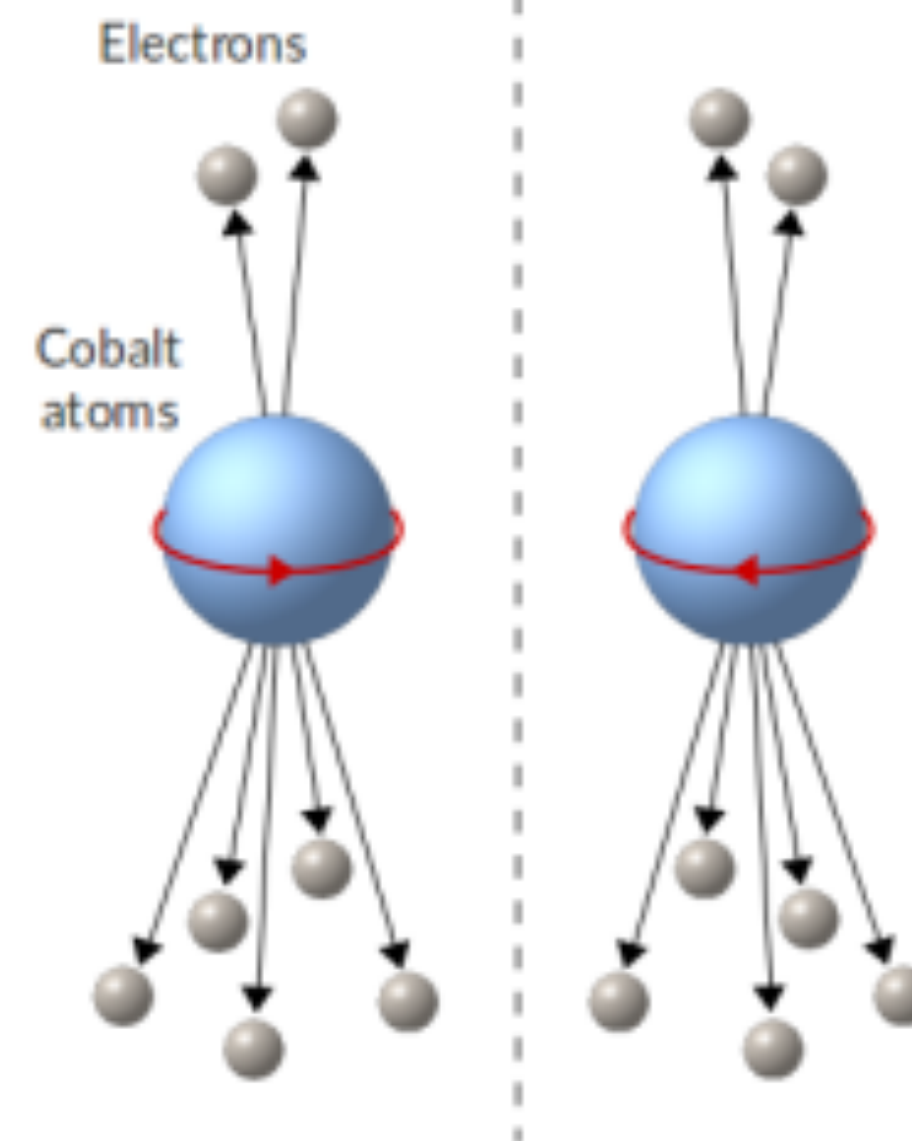


Wu experiment

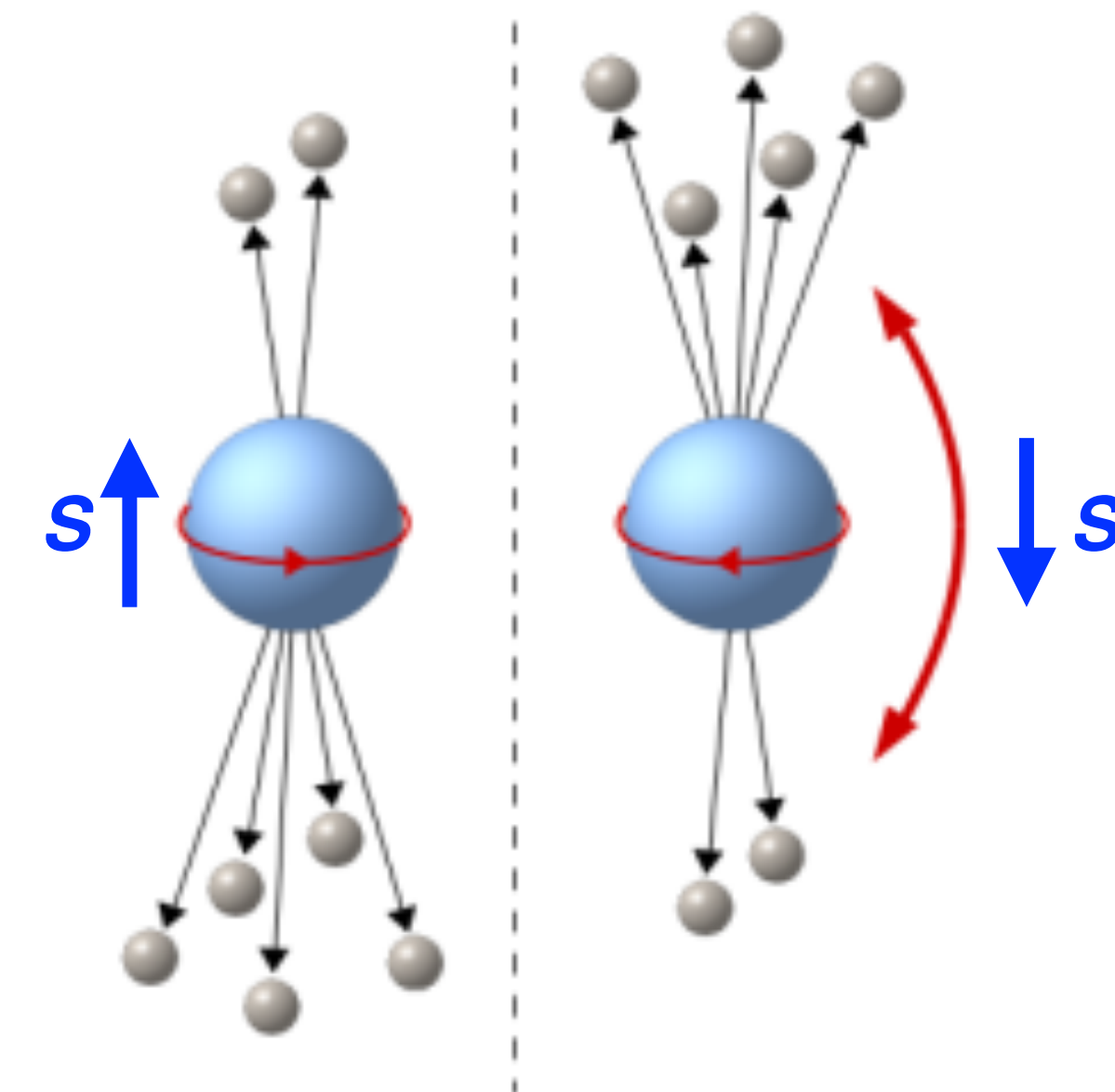
C.S. Wu et al., Phys.Rev.105, 10413(1957)



If parity conservation held true



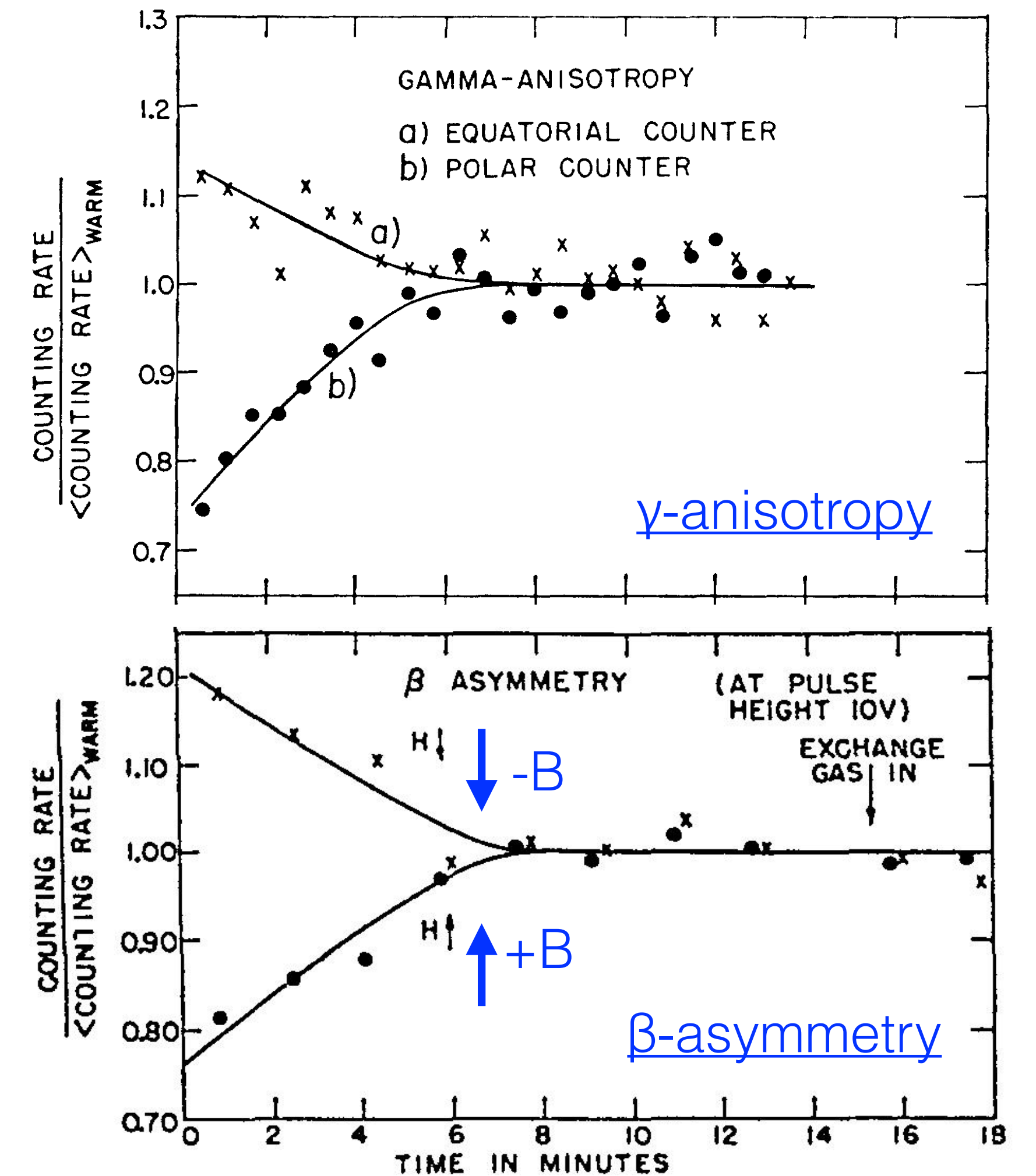
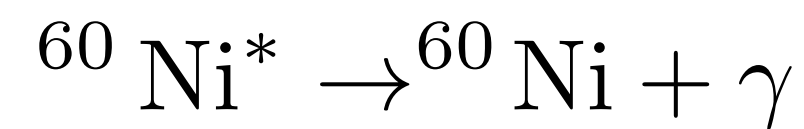
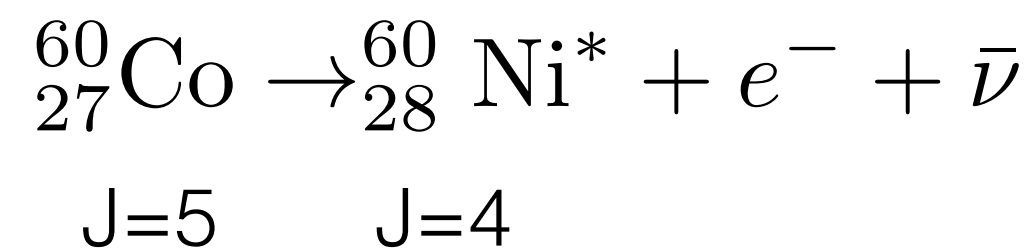
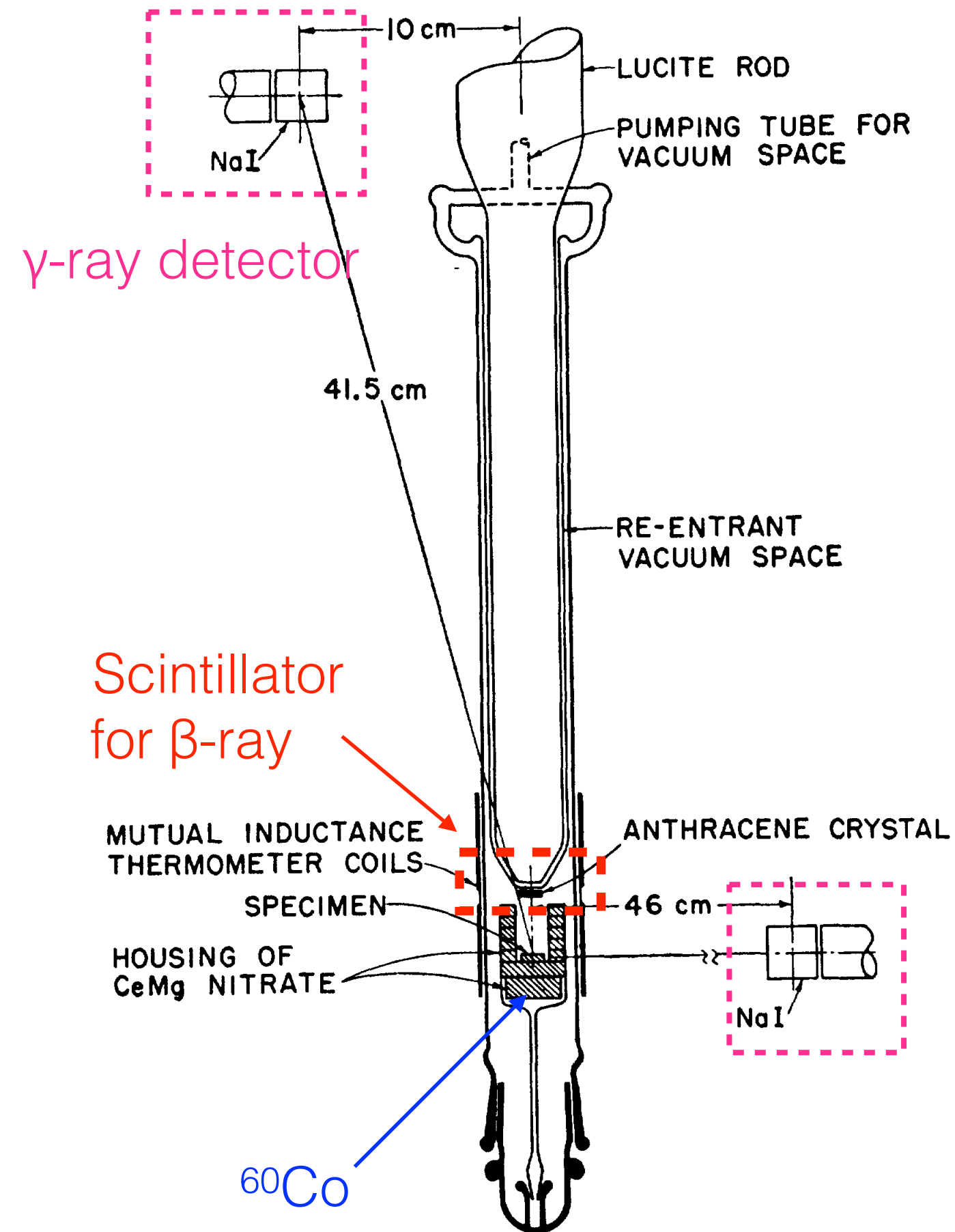
Experimental results



- ^{60}Co spin aligned by external B-field (Cool to 0.01 K)
- checked by the measurement of emitted γ
- Measurement of emitted electrons

Wu experiment

C.S. Wu et al., Phys.Rev.105, 10413(1957)



→ warm up with time
nuclear spins gradually randomized
(due to thermal fluctuations)

Electrons are preferentially emitted opposite to ${}^{60}\text{Co}$ spin direction!
→ Parity violation in β -decay

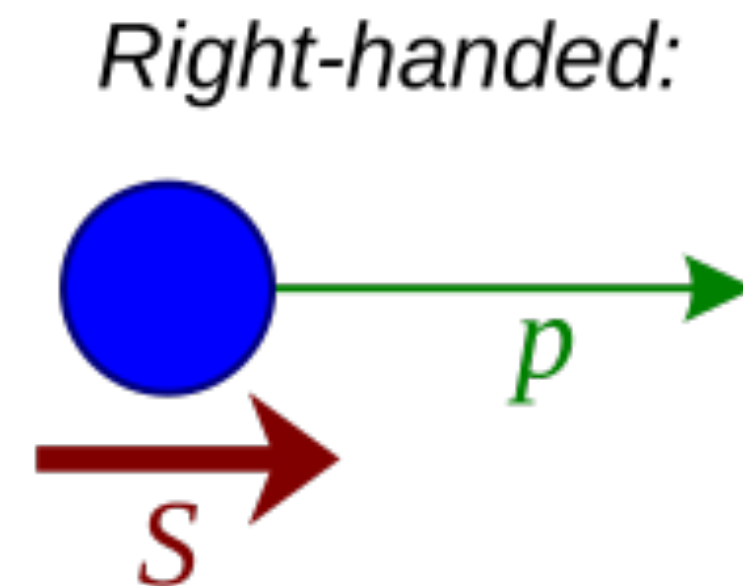
Helicity and Chirality

- Helicity is the projection of spin onto the momentum direction

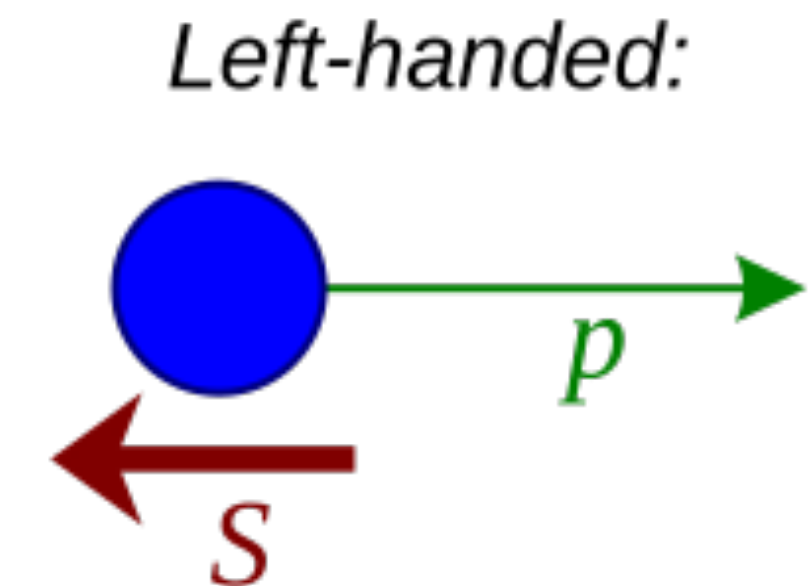
$$h = \frac{\vec{S} \cdot \vec{p}}{|\vec{p}|}$$

$\vec{S}$: spin vector

$\vec{p}$: momentum vector

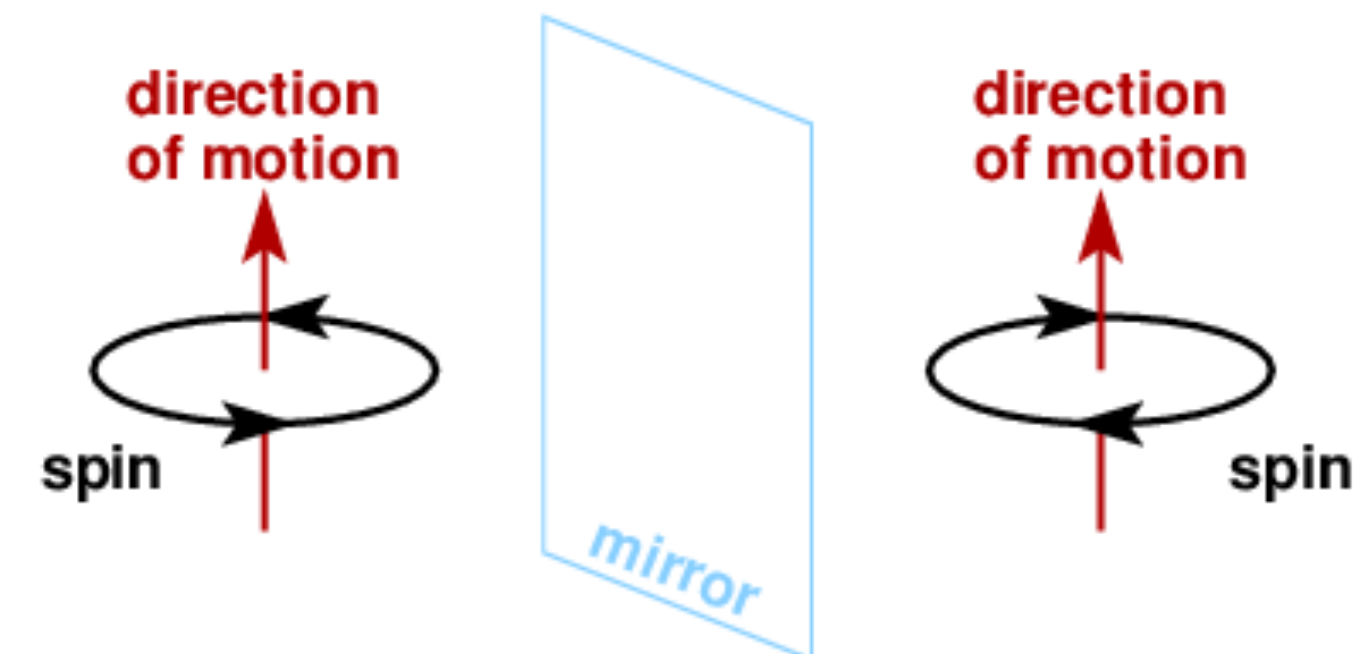


positive helicity

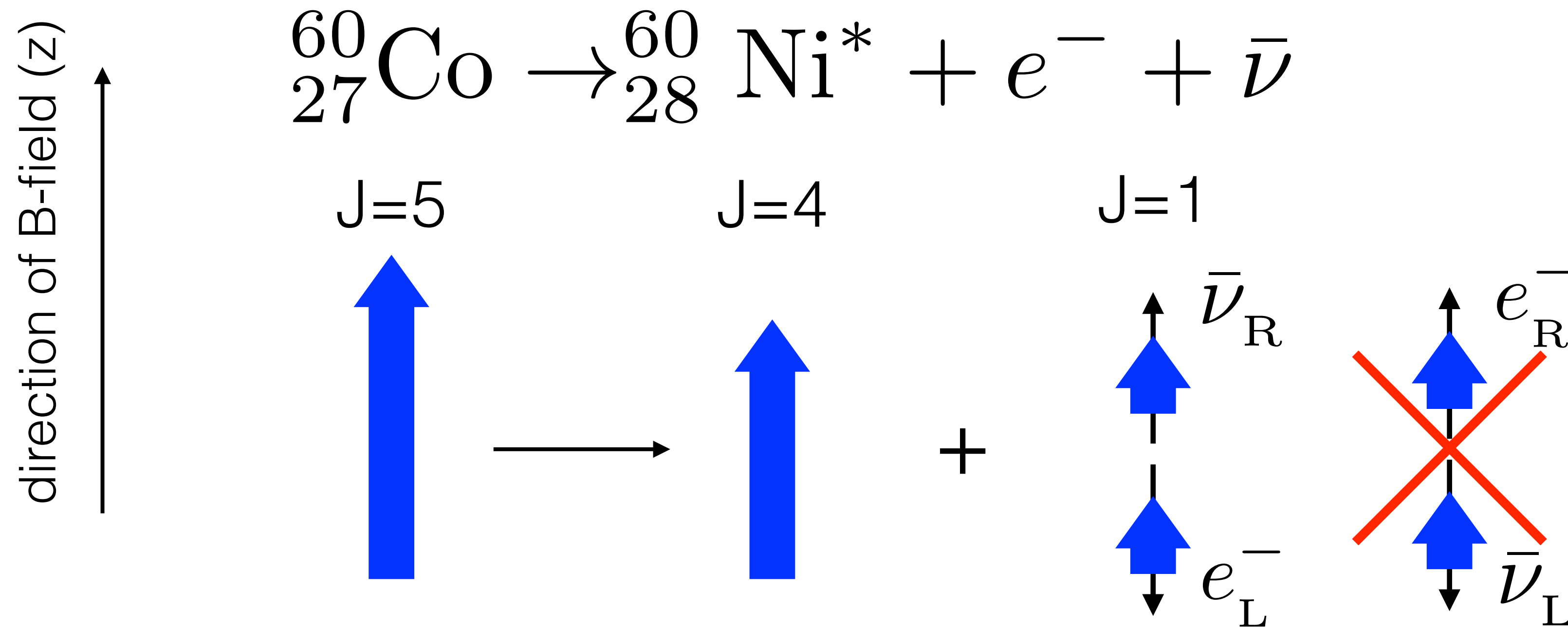


negative helicity

- Chirality is a quantum property that classifies particles into left-handed and right-handed states
- whether the object is distinguishable from its mirror image
 - For massless particles, chirality = helicity

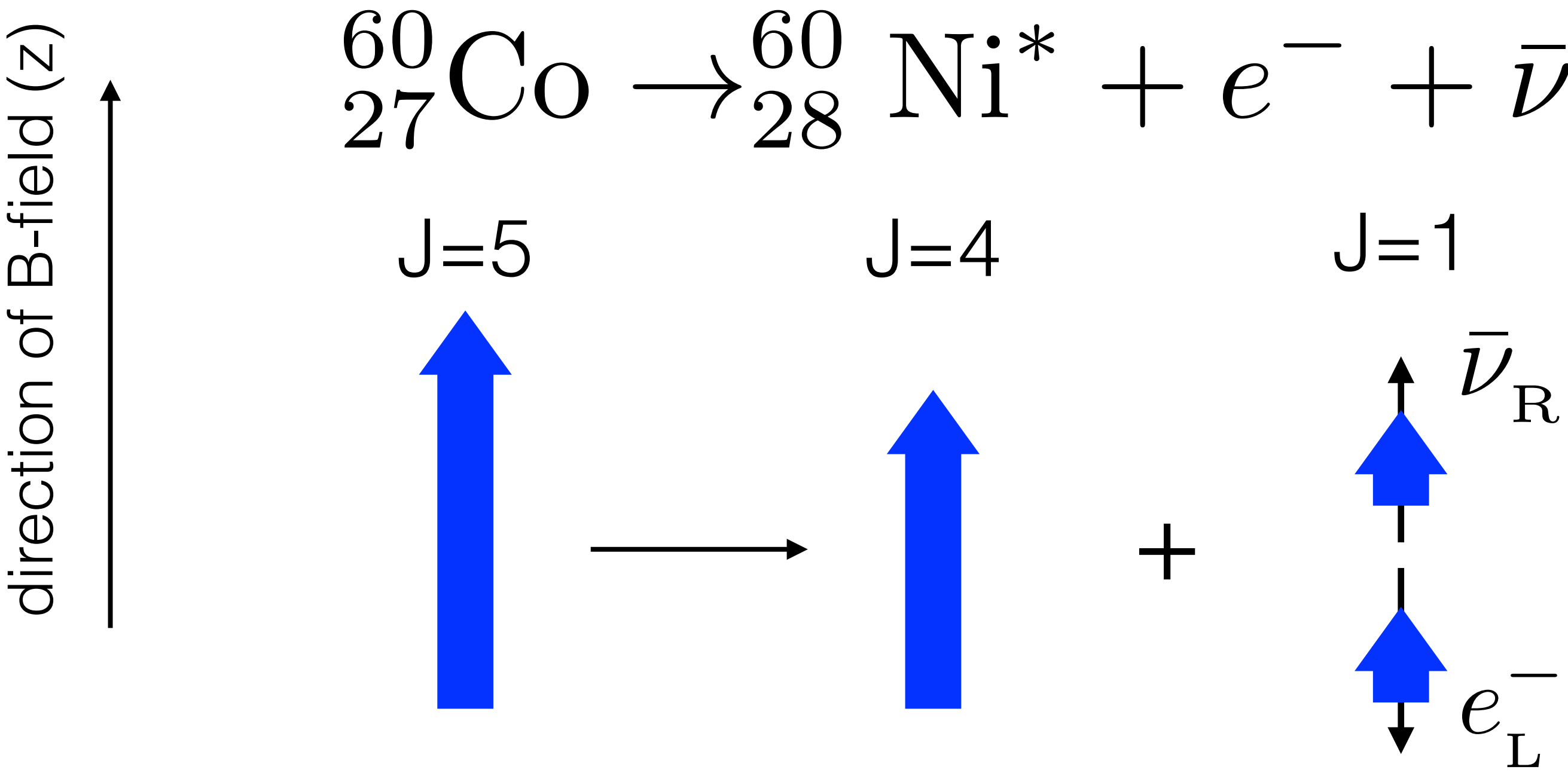


Qualitative explanation



Weak interactions (via $W^\pm$) couple only to left-chiral fermions and right-chiral anti-fermions
 For a massless or ultra-relativistic particle, chirality corresponds to helicity

Qualitative explanation



M. Goldhaber, L. Grodzins, and A.W. Sunyar
 Phys.Rev.109, 1015 (1958)

Helicity of Neutrinos*

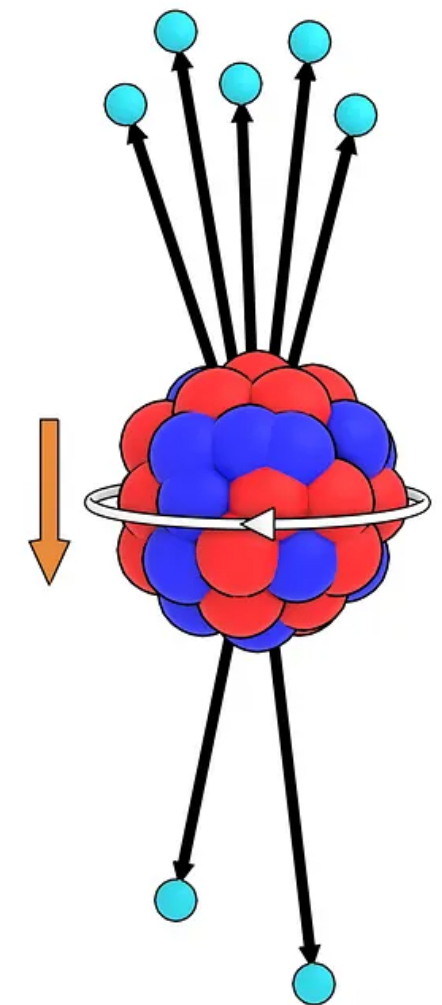
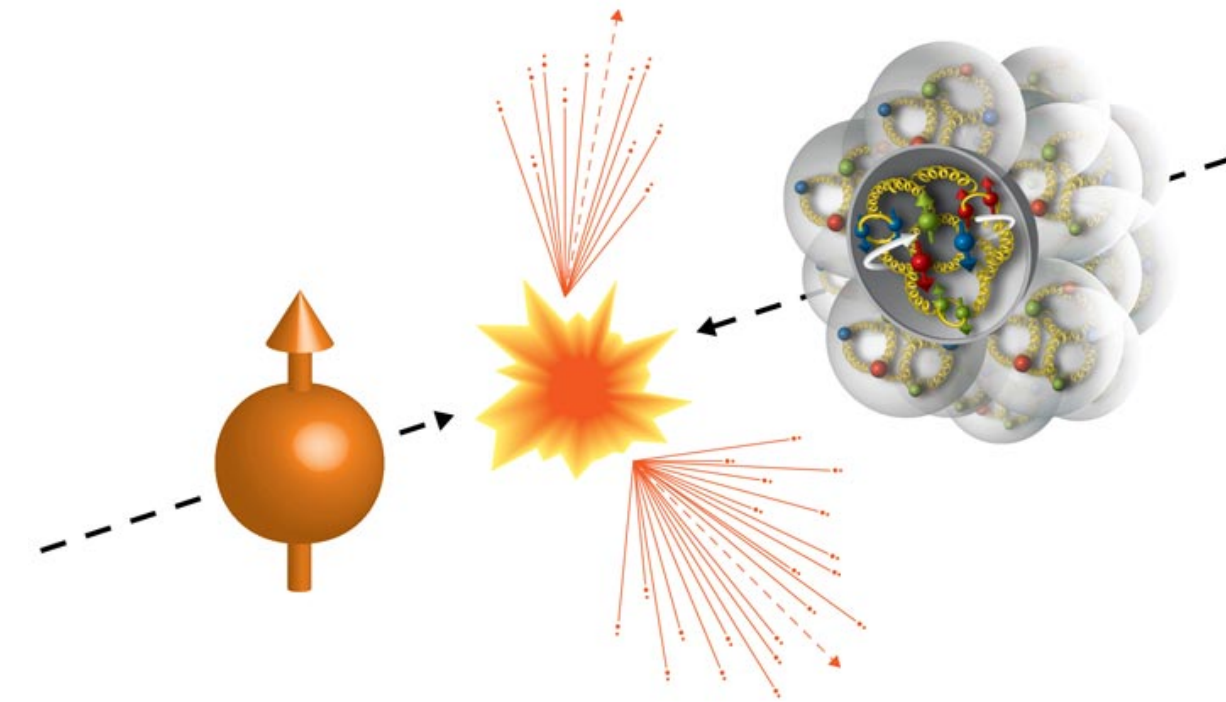
M. GOLDHABER, L. GRODZINS, AND A. W. SUNYAR
Brookhaven National Laboratory, Upton, New York
 (Received December 11, 1957)

A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m} , which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,¹ 0^- , we find that the neutrino is “left-handed,” i.e., $\sigma_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).

Goldhaber experiment shows the existence of only left-handed neutrinos (right-handed anti-neutrinos) in electron capture mediated by the weak interaction

Polarization measurement

- ▶ Spin itself is not directly observable.
- ▶ We infer spin information from spin-dependent angular distributions in scattering or particle decays.
- ▶ Weak decay is particularly useful because parity violation makes them self-analyzing.



Weak decay as a spin analyzer

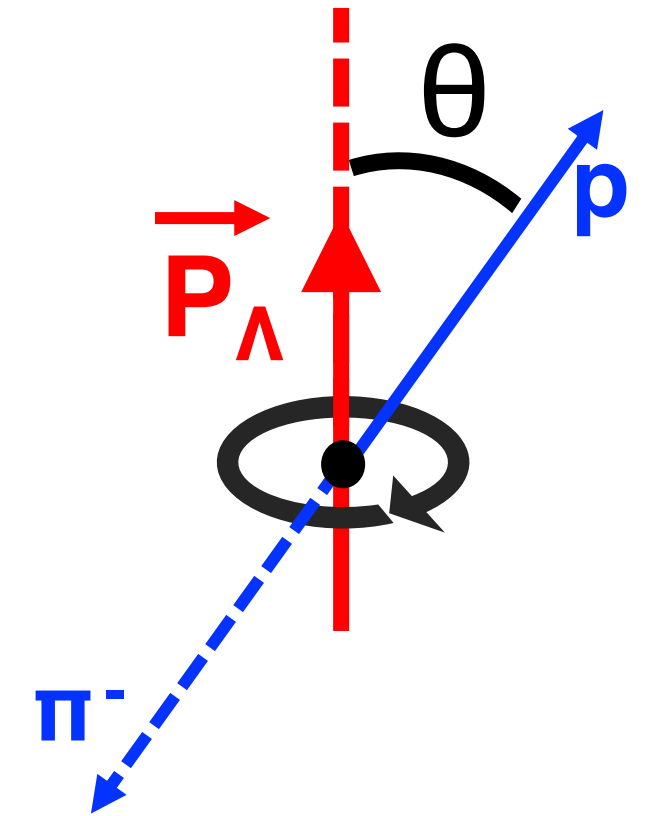
$$\frac{dN}{d\Omega} = \frac{1}{4\pi} (1 + \alpha P \cos \theta)$$

P : polarization in the **parent rest frame**

θ : angle between daughter particle momentum and parent spin direction
in the **parent rest frame**

α : weak decay parameter

$$\Lambda \rightarrow p + \pi^-$$



Correlation between parent's spin and decay product momentum directions

Weak decay as a spin analyzer

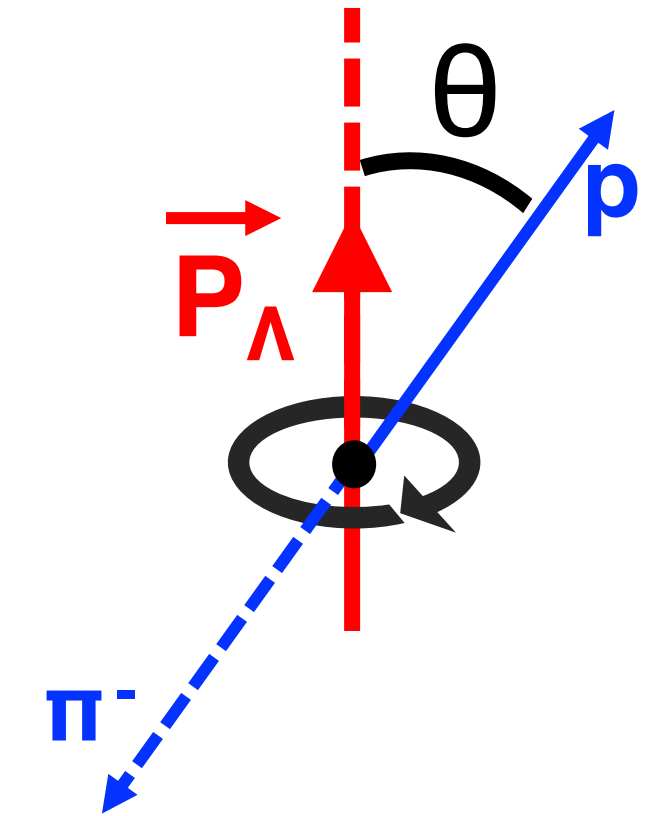
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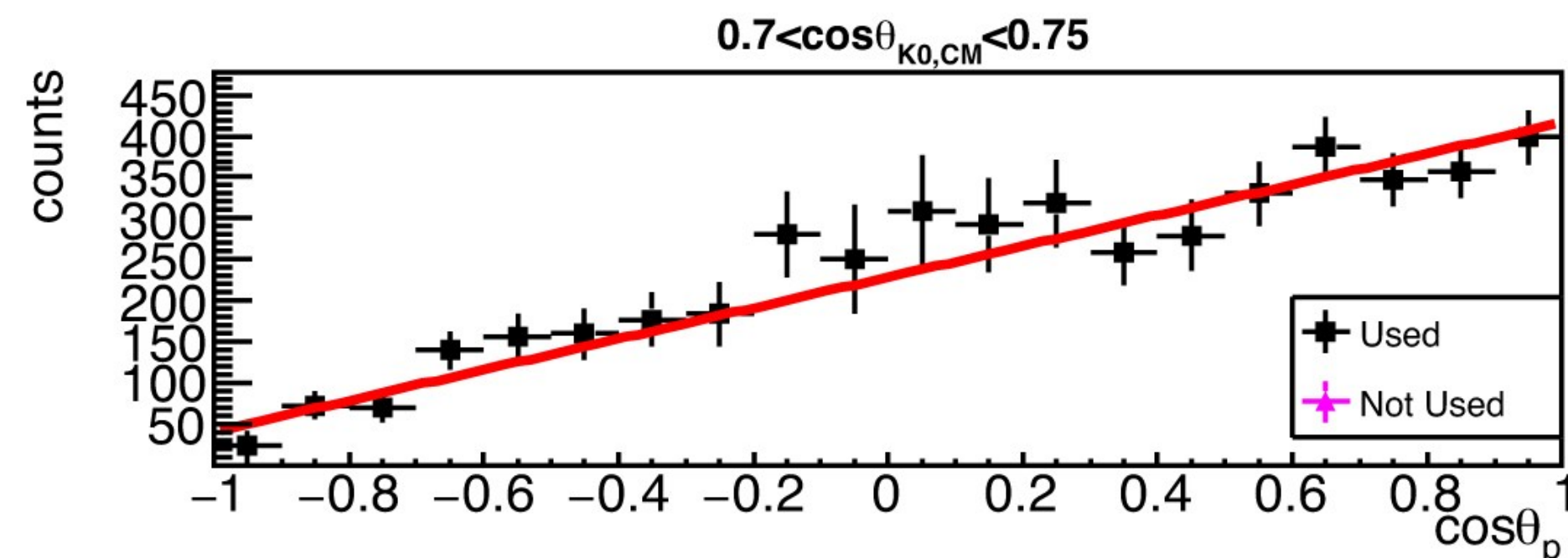
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Correlation between parent's spin and decay product momentum directions

Sakao et al., PTEP2026, 013D01 (2026)



Weak decay as a spin analyzer

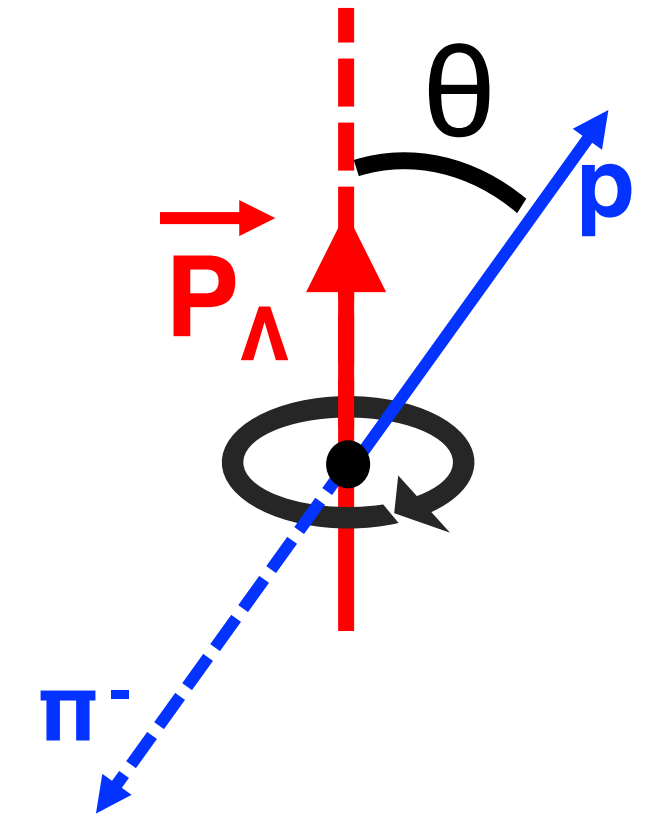
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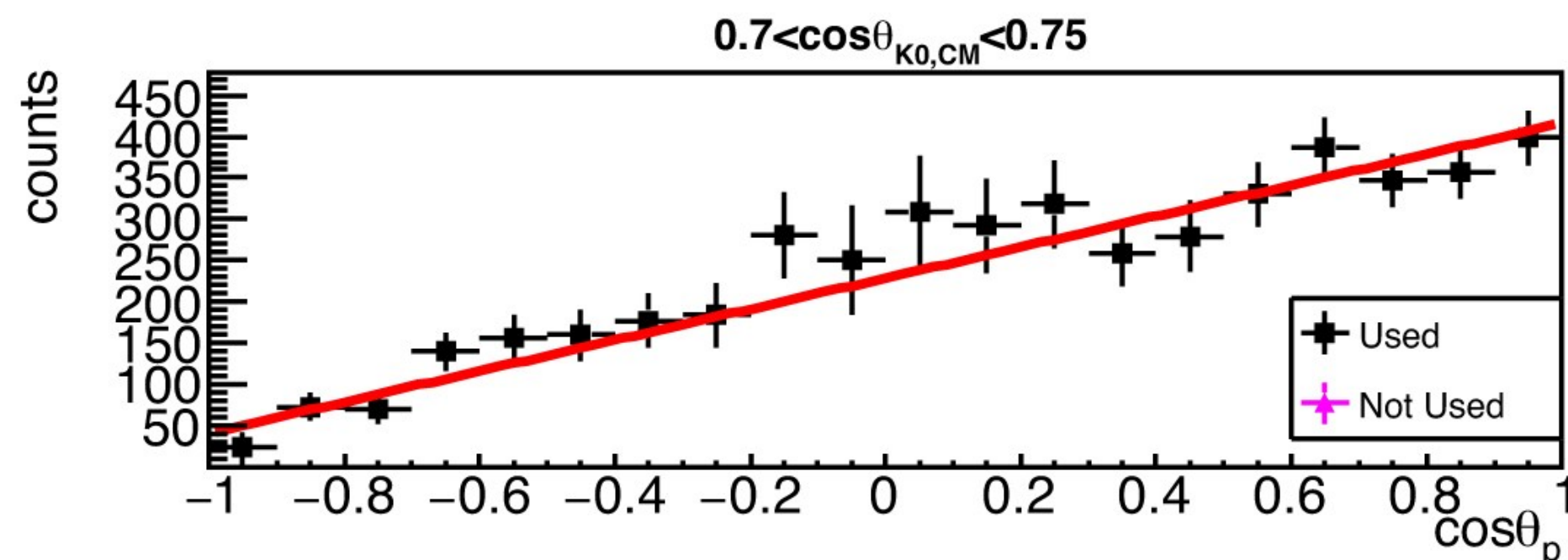
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Correlation between parent's spin and decay product momentum directions

Sakao et al., PTEP2026, 013D01 (2026)



$$\text{slope} = \alpha P$$

α is analyzing power for polarization measurement.

$$\alpha_{\Lambda} = 0.746 \pm 0.008 \text{ (PDG2026)}$$

Λ reconstruction in the experiment

- Λ decays into two particles

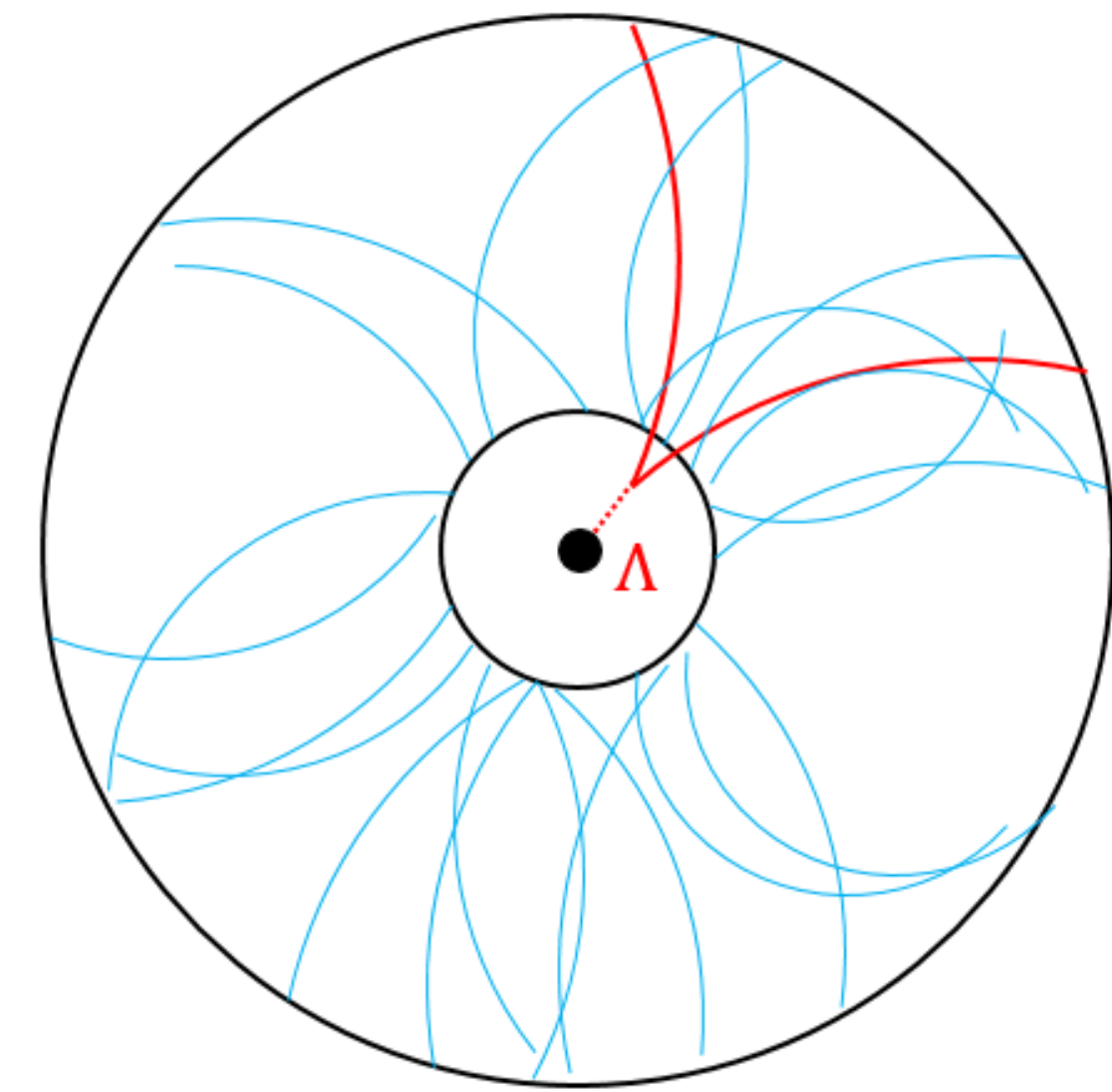
$$\Lambda \rightarrow p + \pi^- \quad (64.1\%)$$

$$\Lambda \rightarrow n + \pi^0 \quad (35.9\%)$$

*difficult to measure neutron

- Charge particles can be reconstructed by using tracking detectors, e.g. TPC
 - ❑ 3D hit positions + Tracking algorithm $\rightarrow$ trajectory
+ magnetic field $\rightarrow$ momentum & charge sign
 - ❑ Energy loss $dE/dx \rightarrow$ particle identification (Bethe-Bloch)

Event display in the transverse plane



Solid lines show charged-particles' trajectories, which are bended due to magnetic field.

Λ reconstruction and its spin

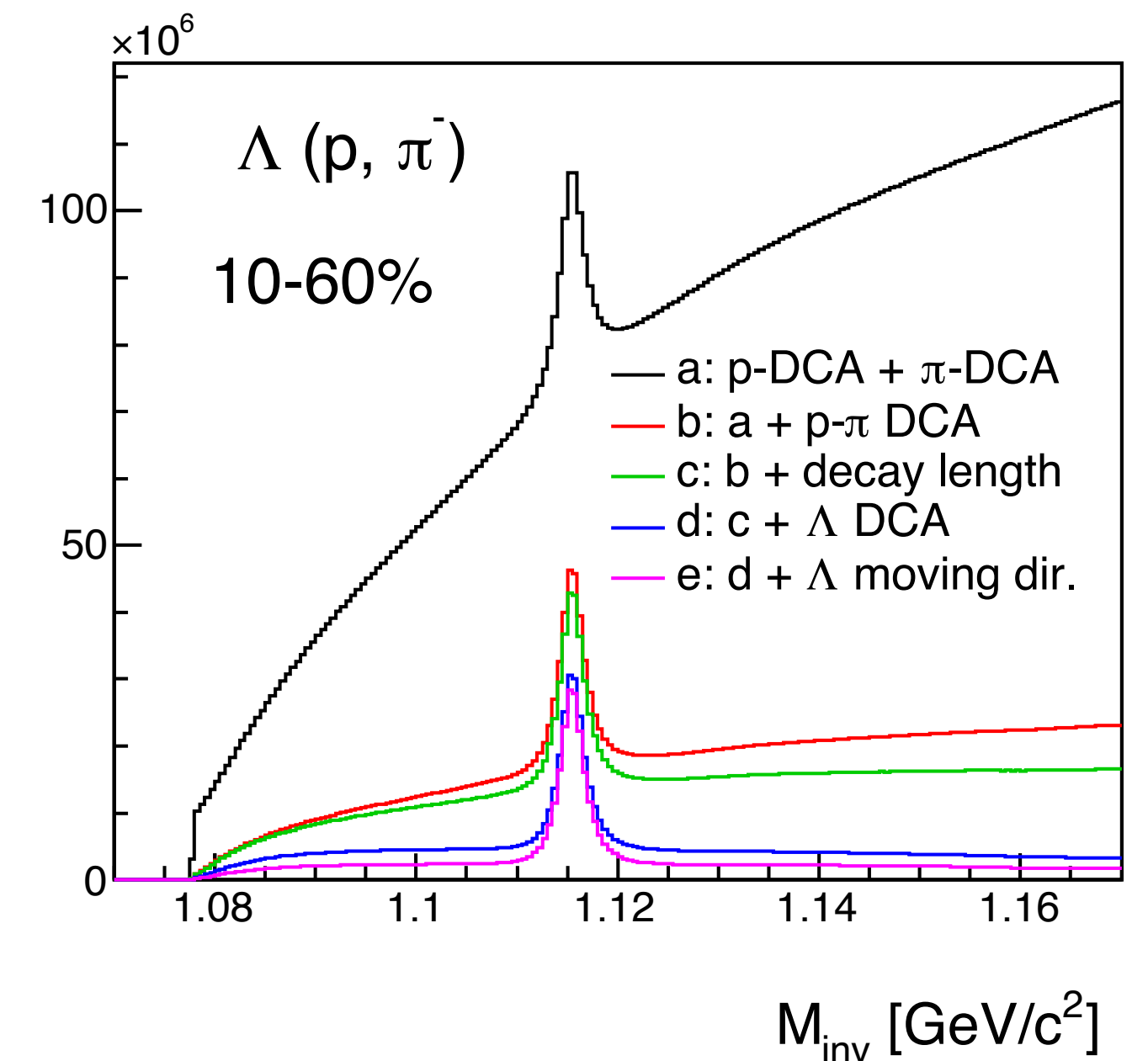
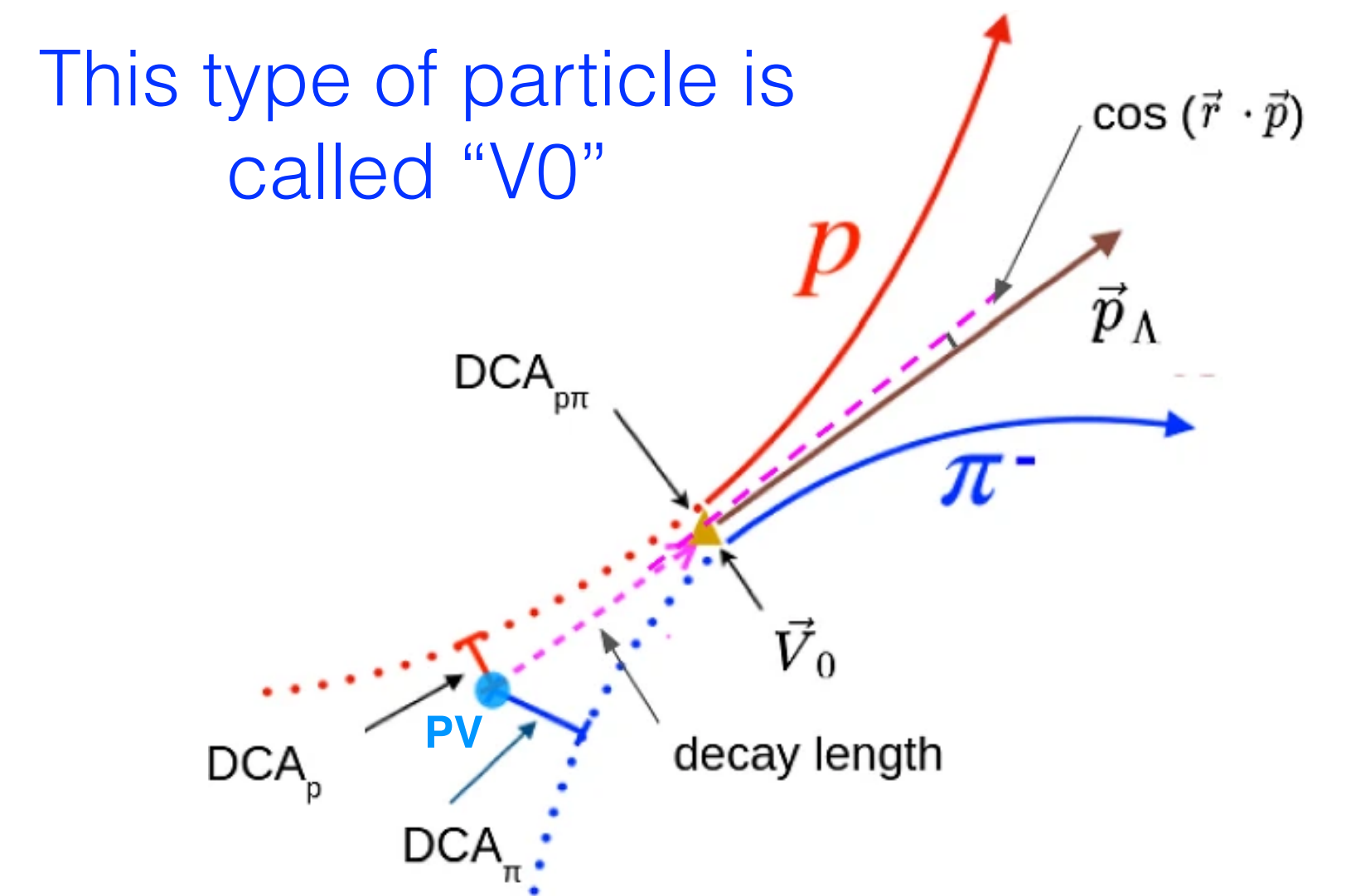
- Make all possible pairs of daughter particles and calculate their invariant mass

$$M^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$$

- Reduce the combinatorial background (random proton-pion pairs) by applying cuts on the decay topology
 - The efficiency and purity change with the cuts.
Usually the cuts are optimized to maximize the statistical significance: square root of purity $\times$ signal-yield

$$S/\sqrt{S+B}$$

- Boost the proton momentum to the Λ rest frame
→ Proton emission direction carries the Λ spin information



Reconstruction of multi-strange hyperons

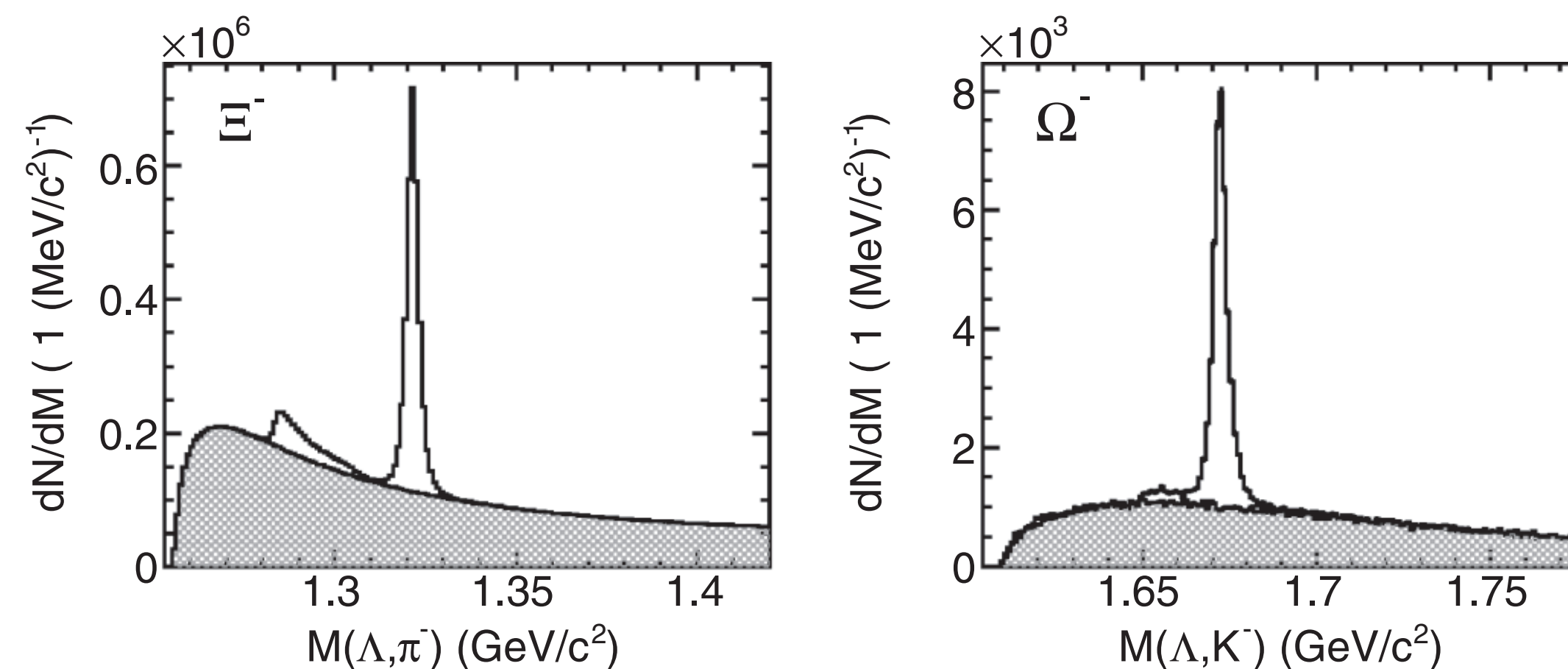
- Multi-strange hyperons such as Ξ (dss) and Ω (sss) have two-step decay

$$\Xi^- \rightarrow \Lambda + \pi^- \rightarrow p + \pi^- + \pi^-$$

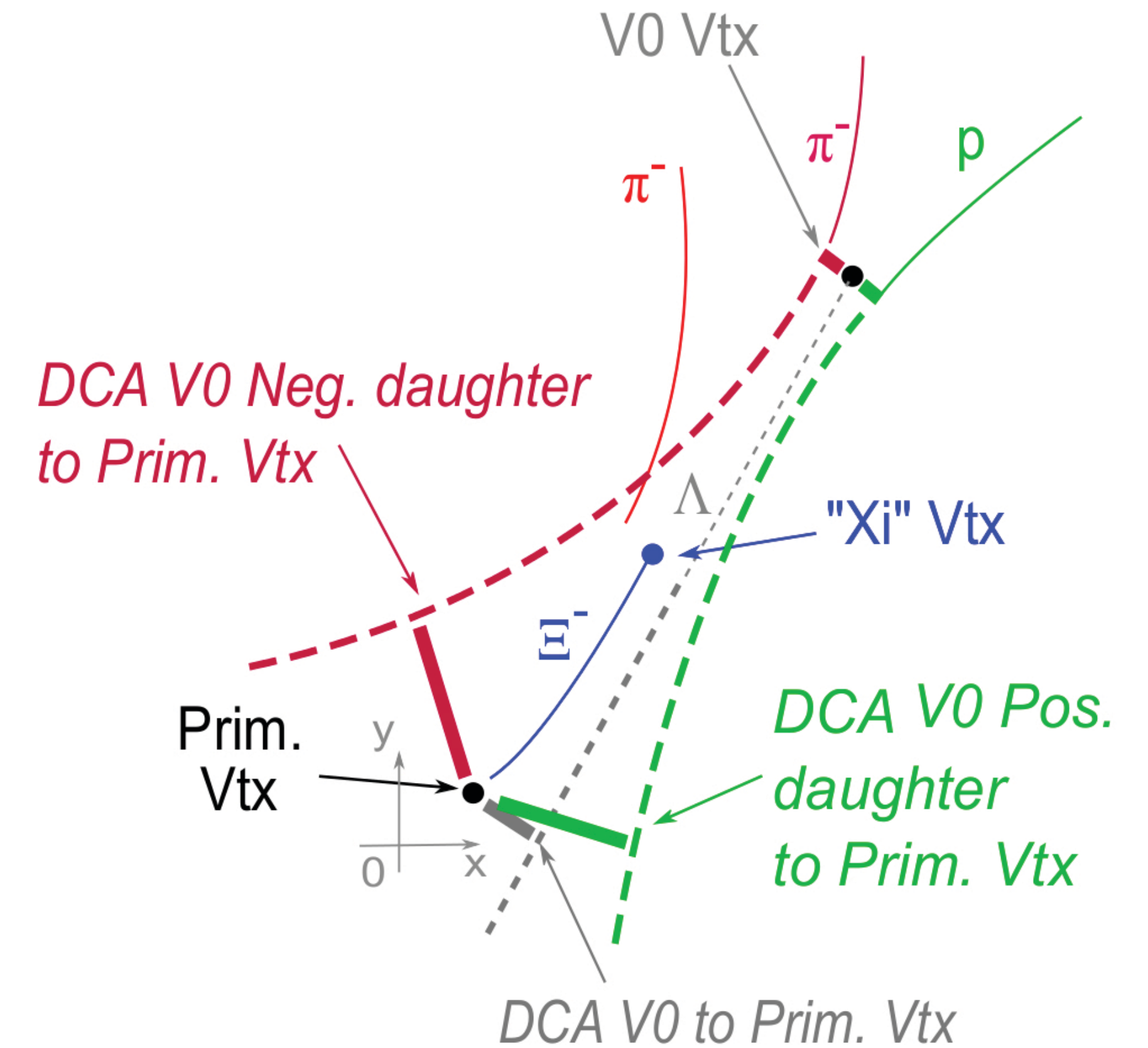
$$\Omega^- \rightarrow \Lambda + K^- \rightarrow p + \pi^- + K^-$$

- First, reconstruct Λ and then make pairs with granddaughter particle (bachelor)

- A bit more complicated topological cuts



STAR, PRC88.014902 (2013)



from CERN

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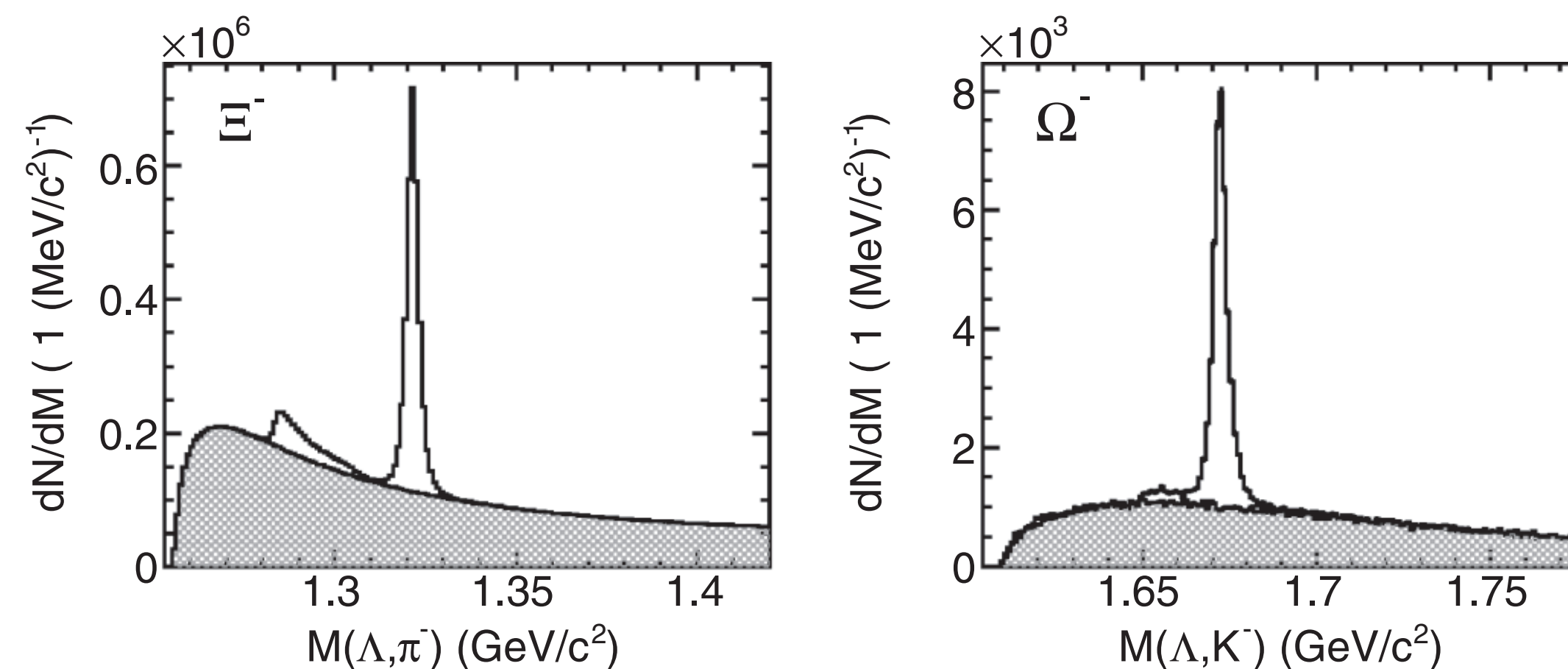
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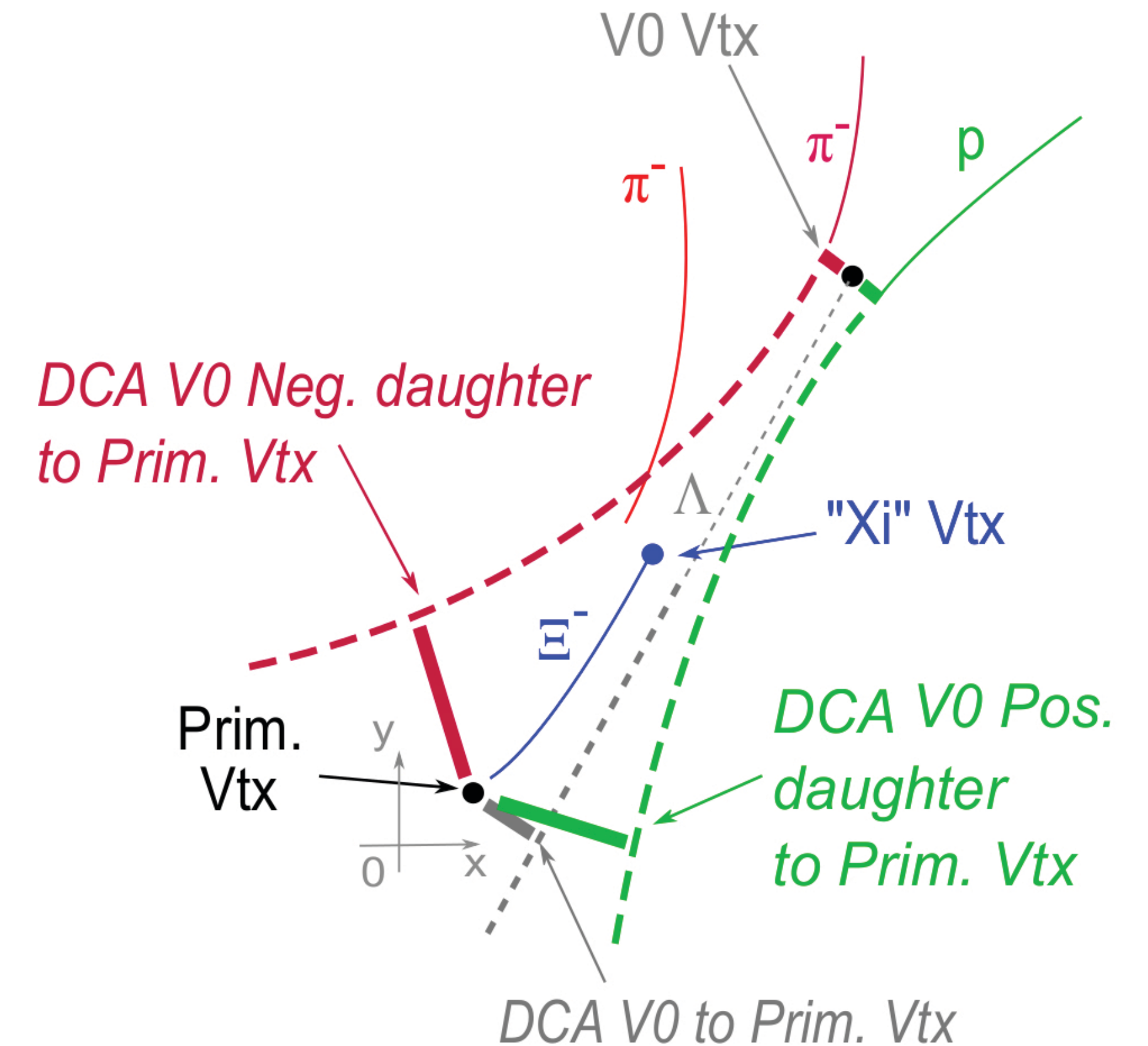
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from CERN

Spin transfer in multi-strange hyperon decay

- Similarly, by analyzing daughter Λ distributions, Ξ polarization can be measured
- Sensitivity becomes lower. Practically impossible for Ω

$$|\alpha_{\Lambda}| > |\alpha_{\Xi}| > |\alpha_{\Omega}|$$

$$\frac{dN}{d\Omega} = \frac{1}{4\pi} (1 + \alpha P \cos \theta)$$

$$\alpha_{\Lambda} = 0.746 \pm 0.008$$

$$\alpha_{\Xi} = -0.367^{+0.005}_{-0.006}$$

$$\alpha_{\Omega} = 0.0154 \pm 0.0020$$

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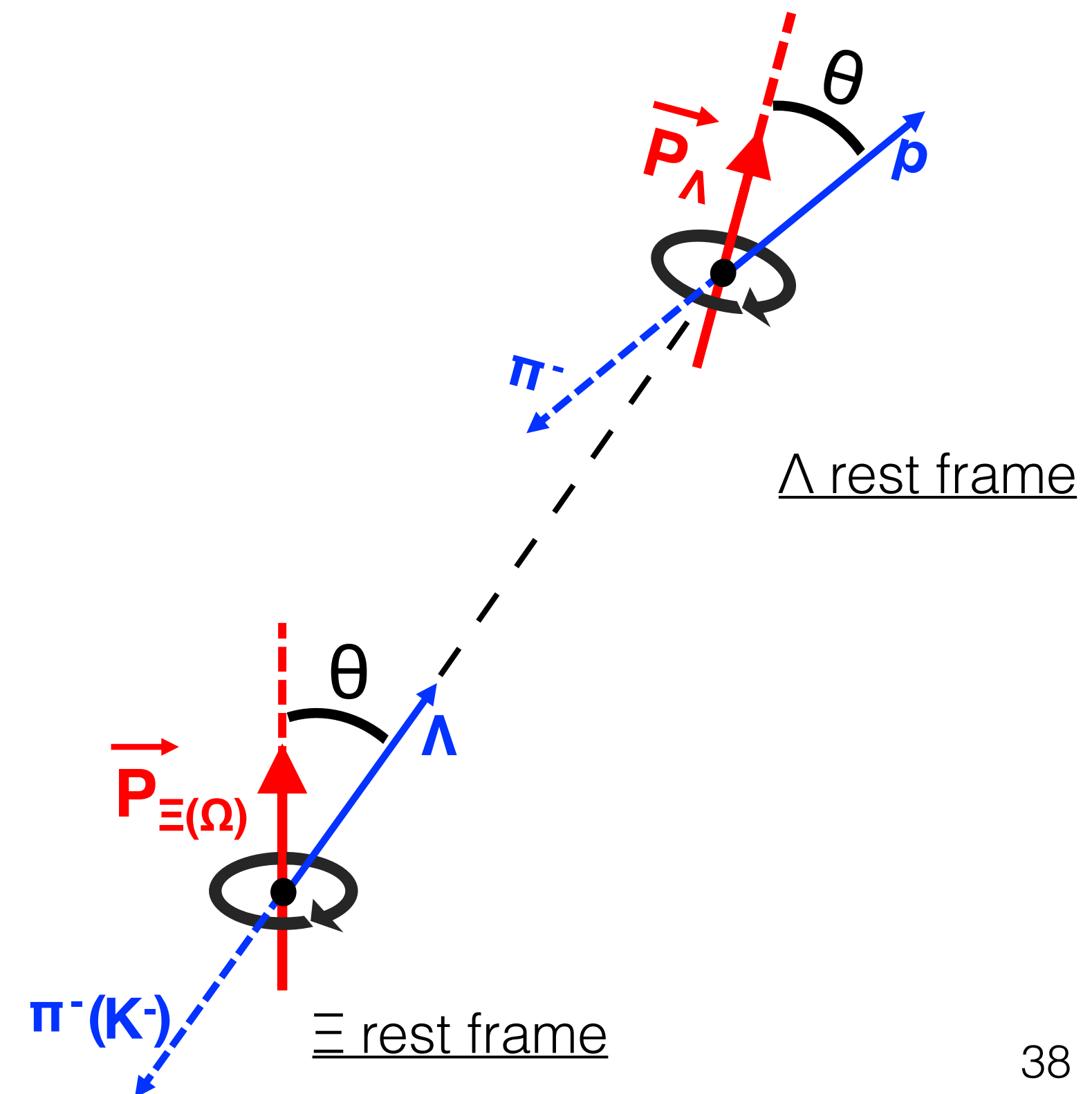
- Another way: spin transfer from Ξ (Ω) to Λ

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- Lee-Yang formula for spin-1/2 particle

T.D. Lee and C.N. Yang, Phys. Rev.108.1645 (1957)

$$\mathbf{P}_\Lambda^* = \frac{(\alpha_\Xi + \mathbf{P}_\Xi^* \cdot \hat{\mathbf{p}}_\Lambda^*)\hat{\mathbf{p}}_\Lambda^* + \beta_\Xi \mathbf{P}_\Xi^* \times \hat{\mathbf{p}}_\Lambda^* + \gamma_\Xi \hat{\mathbf{p}}_\Lambda^* \times (\mathbf{P}_\Xi^* \times \hat{\mathbf{p}}_\Lambda^*)}{1 + \alpha_\Xi \mathbf{P}_\Xi^* \cdot \hat{\mathbf{p}}_\Lambda^*}$$

$$\mathbf{P}_\Lambda^* = C_{\Xi-\Lambda} \mathbf{P}_\Xi^*$$

- Similarly, spin-3/2 Ω has the following relation

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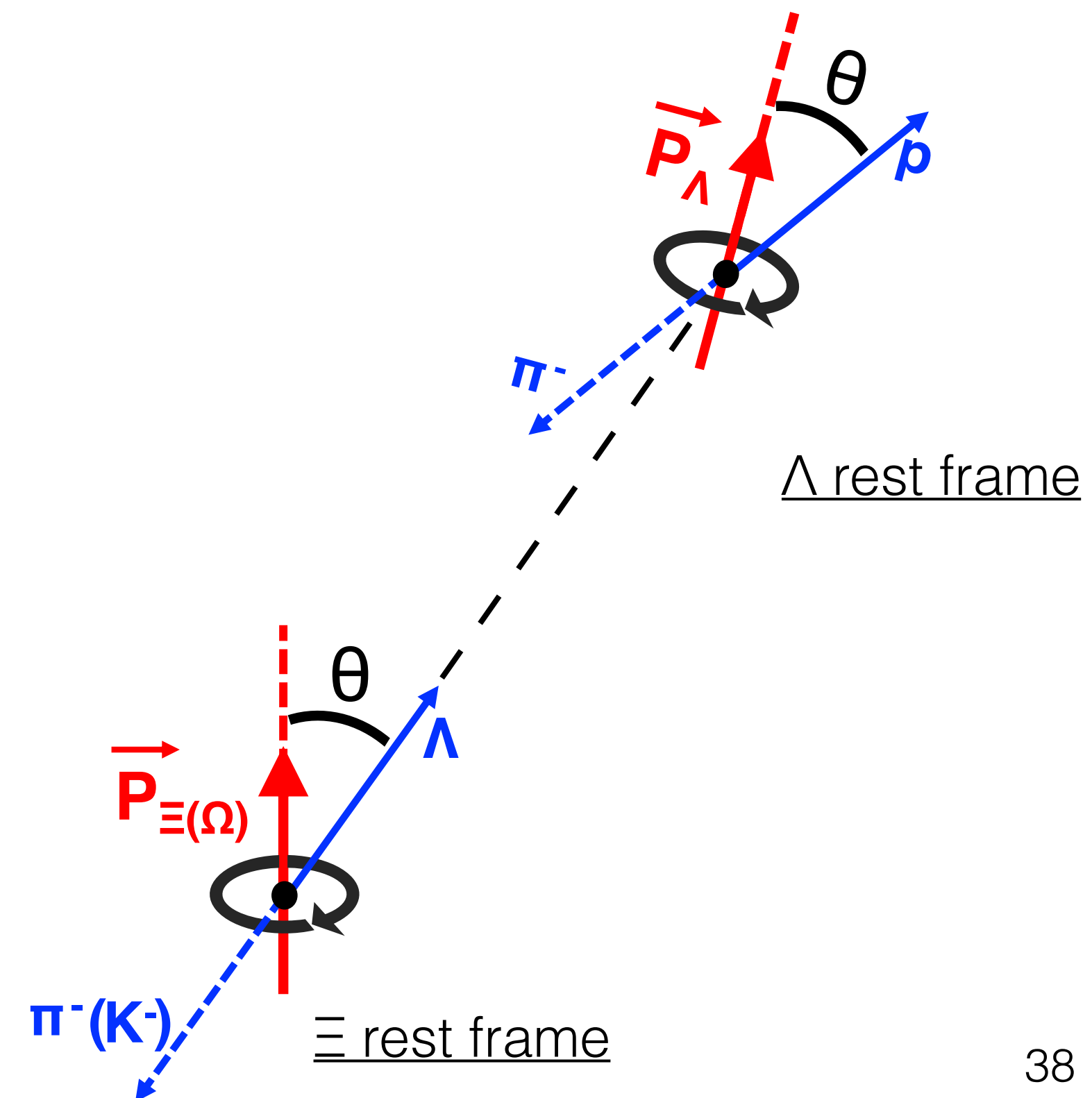
Becattini, Karpenko, Lisa, Upsal, and Voloshin, PRC95.054902 (2017)

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$$\alpha_{\Omega} = 0.0154 \pm 0.0020$$

Polarization transfer factor C

Decay	C
parity-conserving: $1/2^+ \rightarrow 1/2^+ 0^-$	$-1/3$
parity-conserving: $1/2^- \rightarrow 1/2^+ 0^-$	1
parity-conserving: $3/2^+ \rightarrow 1/2^+ 0^-$	$1/3$
parity-conserving: $3/2^- \rightarrow 1/2^+ 0^-$	$-1/5$
$\Xi^0 \rightarrow \Lambda + \pi^0$	$+0.900$
$\Xi^- \rightarrow \Lambda + \pi^-$	$+0.927$
$\Sigma^0 \rightarrow \Lambda + \gamma$	$-1/3$

Vector-meson spin alignment

► Vector-mesons (spin-1) can be used to study the polarization

□ For parity-conserving strong decays, the usual decay angular distribution is sensitive to spin alignment rather than the sign of vector polarization

□ Angular distribution of the decay products can be written with spin density matrix ρ_{nn} .

$$\begin{aligned} \frac{dN}{d\cos\theta^*} &\propto \rho_{0,0}|Y_{1,0}|^2 + \rho_{1,1}|Y_{1,-1}|^2 + \rho_{-1,-1}|Y_{1,1}|^2 \propto \rho_{0,0}\cos^2\theta^* + \frac{1}{2}(\rho_{1,1} + \rho_{-1,-1})\sin^2\theta^* & |s, s_z\rangle = |1, s_z\rangle \\ &\propto (1 - \rho_{0,0}) + (3\rho_{0,0} - 1)\cos^2\theta^* & (s_z = -1, 0, +1) \end{aligned}$$

► The diagonal element ρ_{00} : probability to have spin projection to be 0

□ $\rho_{00} = 1/3$: no spin alignment

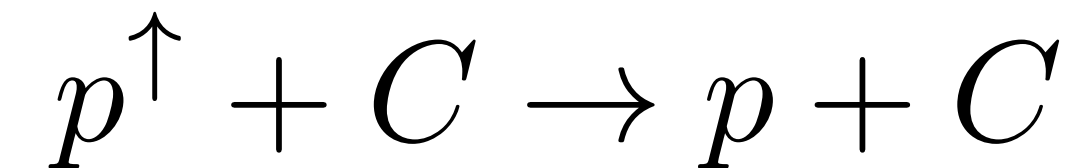
□ $\rho_{00} \neq 1/3$: spin aligned along quantization axis chosen

Polarization measurement of other (stable) particles?

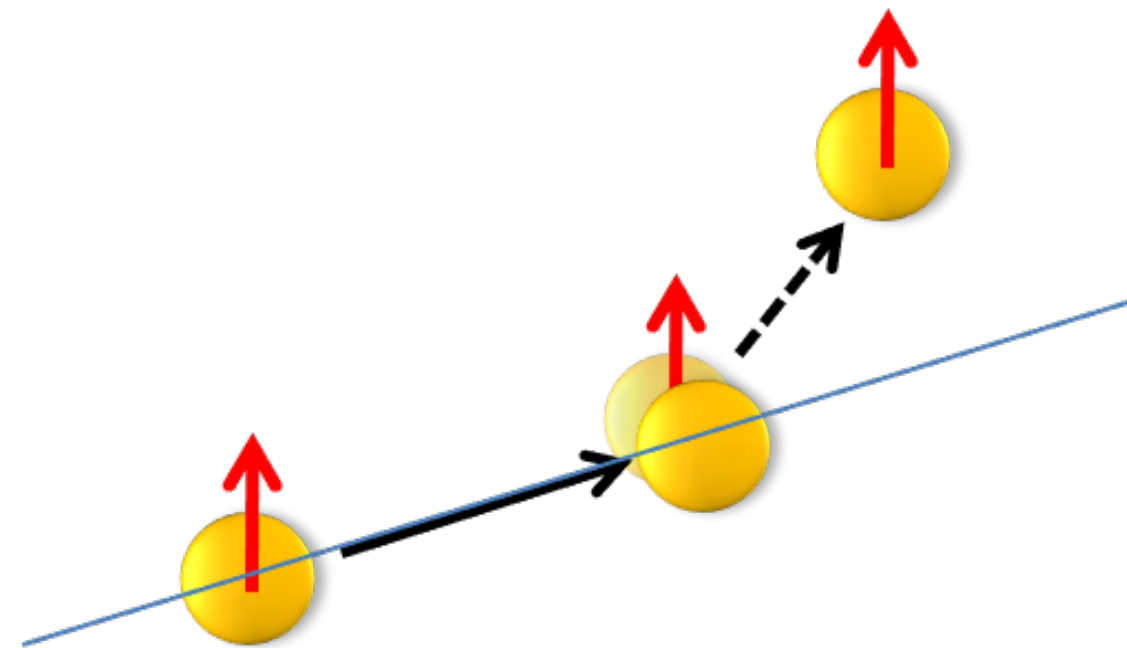
► For example, can we measure proton polarization? If so, how?

Polarization measurement of other (stable) particles?

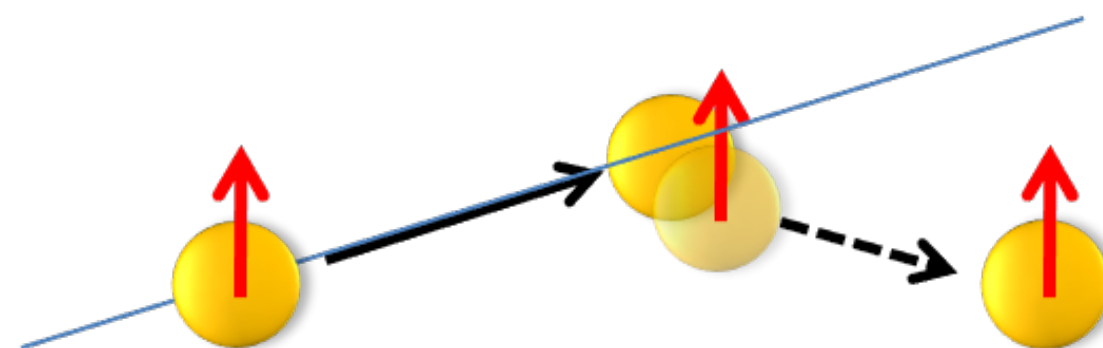
- For example, can we measure proton polarization? If so, how?
- At RHIC, the polarization of proton beam was measured by pC polarimeter (proton-Carbon polarimeter)



scattering to the left



scattering to the right

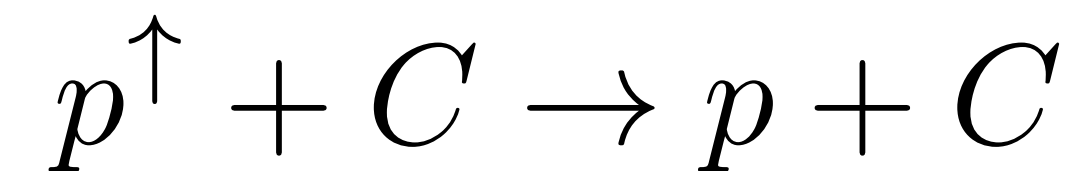


The left-right asymmetry originates from spin-orbit interaction, i.e. interaction between p spin and its orbital motion in EM/nuclear field of Carbon

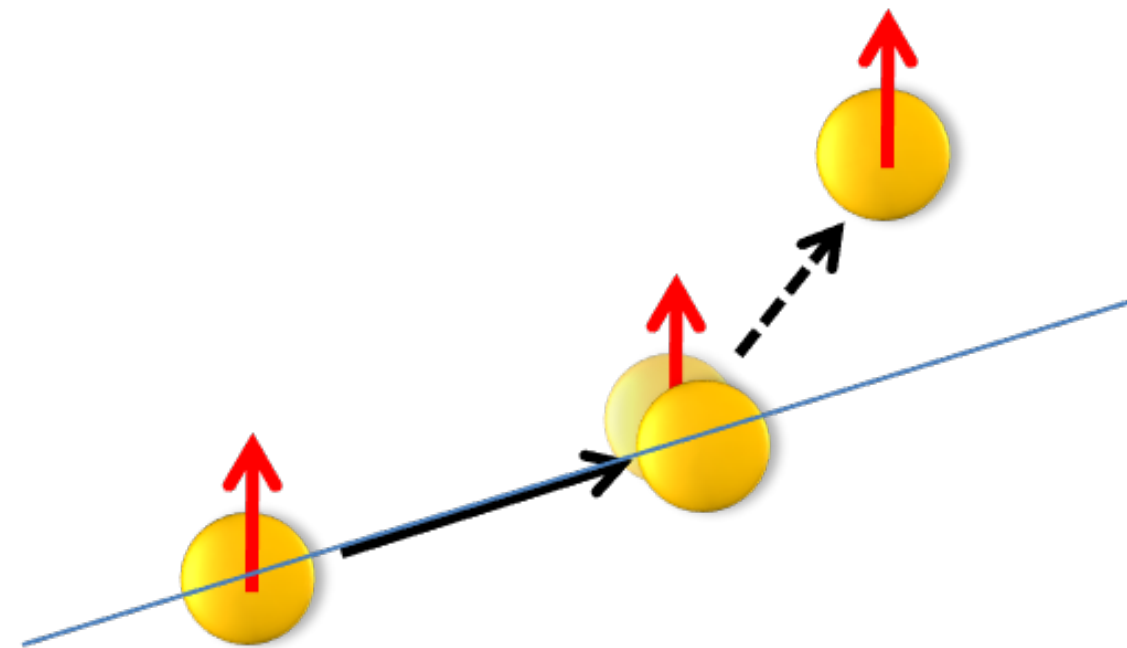
The analyzing power A_N depends on the scattering process and kinematics, and is experimentally calibrated using an absolute polarimeter.

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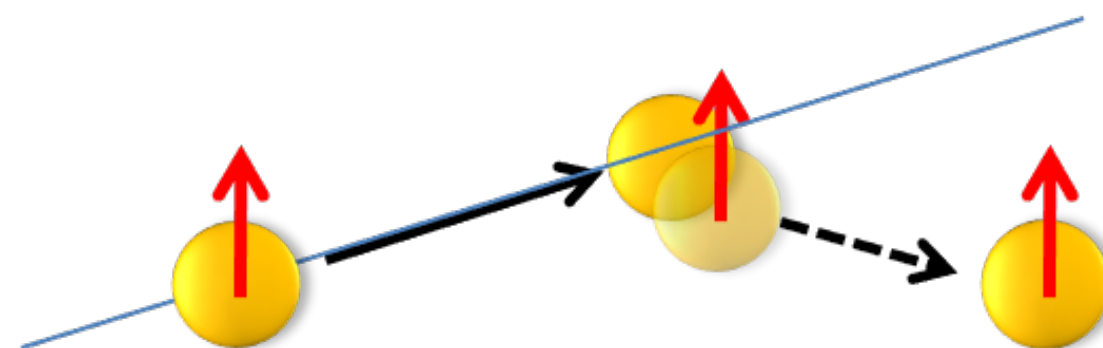
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The left-right asymmetry originates from spin-orbit interaction, i.e. interaction between p spin and its orbital motion in EM/nuclear field of Carbon

$$\frac{N_L - N_R}{N_L + N_R} = P \cdot A_N$$

left-right asymmetry

beam polarization

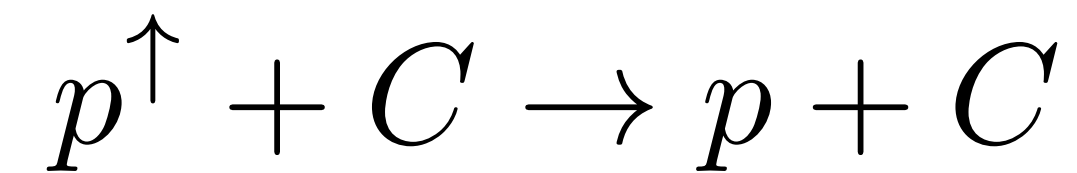
analyzing power
or
single spin asymmetry

P can be determined with known A_N

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Polarization measurement of other (stable) particles?

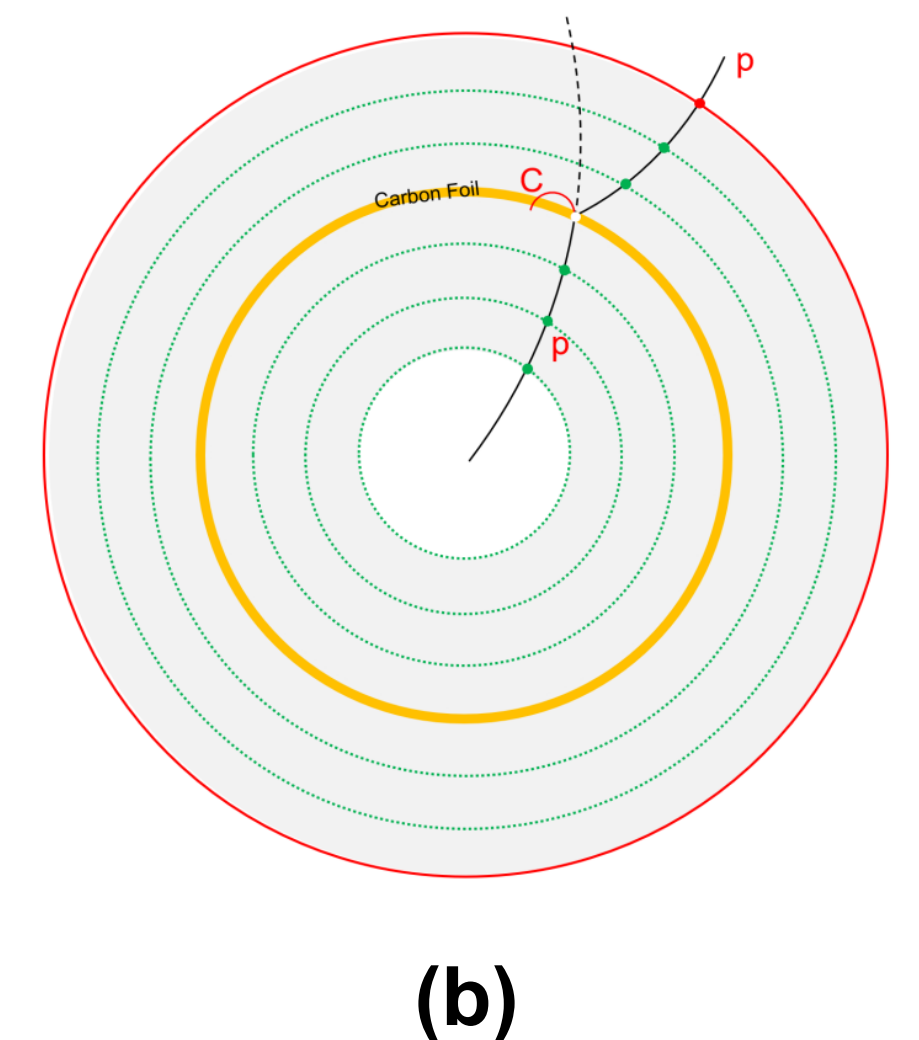
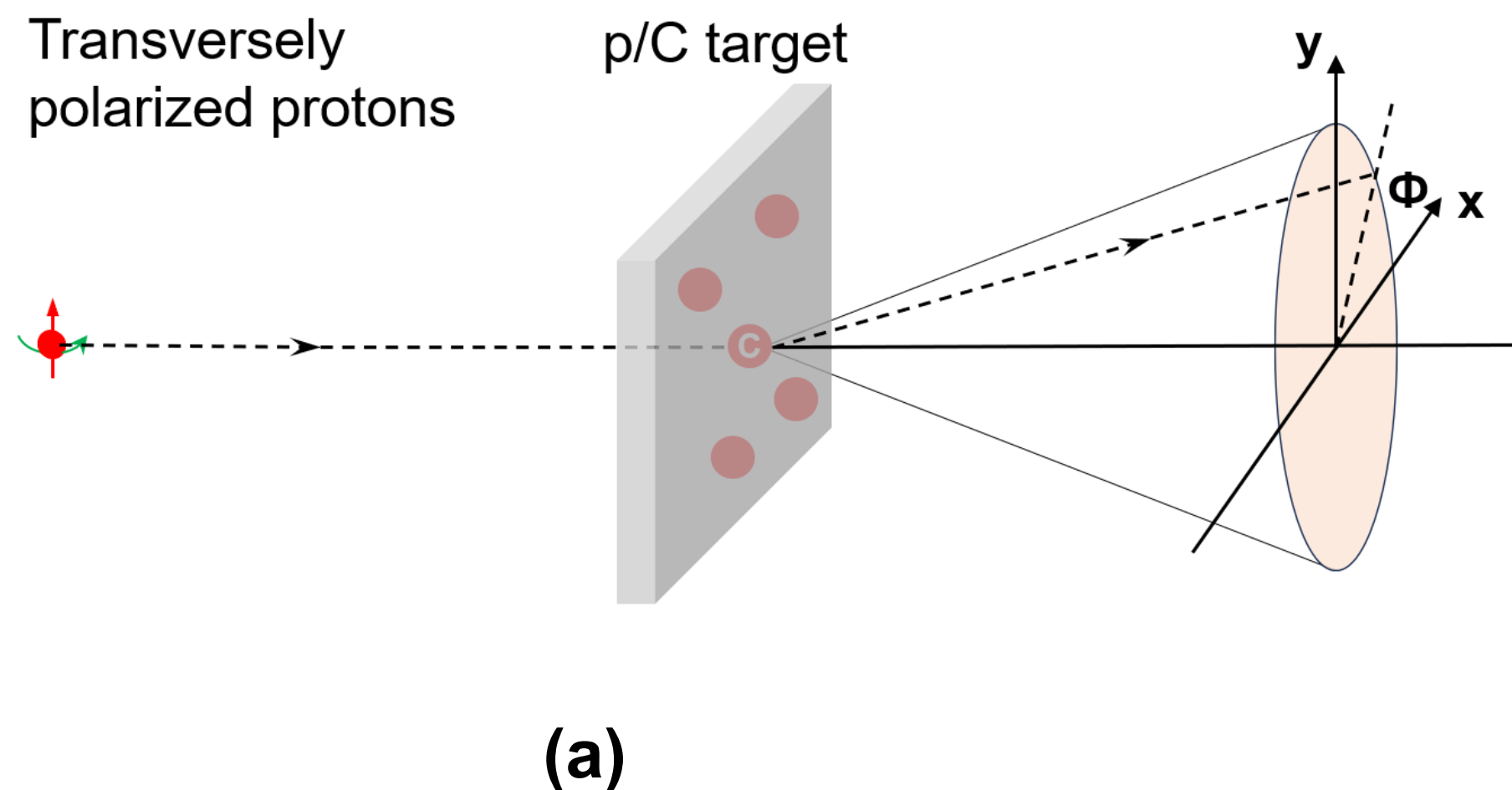
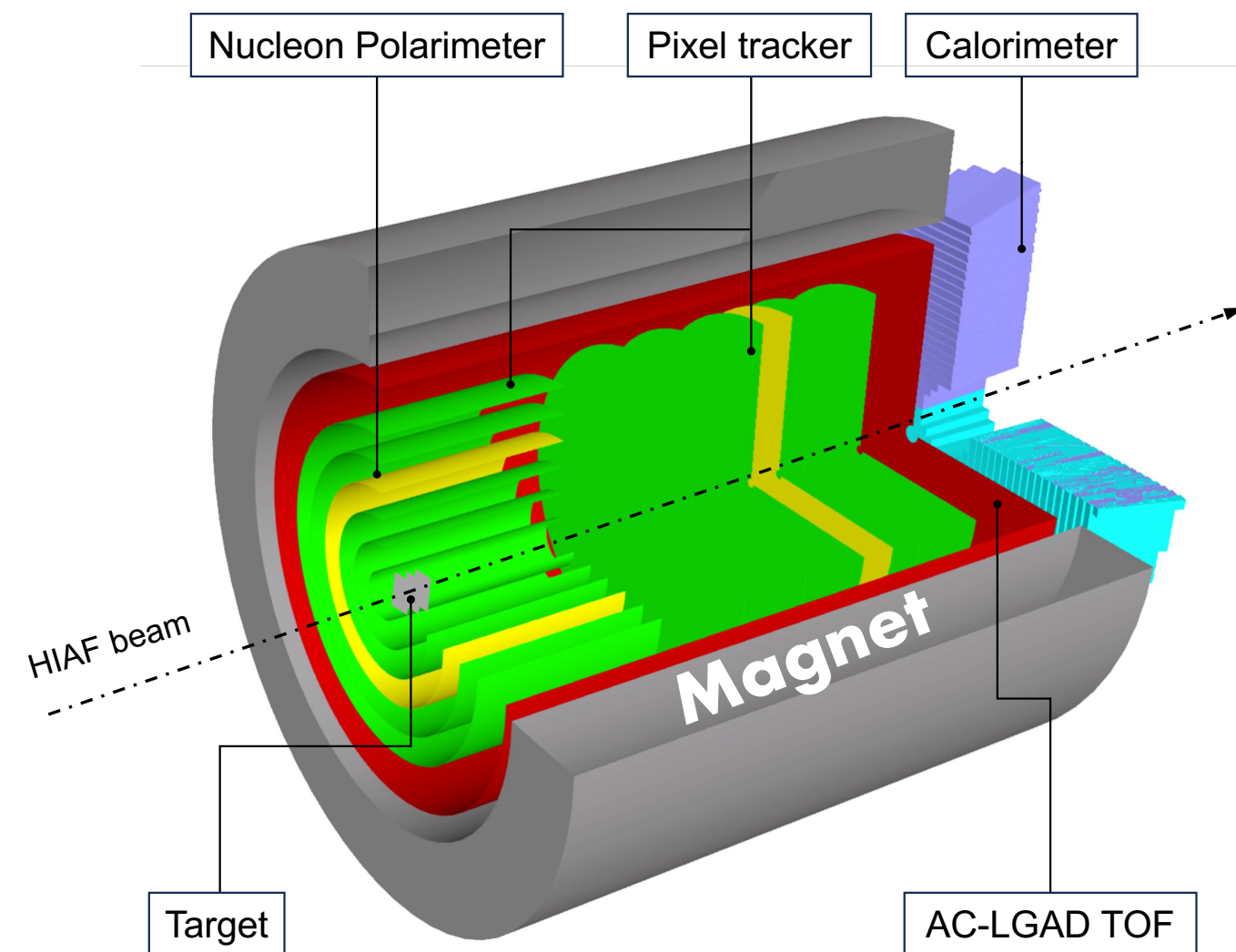
- For example, can we measure proton polarization? If so, how?
- At RHIC, the polarization of proton beam was measured by pC polarimeter (proton-Carbon polarimeter)



The polarized hydrogen jet provides the absolute polarization calibration, while the pC polarimeter provides fast polarization measurements

- The same technique will be used in the future experiment: Hyperon-Nucleon Spectrometer (H-NS) at HIAF

X. Bai et al., arXiv:2606.06553



Summary & Take-home messages

- ▶ Spin is an intrinsic quantum angular momentum
 - Angular momentum projection is quantized, as demonstrated by the Stern-Gerlach experiment
- ▶ Polarization describes the spin state of an ensemble of particles
 - The spin density matrix provides a general description of polarization and spin alignment
- ▶ Weak interactions violate parity
 - The Wu experiment revealed parity violation through a correlation between nuclear spin and electron momentum
- ▶ Spin-dependent interactions allow us to measure polarization
 - Weak decay, strong decay, and scattering can serve as spin analyzers.