Spin Entanglement in QCD

from Parton Scattering and Topological Fluctuations



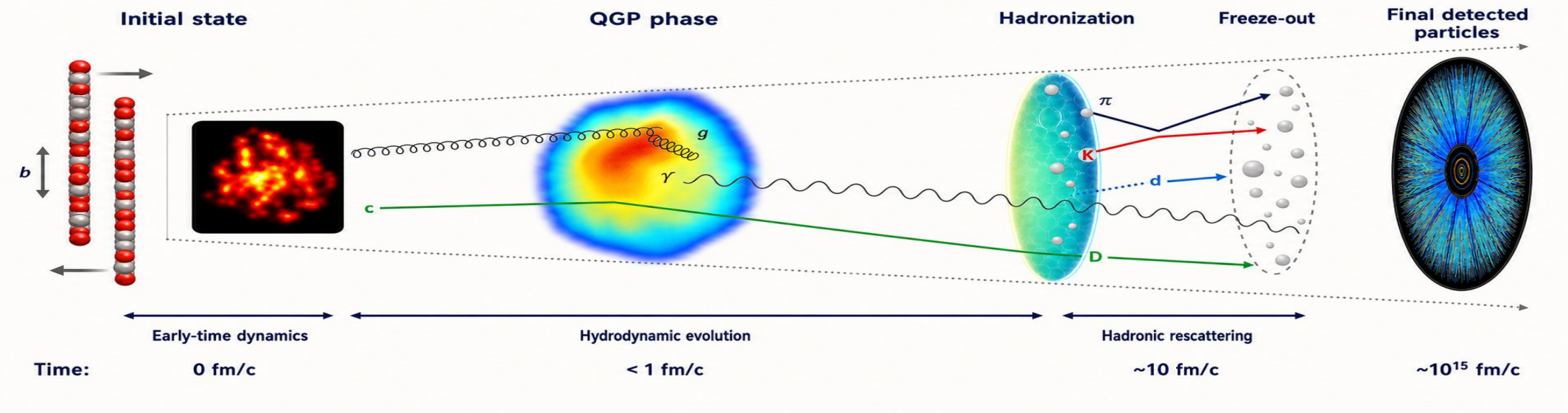

Guowei(国维) Yan(严) [1]

1 School of Physics and Astronamy, SYSU
2 Institute of Modern Physics,Fudan University

with Chenxi(晨曦) Liang(梁) [2], Zhishun(志顺) Chen(陈) [1], Shu(树) Lin(林) [1], Li(力) Yan(严) [2]

11.19 at Zhuhai Chirality 2026

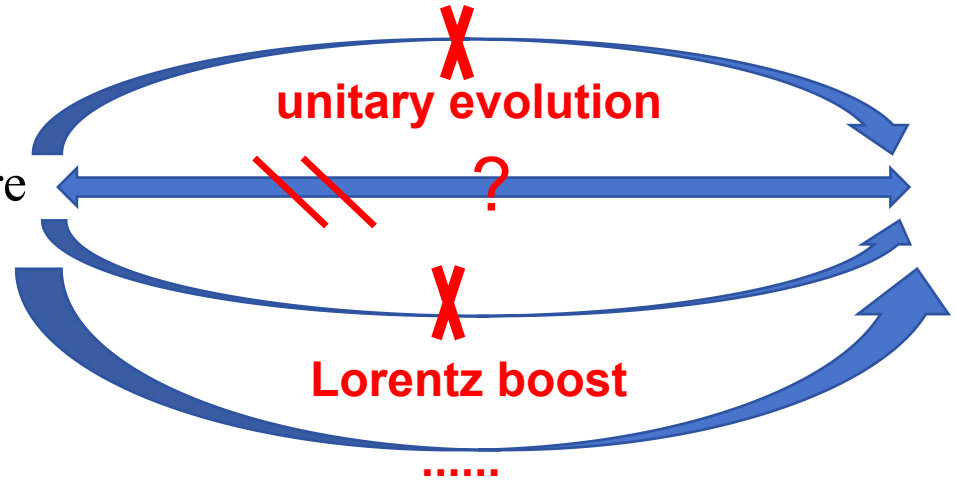
Entropy paradox in QCD matter



Melting nuclei in relativistic heavy ion collision → ***new state of QCD matter*** → ***fast equilibrium~1 fm/c***

Quark or gloun state is pure

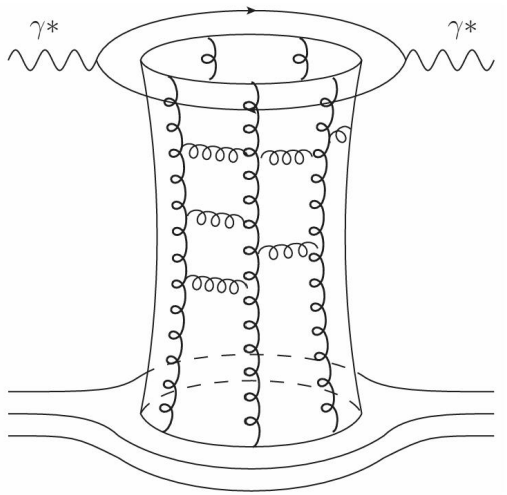
S= $-Tr\ \rho l\ n\ \rho$ = 0



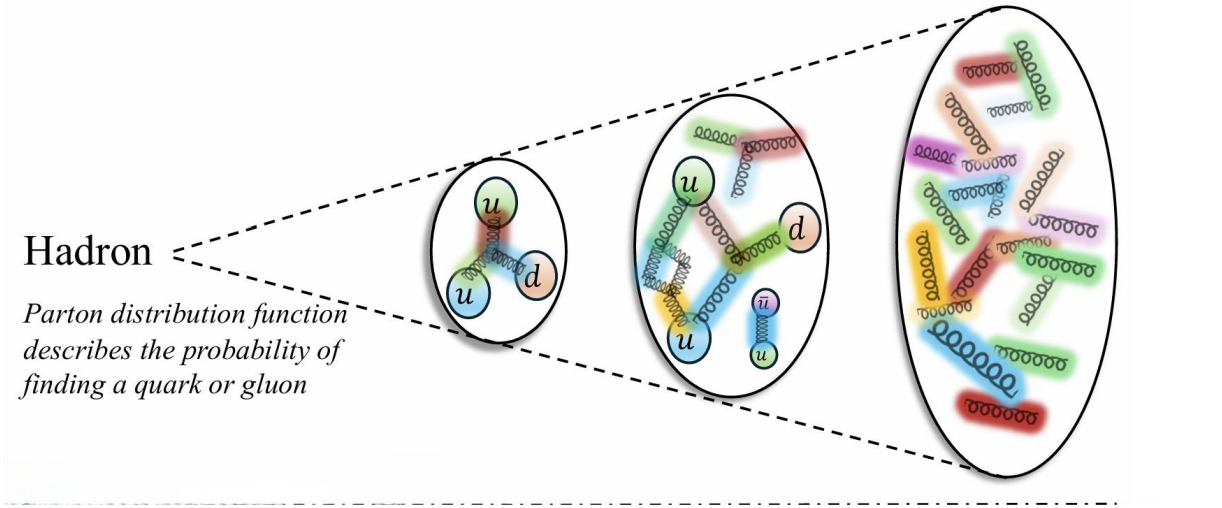
locally thermalized final state with* HUGE *entropy

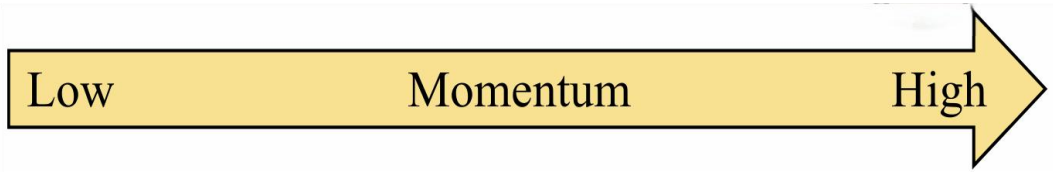
- **A *Mismatch* between Quantum and Classical entropy for an isolated matter.**

Entropy Paradox in QCD Matter



D.Kharzeev e.t. al:2410.22331v2
:1702.03489v3





- Non-linear evolution of QCD gives maximum of entanglement entropy among partons, under limit of high momentum scale and small x.
- Scale-dependent entropy is natural in DIS picture.

$$S(x) = \ln[xG(x)] \simeq \Delta \ln[1/x] \simeq \Delta \ln \frac{L}{\epsilon}$$

CFT

$$S_E = \frac{c}{3} \ln \frac{L}{\epsilon}$$

Entangle Entropy in CFT

$$S_E \sim \frac{c}{3} \ln\left(\frac{1}{\pi T \epsilon}\right) \sinh(\pi T L).$$

finite temperature

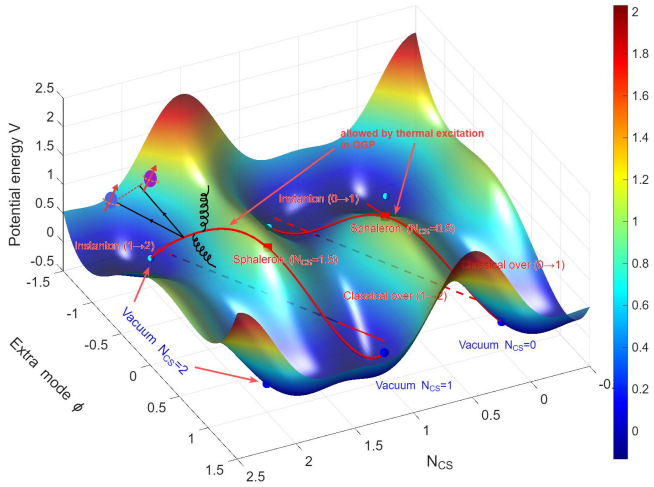
Problems Not Solved: Entanglement entropy ≠ thermal entropy

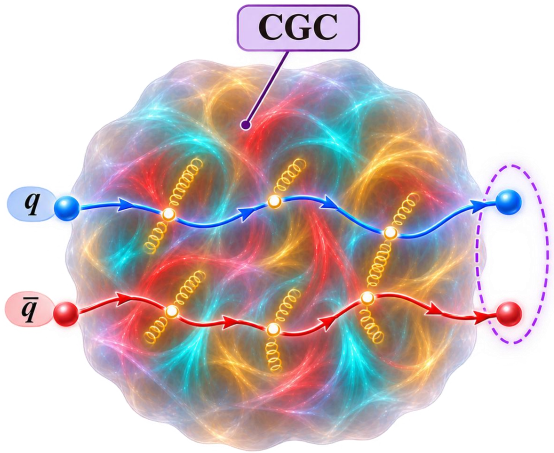
Entropy Paradox in QCD Matter

❖ **To understand the thermalization process with entropy, two questions need to be answered:**

Q1: Source of Entanglement Entropy

- **Topological**
- **Parton Scatter**



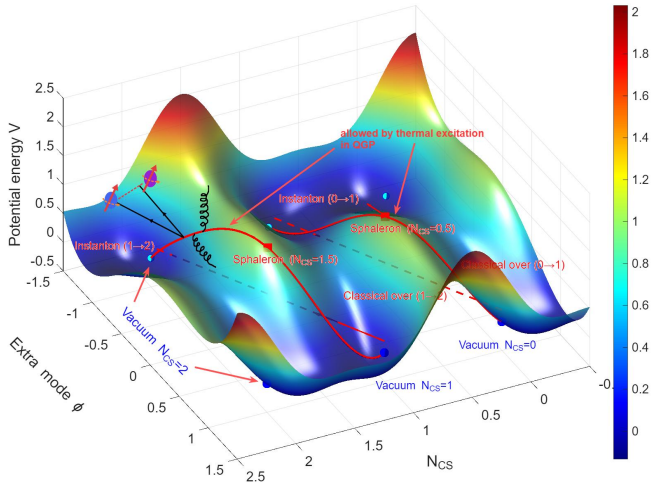


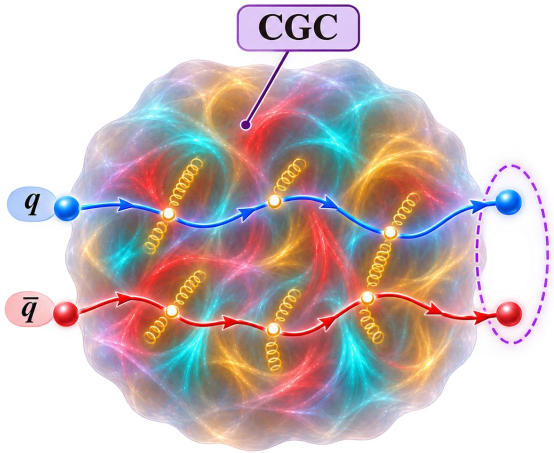
Entropy Paradox in QCD Matter

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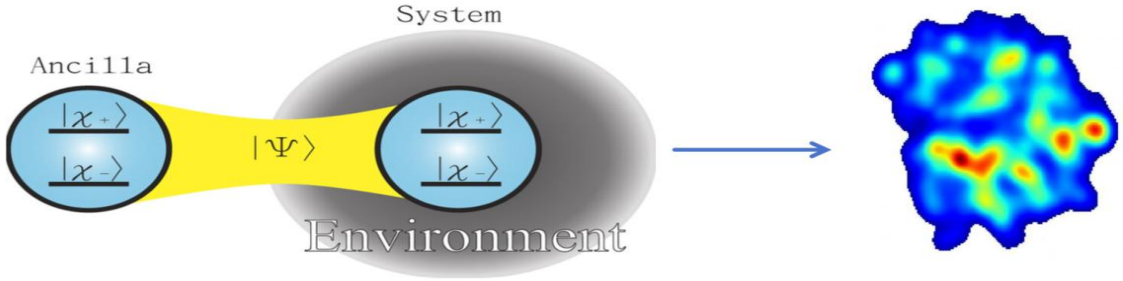
- Topological
- Parton Scatter





Q2: Evolution of Entropy

- Entangle
- Thermal

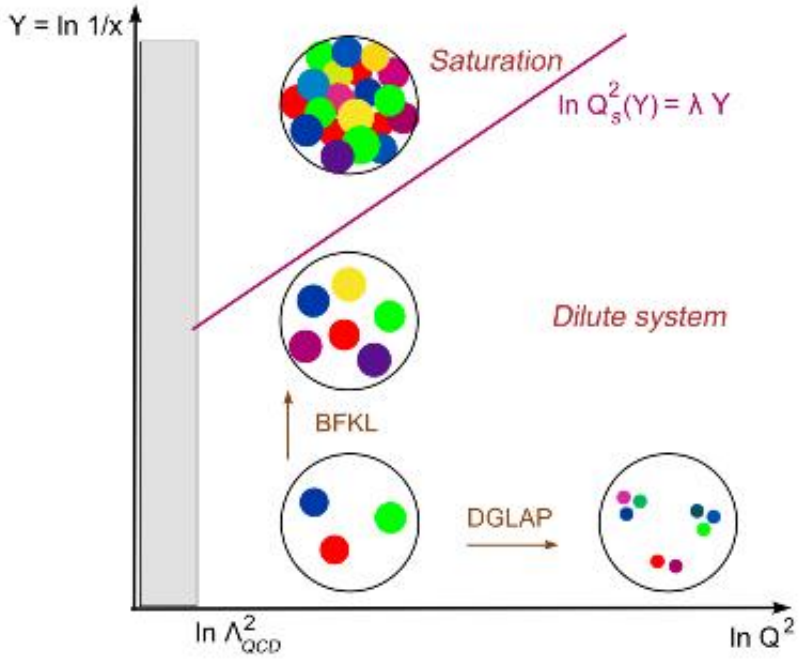


Entanglement → E.T.H. → *Thermalization*

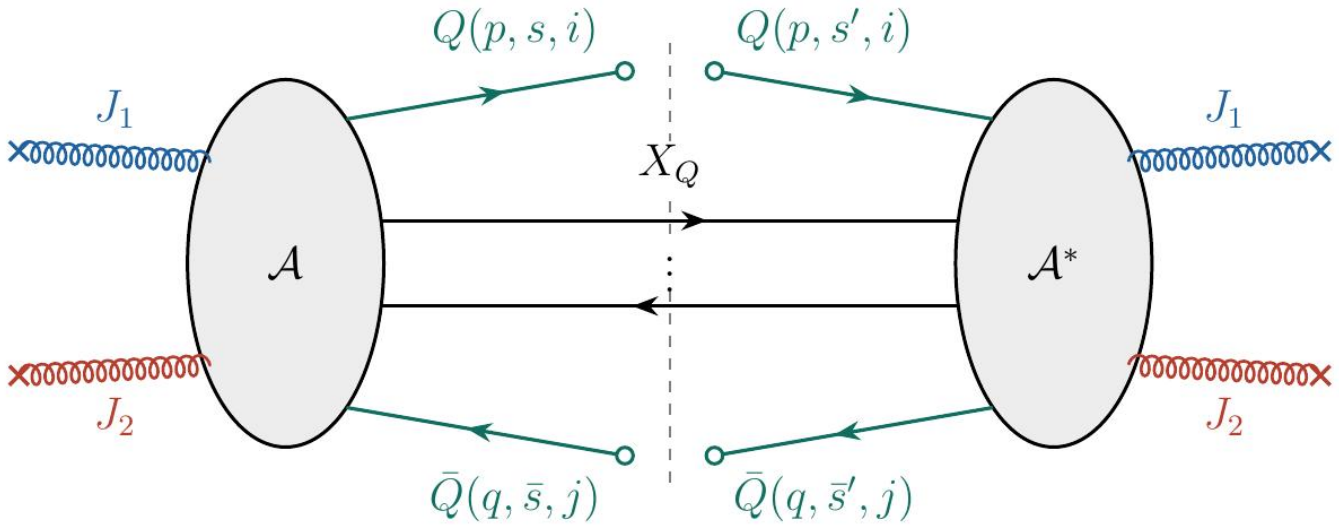
Entanglement From Nucleus Partons

- Two sources of Parton: **Nucleus** & **Sphaleron**

Color Glass Condensate (CGC)



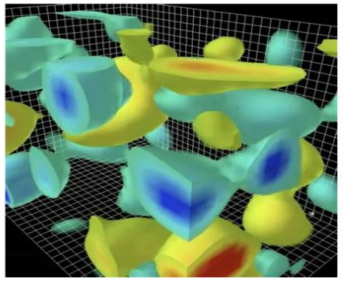
$$J_1[\varrho_1] + J_2[\varrho_2] \longrightarrow Q(p,s) + \bar{Q}(q,\bar{s}) + X_Q$$

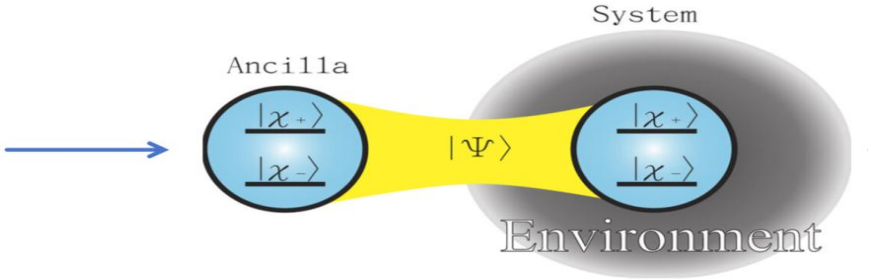


➢ Gluons scatter to form $Q\overline{Q}$ pairs, which scatter with the Background to get entangled.

Entanglement from Topological Effect

- Two sources of Parton: **Nucleus** & **Sphaleron**





Vacuum topology(instanton ensemble) ⟶ ***entanglement***

Prompt Quark Production by exploding Sphalerons

Edward Shuryak and Ismail Zahed

Department of Physics and Astronomy, State University of New York, Stony Brook, NY 11794

(November 7, 2018)

hadron and nucleus-nucleus collisions at large $\sqrt{s}$. Simple arguments indicate that while in hadron-hadron scattering production via sphalerons, it is only a small part of the multiplicity, in nucleus-nucleus collisions it is not so and hundreds of sphalerons may be released, contributing substantially to the total entropy produced [16]. In view of that, the production of $2N_F$ chiral quarks per cluster may make the quark-gluon plasma produced in heavy-ion

E. Shuryak. hep-ph/0205031 hep-ph/0206022

- **Number of sphaleron events**

$$N_{coll} = 8\pi\sigma_{qq}n_q^2\int_0^R dr_t r_t\left(R^2-r_t^2\right)$$

$$= 3^{4/3}2^{-5/3}\pi\sigma_{qq}n_q^2\left(\frac{AN_q}{\pi n_q}\right)^{4/3},$$

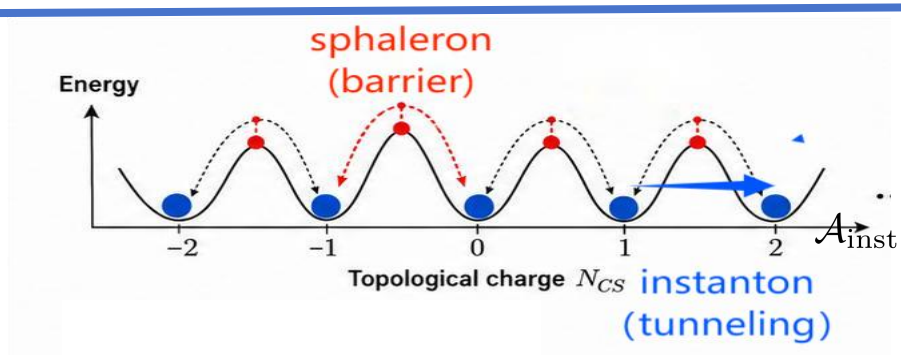
at RHIC energies

$$N_{objects}\approx 400\,,$$

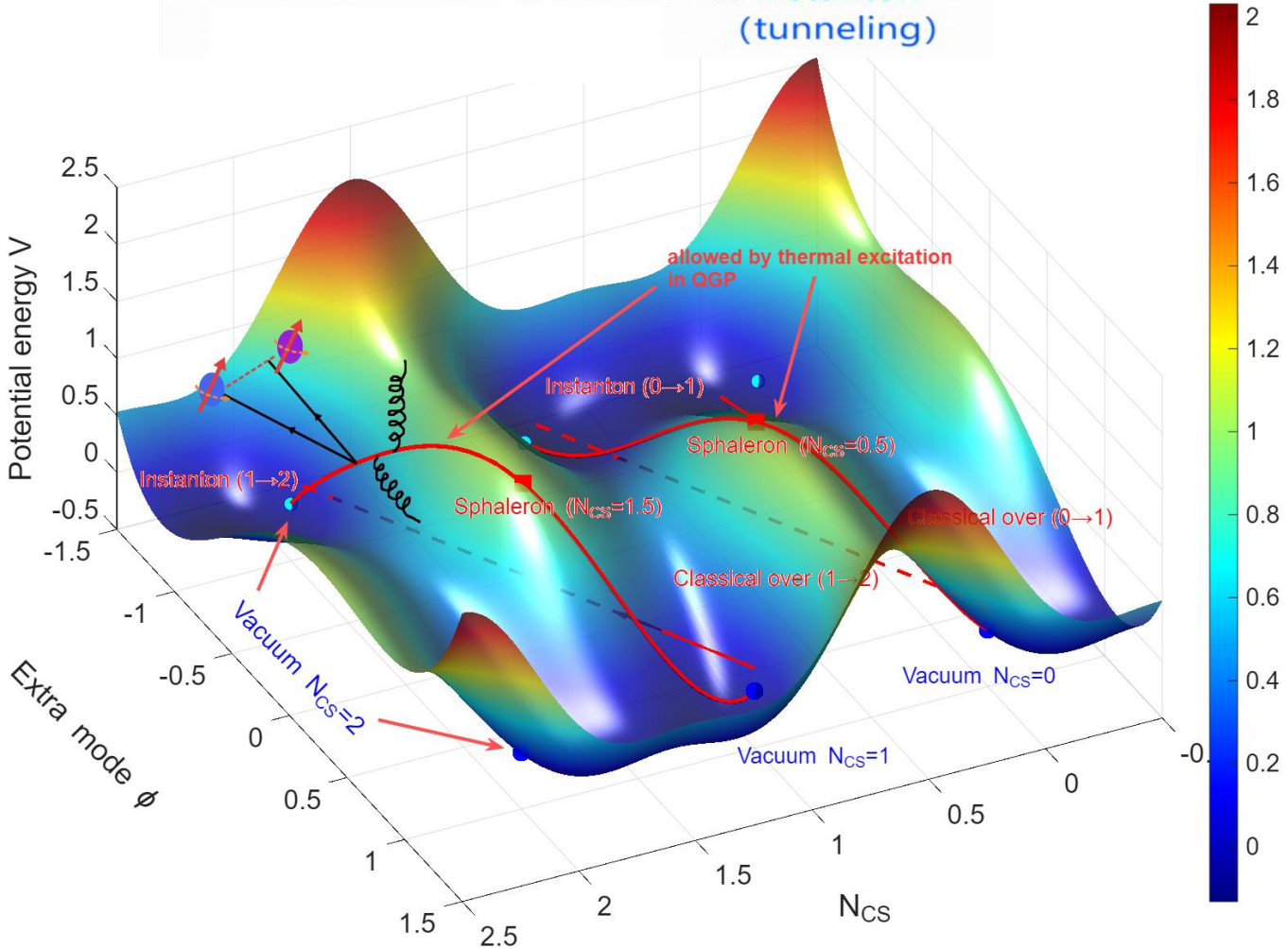
- In evolution of Sphalerons, they explode into many partons, which potentially leads to entropy production

- Entanglement as a novel probe of early QCD matter
  - Not only providing entropy among partons
  - But also reflecting the non-comutative Topo effects

QCD Topo-induced Entanglement



$$\mathcal{A}_{\text{inst}} \propto \exp\left(-\frac{8\pi^2}{g_s^2}\right),$$



- Each field configuration of QCD vacuum is a valley labelled with ***Chern-Simon number***:

$$n = \frac{1}{24\pi^2}\int d^3x\,\epsilon^{ijk}\,\mathrm{Tr}\left[(U^\dagger\partial_i U)(U^\dagger\partial_j U)(U^\dagger\partial_k U)\right] \in \mathbb{Z}.$$

- Spharleron is a static classical solution A_μ^{cl} with non-trivial topo number in Path Integral

$$Z[A_\mu,\bar{q}_f,q_f] = \int \mathcal{D}A^\mu_{inst}\prod_f \mathcal{D}\bar{q}_f\mathcal{D}q_f exp\{-\int d^4x\left[\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}+\sum_{f=1}^{N_f}\bar{q}_f(\not{D})q_f\right]\}$$
$$+\int \mathcal{D}A^\mu_{excited}\prod_f \mathcal{D}\bar{q}_f\mathcal{D}q_f exp\{-\int d^4x\left[\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}+\sum_{f=1}^{N_f}\bar{q}_f(\not{D})q_f\right]\}$$

➢ **Fermi zero mode leads to divergence in second part of path integral, absorbed by external quark line,which gives rise to real quarks production.**

Amplitude to Concurrence WorkFlow

➢ Final state Density Matrix can be formulated with scattering amplitude

$$R^{(\eta_2,\zeta_2)}_{(\eta_1,\zeta_1)} = \frac{1}{N_a N_b}\sum_{s_a,s_b} \mathcal{M}^{s_a,s_b}_{\eta_1,\zeta_1}\mathcal{M}^{\dagger\eta_2\zeta_2}_{s_a s_b}\ , \quad \rho_{\text{out}} = \frac{R}{\text{Tr}\,R}$$

➢ Concurrence can be extracted from Spin Density Matrix

$$\rho = \frac{1}{4}\left(\mathbb{1}\otimes\mathbb{1} + \widetilde{C}_{ij}\hat{\sigma}_i\otimes\hat{\sigma}_j\right)$$

$$\tilde{\rho} = (\sigma_2\otimes\sigma_2)\rho(\sigma_2\otimes\sigma_2)$$

$$\lambda_i \in \sqrt{\text{EigenV}\left(\text{R}_w := \rho\tilde{\rho}\right)}$$

W.K. Wooters, Phys. Rev. Lett. **80**, 2245 (1998)

$$\mathcal{C} = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

$$= \frac{1}{2}\max(0, -C_{nn} + |C_{rr} + C_{kk}|)$$

- Concurrence is measurable

$$t\bar{t} \to (l^+\nu b)\left(l^-\bar{\nu}\bar{b}\right)$$

Nature **633**, 542-547 (2024) ATLAS

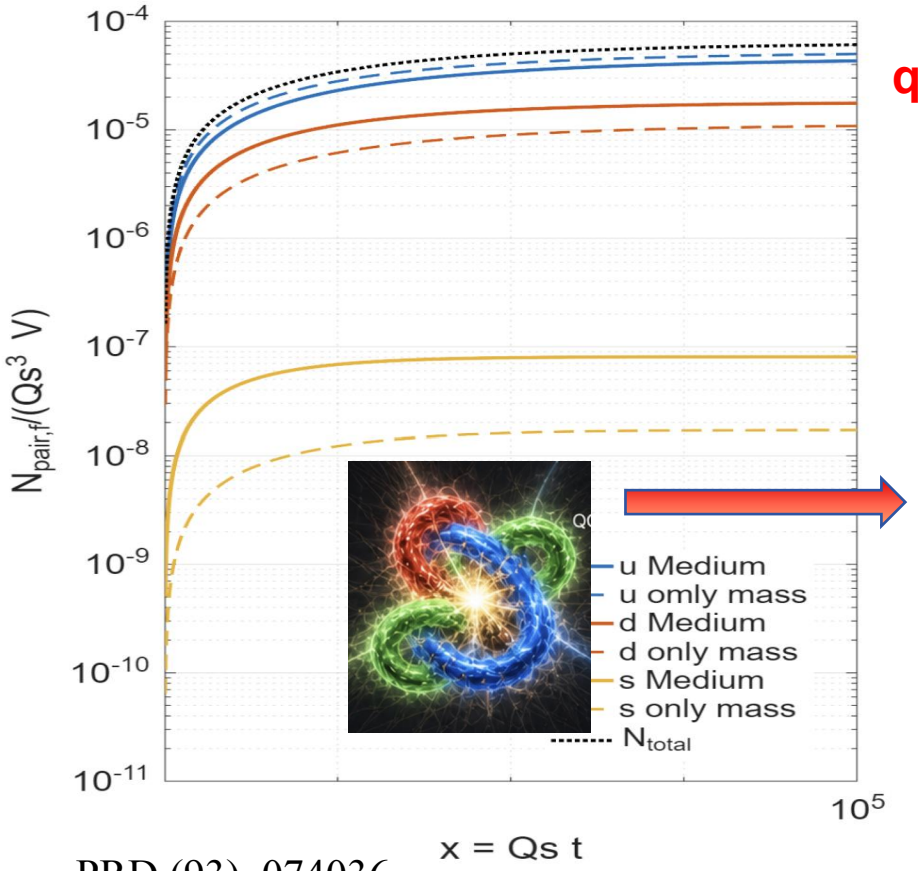
φ: Angles between two leptons

$$D = \langle\cos\varphi\rangle = -\frac{1+2\mathcal{C}[\rho]}{3}$$

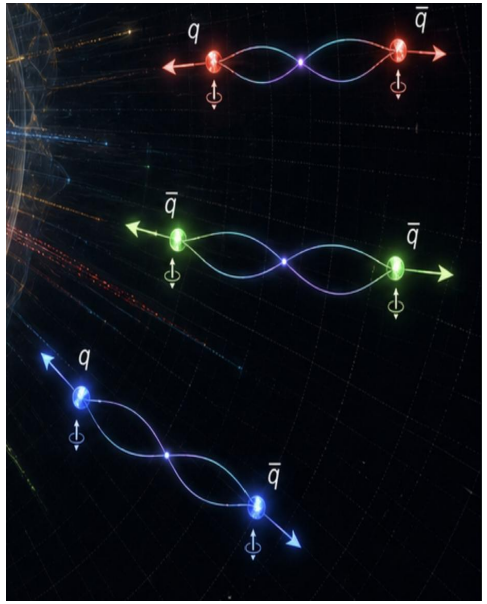
- Entanglement of Formation (Free From Ambience & Projective to trivial knot)

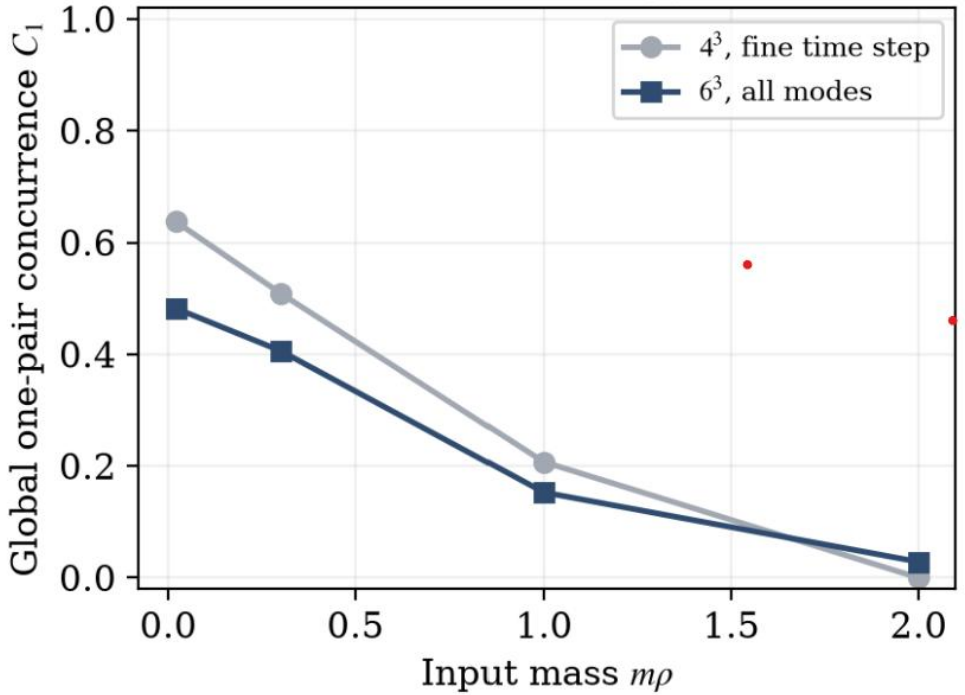
$$E_F(\mathcal{C}(\rho)) := h\left(\frac{1+\sqrt{1-\mathcal{C}^2}}{2}\right), \quad h(x) = -x\log_2 x - (1-x)\log_2(1-x)$$

$Q\overline{Q}$ Entanglement from Sphaleron



quark pair concurrence





PRD (93), 074036

- Mulitplicity from ***Sphaleron*** $\mathcal{N}_f(t) = 2\int d\rho\,\delta(\rho-\rho_*(t))\int d\Phi_2\,C_{\rm color}\left|S^{(f)}_{\rm spec}(\rho,t)\right|^2\left(\frac{4\pi^2\rho^3}{N_c}\right)^2\mathcal{F}_0^2(p\rho/2)\,\mathcal{F}_0^2(q\rho/2)\,2p\cdot q.$
- Spin entanglement can be extracted from quark pair state, with amplitude $\mathcal{M}$ matched using spaleron rate.

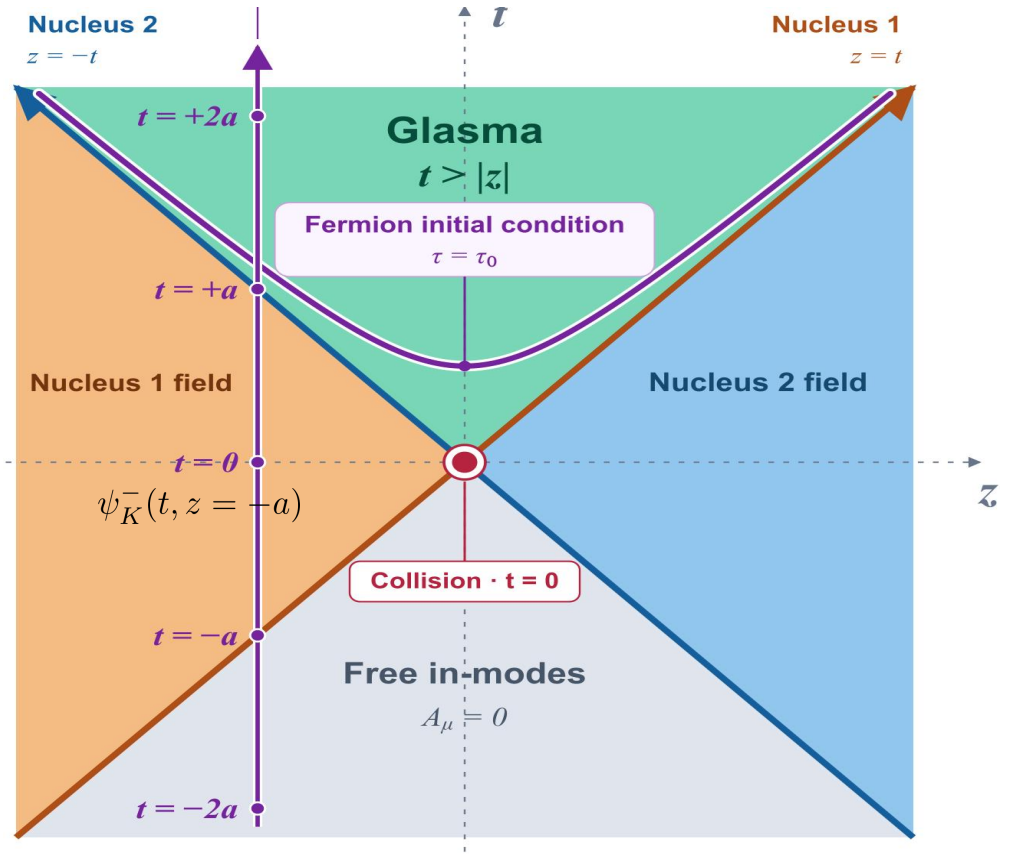
$$dP_f = r_f(t)\,\Gamma^{\rm neq}_{\rm sph}(t)\,d^3X\,dt. \qquad \mathcal{M}_{Q,f;s\bar{s}} = e^{i\delta_Q(t)}\sqrt{\frac{r_f(t)\,\Gamma^{\rm neq}_{\rm sph}(t)}{\mathcal{N}_f(t)}}\,T_{Q,f;s\bar{s}}.$$

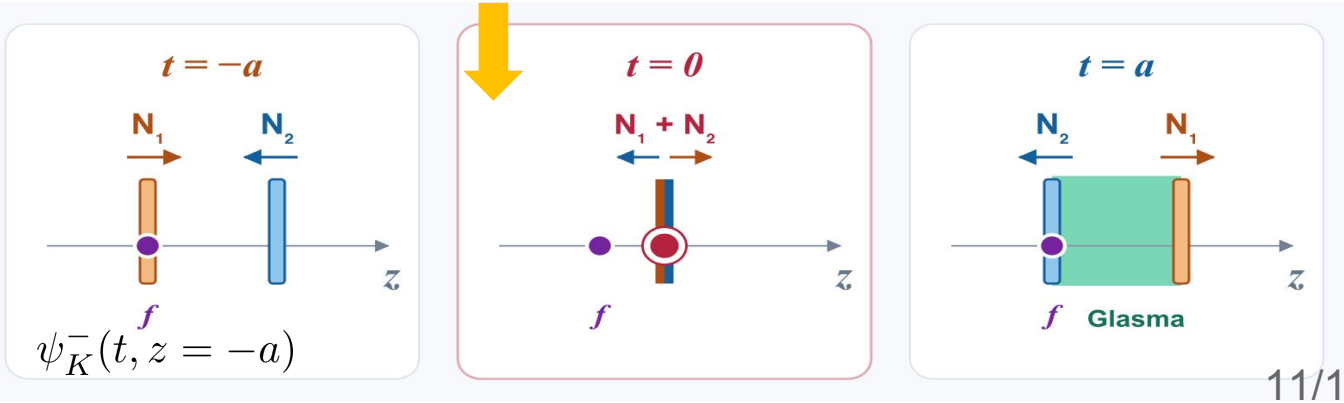
- Spectator flavor and Schwinger–Keldysh propagators are included in T matrix, non-equilibrium medium effect ***enhances*** the pair production.

Entanglement from Scattering in CGC

❖ Based on F. Gelis, N. Taji, 1506.03327

➢ Evolution of fermions under strong nucleus field





Entanglement from Scattering in CGC

❖ Based on F. Gelis, N. Taji, 1506.03327

➢ Evolution of fermions under strong nucleus field

- Fermions field encouters the two thin nucleus, whose colour effects are Wilson Lines under Eikonal Approx.

$$\widehat{\psi}^{-}_{\boldsymbol{k}_\perp \nu s a\,\mathrm{FS}}(x) \underset{\tau\to 0^+}{=} -\frac{e^{i\frac{\pi}{4}}}{\sqrt{M_{\boldsymbol{k}}}}\, e^{i\nu\eta} \int \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\, \frac{e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp}}{M_{\boldsymbol{p}}}$$

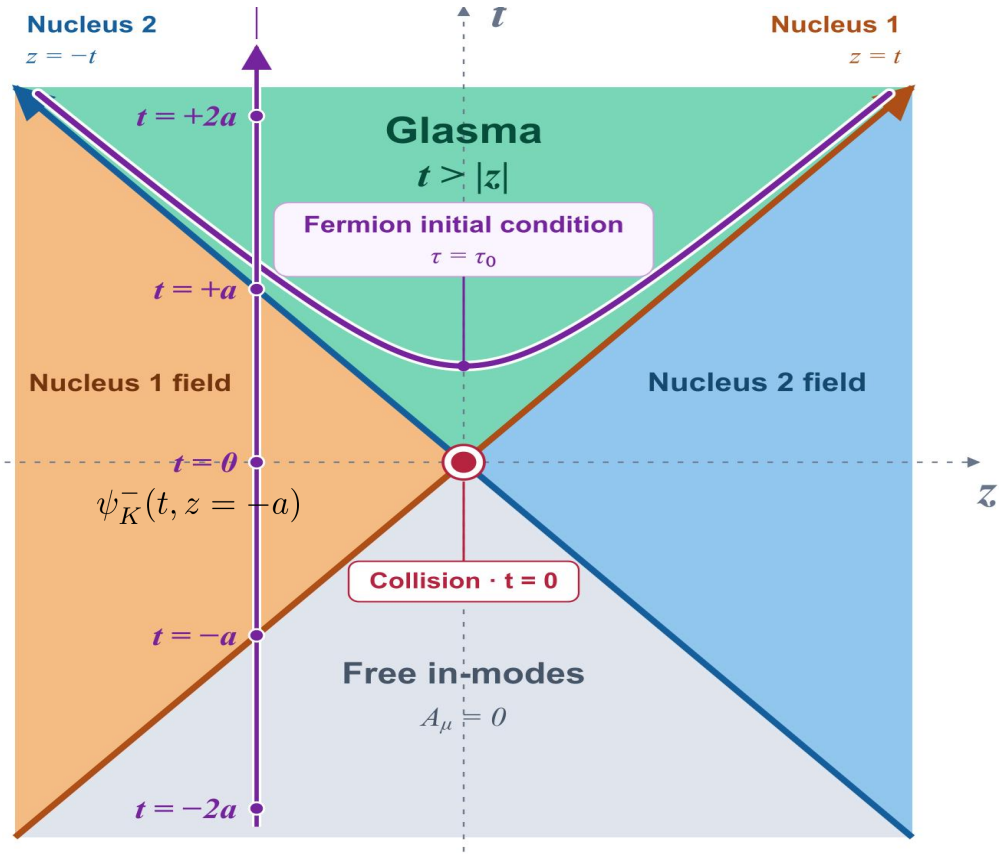
$$\times\Bigg\{ e^{\frac{\pi\nu}{2}} \left(\frac{M_{\boldsymbol{p}}^2\tau}{2M_{\boldsymbol{k}}}\right)^{i\nu} \Gamma(-i\nu+\tfrac{1}{2}) U_2^\dagger(\boldsymbol{x}_\perp)\widetilde{U}_2(\boldsymbol{p}_\perp+\boldsymbol{k}_\perp)\gamma^+$$

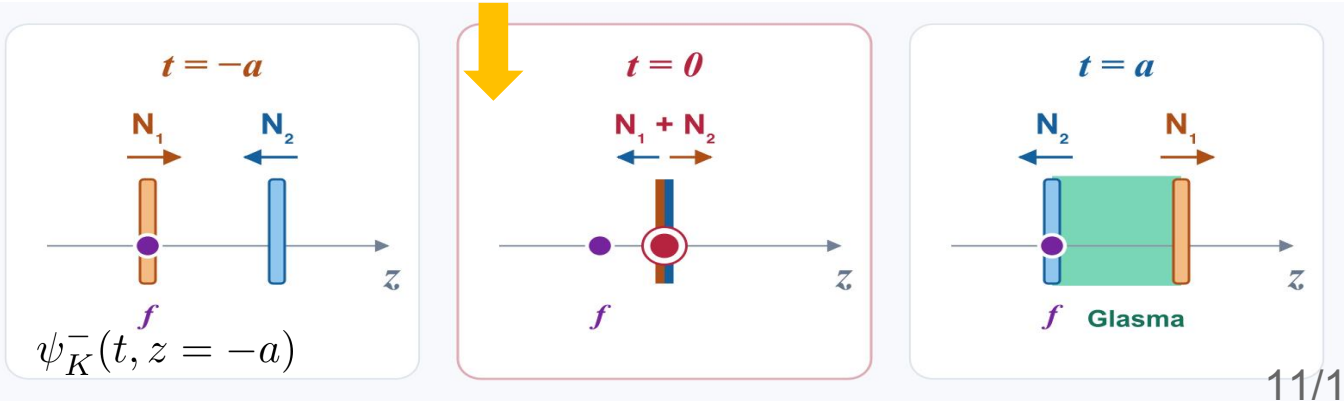
$$+e^{-\frac{\pi\nu}{2}} \left(\frac{M_{\boldsymbol{p}}^2\tau}{2M_{\boldsymbol{k}}}\right)^{-i\nu} \Gamma(i\nu+\tfrac{1}{2}) U_1^\dagger(\boldsymbol{x}_\perp)\widetilde{U}_1(\boldsymbol{p}_\perp+\boldsymbol{k}_\perp)\gamma^-\Bigg\}(p^i\gamma^i+m)v_s(\boldsymbol{k}_\perp, y=0)$$

- Subsequent evolution of fermions in Glasma can be considered within the Dirac Equation

$$i\partial_\tau \psi^-_K =$$

$$\left[\boldsymbol{\alpha}_\perp\cdot(-i\nabla_\perp - \boldsymbol{a}_\perp) + \alpha^3\frac{\nu - a_\eta}{\tau} + \beta_D m_Q\right]\psi^-_K.$$





Entanglement from Scattering in CGC

- Amplitude of producing a single quark pair from the inclusive production

$$R_{IJ;I'J'} = \sum_X \langle X|d_J b_I|\Psi_A\rangle\langle\Psi_A|b^\dagger_{I'} d^\dagger_{J'}|X\rangle = n_{II'}\bar{n}_{JJ'} + \kappa_{IJ}\kappa^*_{I'J'}$$

- Occupation Number in spin basis can be obtained from overlap of fermion modes

$$n_{II'} = \langle b^\dagger_{I'} b_I\rangle = (BB^\dagger)_{II'}$$

$$\kappa_{IJ} = \langle d_J b_I\rangle = (BD^\dagger)_{IJ}$$

$$\psi^-_K = \sum_I \phi^+_I B_{IK} + \sum_J \phi^-_J D_{JK}$$

$$B_{IK} = (\phi^+_I|\psi^-_K),\ D_{JK} = (\phi^-_J|\psi^-_K).$$

ψ^-_K From BG Dirac Eq.

$\phi^\pm_I$ From free Dirac Eq.

Entanglement from Scattering in CGC

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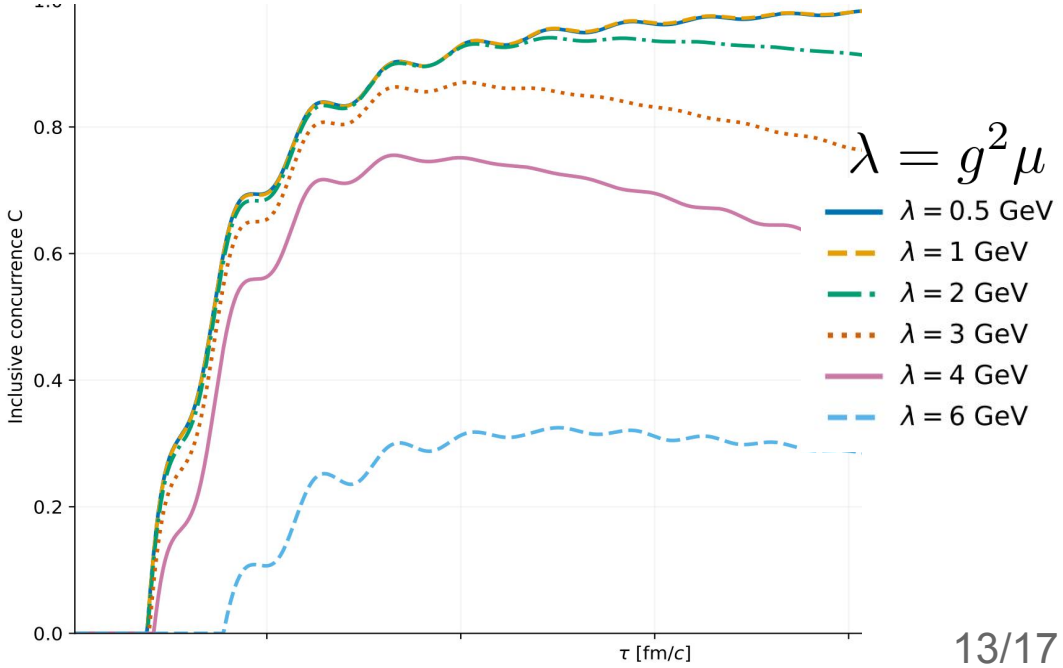
$$B_{IK} = (\phi^+_I|\psi^-_K),\ D_{JK} = (\phi^-_J|\psi^-_K).$$

ψ^-_K From BG Dirac Eq.

$\phi^\pm_I$ From free Dirac Eq.

➢ Evolution of Concurrence for two quarks in CGC

- Entanglment first grows and then decays in the evolution
- The moment of decay starts earlier for stronger colour background from larger size of nucleus.



Where does the generated entanglement go?

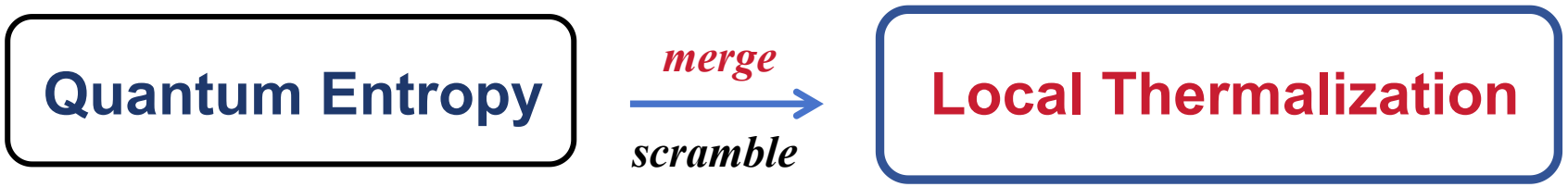
Entropy Relation in quantum thermalization

❖ We still need to answer *Q2* : **Evolution of Entropy**

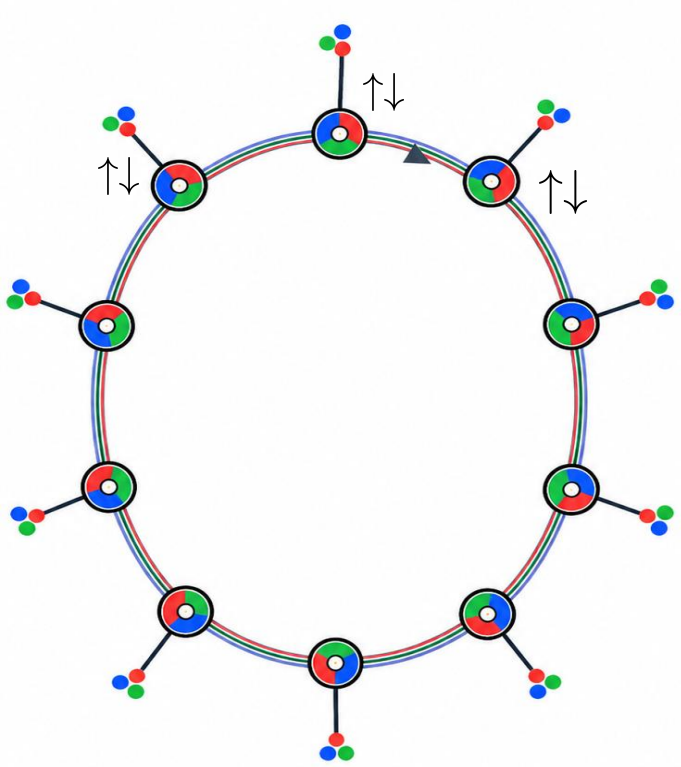
➢ What is the quantitative relation of entropy in thermalization?

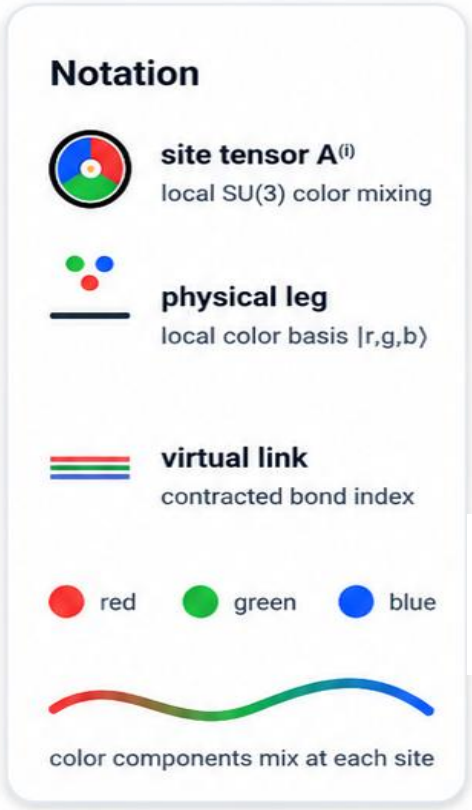
➢ Is the entanglement entropy partial or all of the thermal entropy?

- **Eigenstate Thermalization Hypothesis**



SU(3) link Quantum Simulation





Kogut–Susskind-type Hamiltonian has the structure

$$H_{\mathrm{KS}} = m\sum_i (-1)^i \psi_i^\dagger \psi_i - \kappa \sum_i \left(\psi_i^\dagger U_{i,i+1}\psi_{i+1}\mathrm{h.c.}\right)$$

$$\frac{g^2}{2}\sum_i \boldsymbol{E}_i^2 \frac{1}{2g^2}\sum_{\square}\left(2N_c - \mathrm{Tr}\,U_{\square} - \mathrm{Tr}\,U_{\square}^\dagger\right).$$

effective model:

$$H_{\mathrm{eff}} = m\sum_{i=0}^{N-1}(-1)^i n_i - w\sum_{i,c}\left(c_{i,c}^\dagger c_{i+1,c} + c_{i+1,c}^\dagger c_{i,c}\right) + J\sum_{i=0}^{N-2}\boldsymbol{Q}_i\cdot\boldsymbol{Q}_{i+1}$$

MPS writes the wavefunction as

$$|\Psi\rangle = \sum_{s_0,\ldots,s_{N-1}} A^{[0]s_0}A^{[1]s_1}\cdots A^{[N-1]s_{N-1}}\,|s_0 s_1\cdots s_{N-1}\rangle$$

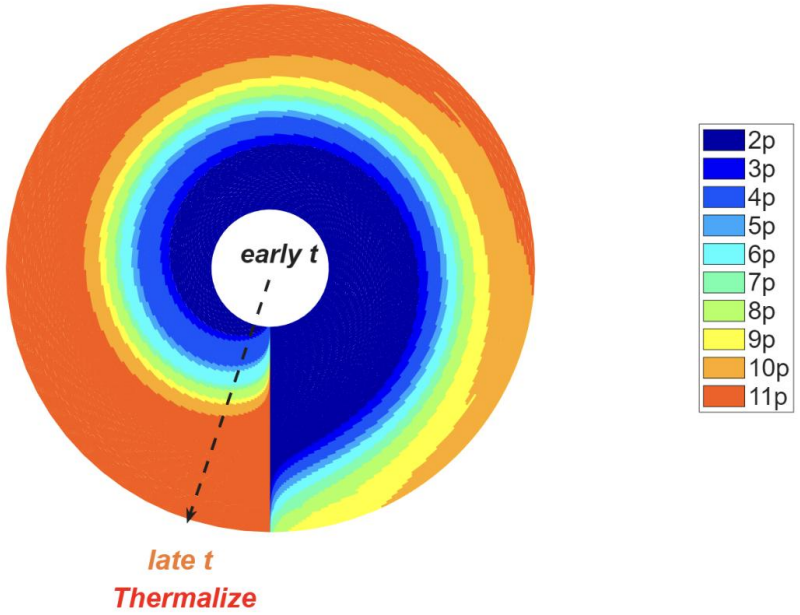
TEBD evolution gate: $\widetilde{\Theta}_{\alpha\gamma}^{s_i' s_{i+1}'} = \sum_{s_i,s_{i+1}} G_{s_i s_{i+1}}^{s_i' s_{i+1}'}\Theta_{\alpha\gamma}^{s_i s_{i+1}}$

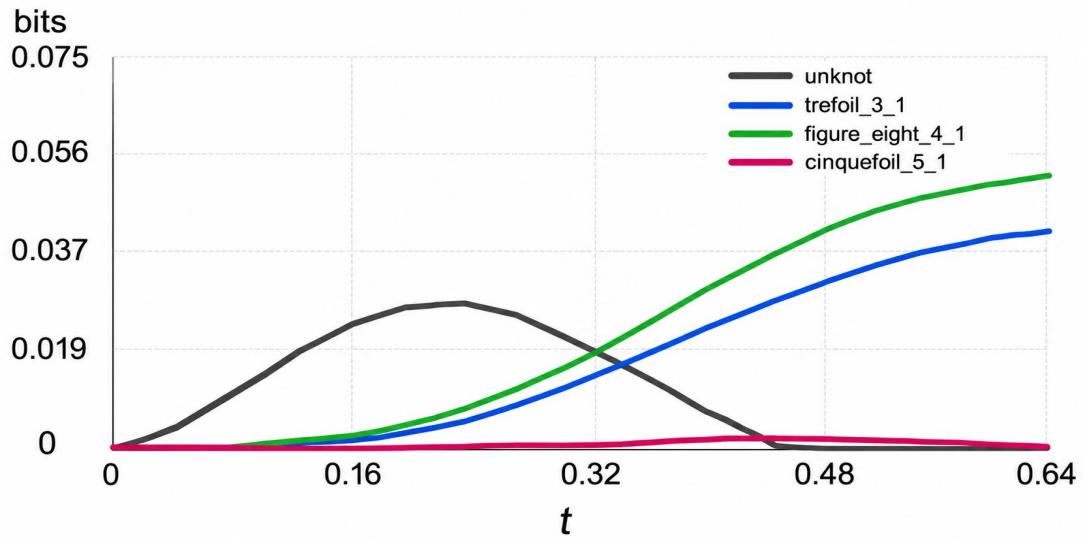
$|\Psi'(+\Delta t)\rangle$

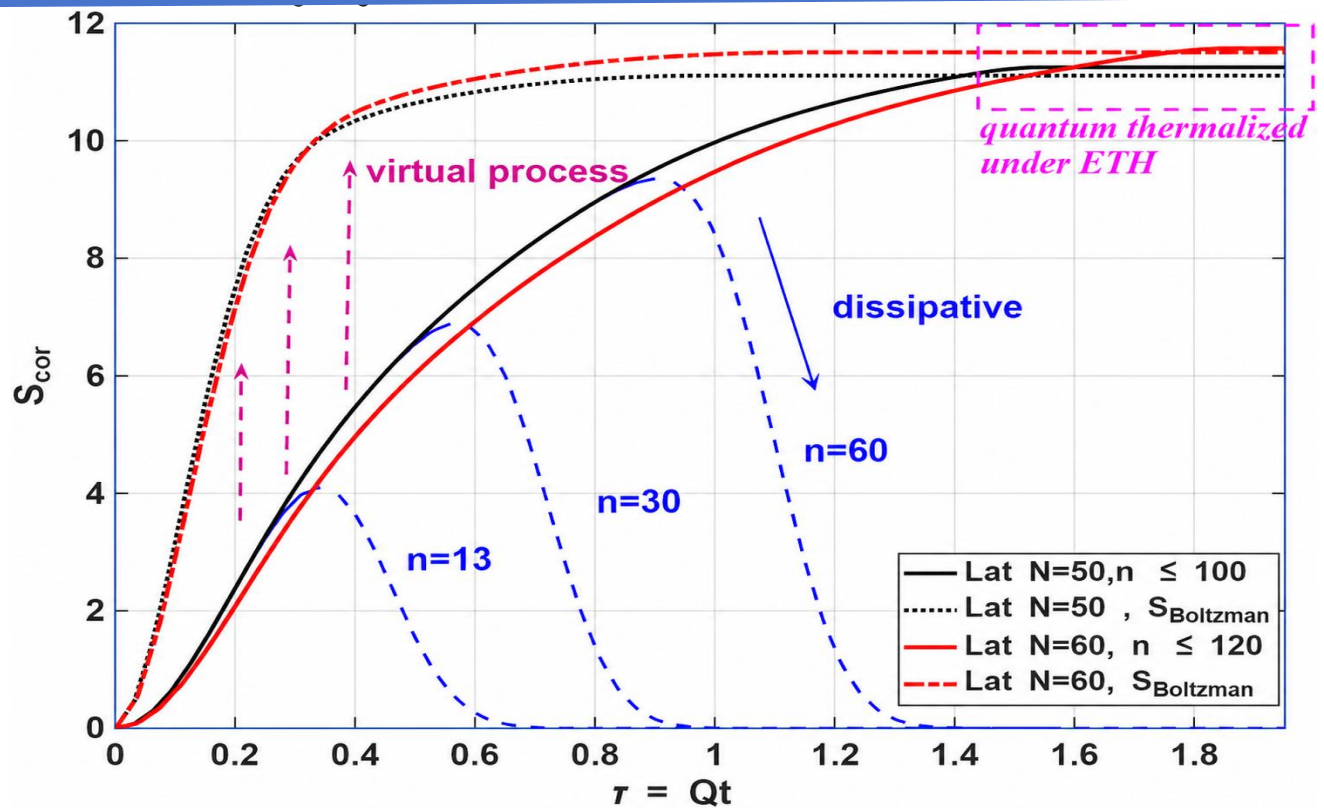
second-order Trotter step is

$$e^{-iH\Delta t} = e^{-iH_m\Delta t/2}e^{-iH_{\mathrm{even}}\Delta t}e^{-iH_{\mathrm{odd}}\Delta t}e^{-iH_m\Delta t/2} + O(\Delta t^3).$$

Quantum & Thermal Entropy Evolution







- In entropy evolution, few-body entanglement *"disperse"* into many-body entanglment by quantum scambling.
- Quantum entropy merges and matches the thermal entropy in window, **a signal of thermalization**

Summary

- **We Propose that *entanglement entropy* can provide a new perspective for the thermlization time problem in HIC.**
- Entanglement entropy can come from QCD topological sphaleron effects and parton scattering at the early stage of collisions.
- By quantum simulation, we find that quantum entanglement entropy can evolve to thermal entropy by quantum scrambling.
- **The time for thermalization can be estimated by quantum scrambling time that merges the many body entangled entropy.**