Tensor spin polarization
of vector mesons within Zubarev formalism

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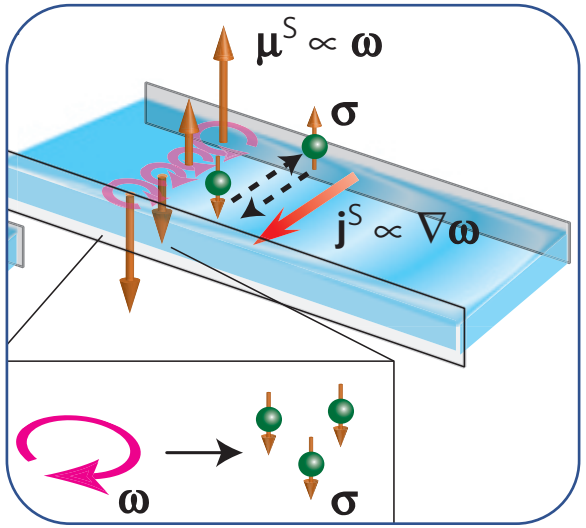
2412.19416 & 2509.20200

With Xu-Guang Huang, Xin-Li Sheng, and Francesco Becattini

18.11.2026 @ Zhuhai, Chirality 2026

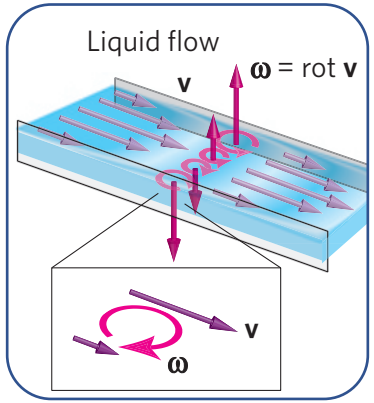
Spin polarization in fluids

Spin hydrodynamic generation



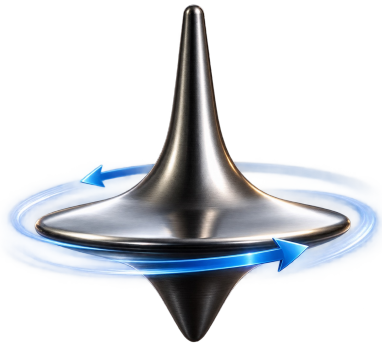
Takahashi etal Nature Physics 12, 52 (2016)

Collective motion of flow

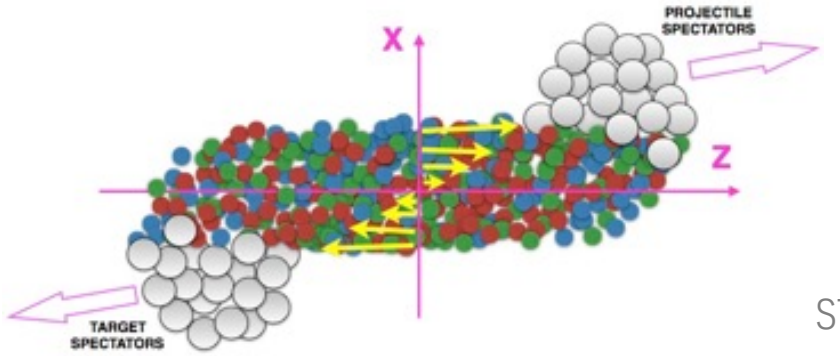


$$H_{\text{spin-rot}} = -\boldsymbol{\omega} \cdot \boldsymbol{S}$$

Spin polarization



Spin — A new probe in HIC to detect the transport of angular momentum



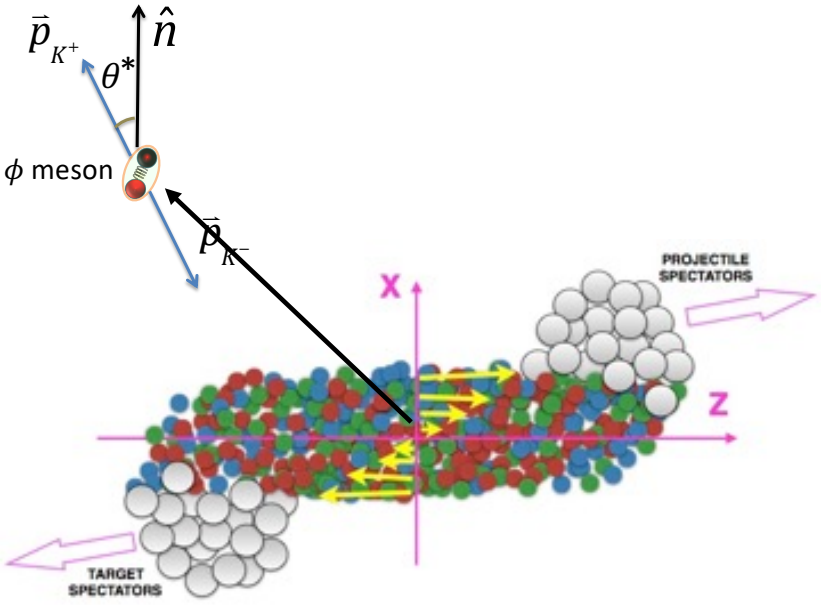
$$\omega \sim 9 \times 10^{21} s^{-1}$$

"Most vortical fluid"

STAR, Nature 548, 62–65 (2017)

Spin alignment puzzle

Non-central heavy-ion collisions



$$\omega \sim 9 \times 10^{21} s^{-1}$$

"Most vortical fluid"

Quantization axis (-y)

$$P = -\langle \sigma^y \rangle \simeq \frac{\omega}{2T} \sim 10^{-2}$$

√

Spin (vector) polarization

Liang, Wang, PRL 94, 102301 (2005)

$$H_{\text{spin-rot}} = -\boldsymbol{\omega} \cdot \boldsymbol{\sigma}/2$$

Spin alignment (tensor polarization)

Liang, Wang, PLB 629, 20–26 (2005)

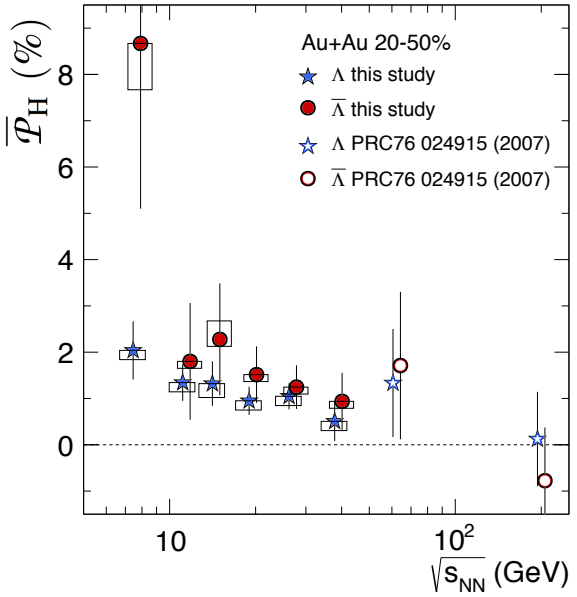
$$\delta\rho_{00} \equiv \rho_{00} - 1/3$$

$$\simeq -\frac{4}{9}\langle \sigma^y \sigma^y \rangle \sim -\left(\frac{\omega}{T}\right)^2 \sim -10^{-4}$$

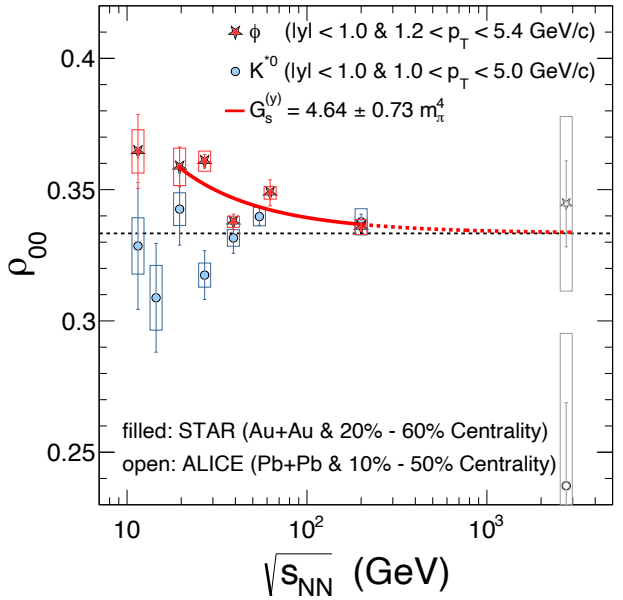
?

Based on quark coalescence

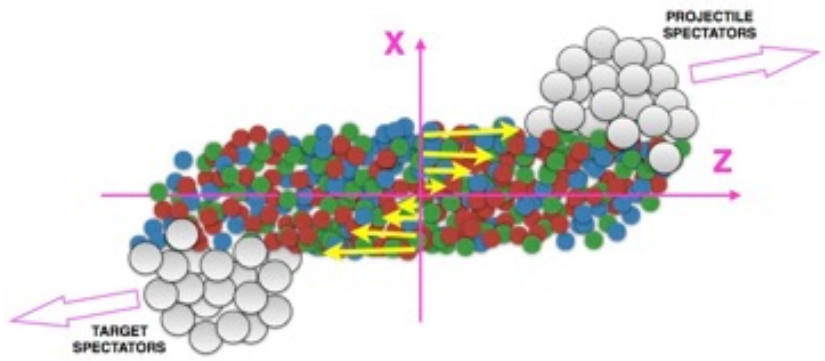
STAR, Nature 548, 62–65 (2017)



STAR, Nature 614, 244-248 (2023)



Local equilibrium density operator



A uniform rotation – a global equilibrium state

$$\widehat{\rho}_{\mathrm{GE}} = \frac{1}{Z_{\mathrm{GE}}} \exp\left\{-\beta \widehat{H} + \beta \boldsymbol{\omega} \cdot \boldsymbol{\sigma}/2\right\}$$

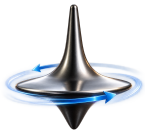
A local Gibbs state for massive spin-1 bosons

Canonical stress tensor Canonical spin tensor

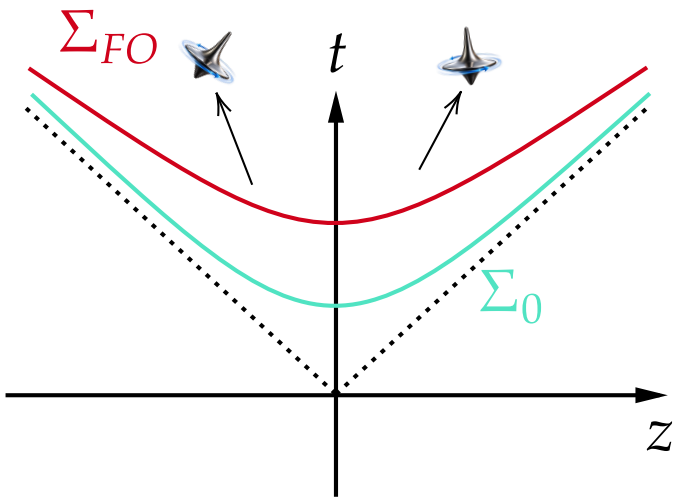
$$\widehat{\rho} = \frac{1}{Z} \exp\left\{-\int_{\Sigma} d\Sigma_{\mu}(y) \left[\widehat{T}^{\mu\nu}(y)\, \beta_{\nu}(y) - \frac{1}{2} \widehat{S}^{\mu\rho\sigma}(y)\, \Omega_{\rho\sigma}(y)\right]\right\}$$

Thermal velocity Spin potential

Zubarev etal 1979, Van Weert 1982, Becattini etal 2013



Particles emitted from $\boldsymbol{\Sigma_{FO}}$ can be treated as free particles



Spin state of vector meson

- Covariant spin operator for single particle

Spin vector $\widehat{\mathcal{S}}^{\mu} = -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\widehat{J}_{\nu\rho}\widehat{P}_{\sigma}$

Spin tensor $\widehat{\mathcal{T}}^{\mu\nu} \equiv \widehat{\mathcal{S}}^{(\mu}\widehat{\mathcal{S}}^{\nu)} + \frac{2}{3}\left(\eta^{\mu\nu} - \frac{1}{m^2}\widehat{P}^{\mu}\widehat{P}^{\nu}\right)$

- Spin state in the rest frame

$\epsilon_i^{\mu}(k)\langle\widehat{\mathcal{S}}^{\mu}\rangle_k$ $\epsilon_{\mu}^{i}(k)\epsilon_{\nu}^{j}(k)\langle\widehat{\mathcal{T}}^{\mu\nu}\rangle_k$

Spin tensor $s^{(i}s^{j)} - \frac{\mathbf{s}^2}{3}\delta^{ij}$

Spin density matrix

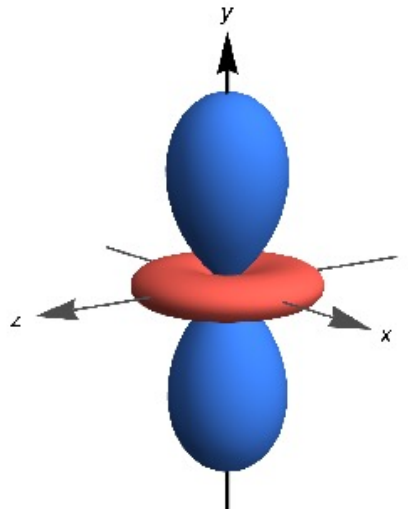
$$\begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1,-1} \\ \rho_{01} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$

$$\rho(k) = \frac{1}{3}I + \frac{1}{2}\sum_i \mathbb{S}^i(k)\, s^i + \sum_{ij} \mathbb{T}^{ij}(k)\, \Sigma^{ij}$$

Adjoint rep. of SO(3)

Symmetric traceless tensor

Quadrupole-type observable



- Spin alignment is yy component of polarization tensor

$$\delta\rho_{00}(k) = \rho_{00}(k) - 1/3 = -\mathbb{T}^{yy}(k)$$

Spin in (pseudo) phase space

- **Wigner function of free Proca field** $W^{\mu\nu}(x,k) = \theta(k^2)\theta(k^0)\int d^4y\, e^{ik\cdot y}\left\langle \widehat{A}^{\nu}\left(x-\frac{y}{2}\right)\widehat{A}^{\mu}\left(x+\frac{y}{2}\right)\right\rangle$

Projected Wigner function $\Delta^{\mu\alpha}_{(k)}\Delta^{\nu\beta}_{(k)}W_{\alpha\beta}(x,k)$

$$W^{\mu\nu}_{\perp}(x,k) = (2\pi) f(x,k)\left\{-\frac{1}{3}\Delta^{\mu\nu}_{(k)} + \frac{i}{2m}\epsilon^{\mu\nu\rho\sigma}k_{\rho}\mathcal{S}_{\sigma}(x,k) - \mathcal{T}^{\mu\nu}(x,k)\right\}$$

Scalar distribution — Polarization vector — Polarization tensor

- **Spin polarization in phase space**

$$\mathcal{S}^{\mu}(x,k) = -\frac{i\epsilon^{\mu\nu\alpha\beta}\,k_{\nu}\,W_{\alpha\beta}(x,k)}{(2\pi)\,m\,f(x,k)} \qquad \mathcal{T}^{\mu\nu}(x,k) = -\frac{W^{\langle\mu\nu\rangle}_{\perp}(x,k)}{(2\pi) f(x,k)}$$

Symmetric traceless part — quadrupole

- **Average over the hypersurface**

$$\mathcal{S}^{\mu}(k) \equiv \left\langle \widehat{\mathcal{S}}^{\mu}\right\rangle_{k} = \frac{\int d\Sigma\cdot k\, f(x,k)\,\mathcal{S}^{\mu}(x,k)}{\int d\Sigma\cdot k\, f(x,k)} \qquad \mathcal{T}^{\mu\nu}(k) \equiv \left\langle \widehat{\mathcal{T}}^{\mu\nu}\right\rangle_{k} = \frac{\int d\Sigma\cdot k\, f(x,k)\,\mathcal{T}^{\mu\nu}(x,k)}{\int d\Sigma\cdot k\, f(x,k)}$$

Evaluating Wigner function — Gradient expansion

- **Decompose density operator by gradients**

$$\widehat{\rho}_{\rm LE} = \frac{e^{\widehat{A}+\widehat{B}}}{{\rm Tr}\, e^{\widehat{A}+\widehat{B}}} \qquad \widehat{A}(x) = -\widehat{P}^\mu \beta_\mu(x) \qquad \widehat{B}(x) = -\int_{\Sigma_{\rm FO}} d\Sigma_\mu(y) \left[\widehat{T}^{\mu\nu}(y)\, \Delta\beta_\nu(y,x) - \frac{1}{2}\widehat{S}^{\mu\rho\sigma}(y)\, \Omega_{\rho\sigma}(y)\right]$$

$$\Delta\beta_\mu(y,x) = \beta_\mu(y) - \beta_\mu(x) = (y-x)^\alpha \partial_\alpha \beta_\mu(x) + \cdots \qquad \Omega_{\rho\sigma}(y) = \Omega_{\rho\sigma}(x) + (y-x)^\alpha \partial_\alpha \Omega_{\rho\sigma}(x) + \cdots$$

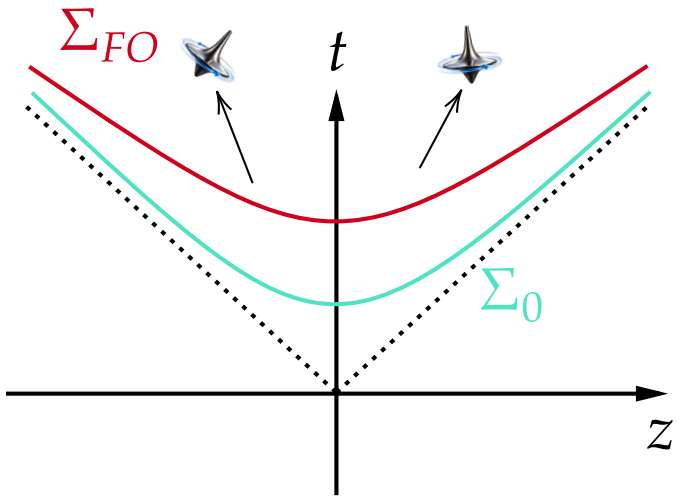
- **Dyson/Kubo series** $\quad e^{\widehat{A}+\widehat{B}} = e^{\widehat{A}}\, \mathcal{P}_\lambda \exp\left[\int_0^1 d\lambda\, \widehat{B}_{(\lambda)}\right] = e^{\widehat{A}}\left(1 + \int_0^1 d\lambda\, \widehat{B}_{(\lambda)} + \cdots\right)$ with $\widehat{B}_{(\lambda)} \equiv e^{-\lambda\widehat{A}}\widehat{B}e^{\lambda\widehat{A}}$

$$W^{\mu\nu}(x,k) = \frac{\left\langle \mathcal{P}_\lambda e^{\int_0^1 d\lambda\, \widehat{B}_{(\lambda)}} \widehat{W}^{\mu\nu}(x,k)\right\rangle_{\beta(x)}}{\left\langle \mathcal{P}_\lambda e^{\int_0^1 d\lambda\, \widehat{B}_{(\lambda)}}\right\rangle_{\beta(x)}} = \left\langle \mathcal{P}_\lambda e^{\int_0^1 d\lambda\, \widehat{B}_{(\lambda)}} \widehat{W}^{\mu\nu}(x,k)\right\rangle_{\beta(x),c}$$

Gaussian distribution + Wick's theorem

Connected correlations

Evaluate Wigner function upto any orders of gradient



- **Results**

Tensor polarization

$\propto \mathcal{O}(\partial^2)$ $\quad \partial^2\beta\,,\ \partial\Omega\,,\ \cdots$ 2412.19416

$\propto \mathcal{O}(\partial) \times$ curvaure 2509.20200

Time-reversal + Parity

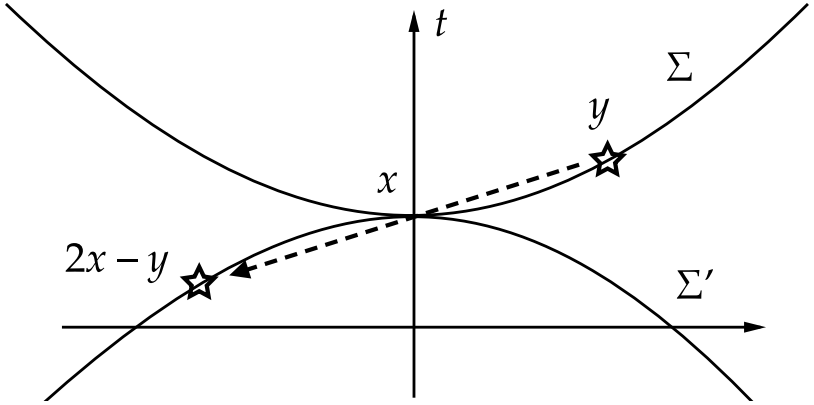
- **Results**

Tensor polarization

(PT even)

$\propto \mathcal{O}(\partial^2)$ $\quad \partial^2\beta\,,\ \partial\Omega\,,\ \cdots$ 2412.19416

$\propto \mathcal{O}(\partial) \times \text{curvaure}$ 2509.20200



- **Space-time reversal w.r.t. x**

PT

$$(PT)\widehat{W}_{\mu\nu}(x,k)(PT)^{-1} = \widehat{W}_{\mu\nu}(x,k)$$

$$W^*_{\mu\nu}(x,k) = \mathrm{Tr}\left[(PT)\hat{\rho}(PT)^{-1}(PT)\widehat{W}_{\mu\nu}(x,k)(PT)^{-1}\right]$$

$$\hat{\rho} = \frac{1}{Z}\exp\left\{-\int_\Sigma d\Sigma_\mu(y)\left[\hat{T}^{\mu\nu}(y)\beta_\nu(y) - \frac{1}{2}\hat{S}^{\mu\rho\sigma}(y)\Omega_{\rho\sigma}(y)\right]\right\}$$

$$(PT)\hat{T}^{\mu\nu}(y)(PT)^{-1} = \hat{T}^{\mu\nu}(y') \qquad (PT)\hat{S}^{\mu\rho\sigma}(y)(PT)^{-1} = -\hat{S}^{\mu\rho\sigma}(y')$$

$$(PT)\hat{\rho}(PT)^{-1} = \frac{1}{Z}\exp\left\{-\int_{\Sigma'} d\Sigma_\mu(y')\left[\hat{T}^{\mu\nu}(y')\beta'_\nu(y') - \frac{1}{2}\hat{S}^{\mu\rho\sigma}(y')\Omega'_{\rho\sigma}(y')\right]\right\}$$

$$y' = 2x - y, \qquad \beta'_\nu(y') = \beta_\nu(y), \qquad \Omega'_{\rho\sigma}(y') = -\Omega_{\rho\sigma}(y)$$

Hermitian $\quad W^*_{\mu\nu}(x,k) = W_{\nu\mu}(x,k)$

$$W_{\mu\nu}(x,k)\big|_{\Sigma,\beta,\Omega} = W_{\nu\mu}(x,k)\big|_{\Sigma',\beta',\Omega'}$$

Hyperplane $\quad W^{(n)}_{\mu\nu}(x,k) = (-1)^n W^{(n)}_{\nu\mu}(x,k)$

$$\beta_\nu \sim \mathcal{O}(1) \qquad \Omega_{\rho\sigma} \sim \mathcal{O}(\partial)$$

Gradient expansion with hyperplane

- **Decompose density operator by gradients**

$$W^{\mu\nu}(x,k)=\frac{\left\langle \mathcal{P}_\lambda e^{\int_0^1 d\lambda\,\widehat{B}_{(\lambda)}}\widehat{W}^{\mu\nu}(x,k)\right\rangle_{\beta(x)}}{\left\langle \mathcal{P}_\lambda e^{\int_0^1 d\lambda\,\widehat{B}_{(\lambda)}}\right\rangle_{\beta(x)}}=\left\langle \mathcal{P}_\lambda e^{\int_0^1 d\lambda\,\widehat{B}_{(\lambda)}}\widehat{W}^{\mu\nu}(x,k)\right\rangle_{\beta(x),c}$$

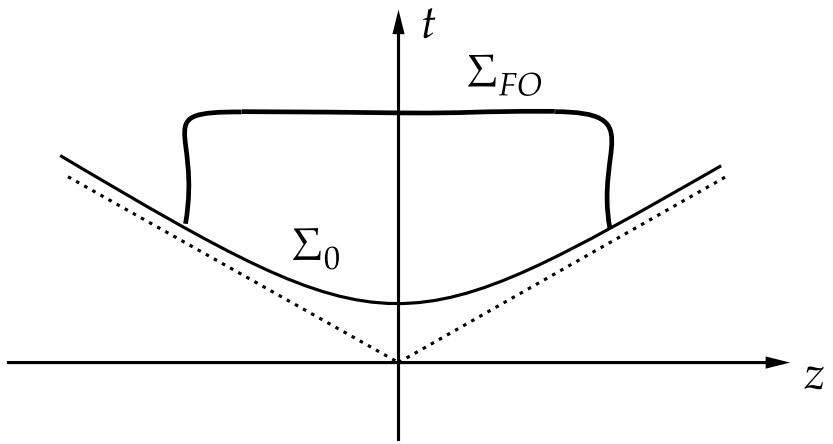
Connected correlations

- **Second-order correlations**

$$W^{(2)}_{\mu\nu}(x,k)=\left\langle \hat{\mathcal{B}}_2\hat{W}_{\mu\nu}(x,k)\right\rangle_{0,c}$$

2412.19416

$$\begin{aligned}\hat{\mathcal{B}}_2=&\int_0^1 d\lambda_1\int_0^{\lambda_1}d\lambda_2\int_{\Sigma_{\rm FO}}d\Sigma_{\mu_1}(y_1)d\Sigma_{\mu_2}(y_2)\\&\times\Big[\partial_{\alpha_1}\beta_{\nu_1}(x)\partial_{\alpha_2}\beta_{\nu_2}(x)(y_1-x)^{\alpha_1}(y_2-x)^{\alpha_2}\hat{T}^{\mu_1\nu_1}(y_1^{(\beta)})\hat{T}^{\mu_2\nu_2}(y_2^{(\beta)})\\&-\frac{1}{2}\partial_{\alpha_1}\beta_{\nu_1}(x)\Omega_{\rho_2\sigma_2}(x)(y_1-x)^{\alpha_1}\hat{T}^{\mu_1\nu_1}(y_1^{(\beta)})\hat{S}^{\mu_2,\rho_2\sigma_2}(y_2^{(\beta)})\\&-\frac{1}{2}\Omega_{\rho_1\sigma_1}(x)\partial_{\alpha_2}\beta_{\nu_2}(x)(y_2-x)^{\alpha_2}\hat{S}^{\mu_1,\rho_1\sigma_1}(y_1^{(\beta)})\hat{T}^{\mu_2\nu_2}(y_2^{(\beta)})\\&+\frac{1}{4}\Omega_{\rho_1\sigma_1}(x)\Omega_{\rho_2\sigma_2}(x)\hat{S}^{\mu_1,\rho_1\sigma_1}(y_1^{(\beta)})\hat{S}^{\mu_2,\rho_2\sigma_2}(y_2^{(\beta)})\Big]\\&+\int_0^1 d\lambda_1\int_{\Sigma_{\rm FO}}d\Sigma_{\mu_1}(y_1)\Big[-\frac{1}{2}\partial_{\alpha_1}\partial_{\alpha_2}\beta_{\nu_1}(x)(y_1-x)^{\alpha_1}(y_1-x)^{\alpha_2}\hat{T}^{\mu_1\nu_1}(y_1^{(\beta)})\\&+\frac{1}{2}\partial_{\alpha_1}\Omega_{\rho_1\sigma_1}(x)(y_1-x)^{\alpha_1}\hat{S}^{\mu_1,\rho_1\sigma_1}(y_1^{(\beta)})\Big].\end{aligned}$$



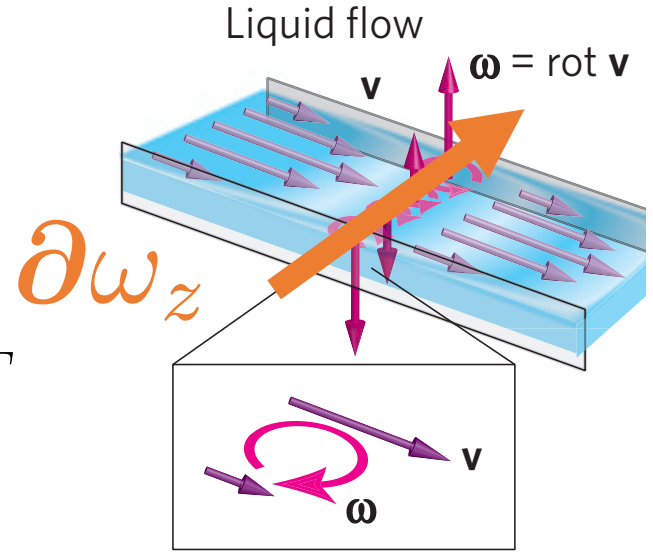
Tensor polarization by gradient of vorticity

$$\delta\rho_{00}(x,k) \simeq \left[\epsilon_\mu^y(k)\epsilon_\nu^y(k) + \frac{1}{3}\Delta_{\mu\nu}^{(k)}\right] \frac{n^{[\rho}\partial^{\mu]}\varpi_{\rho\sigma}}{6E_k}\left(2\eta^{\sigma\nu} - \hat{k}^\sigma n^\nu\right)$$

Vorticity ⟶ Spin polarization

Grad of vorticity ⟶ Spin alignment

Thermal vorticity $\partial_{[\sigma}\beta_{\rho]} \sim \omega/T$



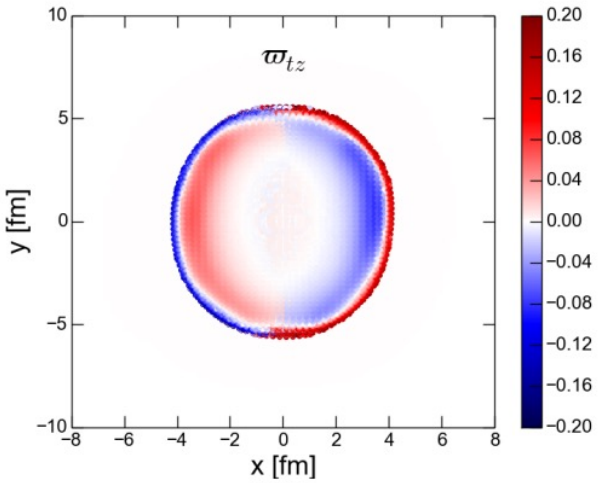
- A special case

In the rest frame of normal vector n At the low momentum limit

$$\delta\rho_{00}(x,k) \simeq \frac{1}{6m}\left(\partial_y\tilde{a}_y - \frac{1}{3}\nabla\cdot\tilde{\boldsymbol{a}}\right)$$

$$= \tilde{a}_z$$

$$(\partial\tilde{a})/m \sim 0.03$$

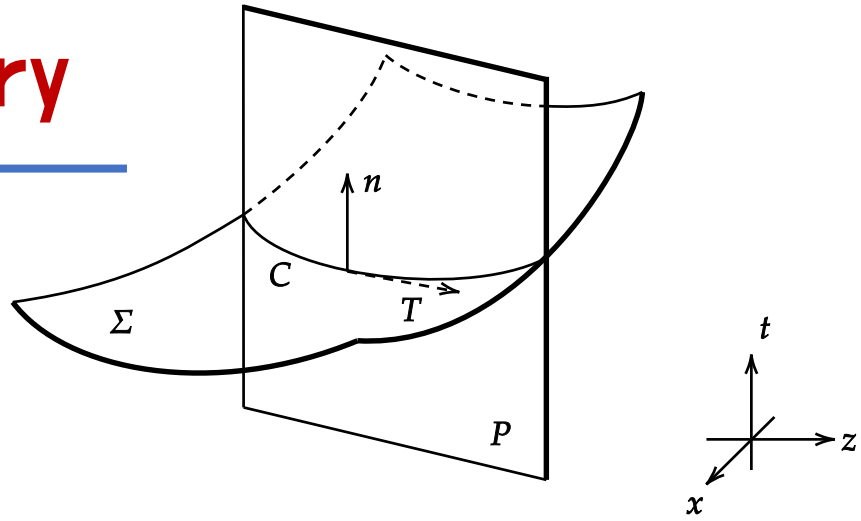


Spin alignment detects the anisotropy of grad acceleration

2412.19416

Karpenko etal. Eur. Phys. J. C (2017) 77: 213

Tensor polarization by freeze-out geometry



Curvature in the tangent space $B^{\mu\nu}(x) \equiv -\Delta^{\mu\alpha}_{(n)} \frac{\partial n^\nu(x)}{\partial x^\alpha}$

$$\mathcal{T}^{\mu\nu}(x,k) \simeq \frac{1+n_B}{6(n\cdot k)} B_{\alpha\beta}(x) \Bigg\{ -\xi_{\rho\sigma}\left(\eta^{\rho\sigma} + \frac{k^\rho k^\sigma}{2m^2}\right)\left[\Xi^{\alpha\mu}\Xi^{\beta\nu} - \frac{1}{3}\Delta^{\mu\nu}_{(k)}\left(\eta^{\alpha\beta} + \hat{k}^\alpha \hat{k}^\beta\right)\right]$$
$$+ \xi_{\rho\sigma}\left[\Xi^{\alpha\rho}\Xi^{\beta(\mu}\Delta^{\nu)\sigma}_{(k)} - \frac{1}{3}\Delta^{\mu\nu}_{(k)}\Xi^{\alpha\rho}\Xi^{\beta\sigma}\right] - \delta\Omega_{\rho\sigma}\Xi^{\alpha\rho}\Xi^{\beta(\mu}\Delta^{\nu)\sigma}_{(k)} \Bigg\}$$

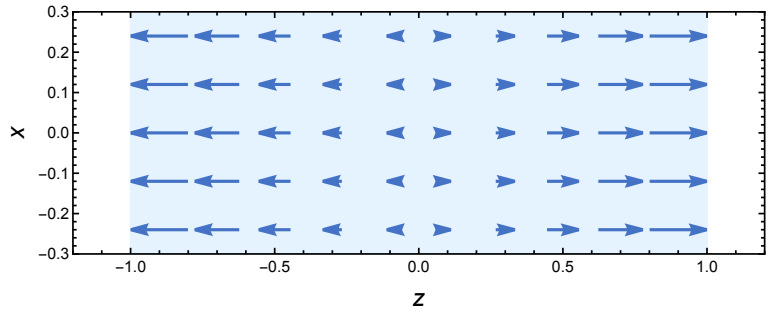
Thermal shear $\partial_{(\rho}\beta_{\sigma)}$

- **Bjorken flow** Static particle inside fluid cell

Bjorken PRD 27, 140 (1983)

$$\mathcal{T}^{\mu\nu}(x,k) \simeq -\frac{c_s^2 B^{\langle\mu\nu\rangle}}{4mT_f\tau_f}$$

Anisotropic freeze-out surface ➡ Tensor polarization

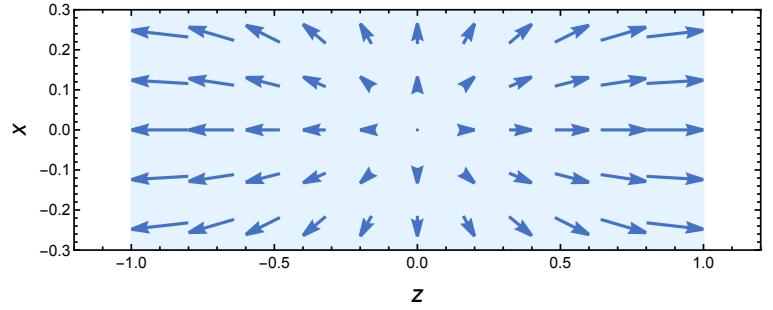


- **Gubser flow** – Bjorken longitudinal expansion + conformal **transversal** expansion

Gubser PRD 82, 085027 (2010)

$$\delta\rho_{00} = -\frac{7q}{36m\tilde{T}_0} \sim L^{-2} \qquad q \sim L^{-1} \qquad T \sim \tilde{T}_0 q \sim L^0$$

Au-Au $\sim -3\times10^{-3}$ **O-O** $\sim -10^{-2}$



2509.20200

Tensor polarization by memory effect

- **Real density operator** $\widehat{\rho}_0 = \frac{1}{Z_0}\exp\left\{-\int_{\Sigma_0} d\Sigma_\mu(y)\left[\widehat{T}^{\mu\nu}(y)\,\beta_\nu(y) - \frac{1}{2}\widehat{S}^{\mu\rho\sigma}(y)\,\Omega_{\rho\sigma}(y)\right]\right\}$

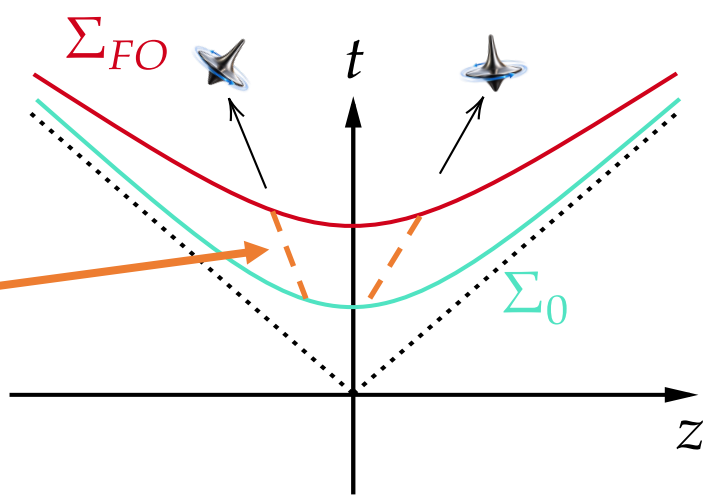
Tensor polarization by **dissipative** effect: $\mathcal{T}^{\mu\nu} \propto \Delta^{\langle\mu}_{(k)\alpha}\Delta^{\nu\rangle}_{(k)\beta}\xi^{\alpha\beta}$ Feng Li, Shuai Y. F. Liu, 2206.11890

- **Memory effect** Scalar particles: talk by D. Roselli

$$\mathcal{T}^{\mu\nu}(x,k) \propto \kappa \times \Delta^{\langle\mu}_{(k)\xi}\Delta^{\nu\rangle}_{(k)\zeta}\beta^\xi\beta^\zeta(x)$$

Spin alignment in k direction

$$\kappa = \int d\theta\,\Gamma(\theta,k,\beta)\left(\cos\theta\hat{k} + \sin\theta\hat{\beta}\right)\cdot\Delta\beta$$



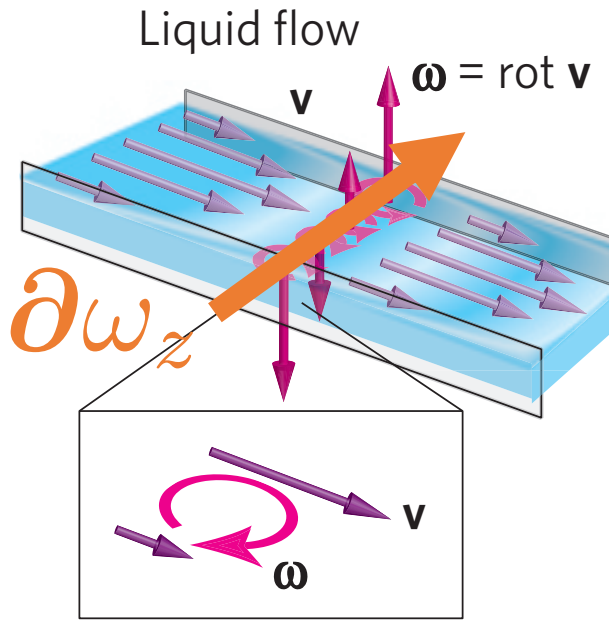
ZZ, Becattini, in prep

Conclusion

Vorticity → Spin polarization

Grad of vorticity → Spin alignment

$$\delta\rho_{00}(x,k) \simeq \frac{1}{6m}\left(\partial_y \tilde{a}_y - \frac{1}{3}\nabla\cdot\tilde{\boldsymbol{a}}\right)$$



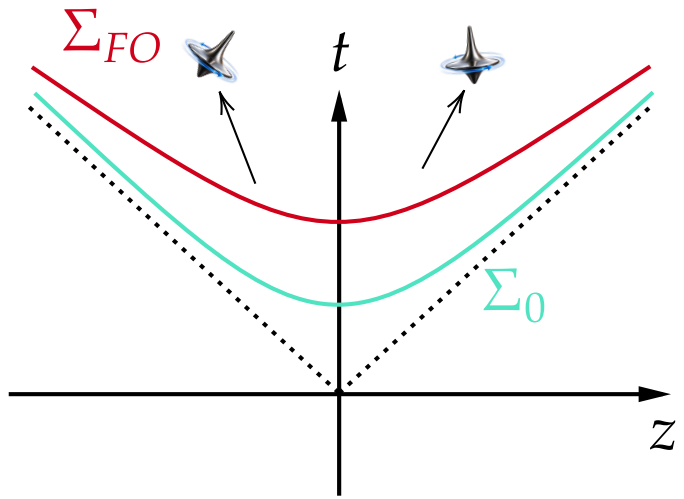
Anisotropic freeze-out surface

↓

Spin alignment

$$\delta\rho_{00} \simeq -\frac{7q}{36m\widetilde{T}_0} \sim L^{-2}$$

May provide a clean probe of this geometric effect



Thank you!

18.11.2026 @ Zhuhai