Axial Symmetry Breaking and Chiral Separation Effect in SU(3) NJL Model

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Outline

01 Introduction

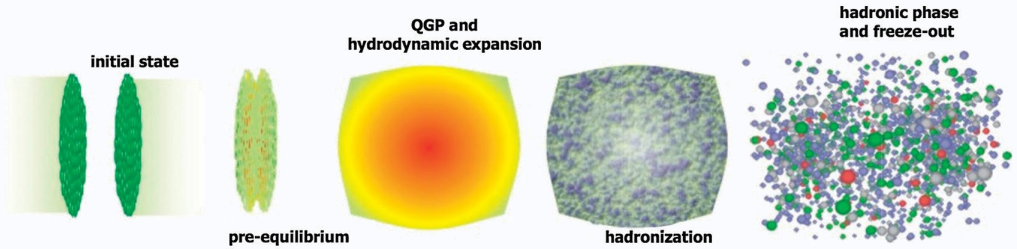
02 SU(3) NJL model

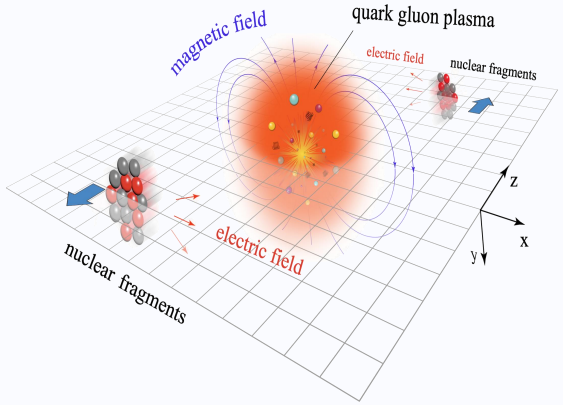
03 Chiral separation effect

04 Susceptibility splittings

Extremely strong magnetic field

Heavy-ion collision

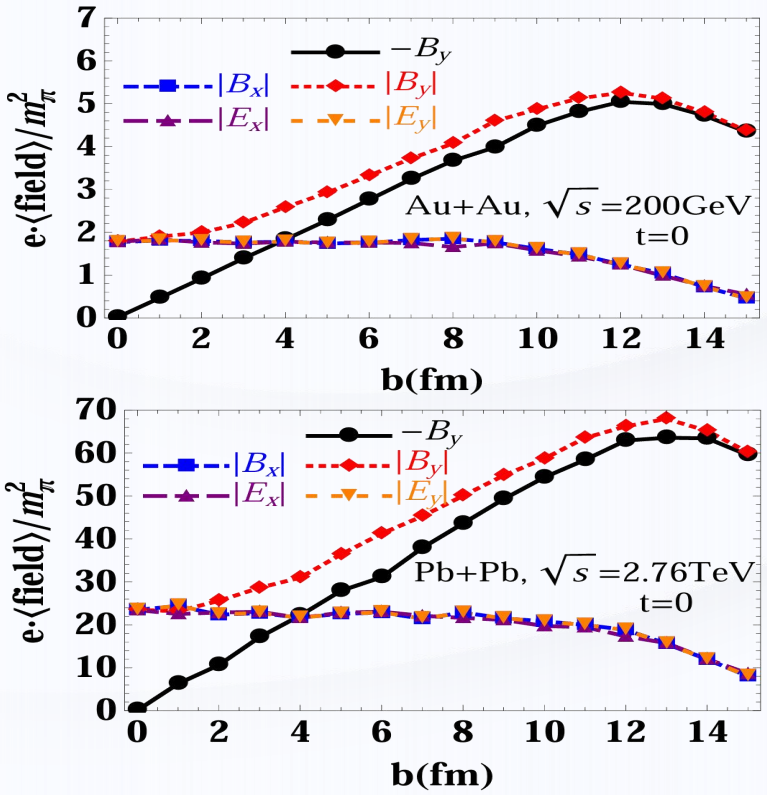




- Non-central heavy-ion collision
- Generate strong but short-lived magnetic fields

M. I. Abdulhamid et al., Phys. Rev. X 14, 011028 (2024), arXiv:2304.03430.

Initial magentic field



W.-T. Deng and X.-G. Huang, Phys. Rev. C 85, 044907 (2012), arXiv: 1201.5108.

$B \sim 10^{18} - 10^{20}$ Gauss

Chiral separation effect (CSE)

Physical picture

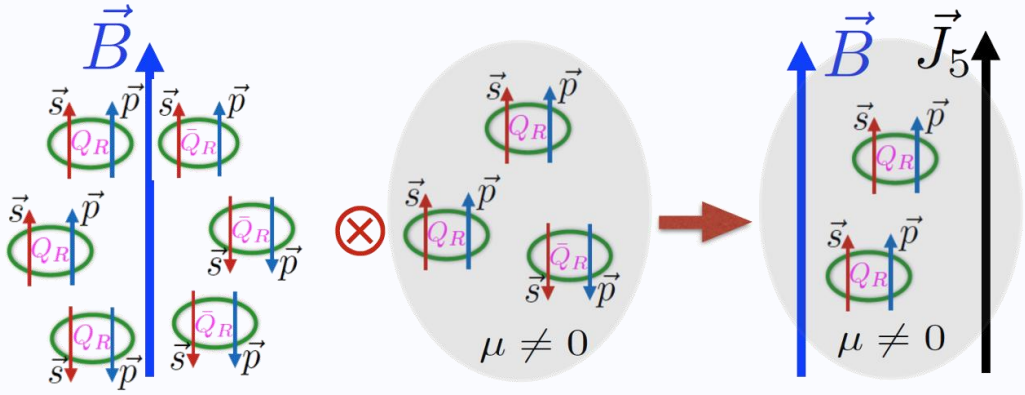


Illustration of chiral separation effect

D. E. Kharzeev, J. Liao, et al., Prog. Part. Nucl. Phys. 88 (2016) 1–28.

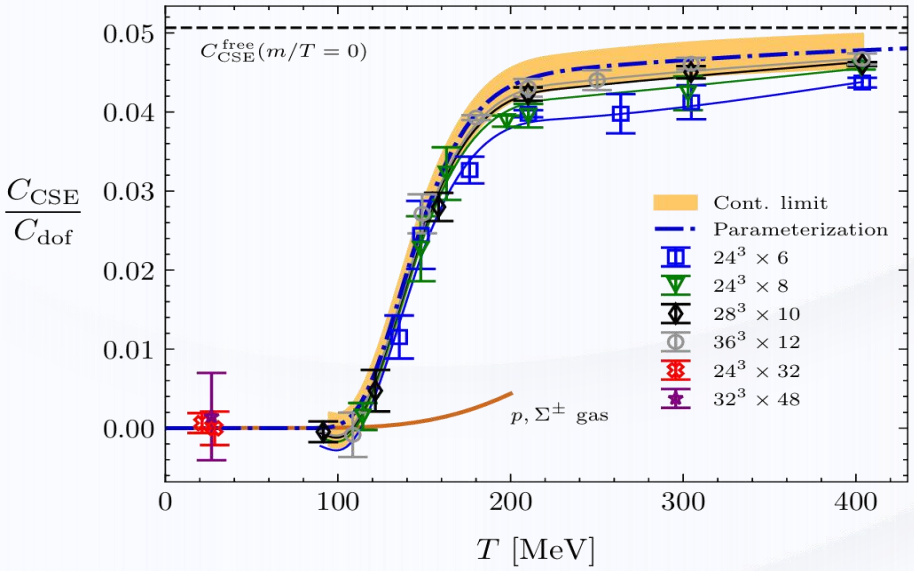
- **Charge-weighted axial current** :

$$J_{5z}^{Q} = \sum_f Q_f \frac{T}{V} \int d^4x \left\langle \bar{\psi}(x)\gamma_5\gamma_3\psi(x) \right\rangle$$

- **CSE conductivity**:

$$C_{\mathrm{CSE}}|eB| = \left. \frac{\partial J_{5z}^{Q}}{\partial \mu_q} \right|_{\mu_q=0}$$

Lattice-QCD result



B. B. Brandt et al., JHEP 02, 142 (2024), arXiv:2312.02945.

- **Weight factor**:

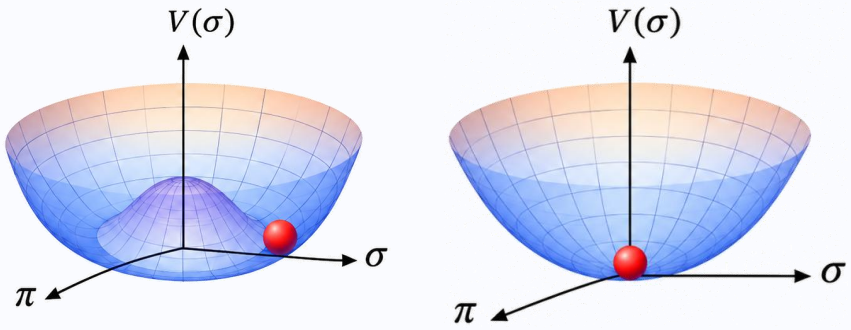
$$C_{\mathrm{dof}} = \frac{N_c}{3} \sum_{f=u,d,s} Q_f^2 = \frac{2}{3}$$

The CSE conductivity is suppressed at low *T* and approaches the chiral limit at high *T*.

From QCD Symmetries to the CSE

Chiral symmetry

$$SU(3)_L \times SU(3)_R \xrightarrow{\langle \bar{q}q \rangle \neq 0} SU(3)_V$$



low T (broken) | high T (restored)

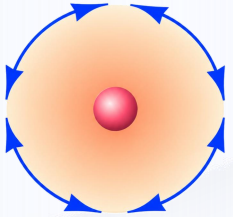
Singlet axial symmetry

$$U(1)_A^{(0)}$$

- **Quantum level**

$$\partial_\mu J_5^{\mu(0)} = \frac{N_f g^2}{16\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} \neq 0$$

Broken by the gluonic topology (axial anomaly).



Instanton (gluonic topology)

Magnetic field effect



$$[\lambda_3, Q_f] = 0$$
$$[\lambda_8, Q_f] = 0$$
$$Q_f = \mathrm{diag}\,[2/3, -1/3, -1/3]$$

Surviving

$$U(1)_A^{(3)}$$
$$U(1)_A^{(8)}$$

Connection to CSE

$$J^\mu_{5,Q} = \sum_f Q_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f = J^\mu_{5,3} + \frac{1}{\sqrt{3}} J^\mu_{5,8}$$

The axial current for the CSE is the linear combination of Noether currents corresponding to the $U(1)_A^{(3)}$ and $U(1)_A^{(8)}$ symmetries.

Motivation: A joint analysis of the CSE and the related axial symmetries in hot and dense magnetized matter

SU(3) NJL model with IMC

SU(3) NJL model

- Effective Lagrangian density with magnetic field

$$\mathcal{L}_{\text{eff}} = \sum_{f=u,d,s} \bar{\psi}_f (i\gamma_\mu D_f^\mu - m_f)\psi_f + G_S \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi} i\gamma_5 \lambda_a \psi)^2 \right]$$

$$- K\left\{ \det{}_f \left[\bar{\psi}(1+\gamma_5)\psi\right] + \det{}_f \left[\bar{\psi}(1-\gamma_5)\psi\right] \right\}$$

KMT interaction

- Thermodynamic potential under MFA

$$\Omega = \sum_{f=u,d,s} (2G_S\sigma_f^2 - \Omega_f) - 4K\sigma_u\sigma_d\sigma_s,$$

- Constituent quark mass (determined by $\partial\Omega/\partial\sigma_f = 0$.)

$$M_f \equiv m_f - 4G_S\sigma_f + 2K \prod_{f'\neq f} \sigma_{f'}$$

- **Non-renormalizable model**

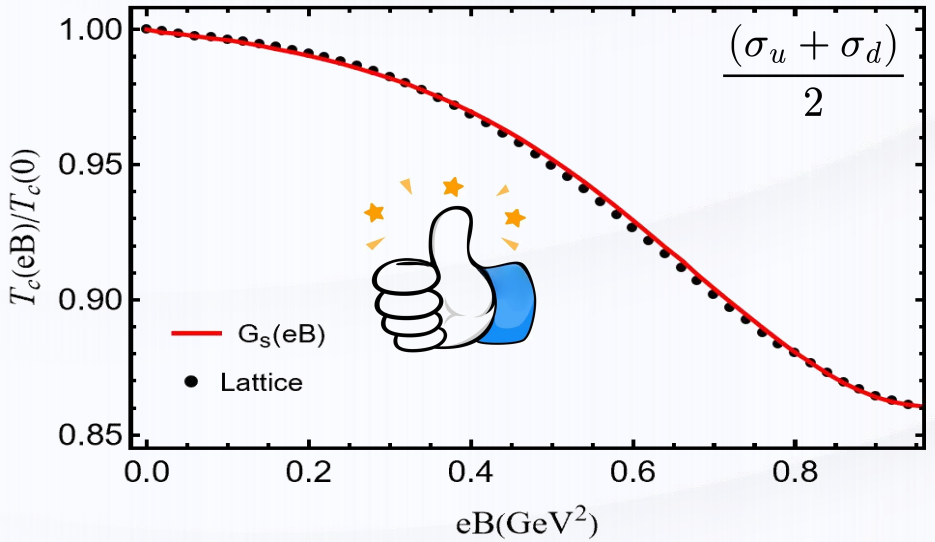
Pauli-Villars regularization → $f(M_f) \to \sum_{j=0}^{3} c_j f(\sqrt{M_f^2 + j\Lambda^2})$

Magnetic catalysis vs. IMC

MC: $G_S^0 = 4.59/\Lambda^2$

IMC: $G_S(|eB|) = \dfrac{1 + a(eB/m_\pi^2)^2 + b(eB/m_\pi^2)^3}{1 + c(eB/m_\pi^2)^2 + d(eB/m_\pi^2)^4} G_S^0$

$a = -5.64541 \times 10^{-4}$, b $= 7.10729 \times 10^{-6}$
$c = 8.34021 \times 10^{-6}$, d $= 5.94379 \times 10^{-8}$



S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992)

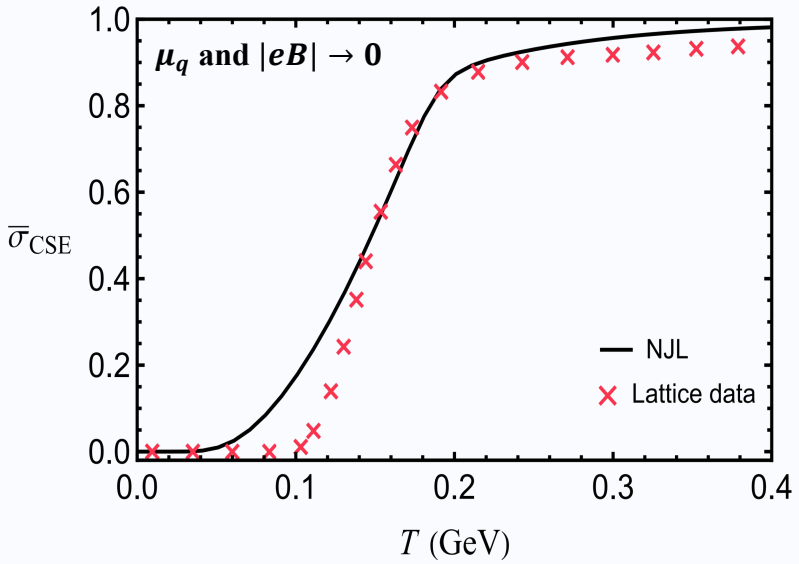
G. S. Bali, F. Bruckmann, G. Endrődi, et al., JHEP 02, 044 (2012), arXiv:1111.4956.

Chiral separation effect

CSE: $J_{5z}^{Q} = \sigma_{\mathrm{CSE}}(T, \mu_q, |eB|)\mu_q |eB| \longrightarrow \sigma_{\mathrm{CSE}}(T, \mu_q, |eB|) = \sum_{f=u,d,s} \frac{N_c Q_f^2}{4\pi^2 \mu_q} \int dp_z \left[n_f^+ \left(E_f^{(0)} \right) - n_f^- \left(E_f^{(0)} \right) \right]$

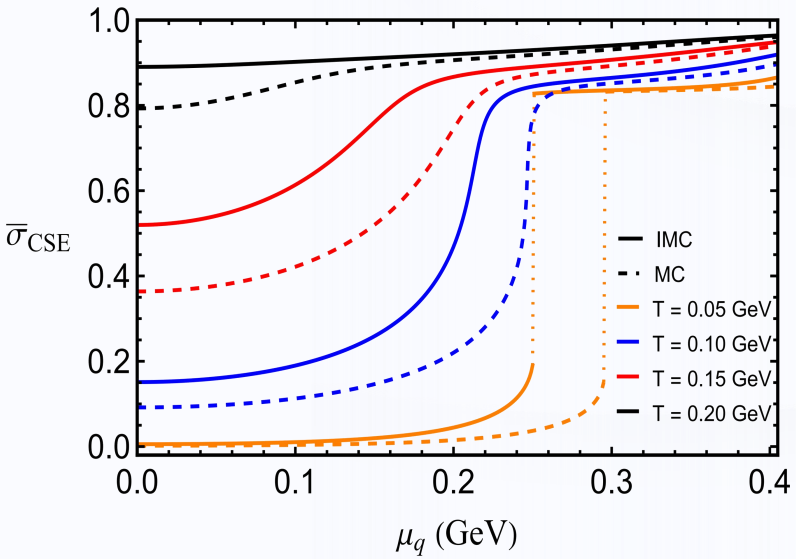
Normalized CSE conductivity: $\bar{\sigma}_{\mathrm{CSE}} \equiv 2\pi^2 \sigma_{\mathrm{CSE}} / N_c \sum_f Q_f^2$

(a) Temperature dependence



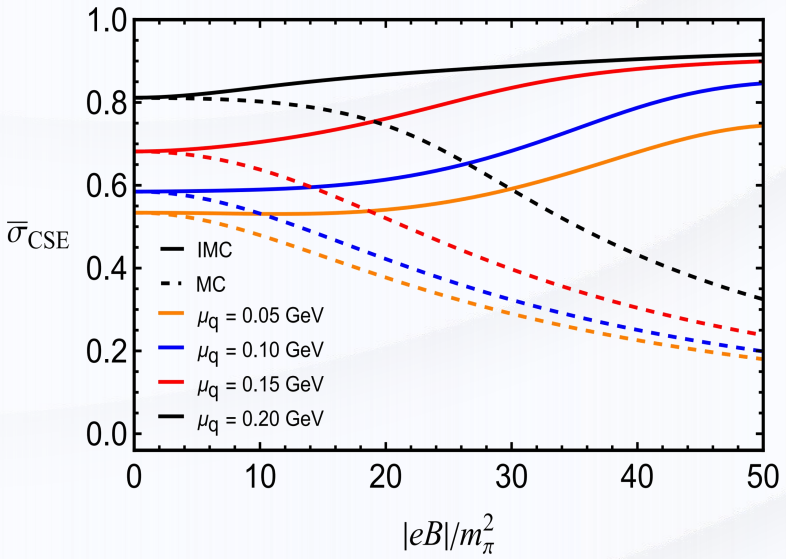
Suppressed at low T, approaching its chiral limit at high T.

(b) Chemical potential dependence



A discontinuity occurs at low T, related to first-order transition.

(c) Magnetic-field dependence



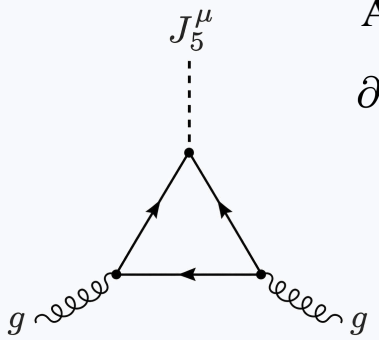
MC and IMC show opposite magnetic-field dependences.

X.-N. Zhu, X.-L. Sheng, et al., arXiv:2609.39886 (2026).

B. B. Brandt et al., JHEP 02, 142 (2024), arXiv:2312.02945.

Axial anomaly and KMT interaction

QCD Picture



Axial anomaly

$$\partial_\mu J_5^{\mu(0)} = 2i\sum_f m_f \bar{q}_f \gamma_5 q_f + \underbrace{\frac{N_f g^2}{16\pi^2} G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}}_{\text{gluonic anomaly}}$$

effective description

SU(3) NJL Picture

KMT interaction

$$\mathcal{L}_{\text{KMT}} = K\left\{\det_f\left[\bar{\psi}_f(1+\gamma_5)\psi_f\right] + \det_f\left[\bar{\psi}_f(1-\gamma_5)\psi_f\right]\right\}$$

KMT interaction effectively mimics the $U(1)_A$ breaking.

Singlet axial current $U(1)_A^{(0)}$

$$J_5^{\mu(0)} = \bar{\psi}\gamma_5\gamma^\mu T^0\psi$$

$$\partial_\mu J_5^{(0)\mu} = 2i\bar{\psi}\hat{m}\gamma_5\psi + 6iK\left\{\det\left[\bar{\psi}(1+\gamma_5)\psi\right] - \det\left[\bar{\psi}(1-\gamma_5)\psi\right]\right\}$$

quark mass | **KMT-induced $U(1)_A$ breaking**

Non-singlet axial current $U(1)_A^{(a)}$

$$J_5^{\mu(a)} = \bar{\psi}\gamma_5\gamma^\mu T^a\psi, \quad T^a = \frac{\lambda_a}{2}, a = 1, 2, \ldots 8$$

$$\partial_\mu J_5^{(a)\mu} = \underbrace{i\bar{\psi}\left\{\hat{m}, \frac{\lambda_a}{2}\right\}\gamma_5\psi}_{\text{quark mass only}}$$

M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 44, 1422 (1970); G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976); T. Kunihiro, Prog. Theor. Phys. 122, 255 (2009).

Susceptibility splittings

Magnetic field

$q_u \neq q_d$ ➡ $[\mathbf{1}, Q] = [\tau^3, Q] = 0,$ $[\tau^1, Q] \neq 0, \quad [\tau^2, Q] \neq 0.$ ➡ $SU(2)_V \longrightarrow U(1)_V^{(3)}$ $SU(2)_A \longrightarrow U(1)_A^{(3)}$ ➡ **Neutral channel as symmetry probe**

In the presence of magnetic field, the full flavor classification is no longer applicable.

Axial partners

Neutral mesons are connected by singlet and non-singlet axial transformations.

$$\delta^0 = \frac{(\bar{u}u - \bar{d}d)}{\sqrt{2}} \xleftrightarrow[U(1)_A^{(8)}]{U(1)_A^{(0)}} \pi^0 = \frac{(\bar{u}i\gamma^5 u - \bar{d}i\gamma^5 d)}{\sqrt{2}} \xleftrightarrow{U(1)_A^{(3)}} \sigma_l = \frac{(\bar{u}u + \bar{d}d)}{\sqrt{2}}$$

Susceptibility splittings

To characterize the restoration of the $U(1)_A^{(0)}(U(1)_A^{(8)})$ and non-singlet $U(1)_A^{(3)}$ symmetries.

Singlet axial splitting

$$\Delta_A \equiv \chi_{\pi^0} - \chi_{\delta^0}$$

Non-singlet axial splitting

$$\Delta_\chi \equiv \chi_{\pi^0} - \chi_{\sigma_l}$$

Difference

$$\Delta_{\text{disc}} \equiv \Delta_A - \Delta_\chi$$

X.-N. Zhu, X.-L. Sheng, et al., arXiv:2609.39886 (2026).

Axial-partner susceptibility

RPA resummation

- **Mesonic susceptibility**

$$\langle X_{f_1} X_{f_2} \rangle_c = \left[(1 - \Pi_X V_X)^{-1} \Pi_X \right]_{f_1 f_2}, \quad X = S, P$$

One-loop polarization function

- **Quark-antiquark correlator** (neutral flavor basis)

$$\Pi^X_{ff'}(p) = \delta_{ff'} \Pi^X_f(p)$$

quark propagator with magnetic field

$$\Pi^X_f(p) = -iN_c \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}[\Gamma_X G_f(q) \Gamma_X G_f(q-p)]$$

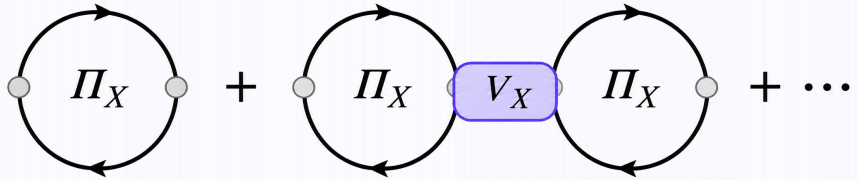
KMT interaction and flavor mixing

- **Interaction kernels (three flavors)**

$$V_S = \begin{pmatrix} 4G_S & -2K\sigma_S & -2K\sigma_d \\ -2K\sigma_S & 4G_S & -2K\sigma_u \\ -2K\sigma_d & -2K\sigma_u & 4G_S \end{pmatrix}, \; V_P = \begin{pmatrix} 4G_S & 2K\sigma_S & 2K\sigma_d \\ 2K\sigma_S & 4G_S & 2K\sigma_u \\ 2K\sigma_d & 2K\sigma_u & 4G_S \end{pmatrix}$$

MC: G_S^0 **IMC:** $G_S(|eB|)$

RPA diagram



One-loop RPA: flavor mixing

Susceptibility splitting

$$\begin{aligned} \Delta_A &= \langle P_u P_u + P_d P_d - 2P_u P_d \rangle_c / 2 \\ &\quad - \langle S_u S_u + S_d S_d - 2S_u S_d \rangle_c / 2, \\ \Delta_\chi &= \langle P_u P_u + P_d P_d - 2P_u P_d \rangle_c / 2 \\ &\quad - \langle S_u S_u + S_d S_d + 2S_u S_d \rangle_c / 2, \\ \Delta_{\mathrm{disc}} &= 2 \langle S_u S_d \rangle_c \end{aligned}$$

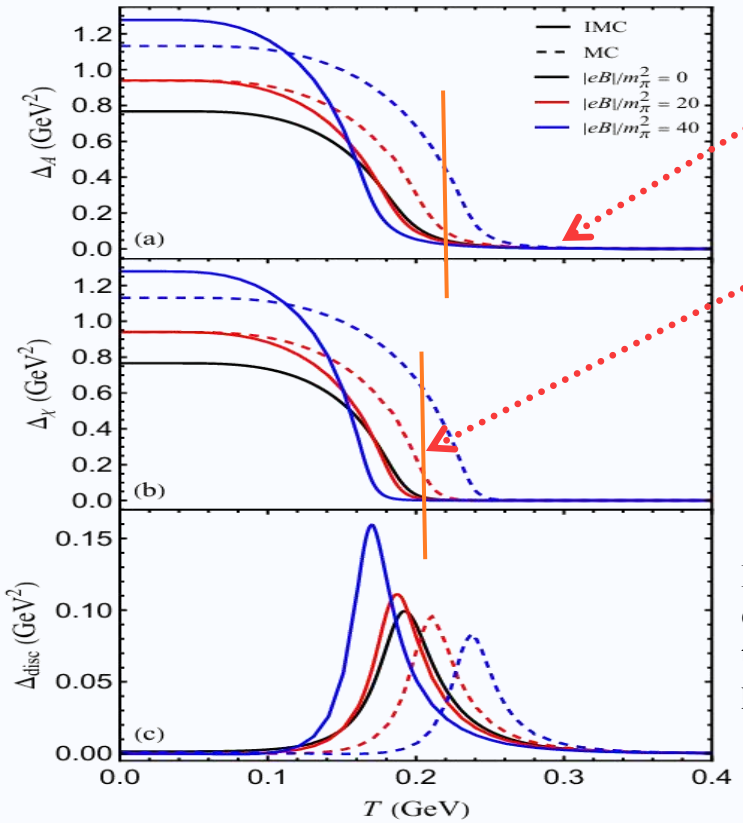
*The KMT interaction generates **off-diagonal flavor mixings** through RPA resummation.*

X.-N. Zhu, X.-L. Sheng, et al., arXiv:2609.39886 (2026).

Results of Susceptibility splittings

NJL result

$$\Delta_{A(\chi)} \equiv \chi_{\pi^0} - \chi_{\delta^0(\sigma_l)},$$
$$\Delta_{\rm disc} \equiv \Delta_A - \Delta_\chi.$$



Approximate restoration

Earlier restoration tendency

Highlight the difference of the restoration patterns

IMC scenario

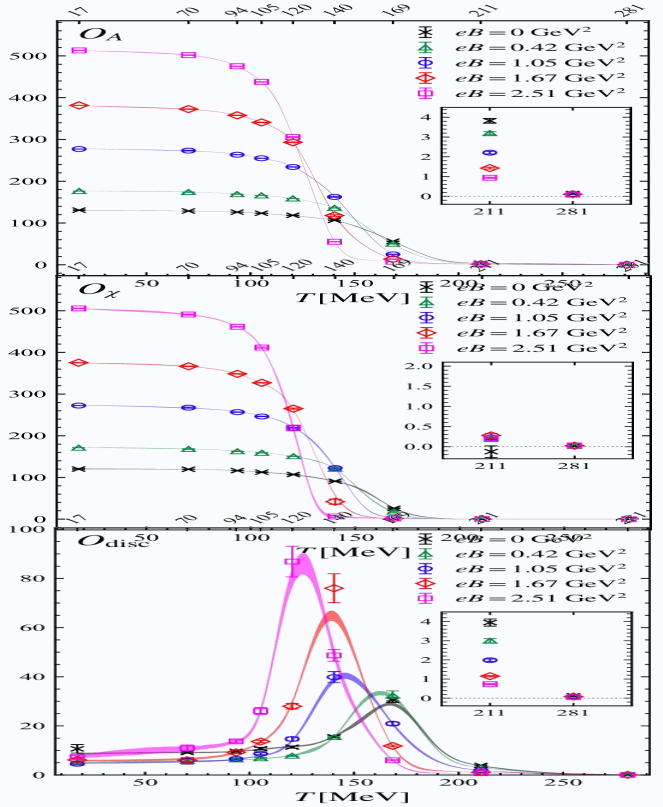
Qualitative agreement

Lattice-QCD result

dimensionless quantities

$$\begin{cases} O_{A(\chi)}(B,T) \equiv \boxed{m_s^2}\left[\chi_{\pi^0}(B,T) - \chi_{\delta^0(\sigma)}(B,T)\right]/f_K^4, \\ O_{\rm disc}(B,T) \equiv \boxed{m_s^2}\left[\chi^{(+)}_{\rm disc}(B,T) - \chi^{(-)}_{\rm disc}(B,T)\right]/f_K^4. \end{cases}$$

???



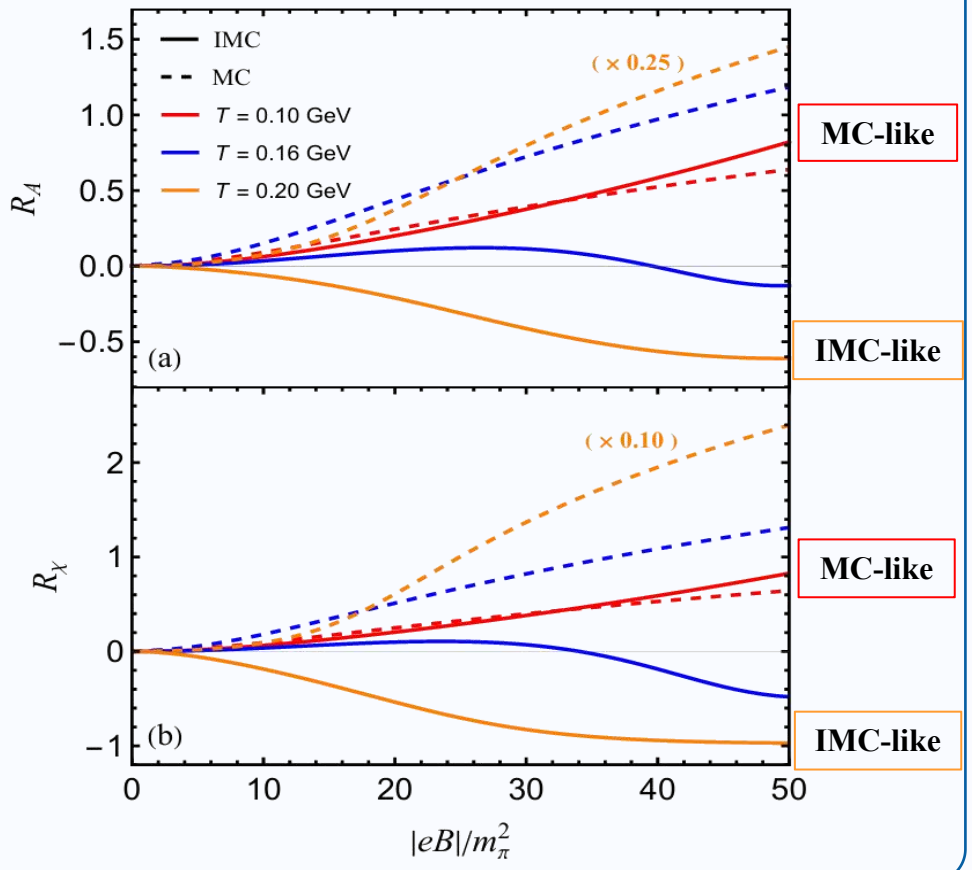
X.-N. Zhu, X.-L. Sheng, et al., arXiv:2609.39886 (2026).

H.-T. Ding, J. J. Hern´andez Hern´andez, and D. Zhang (2026), 2607.11625.

Results of Susceptibility splittings

NJL result

$$R_A = \frac{\Delta_A(|eB|,T)}{\Delta_A(0,T)} - 1 \qquad R_\chi = \frac{\Delta_\chi(|eB|,T)}{\Delta_\chi(0,T)} - 1$$

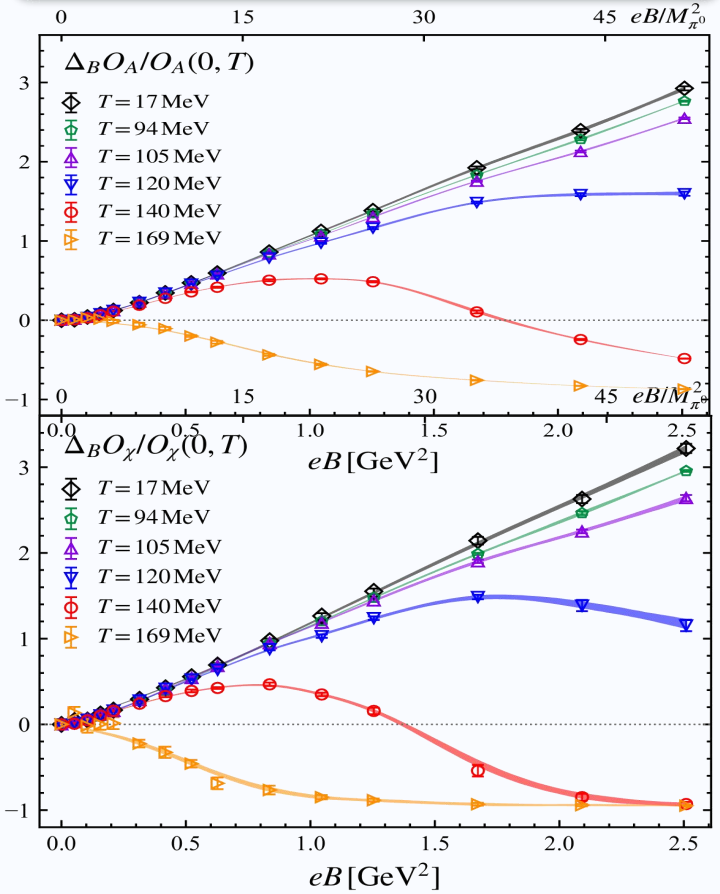


IMC scenario

Qualitative agreement

Lattice-QCD result

$$\frac{\Delta_B O_i(eB,T)}{O_i(eB=0,T)} = \frac{O_i(eB,T) - O_i(eB=0,T)}{O_i(eB=0,T)}.$$

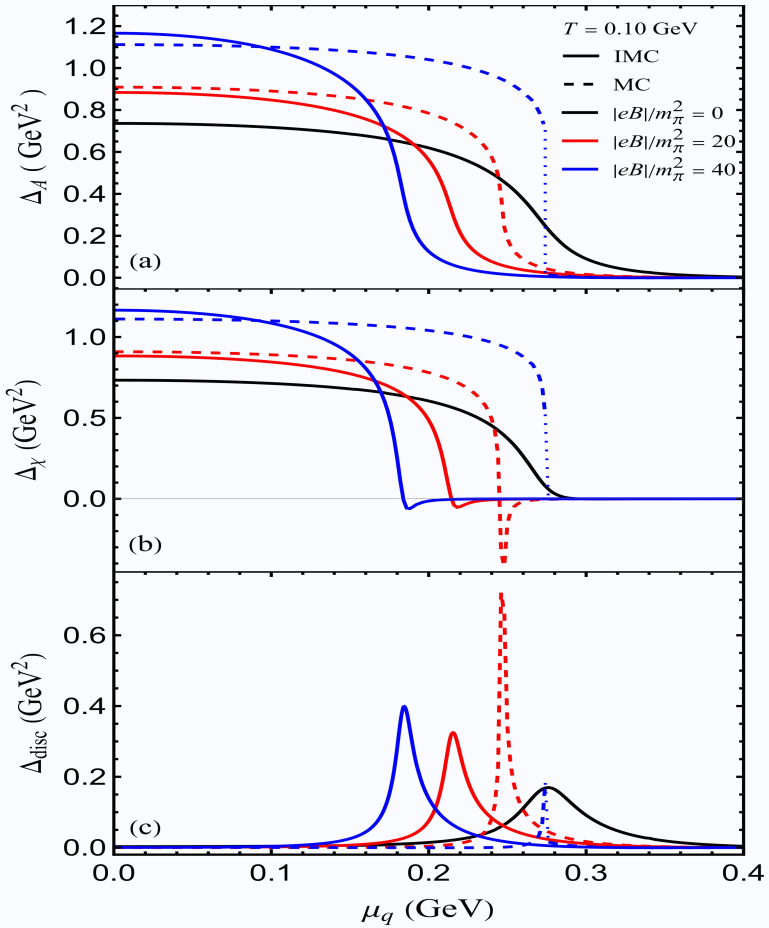


X.-N. Zhu, X.-L. Sheng, et al., arXiv:2609.39886 (2026).

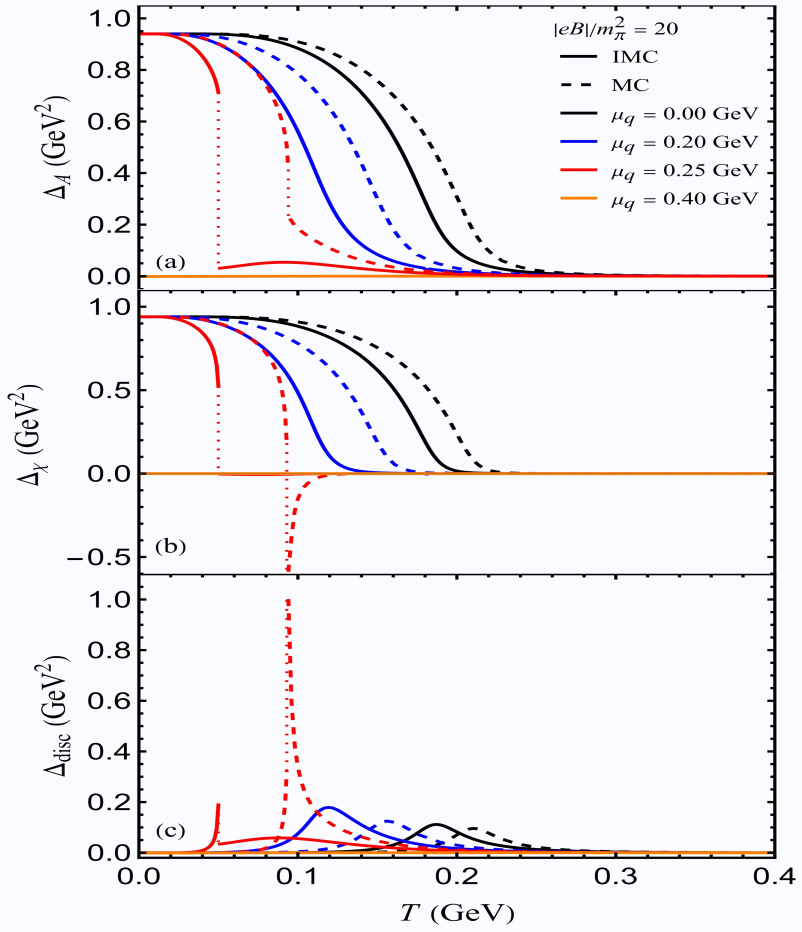
H.-T. Ding, J. J. Hern´andez Hern´andez, and D. Zhang (2026), 2607.11625.

Results of Susceptibility splittings

Chemical-potential dependence



Temperature dependence at finite density



X.-N. Zhu, X.-L. Sheng, et al., arXiv:2609.39886 (2026).

Summary

Chiral separation effect

- The CSE conductivity is suppressed at low T and approaches its chiral-limit as T increases, quantitatively consistent with lattice-QCD at $T > 0.16$ GeV.
- A first-order chiral phase transition induces a discontinuity in chemical potential dependence of the CSE conductivity. MC and IMC exhibit opposite magnetic-field dependences.

Susceptibility splitting

- In the IMC scenario, magnetic fields enhance the splittings at low T but suppress them at high T.
- The peak of Δ_{disc} signals distinct axial-symmetry restoration patterns and shifts toward lower temperatures with increasing $|eB|$ or μ_q in IMC scenario.

Thanks!