Spatial confinement-deconfinement transition in rotating and accelerating QGP within lattice simulation

Artem Roenko[1],

in collaboration with

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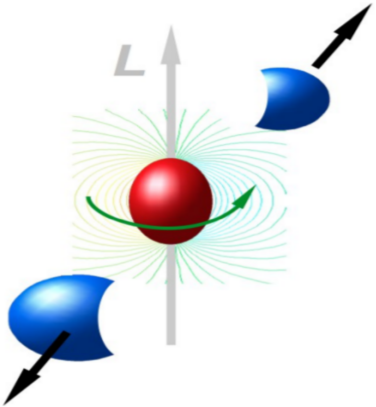
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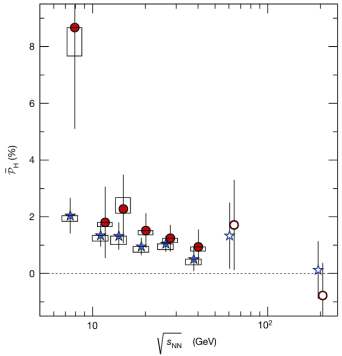
The 10th International Conference on Chirality, Vorticity, and Magnetic Field in Quantum Matter (Chirality 2026), Zhuhai, 16-20 November 2026

based on PLB **855**, 138783 (2024), JHEP **09**, 079 (2025), PRD **113**, 094507 (2026)

Introduction

- HIC allow us to test QCD under extreme conditions
- Relativistic rotation: $\omega \sim 10$ MeV ~ 0.05 fm^{-1} $v \sim c$ at $r \sim 20$ fm [Jiang, Lin, Liao'16; STAR'17]

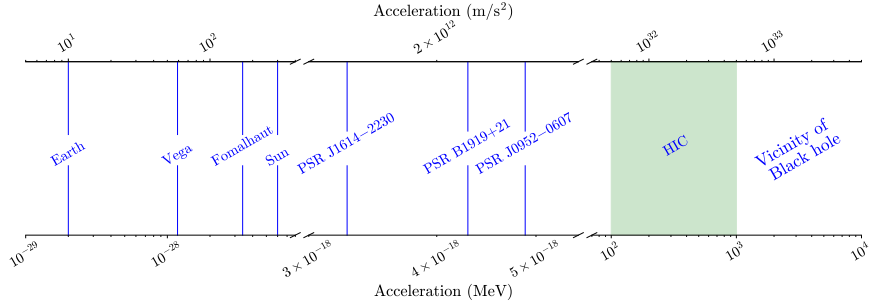




[STAR'17]

Introduction

- HIC allow us to test QCD under extreme conditions
- Relativistic rotation: $\omega \sim 10$ MeV ~ 0.05 fm^{-1} $v \sim c$ at $r \sim 20$ fm [Jiang, Lin, Liao'16; STAR'17]
- Large acceleration: $a \sim 0.1 - 1$ GeV [Kharzeev, Tuchin'05; Prokhorov, et al.'25]



- A lot of models were applied to study rotation/acceleration [See Talk by Xu-Guang Huang]
- Lattice studies of other groups: Yang, Huang'23; Ding, et al.'26 and Chernodub, et al.'25; [See also Talks by Xu-Guang Huang and Eduardo Garnacho-Velasco]

- It is convenient to describe the system in the co-rotating reference frame, $x^\mu = (t, x, y, z)$,

$$\varphi = \left[\varphi_{\rm lab} - \Omega t\right]_{2\pi}, \quad t = t_{\rm lab}, \quad z = z_{\rm lab}, \quad r = r_{\rm lab}, \tag{1}$$

where the effects of rotation are reduced to the metric [Yamamoto, Hirono'13]

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{2}$$

- Lattice study is conducted in Euclidean space $(t \to -i\tau)$, $x^\mu = (x, y, z, \tau)$,

$$g^E_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & -y\Omega_I \\ 0 & 1 & 0 & x\Omega_I \\ 0 & 0 & 1 & 0 \\ -y\Omega_I & x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix}, \tag{3}$$

  - The angular velocity is imaginary: $\Omega_I = \partial_\tau \varphi_{\rm lab} = -i\partial_t \varphi_{\rm lab} = -i\Omega$, to avoid the sign problem.

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  - The angular velocity is imaginary: $\Omega_I = \partial_\tau \varphi_{\rm lab} = -i\partial_t \varphi_{\rm lab} = -i\Omega$, to avoid the sign problem.
  - The causality restriction takes place in Minkowski space: $\boxed{\Omega r < 1}$

The quark action is a linear function in angular velocity:

$$S_\psi = \int d^4x\,\sqrt{g_E}\,\bar\psi\left(\gamma^\mu\left(\partial_\mu + \Gamma_\mu\right) + m\right)\psi = S_0^\psi + S_1^\psi\,\Omega_I =$$

$$= \int d^4x\,\bar\psi\Big(\left(\gamma^1 + y\Omega_I\gamma^4\right)D_x + \left(\gamma^2 - x\Omega_I\gamma^4\right)D_y + \gamma^3 D_z + \gamma^4\left(D_\tau + i\Omega_I\frac{\sigma^{12}}{2}\right) + m\Big)\psi\,, \quad (4)$$

The gluon action is a quadratic function in angular velocity:

$$S_G = \frac{1}{4g_{YM}^2}\int d^4x\,\sqrt{g_E}\,g_E^{\mu\nu}g_E^{\alpha\beta}F^a_{\mu\alpha}F^a_{\nu\beta} \equiv S_0 + S_1\Omega_I + S_2\frac{\Omega_I^2}{2} =$$

$$= \frac{1}{g_{YM}^2}\int d^4x\Big(\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \Omega_I\left[-yF^a_{xy}F^a_{y\tau} - yF^a_{xz}F^a_{z\tau} + xF^a_{yx}F^a_{x\tau} + xF^a_{yz}F^a_{z\tau}\right] +$$

$$+ \Omega_I^2\left[r^2(F^a_{xy})^2 + y^2(F^a_{xz})^2 + x^2(F^a_{yz})^2 + 2xyF^a_{xz}F^a_{zy}\right]\Big) \quad (5)$$

So, for quarks $\mathcal{L}^{(1)}_\psi = \bar\psi(\boldsymbol{\Omega}\cdot\hat{\boldsymbol{J}})\psi$ (note $\hat{\boldsymbol{J}} = \hat{\boldsymbol{L}} + \hat{\boldsymbol{S}}$), whereas for gluons $\mathcal{L}^{(1)}_G = \boldsymbol{\Omega}\cdot\boldsymbol{J}_G$ and $\mathcal{L}^{(2)}_G \propto B^2$.

▷ sign problem ▷ inhomogeneous action ▷ asymmetry between E^2 and H^2

- The system is considered in the reference frame of uniformly accelerated observer (Rindler coordinates), $x^\mu = (t, x, y, z)$,

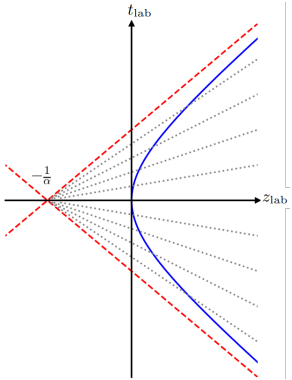
$$t_{\rm lab} = \left(\frac{1}{\alpha} + z\right)\sinh\alpha t, \quad z_{\rm lab} = \left(\frac{1}{\alpha} + z\right)\cosh\alpha t - \frac{1}{\alpha}, \quad x_{\rm lab} = x, \quad y_{\rm lab} = y\,,$$

with observer located at $z_0 = 0$, and

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (1+\alpha z)^2 dt^2 - dx^2 - dy^2 - dz^2\,,$$

- Lattice study is conducted in Euclidean space $(t \to -i\tau)$, $x^\mu = (x, y, z, \tau)$,

$$ds^2 = g^E_{\mu\nu}dx^\mu dx^\nu = -(1+\alpha z)^2 d\tau^2 - dx^2 - dy^2 - dz^2\,.$$



- Acceleration is real[1]: $\alpha > 0$
- Rindler horizon: $g_{00}(z_h) = 0 \quad \Rightarrow \quad z_h = -1/\alpha$ [Rindler'66]
- Similar to the Schwarzschild black hole close to the horizon, $ds^2 = (\rho/4M)^2 dt^2 - d\rho^2$

[1] Acceleration is $\sim \partial_t^2 \propto \partial_\tau^2$; it is designated as α to avoid confusion with lattice spacing a

- For the observer at $z_0 = 0$, the gluon action is [Braguta, Goy, Dey, Roenko'26]

$$S_G = \frac{1}{4g_{YM}^2}\int d^4x\,\sqrt{g_E}\,g_E^{\mu\nu}\,g_E^{\alpha\beta}F^a_{\mu\alpha}F^a_{\nu\beta} = \frac{1}{4g_{YM}^2}\int d^3x\int_0^{1/T_0} d\tau\left(\frac{1}{1+\alpha z}(\boldsymbol{E}^a)^2 + (1+\alpha z)\,(\boldsymbol{H}^a)^2\right). \quad (6)$$

- Observer position is not unique. For an observer at z_1, the interval preserve its form

$$ds^2 = -(1+\alpha_1(z-z_1))^2 d\tau_1^2 - dx^2 - dy^2 - dz^2\,,$$

where $\alpha_1 = \alpha/(1+\alpha z_1)$, $d\tau_1 = (1+\alpha z_1)d\tau$, and $T_1 = T_0/(1+\alpha z_1)$. Therefore, the action reads similarly

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with $z_0 \to z_1$, $\alpha \to \alpha_1$, $T_0 \to T_1$.

▷ symmetry w.r.t. change in the observer position, proper acceleration and temperature
▷ no sign problem ▷ inhomogeneous action ▷ asymmetry between E^2 and H^2

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Thermal equilibrium in an external gravitational field is described by Tolman–Ehrenfest (TE) law:

$$T(\boldsymbol{r})\sqrt{g_{00}(\boldsymbol{r})} = T_0 = const\,, \tag{8}$$

- Rotation: $T(r) = \dfrac{T_0}{\sqrt{1-\Omega^2 r^2}} = \dfrac{T_0}{\sqrt{1+\Omega_I^2 r^2}}$ — Ω effectively heats up the periphery; Ω_I effectively cools down the periphery
- Acceleration: $T(z) = \dfrac{T_0}{1+\alpha z}$ — α effectively heats up the system closer to horizon

Can we parameterize acceleration by the TE-temperature gradient? [Chernodub, et al.'25]

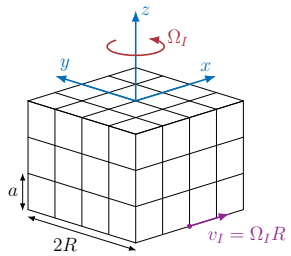
▷ For scalar field, the position-dependent factors are absorbed by $d\tau \to (1+\alpha z)d\tau$ and $\partial_4 \to (1+\alpha z)\partial_4$,

$$S_E^{\varphi} = \frac{1}{2}\int d^3x \int_0^{1/T_0} d\tau \left(\frac{1}{1+\alpha z}(\partial_4\varphi)^2 + (1+\alpha z)\left((\nabla\varphi)^2 + m^2\varphi^2\right)\right)$$
$$= \frac{1}{2}\int d^3x \int_0^{1/T(z)} d\tau \left((\partial_4\varphi)^2 + (\nabla\varphi)^2 + m^2\varphi^2\right). \tag{9}$$

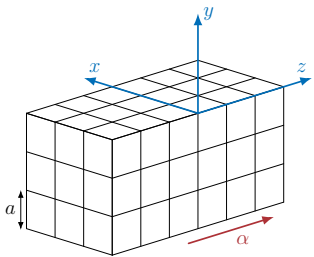
▷ For vector gluon field, it doesn't work: $A_4 \to (1+\alpha z)A_4$ does not commute with $\partial_z A_4$ in E_z, and

$$S_E = \frac{1}{4g_{YM}^2}\int d^3x \int_0^{1/T(z)} d\tau \left((\boldsymbol{E}^a)^2 + (\boldsymbol{H}^a)^2\right) + \mathcal{O}(\alpha)\,. \tag{10}$$

Lattice setups for rotating / accelerated gluodynamics



- Lattice size: $N_t \times N_z \times N_s^2$
- Infinite volume limit $N_z \to \infty$
- PBC in τ, z; PBC/**OBC**/DBC in x, y
- $\sqrt{2}\Omega_I R < 1$; causality respected, a.c. $\Omega_I^2 = -\Omega^2$
- Slow rotation: $\Omega_I \ll \Lambda_{QCD}$, $\Omega_I/T \ll \pi$.
- local Polyakov loop: $L(x, y)$



- Lattice size: $N_t \times N_s^2 \times N_z$
- Infinite volume limit $N_s \to \infty$
- PBC in τ, x, y; PBC/**OBC**/DBC in z
- $L_z/2 \ll 1/\alpha$; system far from horizon
- Small acceleration: $\alpha \ll \Lambda_{QCD}$
- local Polyakov loop: $L(z)$

Tree-level improved Symanzik lattice action is used for gluons in both cases; temperature is $T \equiv T_0 = T(0)$

Inhomogeneous phases for imaginary rotation

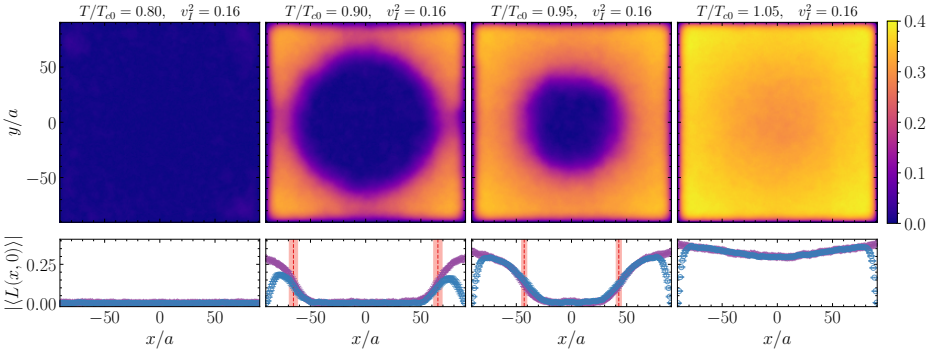
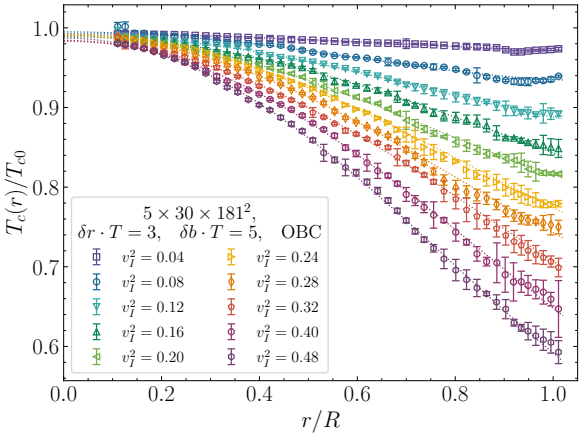


Figure: The local Polyakov loop in x, y-plane for the lattice of size $5 \times 30 \times 181^2$

- Finite temperature confinement-deconfinement transition becomes a spatial transition.
- Mixed inhomogeneous phase takes place at $T \lesssim T_{c0}$.
- For imaginary rotation, deconfinement appears at the periphery; confinement is in the central regions.
- This phases arrangement is supported by some models [Chen, Fukushima, Shimada'24; Jiang'24]

The (on-axis) critical temperature $T_c(r)$ of spatial transition at distance r is calculated from the local Polyakov loop susceptibility.

- Lattice parameters: $4 \times 24 \times 145^2$, $5 \times 30 \times 181^2$, $6 \times 36 \times 216^2$, with $v_I^2 = 0.04, \ldots, 0.48$.



- Results: The critical temperature decreases with distance for imaginary angular velocity.

$$\frac{T_c(r, \Omega_I)}{T_{c0}} = 1 - \kappa_2 (\Omega_I r)^2 \tag{11}$$

- The continuum limit we get (in the bulk)

$$\kappa_2 = 0.902(33)\,, \tag{12}$$

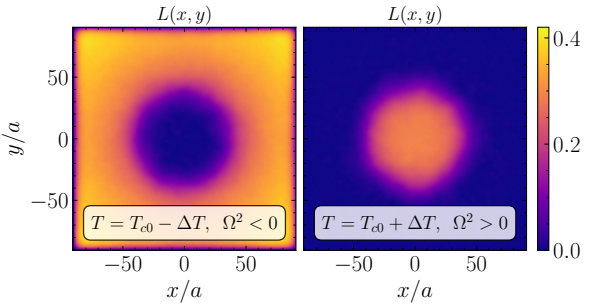
(next terms are affected by b.c.)

- Within the uncertainty,

$$T_c(r = 0, \Omega) = T_{c0} \tag{13}$$

Analytic continuation of the results to real rotation

Analytic continuation: $\Omega_I^2 \to -\Omega^2$



- The critical temperature increases with distance for real angular velocity

$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + \kappa_2 (\Omega r)^2 \qquad (14)$$

- In the leading order, the boundary between phases has the same position at
  - ▷ $T = T_{c0} - \Delta T$, imaginary Ω_I

  and
  - ▷ $T = T_{c0} + \Delta T$, real Ω,

  but the phases has opposite arrangement.
- In disagreement with TE prediction

$$T(r) = T_{c0} \quad \Rightarrow \quad T_c(r) = T_{c0}\sqrt{1 - \Omega^2 r^2}$$

For real rotation,

- Mixed inhomogeneous phase takes place at $T \gtrsim T_{c0}$.
- Deconfinement appears in the central region; confinement is in the periphery.

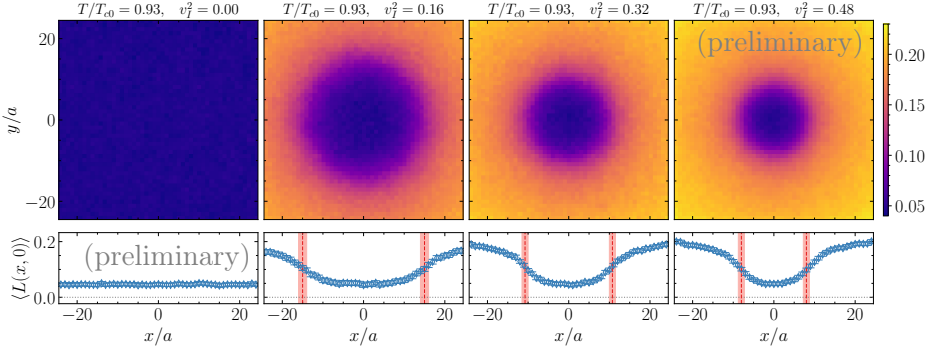


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $4 \times 20 \times 49^2$ at the fixed temperature $T = 0.93\, T_{c0}$ and different v_I; QCD with Wilson fermions (Iwasaki action), $m_\pi/m_\rho = 0.80$.

- Mixed inhomogeneous phase takes place also in QCD! (work in progress ...)
- On the rotation axis, $T_c(0) \approx T_{c0}$, see the results at finite spin density [Braguta, Chernodub, Roenko'25]

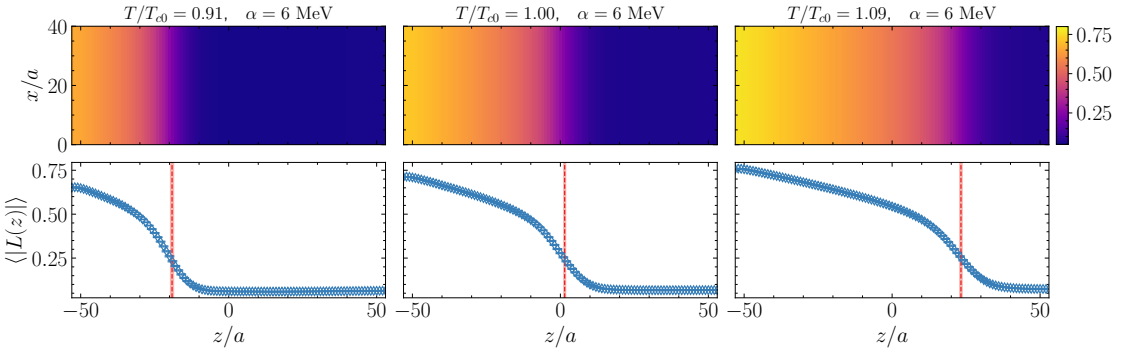


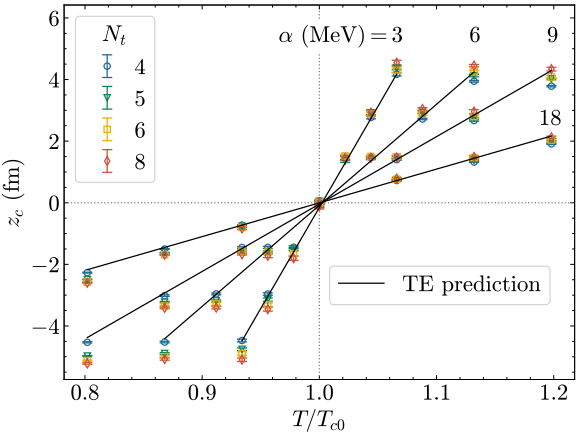
Figure: The local Polyakov loop in x, z-plane for the lattice of size $5 \times 40^2 \times 121$

- Finite temperature confinement-deconfinement transition becomes a spatial transition at $\alpha \neq 0$.
- Mixed inhomogeneous phase takes place both at $T \lesssim T_{c0}$ and $T \gtrsim T_{c0}$.
- Deconfinement appears near the horizon; confinement is in the opposite side.

Critical distance in accelerated system

The critical distance $z_c(T)$ of spatial transition as a function of temperature T were calculated.

- Lattices: $4 \times 32^2 \times 97$, $5 \times 40^2 \times 121$, $6 \times 48^2 \times 145$, $8 \times 64^2 \times 193$, with $\alpha = 3, 6, 9, 18$ MeV.



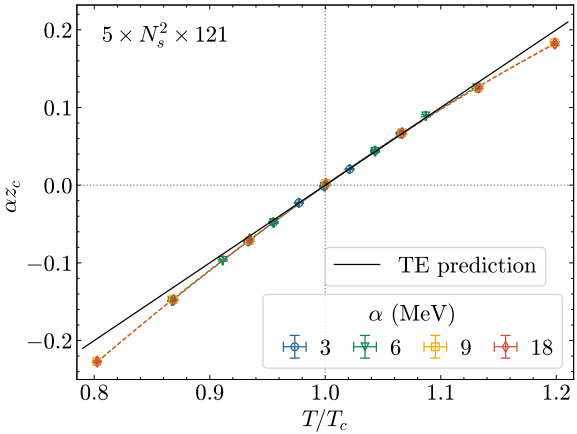
- Rather good agreement with TE prediction

$$T(z_c) = T_{c0} \quad \Rightarrow \quad z_c(T) = \frac{1}{\alpha}\left(\frac{T}{T_{c0}} - 1\right)$$

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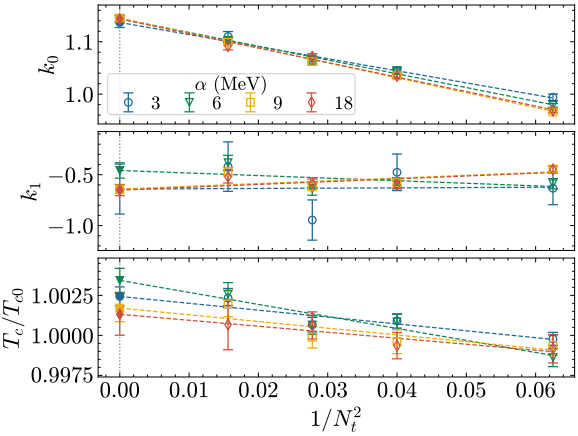
- A minor deviation takes place

$$z_c = \frac{k_0}{\alpha}\left(\frac{T}{T_c(\alpha)} - 1\right) + \frac{k_1}{\alpha}\left(\frac{T}{T_c(\alpha)} - 1\right)^2$$

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- In the continuum limit we get

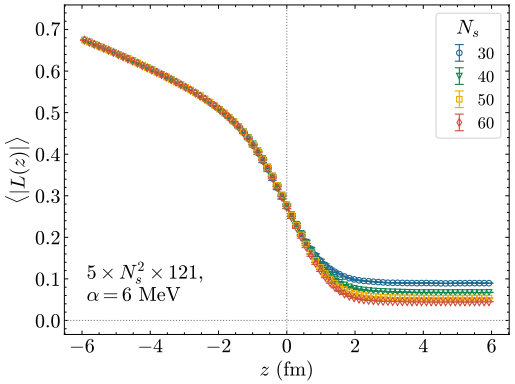
$$k_0 \sim 1.15\,, \tag{15}$$

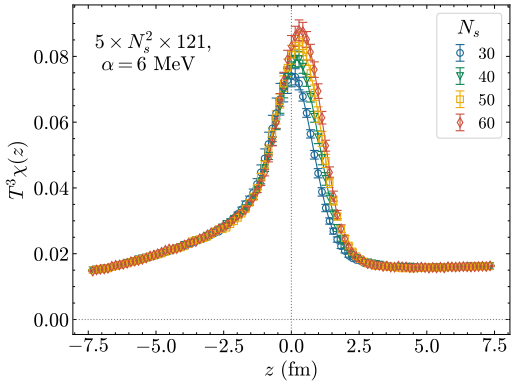
- And within the uncertainty

$$T_c(\alpha) \simeq T_{c0}\,. \tag{16}$$

▷ lattice renormalization of α ?, crossover transition ?, high order α-corrections to TE ?, ...

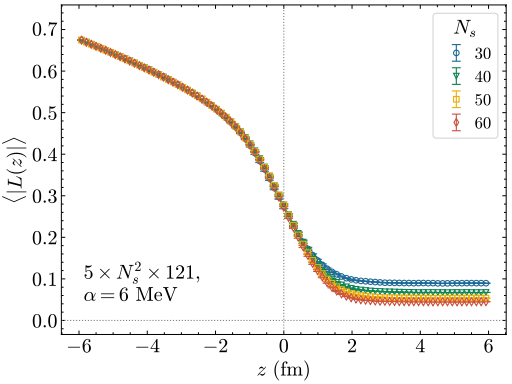
Properties of the spatial transition

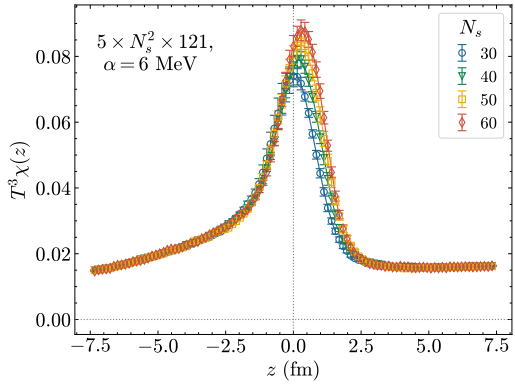




- Type of the transition can be studied from volume scaling of χ_{max} and σ.
- 1st order: $\chi_{max} \sim V \sim N_s^2$, $\sigma \sim 1/V \sim 1/N_s^2$

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- Type of the transition can be studied from volume scaling of χ_{max} and σ.
- 1st order: $\chi_{max} \sim V \sim N_s^2$, $\sigma \sim 1/V \sim 1/N_s^2$ we found $\chi_{max} \to const$, $\sigma \to const$ when $N_s \to \infty$
- Spatial transitions in both cases $\Omega \neq 0$ and $\alpha \neq 0$ are crossovers.
- There is a smallest width $\sigma_0 \sim 1$ fm, due to a finite correlation length.

Local approximation and asymmetry between E and H

- Approximation of local thermalization: coefficients in action are taken at some point, and system with homogeneous local action is simulated.
- $T_c(\Omega r)$ from this approximation demonstrates excellent agreement with full rotating simulations.
- Rotating action has a structure $S_G = S_0 + S_1 + S_2$, and linear term plays subleading role.

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Local action in the rotation-induced metric (at $x = r_0$, $y = 0$)

$$S_G = \int d^4x \left[\beta\left((E^a_x)^2 + (E^a_y)^2 + (E^a_z)^2 + (H^a_y)^2\right) + \tilde{\beta}\left((H^a_x)^2 + (H^a_z)^2\right)\right], \quad \tilde{\beta}/\beta = 1 + (\Omega_I r_0)^2 = 1 - (\Omega r_0)^2$$

- $\tilde{\beta}/\beta > 1$ (imaginary rotation) $\Rightarrow$ $T_{\text{eff}} \nearrow$, $T_c \searrow$;
- $\tilde{\beta}/\beta < 1$ (real rotation) $\Rightarrow$ $T_{\text{eff}} \searrow$, $T_c \nearrow$

Local action in the acceleration-induced metric (at $z = z_0$)

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- $\tilde{\beta}/\beta > 1$ (away from horizon) $\Rightarrow$ $T_{\text{eff}} \nearrow$, $T_c \searrow$;
- $\tilde{\beta}/\beta < 1$ (to horizon) $\Rightarrow$ $T_{\text{eff}} \searrow$, $T_c \nearrow$

Conclusions

- We found new state of gluon plasma: spatially separated confinement and deconfinement phases coexist simultaneously at thermal equilibrium (the same is expected for QCD — in progress).
- For real rotation, mixed state may appear for $T \gtrsim T_{c0}$, with confinement at the periphery and deconfinement in the center (in a disagreement with TE prediction)
- For acceleration, deconfinement appears near the horizon, and confinement is located in the opposite side (in a reasonable agreement with TE prediction)
- Finite temperature confinement/deconfinement transition becomes spatial crossover with minimal finite width of ~ 1 fm.
- Induced by rotation and acceleration metrics modify the gluon action in different ways. For rotation, asymmetry for chromomagnetic fields change dynamics of gluons what can not be accounted for by TE.
- Unexpected phase arrangement in the rotating system may be related to the negative Barnett effect and $I < 0$ below supervortical temperature. [Braguta, et al.'24; Braguta, et al.'24]
- What is the QGP phase near the black hole horizon? — Question for future studies

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Thank you for your attention!

BACKUP

Local approximation for inhomogeneous action

The homogeneous local action (at $x = r_0$, $y = 0$) is

$$S_G = \frac{1}{2g_0^2}\int d^4x \left[F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} + F^a_{z\tau}F^a_{z\tau} + F^a_{xz}F^a_{xz} + \right.$$
$$\left. + \left(1+u_I^2\right)F^a_{yz}F^a_{yz} + \left(1+u_I^2\right)F^a_{xy}F^a_{xy} - 2u_I\left(F^a_{yx}F^a_{x\tau} + F^a_{yz}F^a_{z\tau}\right)\right], \quad (17)$$

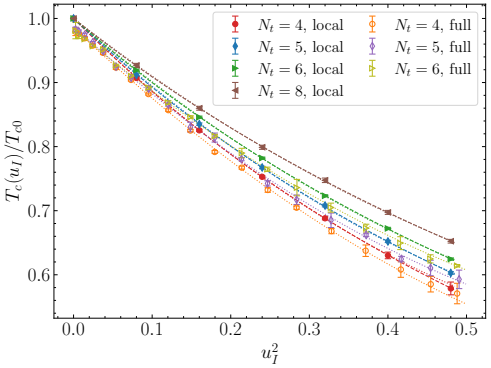
where $u_I = \Omega_I r_0$ is a local velocity. Local system is simulated as usual on small lattices with $N_s = N_z$.

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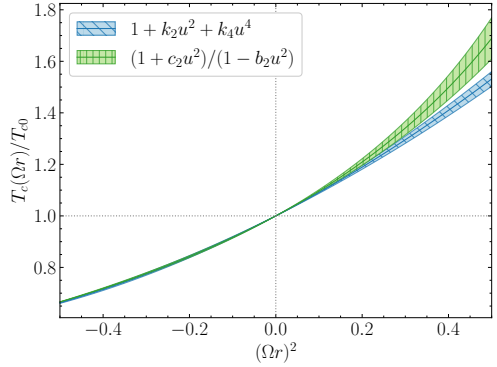
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- $T_c(u)$ from two methods are in agreement.
- Data are fitted by two different functions:

$$\frac{T_c(u)}{T_{c0}} = 1 + k_2u^2 + k_4u^4\,, \quad (18)$$

$$\frac{T_c(u)}{T_{c0}} = \frac{1 + c_2u^2}{1 - b_2u^2}\,. \quad (19)$$

- In continuum limit the coefficients are

$$k_2 = 0.869(31)\,, \qquad k_4 = 0.388(53)\,. \quad (20)$$

$$c_2 = 0.206(66)\,, \qquad b_2 = 0.694(101)\,. \quad (21)$$

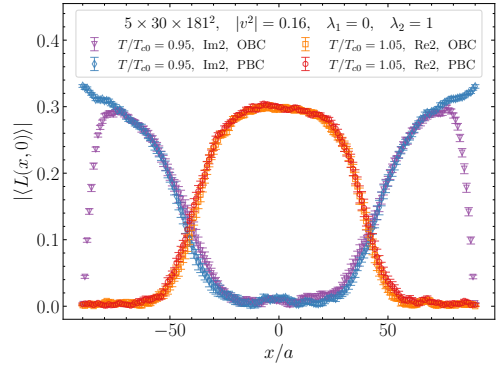
Decomposition of rotating action for gluons

The action of rotating gluons is inhomogeneous and it is a quadratic function in Ω_I,

$$S_G = S_0 + \lambda_1 S_1 \Omega_I + \lambda_2 S_2 \Omega_I^2 \,, \tag{22}$$

where we introduce switching factors λ_1, λ_2; and operators S_1, S_2 are inhomogeneous.

- No sign problem in a case of $\lambda_1 = 0$. Re2: $\lambda_2 \Omega_I^2 = \Omega_I^2 < 0$ or $\Omega^2 > 0$; Im2: $\lambda_2 \Omega_I^2 = \Omega_I^2 > 0$



- Re2: $T = T_{c0} + \Delta T$ vs Im2: $T = T_{c0} - \Delta T$

$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + \kappa(\Omega r)^2 \quad \text{vs} \quad \frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - \kappa(\Omega_I r)^2$$

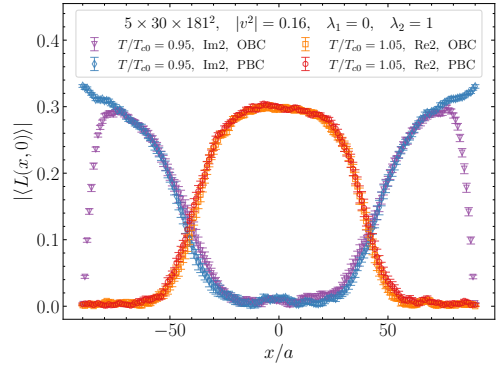
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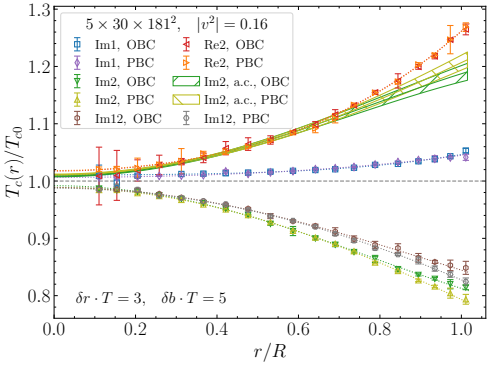
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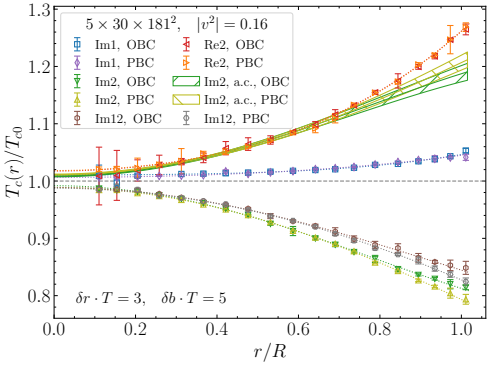
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