Spin contribution to Faraday effect in a non-relativistic magnetized plasma

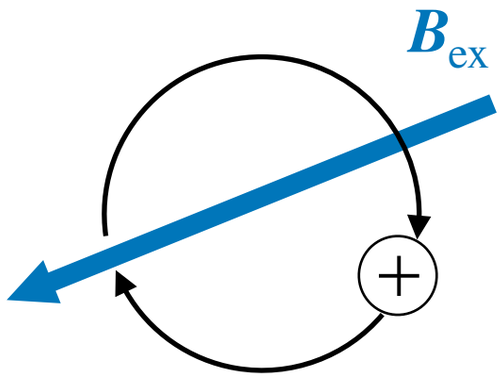
In preparation

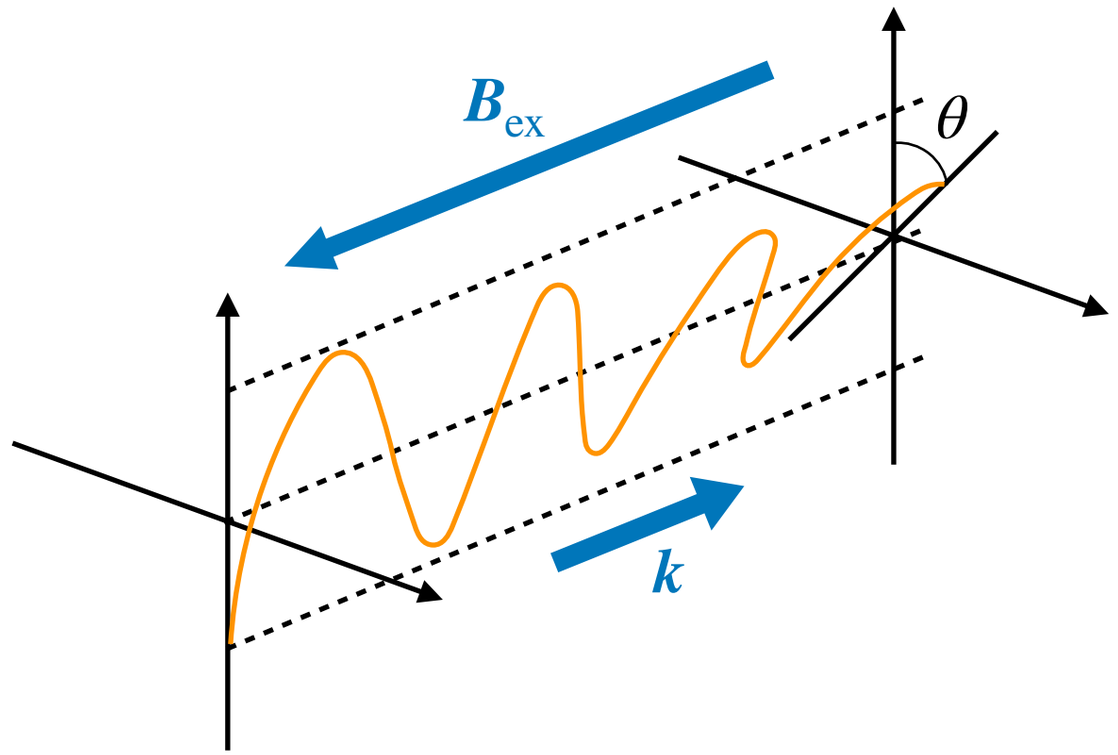
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Faraday effect





- Typical magneto-optical phenomenon seen in many systems

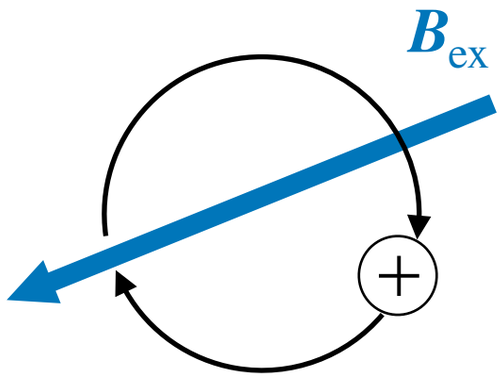
  (plasma physics, condensed-matter physics, astrophysics,…)

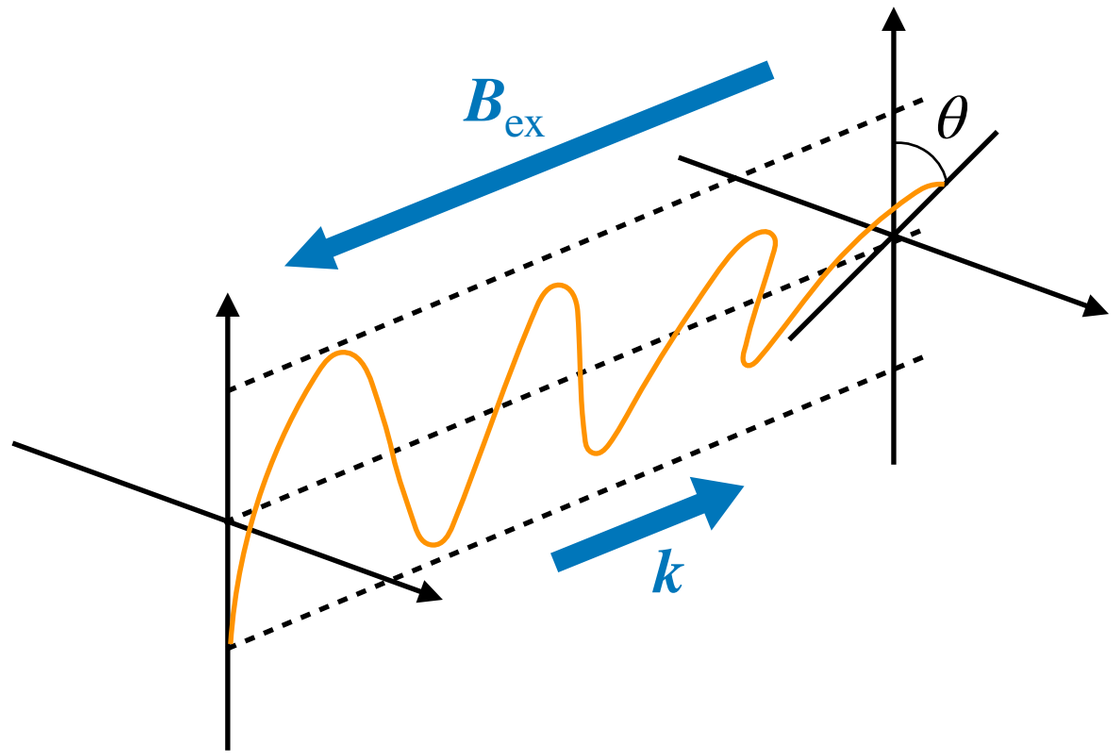
- Faraday rotation angle

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} \simeq -\mathrm{i}\omega\frac{\varepsilon_{xy}+\mu_{xy}}{2}$$

ε_{ij} : permittivity, μ_{ij} : permeability

Faraday effect





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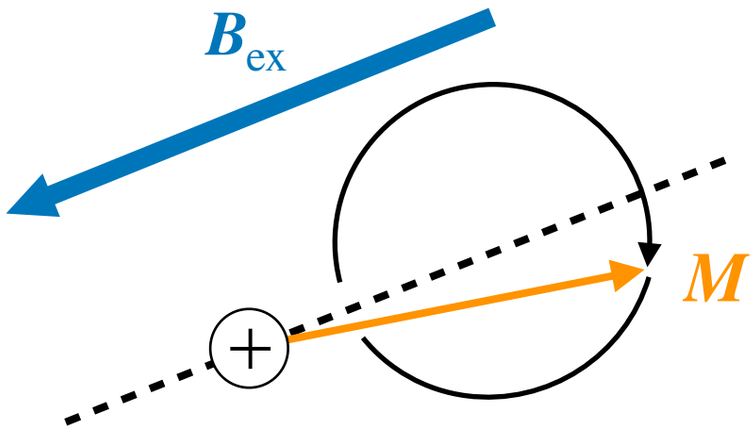
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$$\frac{\mathrm{d}\theta}{\mathrm{d}z} \simeq -\mathrm{i}\omega \frac{\varepsilon_{xy} + \mu_{xy}}{2}$$

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How does spin contribute to the Faraday effect?

Intuitive picture



$$\frac{\mathrm{d}\boldsymbol{M}}{\mathrm{d}t} = \frac{q}{m}\boldsymbol{M}\times\boldsymbol{B}$$

- Spin precession → correction to the rotation angle

- Magnetization under a dynamical magnetic field

$$\delta\boldsymbol{M} = \mathrm{i}\frac{q}{m\omega}\boldsymbol{M}_0\times\delta\boldsymbol{B} \propto \frac{\boldsymbol{B}_{\mathrm{ex}}}{\omega}\delta\boldsymbol{B} \quad\Rightarrow\quad \mu_{xy} \propto \frac{\boldsymbol{B}_{\mathrm{ex}}}{\omega}$$

$$\frac{\mathrm{d}\theta_{\mathrm{spin}}}{\mathrm{d}z} \simeq -\mathrm{i}\omega\frac{\mu_{xy}}{2} \propto -\cancel{\omega}\frac{1}{2}\frac{\boldsymbol{B}_{\mathrm{ex}}}{\cancel{\omega}}$$

$\frac{\mathrm{d}\theta}{\mathrm{d}z} \propto \frac{1}{\omega^2}$: classical result

Spin contribution to the angle is independent of the frequency

SH, Hattori, Yamamoto (in preparation)

Extended phase space

- Construct kinetic theory from the underlying QFT

- Hamiltonian in the non-relativistic limit — Convenient to see spin behavior

$$H = -\frac{\hbar^2}{2m}\boldsymbol{D}^2 + q\phi - \frac{\hbar q}{2m}\boldsymbol{\sigma}\cdot\boldsymbol{B}$$

- Wigner function：quantum analogue of distribution function

$$W_{\alpha\beta}(x,p) \equiv \int \mathrm{d}^4 y\, \mathrm{e}^{\mathrm{i}p\cdot y/\hbar} \left\langle \psi^\dagger_\beta\left(x-\frac{y}{2}\right)\psi_\alpha\left(x+\frac{y}{2}\right)\right\rangle \qquad (\alpha,\beta : \text{spin index})$$

- Scalar Wigner function

$$\mathscr{W}(x,p,\boldsymbol{n}) \equiv \mathrm{Tr}[P(\boldsymbol{n})W(x,p)]\,, \qquad P(\boldsymbol{n}) \equiv \frac{1+\boldsymbol{n}\cdot\boldsymbol{\sigma}}{2}$$

spin in phase space

$$\langle\boldsymbol{\sigma}\rangle = 2\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{\mathrm{d}^2\boldsymbol{n}}{4\pi} 3\boldsymbol{n}\mathscr{W}(x,p,\boldsymbol{n})$$

Kinetic equation

- Equation of motion for the scalar Wigner function

$$\left[p_0 - \frac{|\boldsymbol{p}|^2}{2m} + \frac{\hbar q}{2m}\boldsymbol{B}\cdot\left(\boldsymbol{n} + \frac{D}{D\boldsymbol{n}}\right)\right]\mathscr{W} = 0 \qquad \frac{D}{D\boldsymbol{n}}: \text{tangental derivative}$$

$$\left[\partial_t - qF_{0\nu}\partial_p^\nu + \frac{1}{m}p^i(\partial_i^x - qF_{i\nu}\partial_p^\nu) - \frac{q}{m}\boldsymbol{B}\cdot\left(\boldsymbol{n}\times\frac{\partial}{\partial\boldsymbol{n}}\right) - \frac{\hbar q}{2m}\boldsymbol{B}(\overleftarrow{\partial}_x\cdot\overrightarrow{\partial}_p)\cdot\left(\boldsymbol{n} + \frac{D}{D\boldsymbol{n}}\right)\right]\mathscr{W} = 0$$

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{x}} + \frac{q}{m}(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})\cdot\frac{\partial f}{\partial \boldsymbol{v}} + \underbrace{\frac{q}{m}(\boldsymbol{n}\times\boldsymbol{B})\cdot\frac{\partial f}{\partial \boldsymbol{n}}}_{\text{spin precession}} + \frac{\hbar q}{2m^2}\frac{\partial}{\partial \boldsymbol{x}}\left[\boldsymbol{B}\cdot\left(\boldsymbol{n} + \frac{D}{D\boldsymbol{n}}\right)\right]\cdot\frac{\partial f}{\partial \boldsymbol{v}} = 0$$

Zamanian, Murklund, Brodin (2010)

- Induced current and magnetization

$$q\boldsymbol{j}(\omega,\boldsymbol{k}) = 2q\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{\mathrm{d}^2\boldsymbol{n}}{4\pi}\boldsymbol{v}f(\omega,\boldsymbol{k},\boldsymbol{p},\boldsymbol{n}) \quad\Rightarrow\quad \epsilon^{ij}: \text{permittivity}$$

$$\boldsymbol{M}(\omega,\boldsymbol{k}) = \frac{\hbar q}{m}\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{\mathrm{d}^2\boldsymbol{n}}{4\pi}3\boldsymbol{n}f(\omega,\boldsymbol{k},\boldsymbol{p},\boldsymbol{n}) \quad\Rightarrow\quad \mu^{ij}: \text{permeability}$$

Faraday rotation angle

SH, Hattori, Yamamoto (in preparation)

- Focus on the long-wavelength limit ($\omega \gg |\boldsymbol{k}|$)

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = -\,\mathrm{sgn}(qB_{\mathrm{ex}})\omega_{\mathrm{p}}^2\omega_{\mathrm{c}}\left(\frac{1}{2\omega^2} + \frac{3\hbar^2}{8}\frac{I_2}{I_4}\right)$$

spin contribution

ω_{p} : plasma frequency

ω_{c} : cyclotron frequency

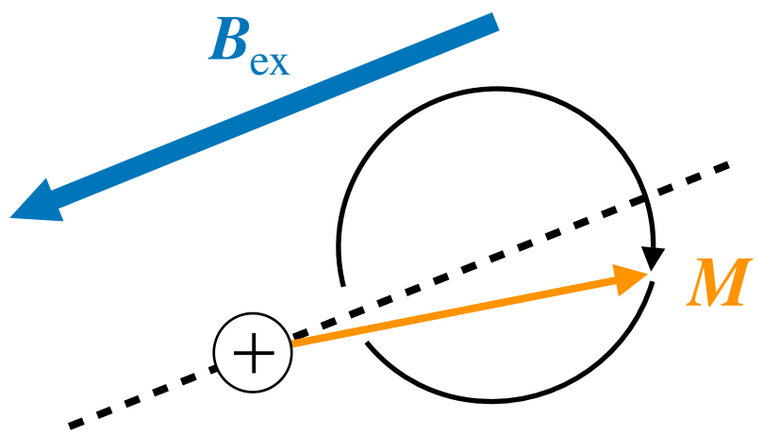
$$I_\alpha \equiv -\int_0^\infty \frac{\mathrm{d}|\boldsymbol{p}|}{\pi^2}|\boldsymbol{p}|^\alpha \frac{\partial f^{(0)}(\boldsymbol{p})}{\partial\varepsilon}$$

→ **Independent of frequency (achromatic)**

- CMB polarization includes this contribution
  - BSM contribution is also achromatic
  - Classical Faraday effect is subtracted by multifrequency observation
- Estimate in the non-degenerate limit

$$\theta_{\mathrm{spin}} \sim 10^{-14}\,\mathrm{rad}\left(\frac{B}{1\,\mathrm{nG}}\right)\left(\frac{T}{10^5\,\mathrm{K}}\right)^{-1}\left(\frac{n}{0.1/\mathrm{cm}^3}\right)\left(\frac{L}{1\,\mathrm{Gpc}}\right)$$

Conclusion and outlook



$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = -\,\mathrm{sgn}(qB_{\mathrm{ex}})\omega_{\mathrm{p}}^2\omega_{\mathrm{c}}\left(\frac{1}{2\omega^2} + \underline{\frac{3\hbar^2}{8}\frac{I_2}{I_4}}\right)$$

spin contribution

- Derived the spin contribution in kinetic theory

- Spin-induced Faraday rotation angle is achromatic

- Spin contribution in relativistic plasmas?
  - e.g. High density effective theory $m \to \mu$?

- Other phenomenological implications?

Appendix

Setup

- Faraday configuration

$$\boldsymbol{E} = \boldsymbol{E}_\perp\,, \quad \boldsymbol{B} = \boldsymbol{B}_{\mathrm{ex}} + \boldsymbol{B}_\perp \qquad (\boldsymbol{B}_{\mathrm{ex}} = B_{\mathrm{ex}}\boldsymbol{e}_z,\ \boldsymbol{k} = k\boldsymbol{e}_z)$$

- Power-counting scheme

$$\partial^x_\mu = \mathcal{O}(\delta)\,, \quad q = \mathcal{O}(\delta)\,, \quad a^\mu = \mathcal{O}(\epsilon_1)\,, \quad A^\mu_{\mathrm{ex}} = \mathcal{O}(\epsilon_2)$$

Need to see response $\propto \epsilon_1\epsilon_2$

- Distribution function

$$f = f^{(0)} + f^{(\delta\epsilon_{1,2})} + f^{(\delta^2\epsilon_1)} + f^{(\delta^3\epsilon_1)} + \underbrace{f^{(\delta^2\epsilon_1\epsilon_2)}}_{\text{classical}} + \underbrace{f^{(\delta^3\epsilon_1\epsilon_2)}}_{\text{spin}}$$

- Induced current and magnetization

$$q\boldsymbol{j} = 2q\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{\mathrm{d}^2\boldsymbol{n}}{4\pi}\boldsymbol{v}f \quad \Rightarrow \quad f = \#\hbar^0 + \underbrace{\#\hbar^2}_{\to 0\ (\omega \gg |\boldsymbol{k}|)}$$

$$\boldsymbol{M} = \frac{\hbar q}{m}\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{\mathrm{d}^2\boldsymbol{n}}{4\pi}3\boldsymbol{n}f \quad \Rightarrow \quad f = \#\hbar$$