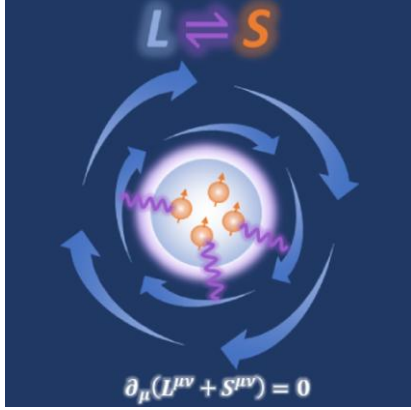


Spin-orbit angular momentum conversion in relativistic spin hydrodynamics with physics-informed neural networks

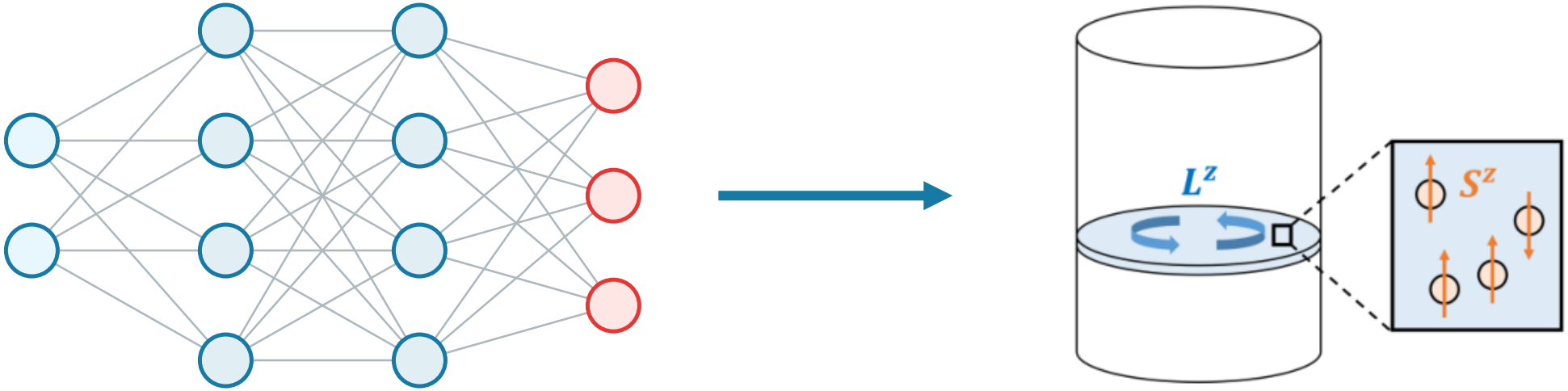
Hidefumi Matsuda (Zhejiang University)
Koichi Hattori (Zhejiang University)
Koichi Murase (The University of Osaka)

Based on PRX Intelligence **1**, 023014 (2026).

Chirality 2026, Zhuhai, China, November 18, 2026

This work

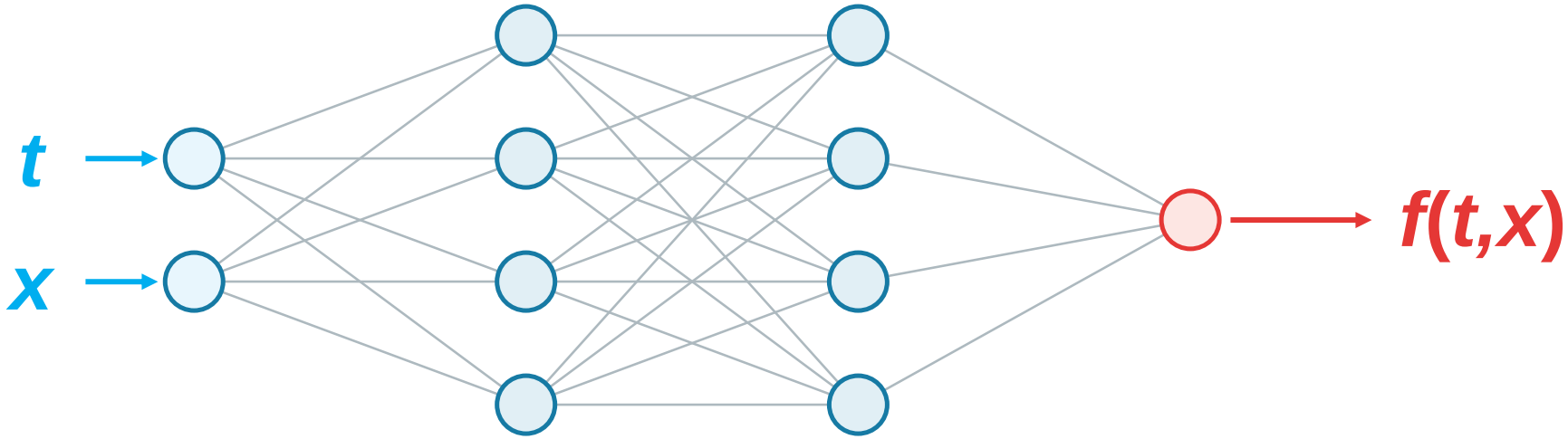
Test a PINN (Physics-informed neural network) solver for relativistic spin hydrodynamics



Background

What is a PINN?

✔ A neural network is a function

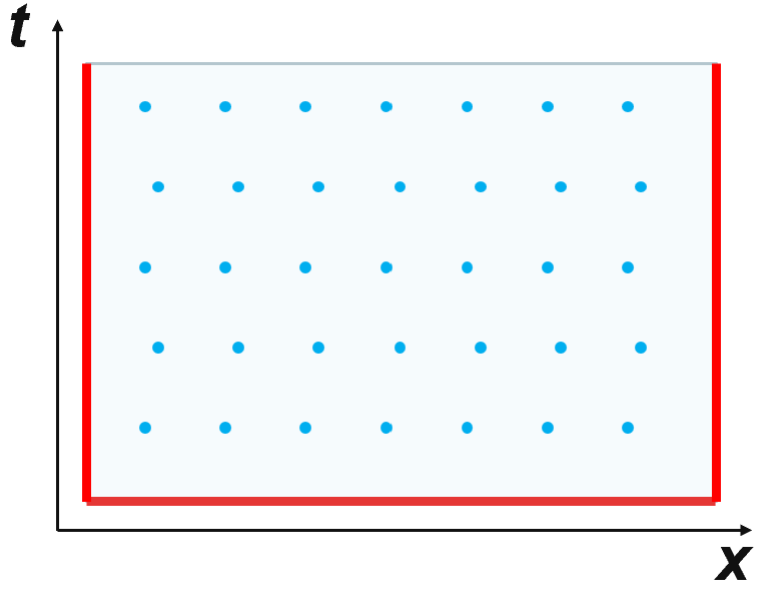


✔ Train the network to approximate a PDE solution

$$F[f] \sim 0$$

Training a PINN

Minimize the combined penalties



Governing equation: $F[f] = 0$

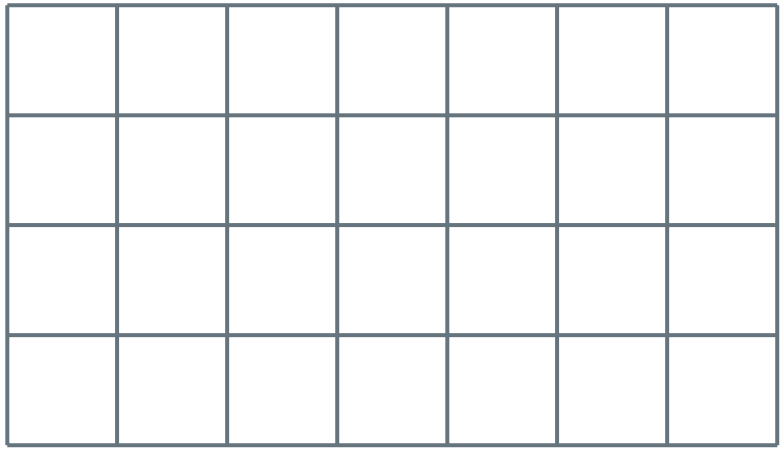
Initial and Boundary conditions:

$$f(t = t_0, x) = f_{\text{I.C.}}(x), \qquad f(t, x = R) = f_{\text{B.C.}}$$

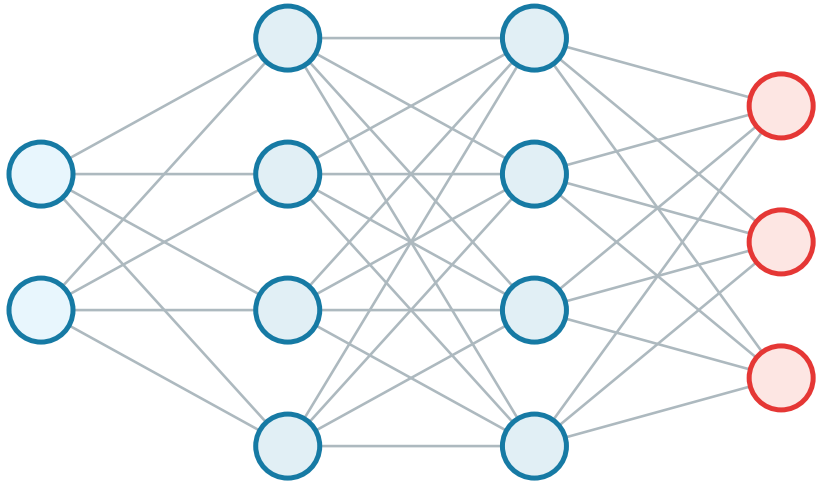
Additional constraints: data, …

Efficiency and flexibility

Mesh-based methods



PINNs



Strong efficiency on standard forward problems

Raissi et al., (2019)

Inverse problems

Raissi et al., (2020)

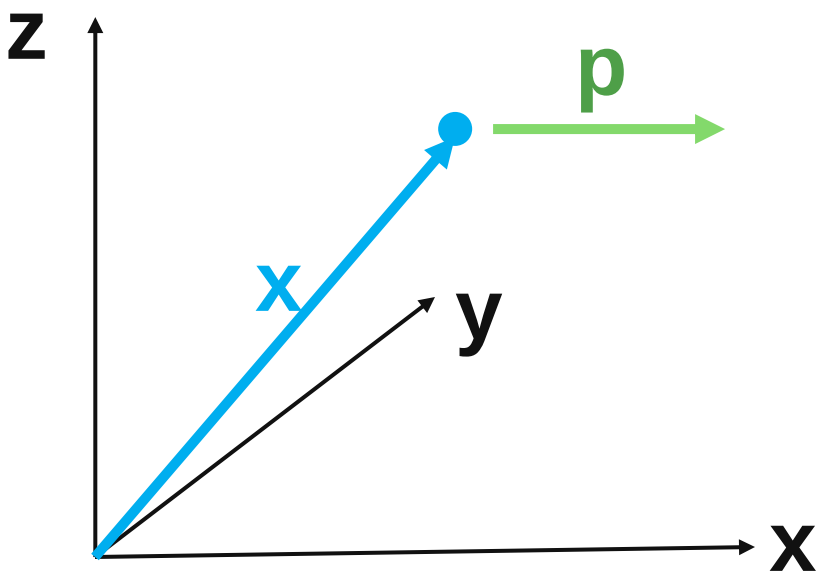
Data + physics

Geometric flexibility

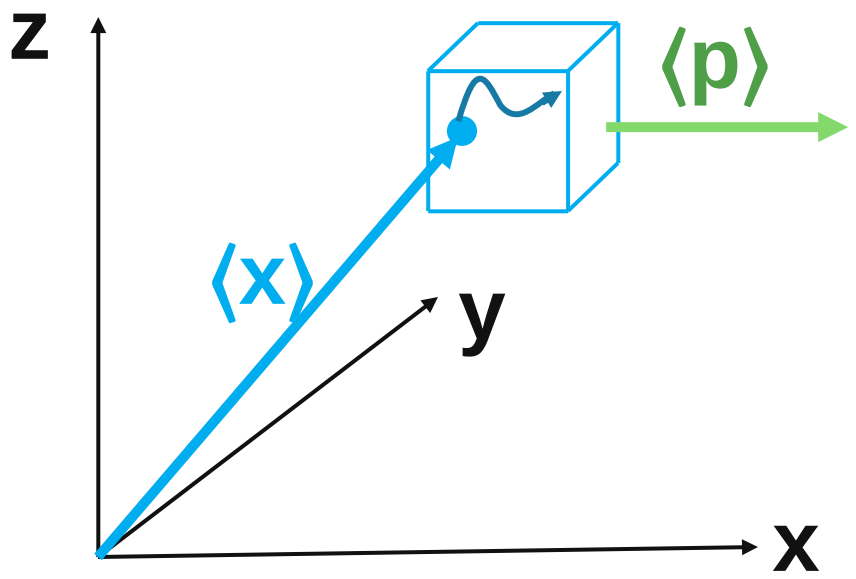
Sukumar & Srivastava, (2022)

Orbital angular momentum on a mesh

Point-like particle



Finite-size fluid mesh cell



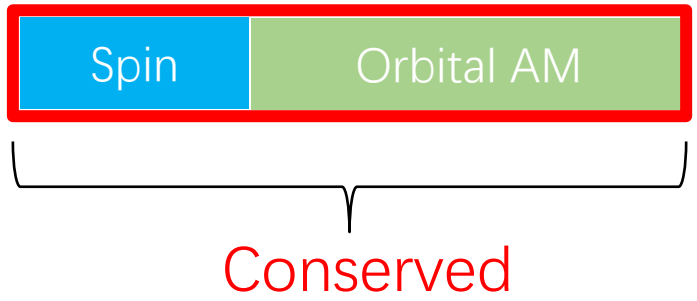
$$L_{\mathrm{mesh}} = \langle \boldsymbol{x} \rangle \times \langle \boldsymbol{p} \rangle \neq \langle \boldsymbol{x} \times \boldsymbol{p} \rangle$$

How to define orbital angular momentum?

Motivation for using PINNs

- ✔ **Mesh-free representation**
- ✔ **Angular momentum conservation in the loss**

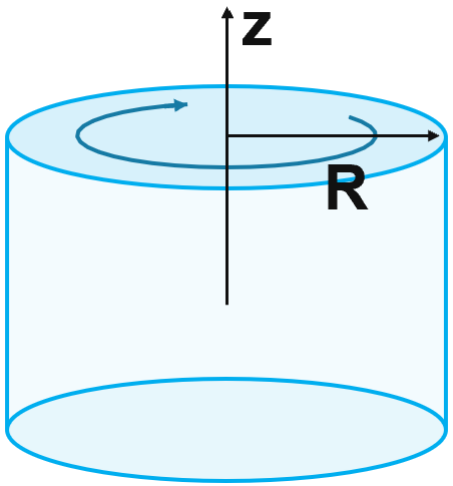
⇨ **Explore PINNs for spin–orbit conversion**



Model

A cylindrical spin fluid

✔ **Model simplified by symmetry**



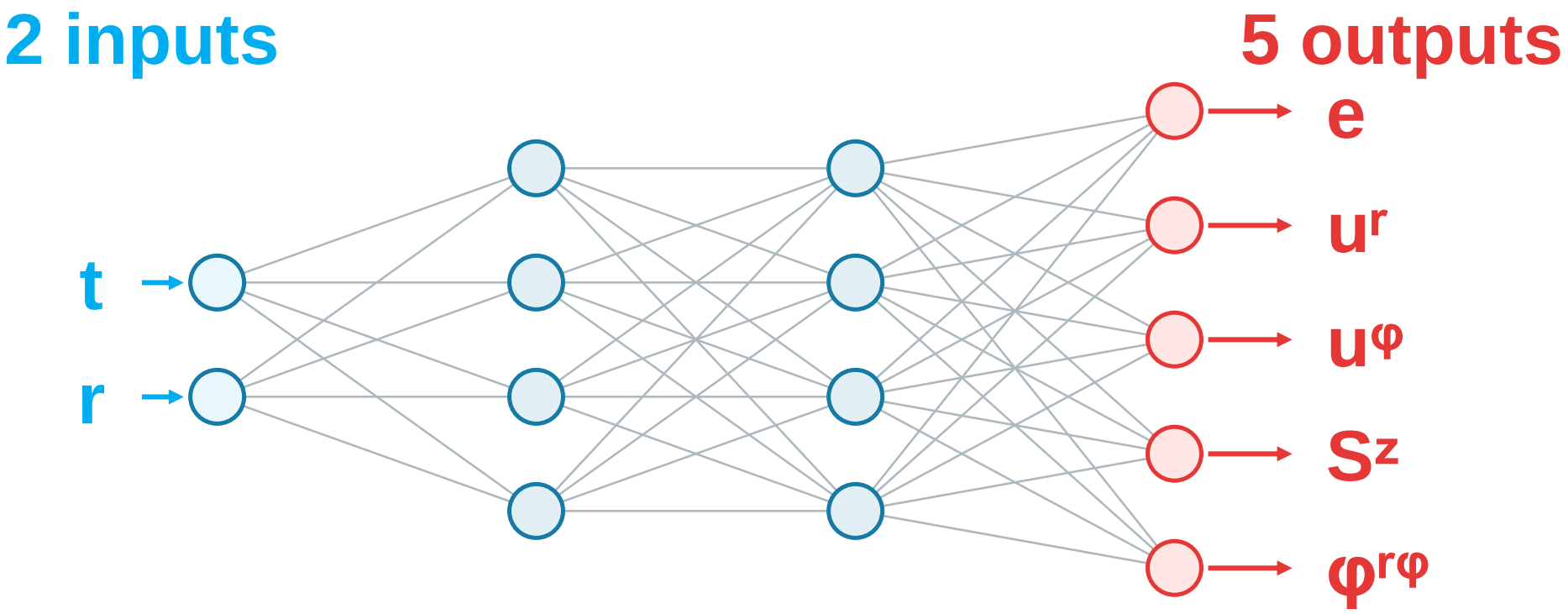
Axial symmetry
Translational symmetry along z
No flow along z

Dynamics in (*t*, *r*)

✔ **Focus on rotational viscosity**

Shear and bulk viscosities set to zero
Rotational viscous correction with MIS relaxation

Our PINN implementation



Loss function

Negative log-likelihood

$$L = -\sum_i \langle \ln p_i(R_i \mid \sigma_i) \rangle$$

$$R_i \in \{ \; R_j^{G.E.} \; , \; R_j^{B.C.} \; , \; R_{local} \; , \; R_{global} \; \}$$

Governing equations

$$R_1^{G.E.}(t, r) = u_\nu \nabla_\mu \Theta^{\mu\nu}$$

$$R_2^{G.E.}(t, r) = \Delta_\nu^r \nabla_\mu \Theta^{\mu\nu}$$

$$R_3^{G.E.}(t, r) = \Delta_\nu^\varphi \nabla_\mu \Theta^{\mu\nu}$$

$$R_4^{G.E.}(t, r) = \nabla_\lambda \Sigma^{\lambda xy} + 2\Theta^{[xy]}$$

$$R_5^{G.E.}(t, r) = \tau_\varphi \Delta_\alpha^\mu \Delta_\beta^\nu D\varphi^{\alpha\beta} + 2\gamma\rho^{\mu\nu} + \varphi^{\mu\nu} + \frac{\tau_\varphi}{2} \Theta\varphi^{\mu\nu}$$

Boundary conditions

$$R_1^{B.C.}(t) = \partial_r u^r(t, R) - \partial_r u^r(0, R)$$

$$R_2^{B.C.}(t) = \partial_r \varphi^{r\varphi}(t, R) - \partial_r \varphi^{r\varphi}(0, R)$$

AM conservation

$$R_{local}(t, r) = \nabla_\lambda J^{\lambda xy}$$

$$R_{global}(t) = \frac{\int dr \, r J^{txy}(t, r) - \int dr \, r J^{txy}(0, r)}{\int dr \, r J^{txy}(0, r)}$$

Initial and Dirichlet boundary conditions are built into the outputs

Results

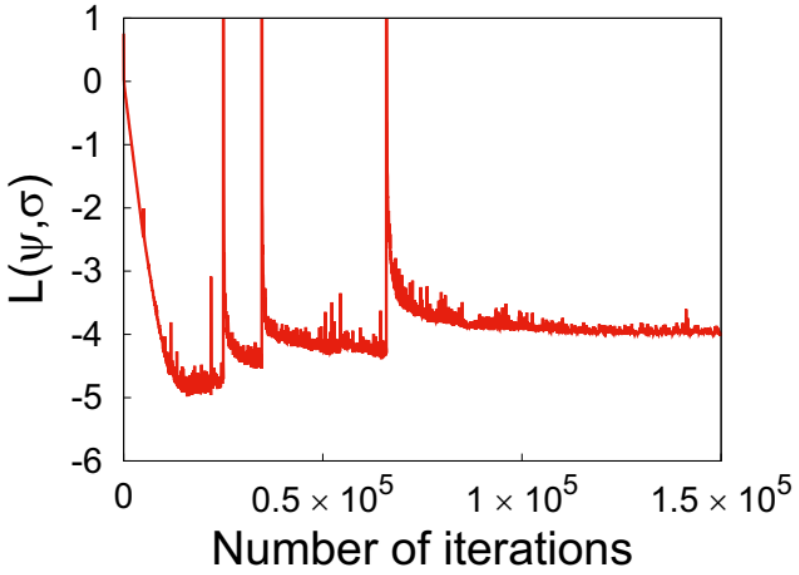
Numerical verification

$\gamma = 2,\ \delta_1 = 0.2,\quad (R = 1 \text{ unit})$

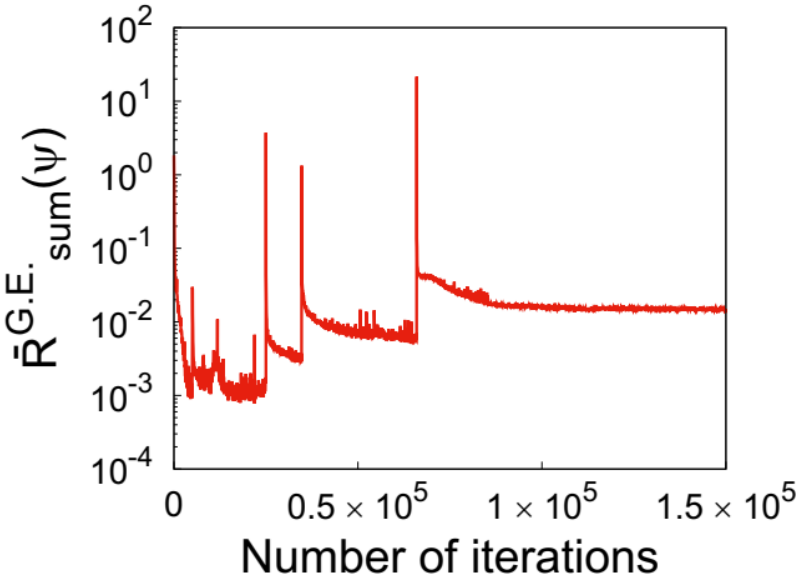
Initial conditions

$$e(t=0,r) = \bar{e},$$
$$u^r(t=0,r) = 0,$$
$$u^\varphi(t=0,r) = \delta_1 \sin^4\left(\frac{\pi r}{R}\right),$$
$$S^z(t=0,r) = 0,$$
$$\phi^{r\varphi}(t=0,r) = 0,$$

Training loss



PDE residual



Both decrease during training

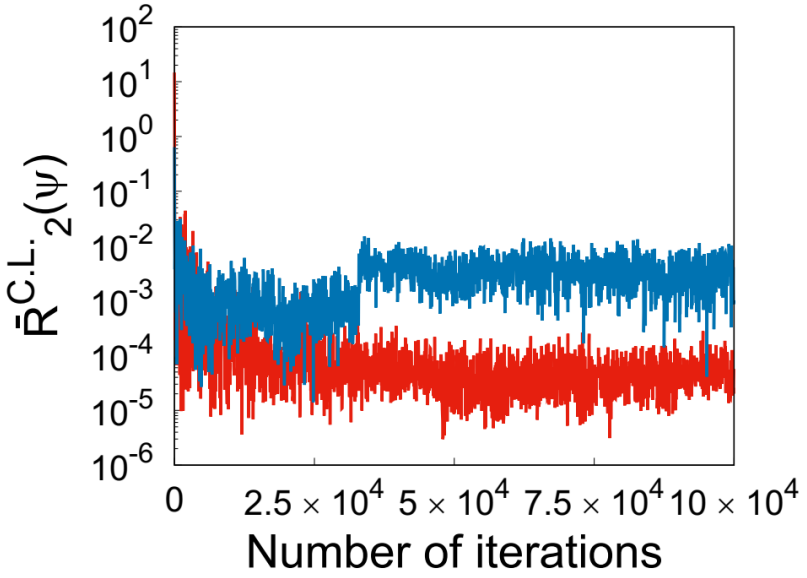
Angular momentum conservation

$\gamma = 2,\ \delta_1 = 0.2,\quad (R = 1\ \text{unit})$

Initial conditions

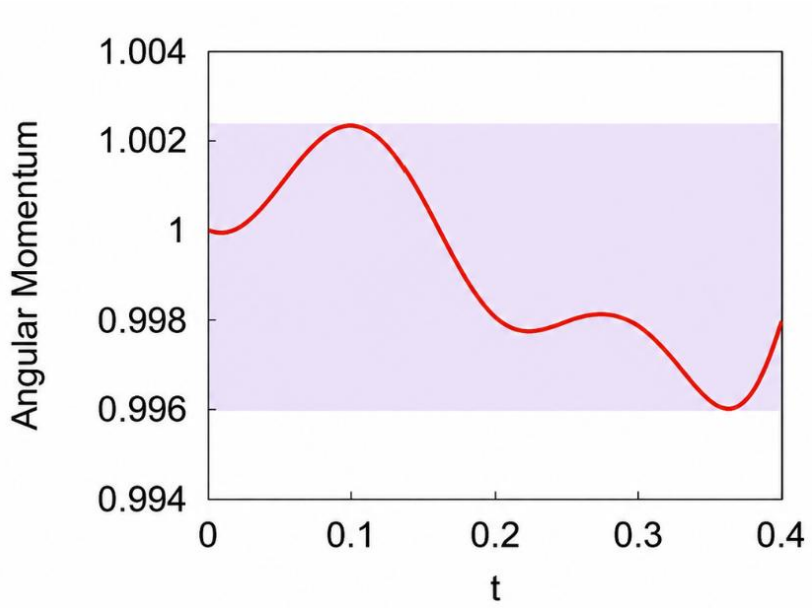
$$e(t=0,r) = \bar{e},$$
$$u^r(t=0,r) = 0,$$
$$u^\varphi(t=0,r) = \delta_1 \sin^4\left(\frac{\pi r}{R}\right),$$
$$S^z(t=0,r) = 0,$$
$$\phi^{r\varphi}(t=0,r) = 0,$$

With / without penalty



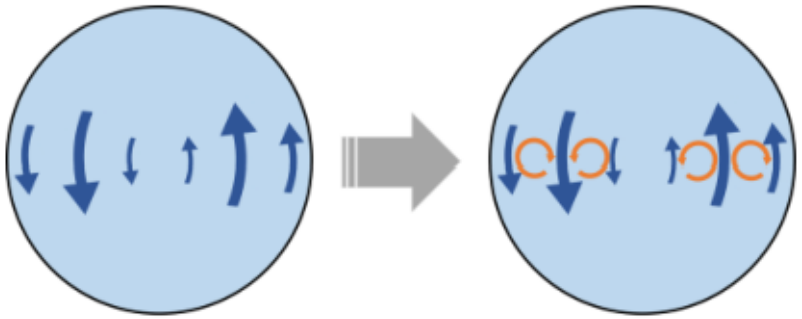
With **Without**

J(t) / J(0)



Deviation < 0.4%

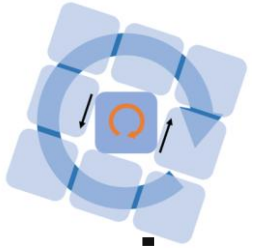
Barnett-like: orbital → spin



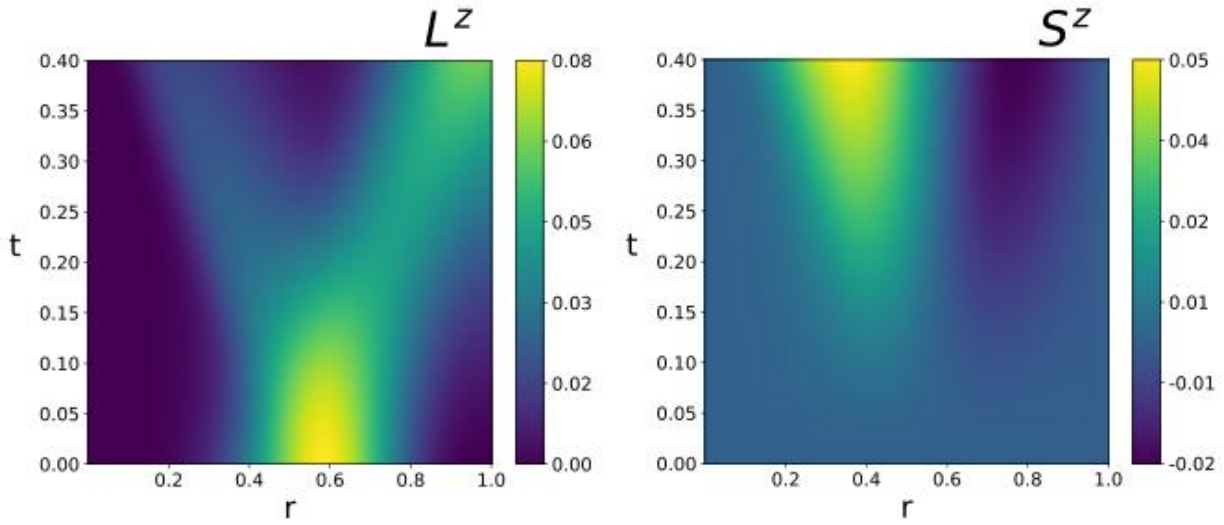
t = 0 t > 0

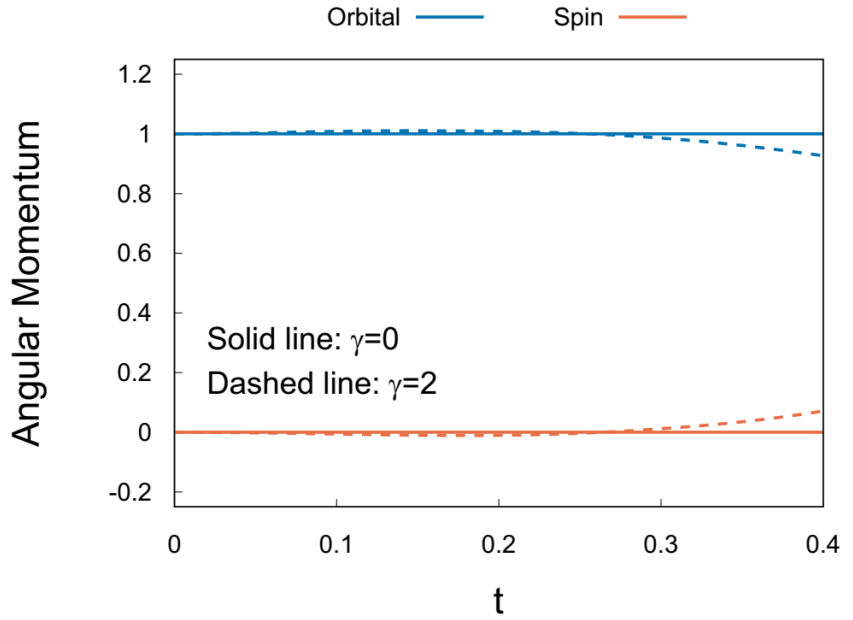
Rotation-rate mismatch

$$\rho^{\mu\nu} = \beta^{-1}\varpi_{\perp}^{\mu\nu} - 2\omega^{\mu\nu}$$



Drives spin–orbit conversion

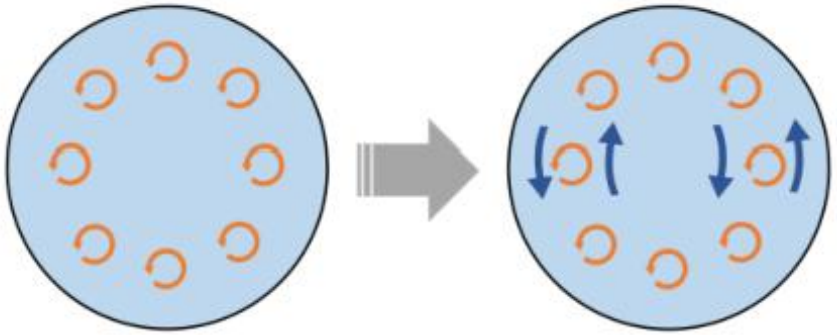




Conversion ≈ 7%

$|J(t)/J(0) - 1| < 0.4\%$

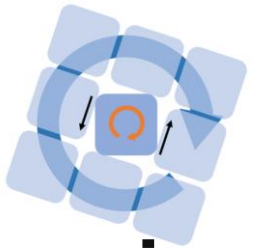
Einstein–de Haas-like: spin → orbital



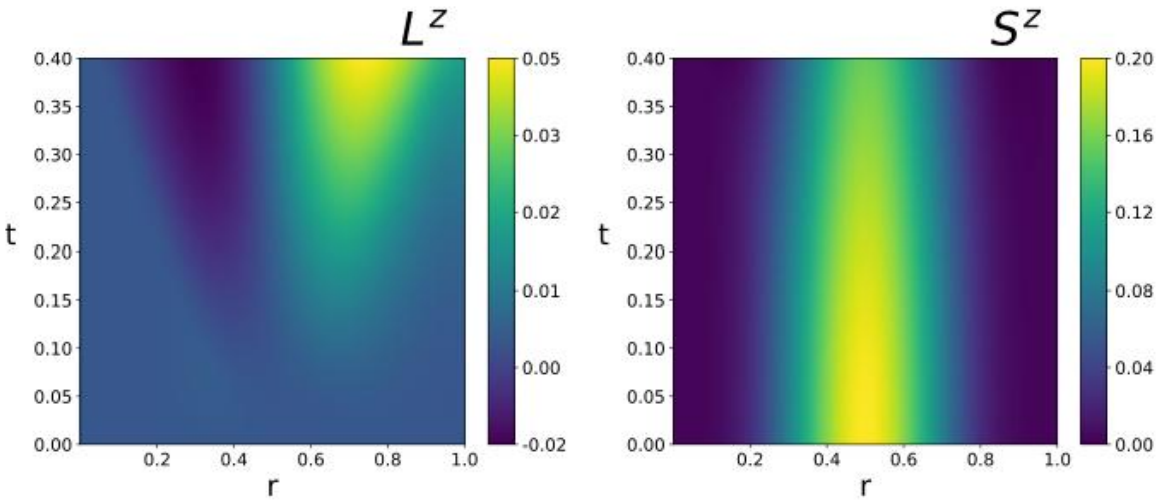
t = 0 t > 0

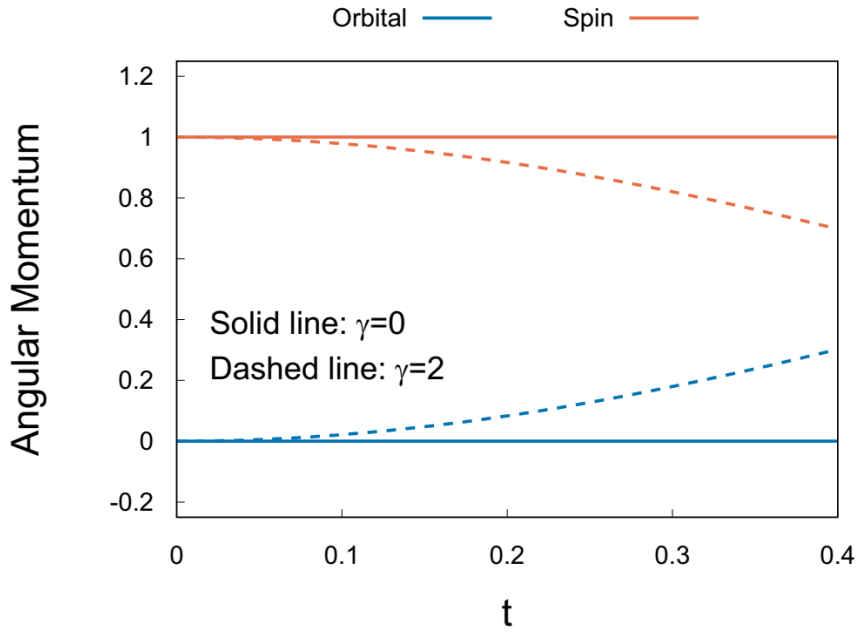
Rotation-rate mismatch

$$\rho^{\mu\nu} = \beta^{-1}\varpi_{\perp}^{\mu\nu} - 2\omega^{\mu\nu}$$



Drives spin–orbit conversion





Conversion ≈ 30%

|J(t)/J(0) − 1| < 0.05%

Summary & Comments

PINN solver for relativistic spin hydrodynamics

Accurate total angular momentum conservation

Resolved orbital ↔ spin conversion

Phenomenological applications require further work