Polarization of massive spin-3/2 particles in a relativistic fluid at local thermodynamic equilibrium

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Why study spin-3/2 polarization?

A probe of the relativistic fluid in heavy-ion collisions

Hyperon polarization probes the vortical structure and thermo-hydrodynamic gradients of the fluid produced in heavy-ion collisions.

Rich polarization structure

The spin density matrix of massive spin-3/2 particles includes rank-2 and rank-3 multipoles beyond the vector component.

The Ω^- hyperon opens access to spin-3/2 polarization

Experimental measurements and prospects:

- **STAR at RHIC** Global Ω polarization measured in Au–Au at $\sqrt{s_{\mathrm{NN}}} = 200$ GeV.
  J. Adam et al. (STAR), *Phys. Rev. Lett.* **126**, 162301 (2021).
- **ALICE at LHC** Prospects for longitudinal Ω polarization in Pb–Pb at $\sqrt{s_{\mathrm{NN}}} = 5.36$ TeV.
  C. De Martin (ALICE), Rencontres de Moriond 2026.
- **CBM at FAIR** Prospects for high-statistics measurements of Ω polarization at low energies.
  Y. Leifels, Strange Quark Matter 2024.

From global to local equilibrium

Global equilibrium: linear order in thermal vorticity

$$S^{\mu}_{\varpi,(1)}(p) = -\frac{S(S+1)}{6}\varepsilon^{\mu\nu\rho\sigma}\frac{p_\sigma}{m}\varpi_{\nu\rho}\left[1+(-1)^{2S}n_{\mathrm{F/B}}\right]$$

A. Palermo and F. Becattini, *Eur. Phys. J. Plus* **138**, 547 (2023).

For spin $S = 3/2$, the tensor multipoles vanish at linear order

$$M^{\mu\nu}_{\varpi,(1)}(p) = 0 \qquad R^{\mu\nu\rho}_{\varpi,(1)}(p) = 0$$

Our objectives

Develop the covariant Wigner-function formalism for the calculation of all spin-3/2 polarization components.

Calculate the spin-3/2 polarization components at local equilibrium, to first-order in gradients of $\beta^\mu = u^\mu/T$.

Polarization components for a massive spin-3/2 particle

Definition of the spin density matrix

$$\Theta_{\sigma\sigma'}(p) = \frac{\mathrm{Tr}\Big(\widehat{\rho}\,\widehat{a}^\dagger(p,\sigma')\,\widehat{a}(p,\sigma)\Big)}{\sum_\tau \mathrm{Tr}\Big(\widehat{\rho}\,\widehat{a}^\dagger(p,\tau)\,\widehat{a}(p,\tau)\Big)} \qquad \sigma,\sigma' = \pm\frac{3}{2}, \pm\frac{1}{2}$$

Irreducible decomposition under rotations

$$\Theta(p) = \frac{1}{4}\left(\mathbb{1} + \frac{4}{5}S^i\Sigma^i + \frac{2}{3}M^{ij}\Sigma^{ij} + \frac{8}{9}R^{ijk}\Sigma^{ijk}\right)$$

Vector

$$\Sigma^i = D^{\frac{3}{2}}(J^i)$$
$$S^i = \mathrm{tr}_s\big(\Theta\Sigma^i\big)$$

3 components

Rank-2 tensor

$$\Sigma^{ij} = \frac{1}{2}\Sigma^{\{i}\Sigma^{j\}} - \frac{5}{4}\delta^{ij}\mathbb{1}$$
$$M^{ij} = \mathrm{tr}_s\big(\Theta\Sigma^{ij}\big)$$

5 components

Rank-3 tensor

$$\Sigma^{ijk} = \frac{1}{6}\Sigma^{\{i}\Sigma^j\Sigma^{k\}} - \frac{41}{60}\delta^{\{ij}\Sigma^{k\}}$$
$$R^{ijk} = \mathrm{tr}_s\big(\Theta\Sigma^{ijk}\big)$$

7 components

Free massive Rarita-Schwinger field

Lorentz group representation

$$\Psi(x) \in \left(\frac{1}{2},\frac{1}{2}\right) \otimes \left[\left(\frac{1}{2},0\right) \oplus \left(0,\frac{1}{2}\right)\right] \implies \widehat{U}(\Lambda)\Psi^{\mu a}(x)\widehat{U}^{\dagger}(\Lambda) = \left(\Lambda^{-1}\right)^{\mu}_{\ \nu} D\left(\Lambda^{-1}\right)^{a}_{\ b} \Psi^{\nu b}(\Lambda x)$$

Spin-3/2 sector

$$(\mathrm{i}\gamma^{\nu}\partial_{\nu} - m)\Psi^{\mu} = 0 \qquad \gamma_{\mu}\Psi^{\mu} = 0 \qquad \partial_{\mu}\Psi^{\mu} = 0$$

Particle and antiparticle modes

$$\Psi^{\mu a}(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma} \int \frac{\mathrm{d}^3 p}{\sqrt{2\varepsilon_{\boldsymbol{p}}}} \left[u^{\mu a}(p,\sigma)\widehat{a}(p,\sigma)\mathrm{e}^{-\mathrm{i}p\cdot x} + v^{\mu a}(p,\sigma)\widehat{b}^{\dagger}(p,\sigma)\mathrm{e}^{\mathrm{i}p\cdot x}\right]$$

How to connect the field to the polarization?

$$\Psi^{\mu} \longrightarrow W^{\mu\nu} = \mathrm{Tr}\left(\widehat{\rho}\,\widehat{W}^{\mu\nu}\right) \longrightarrow \Theta \longrightarrow S, M, R$$

The Wigner function determines the spin density matrix

Covariant Wigner operator and function

$$\widehat{W}^{\mu\nu}(x,p) = -\frac{1}{(2\pi)^4}\int \mathrm{d}^4 y\, \mathrm{e}^{-\mathrm{i}p\cdot y} : \Psi^{\mu}(x-y/2)\overline{\Psi}^{\nu}(x+y/2) :$$

$$W^{\mu\nu}(x,p) = \mathrm{Tr}\Big(\widehat{\rho}\,\widehat{W}^{\mu\nu}(x,p)\Big)$$

On-shell integrated Wigner function for particles

$$0 = (p\cdot\partial_x)\widehat{W}^{\mu\nu}(x,p) \quad\Longrightarrow\quad \mathcal{W}^{\mu\nu}(p) = \int \mathrm{d}p^0 \int_{\Sigma} \mathrm{d}\Sigma(x)\cdot p\; W_{+}^{\mu\nu}(x,p)$$

Projection onto physical spin states

$$\Theta_{\sigma\sigma'}(p) = \frac{\overline{u}_{\mu}(p,\sigma)\mathcal{W}^{\mu\nu}(p)u_{\nu}(p,\sigma')}{\sum_{\tau}\overline{u}_{\mu}(p,\tau)\mathcal{W}^{\mu\nu}(p)u_{\nu}(p,\tau)}$$

Covariant vector and rank-2 tensor components from the Wigner function

$$S^\mu(p) = L(p)^\mu{}_i S^i \qquad M^{\mu\nu}(p) = L(p)^\mu{}_i L(p)^\nu{}_j M^{ij}$$

Mean spin vector

$$S^\mu(p) = \frac{\frac{\mathrm{i}}{m}\varepsilon^{\mu\alpha\beta\xi} p_\xi \,\mathrm{tr_D}\Big(\mathcal{W}_{\alpha\beta}(p)\Big) + \frac{1}{2}\,\mathrm{tr_D}\Big(\gamma^\mu\gamma^5\mathcal{W}^\lambda_\lambda(p)\Big)}{\mathrm{tr}\Big(\mathcal{W}^\lambda_\lambda(p)\Big)}$$

Also obtained in M. Buzzegoli et al. arXiv:2610.37382 (2026)

Covariant rank-2 tensor component

$$M^{\mu\nu}(p) = \frac{5}{4}\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{m^2}\right) + \frac{p_\xi p_\chi \varepsilon^{\xi\mu\alpha\beta}\varepsilon^{\chi\nu\gamma\delta}}{2m^2\,\mathrm{tr_D}\big(\mathcal{W}^\lambda_\lambda(p)\big)}\Big[\eta_{\alpha\gamma}\mathrm{tr_D}\Big(\mathcal{W}_{\delta\beta}(p) + \mathcal{W}_{\beta\delta}(p)\Big) + - \mathrm{i}\,\mathrm{tr_D}\Big(\Sigma_{\gamma\delta}\mathcal{W}_{\alpha\beta}(p) + \Sigma_{\alpha\beta}\mathcal{W}_{\gamma\delta}(p)\Big) + \frac{1}{4}\mathrm{tr_D}\Big(\big\{\Sigma_{\alpha\beta},\Sigma_{\gamma\delta}\big\}\mathcal{W}^\lambda_\lambda(p)\Big)\Big]$$

Rank-3 tensor component from the Wigner function

Covariant rank-3 tensor component

$$R^{\mu\nu\rho}(p) = L(p)^{\mu}{}_{i} L(p)^{\nu}{}_{j} L(p)^{\rho}{}_{k} R^{ijk} \qquad R^{ijk} = \mathrm{tr}_s\big(\Theta \Sigma^{ijk}\big)$$

Expression in terms of the Wigner function

$$R^{\mu\nu\rho}(p) = \frac{1}{6} S^{\{\mu\nu\rho\}}(p) + \frac{41}{60}\Big[\Gamma^{\mu\nu}(p) S^{\rho}(p) + \Gamma^{\nu\rho}(p) S^{\mu}(p) + \Gamma^{\rho\mu}(p) S^{\nu}(p)\Big] \qquad \Gamma^{\mu\nu}(p) \equiv \eta^{\mu\nu} - \frac{p^{\mu} p^{\nu}}{m^2}$$

$$\begin{aligned} S^{\mu\nu\rho}(p) = -\frac{\varepsilon^{\xi\mu\alpha\beta}\varepsilon^{\chi\nu\gamma\delta}\varepsilon^{\omega\rho\kappa\tau}}{\mathrm{tr}\big(\mathcal{W}(p)^{\lambda}_{\lambda}\big)} \frac{p_{\xi} p_{\chi} p_{\omega}}{2m^3} \Big[& 2\mathrm{i}\, \eta_{\alpha\delta}\eta_{\gamma\kappa}\mathrm{tr}\big(\mathcal{W}_{\tau\beta}(p)\big) + \eta_{\alpha\delta}\mathrm{tr}\big(\mathcal{W}_{\gamma\beta}(p)\Sigma_{\kappa\tau}\big) \\ & - \eta_{\alpha\kappa}\mathrm{tr}\big(\mathcal{W}(p)_{\tau\beta}\Sigma_{\gamma\delta}\big) - \eta_{\gamma\kappa}\mathrm{tr}\big(\mathcal{W}_{\tau\delta}(p)\Sigma_{\alpha\beta}\big) + \frac{\mathrm{i}}{2}\mathrm{tr}\big(\mathcal{W}_{\alpha\beta}(p)\Sigma_{\gamma\delta}\Sigma_{\kappa\tau}\big) \\ & + \frac{\mathrm{i}}{2}\mathrm{tr}\Big(\mathcal{W}_{\gamma\delta}(p)\Sigma_{\alpha\beta}\Sigma_{\kappa\tau}\Big) - \frac{\mathrm{i}}{2}\mathrm{tr}\big(\mathcal{W}_{\tau\kappa}(p)\Sigma_{\alpha\beta}\Sigma_{\gamma\delta}\big) - \frac{1}{4}\mathrm{tr}\big(\mathcal{W}^{\lambda}_{\lambda}(p)\Sigma_{\alpha\beta}\Sigma_{\gamma\delta}\Sigma_{\kappa\tau}\big)\Big] \end{aligned}$$

Wigner function at local thermodynamic equilibrium

Local equilibrium density operator (Belinfante pseudo-gauge)

$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z_{\mathrm{LE}}} \exp\Bigg\{ - \int_{\Sigma} \mathrm{d}\Sigma_{\mu}(y)\, \widehat{T}^{\mu\nu}_{\mathrm{B}}(y)\, \underbrace{\beta_{\nu}(y)}_{u_{\nu}(y)/T(y)} \Bigg\}$$

Linear response theory and gradient expansion

Take the linear contribution in $\Delta\beta_{\nu} = \beta_{\nu}(y) - \beta_{\nu}(x)$ and expand $\Delta\beta_{\nu}$ in gradients about x

$$W_{\mathrm{LE}}(x,p) = \mathrm{Tr}\Big(\widehat{\rho}_{\mathrm{LE}}\, \widehat{W}(x,p)\Big) = W_{\mathrm{GE}}(x,p) + W_{(1)}(x,p) + \ldots \quad \Longrightarrow \quad \Theta = \Theta_{\mathrm{GE}} + \Theta_{(1)} + \ldots$$

Global equilibrium contribution

$$\widehat{\rho}_{\mathrm{GE}} = \frac{1}{Z}\, \mathrm{e}^{-\beta(x)\cdot\widehat{P}} \quad \Longrightarrow \quad \Theta_{\mathrm{GE}} = \frac{1}{4}\, \mathbb{1}$$

First-order correction to the Wigner function

First order in gradient expansion

$$W_{(1)}(x,p)_{\alpha\beta} = -\mathrm{i}\left(\frac{\partial}{\partial k^\rho} G^{\mu\nu}_{\alpha\beta}(x,p,k)\right)\Big|_{k=0} \frac{n_\mu(x)}{|p\cdot n(x)|}\Delta^{\rho\lambda}(x)\partial_\lambda\beta_\nu(x)$$

X.-L. Sheng, F. Becattini, D. Roselli *Phys. Rev. C* **114**, 034903 (2026)

where

$$\Delta^{\rho\lambda}(x) = \eta^{\rho\lambda} - \frac{n^\rho(x)p^\lambda}{p\cdot n(x)}$$

$$G^{\mu\nu}_{\alpha\beta}(x,p,k) = \sqrt{\varepsilon_{\boldsymbol{q}}\varepsilon_{\boldsymbol{q}'}}\,\delta\left(p^2+\frac{k^2}{4}-m^2\right)\theta(q^0)\theta(q'^0)\frac{1-\mathrm{e}^{\beta(x)\cdot k}}{\beta(x)\cdot k}\times$$
$$\times\sum_{\sigma,\sigma'} u_\alpha(\boldsymbol{q},\sigma)\overline{u}_\beta(\boldsymbol{q}',\sigma')\left\langle \widehat{a}^\dagger(\boldsymbol{q}',\sigma')a(\boldsymbol{q},\sigma), \widehat{T}^{\mu\nu}_{\mathrm{B}}(0)\right\rangle_{\mathrm{c,GE}}$$

$$p = \frac{q+q'}{2} \qquad k = q'-q \qquad \text{with} \quad q^2 = q'^2 = m^2$$

Mean spin vector: thermal vorticity and shear contributions

Mean spin vector at first-order local equilibrium

$$S^{\mu}_{(1)}(p) = -\frac{5}{8}\varepsilon^{\mu\rho\tau\sigma}\frac{p_\sigma}{m}\frac{\int_\Sigma \mathrm{d}\Sigma(x)\cdot p\; n_{\mathrm{F}}(1-n_{\mathrm{F}})\,\mathrm{sgn}[p\cdot n(x)]\left(\varpi_{\rho\tau}(x) - 2\frac{p^\nu n_\tau(x)}{p\cdot n(x)}\xi_{\rho\nu}(x)\right)}{\int_\Sigma \mathrm{d}\Sigma(x)\cdot p\; n_{\mathrm{F}}}$$

Global equilibrium limit

$$S^{\mu}_{\varpi}(p) = -\frac{5}{8}\varepsilon^{\mu\rho\tau\sigma}\frac{p_\sigma}{m}\varpi_{\rho\tau}(1-n_{\mathrm{F}}) \quad \Longrightarrow \quad S^{\mu}_{\varpi}(p) \propto \left.\frac{S(S+1)}{6}\right|_{S=\frac{3}{2}} = \frac{5}{8}$$

Scaling of the thermal-shear contribution

Also for the thermal-shear contribution we obtain the scaling $S^{\mu}_{\xi} \propto \frac{S(S+1)}{6}$ with respect to $S = \frac{1}{2}$

Rank-2 and rank-3 tensor components vanish at first order

Rank-2 tensor

$$M^{\mu\nu}_{(1)}(p) = 0$$

Rank-3 tensor

$$R^{\mu\nu\rho}_{(1)}(p) = 0$$

Also occurs for spin $S = 1$

Spin density matrix for $S = 3/2$ at local equilibrium

$$\Theta(p) = \frac{1}{4}\,\mathbb{1} + \frac{1}{5}\,S^i_{(1)}(p)\,\Sigma^i + \mathcal{O}(\partial^2)$$

The polarized part of the spin density matrix is entirely a vector at first order

Why the tensor components vanish at first order

We recall that $W_{(1)}(x,p)_{\alpha\beta} = -\mathrm{i}\Big(\frac{\partial}{\partial k^\rho} G^{\mu\nu}_{\alpha\beta}(x,p,k)\Big)\Big|_{k=0} \mathcal{B}^\rho_{\mu\nu}(x,p)$ and the G tensor depends on

$$\Big\langle \widehat{a}^\dagger(\boldsymbol{q}',\sigma')a(\boldsymbol{q},\sigma), \widehat{T}^{\mu\nu}_{\mathrm{B}}(0)\Big\rangle_{\mathrm{c,GE}} = n_{\mathrm{F}}(\boldsymbol{q}')\big[1-n_{\mathrm{F}}(\boldsymbol{q})\big] \underbrace{\langle \boldsymbol{q},\sigma|\, \widehat{T}^{\mu\nu}_{\mathrm{B}}(0)\,|\boldsymbol{q}',\sigma'\rangle}_{\mathcal{T}^{\mu\nu}_{\sigma\sigma'}} \qquad k = q'-q$$

$\mathcal{T}^{\mu\nu}$ matrix in spin space at small k in the frame where $k=(0,\boldsymbol{k})$, $p=(E_{\boldsymbol{k}},0)$

$$\begin{aligned} &0 = k_\mu \mathcal{T}^{\mu\nu}(k) \\ &\mathcal{T}^{\mu\nu}(k) = \mathcal{P}^\mu_\rho \mathcal{P}^\nu_\lambda \mathcal{T}^{\rho\lambda}(\mathcal{P}k) \\ &D^{\frac{3}{2}}(R^{-1})\mathcal{T}^{\mu\nu}(k)D^{\frac{3}{2}}(R) = R^\mu_\rho R^\nu_\lambda \mathcal{T}^{\rho\lambda}(R^{-1}k) \end{aligned} \quad \Longrightarrow \quad \begin{aligned} \mathcal{T}^{00}(k) &= c_0 \mathbb{1}_4 + \mathcal{O}(k^2) \\ \mathcal{T}^{0i}(k) &= \mathrm{i}c_1 \varepsilon^{ial} k_a \Sigma_l + \mathcal{O}(k^3) \\ \mathcal{T}^{ij}(k) &= \mathcal{O}(k^2) \end{aligned}$$

Only scalar and vector structures in spin space survive at first-order

$$\partial_k \mathcal{T}\big|_{k=0} \quad \Longrightarrow \quad \boxed{\Theta = b_0 \mathbb{1} + b_i \Sigma^i + \mathcal{O}(\partial^2)} \quad \Longrightarrow \quad M_{(1)} = R_{(1)} = 0$$

Implications of the parity–time-reversal invariance of $\widehat{\rho}_{\mathrm{GE}}$

The homogeneous global equilibrium operator is PT-invariant

$$\left[\mathbf{PT}, \widehat{\rho}_{\mathrm{GE}}\right] = 0 \qquad \text{with} \quad \widehat{\rho}_{\mathrm{GE}} = \frac{1}{Z}\, \mathrm{e}^{-\beta(x)\cdot\widehat{P}}$$

Implications for the spin density matrix

The PT-invariance of $\widehat{\rho}_{\mathrm{GE}}$ and the Hermiticity of the spin density matrix Θ yield the identity

$$\Theta^{(n)}_{\tau\tau'} = (-1)^{n-1-\tau-\tau'}\Theta^{(n)}_{-\tau',-\tau}$$

Z.-H. Zhang, X.-G. Huang, F. Becattini and X.-L. Sheng, *JHEP* **07** (2025) 224

Implications for the polarization components

$$S^i_{(n)} = R^{ijk}_{(n)} = 0 \quad \text{for } n \text{ even},$$
$$M^{ij}_{(n)} = 0 \quad \text{for } n \text{ odd}.$$

Summary and outlook

First-order local equilibrium results

$$S^{\mu}_{(1)} = S^{\mu}_{\varpi} + S^{\mu}_{\xi} \qquad M^{\mu\nu}_{\mathrm{LE}} = \mathcal{O}(\partial^2) \qquad R^{\mu\nu\rho}_{\mathrm{LE}} = \mathcal{O}(\partial^3)$$

Scaling of the mean spin vector

The comparison with the spin-1/2 result at first-order local equilibrium suggests the scaling

$$S^{\mu}_{(1)} \propto S(S+1)$$

What comes next

Higher-derivative orders for tensor polarization, dissipative corrections, tensor responses to chemical potential gradients.

The contribution to $S^{\mu}_{(1)}$ from $\partial\zeta$ has already been computed.

[M. Buzzegoli et al., arXiv:2610.37382 (2026)]

Thank you for your attention!

Back-up

Independent polarization components for $S = 3/2$

Notation $\langle \cdot \rangle = \mathrm{tr_s}\big(\Theta \cdot\big)$

Vector components

$$S_\mathrm{L} = \langle \Sigma^z \rangle \qquad S^x_\mathrm{T} = \langle \Sigma^x \rangle \qquad S^y_\mathrm{T} = \langle \Sigma^y \rangle$$

Rank-2 tensor components

$$S_\mathrm{LL} = \langle \Sigma^{zz} \rangle \qquad S^x_\mathrm{LT} = 2\langle \Sigma^{xz} \rangle \qquad S^y_\mathrm{LT} = 2\langle \Sigma^{yz} \rangle \qquad S^{xy}_\mathrm{TT} = 2\langle \Sigma^{xy} \rangle \qquad S^{xx}_\mathrm{TT} = \langle \Sigma^{xx} - \Sigma^{yy} \rangle$$

Rank-3 tensor components

$$S_\mathrm{LLL} = \langle \Sigma^{zzz} \rangle \qquad S^x_\mathrm{LLT} = \langle \Sigma^{xzz} \rangle \qquad S^y_\mathrm{LLT} = \langle \Sigma^{yzz} \rangle \qquad S^{xy}_\mathrm{LTT} = 4\langle \Sigma^{xyz} \rangle$$
$$S^{xx}_\mathrm{LTT} = 2\big(\langle \Sigma^{xxz} - \Sigma^{yyz} \rangle\big) \qquad S^{xxx}_\mathrm{TTT} = \langle \Sigma^{xxx} - 3\Sigma^{xyy} \rangle \qquad S^{yxx}_\mathrm{TTT} = \langle 3\Sigma^{yxx} - \Sigma^{yyy} \rangle$$

J. Zhao et al., *Phys. Rev. D* **106**, 094006 (2022)

Angular distribution of the decay products in the $\Omega^- \longrightarrow \Lambda K^-$ process

Decay of a spin-3/2 particle into a spin-1/2 particle and a pseudoscalar particle

The angular distribution of the spin-1/2 daughter particle in the rest frame of the decaying spin-3/2 particle, taking the z-direction along the spin quantization direction, is

$$\begin{aligned}\frac{\mathrm{d}N}{\mathrm{d}\Omega}(\theta,\phi) \propto \frac{1}{4\pi}\Bigg\{1 &- \frac{1}{2}\sin^2\theta\big(S_{TT}^{xy}\sin(2\phi) + S_{TT}^{xx}\cos(2\phi)\big) - \frac{1}{2}\sin(2\theta)\big(S_{LT}^{y}\sin\phi + S_{LT}^{x}\cos\phi\big) \\ &- \frac{1}{4}S_{LL}\big[1+3\cos(2\theta)\big] + \alpha\Bigg[\frac{2}{5}\boldsymbol{S}\cdot\boldsymbol{n}(\theta,\phi) - \frac{1}{2}\sin^3\theta\big(S_{TTT}^{yxx}\sin(3\phi) + S_{TTT}^{xxx}\cos(3\phi)\big) \\ &- \frac{3}{2}\cos\theta\sin^2\theta\big(S_{LTT}^{xy}\sin(2\phi) + S_{LTT}^{xx}\cos(2\phi)\big) \\ &- \frac{3}{8}\big(S_{LLT}^{y}\sin\phi + S_{LLT}^{x}\cos\phi\big)\big[\sin\theta + 5\sin(3\theta)\big] - \frac{1}{4}S_{LLL}\big[3\cos\theta + 5\cos(3\theta)\big]\Bigg]\Bigg\}\end{aligned}$$

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta} \propto \frac{1}{2}\Bigg\{1 - \frac{1}{4}S_{LL}\big[1+3\cos(2\theta)\big] + \alpha\Bigg[\frac{2}{5}S_z\cos\theta - \frac{1}{4}S_{LLL}\big[3\cos\theta + 5\cos(3\theta)\big]\Bigg]\Bigg\}$$

Z. Zhang, J. J. Song, Y.-j. Zhou, *Phys. Rev. D* **109**, 036005 (2024)

Conserved currents for the Rarita-Schwinger field

Belinfante energy-momentum tensor

$$\widehat{T}^{\mu\nu}_{\mathrm{B}} = -\frac{\mathrm{i}}{4}\overline{\Psi}_{\sigma}\left(\gamma^{\mu}\overset{\leftrightarrow}{\partial^{\nu}} + \gamma^{\nu}\overset{\leftrightarrow}{\partial^{\mu}}\right)\Psi^{\sigma} + \frac{\mathrm{i}}{2}\partial_{\lambda}\left[\overline{\Psi}^{\lambda}\left(\gamma^{\mu}\Psi^{\nu} + \gamma^{\nu}\Psi^{\mu}\right) - \left(\overline{\Psi}^{\nu}\gamma^{\mu} + \overline{\Psi}^{\mu}\gamma^{\nu}\right)\Psi^{\lambda}\right]$$

Expectation value in terms of the Wigner function

$$T^{\mu\nu}_{\mathrm{B}}(x) = \int \mathrm{d}^4p \left\{-\frac{p^{\mu}}{2}\mathrm{tr}\left(\gamma^{\mu}W^{\lambda}_{\lambda}(x,p)\right) + p_{\lambda}\mathrm{tr}\left(\gamma^{\mu}\left[W^{\nu\lambda}(x,p) + W^{\lambda\nu}(x,p)\right]\right)\right\} + (\mu \leftrightarrow \nu)$$

Charge current

$$\widehat{j}^{\mu} = -\overline{\Psi}_{\nu}\gamma^{\mu}\Psi^{\nu}$$

Expectation value in terms of the Wigner function

$$j^{\mu}(x) = -\int \mathrm{d}^4p\,\mathrm{tr}\left(\gamma^{\mu}W(x,p)^{\lambda}_{\lambda}\right)$$

Plane-wave expansion for the Rarita-Schwinger field

Lorentz group representation

$$\left(\frac{1}{2},\frac{1}{2}\right)\otimes\left[\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\right]=\left(1,\frac{1}{2}\right)\oplus\left(\frac{1}{2},1\right)\oplus\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)$$

Plane-wave expansion

$$\Psi^{\mu a}(x)=\frac{1}{(2\pi)^{3/2}}\sum_{\sigma}\int\frac{\mathrm{d}^3p}{\sqrt{2\varepsilon_{\boldsymbol{p}}}}\Big[u^{\mu a}(\boldsymbol{p},\sigma)\widehat{a}(\boldsymbol{p},\sigma)\mathrm{e}^{-\mathrm{i}p\cdot x}+v^{\mu a}(\boldsymbol{p},\sigma)\widehat{b}^{\dagger}(\boldsymbol{p},\sigma)\mathrm{e}^{\mathrm{i}p\cdot x}\Big]$$

$$\Big\{\widehat{a}(\boldsymbol{p},\sigma),\widehat{a}^{\dagger}(\boldsymbol{p}',\sigma')\Big\}=\delta_{\sigma\sigma'}\delta^3(\boldsymbol{p}-\boldsymbol{p}')=\Big\{\widehat{b}(\boldsymbol{p},\sigma),\widehat{b}^{\dagger}(\boldsymbol{p}',\sigma')\Big\}$$

$$u^{\mu a}(\boldsymbol{p},\sigma)=L(p)^{\mu}_{\ \nu}D\big(L(p)\big)^{a}_{\ b}u^{\nu b}(\boldsymbol{0},\sigma)\qquad v^{\mu a}(\boldsymbol{p},\sigma)=L(p)^{\mu}_{\ \nu}D\big(L(p)\big)^{a}_{\ b}v^{\nu b}(\boldsymbol{0},\sigma)$$

Zero-momentum vector-spinors

$$\big(\boldsymbol{J}^{\mu}_{\nu}\delta^{a}_{b}+\delta^{\mu}_{\nu}D(\boldsymbol{J})^{a}_{b}\big)u^{\nu b}(\mathbf{0},\sigma)=\sum_{\sigma'}u^{\mu a}(\mathbf{0},\sigma')D^{\frac{3}{2}}_{\sigma'\sigma}(\boldsymbol{J})$$

$$\big(\boldsymbol{J}^{\mu}_{\nu}\delta^{a}_{b}+\delta^{\mu}_{\nu}D(\boldsymbol{J})^{a}_{b}\big)v^{\nu b}(\mathbf{0},\sigma)=-\sum_{\sigma'}v^{\mu a}(\mathbf{0},\sigma')D^{\frac{3}{2}}_{\sigma'\sigma}{}^{*}(\boldsymbol{J})$$

$$u\left(\mathbf{0},\pm\frac{3}{2}\right)=\sqrt{m}\begin{pmatrix}0 & \frac{\mp\mathrm{i}}{\sqrt{2}}h_{\pm} & \frac{1}{\sqrt{2}}h_{\pm} & 0\\ 0 & \frac{\mp\mathrm{i}}{\sqrt{2}}h_{\pm} & \frac{1}{\sqrt{2}}h_{\pm} & 0\end{pmatrix}\quad u\left(\mathbf{0},\pm\frac{1}{2}\right)=\sqrt{m}\begin{pmatrix}0 & \frac{\mp\mathrm{i}}{\sqrt{6}}h_{\pm} & \frac{1}{\sqrt{6}}h_{\pm} & \frac{2\mathrm{i}}{\sqrt{6}}h_{\pm}\\ 0 & \frac{\mp\mathrm{i}}{\sqrt{6}}h_{\pm} & \frac{1}{\sqrt{6}}h_{\pm} & \frac{2\mathrm{i}}{\sqrt{6}}h_{\pm}\end{pmatrix}$$

$$v\left(\mathbf{0},\pm\frac{3}{2}\right)=\sqrt{m}\begin{pmatrix}0 & \frac{-\mathrm{i}}{\sqrt{2}}h_{\mp} & \frac{\mp 1}{\sqrt{2}}h_{\mp} & 0\\ 0 & \frac{\mathrm{i}}{\sqrt{2}}h_{\mp} & \frac{\pm 1}{\sqrt{2}}h_{\mp} & 0\end{pmatrix}\quad v\left(\mathbf{0},\pm\frac{1}{2}\right)=\sqrt{m}\begin{pmatrix}0 & \frac{\mathrm{i}}{\sqrt{6}}h_{\pm} & \frac{\pm 1}{\sqrt{6}}h_{\pm} & \frac{\pm 2\mathrm{i}}{\sqrt{6}}h_{\mp}\\ 0 & \frac{-\mathrm{i}}{\sqrt{6}}h_{\pm} & \frac{\mp 1}{\sqrt{6}}h_{\pm} & \frac{\mp 2\mathrm{i}}{\sqrt{6}}h_{\mp}\end{pmatrix}$$

Wigner function at local equilibrium linear response theory

Linear response theory

$$W_{\mathrm{LE}}(x,p) = \mathrm{Tr}\Big(\widehat{\rho}_{\mathrm{LE}}\,\widehat{W}(x,p)\Big) = W_{\mathrm{GE}}(x,p) + \Delta W_{\mathrm{LE}}(x,p)$$

$$W^{\alpha\beta}_{\mathrm{GE}}(x,p) = \frac{1}{Z}\mathrm{Tr}\Big(\mathrm{e}^{-\beta(x)\cdot\widehat{P}}\,\widehat{W}^{\alpha\beta}(x,p)\Big) = \frac{\delta(p^0-\varepsilon_{\boldsymbol{p}})}{2\varepsilon_{\boldsymbol{p}}}\frac{n_{\mathrm{F}}(p,\beta)}{(2\pi)^3}P^{\alpha\beta}_{+}(p)$$

$$P^{\alpha\beta}_{+}(p) = \sum_{\sigma} u^{\alpha}(p,\sigma)\overline{u}^{\beta}(p,\sigma) = -(\not{p}+m)\left[\eta^{\alpha\beta} - \frac{\gamma^{\alpha}\gamma^{\beta}}{3} - \frac{\gamma^{\alpha}p^{\beta}-\gamma^{\beta}p^{\alpha}}{3m} - \frac{2}{3}\frac{p^{\alpha}p^{\beta}}{m^2}\right]$$

$$\Delta W^{\alpha\beta}_{\mathrm{LE}}(x,p) = -\int_{\Sigma}\mathrm{d}\Sigma_{\mu}(y)\Delta\beta_{\nu}(y,x)\int_0^1\mathrm{d}z\,\Big\langle\widehat{W}^{\alpha\beta}(x,p),\mathrm{e}^{-z\beta(x)\cdot\widehat{P}}\,\widehat{T}^{\mu\nu}_{\mathrm{B}}(y)\,\mathrm{e}^{z\beta(x)\cdot\widehat{P}}\Big\rangle_{\mathrm{c,GE}}$$

Local equilibrium correction to the Wigner function

$$\Delta W_{\mathrm{LE}}(x,p)_{\alpha\beta} = \frac{1}{(2\pi)^3}\int_\Sigma \mathrm{d}\Sigma_\mu(y)\,\Delta\beta_\nu(y,x)\int \mathrm{d}^4k\,\delta(p\cdot k)\,\mathrm{e}^{\mathrm{i}k\cdot(x-y)}G^{\mu\nu}_{\alpha\beta}(x,p,k)$$

where

$$G^{\mu\nu}_{\alpha\beta}(x,p,k) = \sqrt{\varepsilon_{\boldsymbol{q}}\varepsilon_{\boldsymbol{q}'}}\,\delta\!\left(p^2+\frac{k^2}{4}-m^2\right)\theta(q^0)\theta(q'^0)\frac{1-\mathrm{e}^{\beta(x)\cdot k}}{\beta(x)\cdot k}\times$$
$$\times\sum_{\sigma,\sigma'}u_\alpha(\boldsymbol{q},\sigma)\overline{u}_\beta(\boldsymbol{q}',\sigma')\left\langle \widehat{a}^\dagger(\boldsymbol{q}',\sigma')a(\boldsymbol{q},\sigma),\widehat{T}^{\mu\nu}_{\mathrm{B}}(0)\right\rangle_{\mathrm{c,GE}}$$

$$p=(q+q')/2 \qquad k=q'-q \qquad \text{with} \quad q^2=q'^2=m^2$$

Gradient expansion

$$\Delta W_{\mathrm{LE}}(x,p)_{\alpha\beta} = \sum_{N=0}^{\infty}\sum_{\overline{y}(x,p)}\frac{1}{N!}\left(D_{y,k}(\overline{y})^N\left[G^{\mu\nu}_{\alpha\beta}(x,p,k)\frac{n_\mu(y)}{|p\cdot n(y)|}\Delta\beta_\nu(y,x)\right]\right)\Bigg|_{k=0,\,y=\overline{y}(x,p)}$$

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