Second-order evolution of spin shear tensor for massive spin-1/2 particles

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based on : arXiv 2611.03086

The 10th International Conference on Chirality, Vorticity and Magnetic Fields in Quantum Matter (chirality 2026)

Introduction

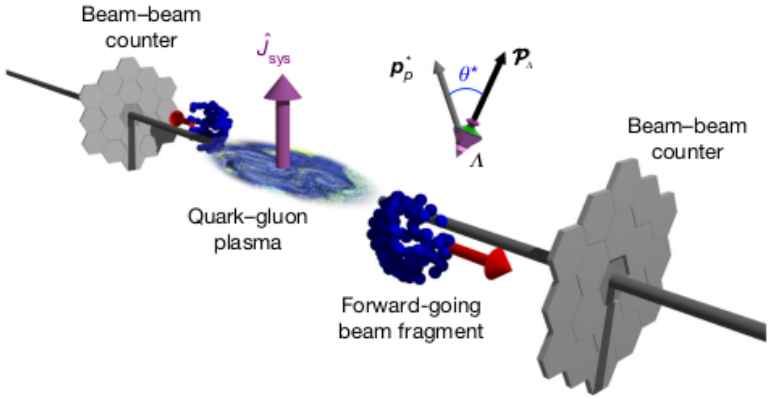


Figure: The initial orbital angular momentum deposited in the system in the initial stage of noncentral relativistic heavy-ion collisions. [L. Adamczyk et al. (STAR), Nature 548, 62 (2017)]

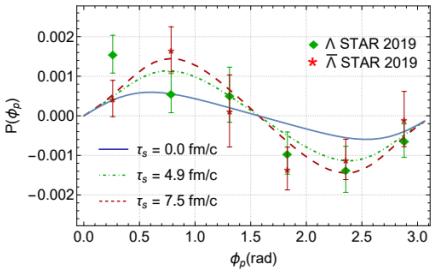
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- Global spin polarization is explained through thermal vorticity [I. Karpenko et al., EPJC 77, 213 (2017), F. Becattini et al., Annals Phys. 323, 2452 (2008)]

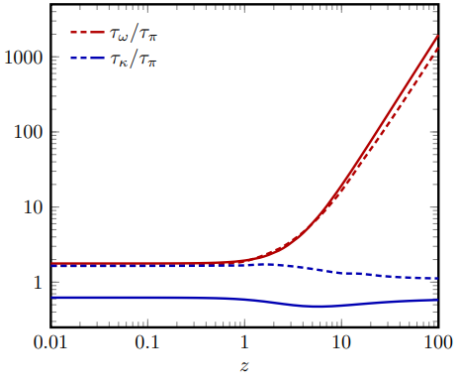
$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) \tag{1}$$

- However, the explanation of longitudinal polarization is non-trivial and thermal vorticity model alone is not sufficient.
- Alongside thermal vorticity, thermal shear model [F. Becattini et al., PRL 127, 272302 (2021)], shear-induced polarization [B. Fu et al., PRL 127, 142301 (2021)] has improved the description.
- Existing polarization models are based on some additional assumptions, and one of the assumptions is the equilibrated spin degrees of freedom.
- Slow spin relaxation compared to the other dissipative modes of the system motivates to treat the spin degrees of freedom dynamically.

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(a) [S. Banerjee et al., PRC 111 (2025) 6, 064912]



(b) [D. Wagner et al., PRR 6 (2024) 4, 4]

- [Sapna et al., Phys.Rev.C 112 (2025) 5, 054902]→ emphasized to focus on evolution of the spin shear component of spin tensor.

Relativistic Spin Hydrodynamics

- In the absence of conserved U(1) charge, the conserved currents for a rotating fluid [A. Daher et al., PRD 109, 114001 (2024)]

$$T^{\mu\nu} = \varepsilon_0 u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}, \tag{2}$$

$$S^{\lambda,\mu\nu} = u^\lambda \mathcal{S}^{\mu\nu} + \left(u^\mu \Delta^{\nu\lambda} - u^\nu \Delta^{\mu\lambda}\right)\Sigma + u^\mu \Sigma^{\langle\lambda\nu\rangle}_{(s)} - u^\nu \Sigma^{\langle\lambda\mu\rangle}_{(s)} + u^\mu \Sigma^{\lambda\nu}_{(a)} - u^\nu \Sigma^{\lambda\mu}_{(a)} + \Sigma^{\lambda,\mu\nu}. \tag{3}$$

$\varepsilon_0 \rightarrow$ energy density, $P_0 \rightarrow$ equilibrium pressure, $\Pi \rightarrow$ bulk pressure, $\pi^{\mu\nu} \rightarrow$ shear stress

$\mathcal{S}^{\mu\nu} \rightarrow$ spin density, $\Sigma \rightarrow$ spin pressure, $\Sigma^{\langle\mu\nu\rangle}_{(s)} \rightarrow$ spin shear, $\Sigma^{\mu\nu}_{(a)} \rightarrow$ spin vorticity, $\Sigma^{\lambda,\mu\nu} \rightarrow$ spin diffusion

Spin dependent distribution function

- The structures of $T^{\mu\nu}$ and $S^{\lambda,\mu\nu}$ can be connected through the moments of the phase-space distribution functions, $f(x, p, s)$
- **Spin-dependent distribution function :** Under the classical treatment to the massive spin-1/2 particles,

$$s^{\alpha\beta} = \frac{1}{m}\epsilon^{\alpha\beta\gamma\delta}p_{\gamma}s_{\delta} \tag{4}$$

- The equilibrium distribution function for particles [W. Florkowski et al., Prog. Part. Nucl. Phys. 108, 103709 (2019)]

$$f_{eq}(x, p, s) = f_{eq}(x, p)\ \exp\left[\frac{1}{2}\omega_{\mu\nu}s^{\mu\nu}\right] \tag{5}$$

$f_{eq}(x, p) = \exp[-p^{\mu}\beta_{\mu}(x)]$; $\omega_{\mu\nu} \rightarrow$ spin polarization tensor

Different components of conserved currents

$$\varepsilon_0 = \left\langle (u\cdot p)^2 \right\rangle_0 , \qquad P_0 = -\left\langle (1/3)(p\cdot\Delta\cdot p)\right\rangle_0 , \tag{6}$$

$$\Pi = -\left\langle (1/3)(p\cdot\Delta\cdot p)\right\rangle_\delta , \qquad \pi^{\mu\nu} = \left\langle p^{\langle\mu} p^{\nu\rangle} \right\rangle , \tag{7}$$

$$\mathcal{S}^{\mu\nu} = \langle (u\cdot p) s^{\mu\nu}\rangle = \langle (u\cdot p) s^{\mu\nu}\rangle_0 + \langle (u\cdot p) s^{\mu\nu}\rangle_\delta , \tag{8}$$

$$\Sigma = \left\langle (1/3)\, u_\mu p_{\langle\nu\rangle} s^{\mu\nu} \right\rangle = 0, \tag{9}$$

$$\Sigma^{\langle\lambda\nu\rangle}_{(s)} = \left\langle u_\mu s^{\mu\langle\nu} p^{\lambda\rangle} \right\rangle = \left\langle u_\mu s^{\mu\langle\nu} p^{\lambda\rangle} \right\rangle_0 + \left\langle u_\mu s^{\mu\langle\nu} p^{\lambda\rangle} \right\rangle_\delta , \tag{10}$$

$$\Sigma^{\lambda\nu}_{(a)} = \left\langle u_\mu s^{\mu[\nu} p^{\lambda]} \right\rangle = \left\langle u_\mu s^{\mu[\nu} p^{\lambda]} \right\rangle_0 + \left\langle u_\mu s^{\mu[\nu} p^{\lambda]} \right\rangle_\delta , \tag{11}$$

$$\Sigma^{\lambda,\mu\nu} = \left\langle p^{\langle\lambda\rangle} s^{\langle\mu\rangle\langle\nu\rangle} \right\rangle = \left\langle p^{\langle\lambda\rangle} s^{\langle\mu\rangle\langle\nu\rangle} \right\rangle_0 + \left\langle p^{\langle\lambda\rangle} s^{\langle\mu\rangle\langle\nu\rangle} \right\rangle_\delta , \tag{12}$$

phase-space integral $\rightarrow \langle(\cdots)\rangle = \int dPdS\,(\cdots)\, f_s$,

Relativistic kinetic theory

- Relativistic Boltzmann transport equation under relaxation time approximation

$$p^{\mu}\partial_{\mu}f_{s} = -\frac{(u\cdot p)}{\tau_{R}}\delta f_{s} \tag{13}$$

$\tau_R \rightarrow$ relaxation time, independent of momentum and spin degrees of freedom

- First-order correction to the distribution function

$$\begin{aligned}
\delta f_{s}^{(1)} &= -\frac{\tau_{\mathrm{R}}}{(u\cdot p)}p^{\mu}\partial_{\mu}f_{0s} \\
&= -\frac{\tau_{\mathrm{R}}}{(u\cdot p)}\left[\left\{\left(\frac{\beta}{3}-\chi_{b}\right)(u\cdot p)^{2}-\frac{\beta m^{2}}{3}\right\}\theta-\beta p^{\mu}p^{\nu}\sigma_{\mu\nu}\right]\left[1+\frac{1}{2}(s:\omega)\right]f_{0\mathbf{p}} \\
&\quad -\frac{\tau_{\mathrm{R}}}{2}s_{\alpha\beta}\left[D_{\Pi}^{\alpha\beta}\theta+D_{\pi}^{\alpha\beta\gamma\phi}\sigma_{\gamma\phi}+D_{\Sigma}^{\alpha\beta\gamma\phi\varphi}(\nabla_{\gamma}\omega_{\phi\varphi})+\frac{p^{\gamma}}{(u\cdot p)}\left(\nabla_{\gamma}\omega^{\alpha\beta}\right)\right]f_{0\mathbf{p}},
\end{aligned} \tag{14}$$

First-order transport coefficients

$$\Pi = -\tau_R \beta_\Pi \theta, \qquad \pi^{\mu\nu} = 2\tau_R \beta_\pi \sigma^{\mu\nu}, \qquad \delta\mathcal{S}^{\mu\nu} = \tau_{\rm eq}\left(B_\Pi^{(1)} u^{[\mu}\omega^{\nu]\lambda}u_\lambda + B_\Pi^{(2)} u^\alpha u^{[\mu}\omega^{\nu]}_{\ \alpha}\right)\theta, \tag{15}$$

$$\delta\Sigma^{\langle\mu\nu\rangle}_{(s)} = \tau_{\rm eq}\left[\frac{B_\pi^{(2)}+B_\pi^{(3)}}{4}(\sigma^\mu_\beta\omega^{\nu\beta}+\sigma^\nu_\beta\omega^{\mu\beta}) + (B_\Sigma^{(1)} - B_\Sigma^{(3)} - \frac{B_\Sigma^{(2)}+B_\Sigma^{(5)}}{2})\Delta^{\eta\langle\mu}\Delta^{\nu\rangle\gamma}u^\beta(\nabla_\eta\omega_{\beta\gamma})\right], \tag{16}$$

$$\delta\Sigma^{\mu\nu}_{(a)} = \tau_{\rm eq}\left[-\frac{1}{2}B_\Pi^{(1)}\omega^{\mu\nu}\theta + \frac{B_\pi^{(2)}-B_\pi^{(3)}}{2}(\sigma^{[\mu}_\gamma\omega^{\nu]\gamma}) + (B_\Sigma^{(1)} + \frac{B_\Sigma^{(5)}-B_\Sigma^{(2)}}{2})\Delta^{\eta[\mu}\Delta^{\nu]\gamma}u^\beta(\nabla_\eta\omega_{\beta\gamma})\right], \tag{17}$$

$$\delta\Sigma^{\lambda,\mu\nu} = \tau_{\rm eq}\Big[B_\Pi^{(3)}\Delta^{\lambda[\mu}\omega^{\nu]\alpha}u_\alpha\theta + B_\pi^{(1)}\sigma^{\lambda[\mu}\omega^{\nu]\alpha}u_\alpha + B_\pi^{(4)}\Delta^{\lambda[\mu}\sigma^{\nu]}_\alpha\omega^{\alpha\gamma}u_\gamma + B_\Sigma^{(3)}(\Delta^{\eta[\mu}\Delta^{\nu]\beta}\Delta^{\lambda\gamma})(\nabla_\eta\omega_{\beta\gamma})$$
$$+ (B_\Sigma^{(1)} - B_\Sigma^{(3)})\Delta^{\lambda\eta}\Delta^{\mu\beta}\Delta^{\nu\gamma}(\nabla_\eta\omega_{\beta\gamma}) + (B_\Sigma^{(3)} + B_\Sigma^{(4)})\Delta^{\lambda[\mu}\Delta^{\nu]\beta}\Delta^{\eta\gamma}(\nabla_\eta\omega_{\beta\gamma})\Big], \tag{18}$$

$$\delta\Sigma = \frac{\tau_{\rm eq}}{6}\left(2B_\Sigma^{(1)} - B_\Sigma^{(2)} - 5B_\Sigma^{(3)} - B_\Sigma^{(5)}\right)u^\alpha(\nabla^\beta\omega_{\alpha\beta}). \tag{19}$$

Second-order transport coefficients

- **Second-order evolution equation :** Taking the co-moving derivative of δf_s

$$\delta \dot{f}_s = -\dot{f}_{0s} - \frac{1}{(u\cdot p)} p^\mu \left(\nabla_\mu f_s\right) - \frac{\delta f_s}{\tau_R} \tag{20}$$

Integrating both sides with weight, $p\cdot\Delta\cdot p$, $p^{\langle\mu}p^{\nu\rangle}$, and $u_\mu s^{\mu\langle\nu}p^{\lambda\rangle}$

$$\dot{\Pi} + \frac{\Pi}{\tau_R} = \mathcal{H}_\Pi\ \theta + \mathcal{H}_\pi^{\alpha\beta}\ \sigma_{\alpha\beta} + \mathcal{H}_\Sigma^{\nu\phi\gamma}\ \left(\nabla_\phi \omega_{\gamma\nu}\right) + \mathcal{H}_{\Pi\Pi}\ \Pi\theta + \mathcal{H}_{\pi\pi}\ \sigma^{\alpha\beta}\pi_{\alpha\beta}, \tag{21}$$

$$\begin{aligned}\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_R} &= \mathcal{G}_\Pi^{\mu\nu}\theta + \mathcal{G}_{\pi,\ \gamma\rho}^{\mu\nu}\sigma^{\gamma\rho} + \mathcal{G}_\Sigma^{\phi\rho\gamma}(\nabla_\phi\omega_{\rho\gamma}) + \mathcal{G}_{\Pi\pi}\ \sigma^{\mu\nu}\Pi + \mathcal{G}_{\pi\pi}\ \pi_\phi^{\langle\mu}\sigma^{\nu\rangle\phi} \\ &\quad + \mathcal{G}_{\pi\Omega}\ \pi_\phi^{\langle\mu}\Omega^{\nu\rangle\phi} + \mathcal{G}_{\pi\Pi}\ \pi^{\mu\nu}\theta,\end{aligned} \tag{22}$$

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$$\dot{\Sigma}^{\langle\lambda\nu\rangle}_{(s)} + \frac{\Sigma^{\langle\lambda\nu\rangle}_{(s)}}{\tau_R} = \mathcal{B}^{\lambda\nu}_{\Pi}\theta + \mathcal{B}^{\lambda\nu\alpha}_{\mathfrak{a}}\dot{u}_\alpha + \mathcal{B}^{\lambda\nu\alpha\beta}_{\pi}\sigma_{\alpha\beta} + \mathcal{B}^{\lambda\nu\alpha\beta\gamma}_{\Sigma}\left(\nabla_\gamma\omega_{\alpha\beta}\right)$$
$$+ \left[\mathcal{B}^{\lambda\nu}_{\Pi\Pi}\theta + \mathcal{B}^{\lambda\nu\alpha}_{\Pi\mathfrak{a}}\dot{u}_\alpha + \mathcal{B}^{\lambda\nu\alpha\beta}_{\Pi\pi}\sigma_{\alpha\beta} + \mathcal{B}^{\lambda\nu\alpha\beta\gamma}_{\Pi\Sigma}\left(\nabla_\gamma\omega_{\alpha\beta}\right) + \mathcal{B}^{\alpha\beta\nu\lambda}_{\Pi\Omega}\Omega_{\alpha\beta}\right]\Pi$$
$$+ \left[\mathcal{B}^{\lambda\nu\rho\phi}_{\pi\Pi}\theta + \mathcal{B}^{\lambda\nu\alpha\rho\phi}_{\pi\mathfrak{a}}\dot{u}_\alpha + \mathcal{B}^{\lambda\nu\alpha\beta\rho\phi}_{\pi\pi}\sigma_{\alpha\beta} + \mathcal{B}^{\lambda\nu\alpha\beta\gamma\rho\phi}_{\pi\Sigma}\left(\nabla_\gamma\omega_{\alpha\beta}\right) + \mathcal{B}^{\nu\lambda\gamma\xi\rho\phi}_{\pi\Omega}\Omega_{\gamma\xi}\right]\pi_{\rho\phi}$$
$$+ \left[\mathcal{B}^{\nu\lambda\rho\alpha\beta}_{\mathfrak{b}\Pi}\theta + \mathcal{B}^{\gamma\nu\lambda\rho\alpha\beta}_{\mathfrak{b}\mathfrak{a}}\dot{u}_\gamma + \mathcal{B}^{\nu\lambda\mu\gamma\rho\alpha\beta}_{\mathfrak{b}\pi}\sigma_{\mu\gamma} + \mathcal{B}^{\nu\lambda\mu\gamma\rho\alpha\beta}_{\mathfrak{b}\Omega}\Omega_{\mu\gamma}\right]\delta S_{\rho\alpha\beta}$$
$$+ \mathcal{B}^{\lambda\nu\alpha}_{\Pi D}\left(\nabla_\alpha\Pi\right) + \mathcal{B}^{\lambda\nu\alpha\beta\gamma}_{\pi D}\left(\nabla_\gamma\pi_{\alpha\beta}\right) + \mathcal{B}^{\lambda\nu\rho\alpha\beta\gamma}_{\Sigma D}\left(\nabla_\gamma\delta S_{\rho,\alpha\beta}\right) + \mathcal{B}^{\lambda\nu\alpha\beta}_{\pi D}\dot{\pi}_{\alpha\beta}, \tag{23}$$

$\Omega^{\mu\nu} \rightarrow$ fluid vorticity tensor

- We find the dependency of the spin shear on the fluid vorticity to be at higher order spacetime gradient corrections.

Comparison with [D. Wagner Phys.Rev.D 111 (2025) 1, 016008]

- We can compare our results with a traceless symmetric rank-two tensor $\mathfrak{t}^{\mu\nu}$, spin-shear stress in the absence of a U(1) conserved charge [D. Wagner, PRD 111 (2025) 1, 016008]

$$\begin{aligned}\tau_{\mathfrak{t}}\dot{\mathfrak{t}}^{\langle\mu\nu\rangle}+\mathfrak{t}^{\mu\nu}=&\frac{\mathfrak{d}}{T}\sigma^{\mu\nu}+\delta_{\mathfrak{tt}}\mathfrak{t}^{\mu\nu}\theta+\lambda_{\mathfrak{tt}}\,\mathfrak{t}_{\lambda}^{\langle\mu}\sigma^{\nu\rangle\lambda}+\frac{5}{3}\tau_{\mathfrak{t}\lambda}\,\mathfrak{t}_{\lambda}^{\langle\mu}\omega_{\kappa}^{\nu\rangle\lambda}\\&+\ell_{\mathfrak{t}\kappa}\nabla^{\langle\mu}\kappa_{0}^{\nu\rangle}+\tau_{\mathfrak{t}\omega}\omega_{\kappa}^{\langle\mu}\omega_{0}^{\nu\rangle}+\lambda_{\mathfrak{t}\omega}\sigma^{\langle\mu}{}_{\lambda}\epsilon^{\nu\rangle\alpha\beta}u_{\alpha}\omega_{0,\beta},\end{aligned}\tag{24}$$

with our [P. Panday et al., arXiv:2611.03086]

$$\begin{aligned}\dot{\Sigma}_{(s)}^{\langle\lambda\nu\rangle}+\frac{\Sigma_{(s)}^{\langle\lambda\nu\rangle}}{\tau_R}=&\mathcal{B}_{\Pi}^{\lambda\nu}\theta+\mathcal{B}_{\mathfrak{a}}^{\lambda\nu\alpha}\dot{u}_{\alpha}+\mathcal{B}_{\pi}^{\lambda\nu\alpha\beta}\sigma_{\alpha\beta}+\mathcal{B}_{\Sigma}^{\lambda\nu\alpha\beta\gamma}\left(\nabla_{\gamma}\omega_{\alpha\beta}\right)\\&+\left[\mathcal{B}_{\Pi\Pi}^{\lambda\nu}\theta+\mathcal{B}_{\Pi\mathfrak{a}}^{\lambda\nu\alpha}\dot{u}_{\alpha}+\mathcal{B}_{\Pi\pi}^{\lambda\nu\alpha\beta}\sigma_{\alpha\beta}+\mathcal{B}_{\Pi\Sigma}^{\lambda\nu\alpha\beta\gamma}\left(\nabla_{\gamma}\omega_{\alpha\beta}\right)+\mathcal{B}_{\Pi\Omega}^{\alpha\beta\nu\lambda}\Omega_{\alpha\beta}\right]\Pi\\&+\left[\mathcal{B}_{\pi\Pi}^{\lambda\nu\rho\phi}\theta+\mathcal{B}_{\pi\mathfrak{a}}^{\lambda\nu\alpha\rho\phi}\dot{u}_{\alpha}+\mathcal{B}_{\pi\pi}^{\lambda\nu\alpha\beta\rho\phi}\sigma_{\alpha\beta}+\mathcal{B}_{\pi\Sigma}^{\lambda\nu\alpha\beta\gamma\rho\phi}\left(\nabla_{\gamma}\omega_{\alpha\beta}\right)+\mathcal{B}_{\pi\Omega}^{\nu\lambda\gamma\xi\rho\phi}\Omega_{\gamma\xi}\right]\pi_{\rho\phi}\\&+\left[\mathcal{B}_{\mathfrak{b}\Pi}^{\nu\lambda\rho\alpha\beta}\theta+\mathcal{B}_{\mathfrak{b}\mathfrak{a}}^{\gamma\nu\lambda\rho\alpha\beta}\dot{u}_{\gamma}+\mathcal{B}_{\mathfrak{b}\pi}^{\nu\lambda\mu\gamma\rho\alpha\beta}\sigma_{\mu\gamma}+\mathcal{B}_{\mathfrak{b}\Omega}^{\nu\lambda\mu\gamma\rho\alpha\beta}\Omega_{\mu\gamma}\right]\delta\mathcal{S}_{\rho\alpha\beta}\\&+\mathcal{B}_{\Pi D}^{\lambda\nu\alpha}\left(\nabla_{\alpha}\Pi\right)+\mathcal{B}_{\pi D}^{\lambda\nu\alpha\beta\gamma}\left(\nabla_{\gamma}\pi_{\alpha\beta}\right)+\mathcal{B}_{\Sigma D}^{\lambda\nu\rho\alpha\beta\gamma}\left(\nabla_{\gamma}\delta\mathcal{S}_{\rho,\alpha\beta}\right)+\mathcal{B}_{\pi D}^{\lambda\nu\alpha\beta}\dot{\pi}_{\alpha\beta},\end{aligned}\tag{25}$$

Conclusions

- We focused on the evolution of the spin shear component of the spin tensor under the GLW pseudogauge.
- We used the semi-classical kinetic theory for spin-1/2 particles to obtain the dissipative transport coefficients.
- The relaxation-type equation for spin shear obtained in this work is induced by the thermodynamic forces such as scalar expansion, shear-flow, acceleration, and the spacelike gradient of spin polarization tensor as well as high order gradient corrections that includes effects from fluid vorticity.
- An important aspect that is to be added in this framework is the inclusion of non- local collision kernel.

Thank you!!

Backup slides

- Integral measure

$$\int dP \equiv \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_{\mathbf{p}}}$$
$$\int dS \equiv \frac{m}{\pi\mathfrak{s}} \int d^4 s \delta(s^2 + \mathfrak{s}^2)\delta(p \cdot s), \tag{26}$$

- hydrodynamic equations

$$\dot{\varepsilon}_0 + (\varepsilon_0 + P_0 + \Pi)\theta + (\pi : \sigma) = 0, \tag{27a}$$
$$(\varepsilon_0 + P_0)\dot{u}^\alpha - (\nabla^\alpha P_0) + \Delta^\alpha_\mu \partial_\nu \pi^{\mu\nu} = 0, \tag{27b}$$
$$\dot{\mathcal{S}}^{\mu\nu} + \mathcal{S}^{\mu\nu}\theta + \left(\Sigma^{\langle\lambda\nu\rangle}_{(s)} + \Sigma^{\lambda\nu}_{(a)}\right)(\partial_\lambda u^\mu) - \left(\Sigma^{\langle\lambda\mu\rangle}_{(s)} - \Sigma^{\lambda\mu}_{(a)}\right)(\partial_\lambda u^\nu)$$
$$+ u^\mu\left(\partial_\lambda \Sigma^{\langle\lambda\nu\rangle}_{(s)} + \partial_\lambda \Sigma^{\lambda\nu}_{(a)}\right) - u^\nu\left(\partial_\lambda \Sigma^{\langle\lambda\mu\rangle}_{(s)} + \partial_\lambda \Sigma^{\lambda\mu}_{(a)}\right) + \left(\partial_\lambda \Sigma^{\lambda,\mu\nu}\right) = 0. \tag{27c}$$