Is the shear induced spin polarization non-dissipative?

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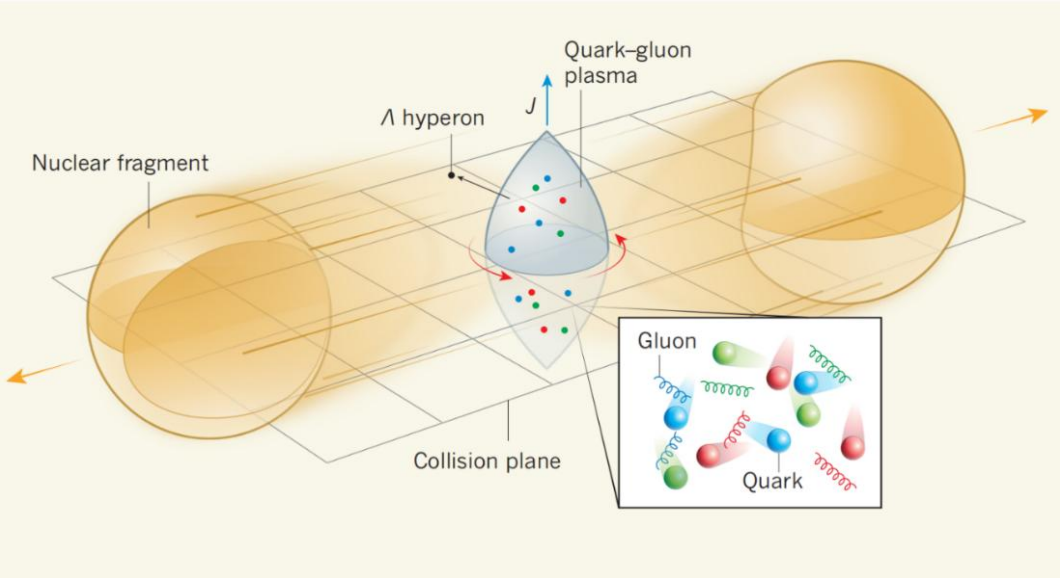
Based on JRW, Shuo Fang, Di-Lun Yang, Shi Pu, Phys. Rev. D 113, 114026 (2026)

Outline

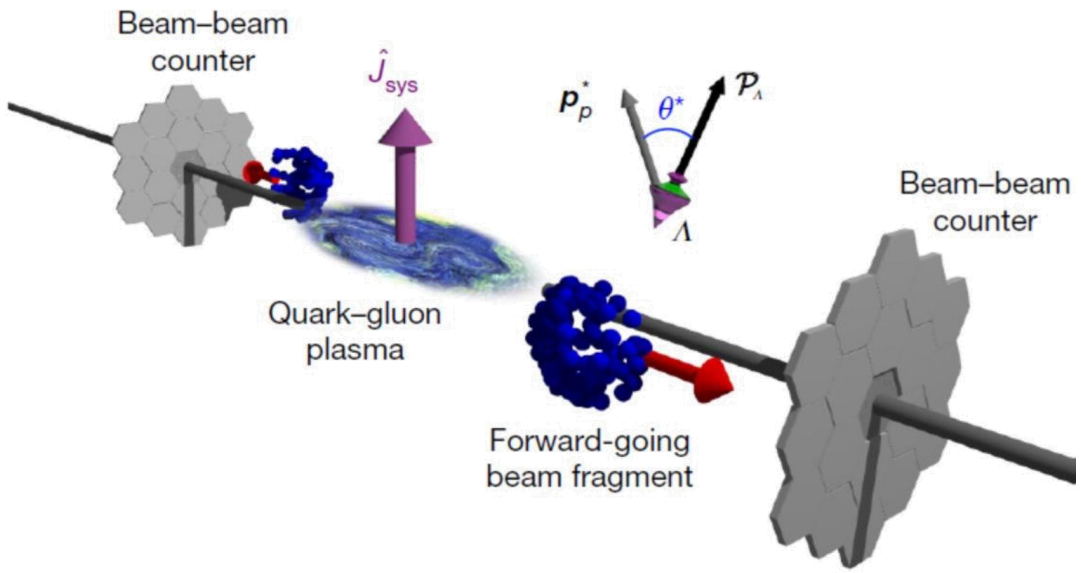
- **Introduction**
- **Entropy analysis for shear-induced polarization**
  - ✓ Entropy in classical kinetic theory
  - ✓ Extension to quantum kinetic theory
- **Summary**

Introduction

Relativistic heavy-ion collisions



Hannah Petersen, Nature 548, 34–35 (2017)

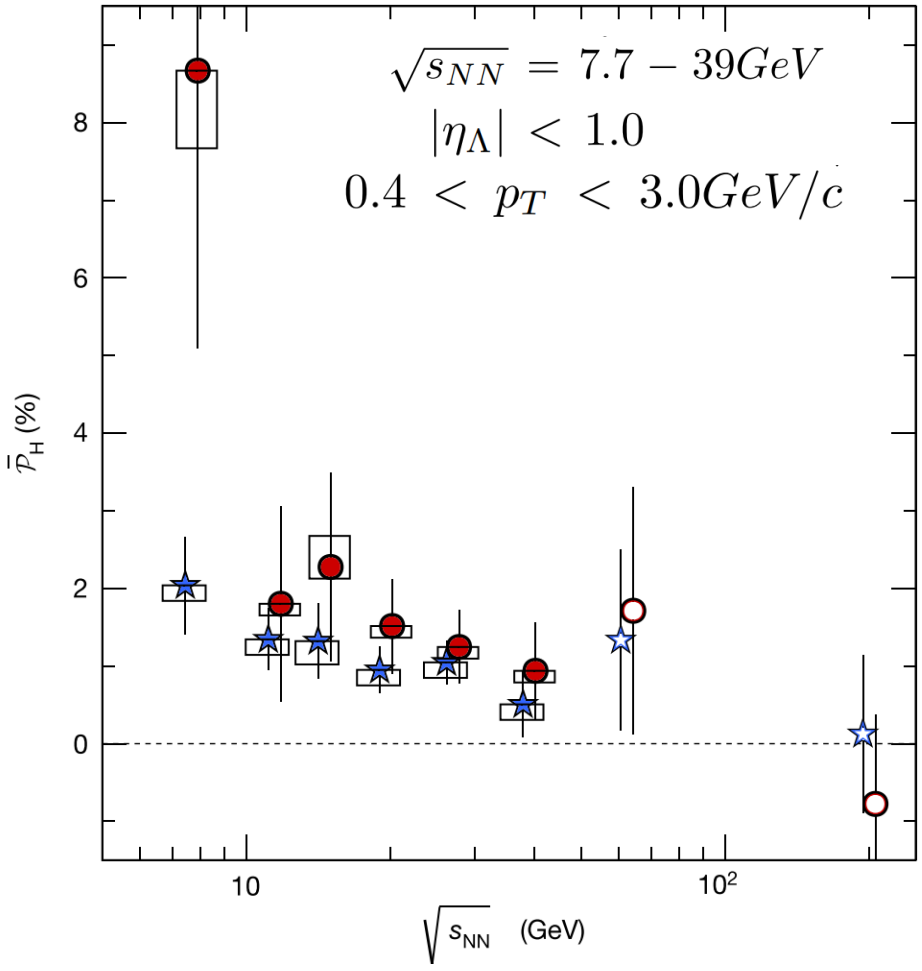


STAR, Nature 548, 62 (2017)

- **QCD matter formed in HIC at finite impact parameter carries a large orbital angular momentum**
- **Final state products could be globally polarized through spin-orbit coupling.**

Zuo-Tang Liang, Xin-Nian Wang, PRL 94, 102301 (2005)

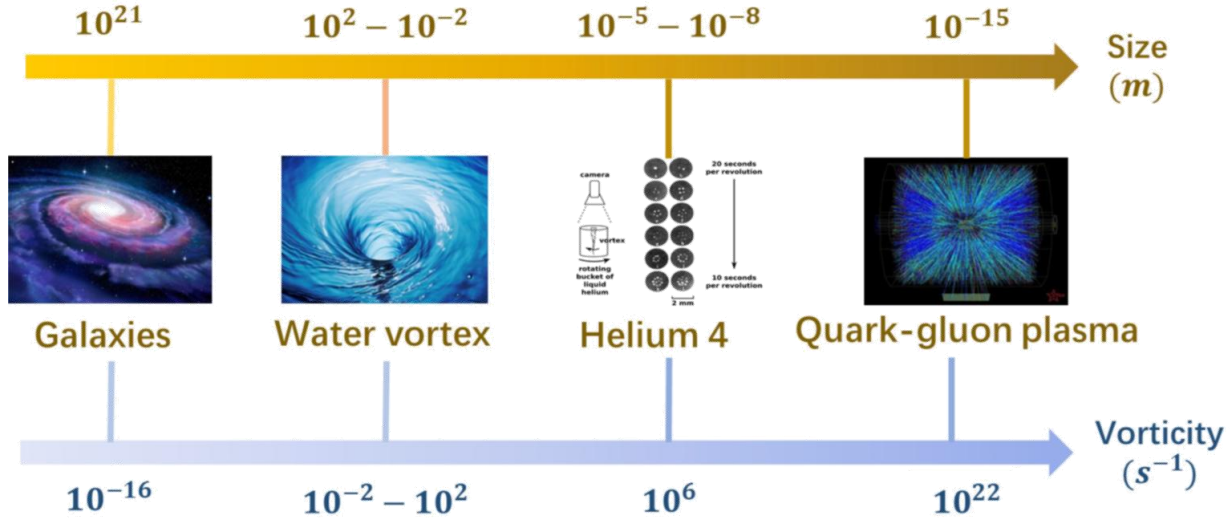
Global polarization of Λ hyperons



STAR, Nature 548, 62 (2017)

- **Vorticity estimating from the polarization**

$$\omega \approx \frac{1}{\hbar} k_B T \left(\bar{P}_\Lambda + \bar{P}_{\bar{\Lambda}}\right) \approx (9 \pm 1) \times 10^{21} s^{-1}$$

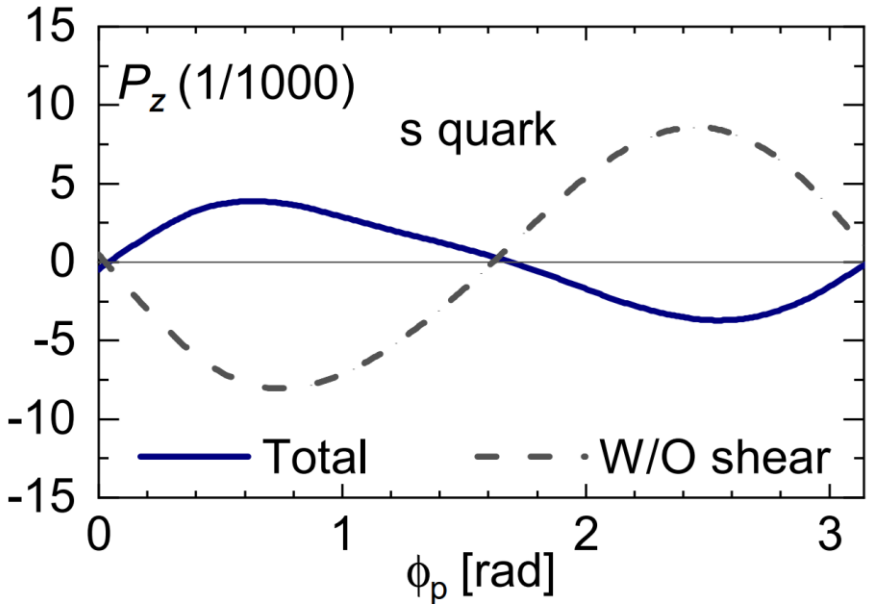


- **QGP is most vortical fluid so far.**

Becattini, et. al, PRC95, 054902 (2017)

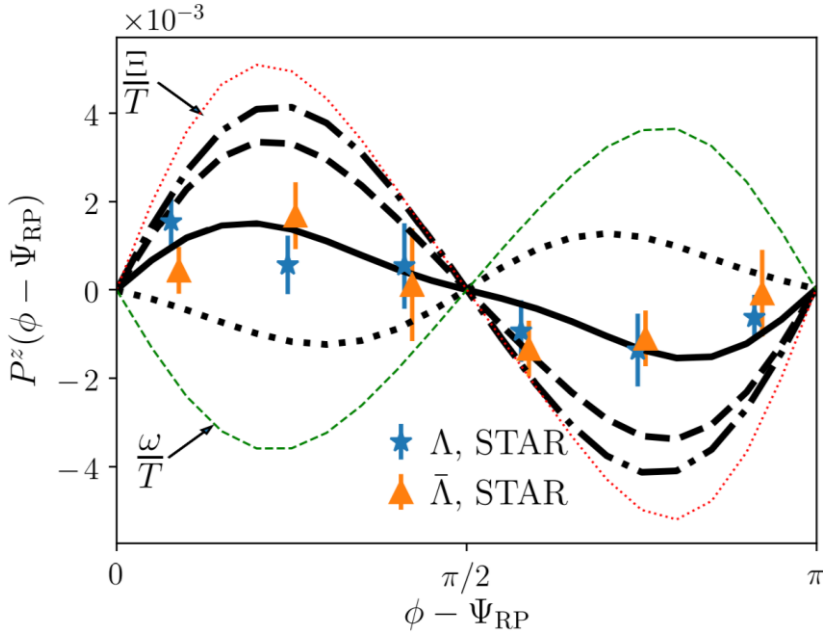
Shear induced polarization in local polarization

Shear-induced polarization plays a crucial role in understanding local spin polarization.



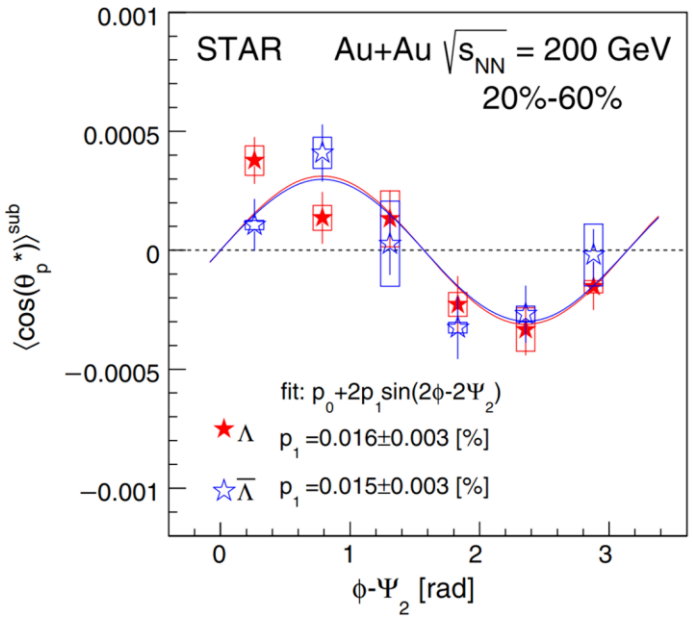
Thermal + Shear, s quark equilibrium

Fu, Liu, Pang, Song, Yin, Phys. Rev. Lett. 127.142301



Thermal + Shear, isothermal equilibrium

Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, Phys. Rev. Lett. 127, 272302



Experimental data

STAR, Phys. Rev. Lett. 123, 132301

Puzzle: is shear induced polarization dissipative?

- **Commonly used:** T-odd/even ⟷ dissipative/non-dissipative

  Example: Ohm law

$$j = \sigma E \quad \xrightarrow{\text{Time reversal}} \quad j \to -j \;\; E \to E$$

σ: T-odd → dissipative

- **Shear-induced polarization**

$$s^i \sim C^{ijk}\partial_{\langle j}u_{k\rangle} \quad \xrightarrow{\text{Time reversal}} \quad s^i \to -s^i \;\; \partial_{\langle j}u_{k\rangle} \to -\partial_{\langle j}u_{k\rangle}$$

C: T-even → non-dissipative?

?

- **But, shear viscous tensor in hydrodynamics is dissipative!**

Our strategy

- **Our strategy:** using H theorem and entropy production rate

Classical kinetic theory:

H theorem,

$$\frac{dH}{dt} \geq 0$$

Entropy production rate

$$\partial_\mu s^\mu \geq 0$$

Well-established



Quantum kinetic theory:

H theorem,

?

Entropy production rate

?

Unknown

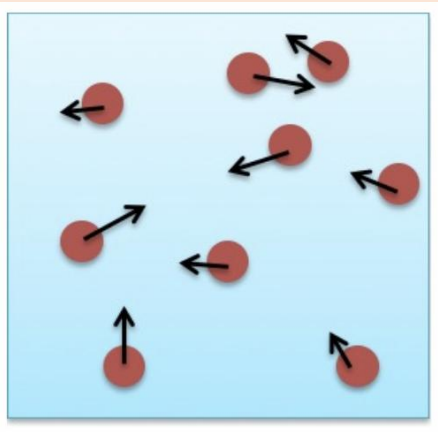
Revisit classical kinetic theory

Relativistic kinetic equation

➢ **Relativistic Boltzmann equation:**

$$\frac{d}{dt}f_p \equiv \frac{\partial}{\partial t}f_p + \dot{\boldsymbol{x}}\cdot\boldsymbol{\nabla}_x f_p + \dot{\boldsymbol{p}}\cdot\boldsymbol{\nabla}_p f_p = C\left[f_p\right]$$

distribution function



Distribution function:
Probability to find particles in a small volume of phase space

Assumptions:
Mean free path >> collision length scaling

Particle's velocity:

$$\dot{\boldsymbol{x}} = \boldsymbol{\nabla}_p \varepsilon$$

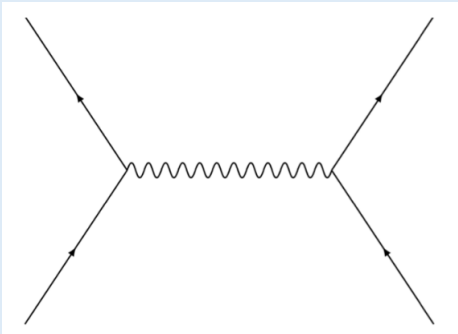
ε **: Particle's energy**

Effective force:

$$\dot{\boldsymbol{p}} = \boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}$$

Collision term

Describe the interaction between particles



Boltzmann H theorem in classical kinetic theory

➢ **Conventional Boltzmann's H function**

$$H[f_p] = \int \frac{d^3p}{(2\pi)^3} \mathcal{H}[f_p]$$

For fermions: $\mathcal{H}[f] = -f \ln f - (1-f)\ln(1-f)$

H function is the entropy density well-known in statistic physics.

H-theorem $\dfrac{dH}{dt} \geq 0$

Second law of thermodynamics in the theoretical framework of classical kinetic theory

Entropy current in a relativistic system

Velocity × distribution function

Conserved current

$$j^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} f_p$$

Entropy current

$$s^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} \mathcal{H}[f_p]$$

Velocity × H function

G. S. Rocha, D. Wagner, G. S. Denicol, J. Noronha, and D. H. Rischke, Entropy 26, 189 (2024)

Test the entropy current (I)

- **It is straightforward to find that, up to $O(\partial)$**

$$s^\mu = \frac{P}{T}u^\mu + \frac{1}{T}u_\nu T^{\mu\nu} - \frac{\mu}{T}j^\mu$$

Current:

$$j^\mu = \int \frac{d^3p}{(2\pi)^3 E_p} p^\mu f_p$$

Energy-momentum:

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 E_p} p^\mu p^\nu f_p$$

✓ **It agrees with the results from relativistic hydrodynamics.**

✓ **It is a relativistic extension of thermodynamic relation.**

$$s = (P + \varepsilon - \mu n)/T$$

W. Israel and J. M. Stewart, Annals Phys. 118, 341 (1979)

Test the entropy current (II)

- **One can reproduce the second law of thermodynamics by using relativistic entropy current in classical kinetic theory.**

$$\partial_\mu s^\mu = \frac{1}{4}\int \frac{d^3p}{2E_p} d\Gamma_{kk'\to pp'} f_k f_{k'}$$

$$(1-f_p)(1-f_{p'})\left[(1-X)\ln X\right] \geq 0$$

$$X = \frac{f_p f_{p'}(1-f_k)(1-f_{k'})}{(1-f_p)(1-f_{p'})f_k f_{k'}}$$

Second law of thermodynamics

Shear viscous tensor in classical kinetic theory

- **Consider a perturbation around the equilibrium**

$$f_p = f_p^{(0)} + \delta f_p$$

Shear viscous tensor

$$\pi^{\mu\nu} = T^{\langle\mu\nu\rangle} = \int \frac{d^3p}{(2\pi)^3 E_p} p^{\langle\mu} p^{\nu\rangle} \delta f_p$$

- **Inserting it to entropy current yields,**

$$\begin{aligned}
\partial_\mu s^\mu_{(1)} &= \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} \delta f_p \partial_\mu \ln\left(\frac{1-f_p^{(0)}}{f_p^{(0)}}\right) \\
&= \frac{\partial_{\langle\mu} u_{\nu\rangle}}{T} \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} p^{\langle\mu} p^{\nu\rangle} \delta f_p \geq 0
\end{aligned}$$

shear viscous tensor

The shear viscous tensor contributes to an increase in entropy → dissipative

Brief summary in classical kinetic theory

Step 1: Construct entropy current

Conserved current $j^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} f_p$ → Entropy current $s^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} \mathcal{H}[f_p]$

Step 2: Test two physical conditions:

(a) Is it satisfied the relativistic hydrodynamics?

$$s^\mu = \frac{P}{T}u^\mu + \frac{1}{T}u_\nu T^{\mu\nu} - \frac{\mu}{T}j^\mu$$

(b) Is entropy production rate positive-define?

$$\partial_\mu s^\mu \geq 0$$

Step 3: Consider perturbation related to shear tensor and compute the entropy production rate

shear viscous tensor

$$\partial_\mu s^\mu_{(1)} = \frac{\partial_{\langle\mu} u_{\nu\rangle}}{T} \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} p^{\langle\mu} p^{\nu\rangle} \delta f_p \geq 0$$

Extension to quantum kinetic theory

Which kinds of quantum kinetic theory we adopted?

	✓ Chiral kinetic theory (massless fermions, right handed fermions only)	✗ Quantum kinetic theory (massive fermions)
Polarization in phase space	**Right-handed Wigner function**	**Axial-vector Wigner function**
Polarization after momentum integration	**Conserved current** $\partial_\mu j_R^\mu = 0$	**Not conserved current** $\partial_\mu j_5^\mu \neq 0$
Contribute to entropy	**contribute to entropy production rate directly** $s_R^\mu = \frac{P}{T}u^\mu + \frac{1}{T}u_\nu T^{\mu\nu} - \frac{\mu_R}{T}j_R^\mu$	**Not contribute to entropy production rate directly**

What is chiral kinetic theory?

Chiral kinetic theory

- **Boltzmann equation** ⟺ $\frac{\partial}{\partial t} f_p + \dot{\boldsymbol{x}} \cdot \boldsymbol{\nabla}_x f_p + \dot{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_p f_p = C[f_p]$
- **Massless fermions**
- **Quantum corrections** ⟺

$$\begin{aligned} \varepsilon &= |\boldsymbol{p}| + \hbar \ldots \\ \dot{\boldsymbol{x}} &= \boldsymbol{\nabla}_p \varepsilon + \hbar \ldots \\ \dot{\boldsymbol{p}} &= \boldsymbol{E} + (\boldsymbol{\nabla}_p \varepsilon) \times \boldsymbol{B} + \hbar \ldots \end{aligned}$$

D. T. Son and N. Yamamoto, Phys. Rev. Lett. 109, 181602 (2012)
M. A. Stephanov and Y. Yin, Phys. Rev. Lett. 109, 162001 (2012)
J.-Y. Chen, S. Pu, Q. Wang, and X.-N. Wang, Phys. Rev. Lett. 113, 182302 (2014)
Y. Hidaka, S. Pu, and D.-L. Yang, Phys. Rev. D 95, 091901 (2017)

Chiral kinetic equation in relaxation-time approximation

- **Chiral kinetic equation**

$$\left[p\cdot\partial+\hbar\left(\partial_\mu S^{\mu\nu}_{(n)}\right)\partial_\nu\right] f_p^{(n)} = \overline{\mathcal{C}}\left[f_p^{(n)}\right] \qquad S^{\mu\nu}_{(n)} = \frac{\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta}{2\,(p\cdot n)}$$

relaxation-time approximation

$$\overline{\mathcal{C}}\left[f_p^{(n)}\right] = -\frac{1}{\tau_R}\left(p\cdot u+\hbar\frac{p\cdot\mathcal{R}}{(p\cdot u)^2}\right)\delta f_p$$

- **Power counting:**

$$\bar{\mathcal{C}}\sim\mathcal{O}\left(\partial^1\right),\ \delta f_p\sim\mathcal{O}\left(\partial^1\right)\rightarrow\mathcal{R}^\mu\sim\mathcal{O}\left(1\right)$$

- **Parameterization:**

$$\mathcal{R}^\mu = c_1 u^\mu + c_2\bar{p}^\mu + c_3\bar{R}^\mu$$

c_1: quantum corrections to RTA form of collision

Entropy flow in chiral kinetic theory

➢ **Entropy current: replace the f in current by H function**

For simplicity, we neglect the electromagnetic fields.

$$J^{\mu}=2\int\frac{d^4p}{(2\pi)^3}\bar{\epsilon}_{(u)}\delta\left(p^2\right)\left\{\left(p^{\mu}+\hbar S^{\mu\nu}_{(u)}\partial_{\nu}\right)f_p^{(u)}-\hbar S^{\mu\nu}_{(u)}\mathcal{C}_{\nu}\left(f_p^{(u)}\right)\right\}$$

$$s^{\mu}=2\int\frac{d^4p}{(2\pi)^3}\bar{\epsilon}_{(u)}\delta\left(p^2\right)\left\{\left(p^{\mu}+\hbar S^{\mu\nu}_{(u)}\partial_{\nu}\right)\mathcal{H}\left[f_p^{(u)}\right]-\hbar S^{\mu\nu}_{(u)}\mathcal{C}_{\nu}\left(\mathcal{H}\left[f_p^{(u)}\right]\right)\right\}$$

quantum corrections **collision kernel**

Also see early discussion in Di-Lun Yang, Phys. Rev. D 98, 076019 (2018)

H theorem in the chiral kinetic theory

- **Using the similar relaxation-time approach to the collision kernel in entropy, we compute the entropy production rate as:**

$$\partial_\mu s^\mu = \frac{1}{\tau_R}\int \frac{d^3p}{(2\pi)^3}\left\{\left[(1+\hbar c_1)\frac{(\delta f_p)^2}{f_p^{eq}\left(1-f_p^{eq}\right)}\right]_{p_0=|\boldsymbol{p}|} + \left[(1+\hbar c_1)\frac{(\delta f_p)^2}{f_p^{eq}\left(1-f_p^{eq}\right)}\right]_{p_0=-|\boldsymbol{p}|}\right\}$$

c_1: quantum corrections to RTA form of collision

$$1+\hbar c_1 > 0$$

Power-counting

$$\partial_\mu s^\mu \geq 0$$

H theorem

Brief summary in chiral kinetic theory

✓ **Step 1**: Construct entropy current

Conserved current

$$j_R^\mu = 2\int \frac{d^4p}{(2\pi)^3}\bar{\epsilon}_{(u)}\delta\left(p^2\right)\left(p^\mu + \hbar S_{(u)}^{\mu\nu}\mathcal{D}_\nu\right)\boxed{f_p^{(u)}}$$

Entropy current

$$\Rightarrow \quad s_R^\mu = 2\int \frac{d^4p}{(2\pi)^3}\bar{\epsilon}_{(u)}\delta\left(p^2\right)\left(p^\mu + \hbar S_{(u)}^{\mu\nu}\mathcal{D}_\nu\right)\boxed{\mathcal{H}\left[f_p^{(u)}\right]}$$

✓ **Step 2**: Test physical condition: Is entropy production rate positive-define?

$$\partial_\mu s^\mu \sim \frac{1}{\tau_R}\int \frac{d^3p}{(2\pi)^3}\left(1+\hbar c_1\right)\frac{\left(\delta f_p\right)^2}{f_p^{eq}\left(1-f_p^{eq}\right)} \qquad 1+\hbar c_1 > 0 \quad \Rightarrow \quad \partial_\mu s^\mu \geq 0$$

Step 3: Compute the entropy production rate related to the shear-induced polarization

Anomalous Hall effect

➢ **Interaction corrections to the axial current:**

$$\delta S_R^{<,\mu} = 2\pi p^\mu \delta\left(p^2\right) \frac{\tau_R}{u\cdot p} f_p^{(0)}\left(1-f_p^{(0)}\right)\beta p^\rho p^\sigma \partial_{\langle\rho} u_{\sigma\rangle}$$

➡ **Classical term**

Anomalous Hall effect ⬅

$$-\frac{4\pi\hbar}{u\cdot p}\frac{\tau_R}{\tau_R'}\delta\left(p^2\right) S_{(u)}^{\mu\nu} f_p^{(0)}\left(1-f_p^{(0)}\right) c_3\beta p^\sigma \partial_{\langle\nu} u_{\sigma\rangle}$$

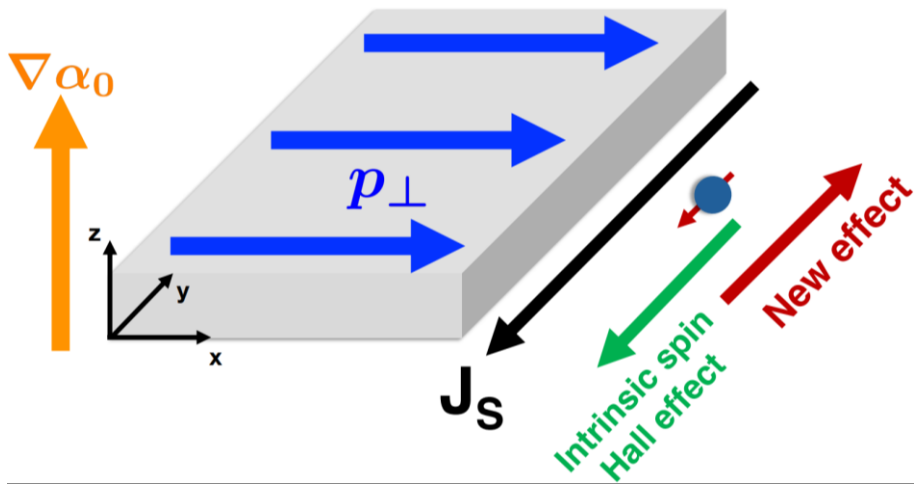


Figure from C.Yi, et al, arXiv:2509.00377

Also see the results from QED in HTL:
Shuo Fang, Shi Pu, Phys. Rev. D 111, 034015

- **Coupling constant independence originated from the "side jump" effect, which is known in condensed matter physics.**

Naoto Nagaosa, et al, Rev. Mod. Phys. 82, 1539

Entropy production rate at $O(\partial^2)$

- **Entropy production rate:**

$$\partial_\mu s_R^\mu = \delta T^{\mu\nu}\,\partial_\mu(\beta u_\nu) + \delta j_R^\mu\,\partial_\mu(\beta\mu_R)$$

Shear-induced polarization vanishes in both $\delta T^{\mu\nu}$ and δj_R^μ.

$$= \frac{4}{15}\tau_R\beta^2\partial_{\langle\mu}u_{\nu\rangle}\partial^{\langle\mu}u^{\nu\rangle}\int\frac{d^4p}{(2\pi)^3}\delta(p^2)(p\cdot u)^3 f_p^{(0)}\left(1-f_p^{(0)}\right)\geq 0$$

- **At $O(\partial^2)$ (1st –order hydro): Total entropy increases due to the classical shear viscous tensor;**
- **Shear-induced polarization and anomalous Hall effect vanish after momentum integral, giving no contribution to $\partial_\mu s^\mu$ at $O(\partial^2)$.**

Go beyond $O(\partial^2)$?

Entropy production rate beyond $O(\partial^2)$

- **Matching to the 2nd –order hydro:**

From Yi Yin's talk at Hefei Spin Workshop

Also see:
Jeremy Hansen's talk,
at 16:30, 11/18/2026, in Parallel 2

Yi Yin's talk, at 10:30, 11/20/2026,
in Plenary

$$T\partial \cdot s = \underbrace{\frac{\eta}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}}_{O(\partial^2)} + \underbrace{\lambda_1\sigma_{\mu\nu}\sigma^{\mu\rho}\sigma^{\nu}_{\rho}}_{O(\partial^3)}$$

- $O(\partial^2)$: Solely from classical shear tensor; shear-induced polarization gives zero contribution, which is consistent with our CKT result;
- $O(\partial^3)$: Does shear-induced polarization contribute to λ_1?

Next for us: Extend CKT to $O(\partial^3)$ for shear-induced polarization!

Summary

- **We introduce entropy flow and the H-theorem in the chiral kinetic theory.**

- **Up to $O(\partial^2)$, the total entropy increases, but the shear-induced polarization does not contribute to the entropy production rate.**

- **It is essential to extend CKT to $O(\partial^3)$ to test whether shear-induced polarization contributes to 2nd-order hydro.**

Thank you for your listening!

Any comments or suggestions are welcome!

Backup

Entropy current in a relativistic system

- **Relativistic Boltzmann equation:**

$$\frac{d}{dt}f_p \equiv \frac{\partial}{\partial t}f_p + \dot{\boldsymbol{x}}\cdot\boldsymbol{\nabla}_x f_p + \dot{\boldsymbol{p}}\cdot\boldsymbol{\nabla}_p f_p = C\left[f_p\right]$$

- **Construct entropy current:**

Conserved current

$$j^\mu = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E_p} f_p$$

Velocity × distribution function

Entropy current

$$s^\mu = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E_p}\mathcal{H}[f_p]$$

Velocity × H function

G. S. Rocha, D. Wagner, G. S. Denicol, J. Noronha, and D. H. Rischke, Entropy 26, 189 (2024)

Test the entropy current

✓ **Test 1: Up to $O(\partial)$, it agrees with the results from relativistic hydrodynamics.**

$$s^\mu = \frac{P}{T}u^\mu + \frac{1}{T}u_\nu T^{\mu\nu} - \frac{\mu}{T}j^\mu$$

W. Israel and J. M. Stewart, Annals Phys. 118, 341 (1979)

✓ **Test 2: It reproduces the second law of thermodynamics.**

$$\partial_\mu s^\mu = \frac{1}{4}\int \frac{d^3p}{2E_p} d\Gamma_{kk'\to pp'} f_k f_{k'} \qquad X = \frac{f_p f_{p'}(1-f_k)(1-f_{k'})}{(1-f_p)(1-f_{p'})f_k f_{k'}}$$

$$(1-f_p)(1-f_{p'})\left[(1-X)\ln X\right] \geq 0$$

H theorem in classical kinetic theory

➢**Positive entropy production rate:**

$$\partial_\mu s^\mu = \frac{1}{4}\int \frac{d^3p}{2E_p} d\Gamma_{kk'\to pp'} f_k f_{k'}$$

$$(1-f_p)(1-f_{p'})\left[(1-X)\ln X\right] \geq 0$$

$$X = \frac{f_p f_{p'}(1-f_k)(1-f_{k'})}{(1-f_p)(1-f_{p'})f_k f_{k'}}$$

Calculation of the entropy production rate

$$\delta f_p = f_p^{(0)}(1 - f_p^{(0)})\chi_p$$

➢ **Insert into** $\frac{d}{dt} f_p^{(0)} = C[\delta f_p]$

$$\chi_p = A(\mathbf{p}) p^i p^j \partial_{\langle i} u_{j\rangle} = A(\mathbf{p}) p^i p^j \frac{\pi_{ij}}{2\eta}$$

With the constraint:

$$f_p^{(0)}(1 - f_p^{(0)}) p^{\langle i} p^{j\rangle} = \frac{T}{2E_p} \int d\Gamma_{kk'\to pp'} f_k^{(0)} f_{k'}^{(0)} f_p^{(0)} f_{p'}^{(0)} \left[B^{ij}(\mathbf{p}) + B^{ij}(\mathbf{p}') - B^{ij}(\mathbf{k}) - B^{ij}(\mathbf{k}')\right]$$

$$B^{ij}(\mathbf{k}) = A(\mathbf{k}) k^{\langle i} k^{j\rangle}$$

Calculation of the entropy production rate

➢ **The correction to entropy flow:**

$$s^{\mu}_{(1)} = \int \frac{d^3p}{(2\pi)^3}\frac{p^{\mu}}{E_p}\ln\left(\frac{1-f_p^{(0)}}{f_p^{(0)}}\right)\delta f_p$$

$$\begin{aligned}\partial_{\mu}s^{\mu}_{(1)} &= \int \frac{d^3p}{(2\pi)^3}\frac{p^{\mu}}{E_p}\delta f_p\partial_{\mu}\ln\left(\frac{1-f_p^{(0)}}{f_p^{(0)}}\right)\\ &= \frac{\partial_{\mu}u_{\nu}}{T}\int \frac{d^3p}{(2\pi)^3}\frac{1}{E_p}p^{\mu}p^{\nu}\delta f_p = \frac{\pi^{\mu\nu}\pi_{\mu\nu}}{2\eta T} \geq 0\end{aligned}$$

Review on the chiral kinetic theory

➢ **Kinetic equation for distribution function:**

on-shell condition

$$\delta\left(p^2-\hbar\frac{B\cdot p}{p\cdot u}\right)\left[p\cdot\Delta+\hbar\frac{S^{\mu\nu}_{(u)}E_\mu}{p\cdot u}\Delta_\nu\right.$$

$$\left.+\hbar S^{\mu\nu}_{(u)}\left(\partial_\mu F_{\rho\nu}\right)\partial_p^\rho+\hbar\left(\partial_\mu S^{\mu\nu}_{(u)}\right)\Delta_\nu\right]f_p^{(u)}$$

$$=\ \delta\left(p^2-\hbar\frac{B\cdot p}{p\cdot u}\right)\overline{C}$$

collision term

Yoshimasa Hidaka et al. Phys. Rev. D 95, 091901 (2017)

Yoshimasa Hidaka et al. Phys. Rev. D 97, 016004 (2018)

Chiral kinetic equation in relaxation-time approximation

- **Chiral kinetic equation**

$$\left[p\cdot\partial+\hbar\left(\partial_\mu S^{\mu\nu}_{(n)}\right)\partial_\nu\right]f_p^{(n)}=\overline{\mathcal{C}}\left[f_p^{(n)}\right] \qquad S^{\mu\nu}_{(n)}=\frac{\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta}{2\left(p\cdot n\right)}$$

relaxation-time approximation

$$\overline{\mathcal{C}}\left[f_p^{(n)}\right]=-\frac{1}{\tau_R}\left(p\cdot u+\hbar\frac{p\cdot\mathcal{R}}{\left(p\cdot u\right)^2}\right)\delta f_p$$

- **Power counting:**

$$\bar{\mathcal{C}}\sim\mathcal{O}\left(\partial^1\right),\ \delta f_p\sim\mathcal{O}\left(\partial^1\right)\rightarrow\mathcal{R}^\mu\sim\mathcal{O}\left(1\right)$$

- **Parameterization:**

$$\mathcal{R}^\mu=c_1u^\mu+c_2\bar{p}^\mu+c_3\bar{R}^\mu \qquad \bar{p}^\mu=p_\nu\Theta^{\mu\nu},\ \bar{R}^\mu=\left(\Theta^{\mu\nu}-\frac{\bar{p}^\mu\bar{p}^\nu}{\bar{p}^2}\right)R_\nu \qquad c_{i=1,2,3}\sim\mathcal{O}\left(1\right)$$

Review on the chiral kinetic theory

➢ **The canonical energy momentum tensor and current:**

$$T^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4}\left(p^\mu \grave{S}^{<\nu} + p^\nu \grave{S}^{<\mu}\right) = u^\mu u^\nu \epsilon - P\Theta^{\mu\nu} + \Pi^{\mu\nu}_{non} + \Pi^{\mu\nu}_{dis}$$

$$J^\mu = 2\int \frac{d^4p}{(2\pi)^4}\grave{S}^{<\mu} = nu^\mu + \nu^\mu_{non} + \nu^\mu_{dis},$$

Yoshimasa Hidaka et al. Phys. Rev. D 97, 016004 (2018)

➢ **The conservation equations with chiral anomaly:**

$$\partial_\mu J^\mu = -\frac{\hbar}{4\pi^2}E_\mu B^\mu,\ \partial_\mu T^{\mu\nu} = F^{\nu\mu}J_\mu$$

D. T. Son and P. Surowka, Phys. Rev. Lett. 103, 191601 (2009)

Di-Lun Yang, Phys. Rev. D 98, 076019 (2018)

Entropy flow in chiral kinetic theory

- **Consistent with the entropy flow in the chiral anomalous hydrodynamics:**

$$s^{\mu}_{R,leq} = \beta\left(u^{\mu}P + T^{\mu\nu}_{leq}u_{\nu} - \mu_R J^{\mu}_{leq} + \xi_B B^{\mu} + 2\xi_{\omega}\omega^{\mu}\right)$$

- **No entropy production in local equilibrium**

$$\partial_{\mu} s^{\mu}_{R,leq} = 0$$

D. T. Son and P. Surowka, Phys. Rev. Lett. 103, 191601 (2009)

Di-Lun Yang, Phys. Rev. D 98, 076019 (2018)

Relaxation time approach

$$\mathcal{C}_\nu[\mathcal{H}(f_p^{(n)})] = \mathcal{C}_\nu^{leq} + \mathcal{C}_\nu^{\delta}$$

$$\mathcal{C}_\nu^{leq} = (b_1 u_\nu + b_2 p_\nu + b_3 \partial_\nu^p) f_p^{eq} + b_{4,\nu} f_p^{(0)}$$

$$\mathcal{C}_\nu^{\delta} = \frac{1}{\tau_R''}(a_1 u_\nu + a_2 p_\nu + a_3 \partial_\nu^p)\delta f_p$$

$$f_p^{eq} = \frac{1}{e^g + 1},\ g = \beta p\cdot u - \beta\mu_R + \frac{\hbar S_{(u)}^{\mu\nu}}{2}\partial_\mu(\beta u_\nu)$$

$$f_p^{(0)} = \frac{1}{\exp(\beta p\cdot u - \beta\mu_R) + 1}$$

- $\mathcal{C}_\nu[\mathcal{H}(f_p^{(n)})]$ **is linear to** $f_p^{(n)}$**;**
- $s^\mu \sim \mathcal{O}(\partial^1) \rightarrow b_i \sim \mathcal{O}(\partial^0),\ b_{4,\nu} \sim \mathcal{O}(\partial^1)$ **;** $a_i \sim \mathcal{O}(\partial^0)$**;**
- **After momentum integration, the terms related to collision vanish.**

Relaxation time approach

- **Insert δf_p to the axial current:**

$$\delta f_p = \frac{\tau_R}{u\cdot p} f_p^{(0)}(1-f_p^{(0)})\beta p^\mu p^\nu \partial_{\langle\mu} u_{\nu\rangle} + ...$$

$$\delta S_R^{<,\mu} = 2\pi p^\mu \delta\left(p^2\right)\delta f_p + 2\pi\hbar\delta\left(p^2\right) S_{(u)}^{\mu\alpha}\left(\partial_\alpha \delta f_p - \mathcal{C}_\alpha\left[\delta f_p\right]\right)$$

$$\mathcal{C}_\nu[\delta f_p] = \frac{1}{\tau_R'}(d_1 u_\nu + d_2 p_\nu + d_3\partial_\nu^p)\delta f_p$$

Entropy production rate

➢ **Entropy flow related to the shear tensor**

$$\delta s^{\mu} = \beta u_{\nu} \delta T^{\mu\nu} - \beta \mu_R \delta J^{\mu}$$

$$\delta T^{\mu\nu} = \frac{4}{15} \tau_R \beta \partial^{\langle \mu} u^{\nu \rangle} \int \frac{d^4 p}{(2\pi)^3} \delta\left(p^2\right) (p \cdot u)^3 f_p^{(0)} \left(1 - f_p^{(0)}\right)$$

$$\delta J^{\mu} = 0$$

shear induced polarization vanish after momentum integration