Rotating free fermions on the lattice

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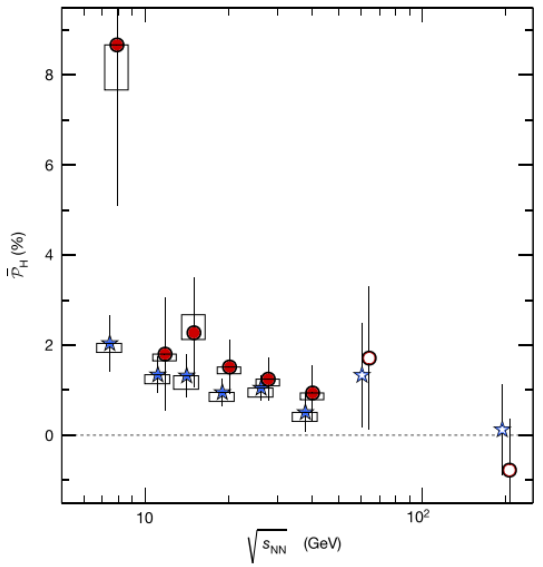




NATIONAL RESEARCH, DEVELOPMENT AND INNOVATION OFFICE

Why is rotation interesting?

- $\Lambda, \bar{\Lambda}$ Polarization Star '17
  Rotating QGP $\Omega \sim 6$ MeV



- Theory

  Thermodynamics of *rigidly* rotating and strongly-interacting system?

- Discrepancy in pure SU(3) theory:
  - Lattice: T_c **increases** with rotation.
  - PT, effective models, holography ...: T_c **decreases** with rotation.

- Same issue when fermions are included $\rightarrow$ needs to be resolved!

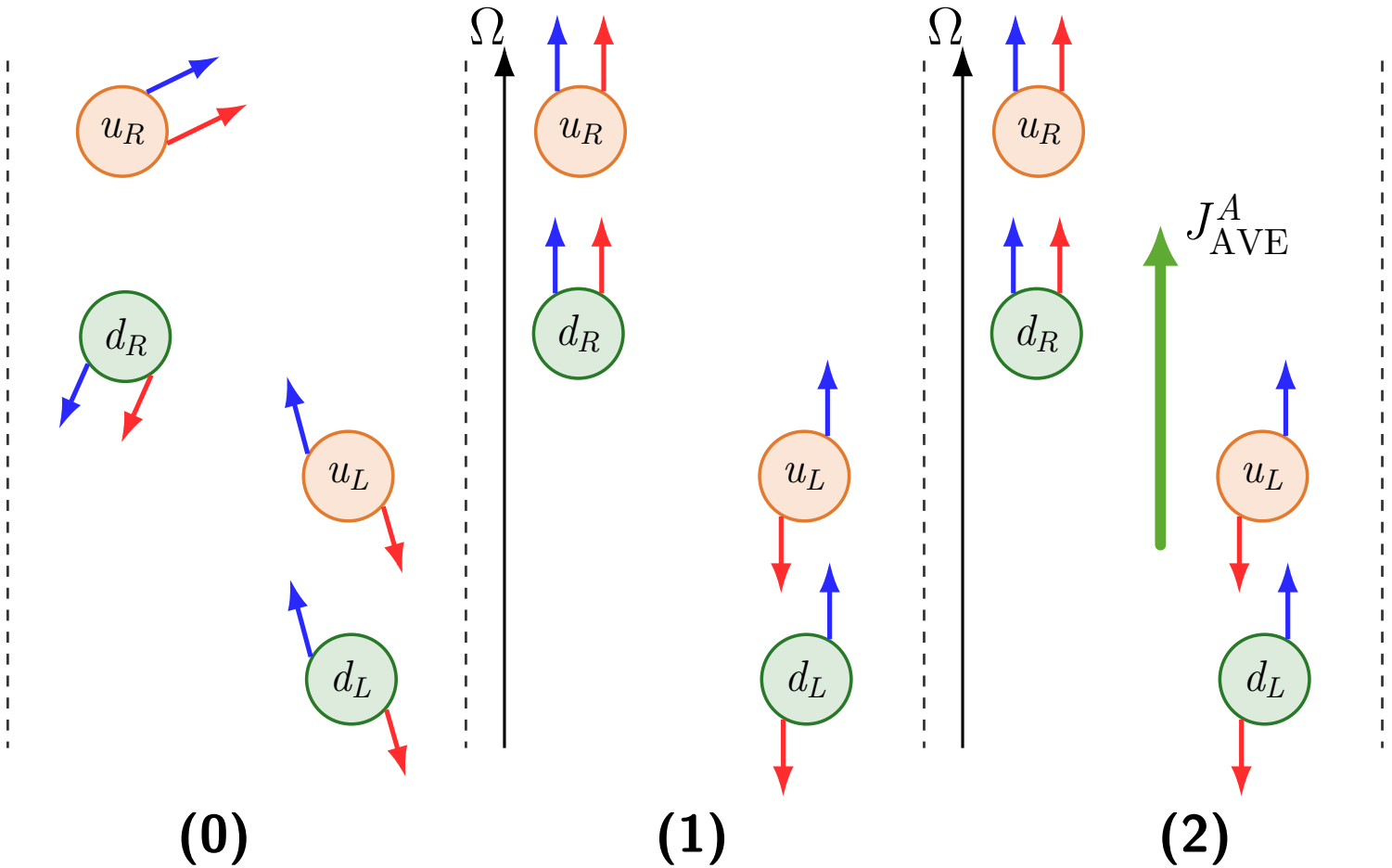
Why focus on fermions?

- Goal: full QCD simulations at the physical point: staggered fermions
- Complicated formalism $\rightarrow$ understand subtleties with Wilson fermions
- Free fermions:
  - No sign problem
  - Can be solved exactly
  - Continuum results known
- Introducing fermions also means *transport effects*:

$$\text{Axial anomaly} + \begin{matrix}\text{EM fields} \\ \text{Vorticity}\end{matrix} \rightarrow \textbf{Anomalous transport phenomena}$$

Axial Vortical Effect

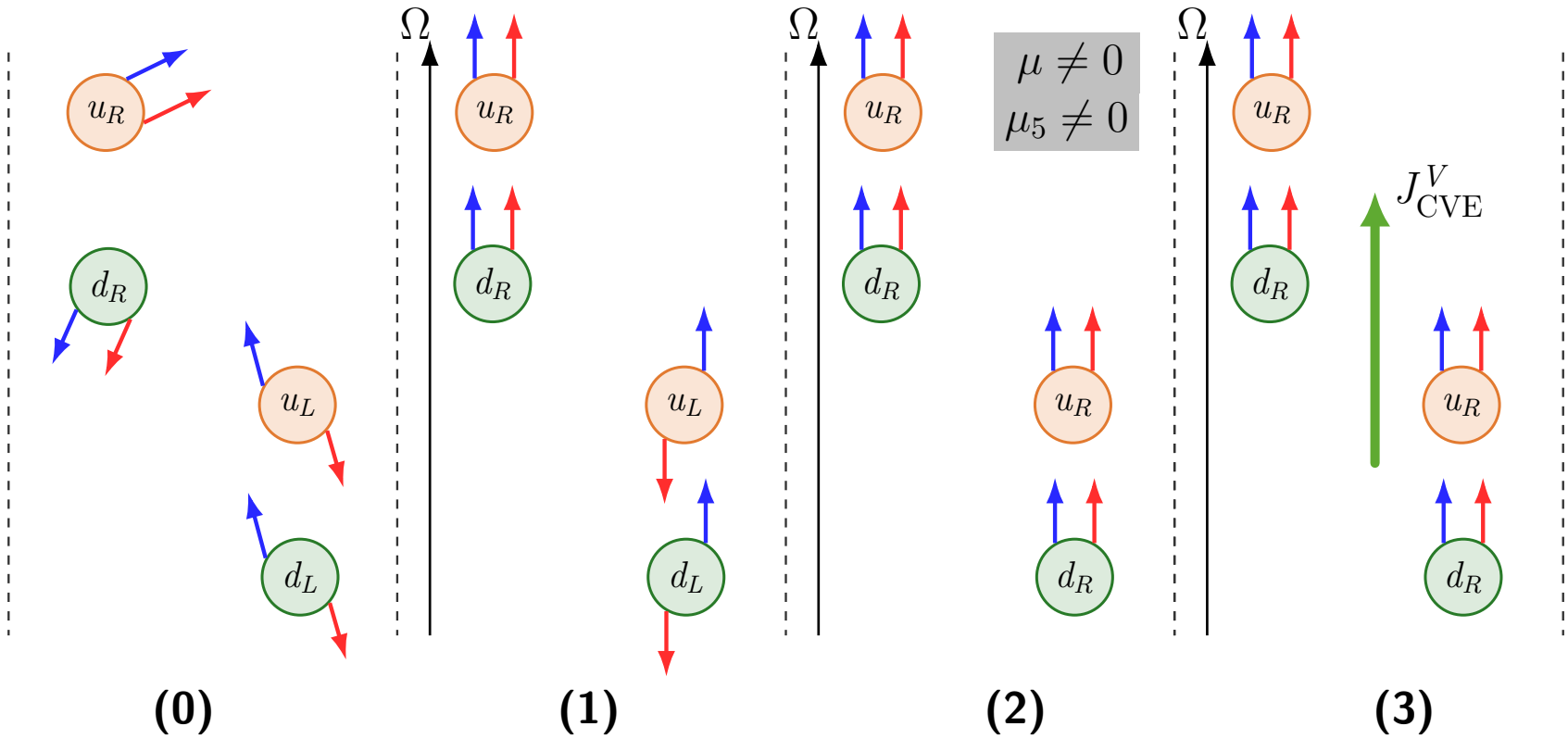
- Rotation + Finite temperature $\rightarrow$ Axial (baryon) current



spin, momentum

Chiral Vortical Effect

- Rotation + Finite density + Finite axial density → Vector (baryon) current



spin, momentum

Conductivities

- Conductivities known for free massless fermions e.g. Ahadi, Sadooghi '25
- **Axial Vortical Effect** (AVE):

$$j_{35} = \frac{1}{6}\Omega T^2$$

- Two-color QCD simulations find largely suppressed conductivity at high T Braguta et al. '14
- **Chiral Vortical Effect** (CVE):

$$j_3 = \frac{1}{\pi^2}\Omega\mu\mu_5$$

- Review for more details Kharzeev, Liao, Liao, Wang '15

Continuum formalism

- Use co-rotating frame and S_F in general metric [Yamamoto, Hirono '13]

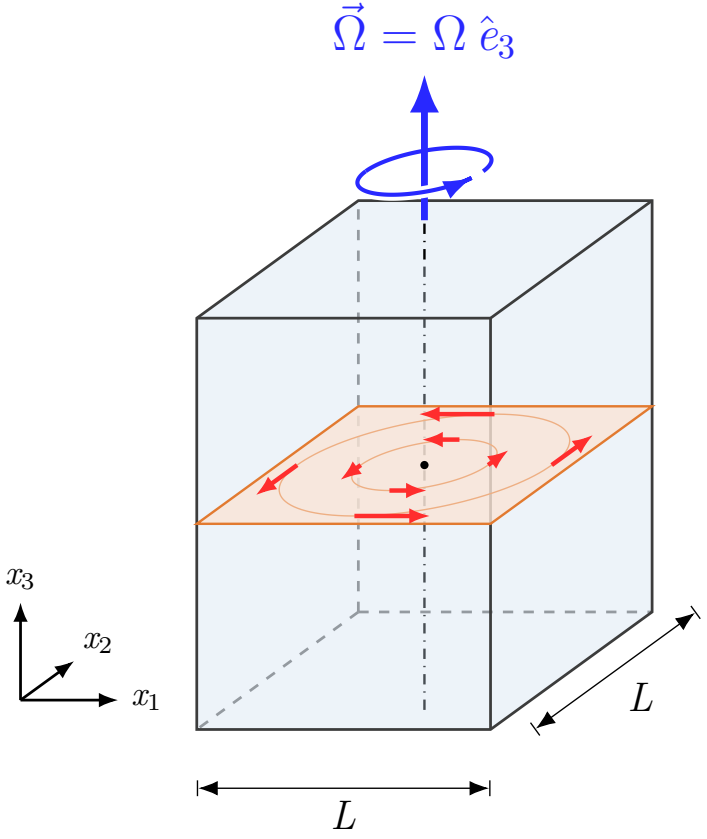
$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & x_2\Omega \\ 0 & 1 & 0 & -x_1\Omega \\ 0 & 0 & 1 & 0 \\ x_2\Omega & -x_1\Omega & 0 & 1 + r^2\Omega^2 \end{pmatrix} .$$

- In Minkowski space-time
  causality $\leftrightarrow$ finite volume in x_1, x_2:

$$v = \Omega\sqrt{x_1^2 + x_2^2} < 1$$

- Constrain on *real* rotation:

$$\Omega L < \sqrt{2}$$



Lattice formalism

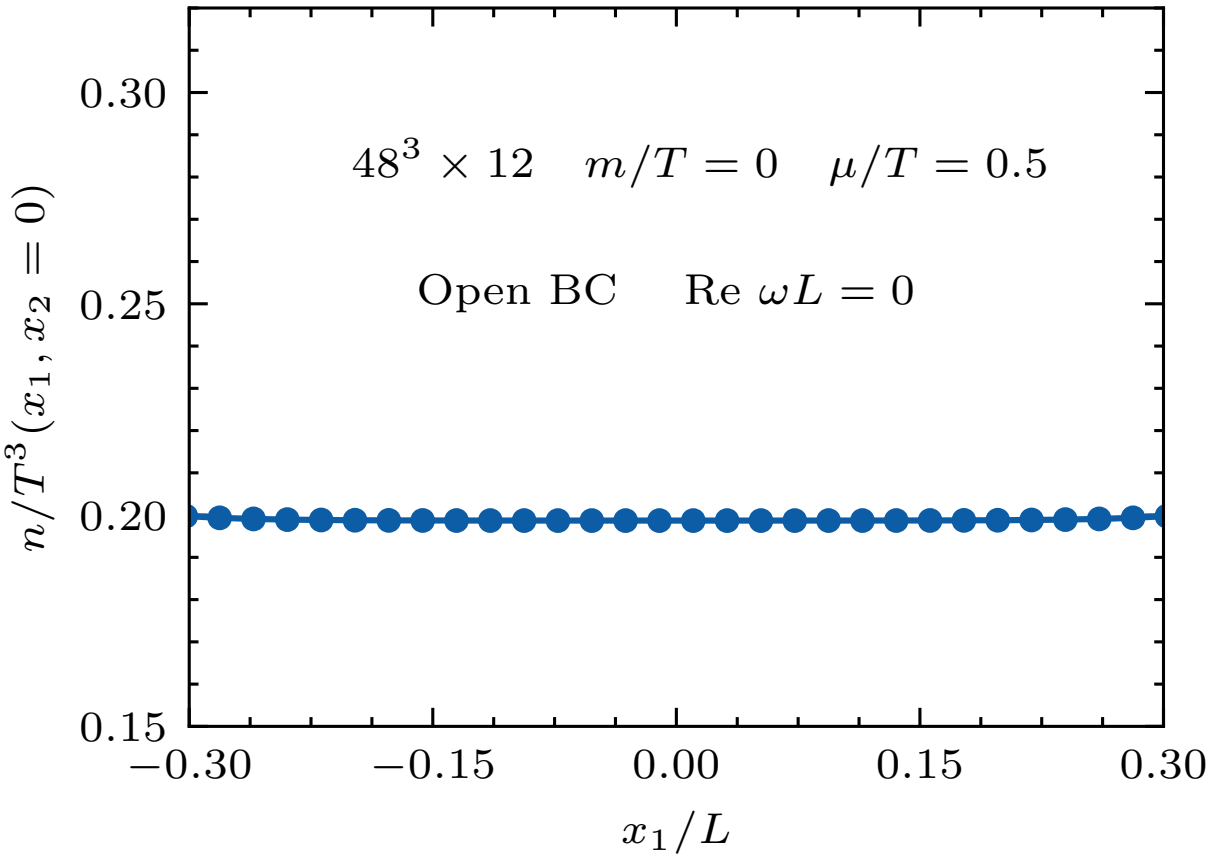
- Wilson fermions with *imaginary* rotation:

$$
\begin{aligned}
D(x,y) =& (4+m)\delta_{x,y} \\
& -\frac{1}{2}\Big[(r-\gamma_1+ x_2\Omega\gamma_4)\delta_{x+\hat{1},y} + (r+\gamma_1- x_2\Omega\gamma_4)\delta_{x-\hat{1},y} \\
& \quad +(r-\gamma_2- x_1\Omega\gamma_4)\delta_{x+\hat{2},y} + (r+\gamma_2+ x_1\Omega\gamma_4)\delta_{x-\hat{2},y} \\
& \quad +(r-\gamma_3)\delta_{x+\hat{3},y} + (r+\gamma_3)\delta_{x-\hat{3},y} \\
& \quad +(r-\gamma_4)\exp\left(i\Omega\frac{\sigma_{12}}{2}\right)\delta_{x+\hat{4},y} + (r+\gamma_4)\exp\left(-i\Omega\frac{\sigma_{12}}{2}\right)\delta_{x-\hat{4},y}\Big]
\end{aligned}
$$

- Orbital and spin angular momentum
- Boundary conditions in $x_1 - x_2$:
  - Periodic: incompatible with velocity profile
  - Open: jumps outside the boundary are removed

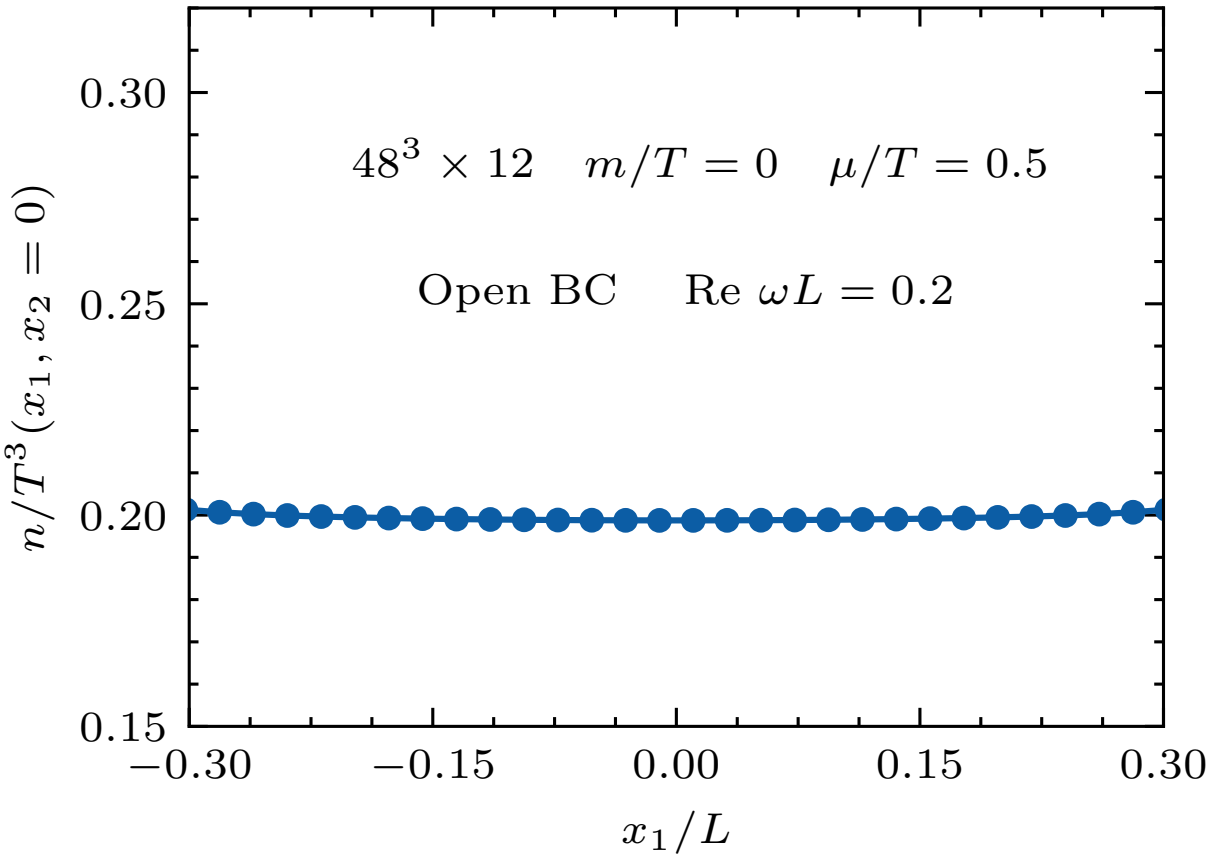
Density

- System is *inhomogeneous* $\rightarrow$ resolve (x_1, x_2) dependence



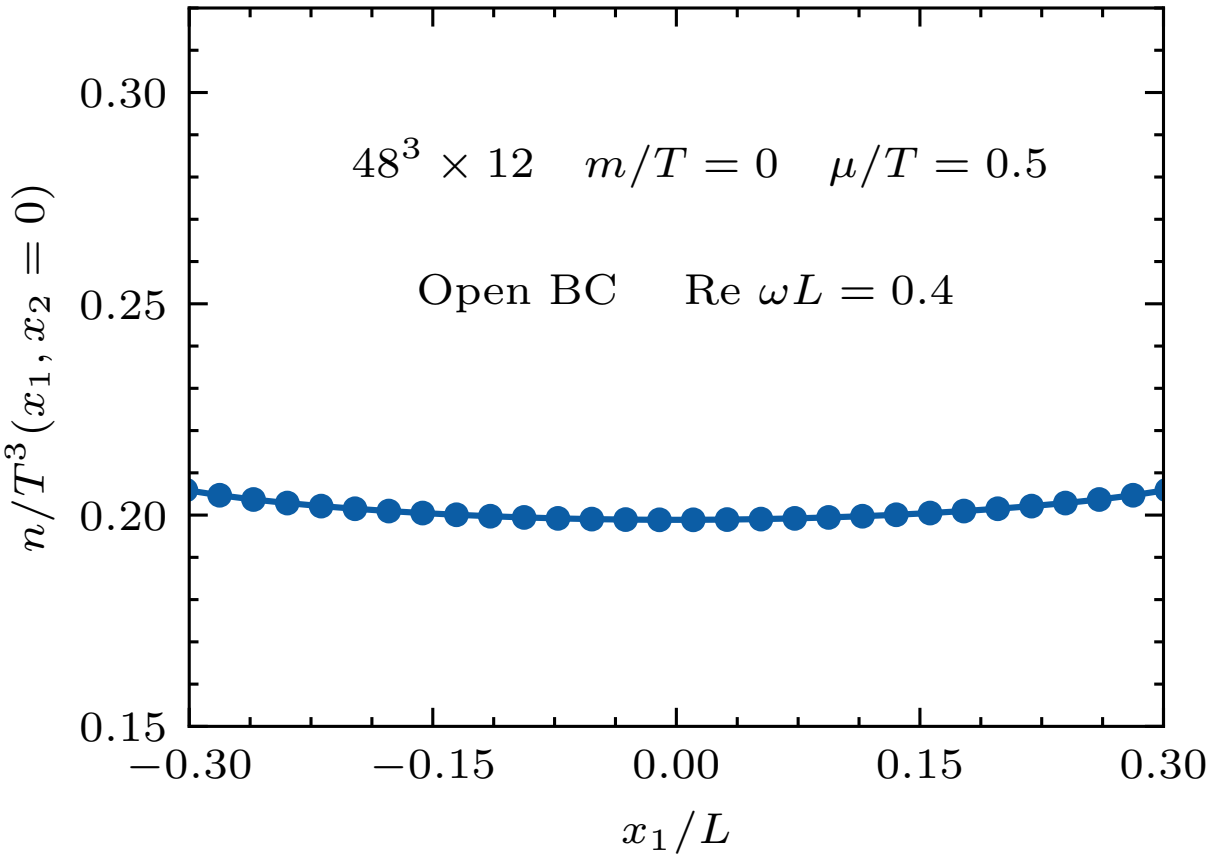
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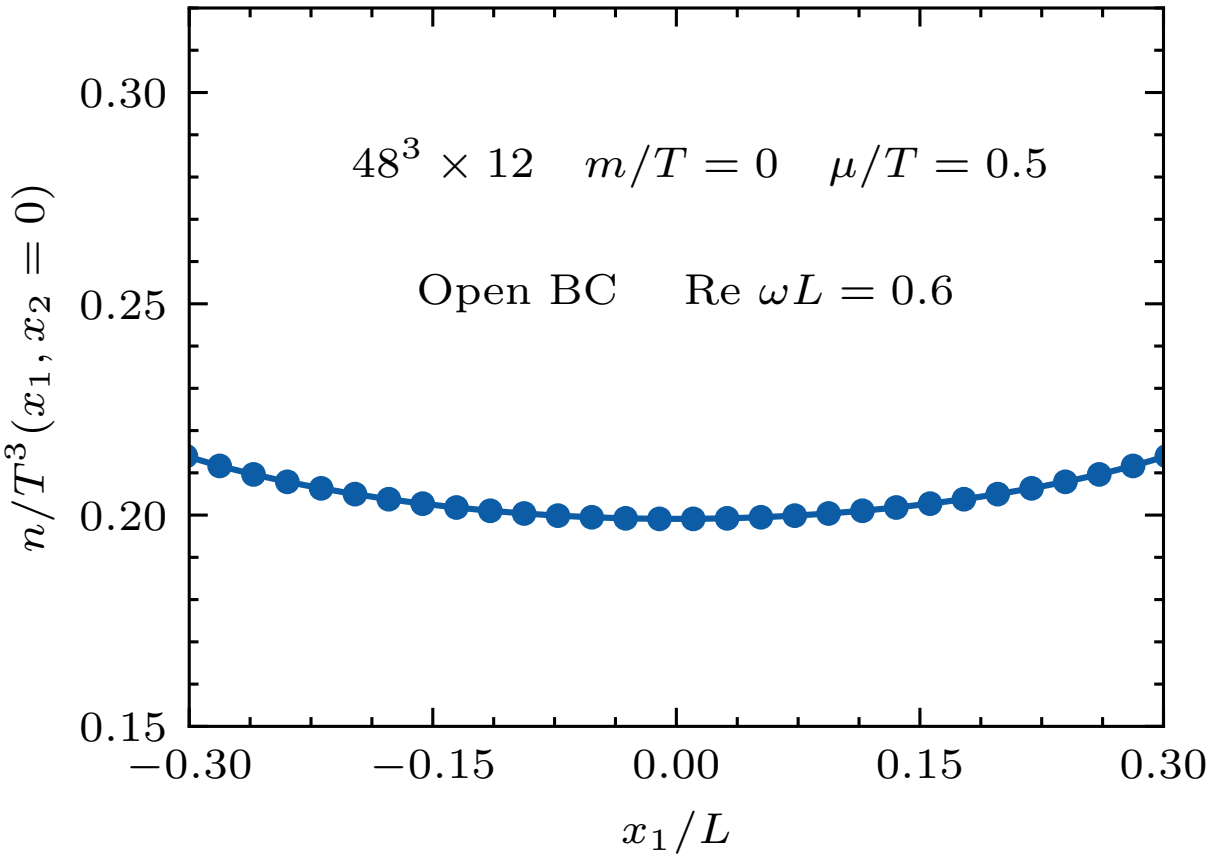
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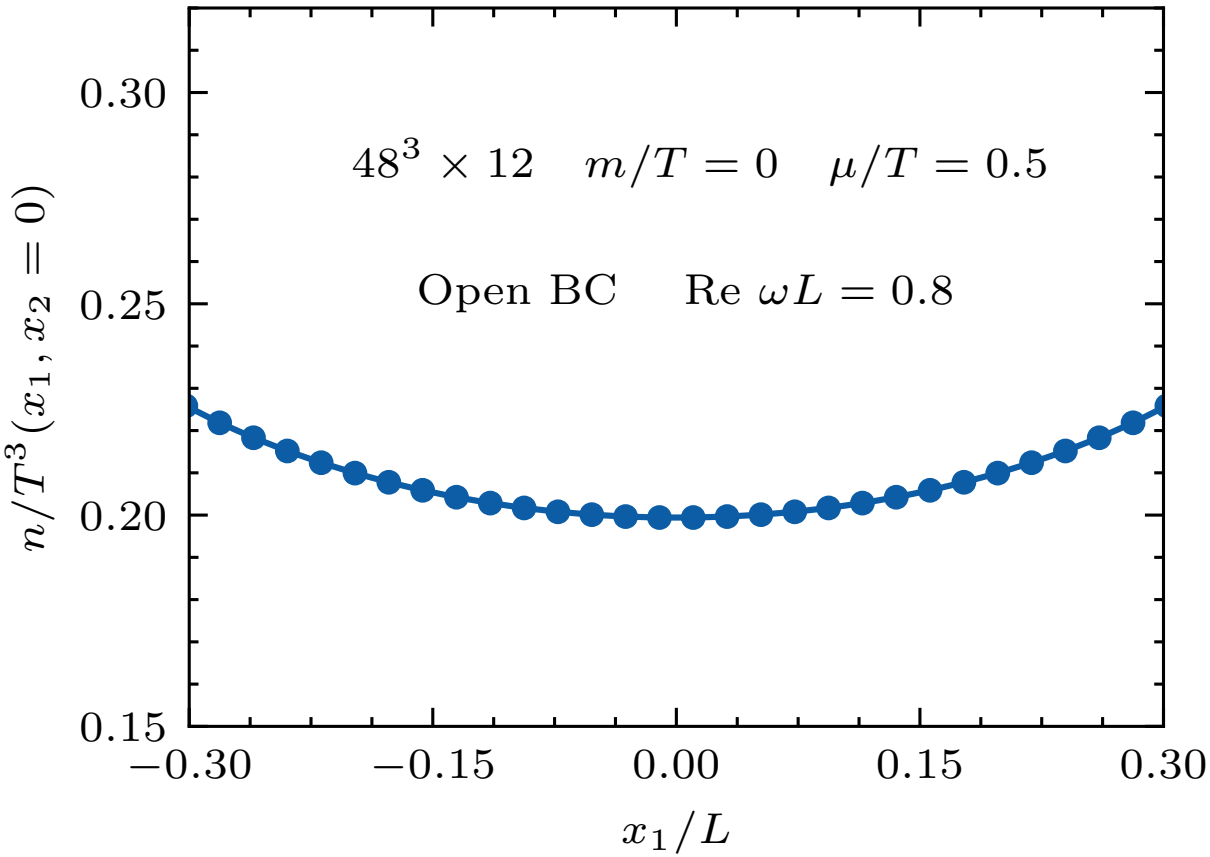
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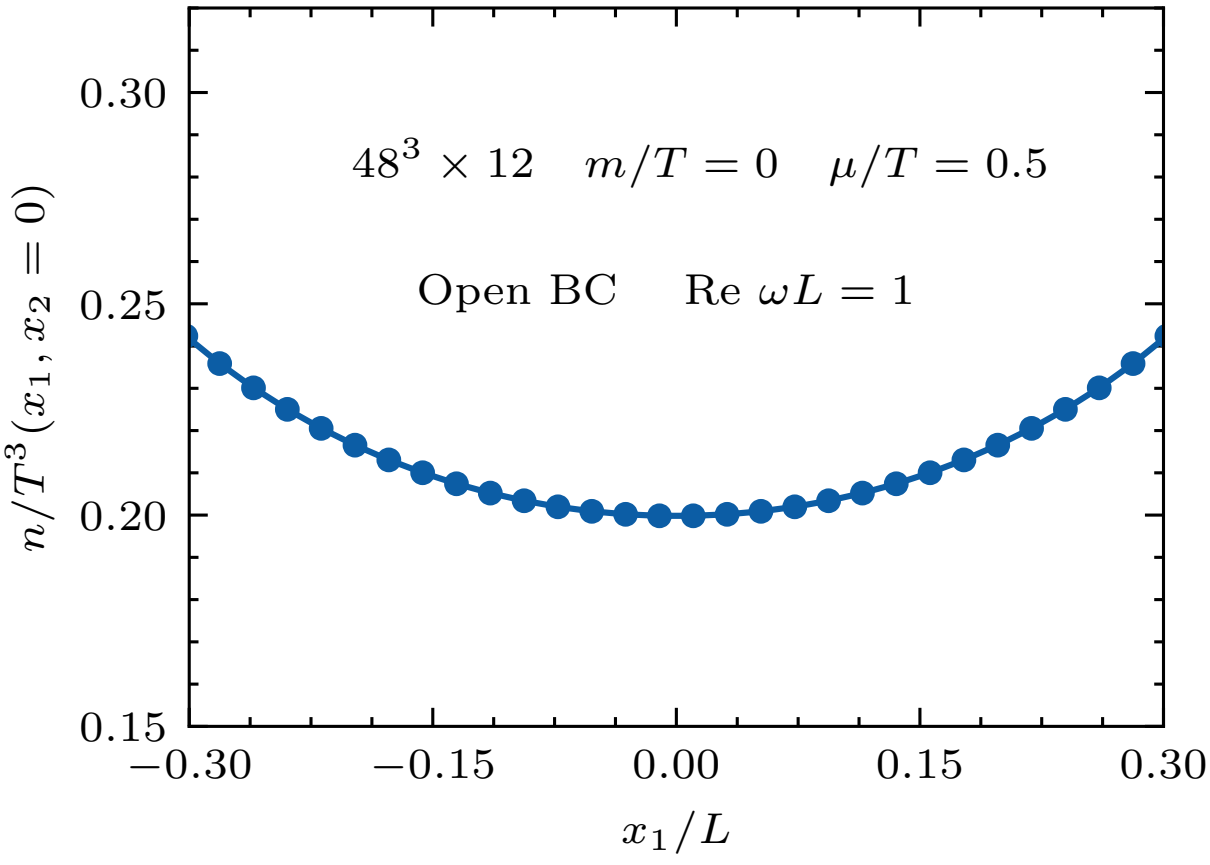
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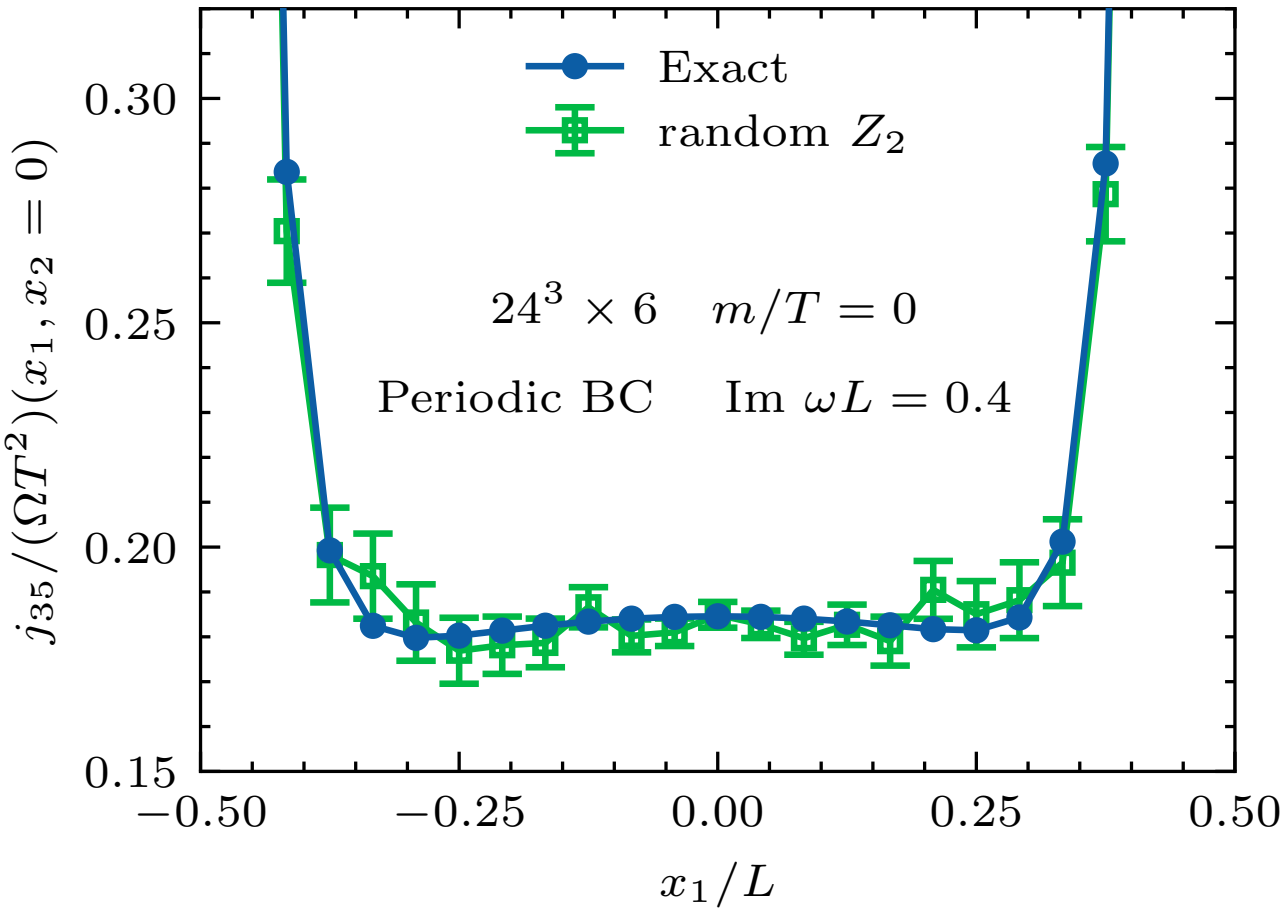
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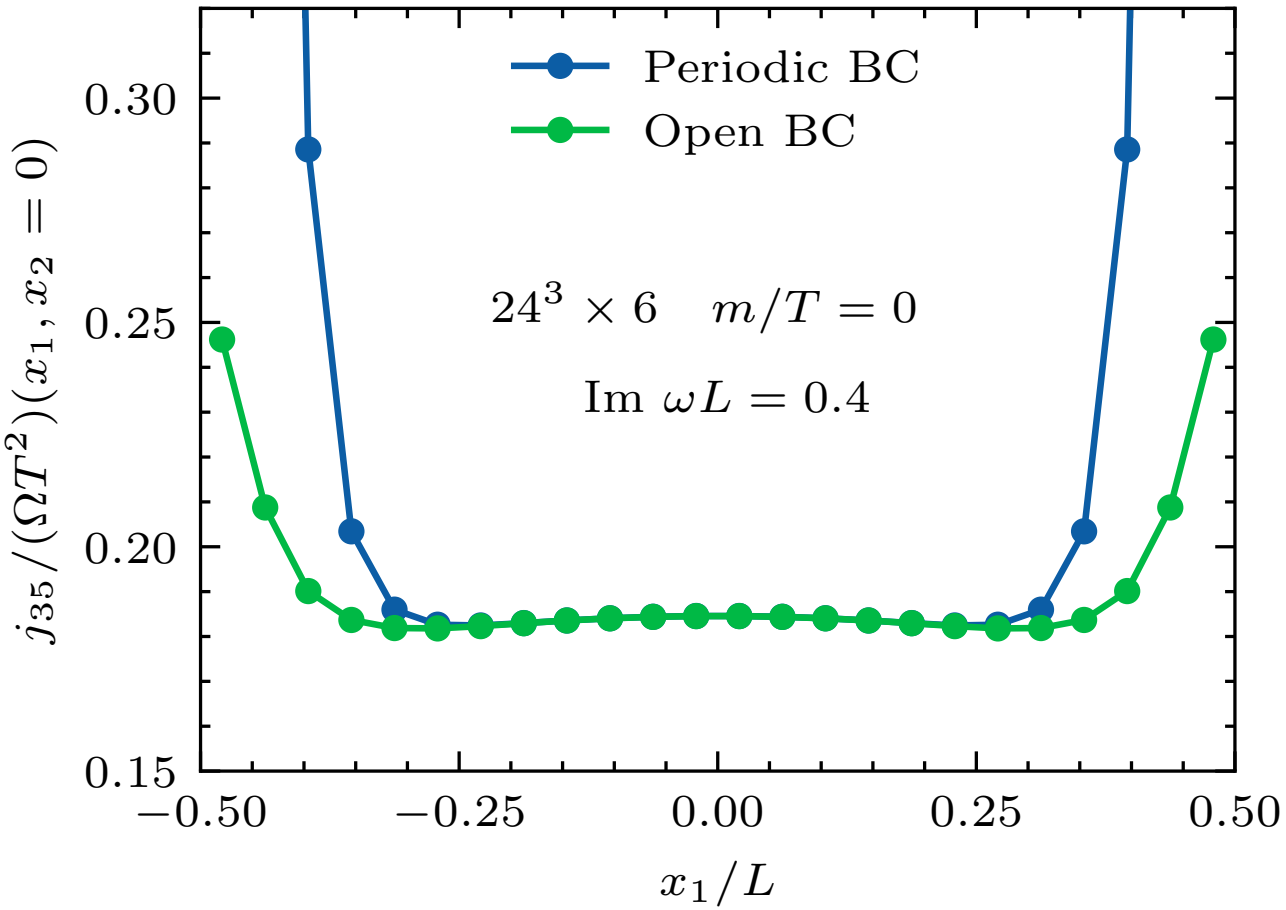
AVE: profile

- Crosscheck of exact trace against random vectors
- Severe BC effects



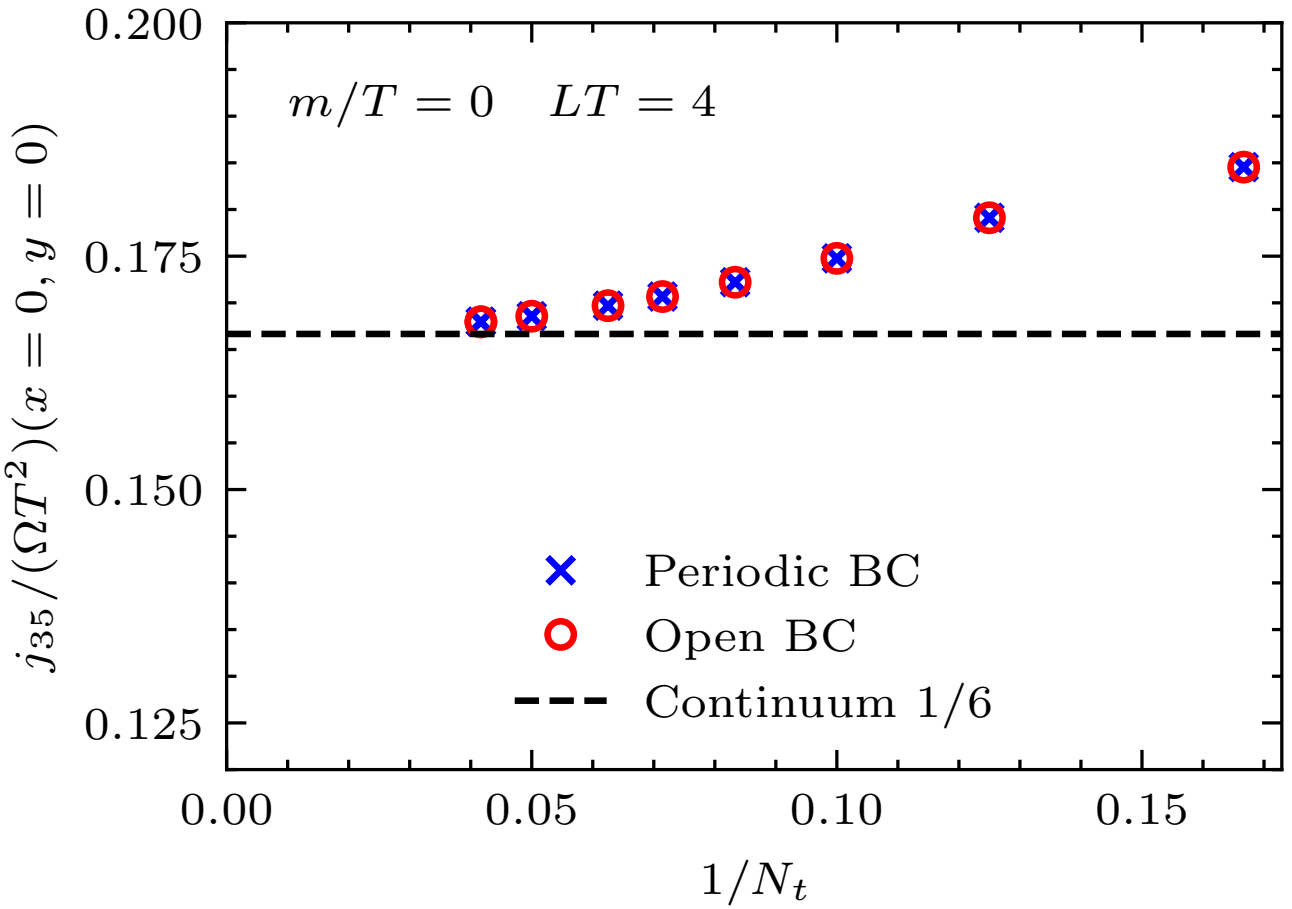
AVE: boundary conditions

- Open BC have much milder effect close to the border
- Around center BCs are equivalent



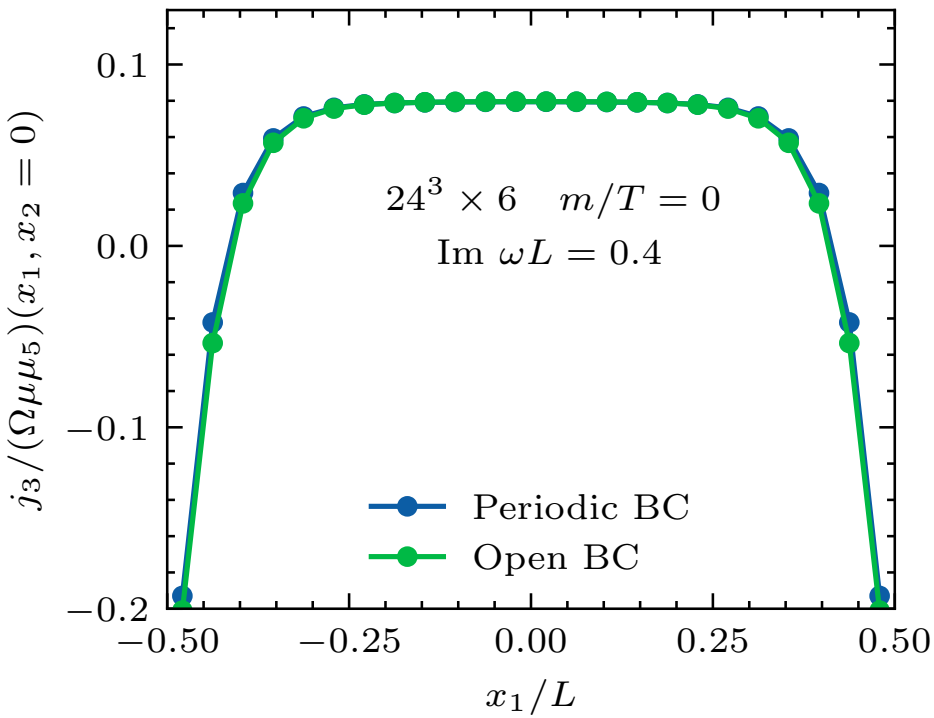
AVE: continuum limit

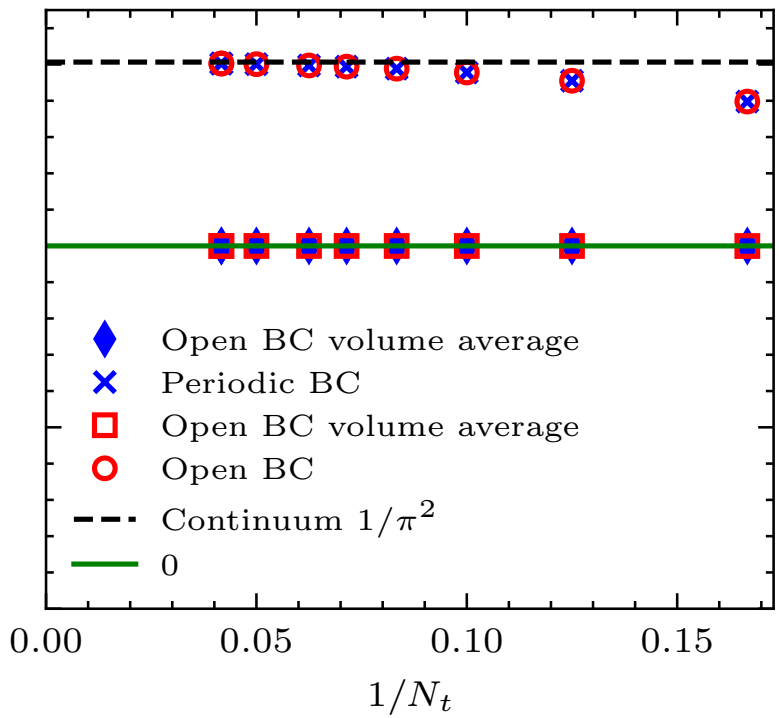
- Correct continuum limit at the center



CVE

- CVE similar continuum behaviour at the center
- Volume average seems constrained to vanish





Exponential spin angular momentum

- The spin angular momentum term is exponentiated like μ
- Staggered fermions do not have Dirac structure

$$\underline{\text{Wilson}} \qquad \longleftrightarrow \qquad \underline{\text{Staggered}}$$

$$\exp\left(i\Omega\frac{\sigma_{12}}{2}\right) \longleftrightarrow \exp\left(i\Omega\frac{\Sigma_{12}(x,y)}{2}\right)$$

- The spin angular momentum term in staggered is highly non-local

  Yang, Huang '23
- Similar issue for μ_5 in the CME Brandt, Endrődi, EGV, Markó '24

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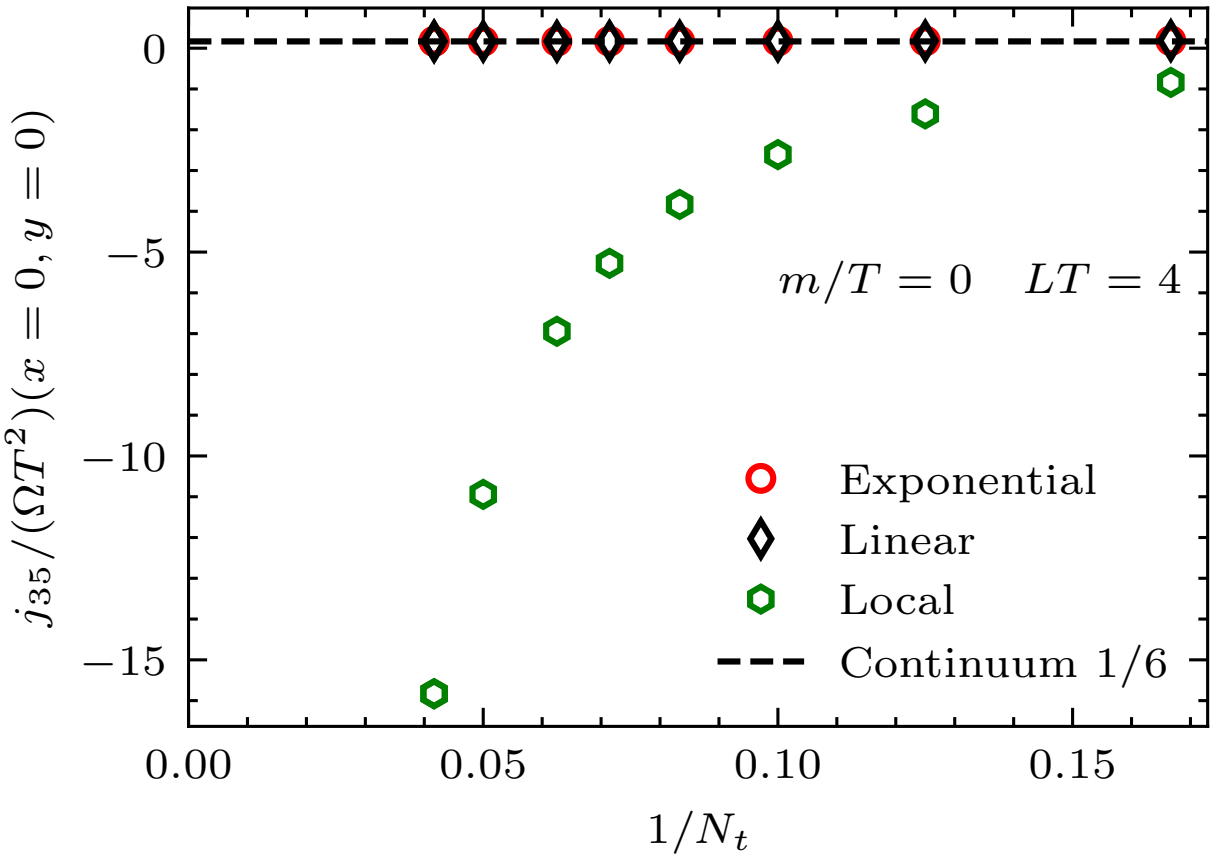
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- The spin angular momentum term in staggered is highly non-local
  Yang, Huang '23
- Similar issue for μ_5 in the CME Brandt, Endrődi, EGV, Markó '24
- Is the exponential necessary? Test with Wilson fermions:
  - Exponential: $(r-\gamma_4)\exp\left(i\Omega\frac{\sigma_{12}}{2}\right)\delta_{x+\hat{4},y} + (r+\gamma_4)\exp\left(-i\Omega\frac{\sigma_{12}}{2}\right)\delta_{x-\hat{4},y}$
  - Linear: $(r-\gamma_4)\left(1+i\Omega\frac{\sigma_{12}}{2}\right)\delta_{x+\hat{4},y} + (r+\gamma_4)\left(1-i\Omega\frac{\sigma_{12}}{2}\right)\delta_{x-\hat{4},y}$
  - Local: $\gamma_4\left(i\Omega\frac{\sigma_{12}}{2}\right)\delta_{x,y}$

Exponential spin angular momentum

- Linear version yields correct continuum limit



- Needs further testing to see if divergence is present

Summary

Summary

- Rotation very interesting experimentally and theoretically
- Boundary conditions play an important role
- Lattice formulation yields correct continuum limits for free fermions at the center

Outlook

- Continuum limit far from the center
- Staggered formalism done carefully
- Understand phase diagram and anomalous transport in QCD

Backup slides