Two-particle correlations from local equilibrium

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Motivation

Identical-particle momentum correlations probe the space-time structure of the source (HBT interferometry). For a chaotic source everything is fixed by the one-particle density matrix

$$G(p_1, p_2) = \langle \hat{a}^\dagger(p_2)\, \hat{a}(p_1) \rangle, \qquad C(p_1, p_2) = 1 + \frac{|G(p_1, p_2)|^2}{G(p_1, p_1)\, G(p_2, p_2)}$$

Hydrodynamic models use a single integral over the freeze-out hypersurface:

$$G(p_1, p_2) = \frac{2}{(2\pi)^3} \int_{\Sigma_{\mathrm{FO}}} d\Sigma_\mu\, K^\mu\, e^{-iq\cdot y}\, n_B(y, K) \qquad K = \tfrac{p_1 + p_2}{2}, \quad q = p_2 - p_1,$$

Smoothness approximation: non-phase relative momentum dependence is neglected

Questions

How does this formula follow from the quantum state of the fluid? How do we compute corrections to it systematically?

Diagonal case ($p_1 = p_2$)

- Using the local-equilibrium density operator gradient corrections are computed systematically, notably for spin polarization

Off-diagonal case ($p_1 \neq p_2$)

- Single-integral representation for free bosons (Makhlin, Sinyukov Yad. Fiz. 46, 637 (1987))
- Single-integral representation through a geometric approximation, corrections in relative momentum q

  (Rennecke, Pisarski, Rischke, Phys. Rev. D 107, 116011 (2023))

This work

An exact representation of $G(p_1, p_2)$, for free complex scalar and Dirac fields, as the flux of a conserved current built from the Wigner function, which allows a systematic expansion in gradients and relative momentum.

Local equilibrium and Wick's theorem

$$\widehat{\rho}_{\rm LTE} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{\rm LTE}} d\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \widehat{j}^\mu\zeta\right)\right], \qquad \beta^\mu = \frac{u^\mu}{T}, \quad \zeta = \frac{\mu}{T}$$

- Dissipation is neglected
- Free fields: exponent quadratic in $\widehat{a} \Rightarrow$ Wick's theorem holds (Gaudin, Nuclear Phys. (1960))
- $[\widehat{\rho}_{\rm LTE}, \widehat{Q}] = 0 \Rightarrow \langle\, \widehat{a}\widehat{a} \,\rangle = 0$: fully chaotic source

$$\langle \widehat{a}_1^\dagger \widehat{a}_2^\dagger \widehat{a}_2 \widehat{a}_1 \rangle = G_{11}G_{22} \pm G_{12}G_{21} \quad \Rightarrow \quad C(p_1,p_2) = 1 \pm \frac{\mathrm{Tr}\,(G_{12}G_{21})}{\mathrm{Tr}\,G_{11}\,\mathrm{Tr}\,G_{22}}$$

Fermions: G is a 2×2 matrix in spin space

$$G = \tfrac{1}{2}\big(\mathcal{F}\,\mathbb{I} + \boldsymbol{\mathcal{P}}\cdot\boldsymbol{\sigma}\big) \qquad C = 1 - \frac{|\mathcal{F}_{12}|^2 + |\boldsymbol{\mathcal{P}}_{12}|^2}{2\,\mathcal{F}_{11}\mathcal{F}_{22}}$$

From fields to the Wigner function

Inversion formulas on a Cauchy surface for $\widehat{a}$ and $\widehat{a}^\dagger$ give

$$G(p_1,p_2)=\frac{1}{(2\pi)^3}\int d\Sigma_\mu(x_1)\int d\Sigma_\nu(x_2)\,e^{i(p_1\cdot x_1-p_2\cdot x_2)}(\partial_1^\mu-ip_1^\mu)(\partial_2^\nu+ip_2^\nu)\langle:\widehat{\phi}^{\dagger-}(x_2)\widehat{\phi}^{+}(x_1):\rangle$$

Introducing $X=\frac{x_1+x_2}{2}$, $y=x_2-x_1$, and the particle Wigner function

$$W^+(x,k)=\frac{2}{(2\pi)^4}\int d^4y\,e^{-ik\cdot y}\langle:\widehat{\phi}^{\dagger-}(x+\tfrac{y}{2})\,\widehat{\phi}^{+}(x-\tfrac{y}{2}):\rangle$$

Double integral representation

$$G(p_1,p_2)=\frac{1}{2(2\pi)^3}\int d\Sigma_\mu(x_1)\int d\Sigma_\nu(x_2)\int d^4k\;e^{-iq\cdot X}e^{i(k-K)\cdot y}\,D^{\mu\nu}(k)\,W^+(X,k)$$

- Exact for free fields
- Integrand is divergence-free in $x_1,x_2\Rightarrow$ the hypersurfaces can be chosen freely.

Single integral on the freeze-out

Pick a constant (independent of $x_{1,2}$, possibly dependent on $p_{1,2}$), unit ($t^2 = 1$), timelike, future-directed vector t^μ. Work in its rest frame, $t^\mu = (1, 0)$.

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1. Hypersurface choice

Perform both integrations on the same hyperplane $t \cdot x = 0$.

$$G(p_1, p_2) = \frac{1}{2(2\pi)^3} \int d^3x_1 \int d^3x_2 \int d^4k \; e^{-iq\cdot X} e^{i(k-K)\cdot y} D_X^{00}(k; K, q)\, W^+(X, k)$$

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2. Integration over the relative coordinate

Change variables $(x_1, x_2) \to (X, y)$ and integrate over $\boldsymbol{y}$:

$$\int d^3y \, e^{-i(\boldsymbol{k}-\boldsymbol{K})\cdot\boldsymbol{y}} = (2\pi)^3 \delta^3(\boldsymbol{k} - \boldsymbol{K}) \quad \Rightarrow \quad G = \int d^3X \, I^{00}(X; K, q), \quad \text{but} \quad \partial_\mu I^{\mu 0} \neq 0$$

The integration over $d^3\boldsymbol{k}$ fixes $\widetilde{K} = (k^0, \boldsymbol{K})$

$$I^{\mu\nu}(X) = \frac{1}{8}\int dk^0\, e^{-iq\cdot X}\left[\partial^\mu - 2i(\widetilde{K}+p_1)^\mu\right]\left[\partial^\nu + 2i(\widetilde{K}+p_2)^\nu\right] W^+(X,\widetilde{K})$$

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3. Symmetrized current

Using the eom and the mass-shell eq. for the Wigner function:

$$\partial_\mu I^{\mu 0} = -\partial_\mu I^{0\mu} \quad \Rightarrow \quad \mathcal{J}^\mu \equiv \tfrac{1}{2}\left(I^{\mu 0} + I^{0\mu}\right) \qquad \partial_\mu \mathcal{J}^\mu = 0, \qquad \mathcal{J}^0 = I^{00}$$

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4. Deform the hypersurface

$$G(p_1,p_2) = \int_{x^0=0} d^3X\, \mathcal{J}^0(X;p_1,p_2) = \int_{\Sigma_{\rm FO}} d\Sigma_\mu(X)\, \mathcal{J}^\mu(X;p_1,p_2)$$

Result: exact single freeze-out integrals

Complex scalar

$$G(p_1,p_2)=\frac{1}{16}\int_{\Sigma_{\rm FO}} d\Sigma_\mu \int dk^0\, e^{-iq\cdot X}\Big\{\big[\partial^\mu-2i(\widetilde{K}+p_1)^\mu\big]\big[\partial^0+2i(\widetilde{K}^0+p_2^0)\big] + \big[\partial^0-2i(\widetilde{K}^0+p_1^0)\big]\big[\partial^\mu+2i(\widetilde{K}+p_2)^\mu\big]\Big\} W^+(X,\widetilde{K})$$

Dirac (same steps, Dirac–Wigner equation and its adjoint)

$$G_{rs}(p_1,p_2)=\frac{1}{2}\int_{\Sigma_{\rm FO}} d\Sigma_\mu \int dk^0\, e^{-iq\cdot X}\,\bar{u}_r(p_1)\big(\gamma^\mu W^+(X,\widetilde{K})\gamma^0+\gamma^0 W^+(X,\widetilde{K})\gamma^\mu\big)u_s(p_2)$$

- Exact for free fields
- Built from W^+ at $X\in\Sigma_{\rm FO}$
- Different choices of t^μ give different local currents but the same integrated flux

A joint expansion in gradients and relative momentum

1. Gradient expansion

$$W^+ = \sum_{n\geq 0} W^+_{(n)} \quad \Longrightarrow \quad G = \sum_{n\geq 0} G^{(n)}$$

2. Relative momentum scale and gradient scale (Makhlin et al. Z. Phys. C 39, 69 (1988))

$$\partial \sim \frac{1}{L} \quad R \sim L\sqrt{\frac{T}{m_T}} \quad \Longrightarrow \quad q \sim R^{-1} \sim L^{-1}\sqrt{\frac{m_T}{T}}$$

3. Combined expansion

$$\boxed{G^{[N]} = \sum_{n+k=N} G^{(n,k)}}$$

$n \rightarrow$ gradient order

$k \rightarrow$ relative-momentum order

$$\Rightarrow \quad C = \sum_N C^{[N]}$$

Expand non-phase factors in q while keeping $e^{-iq\cdot X}$ exact (qR can be $O(1)$)

Leading order for a complex scalar

$$W^{+}_{(0)}(X,k) = \frac{2}{(2\pi)^3}\,\theta(k^0)\,\delta(k^2-m^2)\,n_{\mathrm{B}}(X,k)$$

The mass shell fixes $k^0 = \omega_{\boldsymbol{K}}$ and we obtain the on-shell $P = (\omega_{\boldsymbol{K}}, \boldsymbol{K})$. At all orders in q we have

$$G^{(0)} = \frac{1}{4(2\pi)^3\omega_{\boldsymbol{K}}}\int_{\Sigma_{\mathrm{FO}}} d\Sigma_\mu\, e^{-iq\cdot X}\left[(P+p_1)^\mu(P^0+p_2^0)+(P^0+p_1^0)(P+p_2)^\mu\right] n_{\mathrm{B}}(X,P)$$

Considering the $q \to 0$ limit one gets $[\dots] = 8P^0P^\mu$:

HBT in the smoothness approximation is recovered

$$G^{[0]} = \frac{2}{(2\pi)^3}\int_{\Sigma_{\mathrm{FO}}} d\Sigma_\mu\, P^\mu\, e^{-iq\cdot X}\, n_{\mathrm{B}}(X,P), \qquad C^{[0]} = 1 + \frac{\left|\int d\Sigma\cdot P\, e^{-iq\cdot X} n_{\mathrm{B}}(X,P)\right|^2}{\int d\Sigma\cdot p_1\, n_{\mathrm{B}}(x,p_1)\int d\Sigma\cdot p_2\, n_{\mathrm{B}}(y,p_2)}$$

- Trivially independent of t
- For the free Dirac field fermionic HBT is recovered

First order for a complex scalar

$$G(p_1,p_2)=\frac{1}{8}\int_{\Sigma_{\rm FO}} d\Sigma_\mu \int dk^0\, e^{-iq\cdot X}\Big[\underbrace{4(\widetilde{K}+K)^\mu(\widetilde{K}+K)^0-q^\mu q^0}_{\text{even in } q} + \underbrace{i\big(q^0\partial^\mu+q^\mu\partial^0\big)}_{O(q\,\partial)}+\underbrace{\partial^\mu\partial^0}_{O(\partial^2)}\Big]W^+(X,\widetilde{K})$$

$G^{[1]}=G^{(1,0)}+G^{(0,1)}$ has three possible sources:

- **Linear in q:** non-derivative terms are even in $q \Rightarrow G^{(0,1)}=0$
- **Derivatives of $W^+_{(0)}$:** first derivatives always come with q

- **First-order Wigner function:** using linear response theory

  (Sheng, Becattini, Roselli, Phys. Rev. C 114, 034903 (2026))

$$\Delta W^{+}(x,p) = \frac{1}{(2\pi)^3} \int d^4\ell \; \delta(p\cdot\ell)\, G^{\mu\nu}(\ell)\, F_{\mu\nu}(\ell)$$

where

$$F_{\mu\nu}(\ell) = \int_{\Sigma_{\rm FO}} d\Sigma_\mu(y)\, e^{-i\ell\cdot(y-x)}\, \Delta\beta_\nu(y,x) \qquad G^{\mu\nu}(\ell) = G^{\mu\nu}(-\ell)$$

$$W^{+}_{(1)} \propto (\partial^\ell_\rho G^{\mu\nu}(\ell))|_{\ell=0} \qquad \Rightarrow \qquad W^{+}_{(1)} = 0$$

Result:

No first-order local equilibrium correction to the HBT formula for a free scalar field

$$G^{[1]}(p_1,p_2) = 0 \qquad \Rightarrow \qquad C^{[1]}(p_1,p_2) = 0$$

Summary and outlook

Summary

- We obtain an exact single–integral representation of $G(p_1, p_2)$ for free fields in terms of the particle Wigner function
- The integration hypersurface can be deformed into Σ_{FO}
- A joint expansion in gradients and q (interference phase exact) is considered
- HBT formula is recovered at leading order
- For the complex scalar field no linear order correction appears

Outlook

- Quantum–statistical contribution to spin correlation