Spin Alignment in QCD Matter: From Puzzles to Probes

Nontrivial physics from the "dissipative" term in Zubarev approach

Shuai Liu

Hunan University

Chirality, Vorticity, and Magnetic Field in Quantum Matter (Chirality 2026) ,
Nov 16-20, 2026, Zhuhai, China

Based on work, F. Li, Y-Y Li, Z-Y Sun, Liu on arXiv: 2206.11890 (PRC accepted), 2503.13408 (PRC accepted); Liu & Rapp series research 2018-2022

Outline

1) Spin observables in heavy-ion collisions

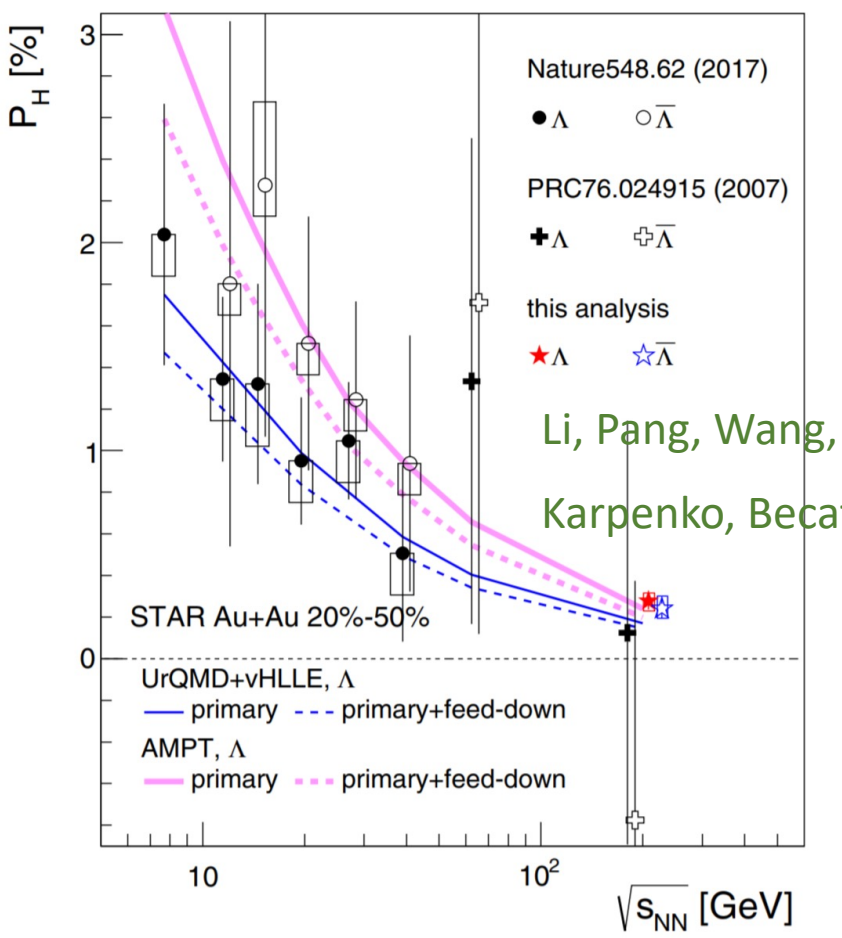
2) Theory for tensor polarization and spin alignment

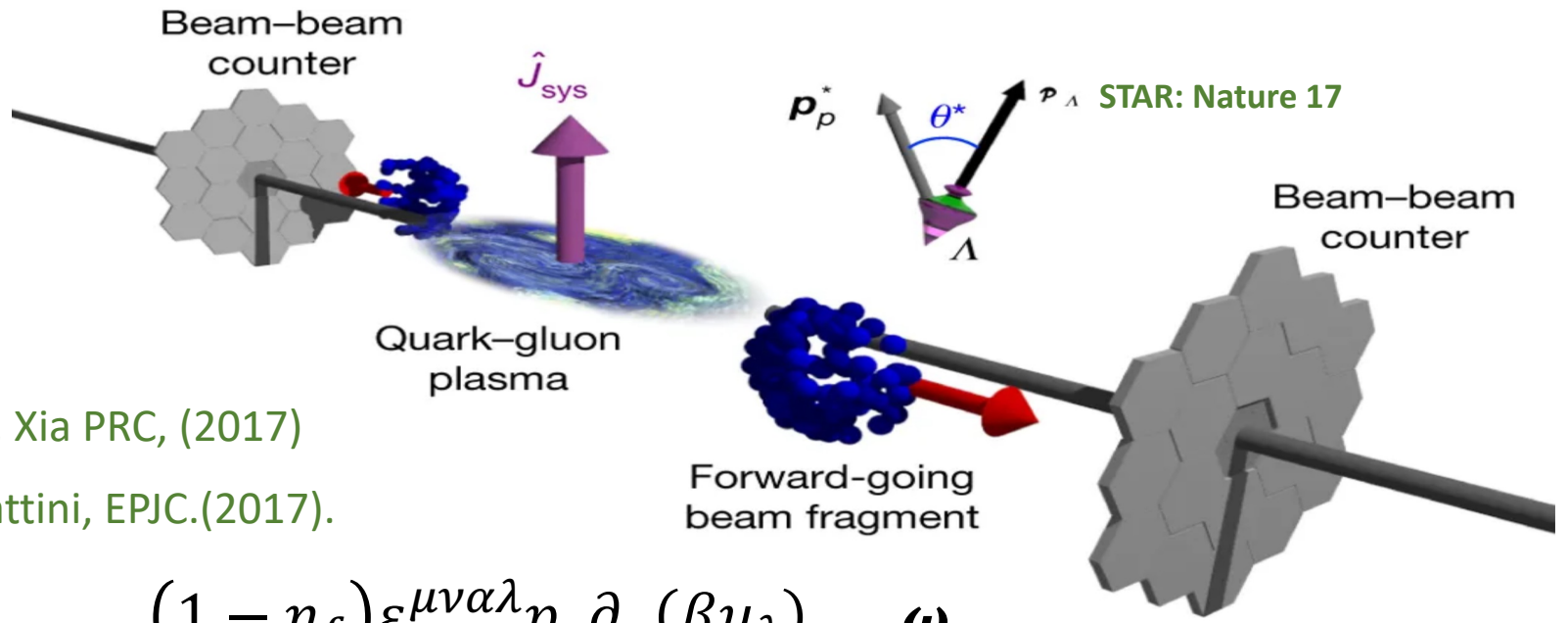
3) Phenomenology and spin alignment as a probe to in-medium spectra

4) Summary and outlooks

Vorticity Induced Polarization

STAR PRC 2018



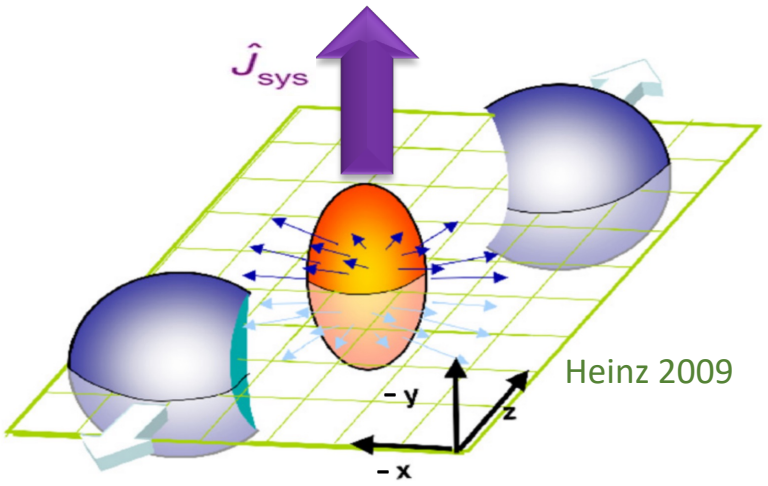


STAR: Nature 17

Li, Pang, Wang, Xia PRC, (2017)

Karpenko, Becattini, EPJC.(2017).

$$\boldsymbol{\mathcal{P}} = \frac{(1-n_f)\varepsilon^{\mu\nu\alpha\lambda}p_\nu\partial_\alpha(\beta u_\lambda)}{4m} \approx \frac{\boldsymbol{\omega}}{2T} \sim 1\%$$

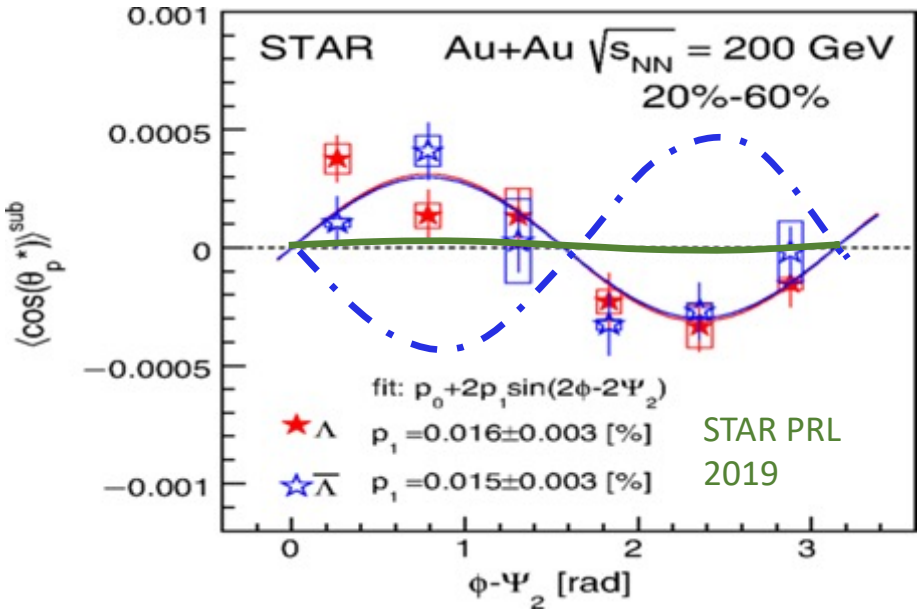


Heinz 2009

- Rotation, vorticity, most vortical fluid
- Vorticities can lead to polarization

Local Polarization and Shear Induced Polarization

Local polarization sign puzzles



STAR PRL 2019

Sketches of theoretical curves

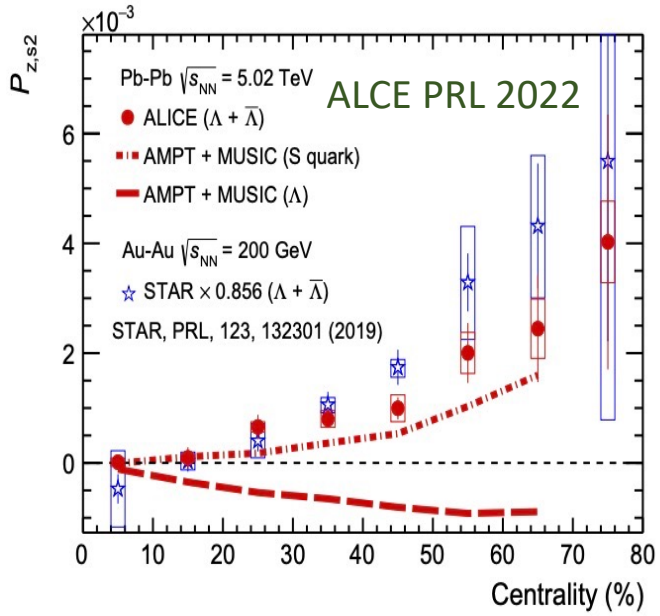
Voloshin, EPJ 2018
Becattini, Karpenko, PRL 2018

❖Previous theory

$$\mathcal{P} = \frac{(1-n_f)\epsilon^{\mu\nu\alpha\lambda}p_\nu\partial_\alpha(\beta u_\lambda)}{4m}$$

❖Sign is opposite!

Progress



ALCE PRL 2022

Liu, Yin, JHEP, 2021;Becatinni, Buzzegoli, Palermo, PLB, 2021

❖Shear-Induced Polarization

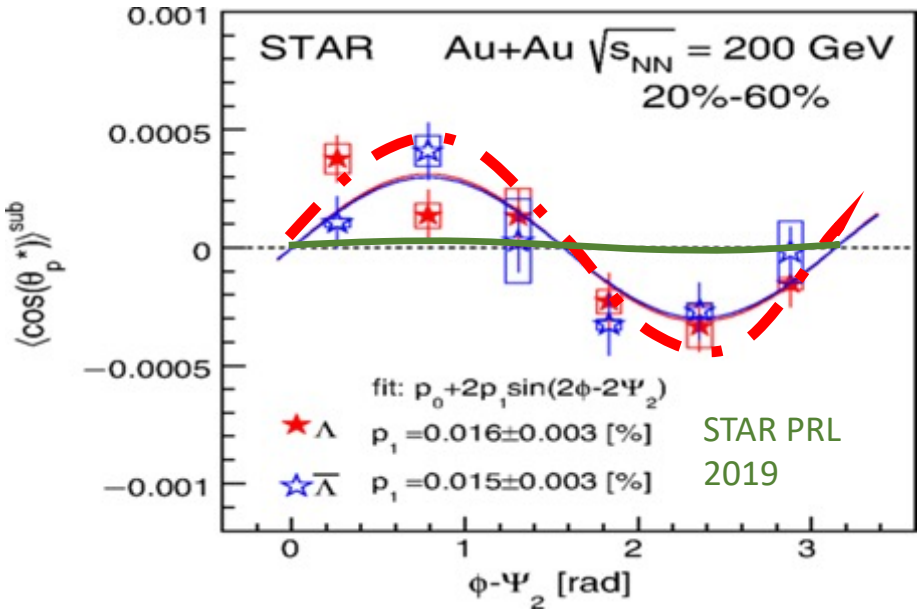
$$-n_0(1-n_0)\frac{1}{\varepsilon_0}\epsilon^{\mu\nu\alpha\rho}u_\nu\, p_\rho p^\lambda\partial_{(\alpha}(\beta u)_{\lambda)}$$

❖Right sign, large phenomenologically

Fu, Liu, Pang, Song, Yin, PRL, 2021;
Becatinni, Buzzegoli, Palermo, Inghirami,Karpenko PRL, 2021

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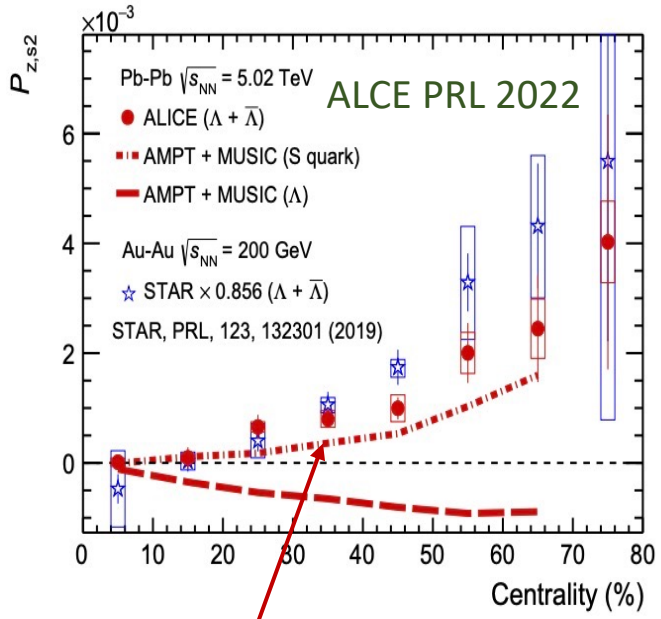
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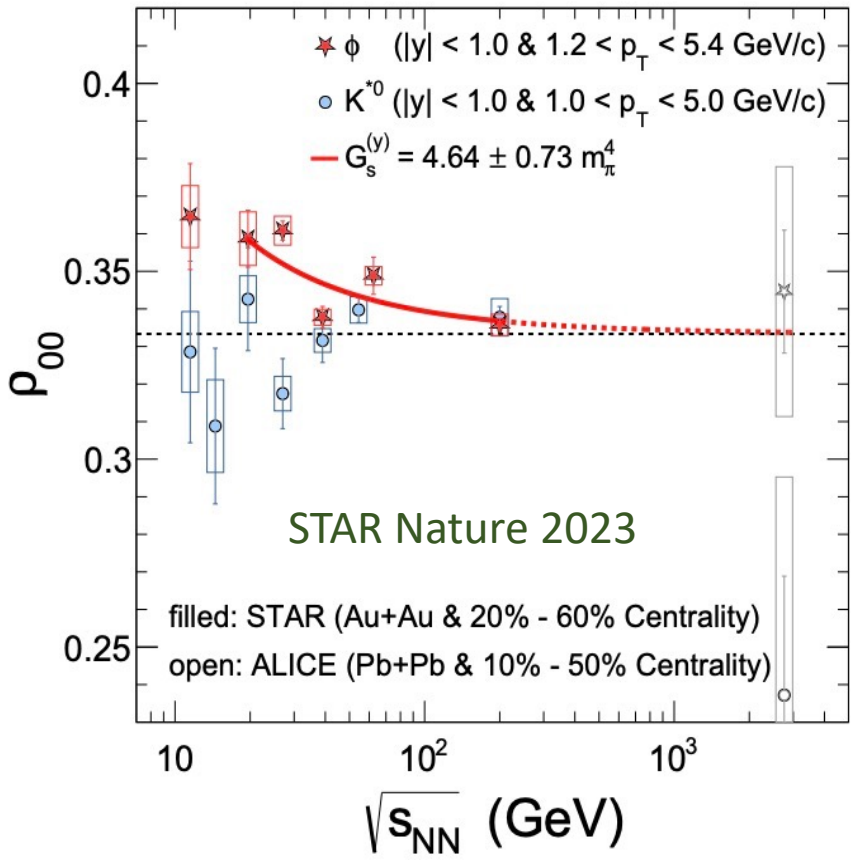
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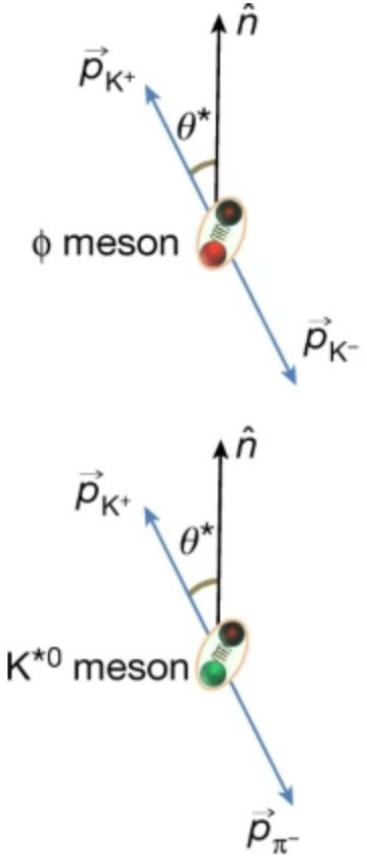
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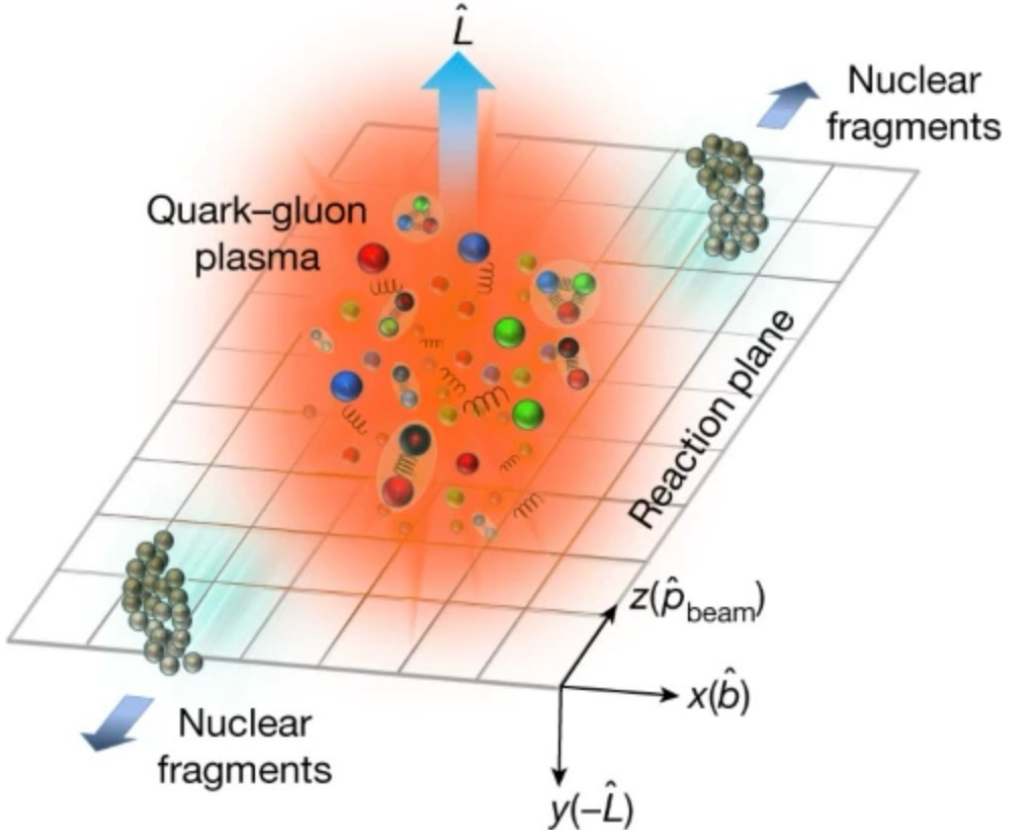
Fu, Liu, Pang, Song, Yin, PRL, 2021;
Becatinni, Buzzegoli, Palermo, Inghirami,Karpenko PRL, 2021

The Spin Alignment in Heavy-ion Collision

❖Experimental findings

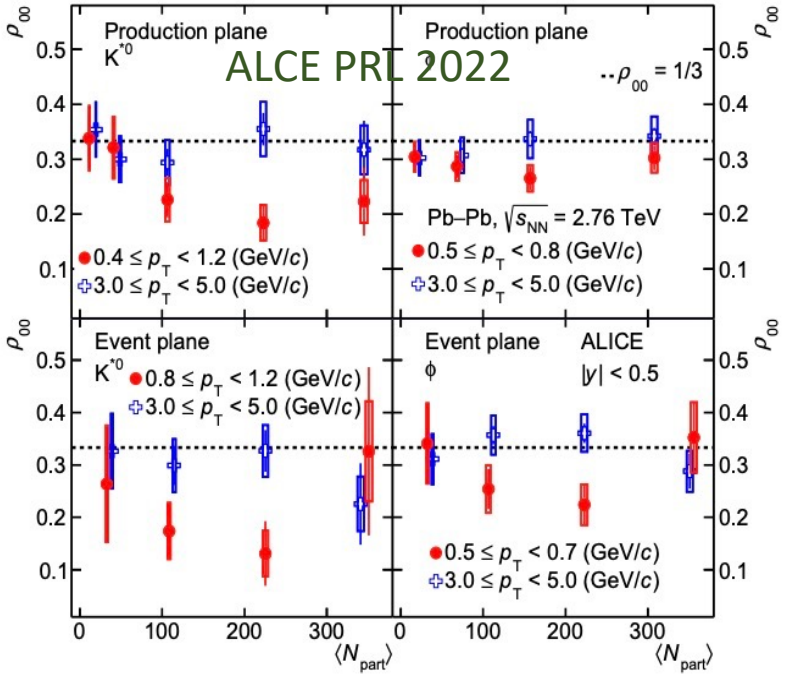


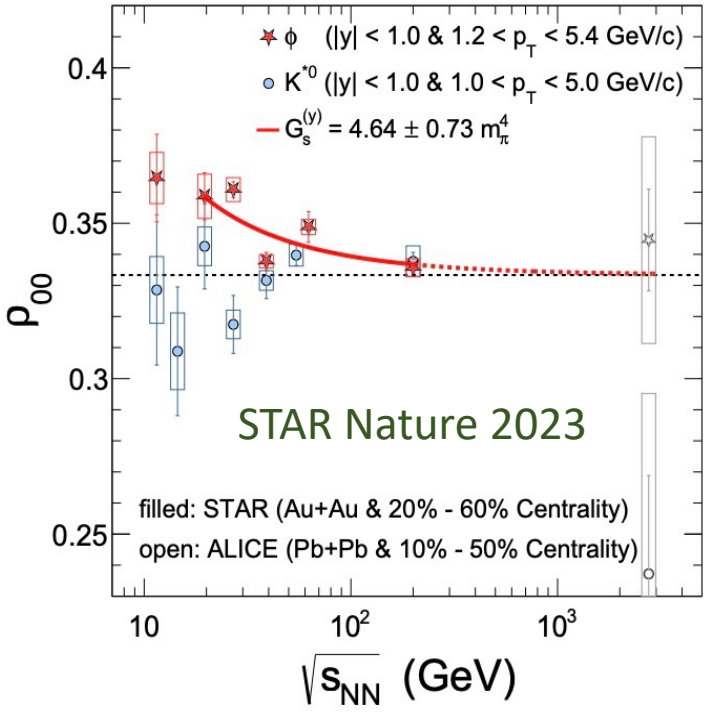




Challenges on Spin Alignment: 1) Large Magnitude

Experiments





1% level spin alignment, magnitude are large

Early Theories

- Coalescence picture — Liang & Wang PLB 2005

$$\frac{1+P_q P_{\bar{q}}}{3+P_q P_{\bar{q}}} \propto \left(\frac{\omega}{T}\right)^2$$

- Thermal picture — Becattini,Piccinini Rizzo,PRC, 2008

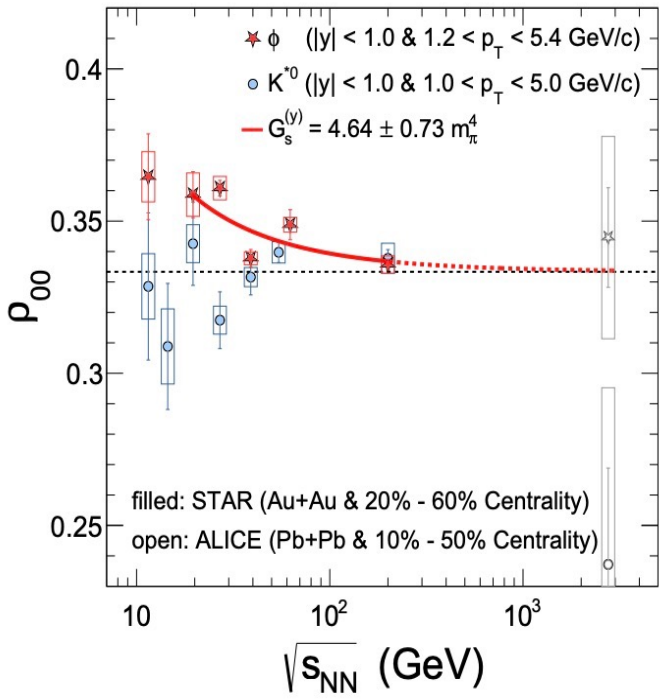
$$\frac{1-(\hat{p}\cdot\hat{\omega})}{18}\left(\frac{\omega}{T}\right)^2 \propto \left(\frac{\omega}{T}\right)^2$$

- $(\omega/T)^2 \sim 1/10^4$, 2nd order in gradient, small

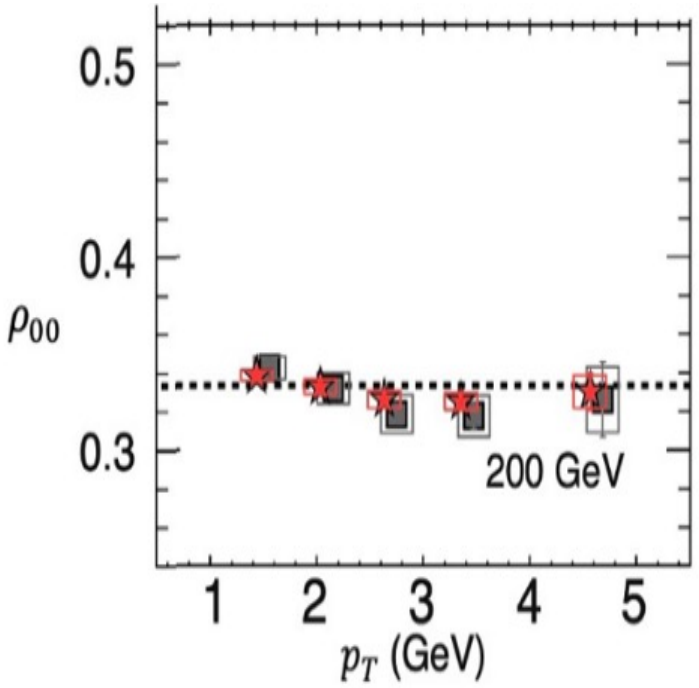
- Problems:
  - Magnitude too small

Challenges on Spin Alignment: 2) Rich behaviors

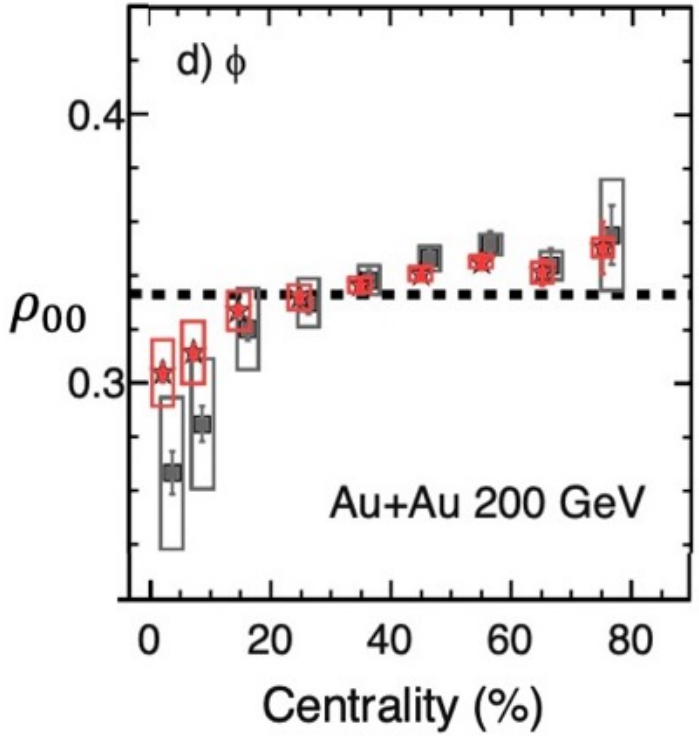
❖The Beam energy dependence



❖The p_T dependence the centrality dependence



❖The centrality dependence



Puzzling sign flip behaviors!

How we progress?

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1) Spin observables in heavy-ion collisions

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Structure of the density matrix

❖The spin-1 boson has (8 degrees of freedom)

$$\rho_{ss'} = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix} = \frac{1}{3}\delta_{ss'} + \frac{1}{2}\mathcal{P}_k(J_k)_{ss'} - \mathcal{T}_{ij}(J_{(i}J_{j)} - \frac{2}{3}\delta_{ij}\mathbf{1})_{ss'}$$

$$J_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, J_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, J_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

❖Need tensor polarization $\mathcal{T}^{\mu\nu}$

$$\delta\rho_{00} \propto T^{zz} \propto \mathrm{Tr}\{\rho(S_x^2 + S_y^2 - 2S_z^2)/3\}.$$

Fluctuation of the spin, relation to the dissipative physics?

Gradient Expansion and Symmetry analysis

❖Expansion up to 1st order gradient expansion

$$\mathcal{T}^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\Big[\kappa_0^u u^\lambda u^\gamma + \kappa_1^u u^\lambda u^\gamma + \kappa_{\rm sh}\sigma^{\lambda\gamma} + \kappa_T u^{(\lambda}\partial_\perp^{\gamma)}\beta + \kappa_{\rm su}u^{(\lambda}\sigma^{\gamma)\alpha}\tilde{p}_\alpha + \kappa_{\rm ou}u^{(\lambda}\Omega^{\gamma)\alpha}\tilde{p}_\alpha + \cdots\Big]$$

T-even

T-even, 0th order

T-odd, Shear Again!

Early theories include terms such as $(\omega/T)^2 \sim (1/100)^2$, 2nd order in gradient

Many Missing BUT Naturally Allowed Contribution at Lower Orders!

- Shear Induced Tensor Polarization(SITP) with $\kappa_{\rm sh}$ to be T-odd and indicate the nature of the dissipative physics
- In-medium spectral properties/interactions, broadening/damping, are required

Could we find these terms in a concrete calculation?

Yes, see later

The Spin alignment from thermal field theory

❖The density matrix can be derived from Wigner function:

$$\mathcal{T}^{\mu\nu} \equiv \mathcal{W}^{\langle\mu\nu\rangle} \equiv \mathcal{W}^{(\mu\nu)} - \frac{1}{3}\tilde{\Delta}^{\mu\nu}\mathcal{S} = 2\tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}W_{+}^{(\lambda\gamma)} \qquad \varrho_{ss'}(x,\boldsymbol{p}) = \epsilon^{\mu}_{s'}(\boldsymbol{p})\epsilon^{\nu *}_{s}(\boldsymbol{p})\mathcal{W}_{\mu\nu}(x,\boldsymbol{p})$$

❖The Wigner function can be related to correlation functions/Green functions that is well-defined in quantum field theory

$$W^{\mu\nu}(x,\boldsymbol{p}) \equiv \varepsilon_{\boldsymbol{p}} \int dp^0 \int d^4y e^{ip\cdot y}\langle V^{\mu}(x_-)V^{\nu}(x_+)\rangle$$

Zero for non-dissipative term in Zubarev approach at leading order

❖**Non-zero results from dissipative term in Zubarev approach**

$$W^{\mu\nu}_{+(1)} = \varepsilon_{\boldsymbol{p}} \lim_{\nu,q\to 0} \frac{\partial}{\partial\nu}[-\mathrm{Im}G^{\mu\nu\lambda\gamma}_{R+}(\nu,\boldsymbol{q},\boldsymbol{p})]\xi_{\lambda\gamma}$$

$$G^{\mu\nu\lambda\gamma}_{R}(\nu,\boldsymbol{q},\boldsymbol{p}) \quad \Leftrightarrow \text{Wigner Trans} \quad (-i)\Theta(t-t')\langle[V^{\mu}(t,\boldsymbol{x}^-)V^{\nu}(t,\boldsymbol{x}^+),T^{\lambda\gamma}(t',\boldsymbol{z})]\rangle,$$

❖We can obtain the coefficients of the general expansion in previous slide

$$\mathcal{T}^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\Big[\kappa^{u}_{0}u^{\lambda}u^{\gamma} + \kappa^{u}_{1}u^{\lambda}u^{\gamma} + \boxed{\kappa_{\mathrm{sh}}\sigma^{\lambda\gamma}} + \kappa_{T}u^{(\lambda}\partial^{\gamma)}_{\perp}\beta$$
$$+\kappa_{\mathrm{su}}u^{(\lambda}\sigma^{\gamma)\alpha}\tilde{p}_{\alpha} + \kappa_{\mathrm{ou}}u^{(\lambda}\Omega^{\gamma)\alpha}\tilde{p}_{\alpha} + \cdots\Big)$$

Total Theory Results

❖Full one loop results:

$$T^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\Big[\kappa_0^u u^\lambda u^\gamma + \kappa_1^u u^\lambda u^\gamma + \kappa_{\rm sh}\sigma^{\lambda\gamma} + \kappa_T u^{(\lambda}\partial_\perp^{\gamma)}\beta + \kappa_{\rm su}u^{(\lambda}\sigma^{\gamma)\alpha}\tilde{p}_\alpha + \kappa_{\rm ou}u^{(\lambda}\Omega^{\gamma)\alpha}\tilde{p}_\alpha + \cdots\Big]$$

$$\kappa_0^u = \frac{\alpha_0}{-\tilde{v}^2}n_0,\ \kappa_1^u = \left[\alpha_{\rm sh}\left(c_s^2 - \frac{1}{3}\right)\theta + \frac{\alpha_{\rm sp}\xi_p}{-\tilde{v}^2}\right]\beta n_0$$

$$\kappa_{\rm sh} = \alpha_{\rm sh}\beta n_0, \quad \kappa_T = 0, \quad \kappa_{\rm su} = 0, \quad \kappa_{\rm ou} = 0$$

$$\alpha_{\rm sh} = \frac{4\varepsilon_{\boldsymbol{p}}\pi}{\beta n(\varepsilon_{\boldsymbol{p}})}\int_0^\infty \frac{\partial n(\omega)}{\partial\omega}d\omega(\omega^2 - \varepsilon_{\boldsymbol{p}}^2)A_{T/L}^2(\omega,\boldsymbol{p})$$

$$\alpha_{\rm sp} = \frac{4\varepsilon_{\boldsymbol{p}}\pi}{\beta n(\varepsilon_{\boldsymbol{p}})}\int_0^\infty \frac{\partial n(\omega)}{\partial\omega}d\omega\varepsilon_{\boldsymbol{p}}^2(A_T^2(\omega,\boldsymbol{p}) - A_L^2(\omega,\boldsymbol{p}))$$

$$\alpha_0 = 2\varepsilon_{\boldsymbol{p}}\int_0^\infty d\omega\,\frac{n(\omega)}{n(\varepsilon_{\boldsymbol{p}})}\left[(A_L - A_T) - \frac{\Delta\omega^2\tilde{v}^2}{p^2}A_L\right]$$

❖Features of the result

- Suggested by symmetry,
- Verified in concrete calculation,
- All have been done is to include dissipative part in Zubarev approach with **more realistic spectral functions, realize the fluctuation-dissipation relation** for spin alignment

Total Theory Results

❖Full one loop results:

$$T^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\left[\kappa_0^u u^\lambda u^\gamma + \kappa_1^u u^\lambda u^\gamma + \kappa_{\rm sh}\sigma^{\lambda\gamma} + \kappa_T u^{(\lambda}\partial_\perp^{\gamma)}\beta\right]$$

$$\rho_{00} = \epsilon_\lambda^0(p)\epsilon_\gamma^0(p)\left(\kappa_0^u u^\lambda u^\gamma + \kappa_1^u u^\lambda u^\gamma + \kappa_{sh}\sigma^{\lambda\gamma}\right)/\text{norm}$$

$$\kappa_0^u = \frac{\alpha_0}{-\tilde{v}^2}n_0,\ \kappa_1^u = \left[\alpha_{\rm sh}\left(c_s^2 - \frac{1}{3}\right)\theta + \frac{\alpha_{\rm sp}\xi_p}{-\tilde{v}^2}\right]\beta n_0$$

$$\kappa_{\rm sh} = \alpha_{\rm sh}\beta n_0, \quad \kappa_T = 0, \quad \kappa_{\rm su} = 0, \quad \kappa_{\rm ou} = 0$$

$$\alpha_{\rm sh} \approx -\frac{2\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} + 2\frac{\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\frac{\Delta\varepsilon_{\boldsymbol{p}}}{T} + \frac{\Gamma_{\boldsymbol{p}}}{2T} \sim \mathcal{O}(1)$$

$$\alpha_{\rm sp} \approx -\frac{\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\left(\frac{\Gamma^\Delta_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} - \frac{\Delta\varepsilon_{\boldsymbol{p}}}{T}\frac{\Gamma^\Delta_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} + \frac{\Gamma_{\boldsymbol{p}}}{T}\frac{\omega^\Delta_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\right) \sim \mathcal{O}(\delta_{\rm qp}^{-1}\delta_{\rm sp}).$$

$$\alpha_0 \approx (\omega^T_{\boldsymbol{p}} - \omega^L_{\boldsymbol{p}})/T$$

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1) Spin observables in heavy-ion collisions

2) Theory for tensor polarization and spin alignment

3) In-medium spectral function and spin alignment as a probe

4) Summary and outlooks

Broadening of hadron in-medium

❖Where the mesonic resonance forms?

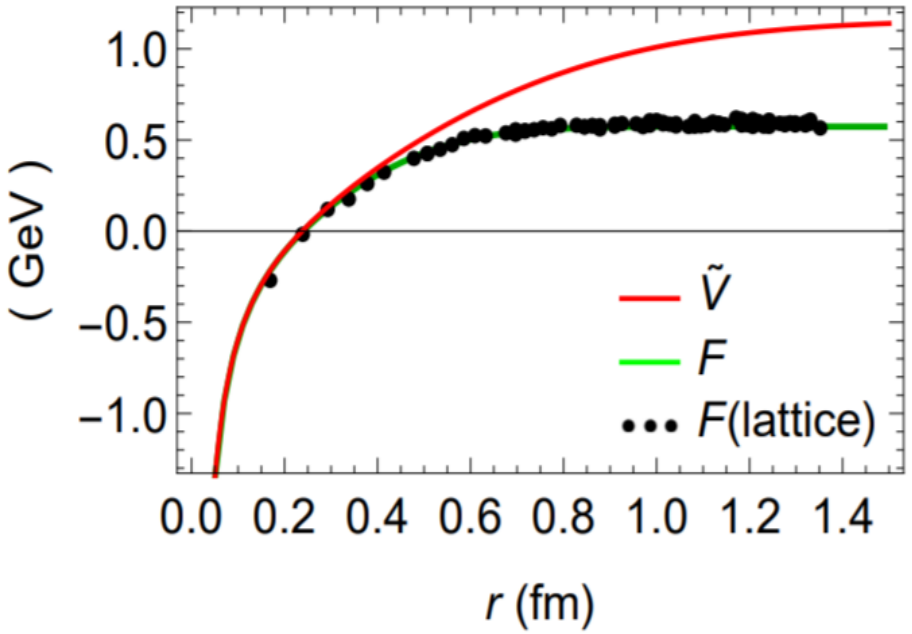
- Chemical freezeout
- Late stage of QGP?

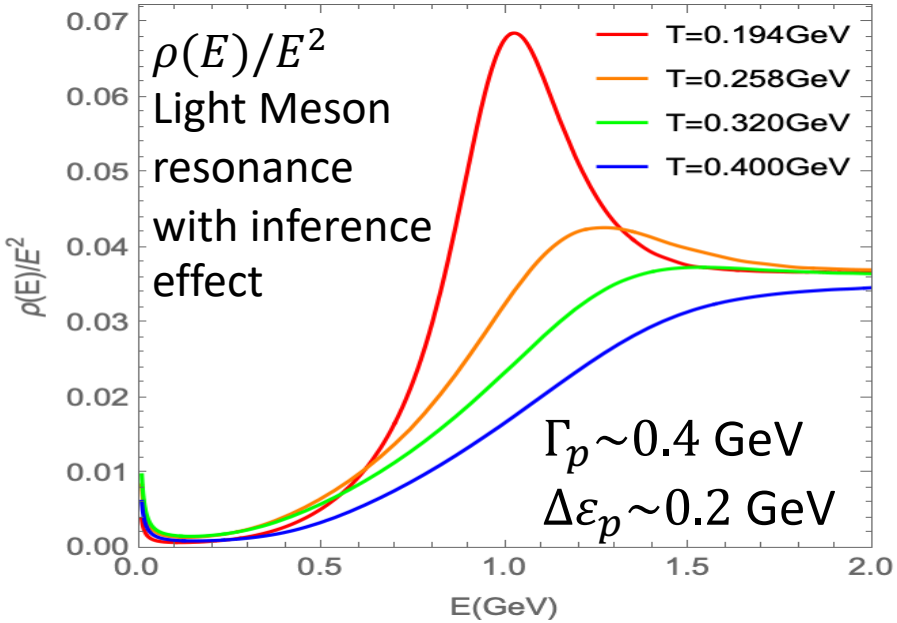
Towards the Theory of Binary Bound States in Quark-Gluon Plasma

Edward V.Shuryak and Ismail Zahed

Shuryak, Zahed, PRD 2004

❖Large confining potential ⟺ mesonic resonance at $T\sim0.2$ GeV





Liu, Rapp, PRC 2018

Spectral properties of in-medium degree of freedom

❖Where the mesons of freedom forms?

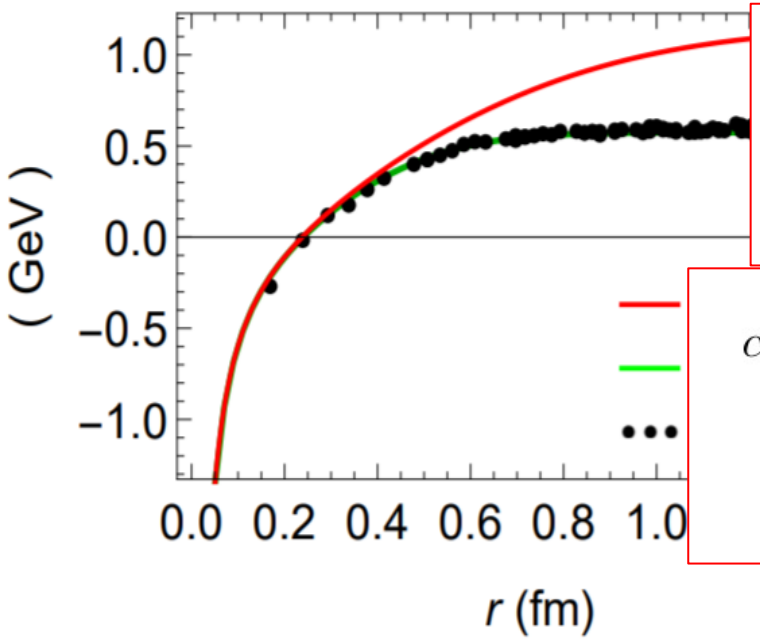
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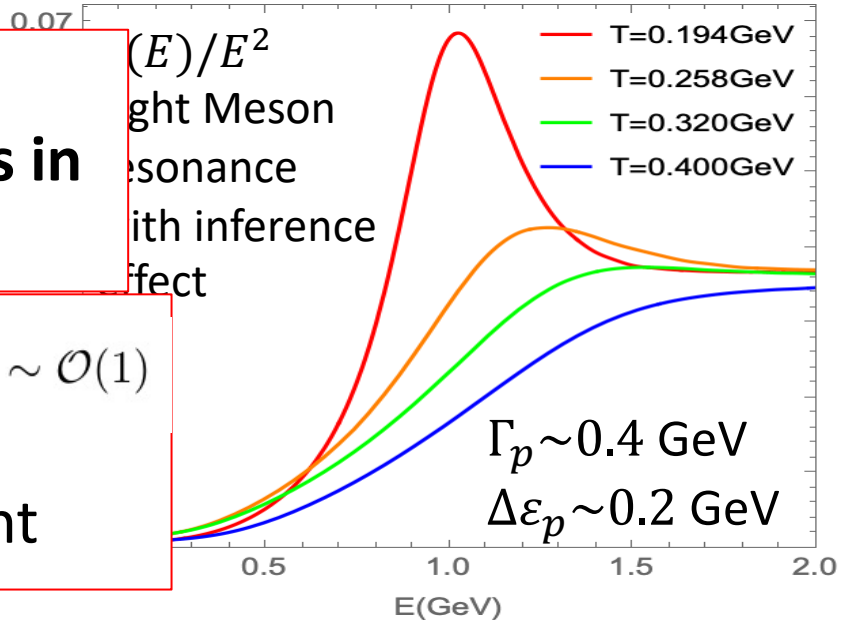
❖Large confining potential ⟺ mesonic resonance at $T\sim0.2$ GeV



Large width can be generated by collisions in strong coupling

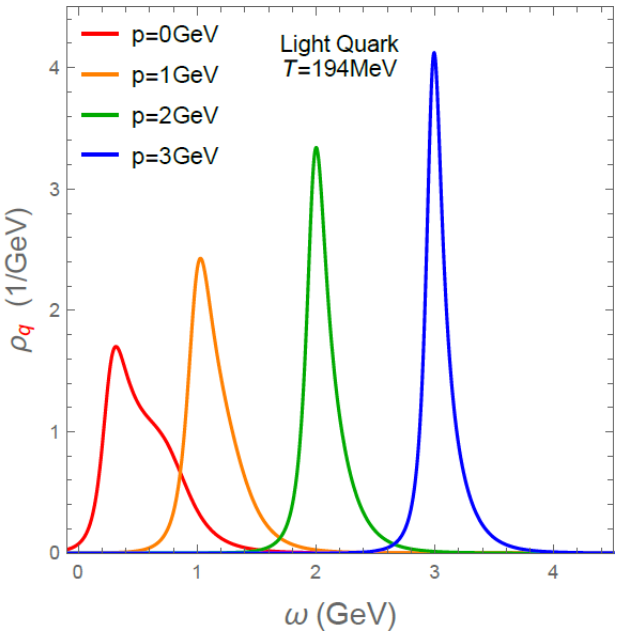
$$\alpha_{\rm sh} \approx -\frac{2\Delta\varepsilon_p}{\Gamma_p} + 2\frac{\Delta\varepsilon_p}{\Gamma_p}\frac{\Delta\varepsilon_p}{T} + \frac{\Gamma_p}{2T} \sim \mathcal{O}(1)$$

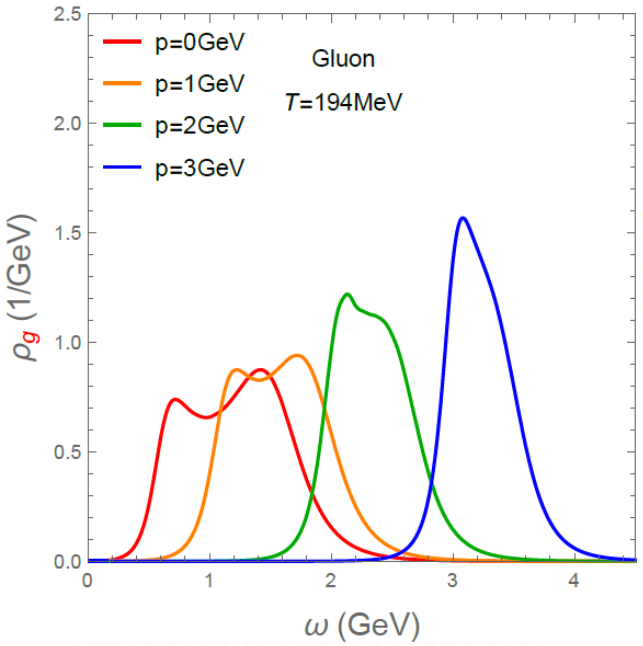
$\alpha_{\rm sh}\sim1$, Large Coefficient

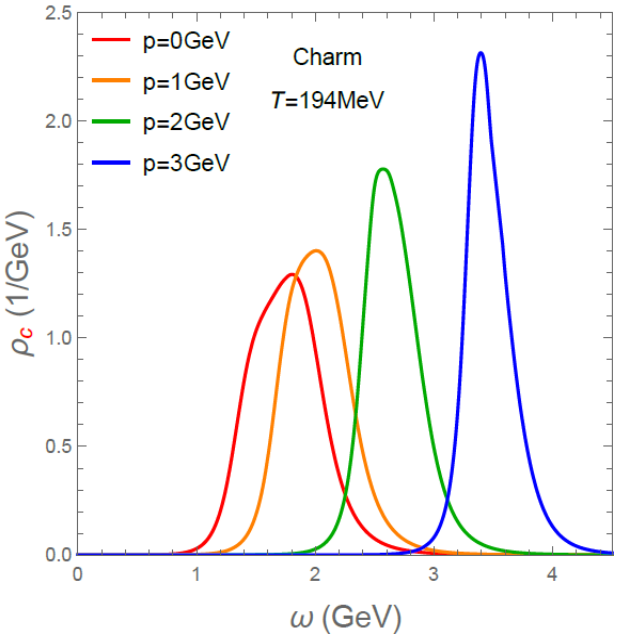


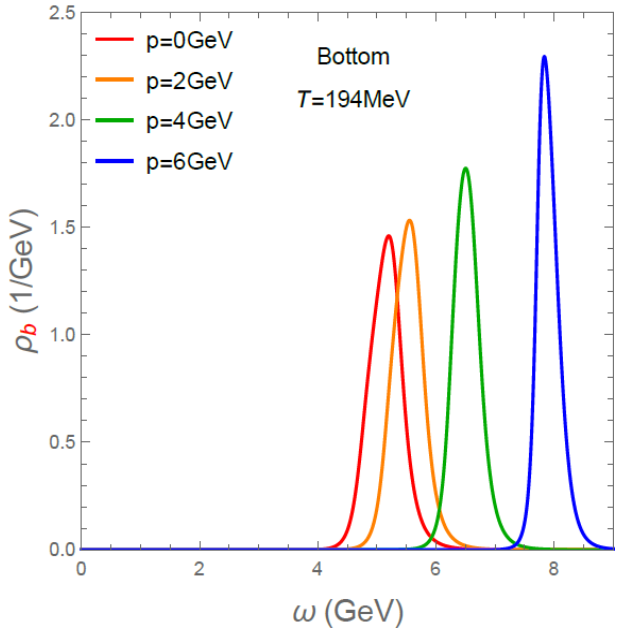
Liu, Rapp, PRC 2018

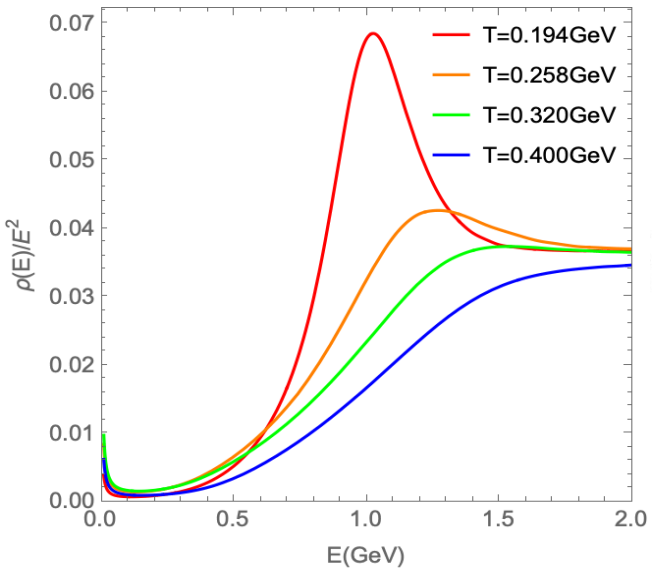
Reminder of Nontrivial In-medium Spectral Function

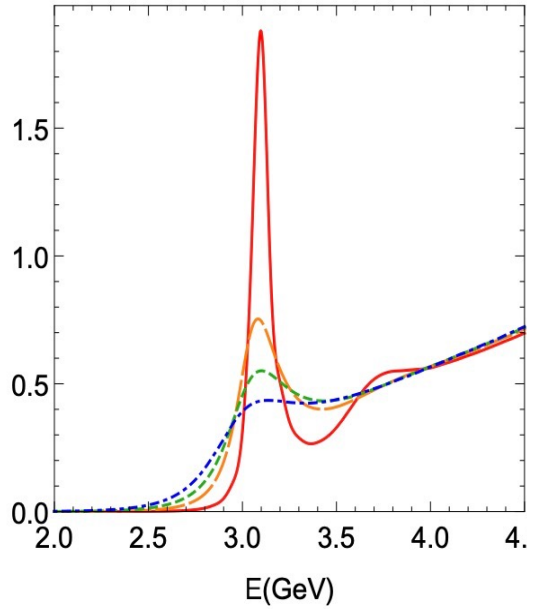


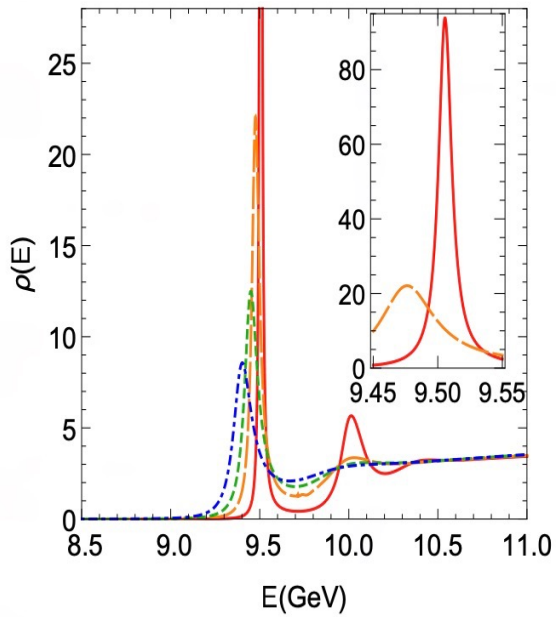










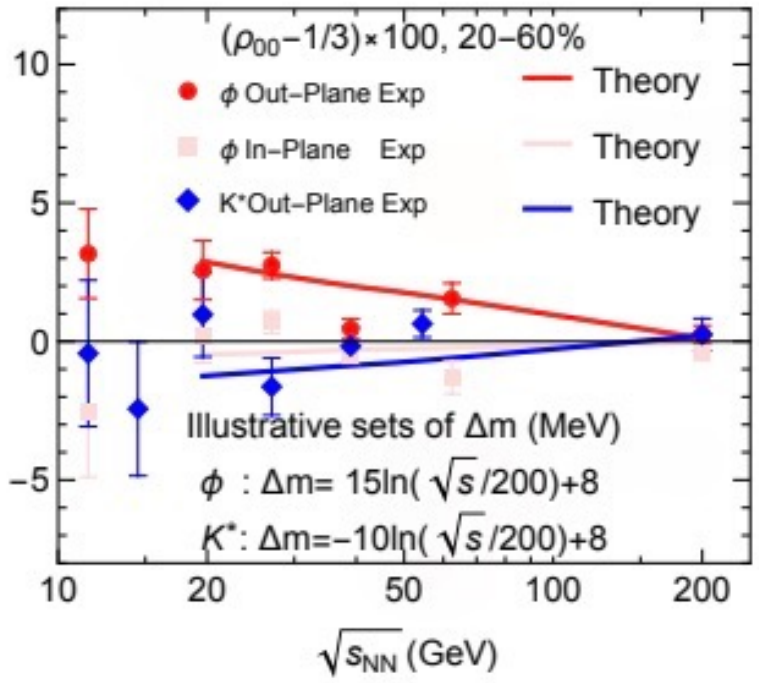


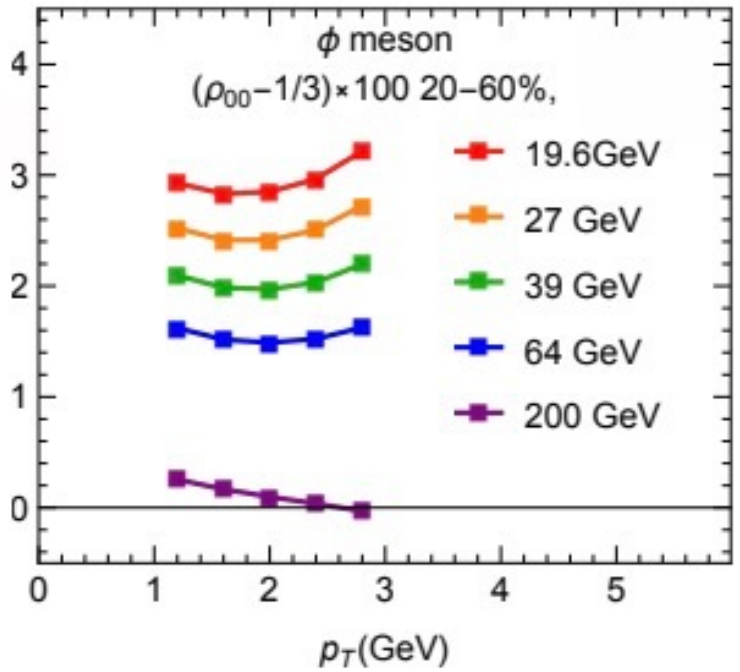
- Width can be large
- Nontrivial dependence on momentum
- Mass shift can be positive or negative

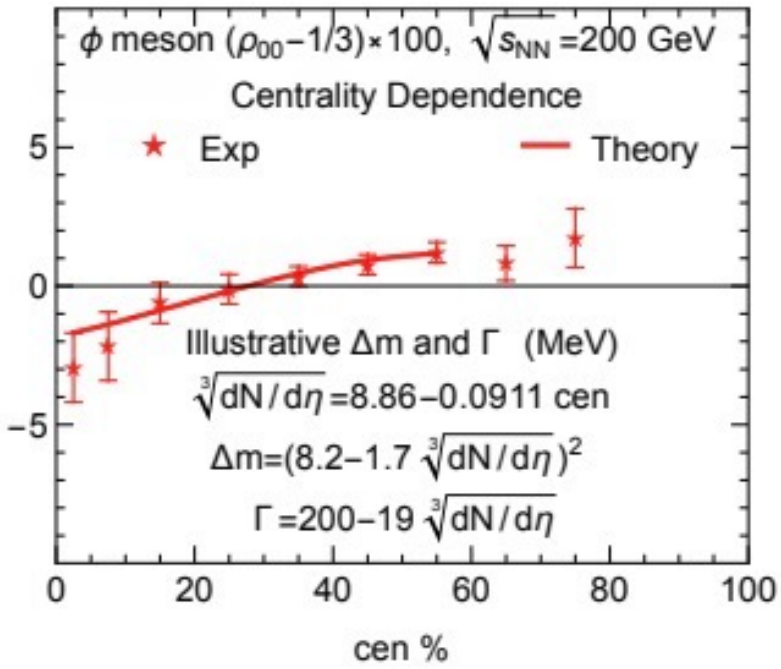
Liu and Rapp 2018

A Tune to Reproduce the Data

- Tune the coefficients $\alpha_{sh}(\Gamma, \Delta m)$ with mass shift and width
- Reproduce the beam energy (ϕ, K^*), p_T, centrality dependence







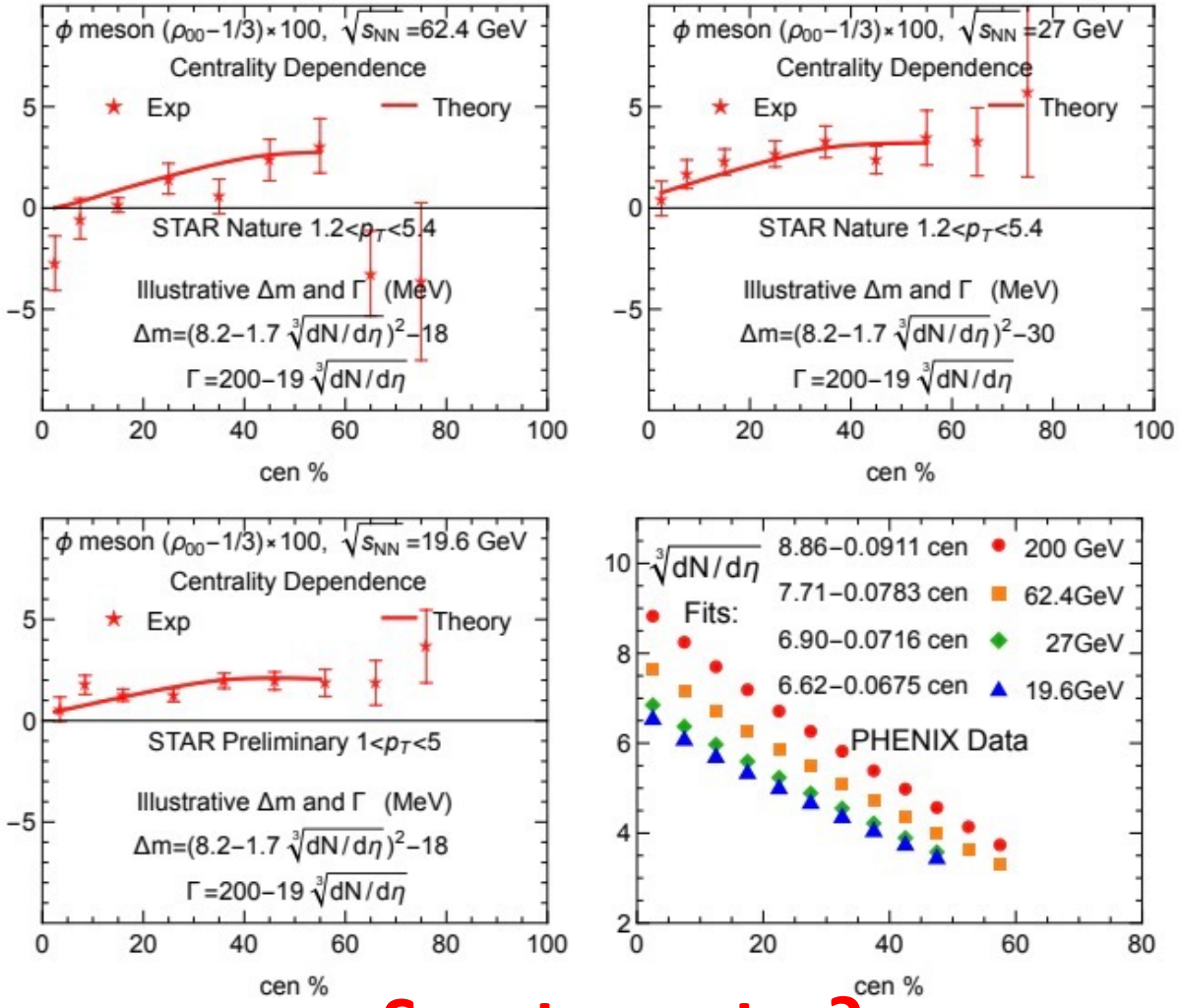
Spectrometer?

Li and Liu, arXiv:2206.11890

More for Centrality Dependence

Centrality Freezeout Correlation

- More central, longer hadronic phase
- Longer hadronic phase, ϕ will re-equilibrate in hadronic phase and get freezeout there
- Short hadronic phase, ϕ do not have time to re-equilibrate so it have memory of spin alignment in QGP or mix phase



Spectrometer?

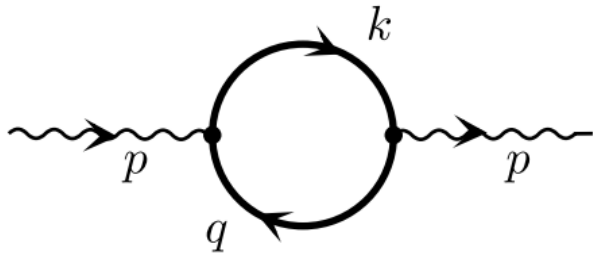
Li and Liu, arXiv:2206.11890

Can we get similar trend it from microscopic model?

A Hybrid Model:Quark-Meson Model and Hadronic Field Theory

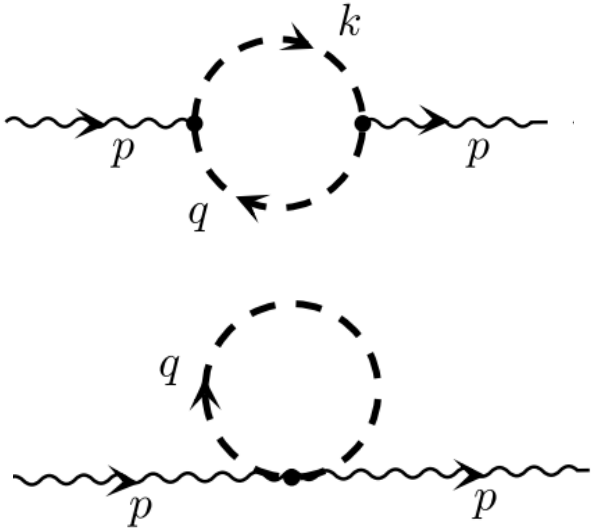
❖The quark meson model in QGP and **around phase boundary**,

$$\mathcal{L} = \bar{\psi}(i\not{D} - m_s)\psi - \frac{1}{4}\phi^{\mu\nu}\phi_{\mu\nu} + \frac{1}{2}m_\phi^2\phi^\mu\phi_\mu,$$



❖Chiral Perturbation Lagrangian in **pure hadronic phase**

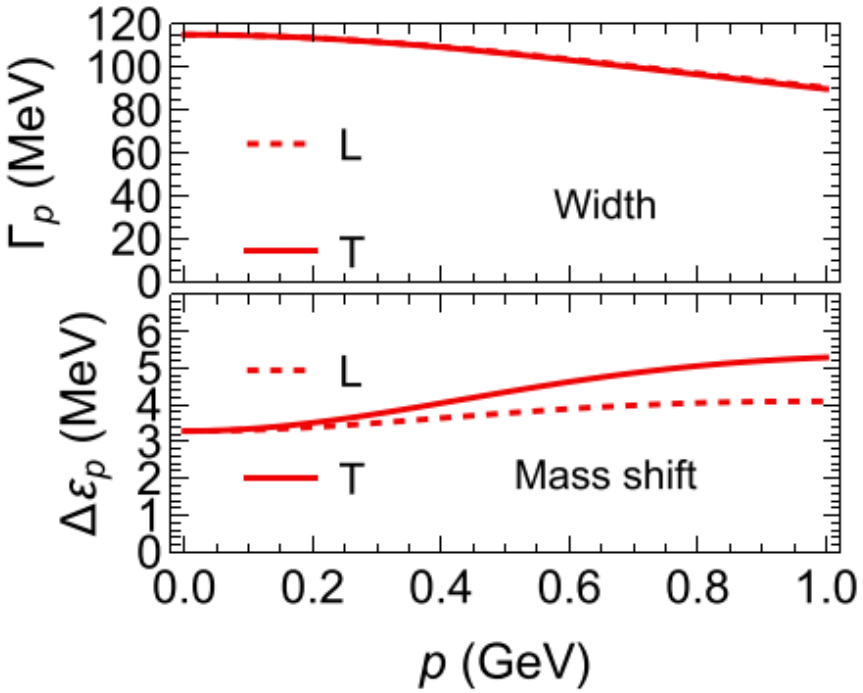
$$\begin{aligned}\mathcal{L} = &|\partial_\mu K|^2 + |\partial_\mu K_0|^2 + ig_\phi\phi^\mu(K^*\overleftrightarrow{\partial}_\mu K + K_0^*\overleftrightarrow{\partial}_\mu K_0)\\ &+ g_\phi^2(|K_0|^2 + |K|^2)\phi^\mu\phi_\mu - \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2}m_\phi^2\phi^\mu\phi_\mu\\ &- m_K^2|K|^2 - m_{K_0}^2|K_0|^2, \qquad (6)\end{aligned}$$



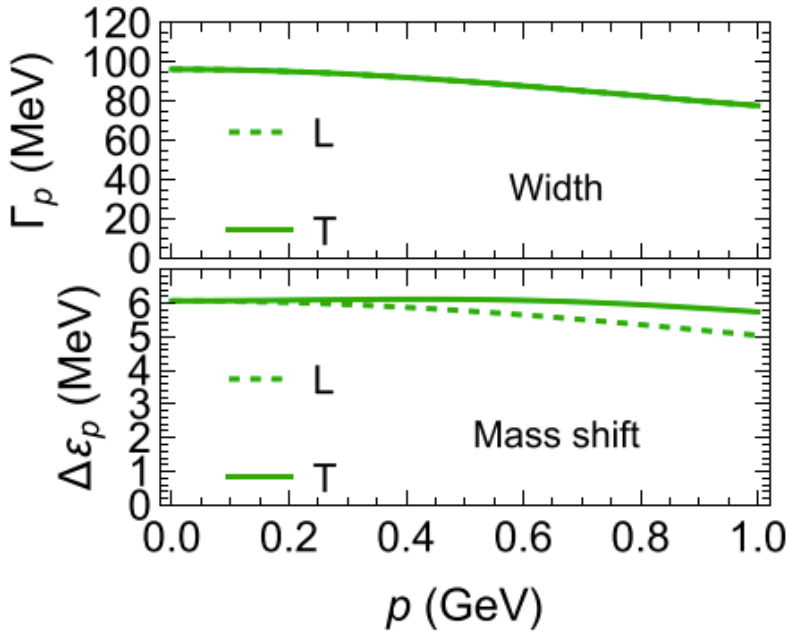
The mass shift and width from QM model or hadronic field theory

❖Width and mas-shift of ϕ with **in-medium** quark or Kaon mass

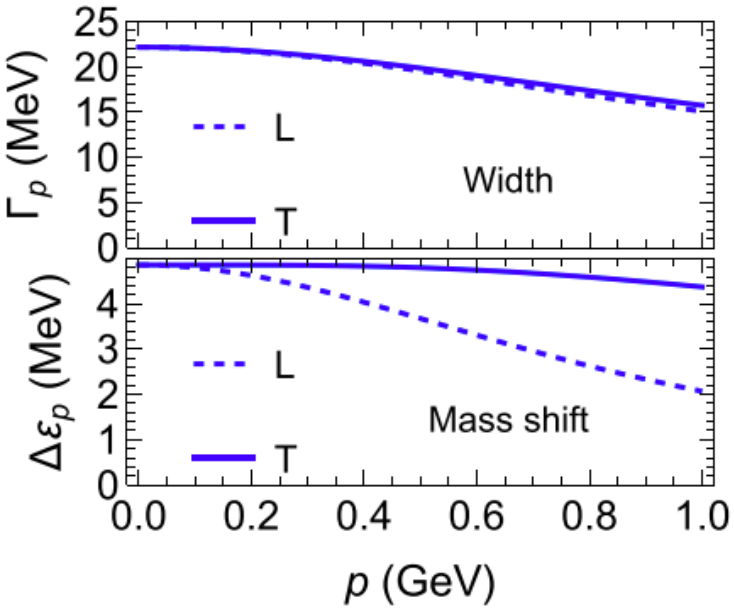
$m_s = 0.3$



$m_s = 0.42$

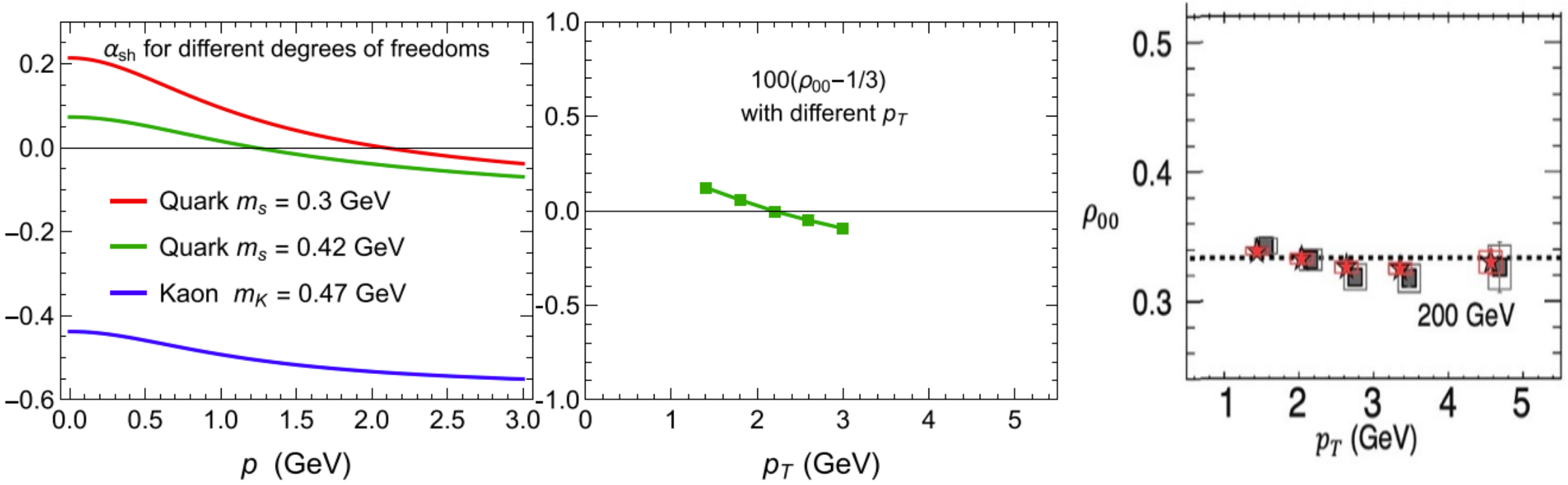


$m_K = 0.47$

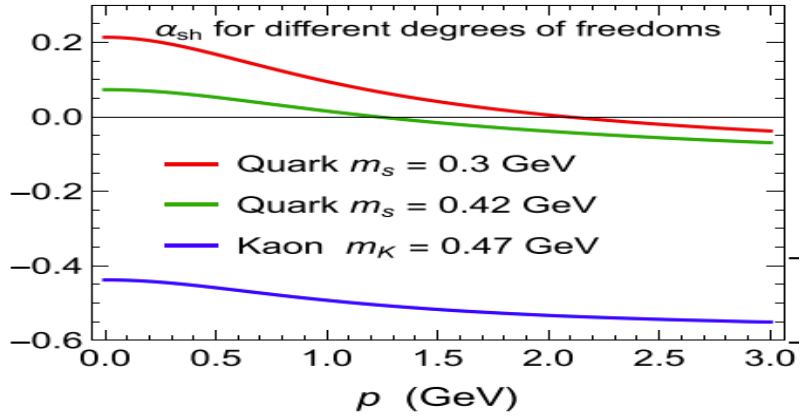


The spin alignment p_T

Can naturally generate rich centrality behavior, such as sign flips

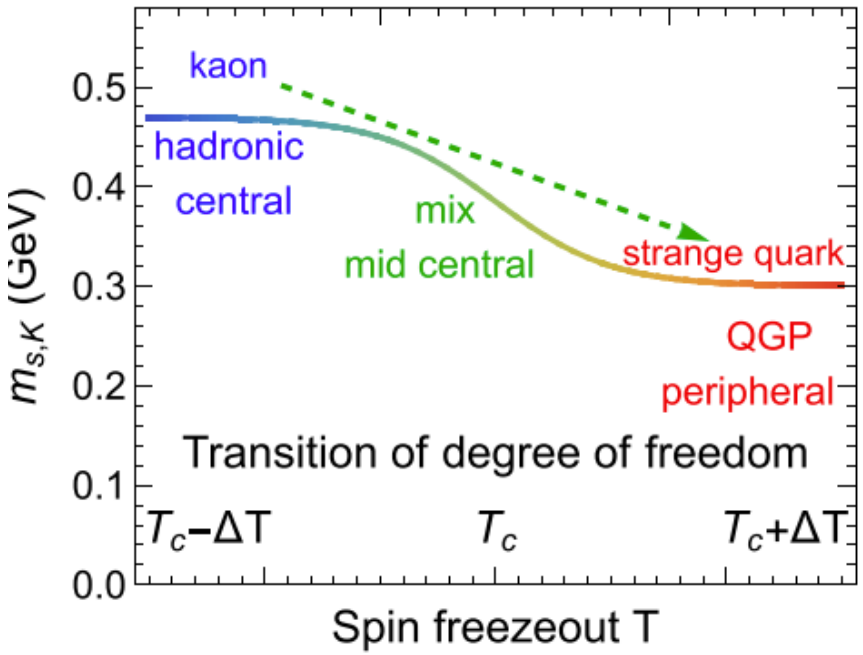


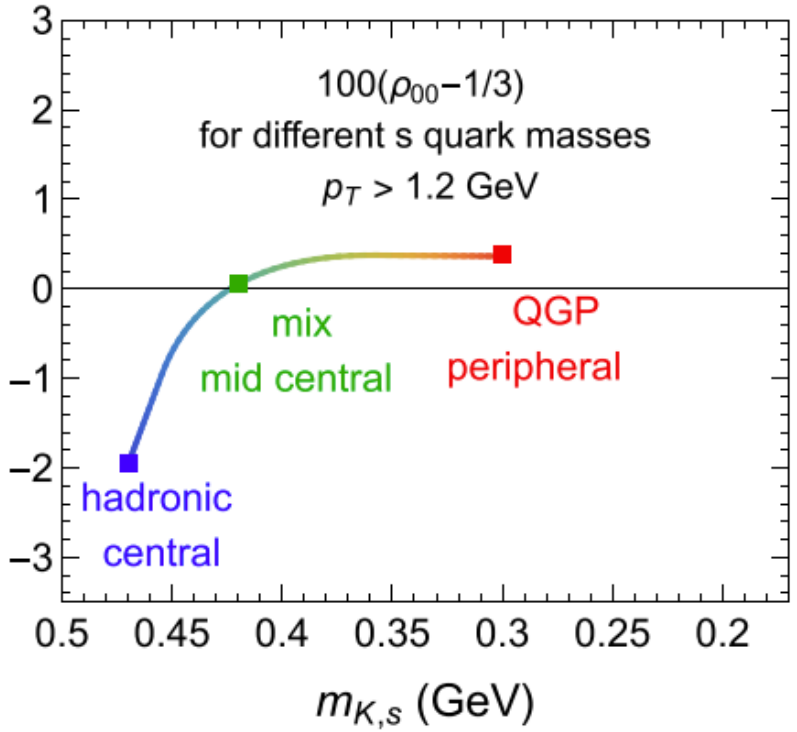
The spin alignment centrality

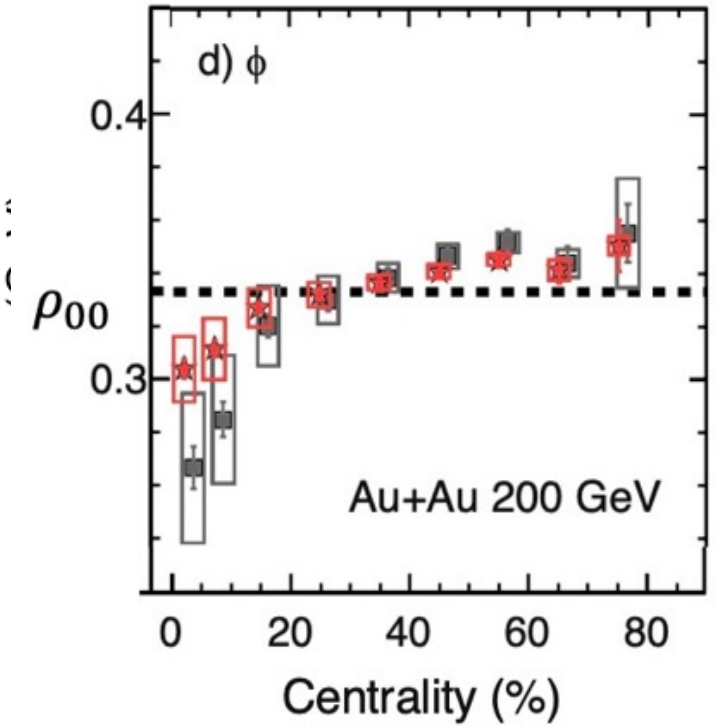


Can naturally generate rich centrality behavior, such as sign flips

Probe spectra at different stages







Spin Alignment Talk Summary

- **Conceptually**: establishment of the **fluctuation-dissipation relation** and discovery of the **SITP effect**
- **Methodologically**: first **QFT framework** to handle **dissipative effects** and evaluate transport coefficients for spin alignment physics
- **Phenomenologically**: qualitatively describing the puzzling "spin alignment" data, revealing its potential as a **"spectrometer"**.



Hypersurface covariance in relativistic local equilibrium

Nontrivial physics from the "dissipative" part in Zubarev approach

Shuai Liu

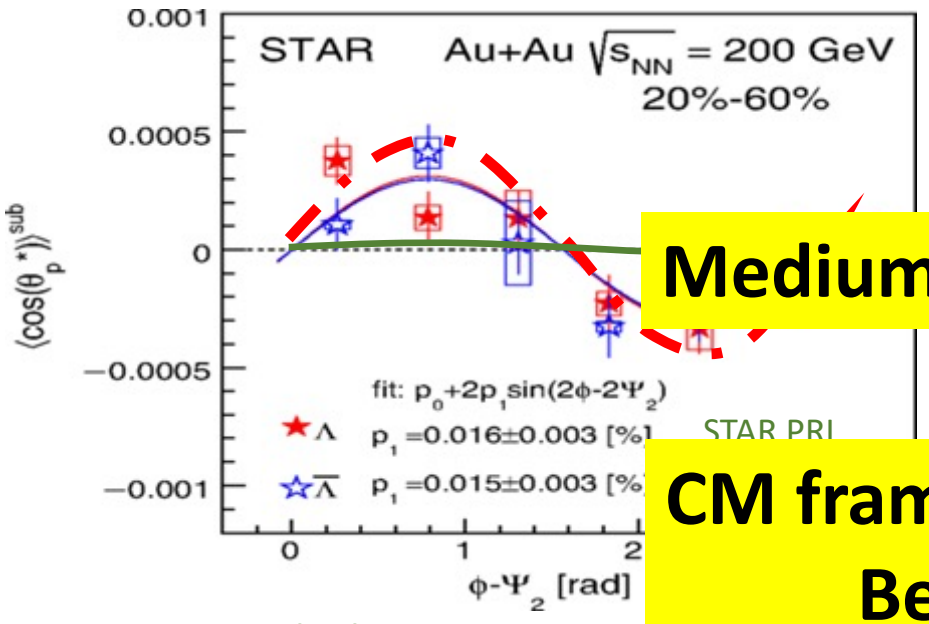
Hunan University

Chirality, Vorticity, and Magnetic Field in Quantum Matter (Chirality 2026) ,
Nov 16-20, 2026, Zhuhai, China

Based on work, Y-Y Li, Liu on arXiv:2611.08482

Local Polarization and Shear-induced Polarization

Local polarization sign puzzle



Voloshin, EPJ 2018
Becattini, Karpenko, PRL 2018

Medium Rest Frame, Liu, Yin

Sketches of theoretical curves

STAR PRL

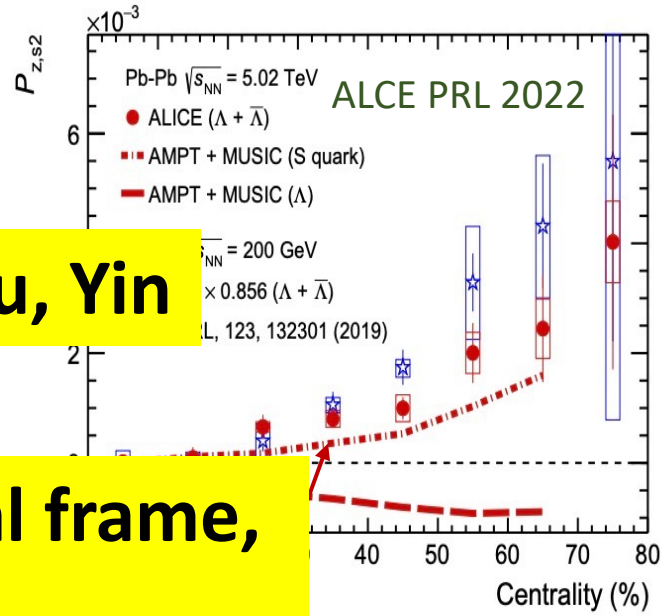
CM frame or Isothermal frame, Becattini et al.

❖Previous theory

$$\mathcal{P} = \frac{(1-n_f)\epsilon^{\mu\nu\alpha\lambda}p_\nu\partial_\alpha(\beta u_\lambda)}{4m}$$

❖Sign is opposite!

Progress



Liu, Yin, JHEP, 2021;Becatinni, Buzzegoli, Palermo, PLB, 2021

❖Shear-Induced Polarization

$$-n_0(1-n_0)\frac{1}{\varepsilon_0}\epsilon^{\mu\nu\alpha\rho}u_\nu p_\rho p^\lambda\partial_{(\alpha}(\beta u)_{\lambda)}$$

❖Right sign, large phenomenologically

Fu, Liu, Pang, Song, Yin, PRL, 2021;
Becatinni, Buzzegoli, Palermo, Inghirami,Karpenko PRL, 2021

Hypersurface covariance of local equilibrium

❖Zubarev formalism with superficial hypersurface dependence

$$\rho = e^{-\mathcal{B}}/Z, \quad Z = \mathrm{Tr}[e^{-\mathcal{B}}] \tag{1}$$

$$\mathcal{B} = \int \mathrm{d}\Sigma_\gamma T^{\gamma\lambda}\beta_\lambda(x') - \int_{\Sigma^0}^{\Sigma} d^4x' \partial_\gamma \beta_\lambda(x') T^{\gamma\lambda}(x'),$$

Two hypersurface involved

$$\mathcal{B} = \int \mathrm{d}\Sigma_\gamma T^{\gamma\lambda}\beta_\lambda(x') - \int^{\Sigma} d^4x' \partial_\gamma \beta_\lambda(x') T^{\gamma\lambda}(x') e^{-\epsilon t(x')}.$$

❖Typically, one only effectively include the non-dissipative term with explicit hypersurface dependence

$$\mathcal{B} = \int \mathrm{d}\Sigma_\gamma T^{\gamma\lambda}\beta_\lambda(x')$$

Source for the hypersurface disagreements:
medium rest frame, CM frame, isothermal frame

Gauss theorem protect the covariance

❖Gauss theorem

$$\Delta\mathcal{B}^{\rm nd} = \int {\rm d}\Sigma_\mu T^{\mu\nu}\beta_\nu - \int {\rm d}\tilde{\Sigma}_\mu T^{\mu\nu}\beta_\nu. \qquad \Delta\mathcal{B}^{\rm d} = -\int_{\tilde{\Sigma}}^{\Sigma} d^4x'\partial_\mu\beta_\nu T^{\mu\nu} \tag{3}$$

$$\Delta\mathcal{B} = \Delta\mathcal{B}^{\rm nd} + \Delta\mathcal{B}^{\rm d} = 0 \rightarrow \rho(\Sigma) = \rho(\tilde{\Sigma})$$

Formally trivial, but not many one calculate volume integral systematically

$$\Delta\mathcal{B}^{\rm d} = -\int_{\tilde{\Sigma}}^{\Sigma} d^4x'\partial_\mu\beta_\nu T^{\mu\nu} \tag{3}$$

Also, Gauss theorem do not remove the initial hypersurface dependence

$$\mathcal{B} = \int {\rm d}\Sigma_\gamma T^{\gamma\lambda}\beta_\lambda(x') - \int_{\Sigma^{\rm o}}^{\Sigma} d^4x'\partial_\gamma\beta_\lambda(x')T^{\gamma\lambda}(x'),$$

Explicit calculation for both "non-dissipative" and "dissipative" part

❖A convenient representation, with original Zubarev damping factor

$$\mathcal{B} = \int \mathrm{d}^4x' K_{\lambda\gamma}(x')T^{\lambda\gamma}(x'),\ K_{\lambda\gamma} = K^{\mathrm{nd}}_{\lambda\gamma} + K^{\mathrm{d}}_{\lambda\gamma}$$

$$K^{\mathrm{nd}}_{\lambda\gamma}(x') = \delta(\phi(x'))\partial_\lambda\phi(x')\beta_\gamma(x')$$

Non-dissipative part

$$K^{\mathrm{d}}_{\lambda\gamma}(x') = -\theta(-\phi(x'))\partial_\lambda\beta_\gamma(x')e^{-\epsilon \mathrm{t}\cdot(x-x')}$$

"dissipative" part

$$\tilde{\mathcal{J}}^\mu_{\mathrm{nd},5} = \frac{1}{2}n_0(1-n_0)\varepsilon^{\mu\rho\sigma\gamma}(\tilde{p}_\rho\tilde{\partial}^\perp_\sigma\beta_\gamma - 2\tilde{p}_\gamma\tilde{l}_\rho\frac{\tilde{p}^\lambda}{\varepsilon_{\tilde{p}}}\tilde{\partial}^\perp_{(\sigma}\beta_{\lambda)})$$

$$\equiv \frac{1}{2}n_0(1-n_0)\varepsilon^{\mu\rho\sigma\gamma}(\tilde{p}_\rho\tilde{\partial}_\sigma\beta_\gamma - 2\tilde{p}_\gamma\tilde{l}_\rho\frac{\tilde{p}^\lambda}{\varepsilon_{\tilde{p}}}\tilde{\partial}_{(\sigma}\beta_{\lambda)}) \qquad (15)$$

$$\tilde{\mathcal{J}}^\mu_{\mathrm{d},5} = n_0(1-n_0)(\frac{\tilde{l}_\sigma}{\varepsilon_{\tilde{p}}} - \frac{\tilde{u}_\sigma}{\tilde{u}\cdot\tilde{p}})\tilde{p}_\rho\varepsilon^{\mu\sigma\gamma\rho}\tilde{p}^\lambda\tilde{\partial}_{(\lambda}\tilde{\beta}_{\gamma)}$$

Non-dissipative contribution from dissipative part

❖**Natural case total ((typical hydro predict $\mathrm{t} = u$ at zero order)**

$$\mathcal{J}^\mu_5 = \frac{1}{2}n_0(1-n_0)\varepsilon^{\mu\rho\sigma\gamma}(p_\rho\partial^\perp_\sigma\beta_\gamma - 2p_\gamma u_\rho\frac{p^\lambda}{u\cdot p}\partial^\perp_{(\sigma}\beta_{\lambda)})$$

❖**If we insist damping time direction is no medium rest $\mathrm{t} \neq u$**

$$\mathcal{J}^\mu_{\mathrm{t},5} = \frac{1}{2}n_0(1-n_0)\varepsilon^{\mu\rho\sigma\gamma}(p_\rho\partial^\perp_\sigma\beta_\gamma - 2p_\gamma \mathrm{t}_\rho\frac{p^\lambda}{\mathrm{t}\cdot p}\partial^\perp_{(\sigma}\beta_{\lambda)})$$

Tricks for to calculate the volume integral

❖Any small dissipation can remove initial hypersurface dependence

$$
\begin{aligned}
&\int d\tilde{t}' K^{\mathrm{d}} e^{-i\tilde{q}_0(\tilde{t}-\tilde{t}')}(-i\tilde{q}_\sigma) \\
&= -\exp[-\epsilon\tilde{u}_\xi^\perp \cdot (\frac{\partial}{i\partial\tilde{q}_\xi^\perp}))]\frac{-i\tilde{q}_\sigma}{i\tilde{q}_0+\tilde{u}_0\epsilon} \\
&= -[\sum_{n=1}\frac{1}{n!}(\tilde{u}_\xi^\perp\frac{\epsilon\,\partial}{-i\partial\tilde{q}_\xi^\perp})^n][\sum_{m=0}\frac{-i\tilde{q}_\sigma}{\epsilon\tilde{u}_0}(\frac{-i\tilde{q}_0}{\epsilon\tilde{u}_0})^m] \\
&= -(\tilde{u}_\sigma^\perp-\frac{\tilde{p}_\perp\cdot\tilde{u}_\perp}{\varepsilon_{\tilde{p}}}\tilde{l}_\sigma)\sum_{m=0}\frac{1}{\tilde{u}_0}(\frac{-\tilde{p}_\perp\cdot\tilde{u}_\perp}{\varepsilon_{\tilde{p}}\tilde{u}_0})^m] \\
&= (\frac{\tilde{l}_\sigma}{\varepsilon_{\tilde{p}}}-\frac{\tilde{u}_\sigma}{\tilde{u}\cdot\tilde{p}})\varepsilon_{\tilde{p}} \qquad (16)
\end{aligned}
$$

- **Expansion need to keep to all order to obtain the final leading q order**
- **keep track of ϵ, take it to zero after take q to zero**

Using physical damping

❖Damping/broadening of the density-density correlation (Includes effects of interacting effects in a leading skeleton expansion scheme)

$$\delta(\omega - \bar{u}\cdot\bar{q}) \to \frac{1}{\pi}\frac{\Gamma}{(\omega-\bar{u}\cdot\bar{q})^2+\Gamma^2}$$

Similar physics like the spin alignment treatment, in-medium broadening

❖Exact same results

$$\begin{aligned}
&-[\theta^+\exp[-\Gamma\tilde{u}_\xi^\perp\cdot(\frac{\partial}{i\partial\tilde{q}_\xi^\perp})]+\theta^- e^{i\tilde{q}_0\delta\tilde{x}_u^\perp/\tilde{u}_0}]\frac{-i\tilde{q}_\sigma}{i\tilde{q}_0+\tilde{u}_0\Gamma}\\
&-\theta^-[\exp[\Gamma\tilde{u}_\xi^\perp\cdot(\frac{\partial}{i\partial\tilde{q}_\xi^\perp})]-e^{i\tilde{q}_0\delta\tilde{x}_u^\perp/\tilde{u}_0}]\frac{-i\tilde{q}_\sigma}{i\tilde{q}_0-\tilde{u}_0\Gamma}\\
&=\exp[-\Gamma\tilde{u}_\perp\cdot(\frac{\partial}{i\partial\tilde{q}_\xi^\perp})))]\frac{(-)(-i\tilde{q}_\sigma)}{i\tilde{q}_0+\tilde{u}_0\Gamma}|_{q=0}\\
&=(\frac{\tilde{l}_\sigma}{\varepsilon_{\tilde{p}}}-\frac{\tilde{u}_\sigma}{\tilde{u}\cdot\tilde{p}})\varepsilon_{\tilde{p}} \qquad (22)
\end{aligned}$$

❖**Leading to medium rest result** $\mathcal{J}_5^\mu=\frac{1}{2}n_0(1-n_0)\varepsilon^{\mu\rho\sigma\gamma}(p_\rho\partial_\sigma^\perp\beta_\gamma-2p_\gamma u_\rho\frac{p^\lambda}{u\cdot p}\partial_{(\sigma}^\perp\beta_{\lambda)})$

Using physical damping, include initial hypersurface and memory effects

❖Extra contribution from memory of initial hypersurface $\mathrm{l} \cdot (x - x') = \mathrm{t}_0$

$$
\begin{aligned}
&[\exp[-\Gamma(\tilde{u}_\xi^\perp)\cdot(\frac{\partial}{i\partial\tilde{q}_\xi^\perp})]]\frac{-i\tilde{q}_\sigma}{i\tilde{q}_0+\tilde{u}_0\Gamma}e^{-iq_0\mathrm{t}_0}e^{-\Gamma(\tilde{u}\cdot\mathrm{l})\mathrm{t}_0}\\
&=\sum_{n=0}^{\infty}\sum_{m=0}^{n}(\tilde{u}_\sigma^\perp-\frac{\tilde{p}_\perp\cdot\tilde{u}_\perp}{\varepsilon_{\tilde{p}}}\tilde{l}_\sigma)\frac{1}{\tilde{u}_0}(\frac{-\tilde{p}_\perp\cdot\tilde{u}_\perp}{\varepsilon_{\tilde{p}}\tilde{u}_0})^m\\
&\quad\times\frac{(-\Gamma\mathrm{t}_0\tilde{p}_\perp\cdot\tilde{u}/\varepsilon_{\tilde{p}})^{n-m}}{(n-m)!}e^{-\Gamma(\tilde{u}\cdot\mathrm{l})\mathrm{t}_0}\\
&=-\varepsilon_{\tilde{p}}(\frac{\tilde{\mathrm{l}}^\sigma}{\varepsilon_{\tilde{p}}}-\frac{\tilde{u}^\sigma}{\tilde{u}\cdot\tilde{p}})e^{-\Gamma(\tilde{u}\cdot\tilde{p}/\varepsilon_{\tilde{p}})\mathrm{t}_0}
\end{aligned}
\tag{31}
$$

❖**Start at initial hypersurface and relax to medium rest frame, it can incorporate some of the physics discussed in Becatinni's talk**

$$\mathcal{J}_5^\mu=\mathcal{J}_5^\mu(u)+(\mathcal{J}_5^\mu(\mathrm{l})-\mathcal{J}_5^\mu(u))e^{-(u\cdot p/\mathrm{l}\cdot p)\Gamma\mathrm{t}_0}$$

Hypersurface Summary

Include both non-dissipative and dissipative part in Zubarev approach

Careful treatment (resum, limit order) to obtain the true leading order

Include the damping effects, give enough time for memory loss

Hypersurface covariant theory

Based on work, Y-Y Li, Liu on arXiv:2611.08482