# Isospin-Driven Splitting of Chemical Potentials in Isobar Collisions from Lattice QCD

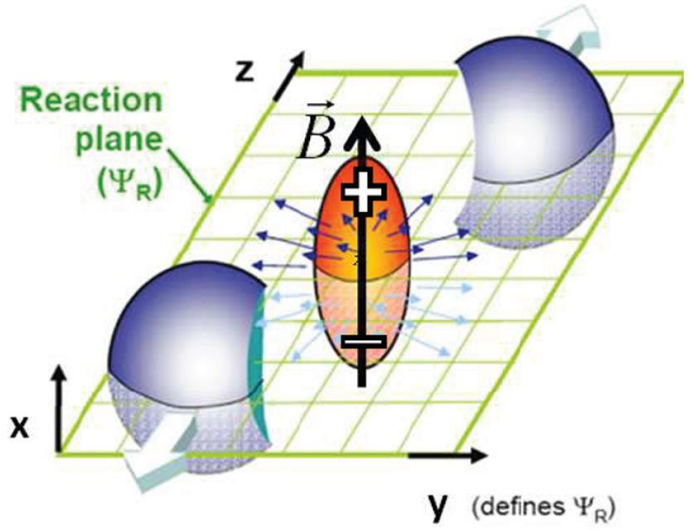
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Central China Normal University

In collaboration with Heng-Tong Ding(丁亨通), Jin-Biao Gu(顾锦彪) and Arpith Kumar

Based on arXiv:2606.30164 [hep-lat]

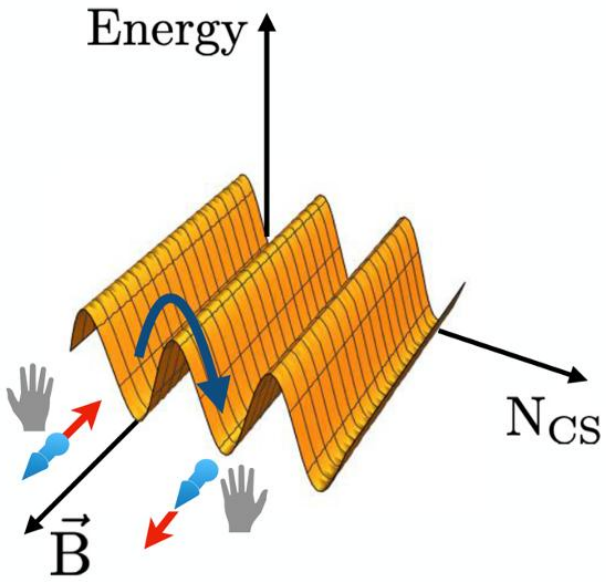
Chirality 2026
Zhuhai · Nov.15-21

# Strong magnetic fields and the chiral magnetic effect

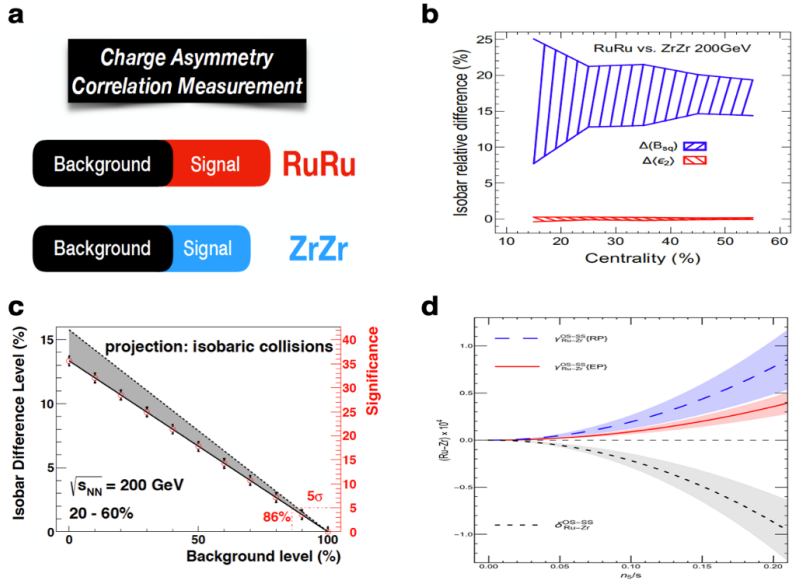



$10^{17} - 10^{18}$G orders of $\Lambda^2_{\mathrm{QCD}}$

V. Skokov, A. Illarionov, V. Toneev, IJMPA 24 (2009) 592




D.E. Kharzeev and J. Liao, Nature Rev. Phys. 3 (2021) 55 [2102.06623].




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# The chemical-potential splittings in STAR isobar data

**Nuclear composition**

$^{96}_{44}\mathrm{Ru}$ $^{96}_{40}\mathrm{Zr}$

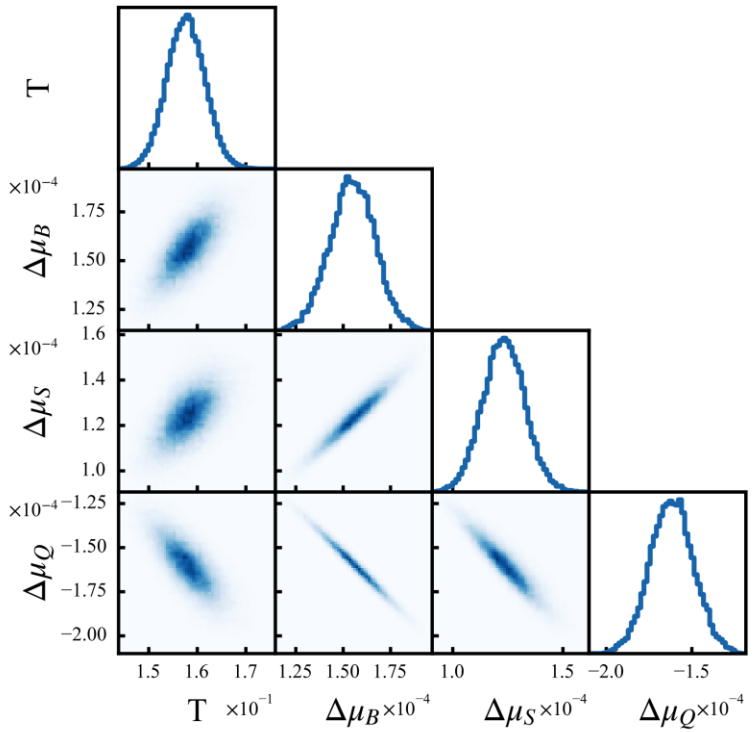
**charge-to-baryon ratio**

$$r \equiv \frac{n_Q}{n_B} \simeq \frac{\mathrm{Z}}{\mathrm{A}} \qquad r_{\mathrm{Ru}} \simeq 0.458 \quad r_{\mathrm{Zr}} \simeq 0.417$$

**chemical potential splittings**

$$\Delta\mu_{\mathrm{B}}, \Delta\mu_{\mathrm{Q}}, \Delta\mu_{\mathrm{S}}$$

Relate to Zr-Ru Splitting Ratios in Isobar Experiments




| | |
|---|---|
| $\Delta\mu_B$ | $0.156^{+0.011}_{-0.012}$ |
| $\Delta\mu_S$ | $0.124^{+0.01}_{-0.01}$ |
| $\Delta\mu_Q$ | $-0.161^{+0.013}_{-0.012}$ |
| $\frac{\Delta\mu_B}{\Delta\mu_Q}$ | $-0.968^{+0.011}_{-0.012}$ |
| $\frac{\Delta\mu_S}{\Delta\mu_Q}$ | $-0.769^{+0.021}_{-0.026}$ |
| $\frac{\Delta\mu_B}{\Delta\mu_S}$ | $1.259^{+0.028}_{-0.032}$ |

0~10% centrality class
Bayesian analyses posterior

J. Grefa, C.Y. Tsang, R. Kumar, V. Dexheimer,C. Ratti, Z. Xu; arXiv:2601.21232 [nucl-th]

# The chemical-potential splittings in STAR isobar data

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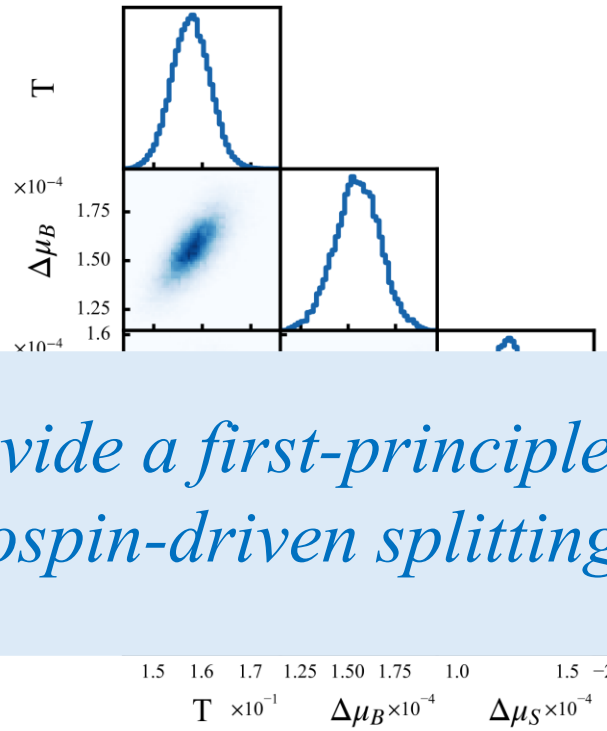
$$r \equiv \frac{n_Q}{n_B} \simeq \frac{\mathrm{Z}}{\mathrm{A}}$$

$r_{\mathrm{Ru}} \simeq 0.458$

**chemical p… splittings**

$\Delta\mu_{\mathrm{B}}, \Delta\mu_{\mathrm{Q}}, \Delta\mu_{\mathrm{S}}$

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| | $59^{+0.028}_{-0.032}$ |

…ality class

Bayesian analyses posterior

*Can we provide a first-principles QCD benchmark for these isospin-driven splittings ?*

J. Grefa, C.Y. Tsang, R. Kumar, V. Dexheimer,C. Ratti, Z. Xu; arXiv:2601.21232 [nucl-th]

# From conserved-charge fluctuations to $q_1$ and $s_1$

Heavy-ion constrains

$$n_S = 0$$
$$n_Q = r n_B$$
$$r \simeq Z/A$$

Lattice-QCD input

$$\chi^{\mathrm{BQS}}_{ijk}(T, eB) = \left. \frac{\partial^{i+j+k}(p/T^4)}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k} \right|_{\hat{\mu}_B=\hat{\mu}_Q=\hat{\mu}_S=0}$$

Solve the constraints at leading order

$$\hat{\mu}_{\mathrm{Q}} = q_1(r,T,eB)\hat{\mu}_{\mathrm{B}} + q_3(r,T,eB)\hat{\mu}_{\mathrm{B}}^3 + \mathcal{O}(\hat{\mu}_{\mathrm{B}}^5)$$
$$\hat{\mu}_{\mathrm{S}} = s_1(r,T,eB)\hat{\mu}_{\mathrm{B}} + s_3(r,T,eB)\hat{\mu}_{\mathrm{B}}^3 + \mathcal{O}(\hat{\mu}_{\mathrm{B}}^5)$$

# From $q_1$ and $s_1$ to splitting ratios

At common $T$ and $eB$

## Zr−Ru convention

$$\Delta\mu_X \equiv \mu_X(r_{\mathrm{Zr}}) - \mu_X(r_{\mathrm{Ru}})$$

## One algebraic step

$$\Delta\mu_X = c_X(r_{\mathrm{Zr}})\Delta\mu_B$$
$$+[c_X(r_{\mathrm{Zr}}) - c_X(r_{\mathrm{Ru}})]$$

$$c_{\mathrm{Q}} = q_1, \quad c_{\mathrm{S}} = s_1$$

## The splitting ratios

$$\frac{\Delta\mu_{\mathrm{Q}}}{\Delta\mu_{\mathrm{B}}} = q_1(r_{\mathrm{Zr}}) + \Delta q_1 x$$

$$\frac{\Delta\mu_{\mathrm{S}}}{\Delta\mu_{\mathrm{B}}} = s_1(r_{\mathrm{Zr}}) + \Delta s_1 x$$

$$\Delta q_1 = q_1(r_{\mathrm{Zr}}) - q_1(r_{\mathrm{Ru}})$$

$$\Delta s_1 = s_1(r_{\mathrm{Zr}}) - s_1(r_{\mathrm{Ru}})$$

**Nuclear composition:**
$r_{\mathrm{Zr}}, r_{\mathrm{Ru}}$

**Lattice QCD:**
$q_1, s_1, \Delta q_1, \Delta s_1$

**Bayesian input:**
$x = \mu_{\mathrm{B}}(r_{\mathrm{Ru}}) / \Delta\mu_{\mathrm{B}}$

# Lattice Setup

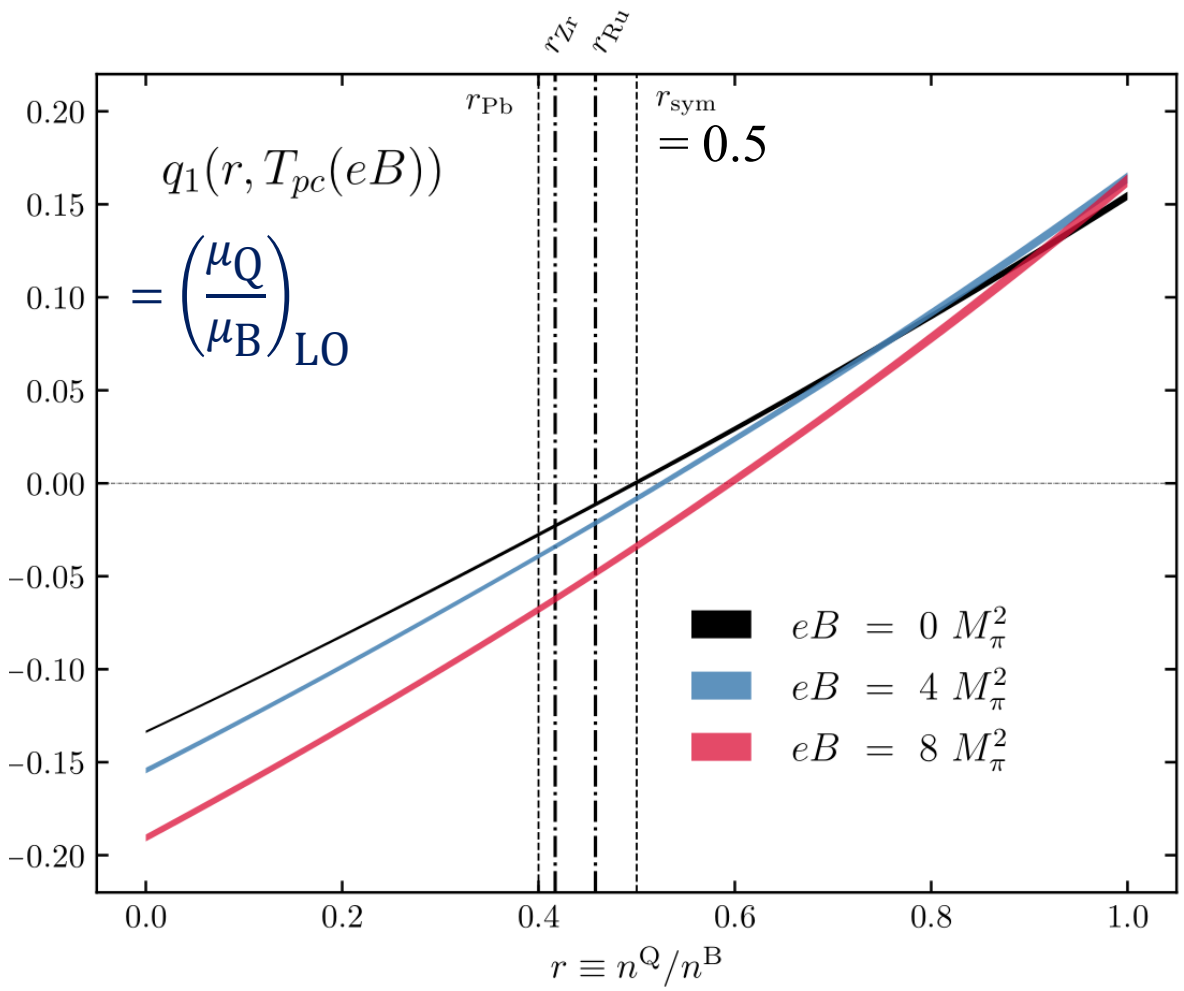
## 2+1 flavor QCD on Lattice

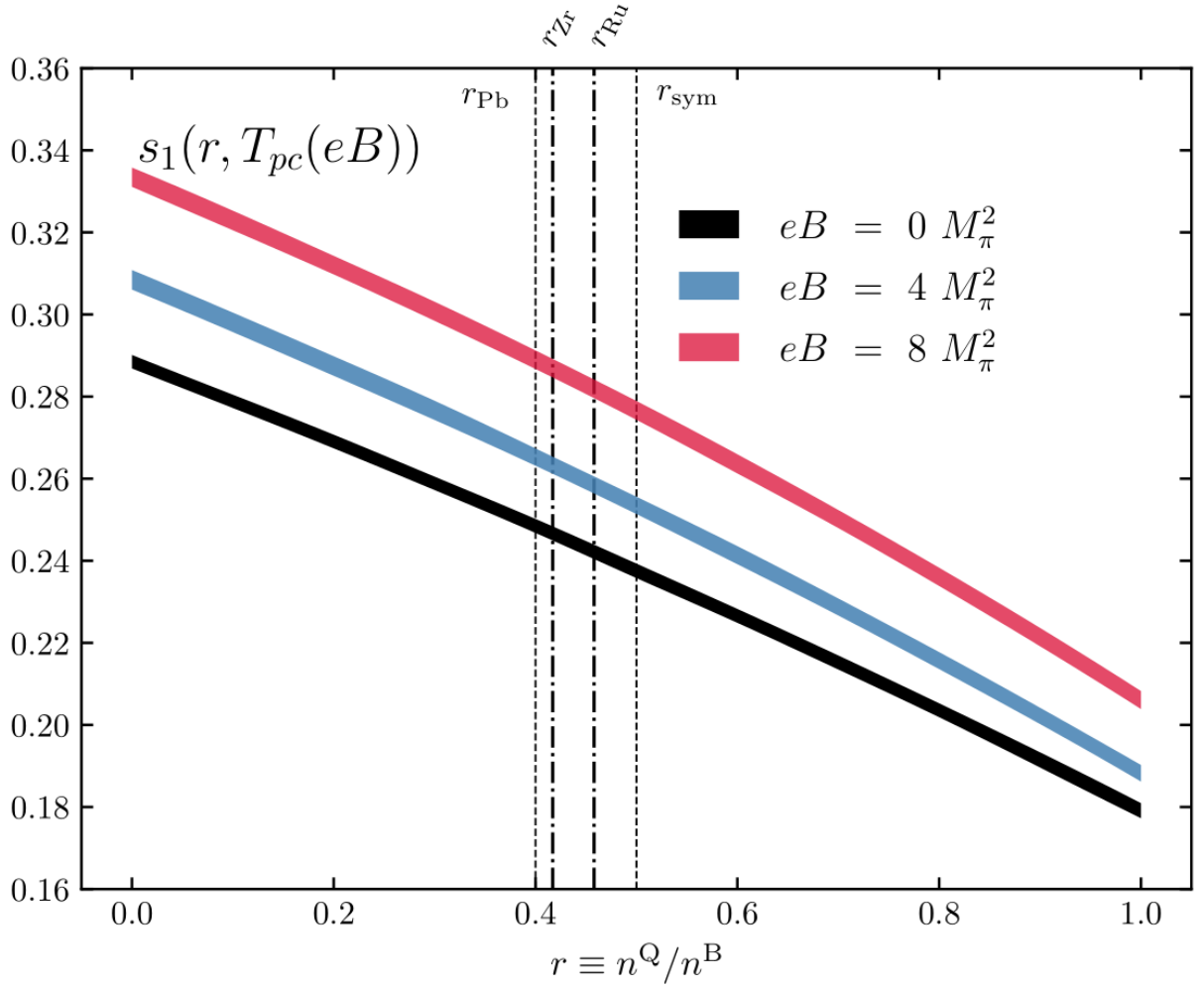
- Highly improved staggered fermions and a tree-level improved Symanzik gauge action
- physical pion mass： $m_l = m_s/27$
- Lattice : $32^3 \times 8$ and $48^3 \times 12$ → cont.est.

## simulated $T$ and $eB$ range

- Temperture: along the $T_{pc}(eB)$ at finite $eB$
- Magnetic field : $0 \leq eB \lesssim 8M_\pi^2 \sim [0, 0.15]\ \mathrm{GeV}^2$

# $r$ –dependence of $q_1$ and $s_1$ along $T_{pc}(eB)$

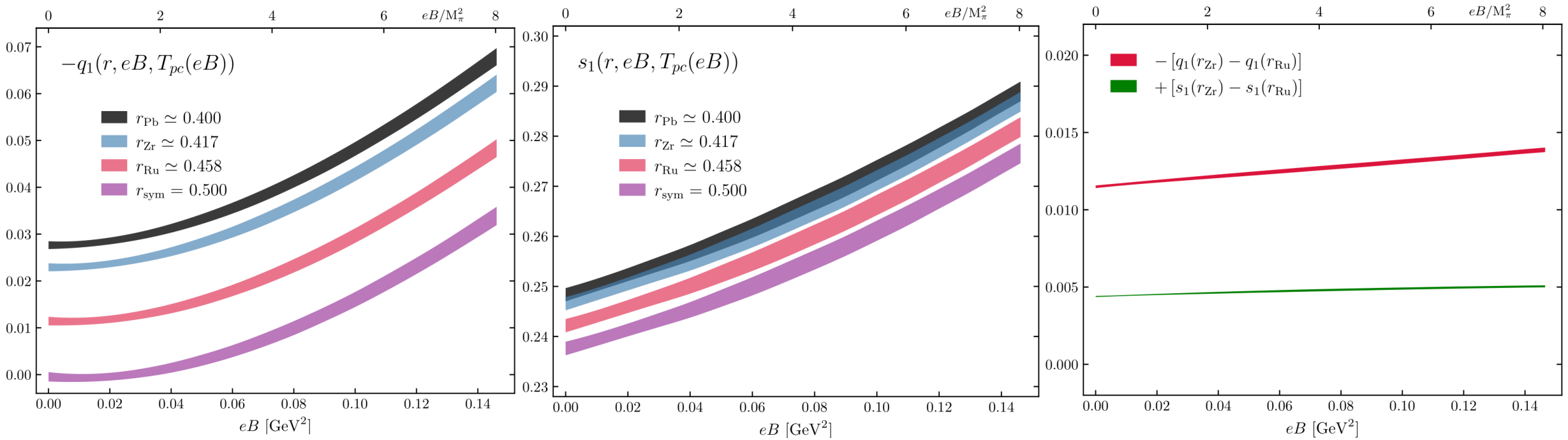



$$|q_1(r_{Pb})| > |q_1(r_{Zr})| > |q_1(r_{Ru})|$$

$$s_1(r_{Pb}) > s_1(r_{Zr}) > s_1(r_{Ru}) > 0$$

# $eB$ –dependence at fixed isospin parameter $r$




$$\frac{\Delta\mu_{\rm Q}}{\Delta\mu_{\rm B}} = q_1(r_{\rm Zr}) + [q_1(r_{\rm Zr}) - q_1(r_{\rm Ru})]\frac{\mu_{\rm B}(r_{\rm Ru})}{\Delta\mu_{\rm B}}$$

$$\frac{\Delta\mu_{\rm S}}{\Delta\mu_{\rm B}} = s_1(r_{\rm Zr}) + [s_1(r_{\rm Zr}) - s_1(r_{\rm Ru})]\frac{\mu_{\rm B}(r_{\rm Ru})}{\Delta\mu_{\rm B}}$$

Intercepts : $-q_1$ grows strongly, while $s_1$ grows mildly.

Slopes : $-\Delta q_1$ increases mildly, while $\Delta s_1$ is nearly $eB$-independent.

# QMHRG baselines & STAR-cut proxies within PDGHRG

Fluctuations expressed in terms of hadron and resonance species R:

$$\chi_{ijk}^{\mathrm{BQS}}\left(T,\hat{\mu}_{\mathrm{B}},\hat{\mu}_{\mathrm{Q}},\hat{\mu}_{\mathrm{S}}\right)=\sum_{R}\mathrm{B}_{R}^{i}\mathrm{Q}_{R}^{j}\mathrm{S}_{R}^{k}\frac{\partial^{i+j+k}p_{R}/T^{4}}{\partial\hat{\mu}_{R}^{i+j+k}}$$

R. Bellwied et al., Phys. Rev. D 101, 034506 (2020)

**QMHRG**

$$q_1^{\mathrm{QMHRG}},\ s_1^{\mathrm{QMHRG}} \Rightarrow \left\{\frac{\Delta\mu_Q}{\Delta\mu_B},\frac{\Delta\mu_S}{\Delta\mu_B},\frac{\Delta\mu_S}{\Delta\mu_Q}\right\}$$

$\pi, K, p$ proxy

$$\tilde{\mathrm{B}}\equiv\tilde{p}$$
$$\tilde{\mathrm{Q}}_{\mathrm{PID}}\equiv\tilde{\pi}+\tilde{K}^{+}+\tilde{p}$$
$$\tilde{\mathrm{S}}\equiv\tilde{K}^{+}$$

Λ-extended proxy

$$\tilde{\mathrm{B}}\rightarrow\tilde{p}+\tilde{\Lambda}$$
$$\tilde{\mathrm{Q}}_{\mathrm{PID}}\equiv\tilde{\pi}+\tilde{K}^{+}+\tilde{p}$$
$$\tilde{\mathrm{S}}\rightarrow\tilde{K}^{+}-\tilde{\Lambda}$$

STAR acceptance

$|\mathrm{y}|<0.5$

species-dependent $p_T$ cut:

$0.2<p_T<2.5\,\mathrm{GeV}/c$ for $\pi, K$,

$0.45<p_T<2.5\,\mathrm{GeV}/c$ for $p$,

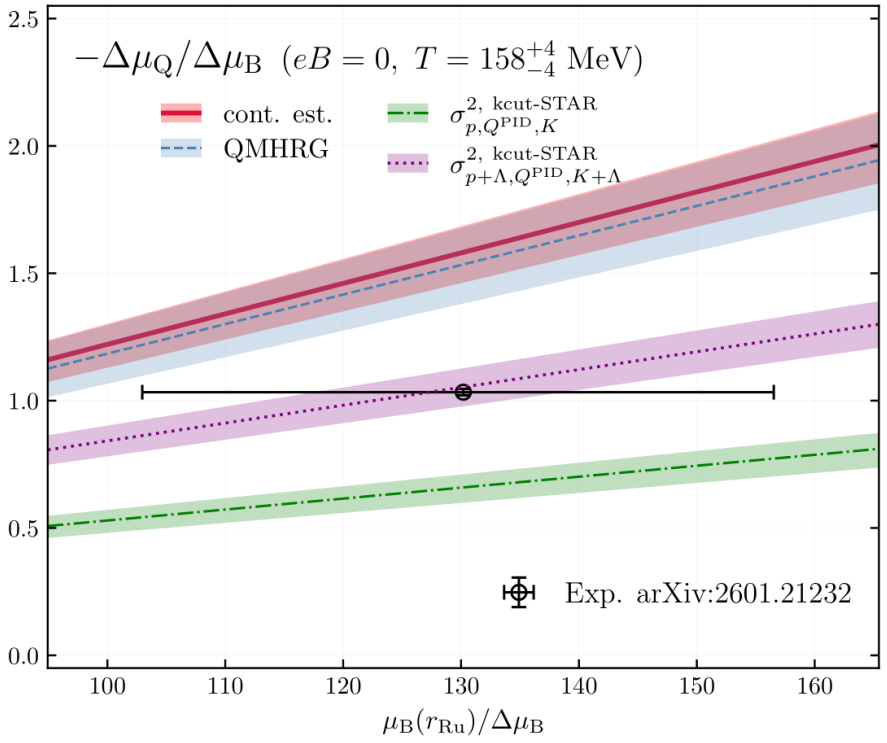
$0.4<p_T<1.6\,\mathrm{GeV}/c$ for $\Lambda$.

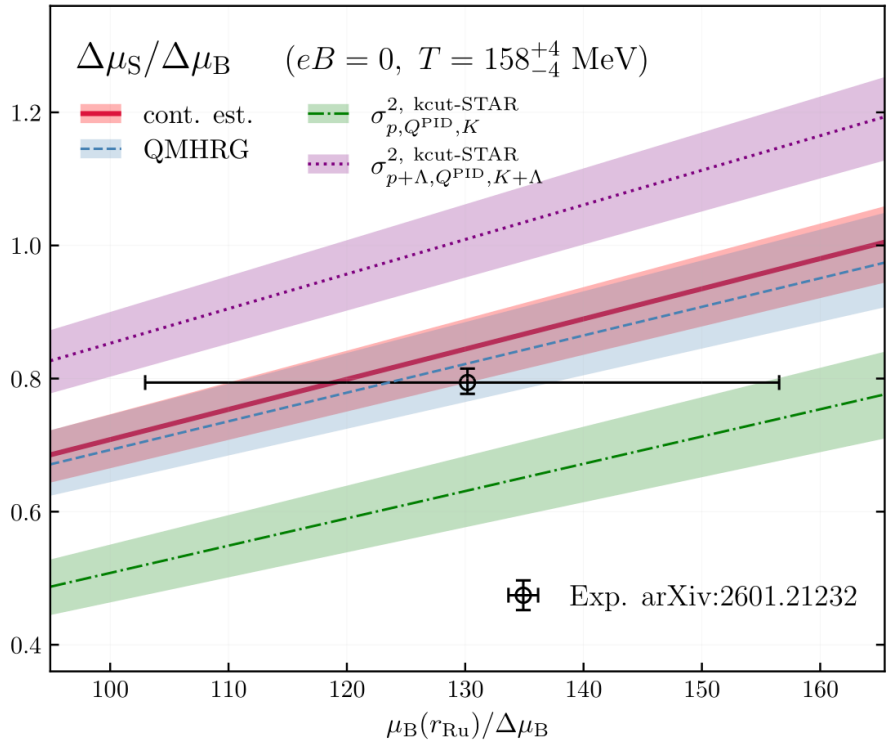
**STAR-cut proxies**

$$\{\pi,K,p\}:\sigma^{2,\mathrm{kcut\text{-}STAR}}_{p,Q^{\mathrm{PID}},K}$$

$$\{\pi,K,p,\Lambda\}:\sigma^{2,\mathrm{kcut\text{-}STAR}}_{p+\Lambda,Q^{\mathrm{PID}},K+\Lambda}$$

# Isospin-driven splitting ratios at vanishing magnetic field




The electric-charge ratio is displayed as $-\Delta\mu_Q/\Delta\mu_B$

*Bayesian thermal-analysis extraction:*

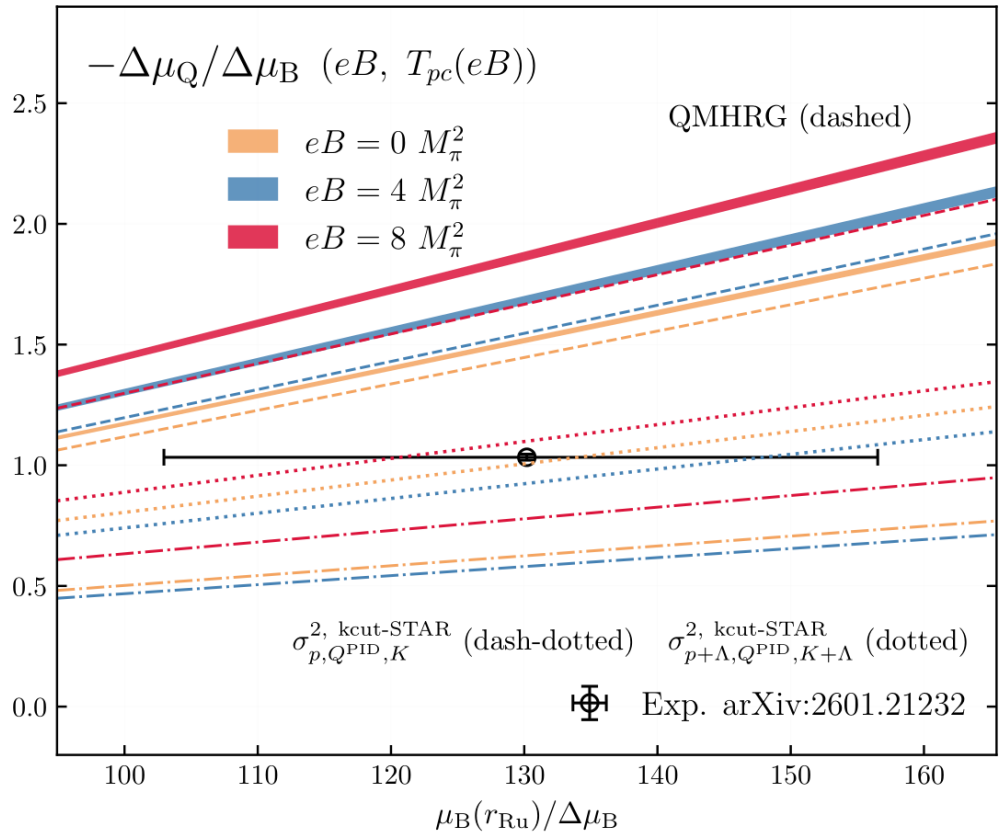
$$\mu_B(r_{Ru}) = 20.31^{+3.8}_{-4.0}\,\text{MeV}$$

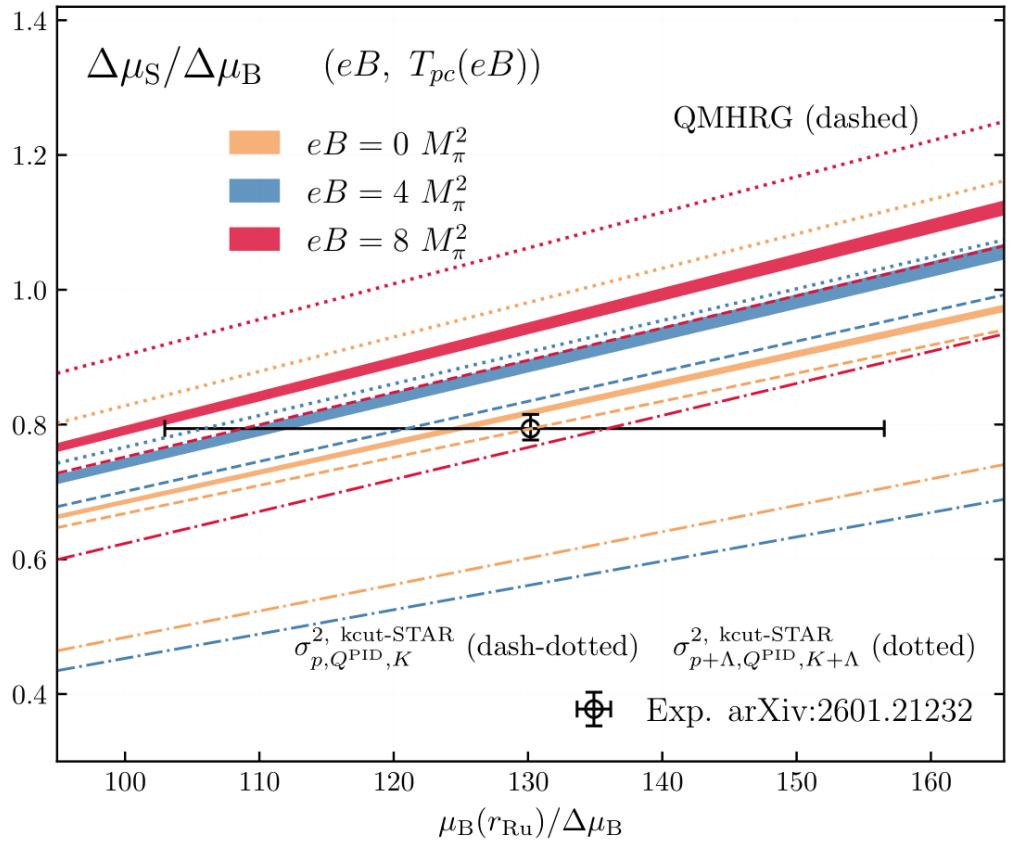
$$\Delta\mu_B = 0.156^{+0.011}_{-0.012}\,\text{MeV}$$

$$\frac{\mu_B(r_{Ru})}{\Delta\mu_B} \simeq 130$$

| | |
|---|---|
| LQCD | $\Delta\mu_Q < 0, \Delta\mu_S > 0$; $\Delta\mu_S/\Delta\mu_B$ is closest to the Bayesian agreement. |
| QMHRG | Follows the lattice results trend in the Bayesian window. |
| Proxies | Adding $\Lambda$ improves the charge ratio, but overshoots the strangeness ratio. |

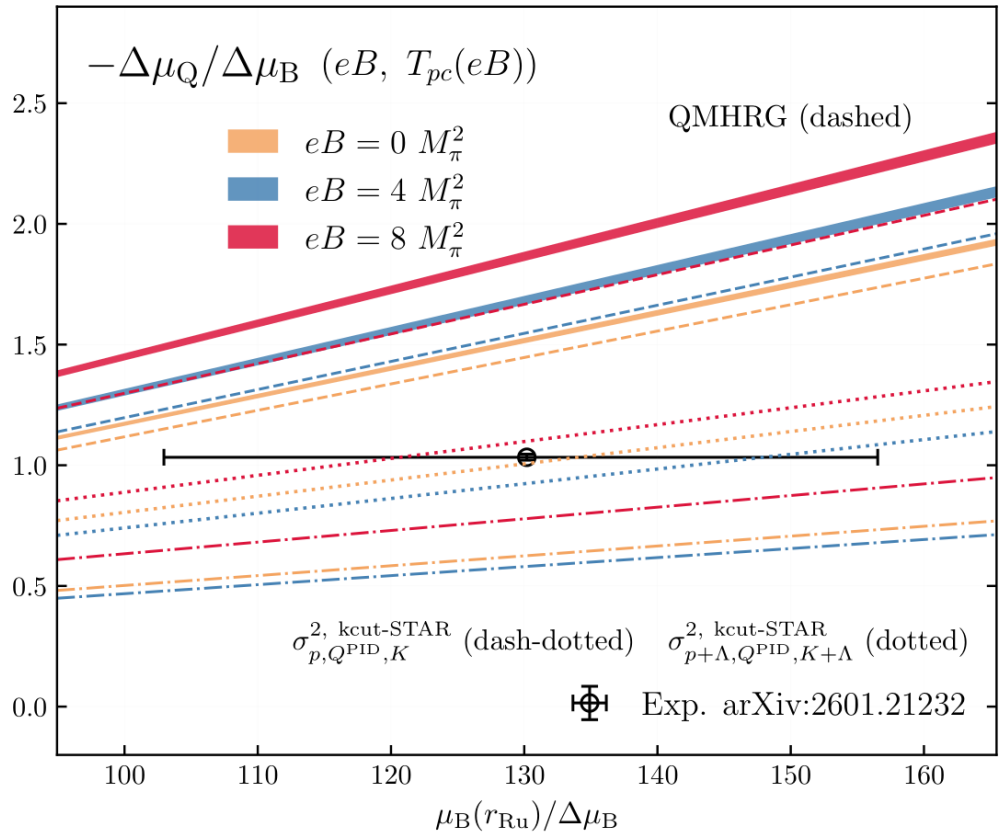
# Magnetic fields modify the splitting ratio

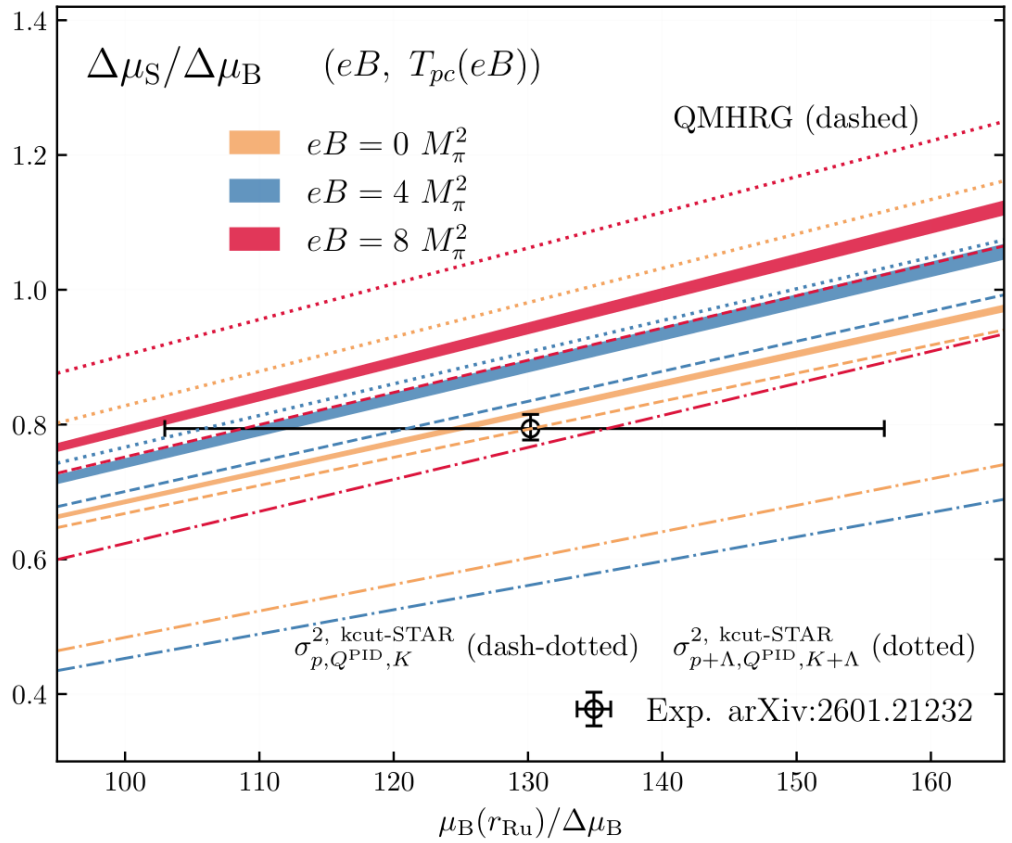



| | |
|---|---|
| **LQCD** | Both ratios increase with $eB$; the charge sector responds more strongly. |
| **QMHRG** | Follows the lattice overall trend. |
| **Proxies** | Show sizeable offsets from full conserved-charge results. |

# Magnetic fields modify the splitting ratio



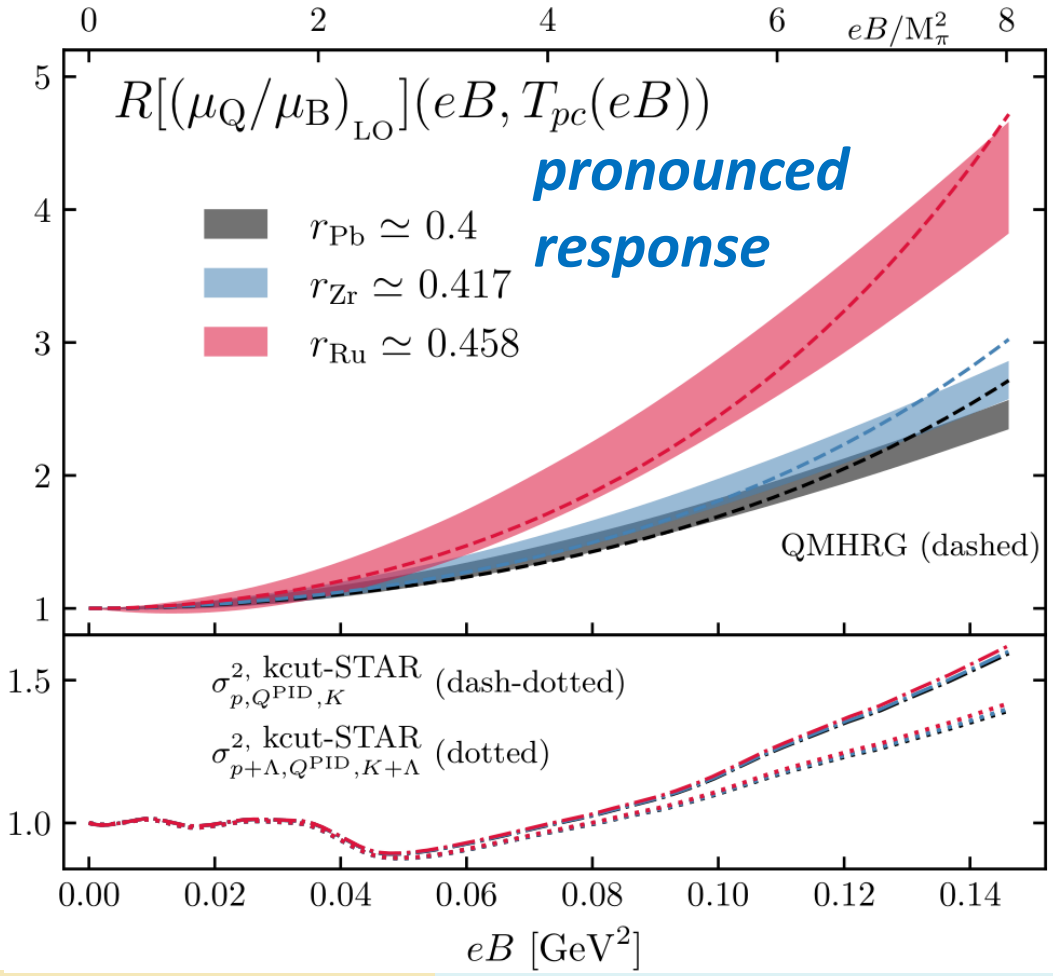

$$\frac{\Delta\mu_Q}{\Delta\mu_B} = q_1(r_{Zr}) + \left[q_1(r_{Zr}) - q_1(r_{Ru})\right]\frac{\mu_B(r_{Ru})}{\Delta\mu_B}$$

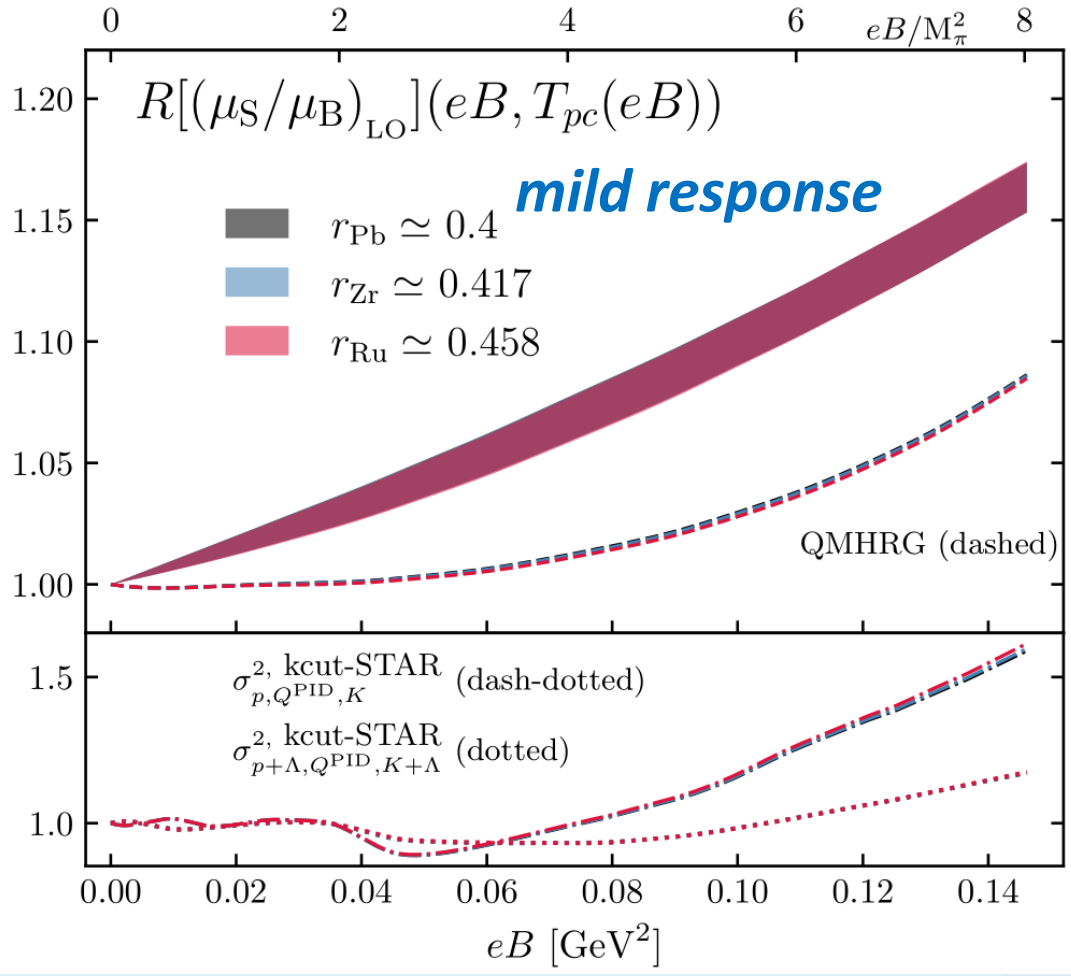
Dominate, but weak eB-dependence

***Can we construct a sensitivity observable ?***

# Normalized ratios reveal a pronounced $eB$ response

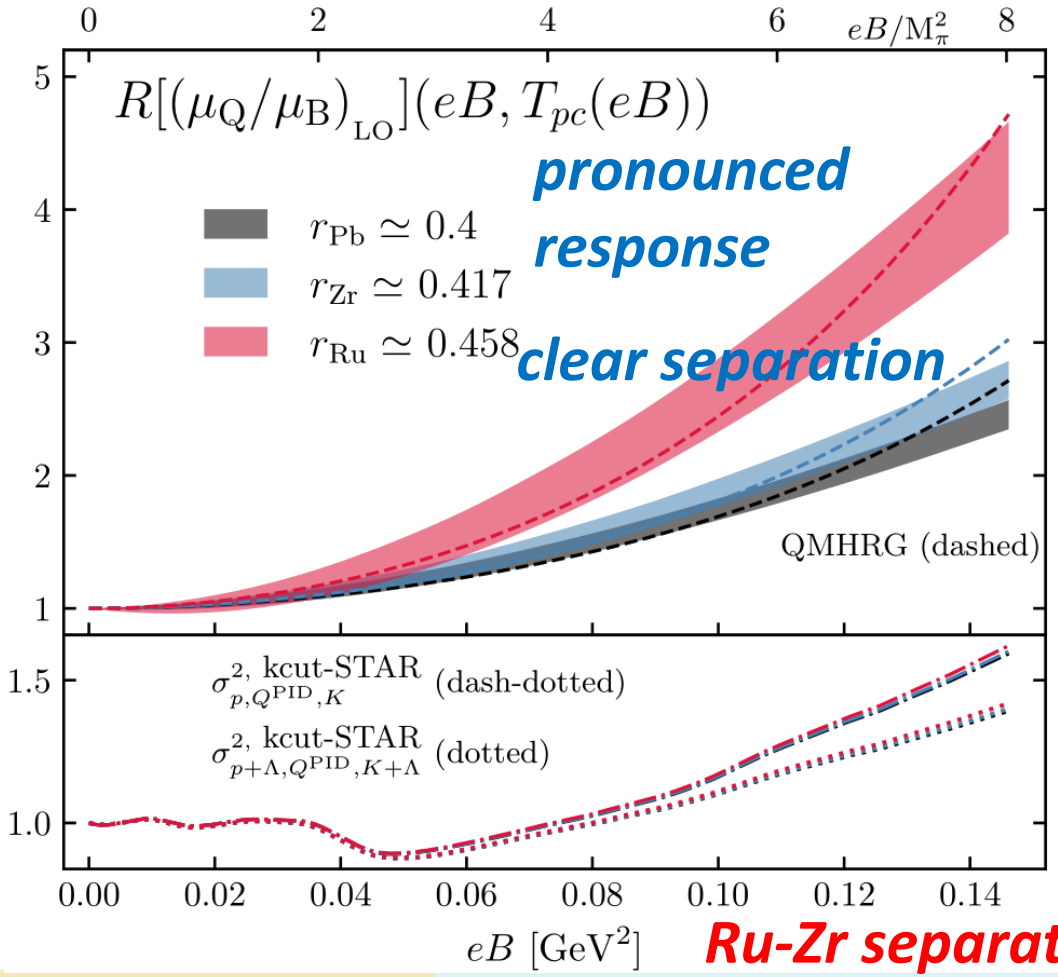
$$R[O](eB) \equiv \frac{O(r, eB, T_{\rm pc}(eB))}{O(r, eB = 0, T_{\rm pc}(0))} \quad , \quad \text{where} \quad O \in \left\{ \left( \frac{\mu_Q}{\mu_B} \right)_{\rm LO} , \left( \frac{\mu_S}{\mu_B} \right)_{\rm LO} \right\}$$

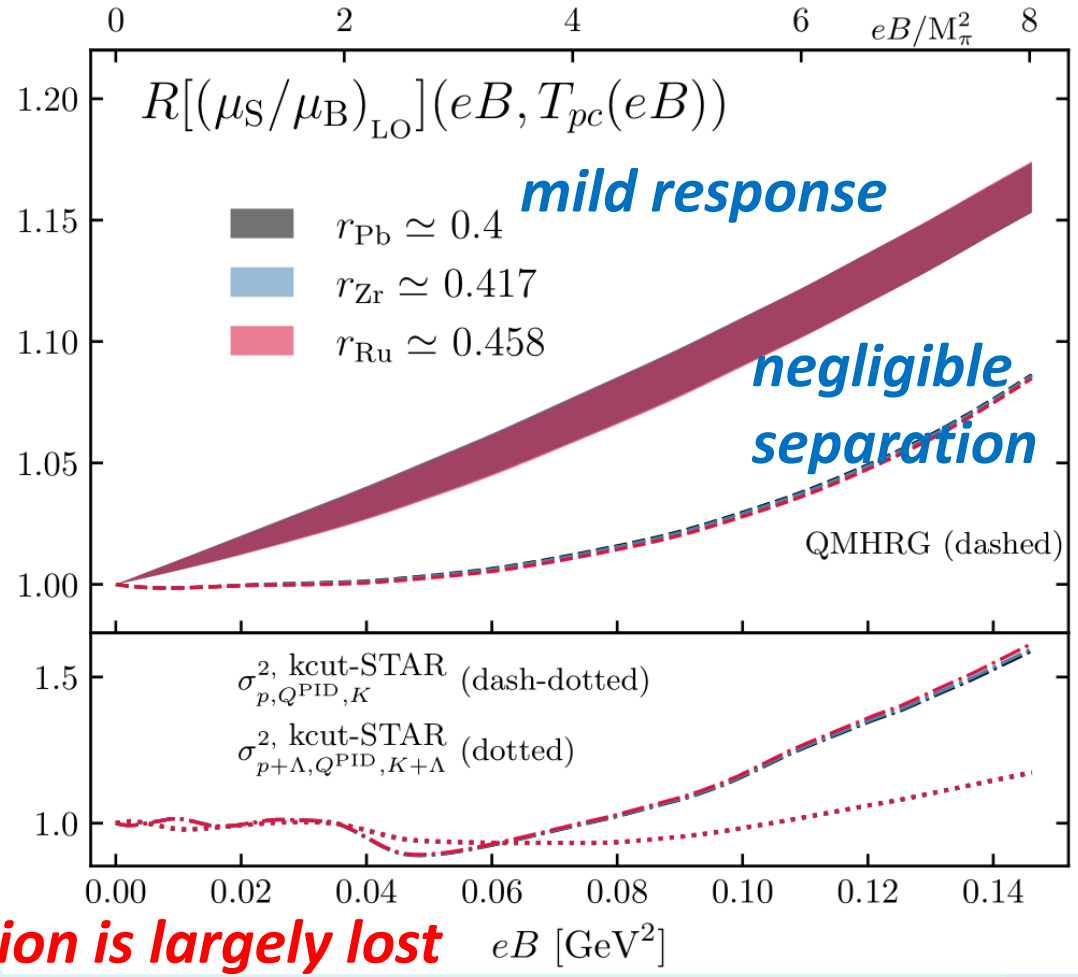


# Normalized ratios reveal a pronounced $eB$ response

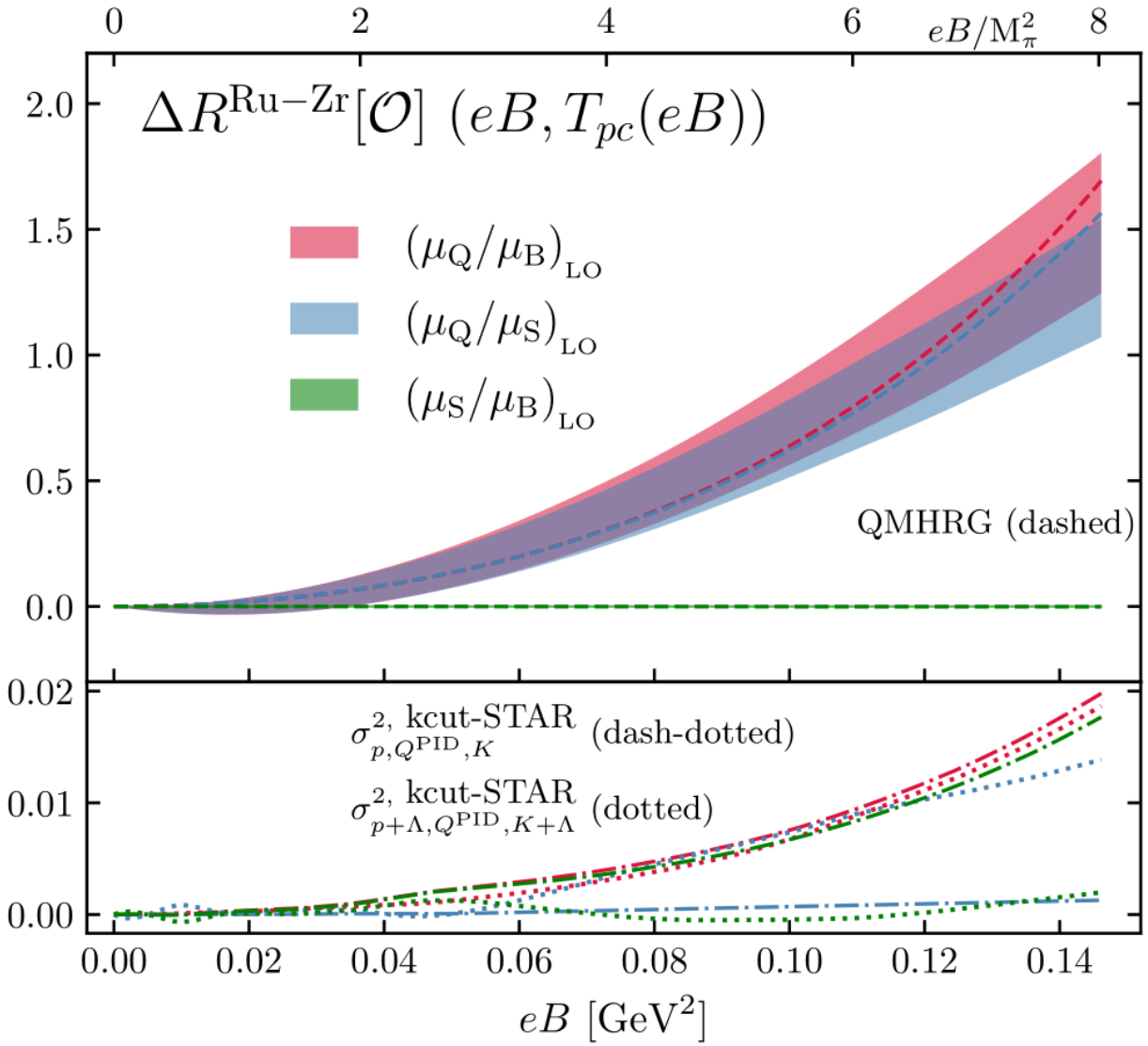
$$R[O](eB) \equiv \frac{O(r, eB, T_{\rm pc}(eB))}{O(r, eB=0, T_{\rm pc}(0))} \quad , \quad \text{where} \quad O \in \left\{ \left(\frac{\mu_Q}{\mu_B}\right)_{\rm LO}, \left(\frac{\mu_S}{\mu_B}\right)_{\rm LO} \right\}$$




**Ru-Zr separation is largely lost**

# Ru-Zr difference of normalized magnetic responses




$$\Delta R^{\mathrm{Ru-Zr}}[O] \equiv R_{\mathrm{Ru}}[O] - R_{\mathrm{Zr}}[O]$$

LQCD/QMHRG:

Charge and mixed differences grow to ~1.5.

Strangeness:

Difference remains near zero.

STAR-cut proxies:

Ru–Zr contrast falls to $O(10^{-2})$.

Restricted charged-hadron content, notably missing the $n$ and $K^0$

# Summary

**Framework** $q_1$ and $s_1$ map isospin asymmetry to chemical-potential splitting ratios.

**At eB = 0** $\Delta\mu_Q < 0, \Delta\mu_S > 0$; $\Delta\mu_S/\Delta\mu_{\mathrm{B}}$ agrees best with the Bayesian extraction.

**At eB > 0** $-\Delta\mu_{\mathrm{Q}}/\Delta\mu_{\mathrm{B}}$ and $\Delta\mu_S/\Delta\mu_{\mathrm{B}}$ increase, while their slopes change only mildly.

**Normalized response** Pronounced Ru-Zr contrast in $\mu_{\mathrm{Q}}$-related ratios, but nearly none in $\mu_S/\mu_{\mathrm{B}}$.

**Proxies** Overall $eB$ response retained, Ru-Zr separation largely lost.

# Thank you!

# Next-to-leading-order correction



The NLO splitting ratio reads:

$$\left(\frac{\Delta\mu_Q}{\Delta\mu_B}\right)_{\mathrm{NLO}} \equiv q_1(r_{\mathrm{Zr}}) + \left(q_1(r_{\mathrm{Zr}}) - q_1(r_{\mathrm{Ru}})\right)\frac{\mu_B(r_{\mathrm{Ru}})}{\Delta\mu_B} \quad LO$$

$$+\left(q_3(r_{\mathrm{Zr}}) - q_3(r_{\mathrm{Ru}})\right)\hat{\mu}_B^2(r_{\mathrm{Ru}})\left(\frac{\mu_B(r_{\mathrm{Ru}})}{\Delta\mu_B}\right)$$

$$+3q_3(r_{\mathrm{Zr}})\hat{\mu}_B^2(r_{\mathrm{Ru}}) \quad NLO$$

Near the isobar freeze out point:

$$\hat{\mu}_B = \frac{20.31}{158} \simeq 0.129,$$

$$\hat{\mu}_B^2 \simeq 0.017 \simeq 2\%.$$

The lattice estimate:

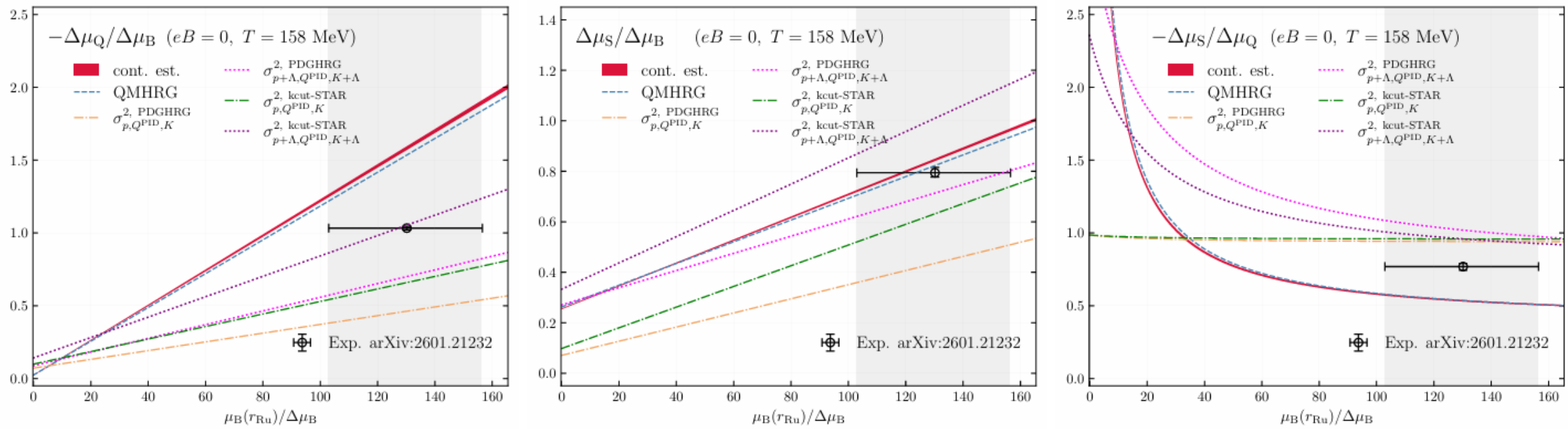
$$\frac{q_3}{q_1} \lesssim 2\%$$

H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li and J.-H. Liu, Phys. Rev. Lett. 132 (2024) 201903 [2312.08860].

- Suppresses the NLO corrections to $\Delta\mu_Q/\Delta\mu_B$ to $O(10^{-3})$ relative to the LO.
- Similarly for $\Delta\mu_S/\Delta\mu_B$, $\Delta\mu_S/\Delta\mu_Q$

# Results in an extended $\mu_B(r_{\mathrm{Ru}})/\Delta\mu_B$ window






- The charge and strangeness splitting ratios grow linearly with $\mu_B(r_{\mathrm{Ru}})/\Delta\mu_B$ , with the isospin-difference terms dominating in the experimental window.
- The mixed ratio begins to saturate near $1/2$ as the common $\mu_B(r_{\mathrm{Ru}})/\Delta\mu_B$ dependence cancels.
- The Λ-extended proxy improves the strangeness sector at small $\mu_B(r_{\mathrm{Ru}})/\Delta\mu_B$ , but current proxies do not reproduce the lattice behavior in the experimental window.

# Magnetic Field Setup on Lattice



$B$ pointing along the $z$ direction:

$$u_x(n_x,n_y,n_z,n_\tau)=\begin{cases}\exp[-iqa^2BN_xn_y] & (n_x=N_x-1)\\ 1 & (\text{otherwise})\end{cases}$$

*No sign problem!*

$$u_y(n_x,n_y,n_z,n_\tau)=\exp[iqa^2Bn_x]$$

$$u_z(n_x,n_y,n_z,n_\tau)=u_t(n_x,n_y,n_z,n_\tau)=1$$

$$q_u=2/3e$$

$$q_d=-1/3e$$

$$q_s=-1/3e$$

Quantization of the magnetic field $qB=\dfrac{2\pi N_b}{N_xN_y}a^{-2}$ ⇒ $eB=\dfrac{6\pi N_b}{N_xN_y}a^{-2}$

The periodicity of U(1) links implies the constrain : $0\le N_b<N_xN_y/4$

In this work, we restrict to $N_b\le 6 \Leftrightarrow eB\simeq 8M_\pi^2$

Landau gauge

G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz, S. Krieg et al., JHEP 02 (2012) 044.

# From chemical potential to coefficient $q_1$ and $s_1$

Taylor expansion $\hat{\mu}_{\{\mathrm{Q,S}\}} \equiv \hat{\mu}_{\{\mathrm{Q,S}\}}(T, eB, \hat{\mu}_{\mathrm{B}})$

$$\hat{\mu}_{\mathrm{Q}} = q_1(r,T,eB)\hat{\mu}_{\mathrm{B}} + q_3(r,T,eB)\hat{\mu}_{\mathrm{B}}^3 + \mathcal{O}(\hat{\mu}_{\mathrm{B}}^5)$$

$$\hat{\mu}_{\mathrm{S}} = s_1(r,T,eB)\hat{\mu}_{\mathrm{B}} + s_3(r,T,eB)\hat{\mu}_{\mathrm{B}}^3 + \mathcal{O}(\hat{\mu}_{\mathrm{B}}^5)$$

Initial nuclei constraint

$$n_Q / n_B = r, n_S = 0$$

$$q_1 = \frac{r\left(\chi_2^{\mathrm{B}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{BS}}\chi_{11}^{\mathrm{BS}}\right) - \left(\chi_{11}^{\mathrm{BQ}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{BS}}\chi_{11}^{\mathrm{QS}}\right)}{\left(\chi_2^{\mathrm{Q}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{QS}}\chi_{11}^{\mathrm{QS}}\right) - r\left(\chi_{11}^{\mathrm{BQ}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{BS}}\chi_{11}^{\mathrm{QS}}\right)}$$

$$s_1 = -\left(\chi_{11}^{\mathrm{BS}} + q_1\chi_{11}^{\mathrm{QS}}\right) / \chi_2^{\mathrm{S}}$$

The fluctuations of conserved charge

$$\chi_{ijk}^{\mathrm{BQS}}(T,eB) = \left.\frac{\partial^{i+j+k}(p/T^4)}{\partial\hat{\mu}_B^i\partial\hat{\mu}_Q^j\partial\hat{\mu}_S^k}\right|_{\hat{\mu}_B=\hat{\mu}_Q=\hat{\mu}_S=0}$$

# Isospin-driven chemical-potential splitting ratios

*Computable in lattice QCD*

The splittings of chemical potential:

$$\Delta\hat{\mu}_X \equiv \hat{\mu}_X(r_{\rm Zr}) - \hat{\mu}_X(r_{\rm Ru}), \quad X \in \{\mathrm{B},\mathrm{Q},\mathrm{S}\}$$

Isospin parameter $r_{\rm Ru}, r_{\rm Zr}$

fluctuations $\chi^{\rm BQS}_{ijk}(T, eB)$

$$q_1 = \left(\frac{\mu_{\rm Q}}{\mu_{\rm B}}\right)_{\rm LO}, s_1 = \left(\frac{\mu_{\rm S}}{\mu_{\rm B}}\right)_{\rm LO}$$

Leading order coefficients $q_1, s_1$

*Bayesian input*

$$\begin{aligned}
\Delta\hat{\mu}_Q &\equiv q_1(r_{\rm Zr})\hat{\mu}_B(r_{\rm Zr}) - q_1(r_{\rm Ru})\hat{\mu}_B(r_{\rm Ru}), \\
&= q_1(r_{\rm Zr})(\hat{\mu}_B(r_{\rm Ru}) + \Delta\hat{\mu}_B) - q_1(r_{\rm Ru})\hat{\mu}_B(r_{\rm Ru}) \\
\Delta\hat{\mu}_S &\equiv s_1(r_{\rm Zr})\hat{\mu}_B(r_{\rm Zr}) - s_1(r_{\rm Ru})\hat{\mu}_B(r_{\rm Ru}) \\
&= s_1(r_{\rm Zr})(\hat{\mu}_B(r_{\rm Ru}) + \Delta\hat{\mu}_B) - s_1(r_{\rm Ru})\hat{\mu}_B(r_{\rm Ru})
\end{aligned}$$

$$\begin{aligned}
\frac{\Delta\mu_{\rm Q}}{\Delta\mu_{\rm B}} &= q_1(r_{\rm Zr}) + \left[q_1(r_{\rm Zr}) - q_1(r_{\rm Ru})\right]\frac{\mu_{\rm B}(r_{\rm Ru})}{\Delta\mu_{\rm B}} \\
\frac{\Delta\mu_{\rm S}}{\Delta\mu_{\rm B}} &= s_1(r_{\rm Zr}) + \left[s_1(r_{\rm Zr}) - s_1(r_{\rm Ru})\right]\frac{\mu_{\rm B}(r_{\rm Ru})}{\Delta\mu_{\rm B}}
\end{aligned}$$

**The splitting ratios**