A microscopic model for global polarization of Λ hyperons

Speaker: Yi-Liang Yin

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Yi-Liang Yin, Wen-Bo Dong, Dirk Rischke, Qun Wang, et al.,

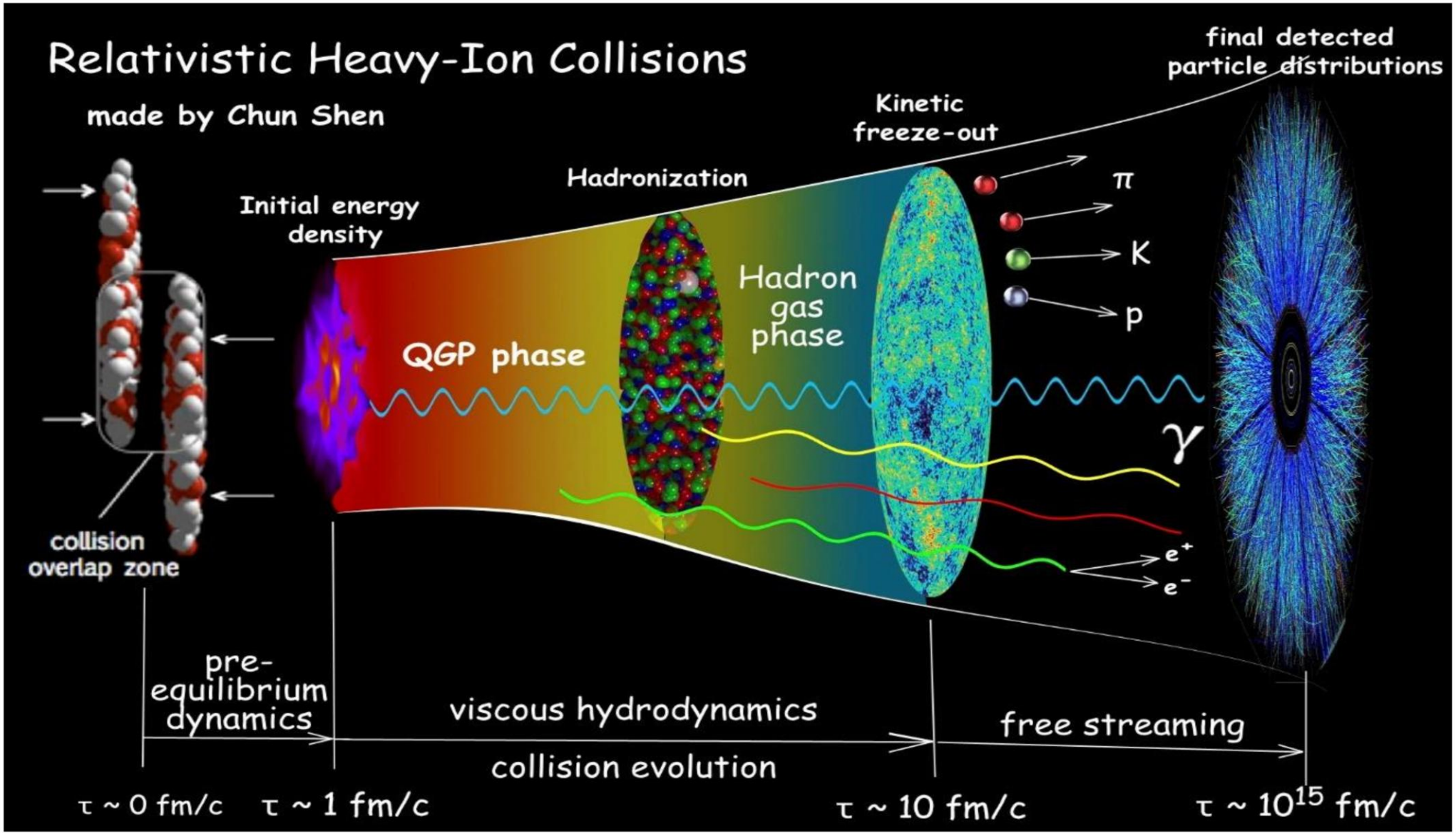
To appear

Chirality 2026

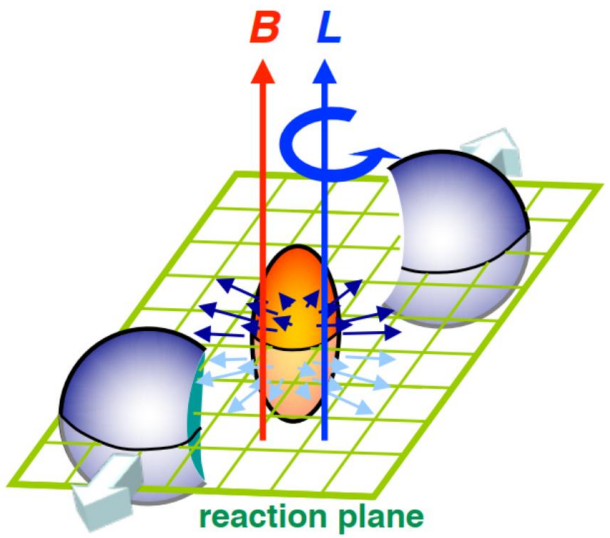
Outline

- Background
- Nonlocal nucleon-nucleon collisions
- Numerical results
- Summary and Outlook

Relativistic heavy ion collision

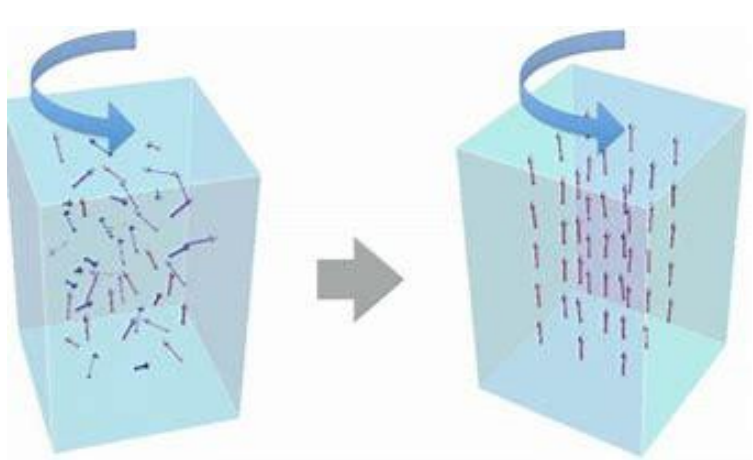


Spin polarization



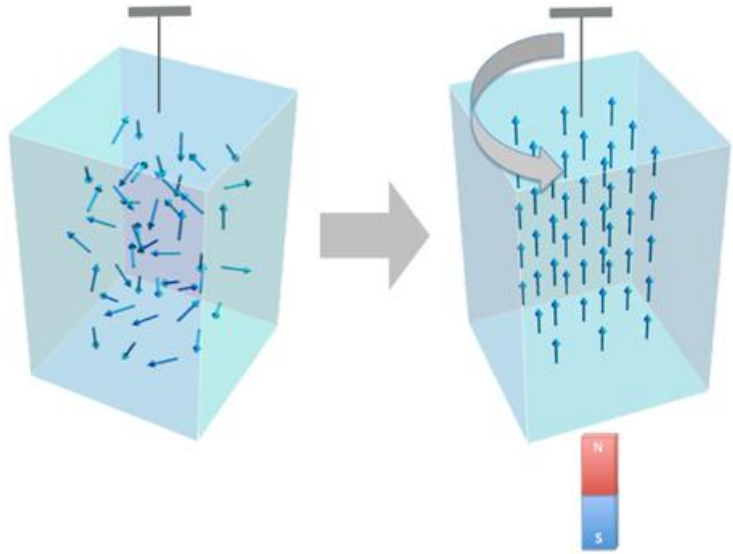
$L \sim 10^5 \hbar$

$B \sim 10^{18}$G



Bernett effect

S. J. Barnett, Rev. Mod. Phys. 7, 129 (1935)



Einstein-de Haas effect

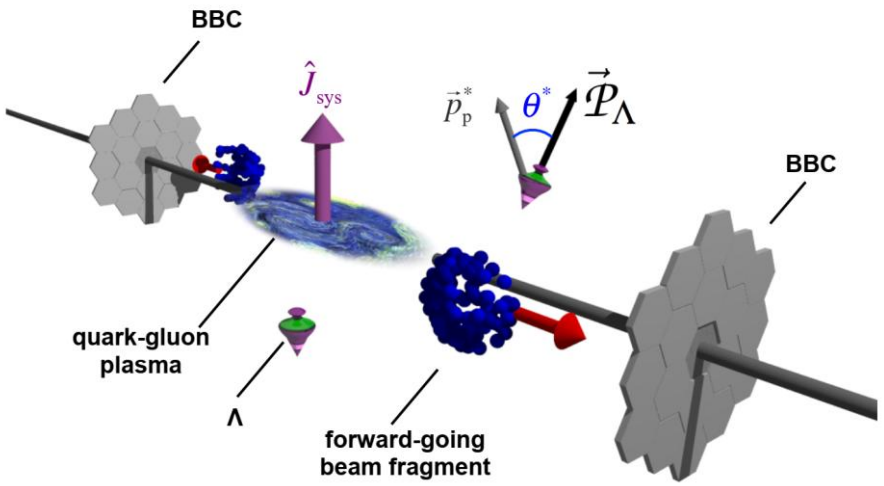
A. Einstein and W. de Haas, Deuts. Physik. Gesells. Verhandlun. 17, 152 (1915)

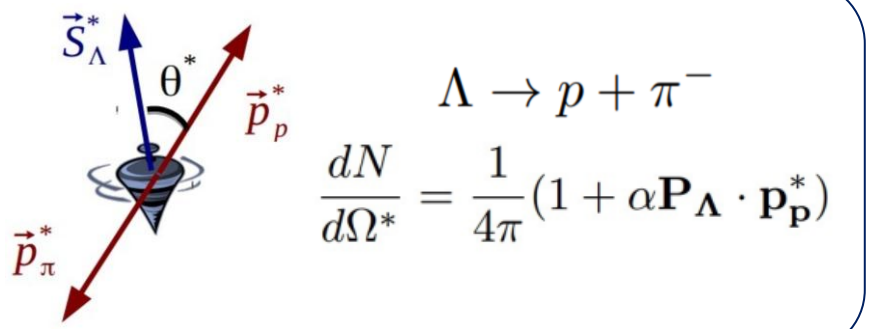
Works on spin polarization

- *Z.-T. Liang et al. Phys. Rev. Lett. 94, 102301 (2005)*
- *Z.-T. Liang et al. Phys. Lett. B 629, 20 (2005)*
- *J.-H. Gao et al. Phys. Rev. C 77, 044902 (2008)*
- *F. Becattini et al. Phys. Rev. C 77, 024906 (2008)*
- *I. Karpenko et al. Eur. Phys. J. C 77, 213 (2017)*
- *H. Li et al. Phys. Rev. C 96, 054908 (2017)*
- *Y. Xie et al. Phys. Rev. C 95, 031901(R) (2017)*
- *S. Shi et al. Phys. Lett. B 788, 409 (2019)*

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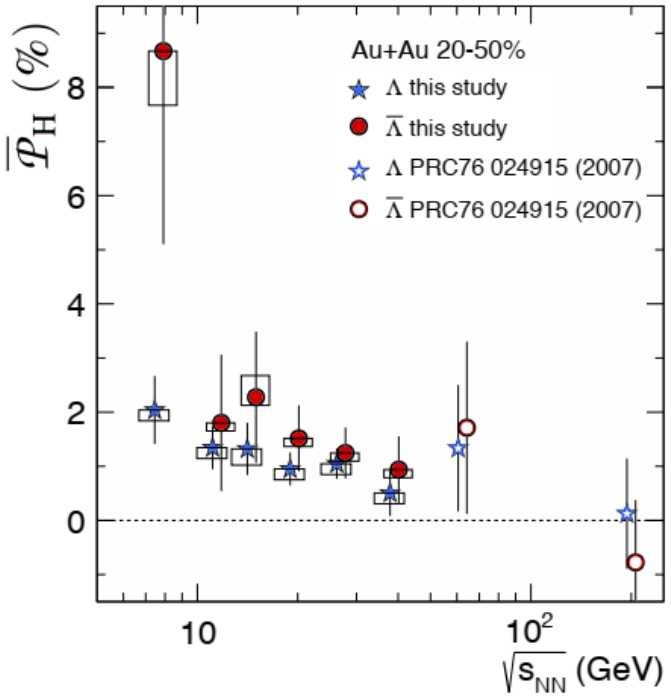
Spin polarization





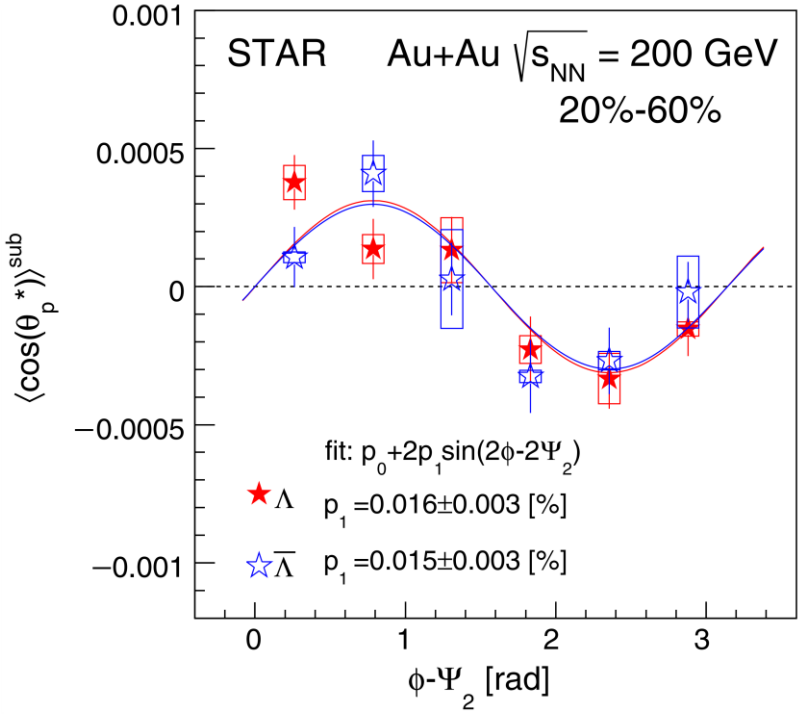
$$\Lambda \to p + \pi^-$$

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}_\Lambda \cdot \mathbf{p}^*_\mathbf{p})$$



Global spin polarization of $\Lambda/\overline{\Lambda}$

STAR, Nature 548 (2017) 62-65



Local spin polarization of $\Lambda/\overline{\Lambda}$

STAR, Phys. Rev. Lett. 123 (2019) 13, 132301

Spin polarization

1. Quantum statistical theory

F. Becattini et al., Phys. Rev. C 77, 024906 (2008)
F. Becattini et al., Annals Phys. 338, 32 (2013)
F. Becattini et al., Phys. Lett. B 789, 419 (2019)
…

$$S^{\mu}(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_{\tau}\frac{\int_{\Sigma_{FO}} \mathrm{d}\Sigma_{\lambda}p^{\lambda}n_F(1-n_F)\varpi_{\rho\sigma}}{\int_{\Sigma_{FO}} \mathrm{d}\Sigma_{\lambda}p^{\lambda}n_F}$$

2. Transport theory

J.-H. Gao et al., Phys. Rev. Lett. 109, 232301 (2012)
J.-W. Chen et al., Phys. Rev. Lett. 110, 262301 (2013)
J.-H. Gao et al., Phys. Rev. D 98, 036019 (2018)
…

$$(\gamma\cdot K - m)\,G^{<}(x,p) = I_{\mathrm{coll}}$$

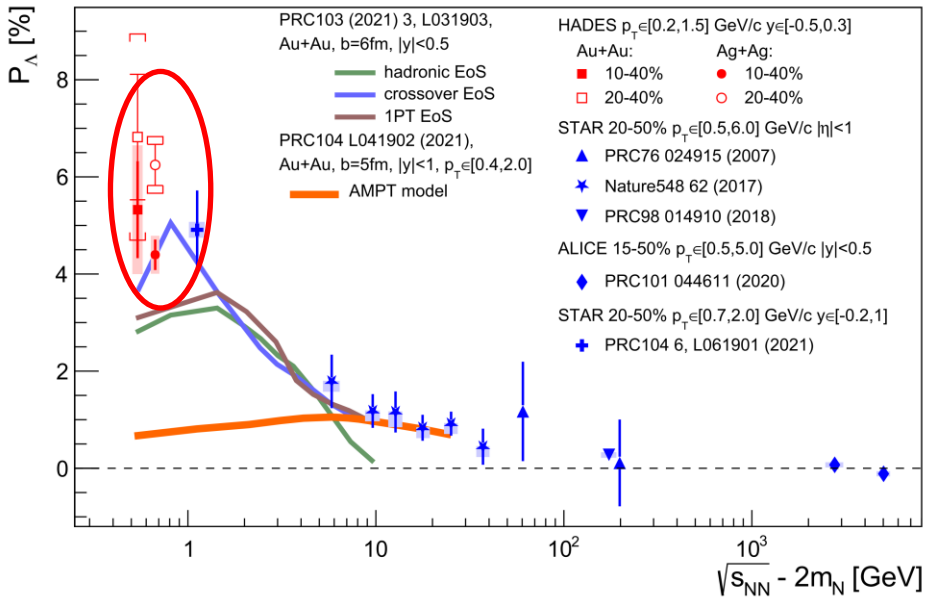
3. Relativistic spin hydrodynamics

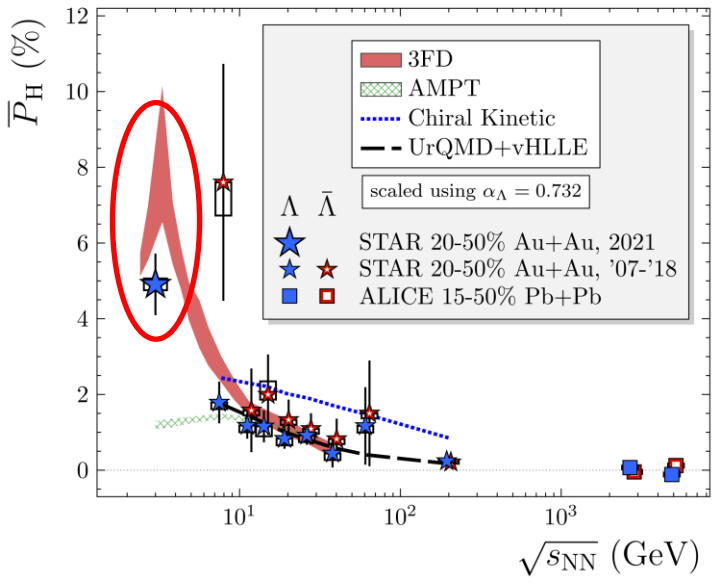
D. Montenegro et al., Phys. Rev. D 96, 056012 (2017)
D.-L. Wang et al., Phys. Rev. D 104, 114043 (2021)
C. Yi et al., Phys. Rev. C 105, 044911 (2022)
…

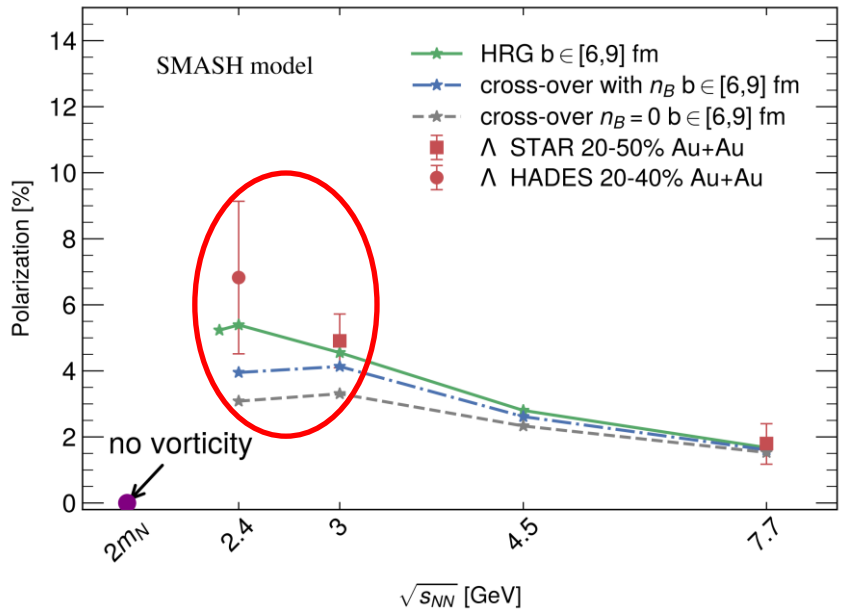
$$\partial_{\mu}T^{\mu\nu} = 0$$
$$\partial_{\mu}N^{\mu} = 0$$
$$\hbar\partial_{\lambda}S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

Collective models

Spin polarization







HADES, Phys.Lett.B 835 (2022) 137506

STAR, Phys.Rev.C 104 (2021) 6, L061901

SMASH model
Cong Yi et.al, 2603.27521

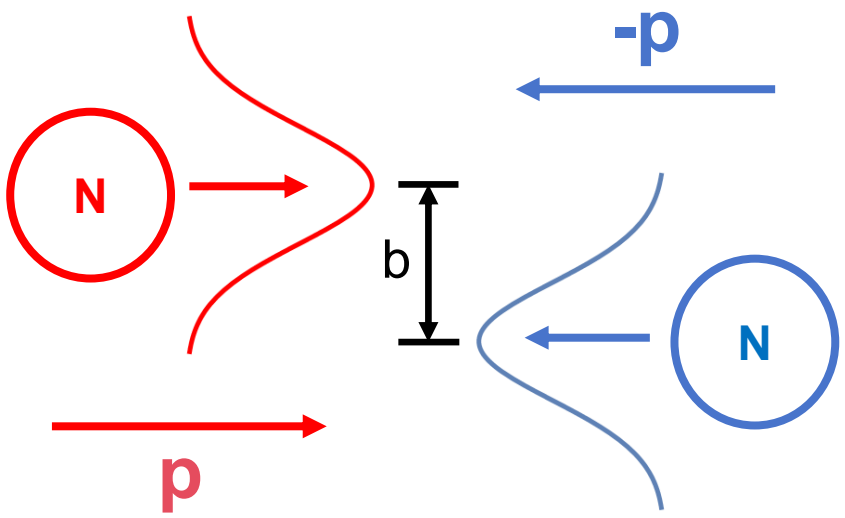
We develop a microscopic model to describe the spin polarization of Λ hyperons.

Outline

- Background
- Nonlocal nucleon-nucleon collisions
  - Wave packet description for $NN \to N\Lambda K$ scattering
  - Lagrangian and polarized cross section
- Numerical results
- Summary and Outlook

Nonlocal scattering

Wave packet description



Nonlocal nucleon-nucleon collisions

The nucleon is described as **Gaussian wave packet**

$$\phi\left(\mathbf{k}_i-\mathbf{p}_i\right) = \frac{(8\pi)^{3/4}}{\alpha^{3/2}}\exp\left[-\frac{\left(\mathbf{k}_i-\mathbf{p}_i\right)^2}{\alpha^2}\right]$$

The **orbital angular momentum** can be converted to **spin polarization**

$$\boldsymbol{L\sim b\times p\rightarrow S}$$

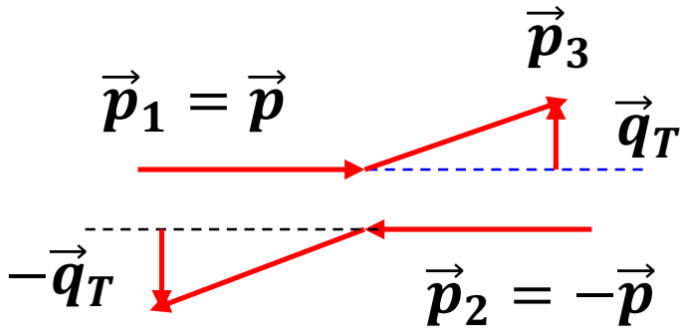
$$|\phi_1\phi_2\rangle_{\text{in}} = \int \frac{d^3k_1}{(2\pi)^3\sqrt{2E_{k_1}}}\frac{d^3k_2}{(2\pi)^3\sqrt{2E_{k_2}}}\phi\left(\mathbf{k}_1-\mathbf{p}_1\right)\phi\left(\mathbf{k}_2-\mathbf{p}_2\right)e^{-i\mathbf{b}\cdot\mathbf{k}_2}|\mathbf{k}_1,s_1;\mathbf{k}_2,s_2\rangle_{\text{in}}$$

Impact parameter

Nonlocal scattering

Wave packet description

$2 \to 2$ scattering



$$d\sigma_{\lambda_3} = \frac{c_{qq}}{F}\frac{1}{4}\sum_{\lambda_1,\lambda_2,\lambda_4}\mathcal{M}(Q)\mathcal{M}^*(Q)(2\pi)^4\delta(P_1+P_2-P_3-P_4)\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}$$

momentum transfer

(Not well-defined in $2 \to 3$ scattering)

$$\frac{d^2\sigma_{\lambda_3}}{d^2\vec{x}_T} = \frac{c_{qq}}{16F}\sum_{\lambda_1,\lambda_2,\lambda_4}\int\frac{d^2\vec{q}_T}{(2\pi)^2}\frac{d^2\vec{k}_T}{(2\pi)^2}e^{i(\vec{k}_T-\vec{q}_T)\cdot\vec{x}_T}\frac{\mathcal{M}(\vec{q}_T)}{\Lambda(\vec{q}_T)}\frac{\mathcal{M}^*(\vec{k}_T)}{\Lambda^*(\vec{k}_T)}$$

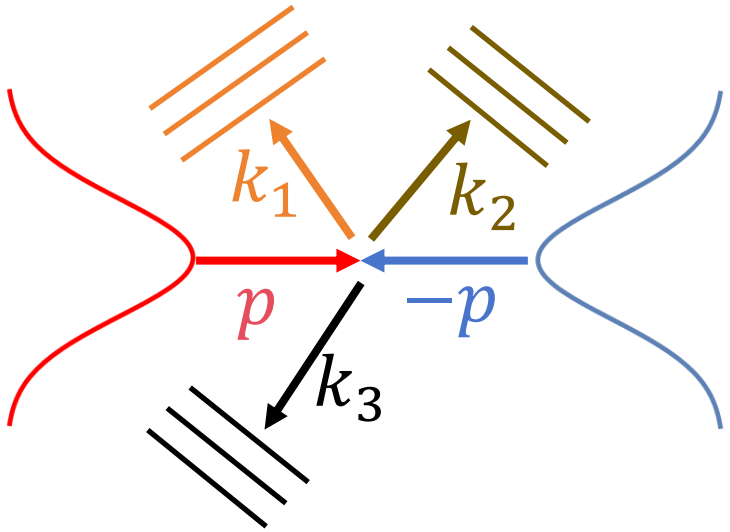
$$\Lambda(\vec{q}_T) = \sqrt{(E_1+E_2)|p_{3z}^+|}.$$

Jian-Hua Gao et al., Phys.Rev.C 77 (2008) 044902

Nonlocal scattering

Wave packet description

$2 \to 3$ scattering



Final state (plane wave) — 13D — Initial state (wave packet)

$$\frac{d\sigma}{d\mathbf{b}}\left(\{r_f\}\right) = \left(\prod_{f=1}^{n}\int\frac{d^3p_f}{(2\pi)^3\,2E_f}\right)\int\frac{d^3k_1}{(2\pi)^3\sqrt{2E_{k_1}}}\frac{d^3k_2}{(2\pi)^3\sqrt{2E_{k_2}}}\frac{d^3k_1'}{(2\pi)^3\sqrt{2E_{k_1'}}}\frac{d^3k_2'}{(2\pi)^3\sqrt{2E_{k_2'}}}$$
$$\times\phi\left(\mathbf{k}_1-\mathbf{p}_1\right)\phi\left(\mathbf{k}_2-\mathbf{p}_2\right)\phi^*\left(\mathbf{k}_1'-\mathbf{p}_1\right)\phi^*\left(\mathbf{k}_2'-\mathbf{p}_2\right)e^{i\mathbf{b}\cdot\left(\mathbf{k}_2'-\mathbf{k}_2\right)}$$
$$\times(2\pi)^4\,\delta^{(4)}\left(\sum p_f-k_1-k_2\right)\times(2\pi)^4\,\delta^{(4)}\left(\sum p_f-k_1'-k_2'\right)$$
$$\times\sum_{s_1,s_2}M\left(k_1,s_1;k_2,s_2\to\{p_f,r_f\}\right)M^*\left(k_1',s_1;k_2',s_2\to\{p_f,r_f\}\right)$$

Jun-jie. Zhang et al., Phys. Rev. C 100, 064904 (2019)

The **wave packet description** can describe any $n \to m$ scattering, but **the integral dimension is very high**.

Nonlocal scattering

Wave packet description

We assume $\boldsymbol{\alpha \ll m}$, and only keep the **leading order** of the packet width α.

$$\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\text{pol}} = \frac{\alpha^2}{4\pi} \frac{e^{-\frac{1}{4}\alpha^2\mathbf{b}^2}}{(2E_p)^2 |\mathbf{v}_1 - \mathbf{v}_2|} \int \frac{d^3p_N}{(2\pi)^3 2E_N} \frac{d^3p_K}{(2\pi)^3 2E_K} \frac{d^3p_\Lambda}{(2\pi)^3 2E_\Lambda}$$
$$\times (2\pi)^4 \delta^{(4)}(p_N + p_K + p_\Lambda - k_1 - k_2)$$
$$\times \sum_{s_1,s_2,r_N,r_\Lambda} r_\Lambda M(k_1, s_1; k_2, s_2 \to p_N, r_N; p_\Lambda, r_\Lambda; p_K)$$
$$\times M^*(k_1', s_1; k_2', s_2 \to p_N, r_N; p_\Lambda, r_\Lambda; p_K),$$

$$\mathbf{k}_1 = \mathbf{p}_1 + \tfrac{1}{2}i\alpha^2\mathbf{b},\ \mathbf{k}_1' = \mathbf{p}_1 - \tfrac{1}{2}i\alpha^2\mathbf{b},$$
$$\mathbf{k}_2 = \mathbf{p}_2 - \tfrac{1}{2}i\alpha^2\mathbf{b},\ \mathbf{k}_2' = \mathbf{p}_2 + \tfrac{1}{2}i\alpha^2\mathbf{b}$$

5D

$$\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\text{tot}} = \frac{\alpha^2}{4\pi} \frac{e^{-\frac{1}{4}\alpha^2\mathbf{b}^2}}{(2E_p)^2 |\mathbf{v}_1 - \mathbf{v}_2|} \int \frac{d^3p_N}{(2\pi)^3 2E_N} \frac{d^3p_K}{(2\pi)^3 2E_K} \frac{d^3p_\Lambda}{(2\pi)^3 2E_\Lambda}$$
$$\times (2\pi)^4 \delta^{(4)}(p_N + p_K + p_\Lambda - k_1 - k_2)$$
$$\times \sum_{s_1,s_2,r_N,r_\Lambda} |M(p_1, s_1; p_2, s_2 \to p_N, r_N; p_\Lambda, r_\Lambda; p_K)|^2,$$

$$\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\text{pol}} \equiv \frac{d\sigma}{d\mathbf{b}}(r_\Lambda = +1) - \frac{d\sigma}{d\mathbf{b}}(r_\Lambda = -1)$$
$$\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\text{tot}} \equiv \frac{d\sigma}{d\mathbf{b}}(r_\Lambda = +1) + \frac{d\sigma}{d\mathbf{b}}(r_\Lambda = -1)$$

Lagrangian and polarized cross section

Chiral perturbation theory (ChPT)

ChPT Lagrangian for **baryon-meson interaction**

$$\mathcal{L}_{\mathrm{ChPT}} = \mathrm{Tr}\left[\overline{B}\left(i\not{D} - M_0\right)B\right] - \frac{D}{2}\mathrm{Tr}\left[\overline{B}\gamma^\mu\gamma_5\left\{u_\mu, B\right\}\right] - \frac{F}{2}\mathrm{Tr}\left[\overline{B}\gamma^\mu\gamma_5\left[u_\mu, B\right]\right]$$

Leading order and next-to-leading order

$$\mathcal{L}_{\mathrm{int}} = \frac{D}{2F_0}\mathrm{Tr}\left(\overline{B}\gamma^\mu\gamma_5\left\{\partial_\mu\Pi, B\right\}\right) + \frac{F}{2F_0}\mathrm{Tr}\left(\overline{B}\gamma^\mu\gamma_5\left[\partial_\mu\Pi, B\right]\right) + \frac{i}{8F_0^2}\mathrm{Tr}\left(\overline{B}\gamma^\mu\left[\left[\Pi, \partial_\mu\Pi\right], B\right]\right).$$

$BB\Pi$ vertex

$BB\Pi\Pi$ vertex

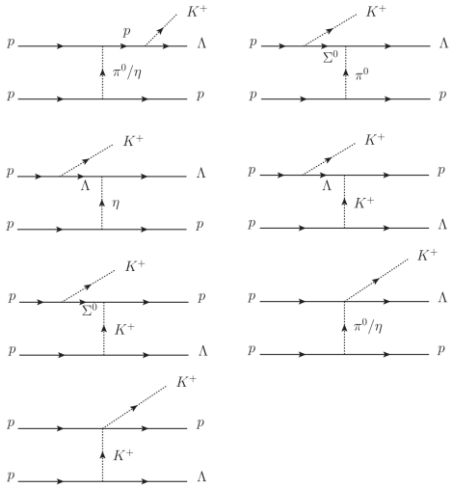
$$\begin{aligned} D_\mu B &= \partial_\mu B + \left[\Gamma_\mu, B\right], \\ \Gamma_\mu &= \frac{1}{2}\left[u^\dagger\left(\partial_\mu - ir_\mu\right)u + u\left(\partial_\mu - il_\mu\right)u^\dagger\right], \\ u_\mu &= i\left[u^\dagger\left(\partial_\mu - ir_\mu\right)u - u\left(\partial_\mu - il_\mu\right)u^\dagger\right]. \end{aligned}$$

S. Scherer and M. R. Schindler, A Primer for Chiral Perturbation Theory

Lagrangian and polarized cross section

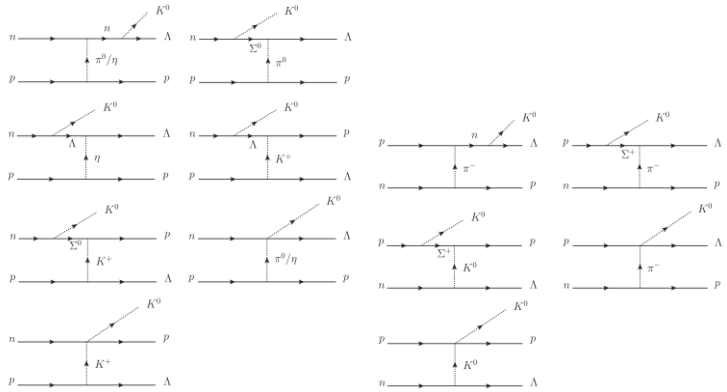
$NN \to N\Lambda K$ cross section

$pp \to p\Lambda K^+$ (7 diagrams)



$$
\begin{aligned}
&iM'_{\mathrm{pp}}\left(k_1,s_1;k_2,s_2\to p_N,r_N;p_\Lambda,r_\Lambda;p_K\right)\\
=&\frac{1}{2F_0^3}\left[\overline{u}(p_N,r_N)\gamma_5 u_p(k_1,s_1)\overline{u}(p_\Lambda,r_\Lambda)\left(G_{1,\mathrm{pp}}\not{p}_K+H_{1,\mathrm{pp}}\right)u_p(k_2,s_2)\right.\\
&\left.+\overline{u}(p_\Lambda,r_\Lambda)\gamma_5 u(k_1,s_1)\overline{u}(p_N,r_N)\left(G_{2,\mathrm{pp}}\not{p}_K+H_{2,\mathrm{pp}}\right)u_p(k_2,s_2)\right]-k_1\leftrightarrow k_2
\end{aligned}
$$

$pn \to p\Lambda K^0$ (12 diagrams)



$$
\begin{aligned}
&iM'_{\mathrm{pn}}\left(k_1,s_1;k_2,s_2\to p_N,r_N;p_\Lambda,r_\Lambda;p_K\right)\\
=&\frac{1}{2F_0^3}\left[\overline{u}(p_N,r_N)\gamma_5 u_p(k_1,s_1)\overline{u}(p_\Lambda,r_\Lambda)\left(G_{1,\mathrm{pn}}\not{p}_K+H_{1,\mathrm{pn}}\right)u_n(k_2,s_2)\right.\\
&+\overline{u}(p_N,r_N)\gamma_5 u_n(k_2,s_2)\overline{u}(p_\Lambda,r_\Lambda)\left(G_{2,\mathrm{pn}}\not{p}_K+H_{2,\mathrm{pn}}\right)u_p(k_1,s_1)\\
&+\overline{u}(p_\Lambda,r_\Lambda)\gamma_5 u_p(k_1,s_1)\overline{u}(p_N,r_N)\left(G_{3,\mathrm{pn}}\not{p}_K+H_{3,\mathrm{pn}}\right)u_n(k_2,s_2)\\
&\left.+\overline{u}(p_\Lambda,r_\Lambda)\gamma_5 u_n(k_2,s_2)\overline{u}(p_N,r_N)\left(G_{4,\mathrm{pn}}\not{p}_K+H_{4,\mathrm{pn}}\right)u_p(k_1,s_1)\right]
\end{aligned}
$$

Because of the **isospin symmetry**, the cross section of $pn \to n\Lambda K^+$ and $nn \to n\Lambda K^0$ are exactly the same as the cross section of $pn \to p\Lambda K^0$ and $pp \to p\Lambda K^+$ respectively.

Lagrangian and polarized cross section

$NN \to N\Lambda K$ cross section

In **near-threshold approximation (NTA)**, we have $\mathbf{p}_N, \mathbf{p}_\Lambda, \mathbf{p}_K \ll m$

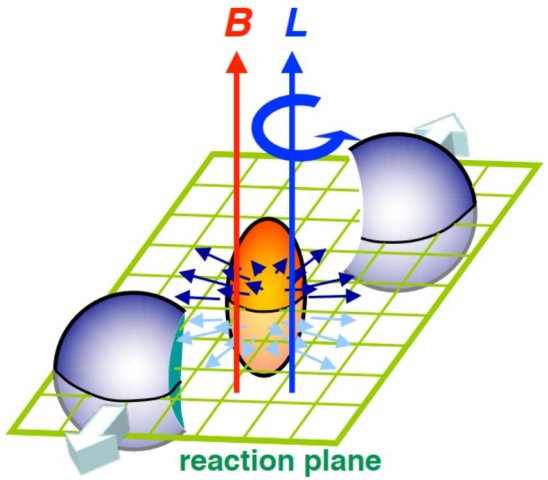
$$\left(\frac{d\sigma}{d\mathbf{b}}\right)^{pp\to p\Lambda K^+}_{\rm pol} = C^{pp\to p\Lambda K^+}_{\rm pol}\sqrt{E_p^2-m_N^2}\,I_{\rm space}e^{-\frac{1}{4}\alpha^2\mathbf{b}^2}\times\frac{1}{2}\alpha^2\mathbf{b}_x$$

$$\left(\frac{d\sigma}{d\mathbf{b}}\right)^{pp\to p\Lambda K^+}_{\rm tot} = C^{pp\to p\Lambda K^+}_{\rm tot}\left(E_p^2-m_N^2\right)I_{\rm space}e^{-\frac{1}{4}\alpha^2\mathbf{b}^2}$$

Constant

$$\left(\frac{d\sigma}{d\mathbf{b}}\right)^{pn\to p\Lambda K^0}_{\rm pol} = C^{pn\to p\Lambda K^0}_{\rm pol}\sqrt{E_p^2-m_N^2}\,I_{\rm space}e^{-\frac{1}{4}\alpha^2\mathbf{b}^2}\times\frac{1}{2}\alpha^2\mathbf{b}_x,$$

$$\left(\frac{d\sigma}{d\mathbf{b}}\right)^{pn\to p\Lambda K^0}_{\rm tot} = C^{pn\to p\Lambda K^0}_{\rm tot}\left(E_p^2-m_N^2\right)I_{\rm space}e^{-\frac{1}{4}\alpha^2\mathbf{b}^2},$$



Spin-orbit coupling

$\boldsymbol{L\sim b\times p\to S}$

$$P_\Lambda = \frac{\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\rm pol}}{\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\rm tot}} = \frac{\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\rm r_\lambda=+1}-\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\rm r_\lambda=-1}}{\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\rm r_\lambda=+1}+\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\rm r_\lambda=-1}}$$

Choosing +y as the quantized direction

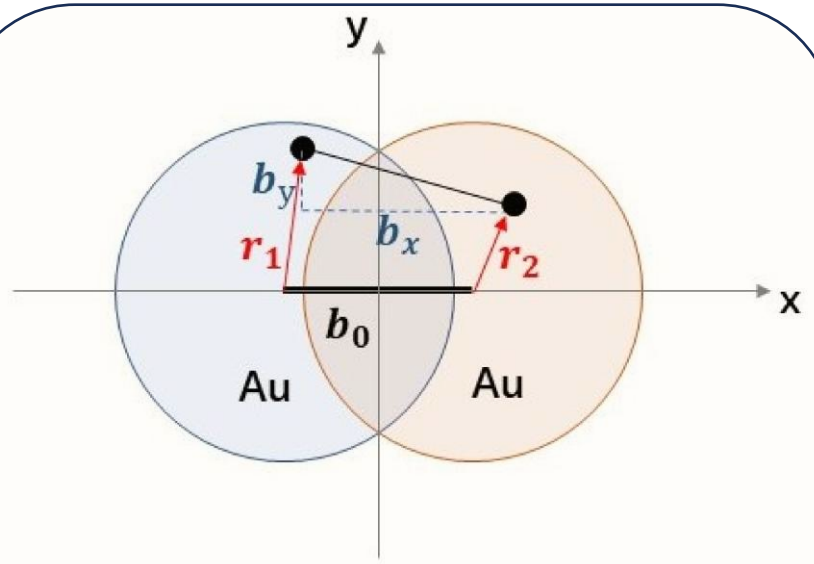
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Numerical results

Structure in the Au-Au collision

Sum over all the nucleon-nucleon collisions in an Au-Au collision



$$P_\Lambda = \frac{\mathcal{N}_{\rm pol}}{\mathcal{N}_{\rm tot}}$$

$$\mathcal{N}_{\rm pol} = \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \left[f_p(\mathbf{r}_1) f_p(\mathbf{r}_2) \frac{d\sigma_{\rm pol}}{d\mathbf{b}}(pp \to p\Lambda K^+) + f_n(\mathbf{r}_1) f_n(\mathbf{r}_2) \frac{d\sigma_{\rm pol}}{d\mathbf{b}}(nn \to n\Lambda K^0) \right.$$
$$\left. + (f_p(\mathbf{r}_1) f_n(\mathbf{r}_2) + f_n(\mathbf{r}_1) f_p(\mathbf{r}_2)) \left(\frac{d\sigma_{\rm pol}}{d\mathbf{b}}(pn \to p\Lambda K^0) + \frac{d\sigma_{\rm pol}}{d\mathbf{b}}(pn \to n\Lambda K^+) \right) \right],$$

$$\mathcal{N}_{\rm tot} = \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \left[f_p(\mathbf{r}_1) f_p(\mathbf{r}_2) \frac{d\sigma_{\rm tot}}{d\mathbf{b}}(pp \to p\Lambda K^+) + f_n(\mathbf{r}_1) f_n(\mathbf{r}_2) \frac{d\sigma_{\rm tot}}{d\mathbf{b}}(nn \to n\Lambda K^0) \right.$$
$$\left. + (f_p(\mathbf{r}_1) f_n(\mathbf{r}_2) + f_n(\mathbf{r}_1) f_p(\mathbf{r}_2)) \left(\frac{d\sigma_{\rm tot}}{d\mathbf{b}}(pn \to p\Lambda K^0) + \frac{d\sigma_{\rm tot}}{d\mathbf{b}}(pn \to n\Lambda K^+) \right) \right].$$

$f_p(\mathbf{r}_1)$: Proton density; $f_n(\mathbf{r}_1)$: Neutron density

Hard-sphere model

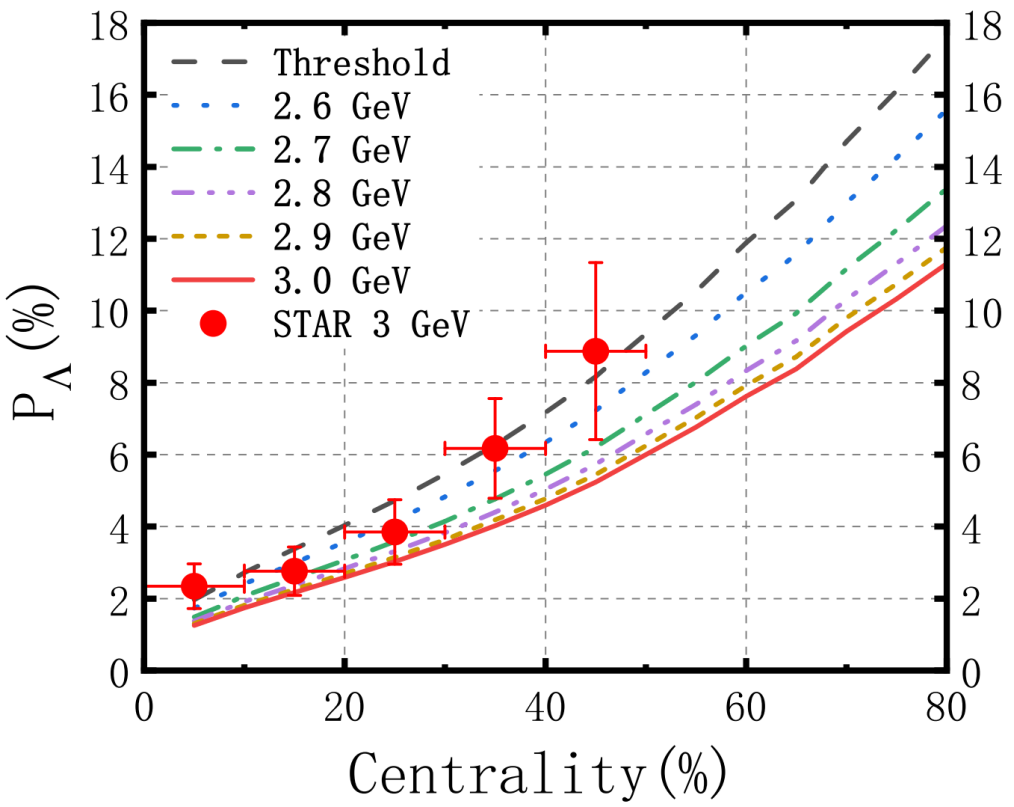
$$f_{p/n}(\mathbf{r}) = \frac{N_{p/n}}{V} \Theta(r - R)$$

Woods-Saxon model (Glauber model)

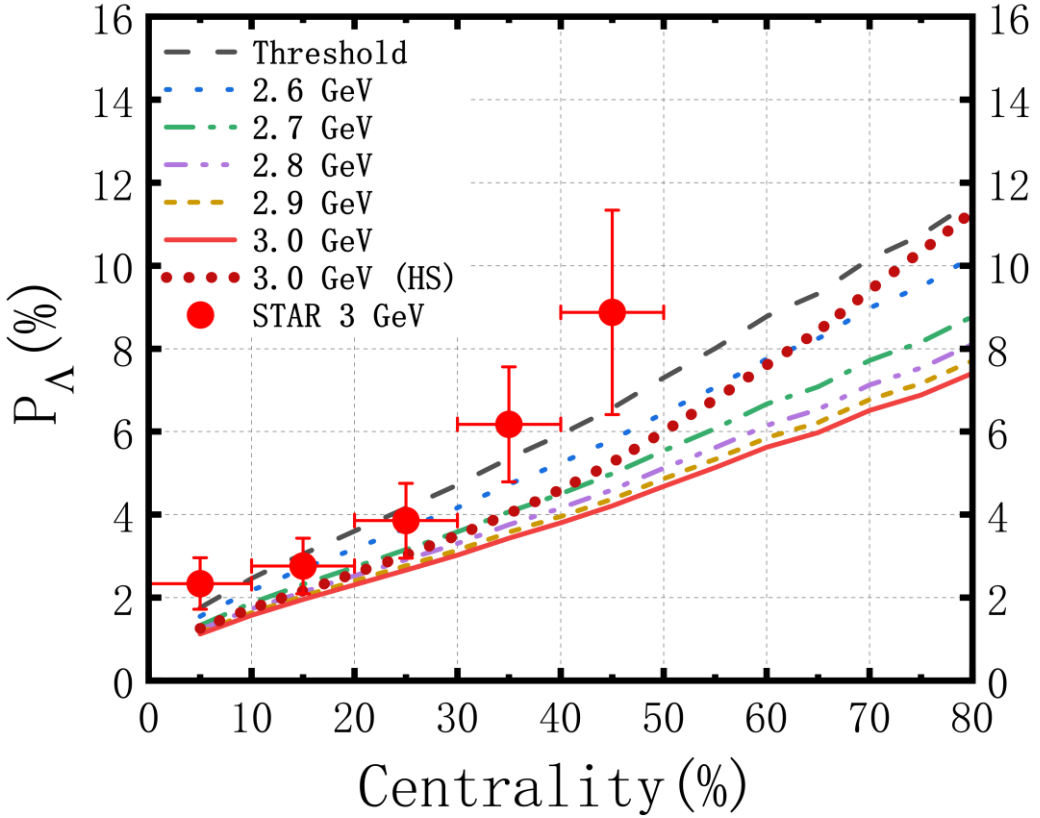
$$f_{p/n}(\mathbf{r}) = \frac{\rho_{p/n}}{1 + \exp\left(\frac{r-R}{a_0}\right)}$$

Numerical results

Hard-sphere model



Woods-Saxon model (Glauber model)



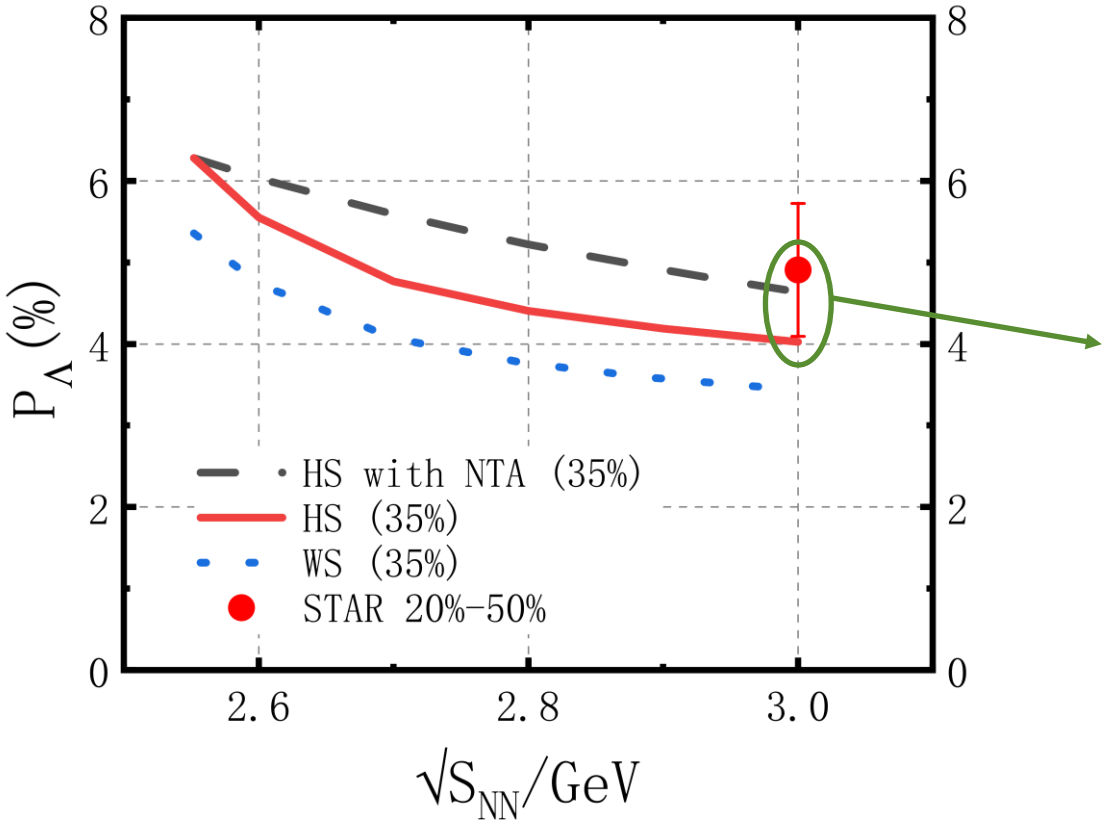
Parameters:

$\alpha = 1\text{fm}^{-1}$, $R = 6.38\text{fm}$, $a_0 = 0.535\text{fm}$,
$n_p = 79$, $n_n = 118$, $D = 0.8$, $F = 0.5$.

S. Scherer and M. R. Schindler, A Primer for Chiral Perturbation Theory
S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 255 (1998)
C. Loizides et al., SoftwareX 1-2, 13 (2015)
C. Loizides et al., Phys. Rev. C 97, 054910 (2018), [Erratum: Phys.Rev.C 99, 019901 (2019)]
STAR, M. S. Abdallah et al., Phys. Rev. C 104, L061901 (2021)

Numerical results

Near-Threshold Approximation (NTA)



About 1-σ deviation

$$\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\rm pol}^{pp\to p\Lambda K^+} = C_{\rm pol}^{pp\to p\Lambda K^+}\sqrt{E_p^2-m_N^2}\,I_{\rm space}e^{-\frac{1}{4}\alpha^2\mathbf{b}^2}\times\frac{1}{2}\alpha^2\mathbf{b}_x$$

$$\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\rm tot}^{pp\to p\Lambda K^+} = C_{\rm tot}^{pp\to p\Lambda K^+}\left(E_p^2-m_N^2\right)I_{\rm space}e^{-\frac{1}{4}\alpha^2\mathbf{b}^2}$$

$$\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\rm pol}^{pn\to p\Lambda K^0} = C_{\rm pol}^{pn\to p\Lambda K^0}\sqrt{E_p^2-m_N^2}\,I_{\rm space}e^{-\frac{1}{4}\alpha^2\mathbf{b}^2}\times\frac{1}{2}\alpha^2\mathbf{b}_x,$$

$$\left(\frac{d\sigma}{d\mathbf{b}}\right)_{\rm tot}^{pn\to p\Lambda K^0} = C_{\rm tot}^{pn\to p\Lambda K^0}\left(E_p^2-m_N^2\right)I_{\rm space}e^{-\frac{1}{4}\alpha^2\mathbf{b}^2},$$

The near-threshold approximation works well enough for collision energies less than 3 GeV.

Summary

1. The Λ hyperons in low-energy heavy ion collision may be produced by **nucleon-nucleon collisions** directly. We develop a **microscopic model** to calculate the spin polarization of Λ hyperon in **nonlocal nucleon-nucleon collision**.

2. The nonlocal collision can be described with **wave packet.**

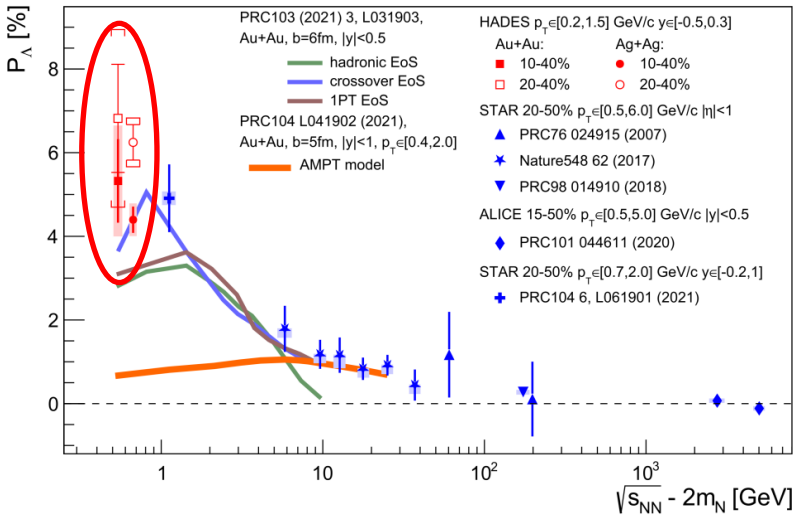
3. The numerical results with **Woods-Saxon model and hard-sphere model** are consistent with the experimental data **within 1~2 σ deviation**.

4. The **near-threshold approximation** works well enough for collision energies less than 3 GeV.

Outlook

1. For more precise result, the **vertex parameters** should be corrected, and contributions from **vector mesons** (ρ, σ…) and **resonance states** (N^*, Δ^*…) should be included.

2. For Au-Au collision at $\sqrt{S_{NN}} = 2.4$ GeV, the process $\boldsymbol{NN \to N\Lambda K}$ is forbidden. We should probably consider the multi-body scattering process, like $\boldsymbol{NNN \to NN\Lambda K}$.



Thanks

Back up

The constants in the cross section

$$
\begin{aligned}
I_{\text{space}} \equiv & \frac{1}{F_0^6}\int \frac{d^3p_N}{(2\pi)^3\,2E_N}\frac{d^3p_K}{(2\pi)^3\,2E_K}\frac{d^3p_\Lambda}{(2\pi)^3\,2E_\Lambda}\sum_{s_1,s_2}\int \frac{d^3\mathbf{K}_1}{(2\pi)^3\,2E_p}\frac{d^3\mathbf{K}_2 d^3\delta\mathbf{k}_2}{(2\pi)^6\,2E_p} \\
& \times\frac{(8\pi)^3}{\alpha^6}\exp\left[-\frac{2\left(\mathbf{K}_1-\mathbf{p}_1\right)^2+2\left(\mathbf{K}_2-\mathbf{p}_2\right)^2+\left(\delta\mathbf{k}_2+\frac{1}{2}i\alpha^2\mathbf{b}\right)^2}{\alpha^2}\right] \\
& \times(2\pi)\,\delta\left(\delta k_1^0+\delta k_2^0\right)(2\pi)^4\,\delta^{(4)}\left(p_N+p_K+p_\Lambda-K_1-K_2\right),
\end{aligned}
$$

$$
\begin{aligned}
C_{\text{pol}}^{pp\to p\Lambda K^+} &= 4m_N m_\Lambda\left[-\left(A_1^{pp}\right)^2+3\left(A_2^{pp}\right)^2+2A_1^{pp}A_2^{pp}\right] \\
C_{\text{tot}}^{pp\to p\Lambda K^+} &= 4m_\Lambda m_N\left[3\left(A_1^{pp}\right)^2+3\left(A_2^{pp}\right)^2+2A_1^{pp}A_2^{pp}\right] \\
C_{\text{pol}}^{pn\to p\Lambda K^0} &= 4m_N m_\Lambda\left[A_1^{pn}A_2^{pn}+\left(A_3^{pn}\right)^2+\left(A_4^{pn}\right)^2-A_3^{pn}A_4^{pn}+A_1^{pn}A_3^{pn}+A_2^{pn}A_4^{pn}\right], \\
C_{\text{tot}}^{pn\to p\Lambda K^0} &= 4m_\Lambda m_N\left[\left(A_1^{pn}\right)^2+\left(A_2^{pn}\right)^2+\left(A_3^{pn}\right)^2+\left(A_4^{pn}\right)^2-A_1^{pn}A_2^{pn}-A_3^{pn}A_4^{pn}+A_1^{pn}A_3^{pn}+A_2^{pn}A_4^{pn}\right]
\end{aligned}
$$