Critical fluctuations in relativistic heavy-ion collisions

Shanjin Wu (吴善进)

大连理工大学
Dalian University Of Technology

10th International Conference on Chirality, Vorticity, and Magnetic Field in Quantum Matter (Chirality 2026)

QCD phase diagram

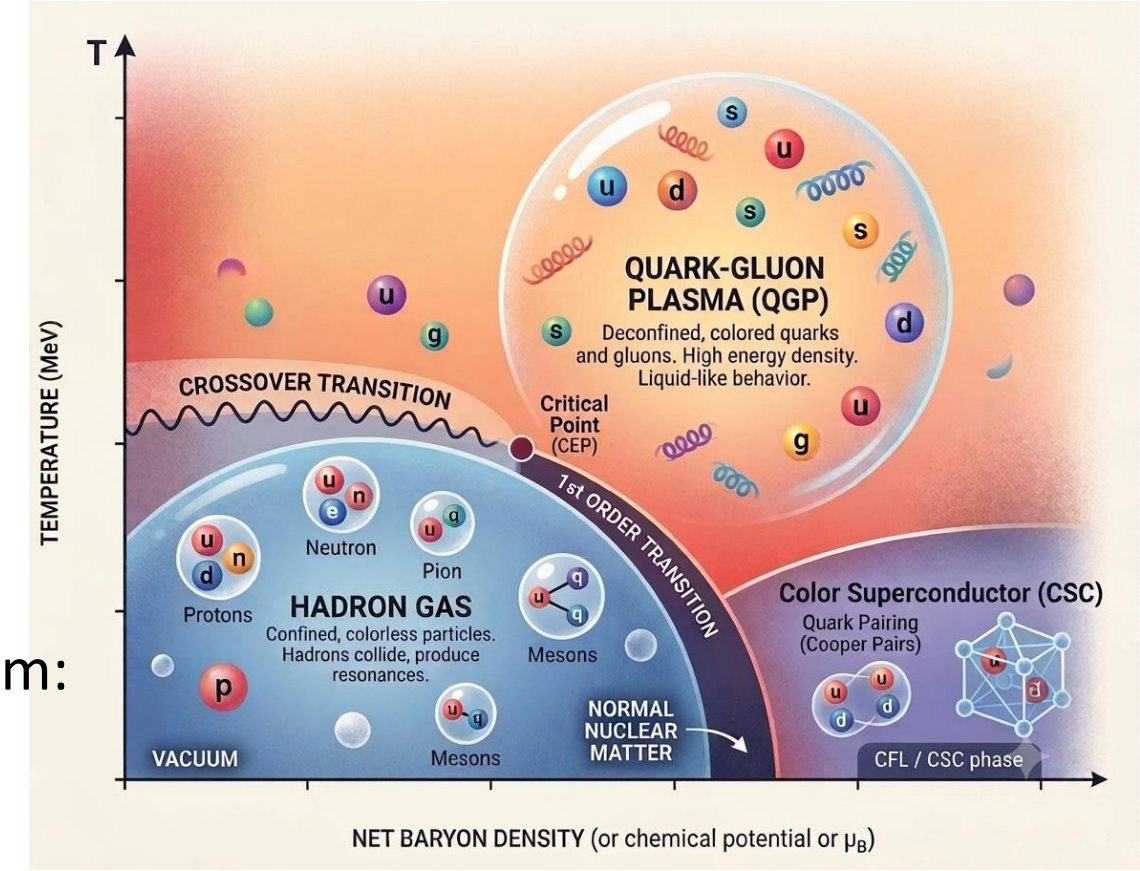
- **Lattice QCD** (small μ_B finite T): Y. Aoki et al, Nature 443, 675 (2006).
  - Crossover (2nd order phase transition)
- **Effective models** (large μ_B)
  - 1st order phase transition

→ Critical End Point

W.J.Fu et al., PRD 77, 014006
S.X.Qin et al., PRL. 106, 172301
K. Fukushima et al, PPNP. 72, 99

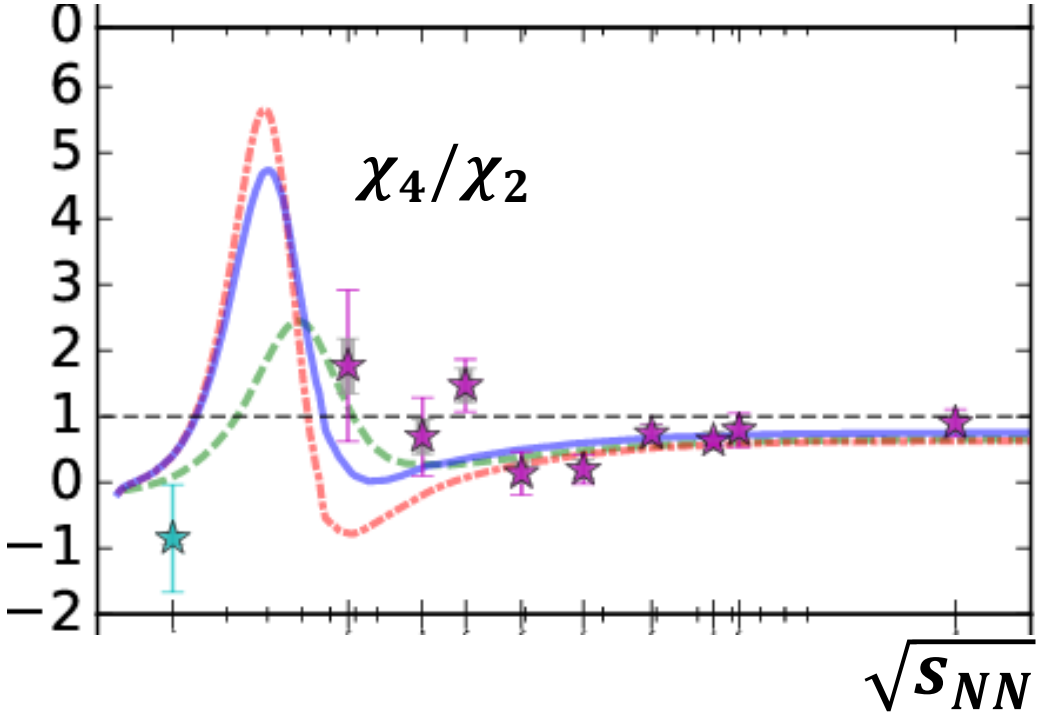
→ **Heavy-ion collisions :**

- tuning $\sqrt{s_{NN}}$,mapping QCD phase diagram:
  RHIC(BES),NICA,FAIR,J_PARC,HIAF….



Net-proton fluctuations with Beam Energy Scan

- Characteristic feature of critical point:
  - long range correlation
  - large fluctuations
- **Non-monotonicity** net-proton fluctuations



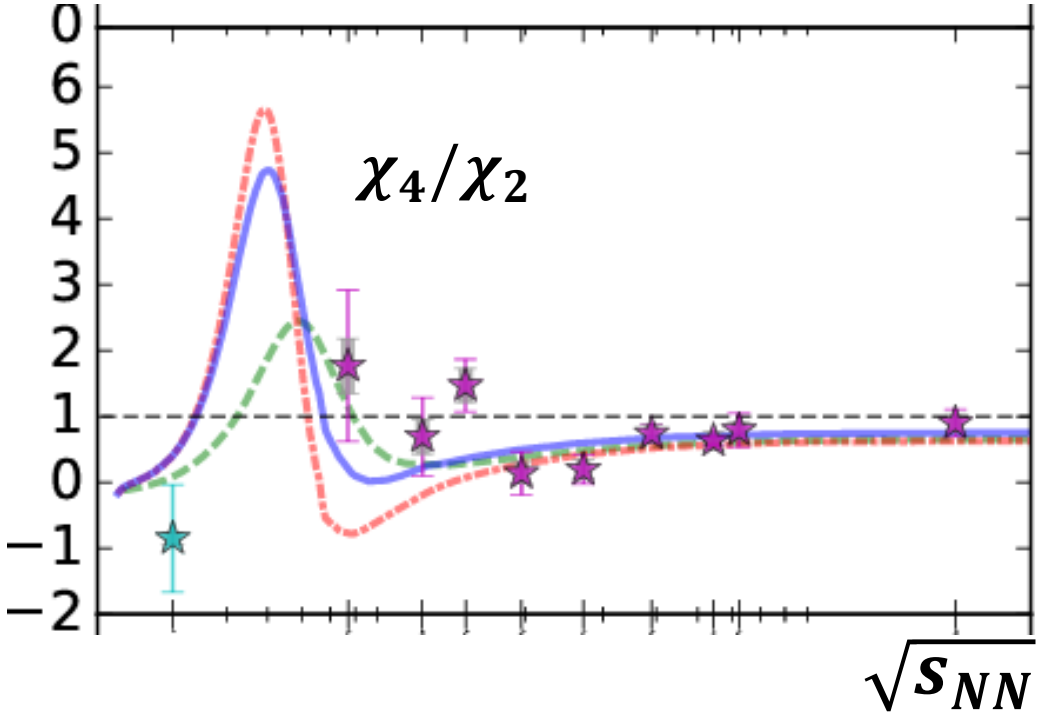
STAR, Phys.Rev.Lett. 135, 142301

M.Stephanov, 2410.02861

Wei-jie Fu, et al., PRD 111, L031502 (2025)

Net-proton fluctuations with Beam Energy Scan

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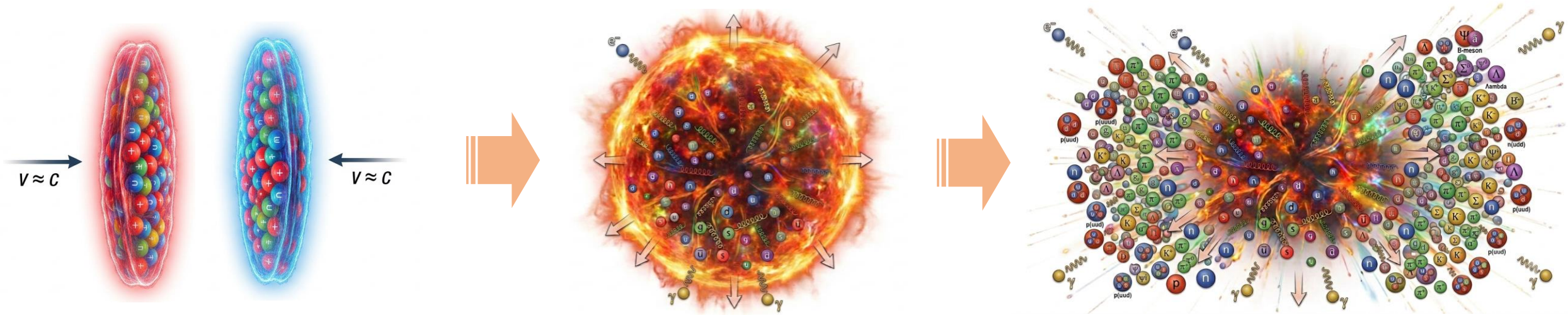
STAR, Phys.Rev.Lett. 135, 142301

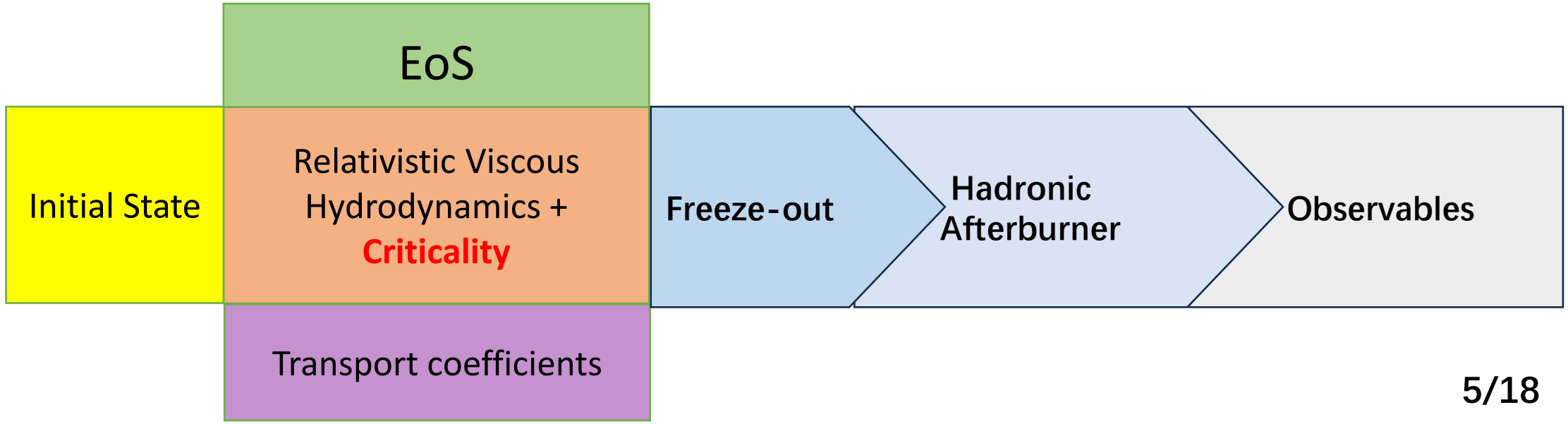
M.Stephanov, 2410.02861

Wei-jie Fu, et al., PRD 111, L031502 (2025)

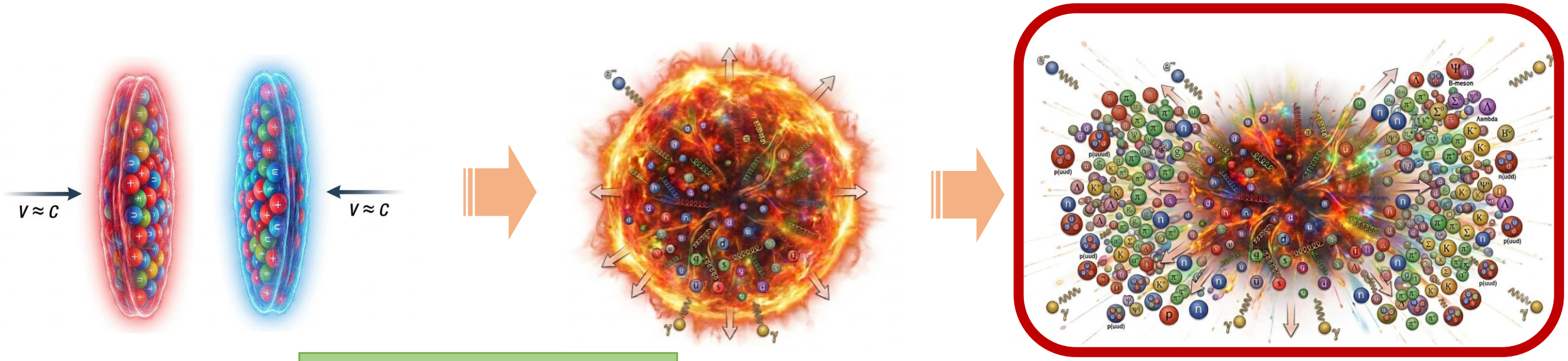
Ideal!

HIC is a hybrid process





Critical fluctuations at Freezeout



EoS

Initial State | Relativistic Viscous Hydrodynamics + Criticality | Freeze-out | Hadronic Afterburner | Observables

Transport coefficients

Particlization from hydro

R-z.Zhao, S.Yin, **SW**, L.Du, X.Luo,W.-j. Fu, 2607.26912

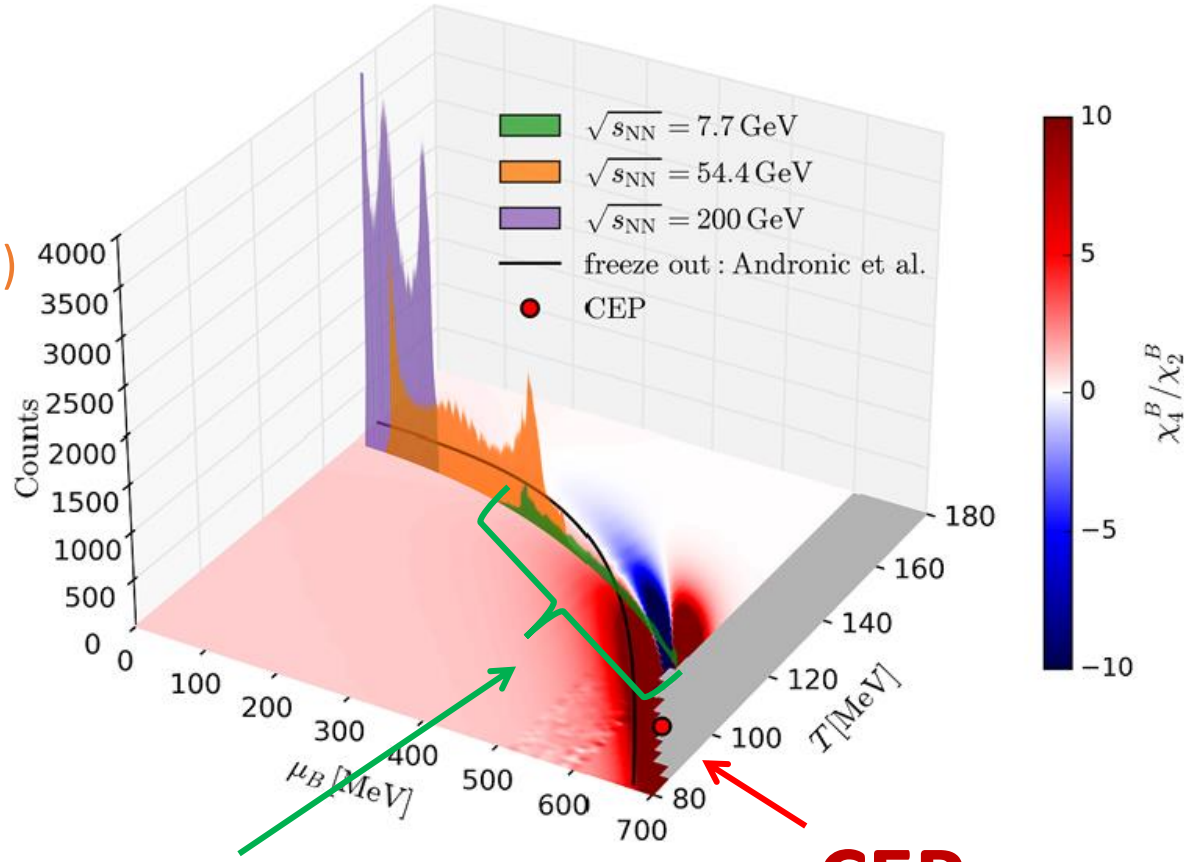
- **Freeze-out:** fluid to particles

$$e(x,t), n(x,t), u^{\mu}(x,t) \rightarrow \frac{d^3N}{dy\, d^2p_{\perp}}$$

- **BES hydro** (Du, Gao, Jeon, Gale, PRC 109 (2024) 014907)

Inhomogeneous QGP fireball, Fluid cells occupy a large area of phase diagram

- **Equil. Fluctuations at each fluid cell:** fRG (Wei-jie Fu, et al., PRD 111, L031502)



Freezeout profile

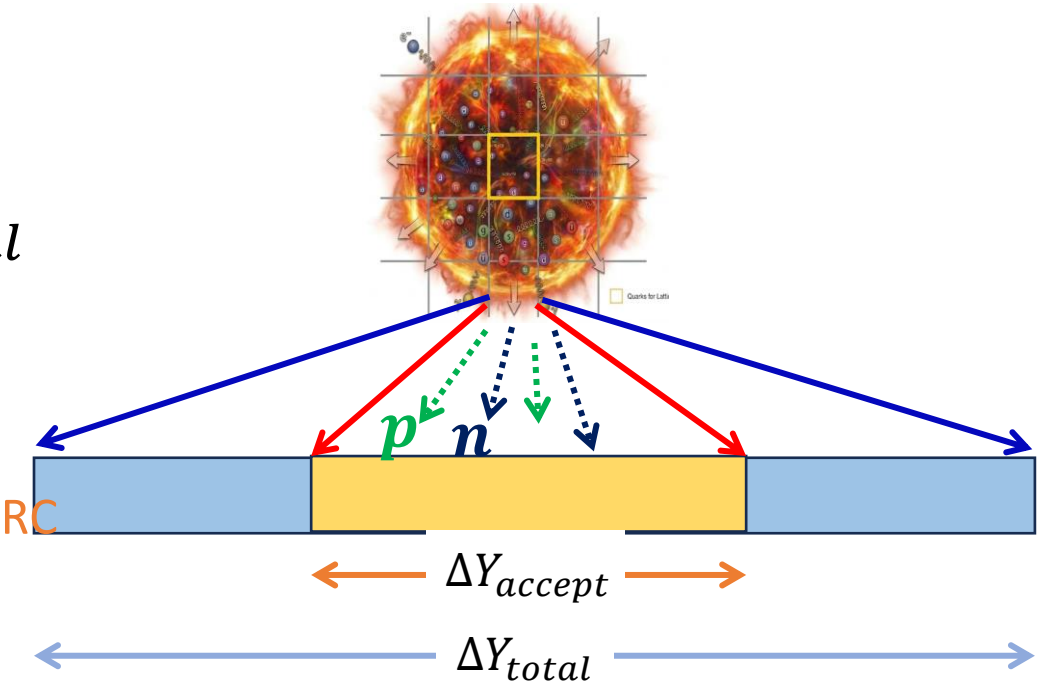
CEP

Fluctuations at each fluid cell

R-z.Zhao, S.Yin, **SW**, L.Du, X.Luo,W.-j. Fu, 2607.26912

- **Fluctuations χ_n** from fRG at fluid cell,

With the following detector correction:



- **Finite acceptance cut**: $\Delta Y_{accept} < \Delta Y_{total}$

$$P(\Delta Y_{accept}) = Binomial * P(\Delta Y_{total})$$

- **Proton-baryon cut**:proton detected, Isospin symmetry(M. Kitazawa and M. Asakawa, PRC 85, 021901)

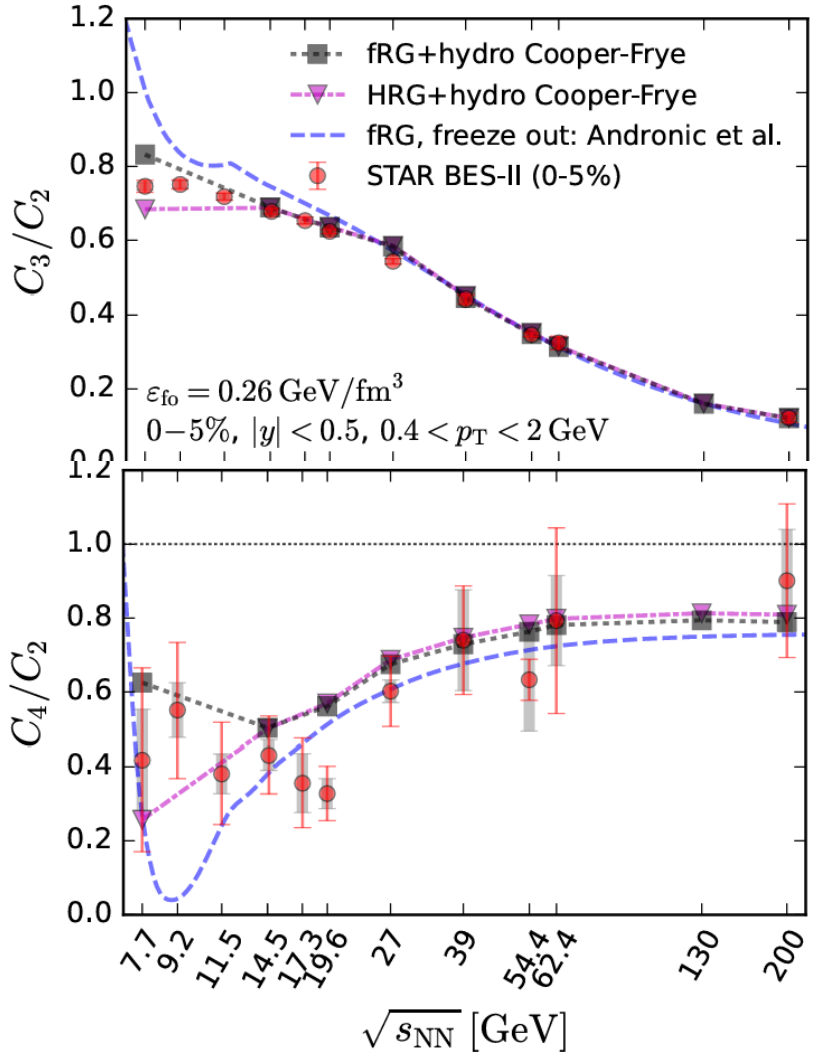
$$P(p) = Binomial * P(p+n)$$

- **Global conservation**: subensemble acceptance method (Vovchenko, Koch, Shen, PRC 105, 014904)

$$P_{in}^{ce} = \sum P_{in}^{gce} P_{out}^{gce} \delta_{in+out,\,baryon}$$

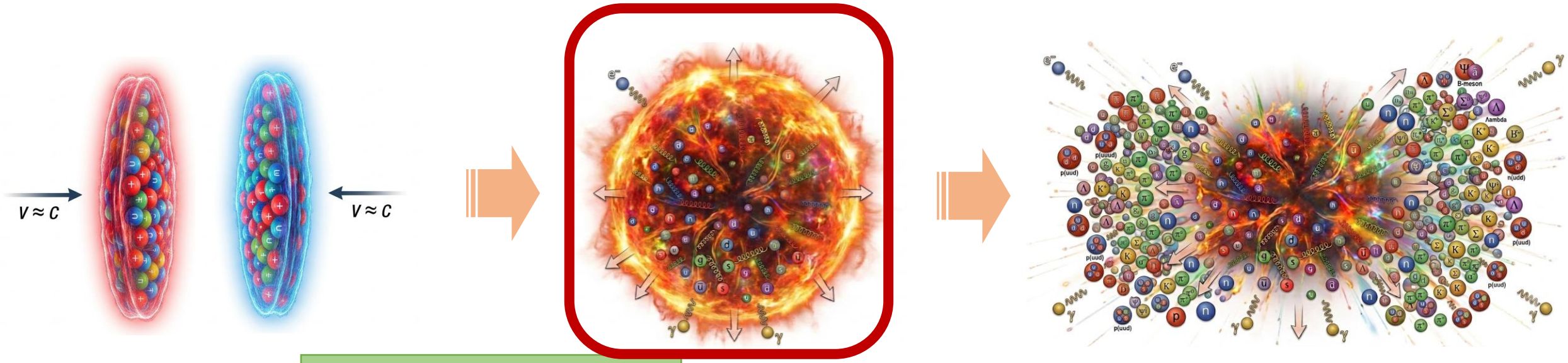
Net-proton fluctuations after Particlization

R-z.Zhao, S.Yin, **SW**, L.Du, X.Luo,W.-j. Fu, 2607.26912



- Particlizations process do impacts on the cumulants
- Better agreement with measurements

Dynamics of critical fluctuations



EoS

Initial State | **Relativistic Viscous Hydrodynamics + Criticality** | Freeze-out | Hadronic Afterburner | Observables

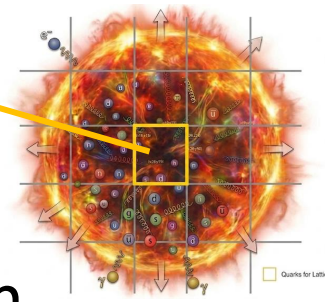
Transport coefficients

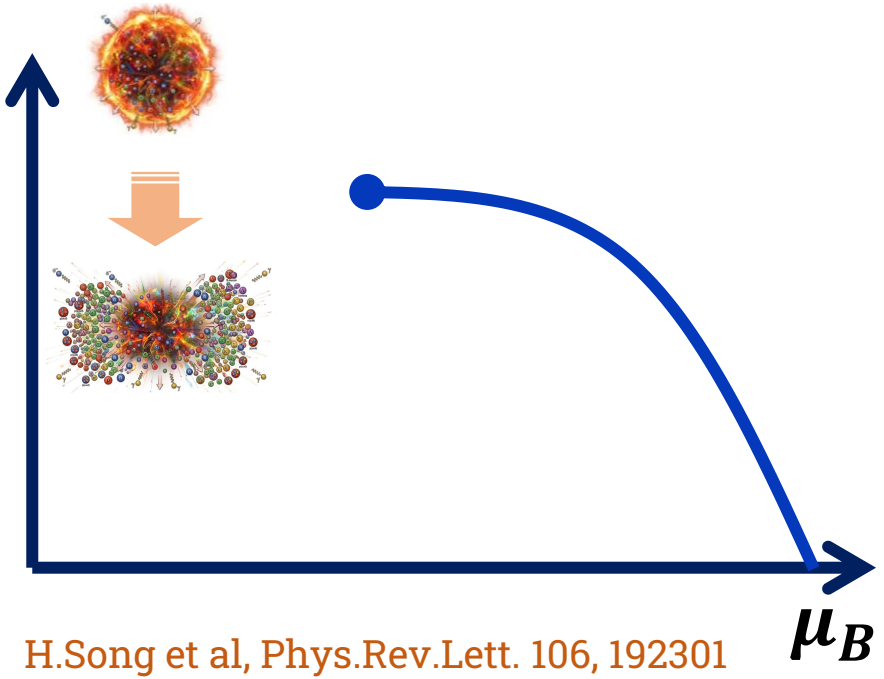
Theory far away from QCD critical point

Hydrodynamics:

- d.o.f: fluid cell with

$$e(x,t),\ n(x,t),\ u^{\mu}(x,t)$$





H.Song et al, Phys.Rev.Lett. 106, 192301

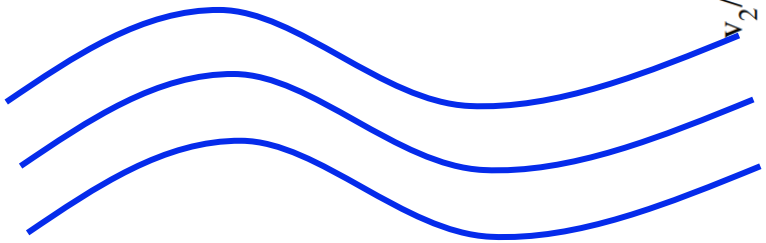
Pure hydro is Macroscopic

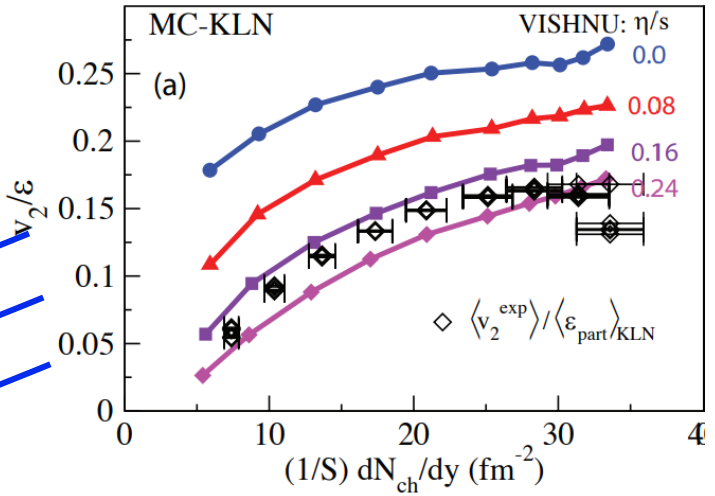
- Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

- Charges conservation

$$\partial_\mu N^{\mu} = 0$$





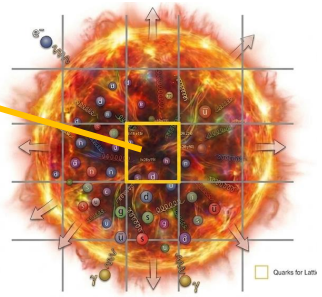
Great success in heavy-ion collisions

Theory CLOSE to QCD critical point

Hydrodynamics + fluctuations:

- d.o.f: fluid cell with

$$e(x,t)+\delta e,\ \ n(x,t)+\delta n,\ \ u^{\mu}(x,t)+\delta u^{\mu}$$



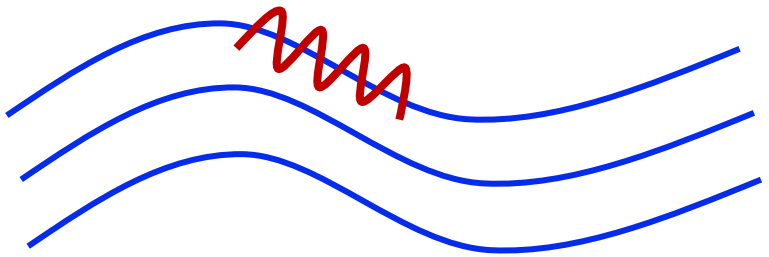
Correlation length comparable, requires Critical Fluctuations d.o.f

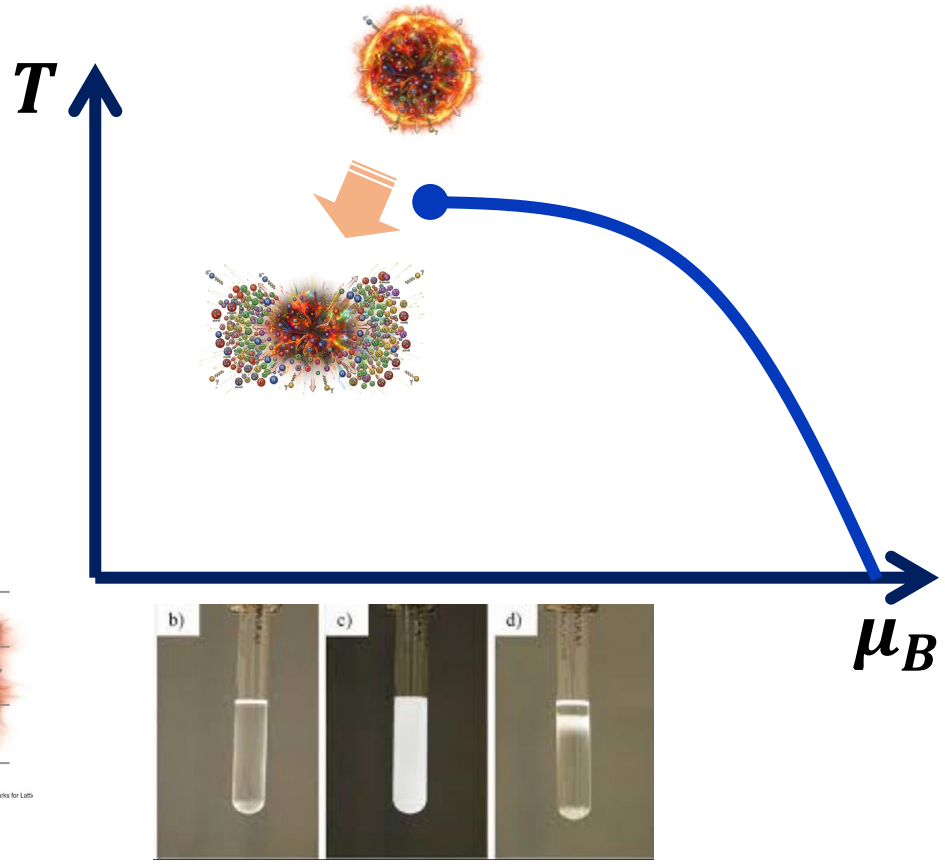
- Energy-momentum conservation

$$\partial_{\mu}(T^{\mu\nu}+\delta T^{\mu\nu})=0$$

- Charges conservation

$$\partial_{\mu}(N^{\mu}+\delta N^{\mu})=0$$





Example: Charge conservation ($\partial_\mu N^\mu = 0$)

- **Charge current:**

$$N^\mu = nu^\mu + D\Delta^\mu n$$

$$\Rightarrow \partial_t n = D\nabla^2 n$$

diffusion

- **Charge current near critical point**

$$N^\mu = nu^\mu + D\Delta^\mu n + \delta N^\mu$$

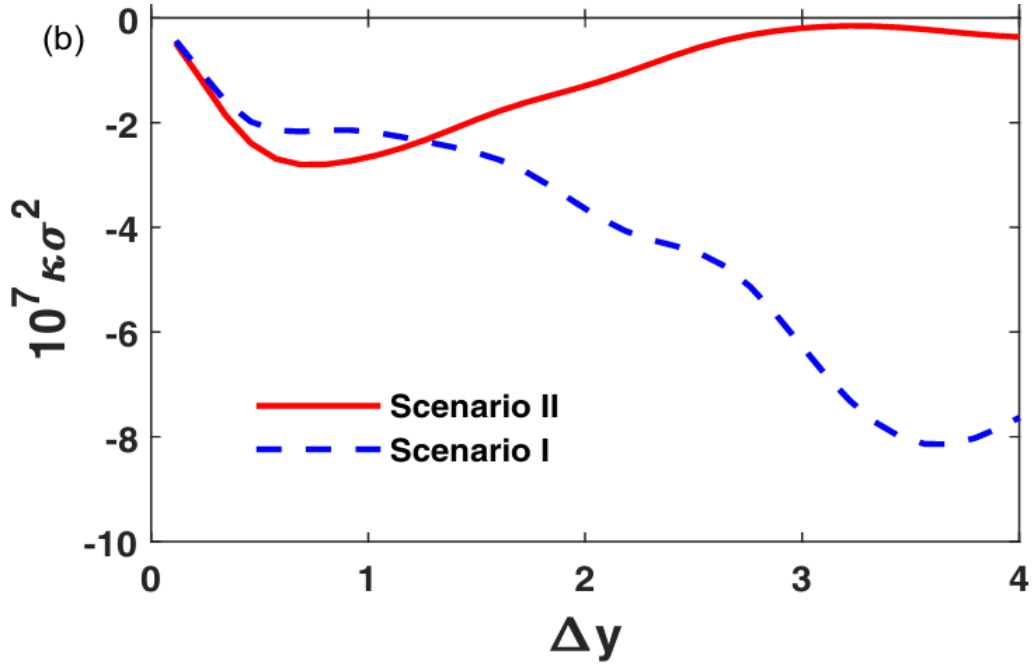
$$\Rightarrow \partial_t n = D\nabla^2 n + noise$$

Stochastic diffusion (hydro fluctuations)

- **Correlator equations:**

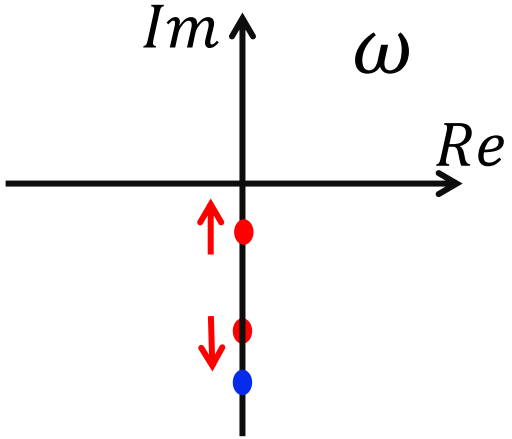
$$\partial_t W_2(\boldsymbol{q}) = -2Dq^2[W_2(\boldsymbol{q}) - W_2^{eq}]$$

X.An et al, Phys.Rev.Lett.127,072301



Sakaida et al, PRC.95.064905(2017)
G.Pihan et al., PRC.107.014908(2022)
SW PRC 111,014915(2025)

Fick to Maxwell-Cattaneo diffusion



- **Non-hydro mode** relevant near critical point

- $N^\mu = (n, \boldsymbol{J})$: **Fick current** → **MC current:**

$$N^\mu = nu^\mu + D\Delta^\mu n + \delta N^\mu \rightarrow \tau\partial_\mu N^\mu + N^\mu = nu^\mu + D\Delta^\mu n + \delta N^\mu$$

N.Abbasi, X.An, **SW**, Phys.Rev.D 114.054033

- Fick diffusion (2pt.) → MC diffusion (2pt.):

$$\partial_t W_{nn}(\boldsymbol{q}) = -2Dq^2[W_{nn}(\boldsymbol{q}) - W_{nn}^{eq}]$$

$$\partial_t W_{nn} = 2W_{gn};$$

$$\partial_t W_{gn} = -\tau^{-1}W_{gn} - \frac{\gamma}{\tau}\boldsymbol{q}_1^2 W_{nn} + W_{gg};$$

$$\partial_t W_{gg} = -2[\tau^{-1}W_{gg} + \frac{\gamma}{\tau}W_{ng} + \frac{\lambda}{\tau^2}\boldsymbol{q}_1 \cdot \boldsymbol{q}_2]$$

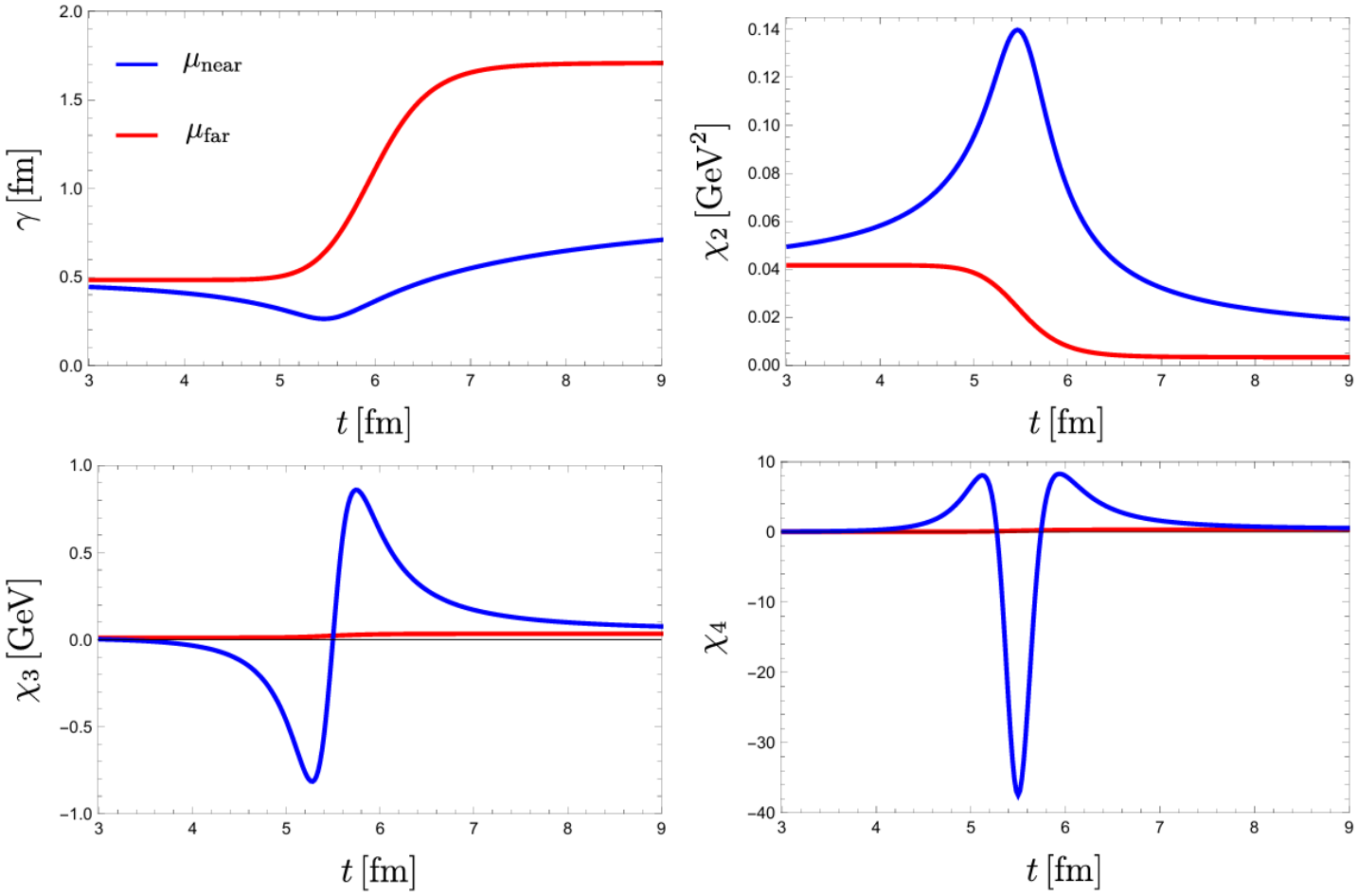
- Similar for 3pt. and 4pt. with non-linear terms

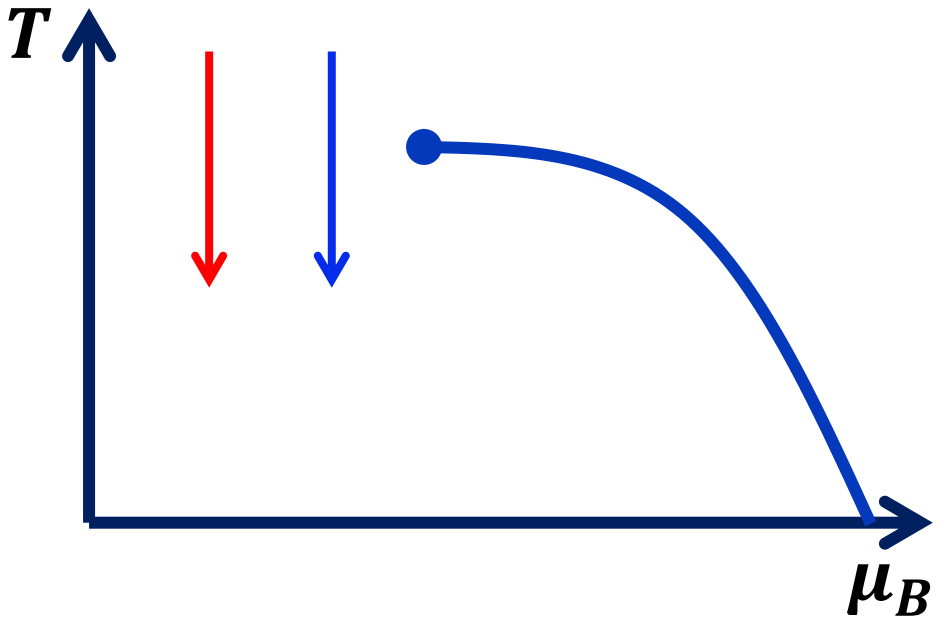
$$\mathrm{W}_{nn} = \langle \delta n \delta n \rangle, W_{gn} = \langle \delta \dot{n} \delta n \rangle$$

Maxwell-Cattaneo diffusion near critical point

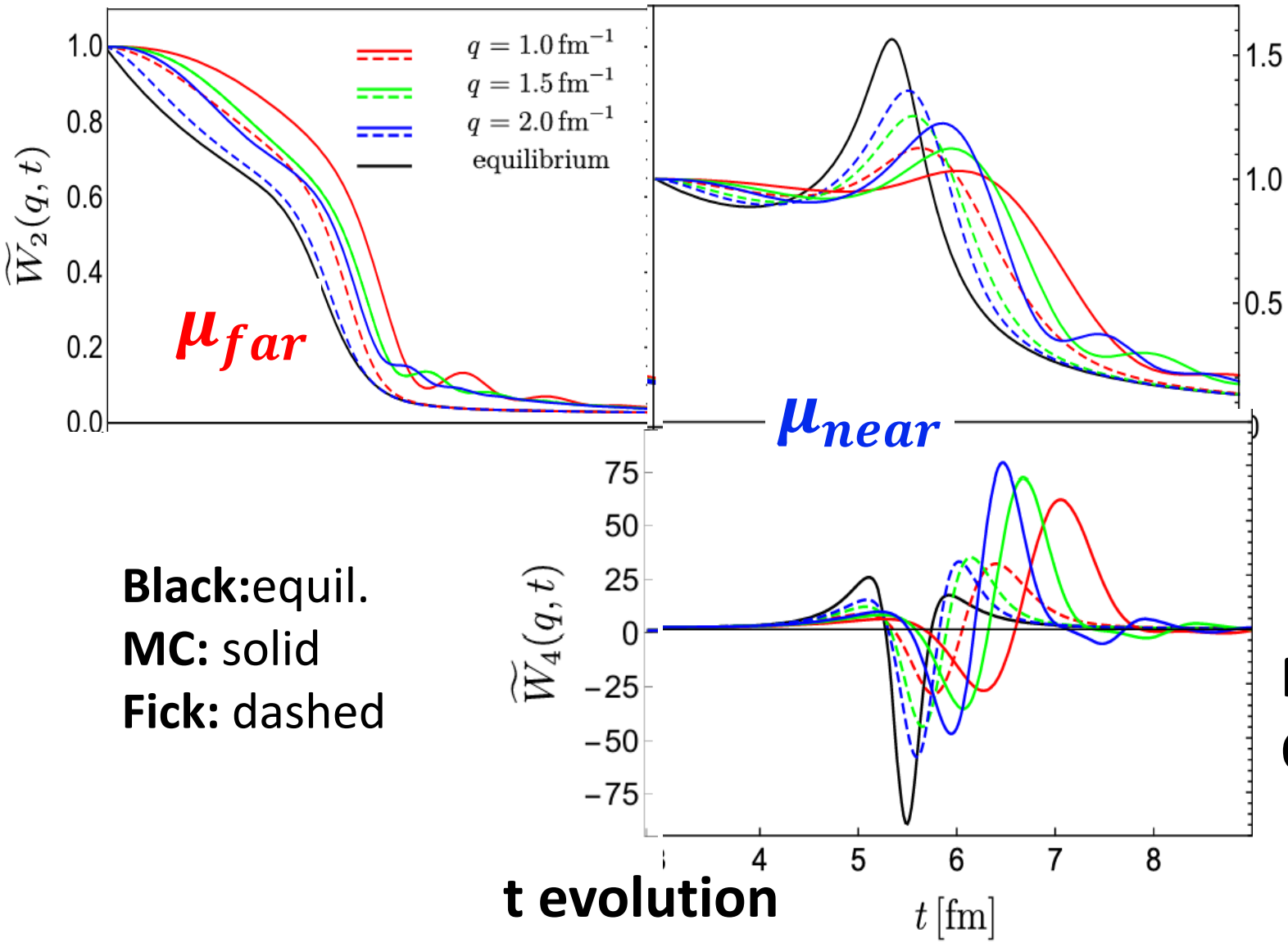
- **Transport coefficients**: from Ising mapping

N.Abbasi, X.An, **SW**, Phys.Rev.D 114.054033





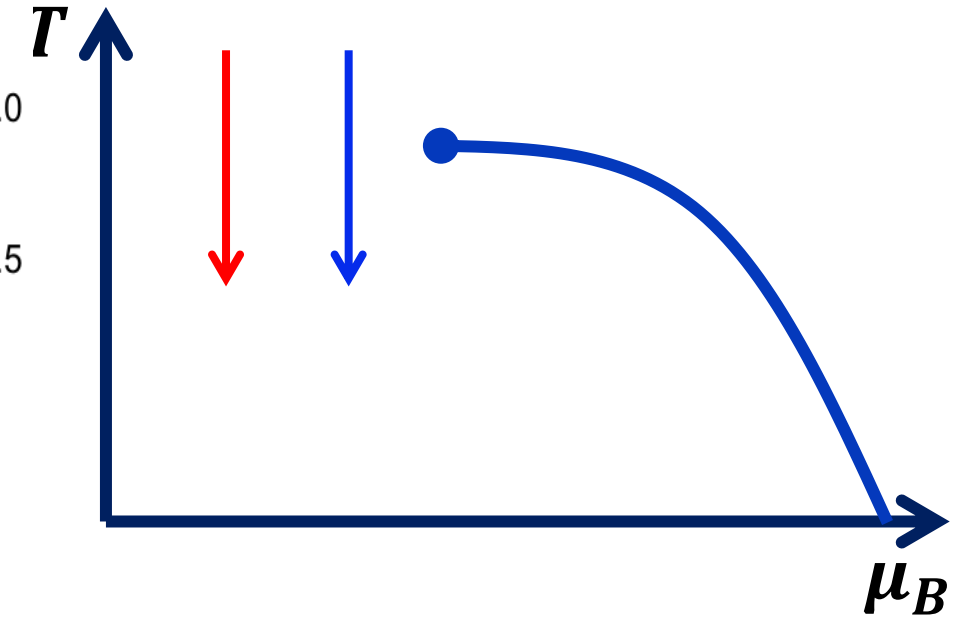
Maxwell-Cattaneo diffusion Wn(q)



Black:equil.
MC: solid
Fick: dashed

t evolution

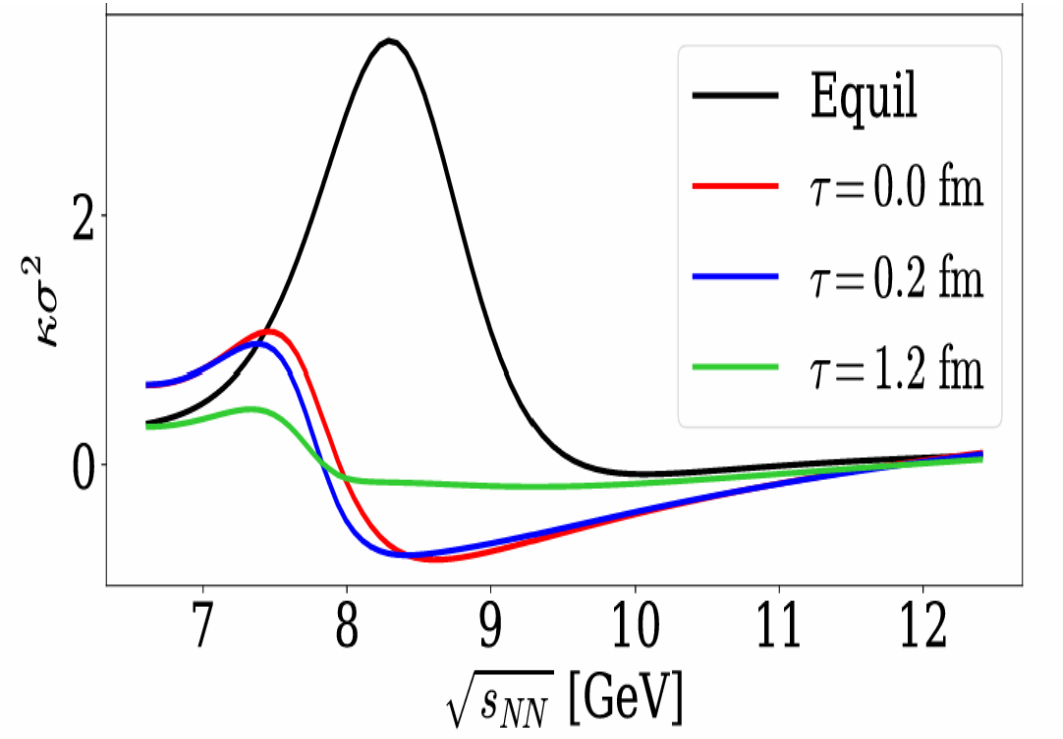
N.Abbasi, X.An, **SW**, Phys.Rev.D 114.054033

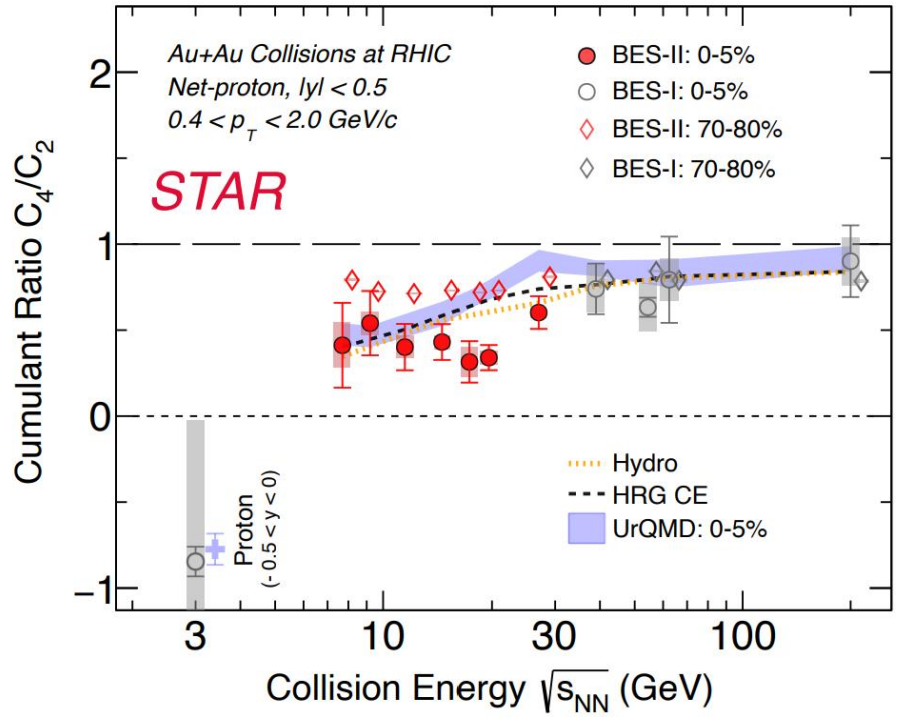


More pronounced and Oscillating behavior near CEP

Maxwell-Cattaneo diffusion on observable

N.Abbasi, X.An, **SW**, Phys.Rev.D 114.054033

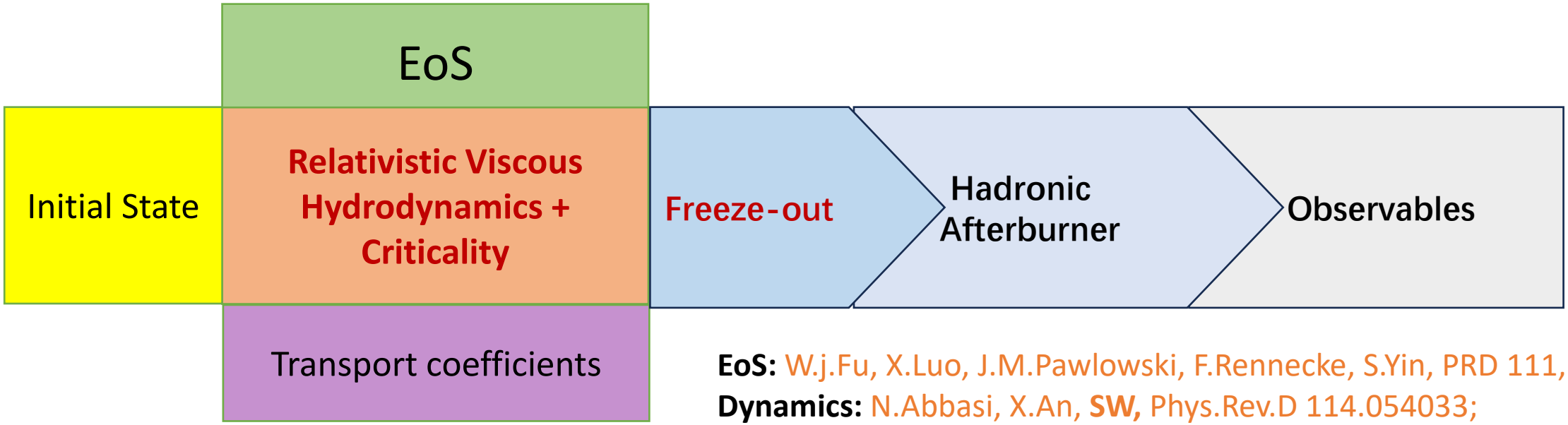




See also A.Sakai, K.Murase, H.Fujii, T.Hirano. Phys.Rev.C 112.024907(2025)

Summary and Outlook

- QGP fireball system in heavy-ion experiments is not an ideal system:



EoS: W.j.Fu, X.Luo, J.M.Pawlowski, F.Rennecke, S.Yin, PRD 111, L031502…
Dynamics: N.Abbasi, X.An, **SW,** Phys.Rev.D 114.054033;
Freeze-out: R-z.Zhao, S.Yin, **SW**, L.Du, X.Luo,W.-j. Fu, 2607.26912
……

Outlook: hybrid model incorporating critical fluctuations

See e.g. Review,
M.Asakawa M. Kitazawa, PPNP, arXiv: 1512.05038
X.Luo, N.Xu, NST, arXiv: 1701.02105
S.Wu, C.Shen, H.Song, CPL, arXiv: 2104.13250

Thank You!

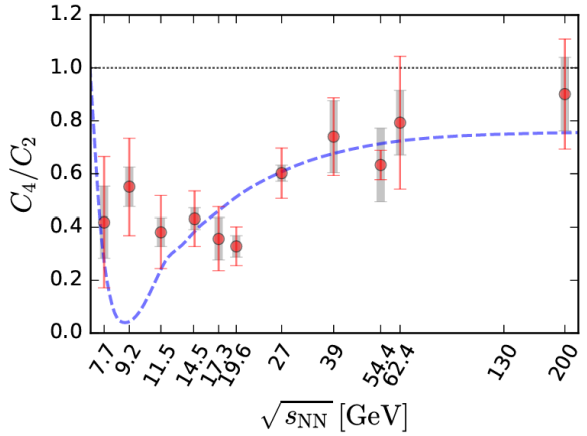
Bkup

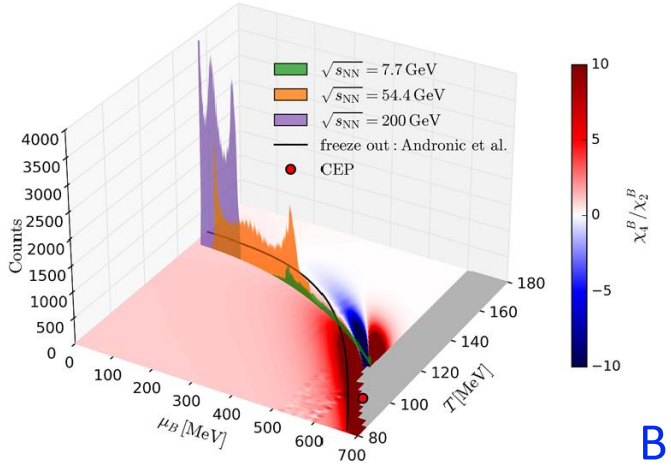
Particlization: inhomogeneous QGP

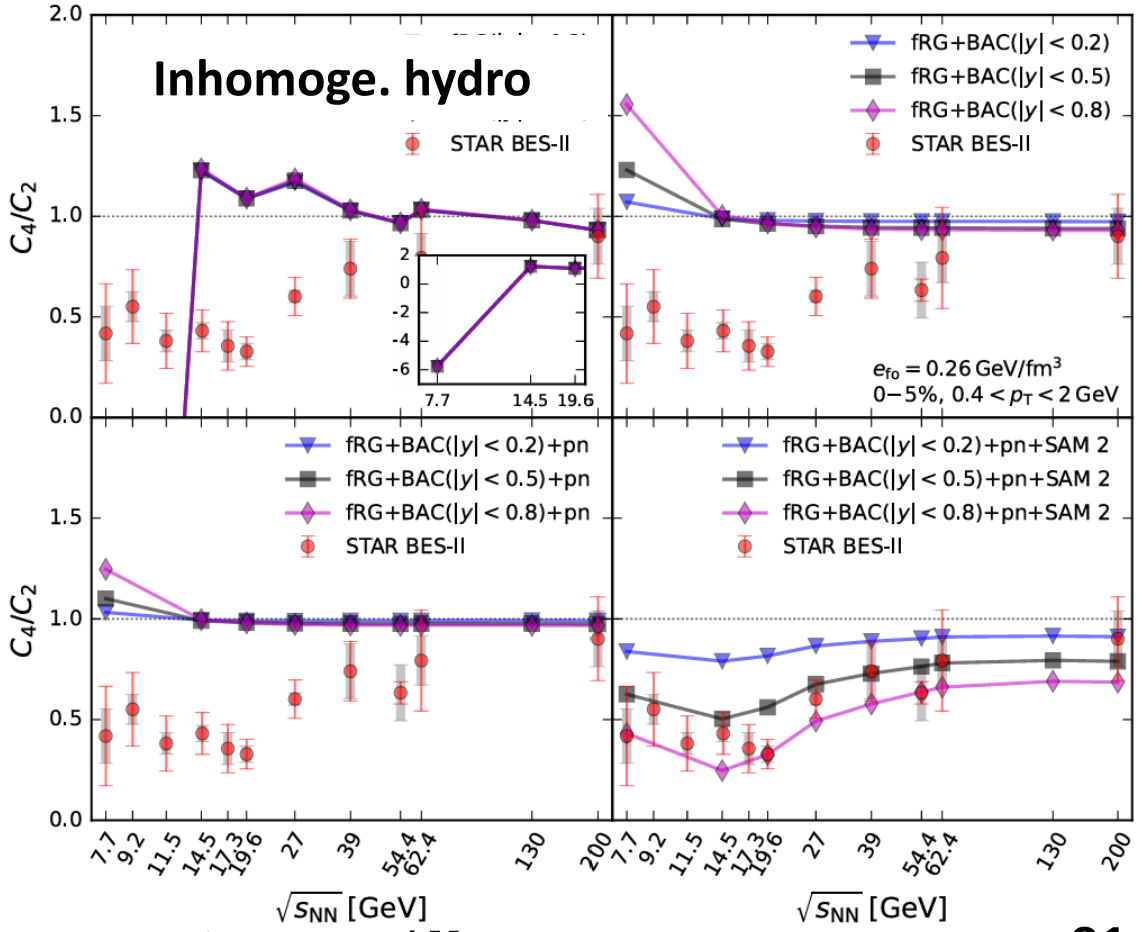
R-z.Zhao, S.Yin, SW, L.Du, X.Luo,W.-j. Fu, 2607.26912

Wei-jie Fu, et al., PRD 111, L031502 (2025)

Homoge. Profile with fRG







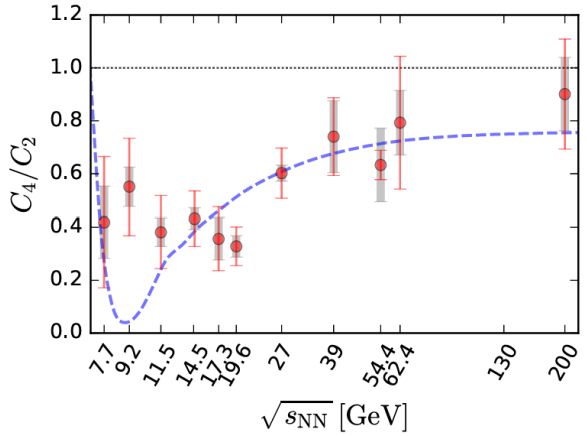
Blue => Magenta: increase ΔY

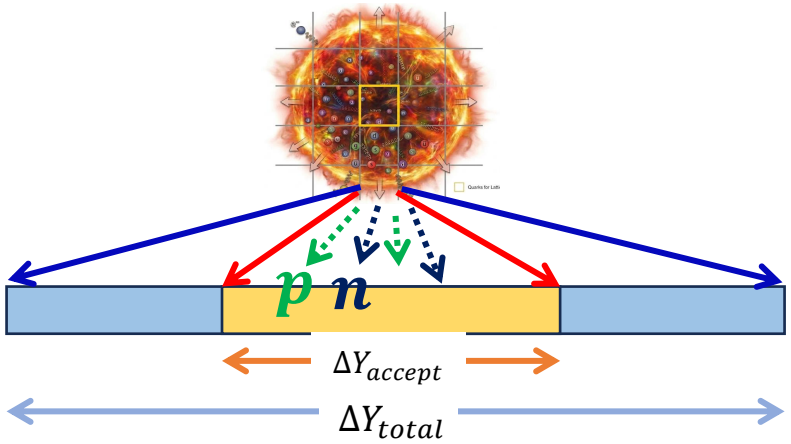
Particlization at freezeout

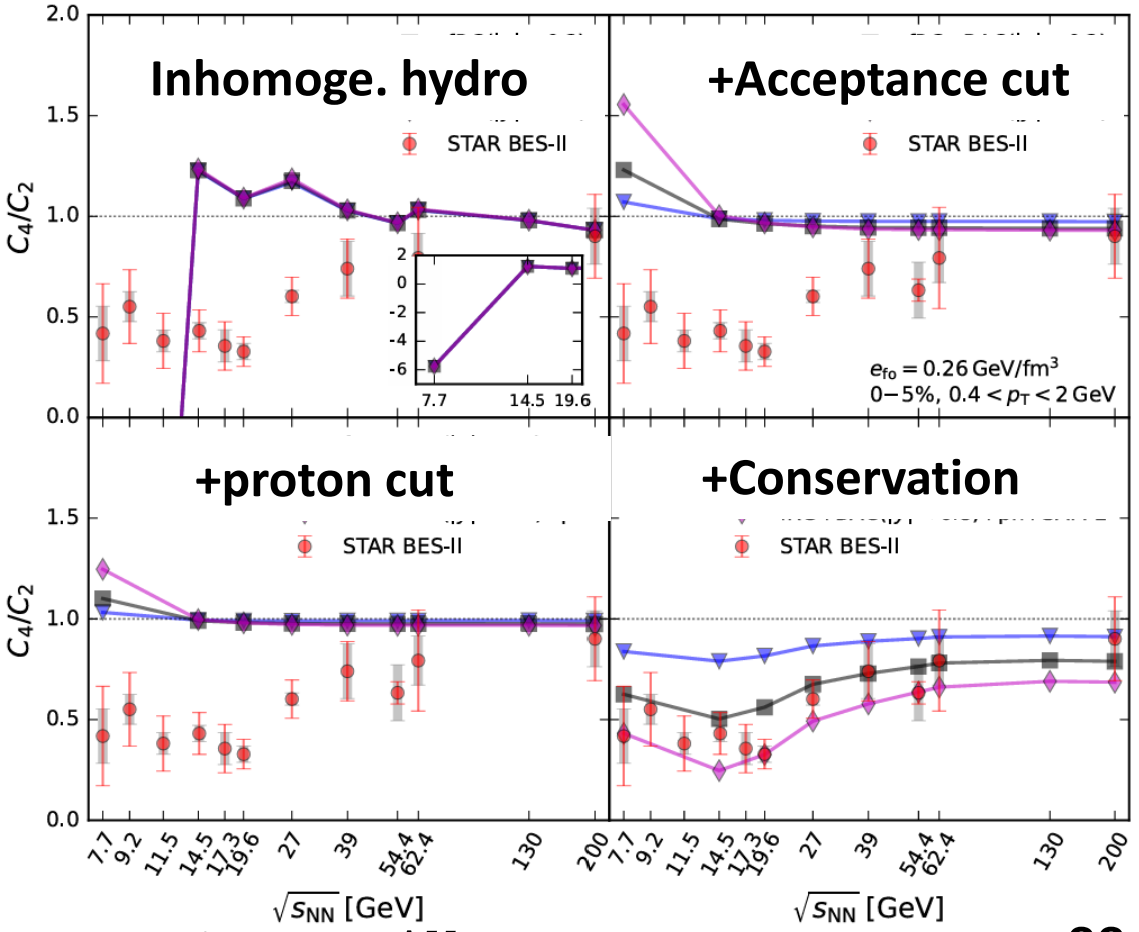
R-z.Zhao, S.Yin, SW, L.Du, X.Luo,W.-j. Fu, 2607.26912

Wei-jie Fu, et al., PRD 111, L031502 (2025)

Homoge. Profile with fRG







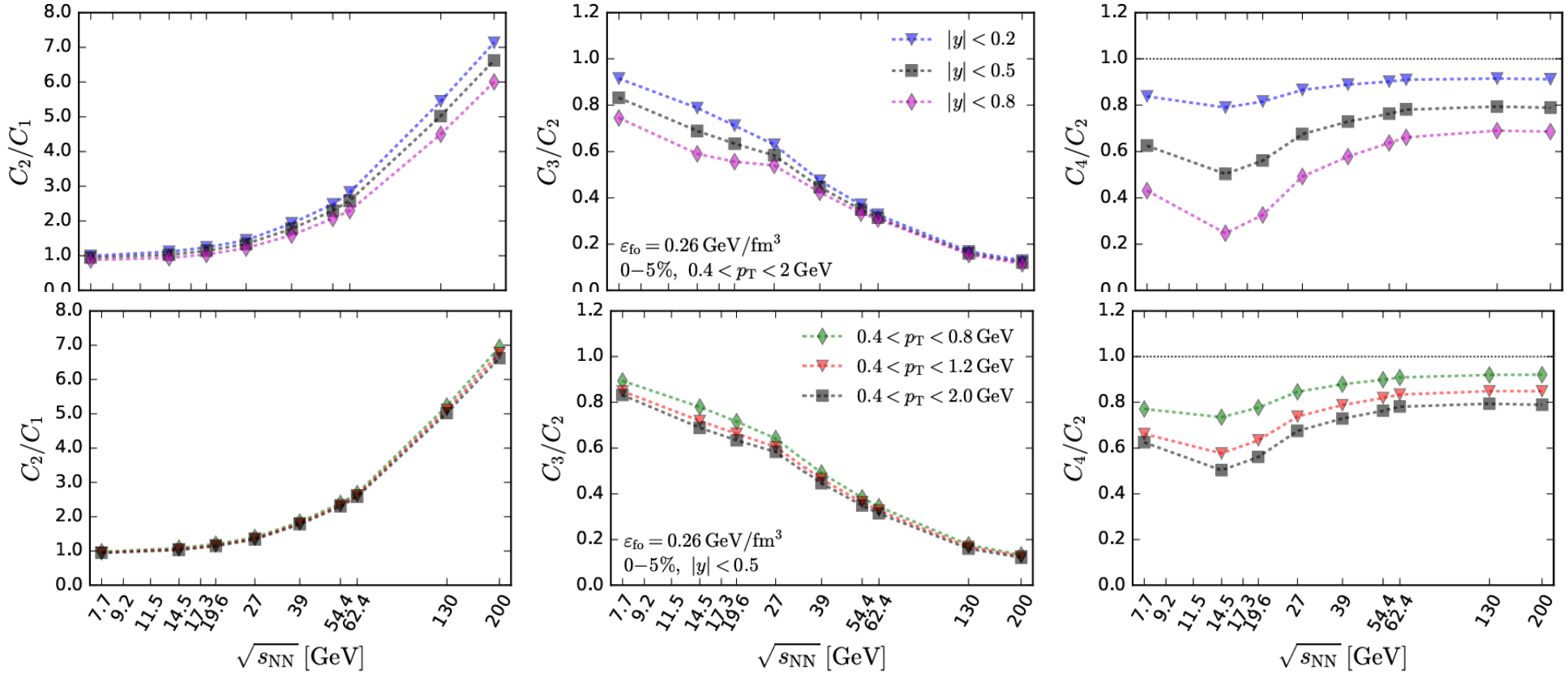
Blue => Magenta: increase ΔY

Particlization at freeze-out

R-z.Zhao, S.Yin, **SW**, L.Du, X.Luo,W.-j. Fu, 2607.26912

Rapidity

Transverse momentum



- More pronounced non-monotonic fluctuations with increasing acceptance

Example: Charge conservation($\partial_\mu N^\mu = 0$)

- **Charge current:**

  $N^\mu = nu^\mu + D\Delta^\mu n$

  $\Rightarrow \partial_\tau n = D\nabla^2 n$

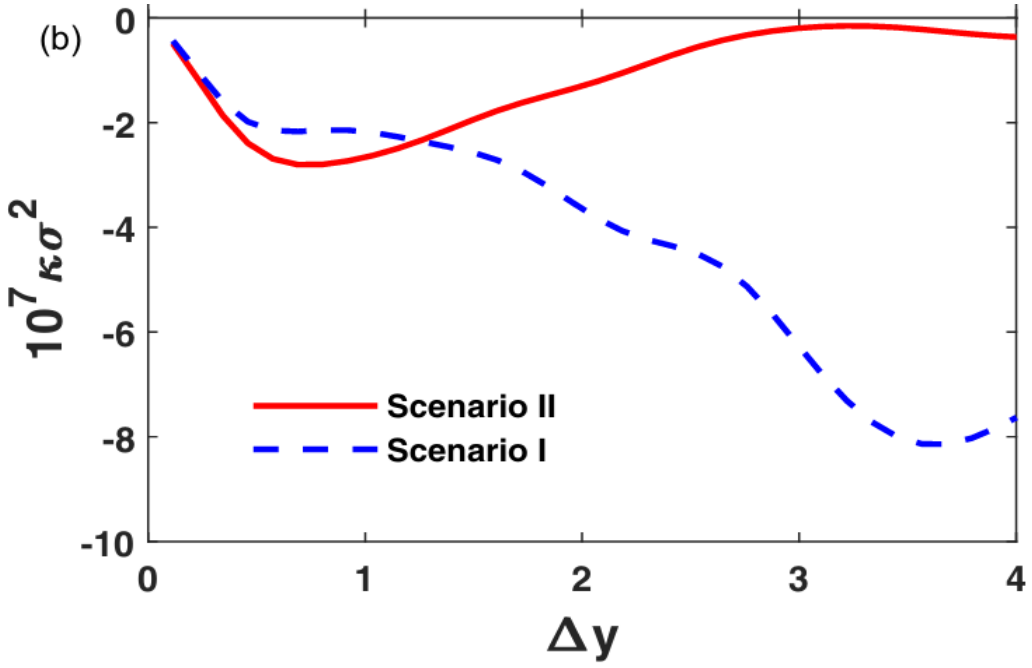
  diffusion

- **Charge current near critical point**

  $N^\mu = nu^\mu + D\Delta^\mu n + I^\mu$

  $\Rightarrow \partial_\tau n = D\nabla^2 n + noise$

  Stochastic diffusion (hydro fluctuations)



Sakaida et al, PRC.95.064905(2017)
G.Pihan et al., PRC.107.014908(2022)
SW PRC 111,014915

Stochastic differential equations for simulation

Simulation of hydro fluctuations

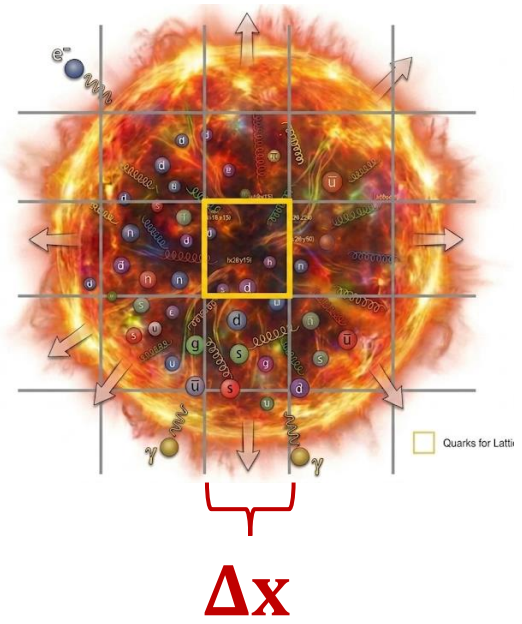
- **Example: stochastic diffusion**

  $\partial_\tau n = D\nabla^2 n + noise$

- **Noise term in simulations:**

$$noise \sim \frac{\delta_{ij}}{\Delta x}$$

  - $\frac{1}{\Delta x} \sim \Lambda$: lattice spacing dependence, renormalization is non-trivial
  - $\xi \to \infty$: diverge fluctuations, high statistics requires



Simulation of stochastic hydro is a non-trivial task

Acceptance cut

$$P(N_{B^\pm}) = \sum_{M_{B^\pm}=N_{B^\pm}}^{\infty} B(M_{B^\pm}, N_{B^\pm}; p_{\rm acc}) P(M_{B^\pm}).$$

$$B(M_{B^\pm}, N_{B^\pm}; p_{\rm acc}) = \frac{M_{B^\pm}!}{N_{B^\pm}!(M_{B^\pm} - N_{B^\pm})!} (p_{\rm acc})^{N_{B^\pm}} (1 - p_{\rm acc})^{M_{B^\pm} - N_{B^\pm}}$$

$$\begin{aligned}
\kappa_1(N_{B^\pm}) &= p_{\rm acc}\, \kappa_1(M_{B^\pm}), \\
\kappa_2(N_{B^\pm}) &= p_{\rm acc}(1 - p_{\rm acc})\kappa_1(M_{B^\pm}) + p_{\rm acc}^2 \kappa_2(M_{B^\pm}), \\
\kappa_3(N_{B^\pm}) &= p_{\rm acc}(1 - p_{\rm acc})(1 - 2p_{\rm acc})\kappa_1(M_{B^\pm}) + 3p_{\rm acc}^2(1 - p_{\rm acc})\kappa_2(M_{B^\pm}) + p_{\rm acc}^3 \kappa_3(M_{B^\pm}), \\
\kappa_4(N_{B^\pm}) &= p_{\rm acc}(1 - p_{\rm acc})(1 - 6p_{\rm acc} + 6p_{\rm acc}^2)\kappa_1(M_{B^\pm}) + p_{\rm acc}^2(1 - p_{\rm acc})(7 - 11p_{\rm acc})\kappa_2(M_{B^\pm}) \\
&\quad + 6p_{\rm acc}^3(1 - p_{\rm acc})\kappa_3(M_{B^\pm}) + p_{\rm acc}^4 \kappa_4(M_{B^\pm}).
\end{aligned}$$

$$p_{\rm acc}(x_i; \Delta p_{\rm acc}) = \frac{\int_{p \in \Delta p_{\rm acc}} \frac{{\rm d}^3 p}{\omega_p} \delta\sigma_\mu(x_i) p^\mu f\left[u^\nu(x_i) p_\nu, T(x_i), \mu_j(x_i)\right]}{\int \frac{{\rm d}^3 p}{\omega_p} \delta\sigma_\mu(x_i) p^\mu f\left[u^\nu(x_i) p_\nu, T(x_i), \mu_j(x_i)\right]}$$

Proton cut

$$P(N_{B^\pm}, N_{p^\pm}) = B(N_{B^\pm}, N_{p^\pm}; q) P(N_{B^\pm}).$$

$$B(M_{B^\pm}, N_{B^\pm}; p_{\rm acc}) = \frac{M_{B^\pm}!}{N_{B^\pm}!(M_{B^\pm} - N_{B^\pm})!} (p_{\rm acc})^{N_{B^\pm}} (1 - p_{\rm acc})^{M_{B^\pm} - N_{B^\pm}}$$

$$\begin{aligned}
\kappa_{0,1}(N_{p^\pm}) &= q\,\kappa_1(N_{B^\pm}), \\
\kappa_{0,2}(N_{p^\pm}) &= q(1-q)\kappa_1(N_{B^\pm}) + q^2\kappa_2(N_{B^\pm}), \\
\kappa_{0,3}(N_{p^\pm}) &= q(1-q)(1-2q)\kappa_1(N_{B^\pm}) + 3q^2(1-q)\kappa_2(N_{B^\pm}) + q^3\kappa_3(N_{B^\pm}), \\
\kappa_{0,4}(N_{p^\pm}) &= q(1-q)(1-6q+6q^2)\kappa_1(N_{B^\pm}) + q^2(1-q)(7-11q)\kappa_2(N_{B^\pm}) \\
&\quad + 6q^3(1-q)\kappa_3(N_{B^\pm}) + q^4\kappa_4(N_{B^\pm}).
\end{aligned}$$

$$q(x_i) = \langle N_{p^\pm}(x_i) \rangle / \langle N_{B^\pm}(x_i) \rangle$$

Conservation law

$$P^{gce}(B_{in}, B_{out}) = P_{in}^{gce}(B_{in}) P_{out}^{gce}(out)$$

$$B_{\rm in} + B_{\rm out} = B\,,$$

$$P_{\rm in}^{\rm ce}(B_{\rm in}) = \sum_{B_{\rm out}} P_{\rm in}^{\rm gce}(B_{\rm in}) P_{\rm out}^{\rm gce}(B_{\rm out}) \delta_{B_{\rm in}+B_{\rm out},B}.$$

$$\begin{aligned}
\kappa_{0,1;\rm in}^{\rm ce} &= \kappa_{0,1;\rm in}^{\rm gce}, \\
\kappa_{0,2;\rm in}^{\rm ce} &= \kappa_{0,2;\rm in}^{\rm gce} - \frac{\left(\kappa_{1,1;\rm in}^{\rm gce}\right)^2}{A}, \\
\kappa_{0,3;\rm in}^{\rm ce} &= \kappa_{0,3;\rm in}^{\rm gce} - \frac{3\kappa_{1,1;\rm in}^{\rm gce}\kappa_{1,2;\rm in}^{\rm gce}}{A} + \frac{3\left(\kappa_{1,1;\rm in}^{\rm gce}\right)^2\kappa_{2,1;\rm in}^{\rm gce}}{A^2} - \frac{B\left(\kappa_{1,1;\rm in}^{\rm gce}\right)^3}{A^3}, \\
\kappa_{0,4;\rm in}^{\rm ce} &= \kappa_{0,4;\rm in}^{\rm gce} - \frac{4\kappa_{1,1;\rm in}^{\rm gce}\kappa_{1,3;\rm in}^{\rm gce} + 3\left(\kappa_{1,2;\rm in}^{\rm gce}\right)^2}{A} + \frac{6\kappa_{1,1;\rm in}^{\rm gce}\left(\kappa_{1,1;\rm in}^{\rm gce}\kappa_{2,2;\rm in}^{\rm gce} + 2\kappa_{1,2;\rm in}^{\rm gce}\kappa_{2,1;\rm in}^{\rm gce}\right)}{A^2} \\
&\quad - \frac{2\left(\kappa_{1,1;\rm in}^{\rm gce}\right)^2\left[2\kappa_{1,1;\rm in}^{\rm gce}\kappa_{3,1;\rm in}^{\rm gce} + 3B\kappa_{1,2;\rm in}^{\rm gce} + 6\left(\kappa_{2,1;\rm in}^{\rm gce}\right)^2\right]}{A^3} + \frac{\left(\kappa_{1,1;\rm in}^{\rm gce}\right)^3\left(C\kappa_{1,1;\rm in}^{\rm gce} + 12B\kappa_{2,1;\rm in}^{\rm gce}\right)}{A^4} \\
&\quad - \frac{3B^2\left(\kappa_{1,1;\rm in}^{\rm gce}\right)^4}{A^5}.
\end{aligned}$$

$$\begin{aligned}
A &= \kappa_{2,0;\rm in}^{\rm gce} + \kappa_{2,0;\rm out}^{\rm gce}, \\
B &= \kappa_{3,0;\rm in}^{\rm gce} + \kappa_{3,0;\rm out}^{\rm gce}, \\
C &= \kappa_{4,0;\rm in}^{\rm gce} + \kappa_{4,0;\rm out}^{\rm gce}.
\end{aligned}$$