Spin Alignment of Moving Quarkonium in a Quark-Gluon Plasma



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Doi : <1>10.1103/PhysRevD.111.074002
<2> In preparation

Chirality 2026

01 Background

Experiment and some basic notions

02 Mechanism

Mechanism we considered

Spin — **Repopulation of spin states**

Previous calculation

Orbital — **Distortion of wave function**

Mechanism we considered

Gluon-dissociation (LO) process

Hard gluon, suppressed by binding energy

Inelastic scattering (NLO) process

Soft gluon, comparable contribution with LO

03 Results

Numerical steady state solution

04 Summary and outlook

Including future Plan

01 Background

research status

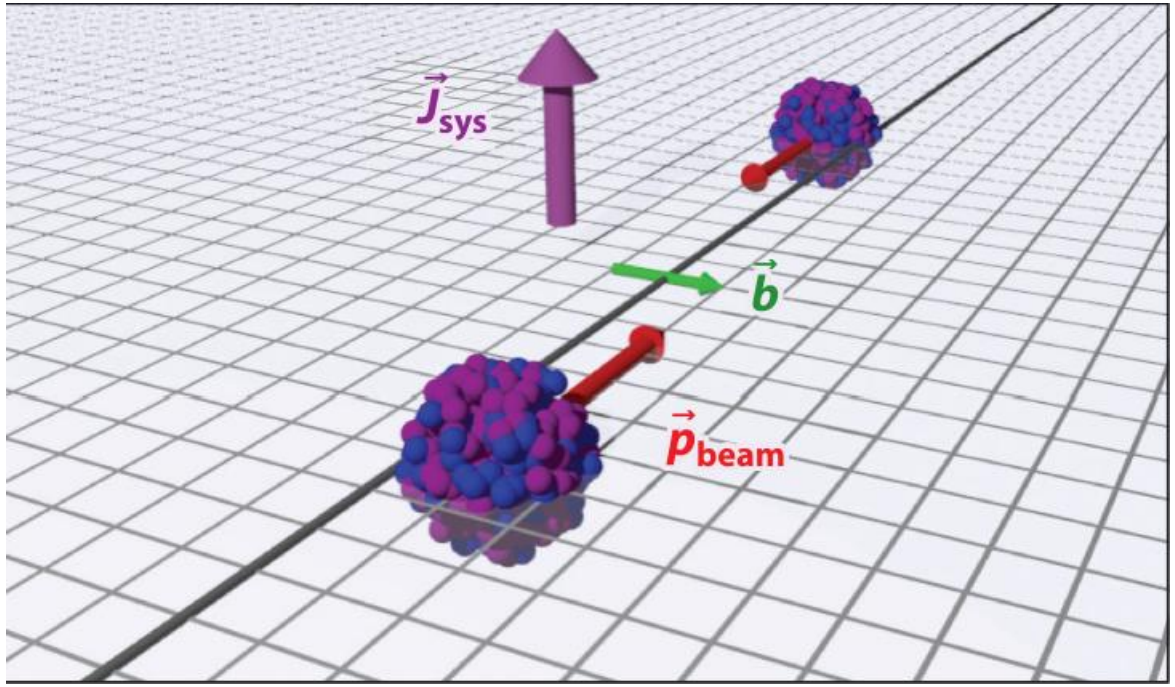


Figure from F. Becattini-Michael A. Lisa, AR 2020

$$\vec{J} = \vec{b} \times \vec{p}$$

- First idea of spin alignment

  Liang-Wang PRL 2005, PLB 2005

- Hyperon (Spin polarization)

  can be nicely described by hydrodynamic and transport-based calculations

- Vector meson (Spin alignment) still not clear…

  Shear
  Vector meson field fluctuation
  Glasma field fluctuation
  Vorticity field
  EM field
  Fragmentation…

01

Background

The measurement of spin alignment in experiment

Measurement

$$J/\psi \rightarrow \ell^+\ell^-$$

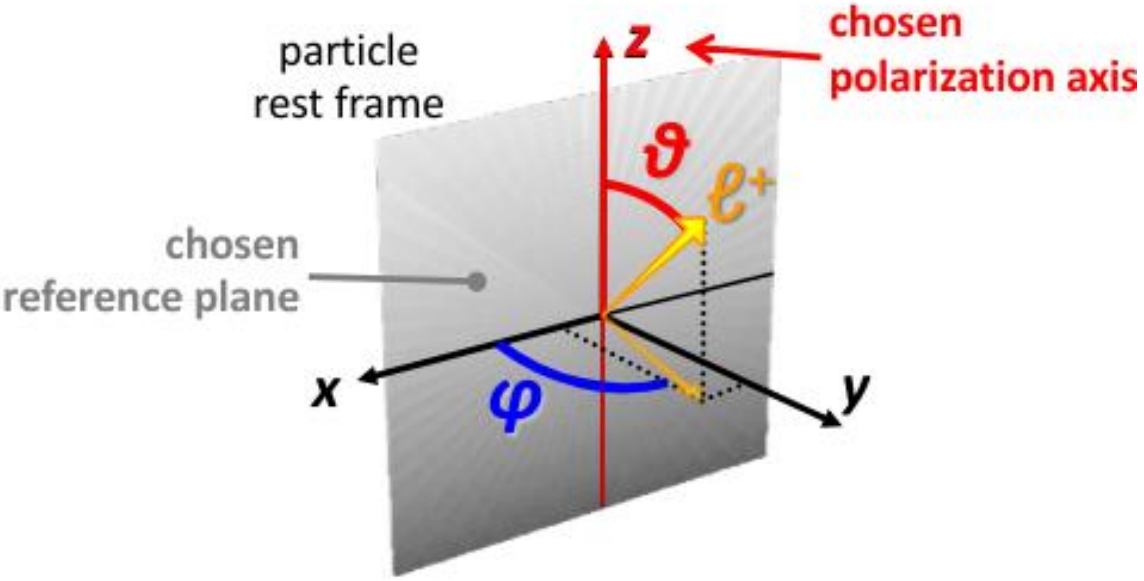
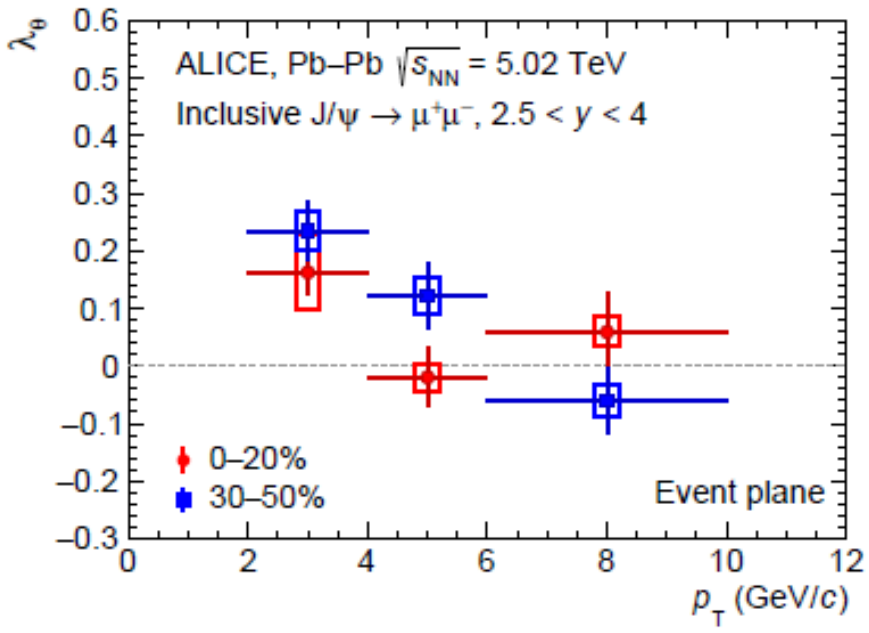


*Figure from P. Faccioli-C.Lourenço, Particle Polarization in High Energy Physics

Angular distribution of dilepton

$$W(\vartheta) = \frac{1 + \lambda_\vartheta \cos^2 \vartheta}{3 + \lambda_\vartheta}$$



$$\lambda_\vartheta > 0$$

ALICE, PRL 2023

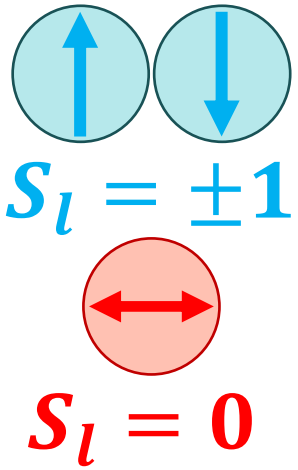
01

Background

Spin density matrix

$$\rho^{spin} = \sum_i P_i \, |\psi_i\rangle\langle\psi_i|$$

$\vec{l}$ → quantization axis



Spin state

$$\rho^{spin} \xrightarrow{\text{no polarization}} \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} +1 \\ 0 \\ -1 \end{matrix}$$

$+1 \quad 0 \quad -1$

Relation between λ_ϑ and ρ^{spin}

$$\lambda_\vartheta = \frac{1 - 3\rho_{00}^{spin}}{1 + \rho_{00}^{spin}}$$

$\rho_{00}^{spin} - 1/3$: spin alignment

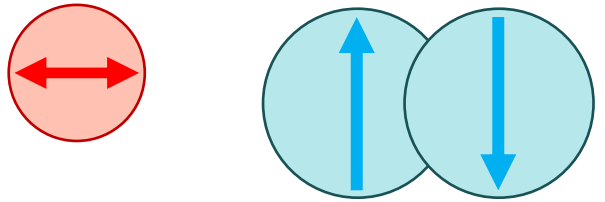
N: Observed Number of particle

$N_{l\perp}$: Observed Number of spin-0 state

$\lambda_\vartheta > 0$

$$\rho_{00}^{spin} < 1/3$$

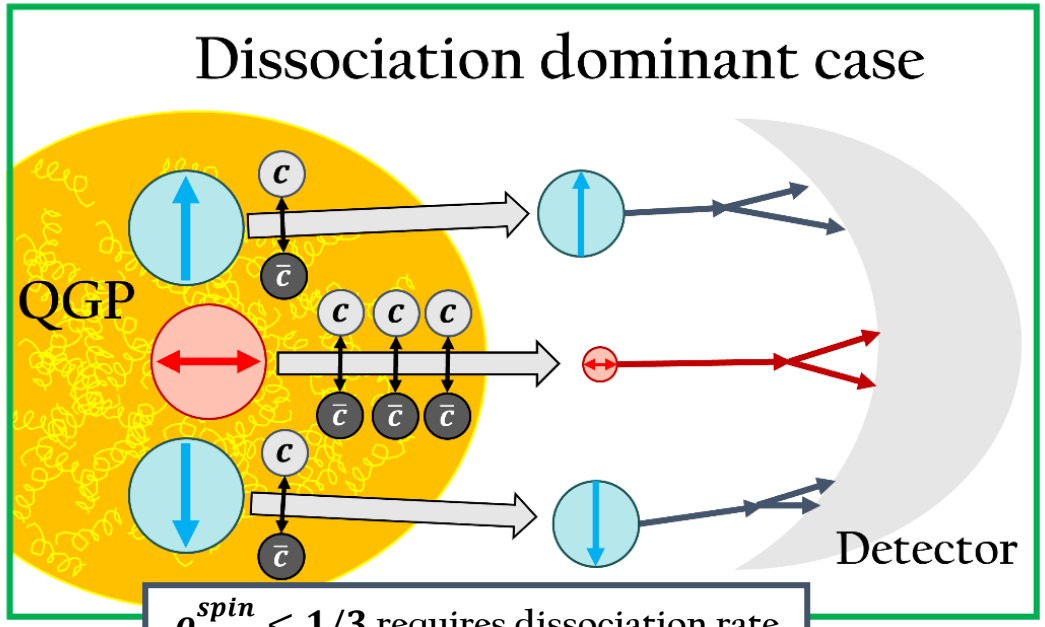
$N_{l\perp} < 1/3\ N$



01 Background

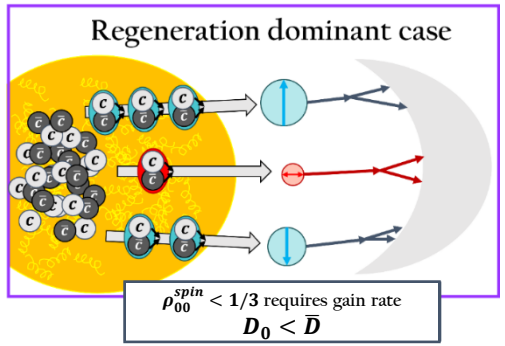
Tow possible origins of λ_θ

Repopulation of spin states (spin)



$\rho_{00}^{spin} < 1/3$ requires dissociation rate $\Gamma_0 > \bar{\Gamma}$

Inverse

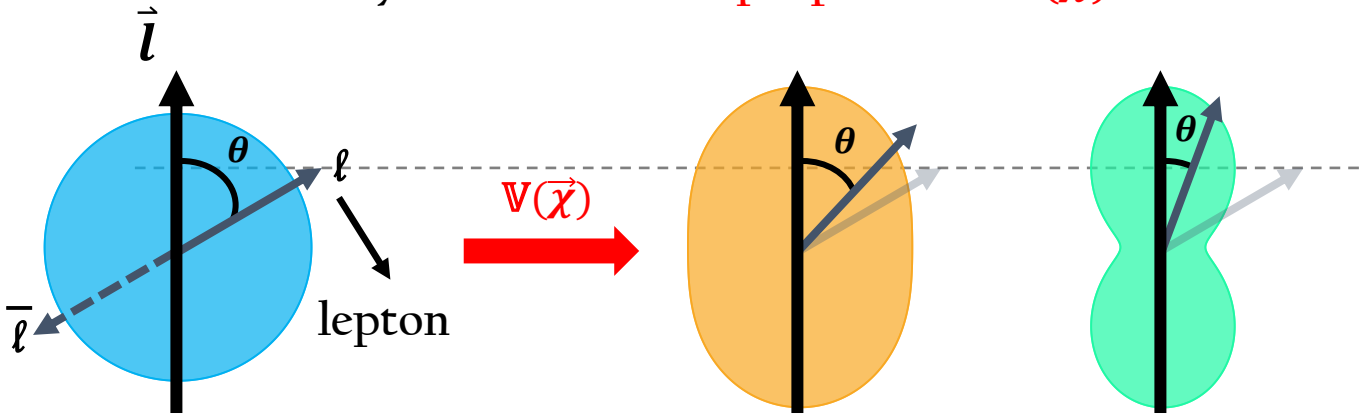


$\rho_{00}^{spin} < 1/3$ requires gain rate $D_0 < \bar{D}$

$$\lambda_\vartheta = \frac{1 - 3\rho_{00}^{spin}}{1 + \rho_{00}^{spin}}$$

Distortion of wave function (orbital)

Quarkonium wave function is deformed by an arbitrary vector $\vec{\chi}$, which yields an anisotropic potential $\mathbb{V}(\vec{\chi})$



$Q\bar{Q} \to \gamma^* \to \ell\bar{\ell}$ process, self energy Π_{γ^*} associates with the S- and D-wave component which affects the measurement

$$\lambda_\theta = \frac{(\Pi_T - \Pi_L)(3\cos^2\theta_{\chi l} - 1)}{\Pi_T(3 - \cos^2\theta_{\chi l}) + \Pi_L(1 + \cos^2\theta_{\chi l})}$$

Yan-Lin PRD 2026

02

Mechanism

Interaction term —— QCD Multipole expansion

Quarkonium rest frame

Hierarchy: $m_Q \gg m_Q v_{rel} \sim \frac{1}{r} \gg T \gg m_Q v_{rel}^2 \sim \epsilon_B \sim m_D$

$\mathrm{s} = s\frac{1_c}{\sqrt{N_c}}$, $\mathrm{o} = o\frac{T^a}{\sqrt{T_F}}$

$$\mathcal{L}_{\mathrm{pNRQCD}} = \mathcal{L}_{\text{light quark}} + \mathcal{L}_{\text{gluon}}$$

Light part

$h = K + V$

$Q\bar{Q}$ potential arise from gluon exchange

color singlet & color octet

$V_s^{(0)} = -C_F\frac{\alpha_s}{r}$ $V_o^{(0)} = \frac{1}{2N_c}\frac{\alpha_s}{r}$

$$+\int d^3\vec{r}\,\mathrm{Tr}\Big\{\mathrm{s}^\dagger[i\partial_0 - h_s]\mathrm{s} + \mathrm{o}^\dagger[iD_0 - h_o]\mathrm{o}$$

Kinetic term

Interaction term

Orbital $\Delta L = 1$

Spin $\Delta S = 1$

$$+\mathrm{o}^\dagger g_s\vec{r}\cdot\vec{E}^a\mathrm{s} + \mathrm{s}^\dagger g_s\vec{r}\cdot\vec{E}^a\mathrm{o} + 1/2\,\mathrm{o}^\dagger\{g_s\vec{r}\cdot\vec{E}^a, \mathrm{o}\}$$

$$+\mathrm{o}^\dagger\vec{\mu}\cdot\vec{B}^a\mathrm{s} + \mathrm{s}^\dagger\vec{\mu}\cdot\vec{B}^a\mathrm{o} + 1/2\,\mathrm{o}^\dagger\{\vec{\mu}\cdot\vec{B}^a, \mathrm{o}\} + \cdots\Big\}$$

Brambilla et al, RMP 2005

Yan, PRD 1980; Kuang-Yan, PRD 1981

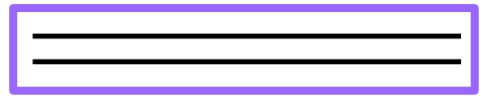
02 Mechanism

Interaction term —— Feynman diagram

Color singlet

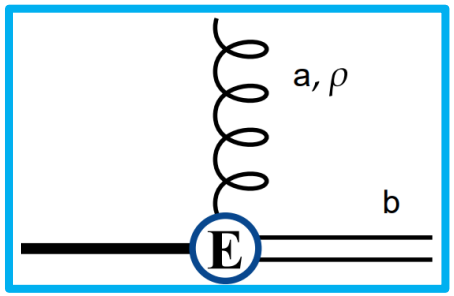


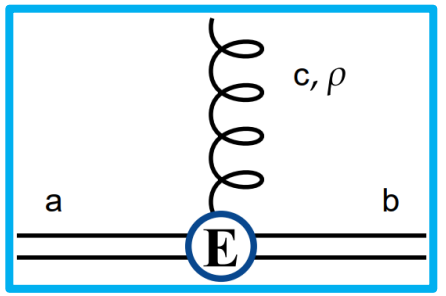
Color octet



Chromoelectric dipole

$$g_s\vec{r}$$

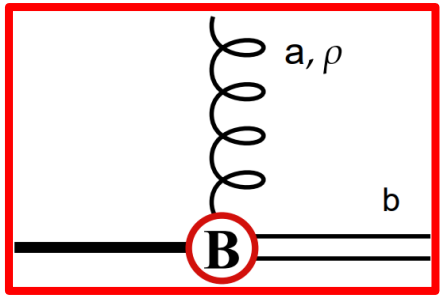


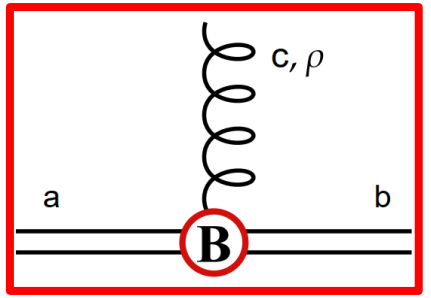


$$+non-Abilian$$

Chromomagnetic dipole

$$\vec{\mu}=\frac{g_s}{2m_Q}\left(\frac{\vec{\sigma}_Q}{2}-\frac{\vec{\sigma}_{\bar{Q}}}{2}\right)$$





02 Mechanism

Unpolarized dissociation

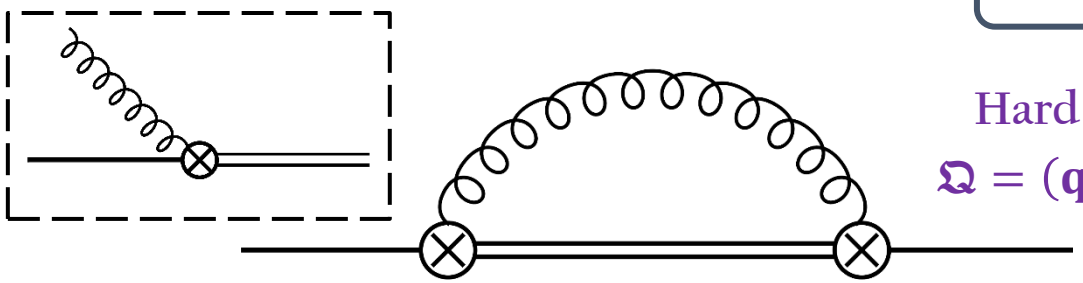
Hierarchy: $m_Q \gg m_Q v_{rel} \sim \frac{1}{r} \gg T \gg m_Q v_{rel}^2 \sim \epsilon_B \sim m_D$

$g + s \leftrightarrow o$

LO

Hard gluon

$\mathfrak{Q} = (q_0, \vec{q}) \sim T$



Peskin, NPB 1979

Chromoelectric

Brambilla et al, RMP 2005

Chen-He, PLB 2017

Chromomagnetic

Chen-He, PLB 2017

Suppressed by $Q\bar{Q}$'s binding energy $\epsilon_B(T)$

When $\epsilon_B(T) \to 0$, kinematically forbidden

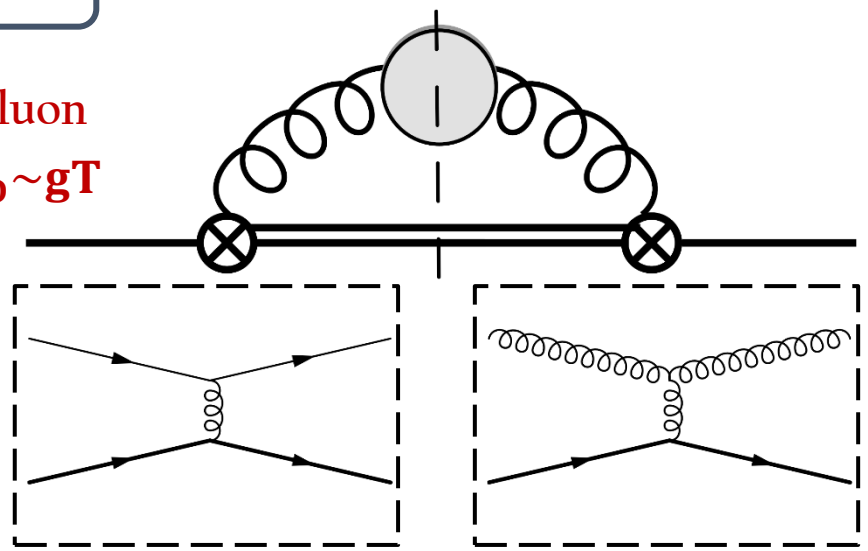
Grandchamp, Rapp, PLB, 2001

NLO

$(q, g) + s \leftrightarrow (q, g) + o$

Soft gluon

$\mathfrak{Q} \sim m_D \sim gT$



Grandchamp, Rapp, PLB, 2001

Chromoelectric

Brambilla et al, JHEP 2013

Yao-Muller, PRD 2019

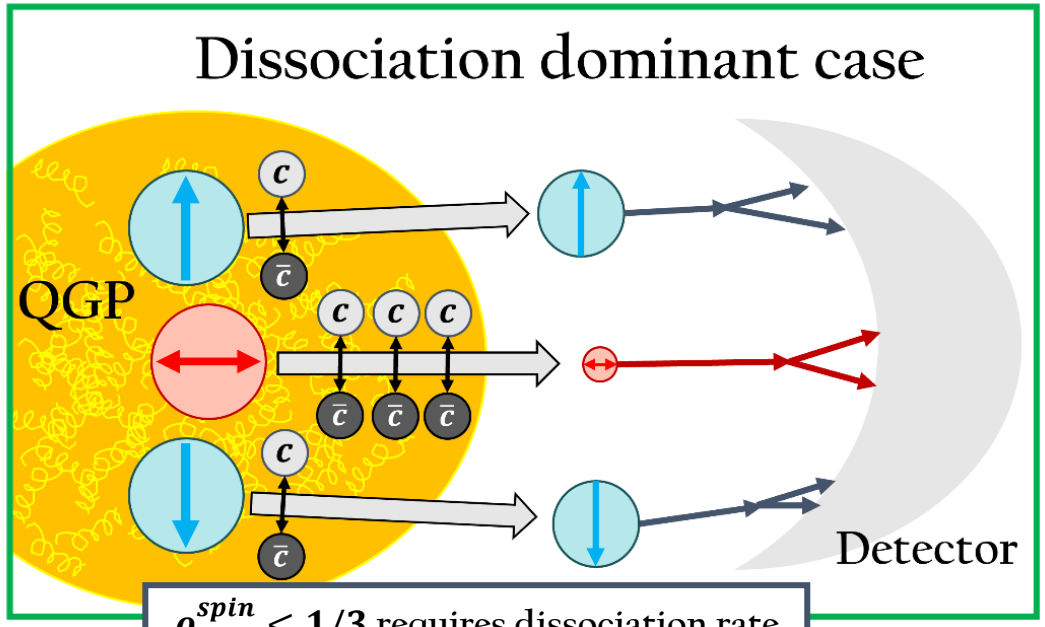
Zhao-He, PRD 2024

01

Background

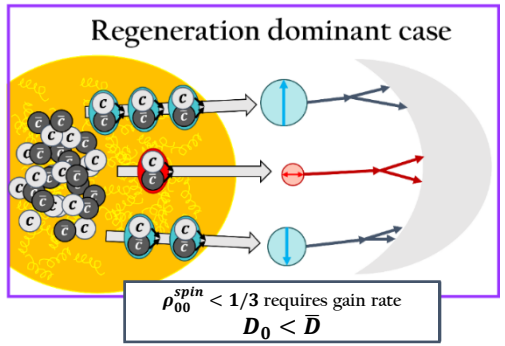
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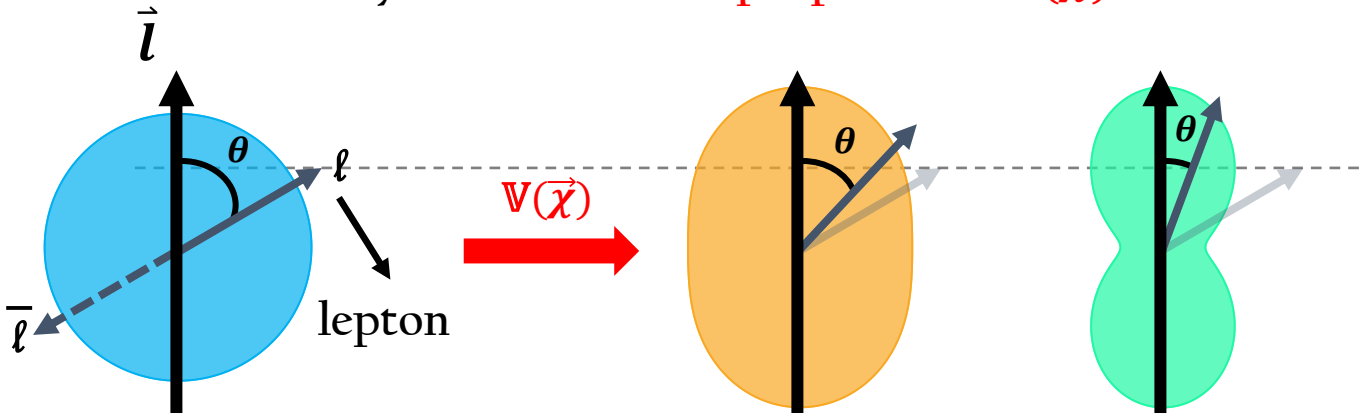


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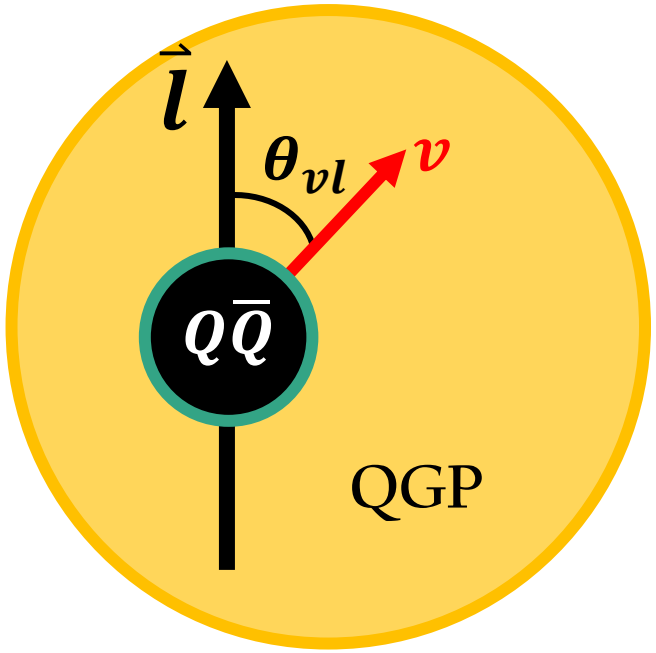
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Yan-Lin PRD 2026

02 Mechanism-Spin

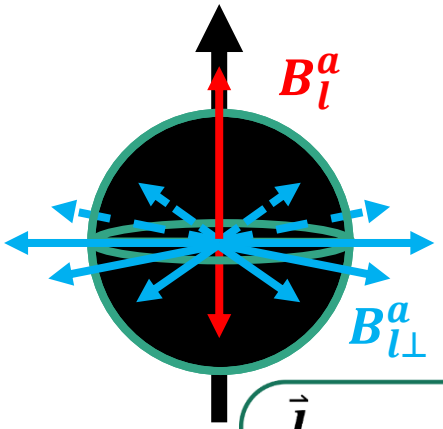
Relative motion between quarkonium and QGP

Quarkonium moves in the QGP with velocity $\vec{v}$



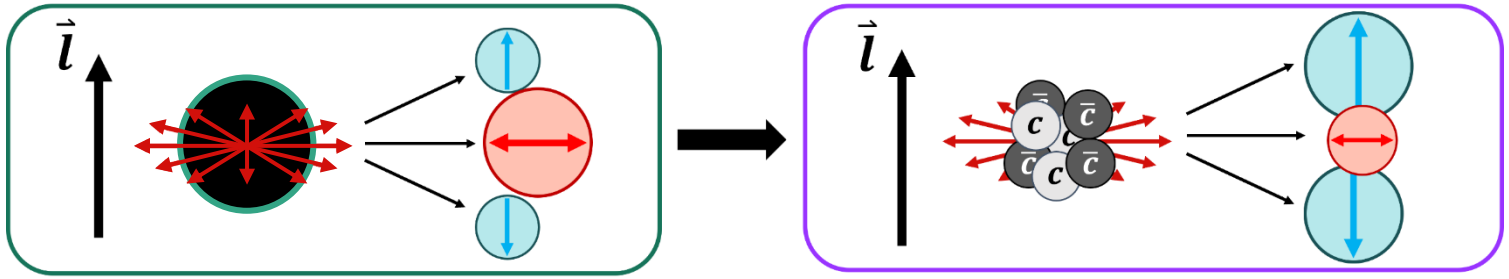
Spin

Quarkonium sees a boosted Chromomagnetic field, which repopulates the spin states



Polarized dissociation/regeneration rates, Applied it to Bjorken flow:

$$\rho_{00}^{spin} - \frac{1}{3} \sim \int_{\tau_0}^{\tau} d\tau' c_2(T, |\vec{v}|) \left(\frac{1}{3} - \langle cos^2 \theta_{vl} \rangle \right)$$



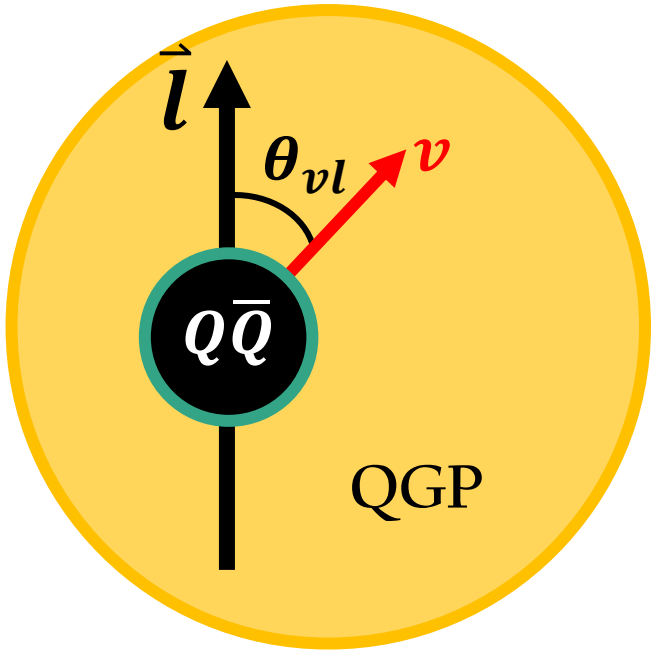
$\lambda_\vartheta < 0$ $\lambda_\vartheta > 0$

Chen-Lin PRD 2025

02 Mechanism-Orbital

Relative motion between quarkonium and QGP

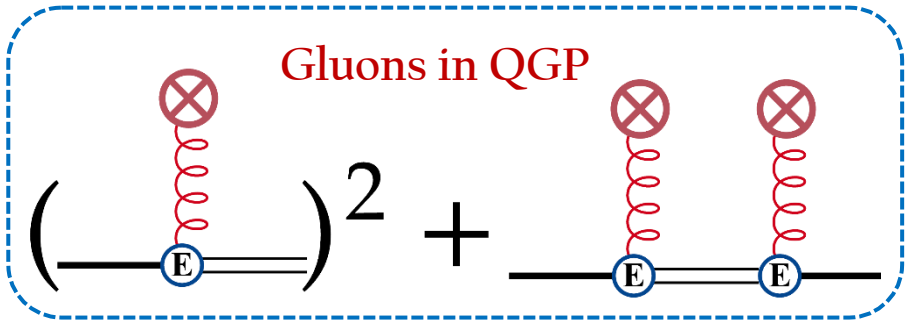
Quarkonium moves in the QGP with velocity $\vec{v}$

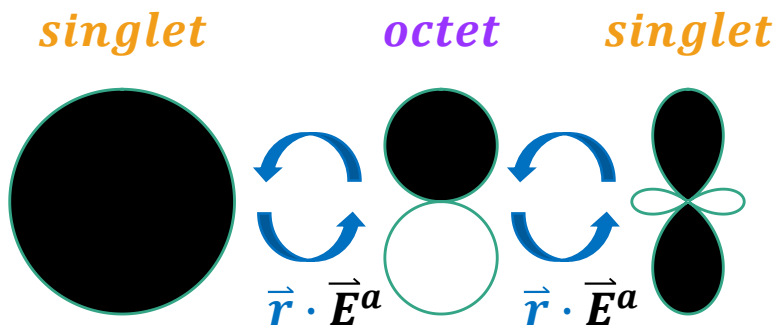


Orbital

$$\Pi_{T/L} = \Pi_{T/L}(\rho_{SS}, \rho_{D^m D^m}, Re[\rho_{SD^0}])$$

Chromo-E dipole sees boosted gluons, which yield anisotropic correlation





S wave $L = 0$ | P wave $L = 1$ | D wave $L = 2$

$$g_s^2 r_i r_j \langle E_i^a E_j^b \rangle_{th} \sim g_s^2 r_i r_j f(\mathfrak{Q})_{ij}^{\mu\nu} D_{\mu\nu}^{>}(\vec{v})$$

$Q\bar{Q}$ transitions among S-,P-,D-wave via $\vec{r} \cdot \vec{E}^a$ interaction $(S_s \leftrightarrow P_o \leftrightarrow D_s)$

anisotropic transitions give non-zero spin alignment $\lambda_\theta \propto \Pi_T - \Pi_L \neq 0$

Chen-Lin In preparation

02 Mechanism-Orbital

Relative motion between quarkonium and QGP

$$\Pi_{T/L} = \Pi_{T/L}(\rho_{SS}, \rho_{D^m D^m}, Re[\rho_{SD^0}])$$

Isotropic correlation/potential

$$\Pi^{(SD^0)} \propto \sum_{n,n'} \left(\rho^{orb}_{nS,n'D^0} + \rho^{orb}_{n'D^0,nS}\right) R_n R_{n'}$$

S-D coherence

zero

$$\Pi^{(D^m)} \propto \sum_{s=\pm|m|} \sum_{n,n'} \left(\rho^{orb}_{nD^s,n'D^s}\right) R_n R_{n'}$$

m splitting

Degenerate

$$2\Pi^{(D^0)} = \Pi^{(D^{\pm 1})} = \Pi^{(D^{\pm 2})}$$

$$\lambda_\theta = \frac{\left(-\Pi^{(SD^0)} - \Pi^{(D^0)} - \frac{1}{2}\Pi^{(D^{\pm 1})} + \Pi^{(D^{\pm 2})}\right)(3\cos^2\theta_{vl} - 1)}{\left(4\Pi^{(SS)} - \frac{1}{3}\Pi^{(SD^0)} + \frac{7}{3}\Pi^{(D^0)} + \frac{5}{2}\Pi^{(D^{\pm 1})} + 3\Pi^{(D^{\pm 2})}\right) + \left(\Pi^{(SD^0)} + \Pi^{(D^0)} + \frac{1}{2}\Pi^{(D^{\pm 1})} - \Pi^{(D^{\pm 2})}\right)\cos^2\theta_{vl}}$$

Chen-Lin In preparation

02 Mechanism-Orbital

Open quantum system

$\rho_{tot} = \rho_{sys} \otimes \rho_{env}$, $H_{tot} = H_{sys} \otimes \mathbb{I}_{env} + \mathbb{I}_{sys} \otimes H_{env} + H_I$

sys: (Quarkonium) system
env: (QGP) environment

Redfield equation Trace env, apply Born- and Markov-approximation

$$\dot{\rho}_{sys} = i[\rho_{sys}, H_{sys}] + \left(\sum_{\omega,\omega'} \sum_{\alpha,\beta=\pm} \Gamma_{ij}(\omega) \left(V_{\beta,j}(\omega) \rho_{sys} V^{\dagger}_{\alpha,i}(\omega') - V^{\dagger}_{\alpha,i}(\omega') V_{\beta,j}(\omega) \rho_{sys} \right) + (h.c.) \right)$$

Self-evolution of system

H_I: Interaction between system and environment

Redfield, IBM J. Res. Dev 1957

Akamatsu PPNP 2022

02 Mechanism-Orbital

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Quarkonium orbital density matrix

$$\rho^{orb}_{sys} = \begin{pmatrix} \rho_{S_s S_s} & 0 & \rho_{S_s D_s} \\ 0 & \rho_{P_o P_o} & 0 \\ \rho_{D_s S_s} & 0 & \rho_{D_s D_s} \end{pmatrix}, \quad H_{sys} = \frac{p^2_{rel}}{m_Q} + \begin{pmatrix} -C_F \frac{\alpha_s}{r} & 0 & 0 \\ 0 & \frac{1}{2N_c} \frac{\alpha_s}{r} + \frac{1(1+1)}{m_Q r^2} & 0 \\ 0 & 0 & -C_F \frac{\alpha_s}{r} + \frac{2(2+1)}{m_Q r^2} \end{pmatrix}$$

System transition operator

$$V_{+,i}(\omega) = \delta_{\omega, E_s - E_o} \sqrt{C_F}\, r_i |o\rangle\langle s| \sim \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, V_{-,i}(\omega) = \delta_{\omega, E_o - E_s} \frac{1}{\sqrt{2N_c}} r_i |s\rangle\langle o| \sim \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

02 Mechanism-Orbital

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Environment correlation functions

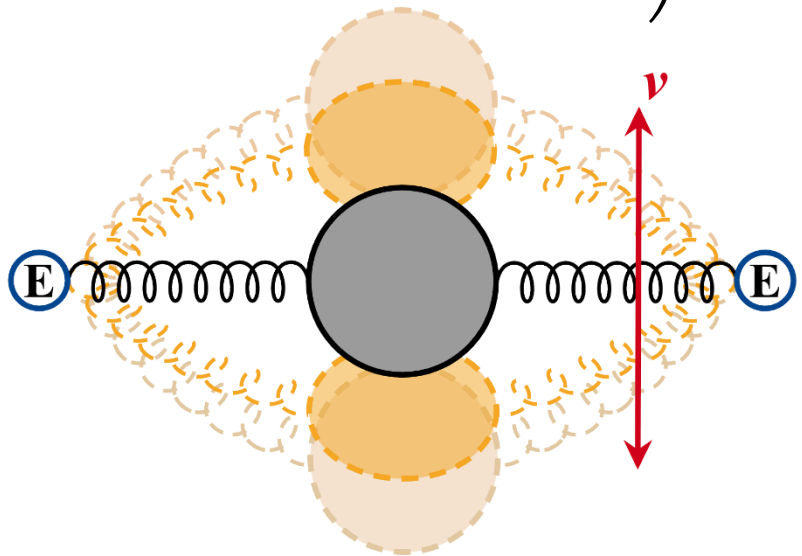
$$\Gamma^{ab}_{ij}(\omega) = \int_0^{\infty} ds\, e^{i\omega s} \left\langle g_s E^a_i(s, \vec{R}) U(s, \vec{0}) g_s E^b_j(0, \vec{R}) \right\rangle_{th}$$

$$\approx g_s^2 \int_0^{\infty} ds \int \frac{d^4\mathfrak{Q}}{(2\pi)^4} e^{i(\omega - q_0)s} \left(q_i q_j D^{>,ab}_{00,th} + 2 q_0 q_i D^{>,ab}_{0j,th} + q_0^2 D^{>,ab}_{ij,th} \right)$$

Decompose

$$\Gamma_{ij}(\omega) = \frac{1}{2}\gamma_{ij}(\omega) + iS_{ij}(\omega)$$

γ_{ij}: dissipative
S_{ij}: dispersive



Relative motion between $Q\bar{Q}$ and QGP gives anisotropic correlation

02 Mechanism-Orbital

Open quantum system

Redfield equation

Find steady states $\dot{\rho}_{sys} = 0$

$$0 = i\left[\rho_{sys}, H_{sys} + \delta H\right] + \mathcal{J}\left[\rho_{sys}\right] - \frac{1}{2}\left\{\mathcal{K}, \rho_{sys}\right\}$$

Recycling

$$\mathcal{J}\left[\rho_{sys}\right] = \sum_{\omega,\omega'} \sum_{\alpha,\beta=\pm} \left(\frac{\gamma_{ij}(\omega) + \gamma_{ij}(\omega')}{2} + i\left(S_{ij}(\omega) - S_{ij}(\omega')\right) \right) V_{\beta,j}(\omega) \rho_{sys} V^{\dagger}_{\alpha,i}(\omega')$$

$\rho_{ss} \leftrightarrow \rho_{oo}$

Hermitian correction

$$\delta H = \sum_{\omega,\omega'} \sum_{\alpha,\beta=\pm} \left(\frac{S_{ij}(\omega) + S_{ij}(\omega')}{2} - \frac{i\left(\gamma_{ij}(\omega) - \gamma_{ij}(\omega')\right)}{4} \right) V^{\dagger}_{\alpha,i}(\omega') V_{\beta,j}(\omega)$$

$\rho_{ss} \leftrightarrow \rho_{ss}$

$nS(D) \leftrightarrow n'S(D), nS \leftrightarrow jD$

Anti-Hermitian correction

$$-\frac{1}{2}\left\{\mathcal{K}, \rho_{sys}\right\} = \sum_{\omega,\omega'} \sum_{\alpha,\beta=\pm} \left(\frac{\gamma_{ij}(\omega) + \gamma_{ij}(\omega')}{2} + i\left(S_{ij}(\omega) - S_{ij}(\omega')\right) \right) \frac{-1}{2} \left\{ V^{\dagger}_{\alpha,i}(\omega') V_{\beta,j}(\omega), \rho_{sys} \right\}$$

Recoilless limit $(\omega, \omega' \to 0)$ they give Real / Imaginary potential contributions

02 Mechanism-Orbital

Open quantum system

Redfield equation

Find steady states $\dot{\rho}_{sys} = 0$

$$0 = i\left[\rho_{sys}, H_{sys} + \delta H\right] + \mathcal{J}\left[\rho_{sys}\right] - \frac{1}{2}\left\{\mathcal{K}, \rho_{sys}\right\}$$

Recycling $\mathcal{J}\left[\rho_{sys}\right] = \sum_{\omega\omega'\alpha\beta=\pm}\left(\frac{\gamma_{ij}(\omega)+\gamma_{ij}(\omega')}{2} + i\left(S_{ij}(\omega) - S_{ij}(\omega')\right)\right)V_{\beta,j}(\omega)\rho_{sys}V^{\dagger}_{\alpha,i}(\omega')$ $\rho_{ss} \leftrightarrow \rho_{oo}$

$\rho_{ss} \leftrightarrow \rho_{ss}$

$nS(D) \leftrightarrow n'S(D), nS \leftrightarrow jD$

$$nS(D) \overset{\omega}{\leftrightarrow} pP \overset{\omega'}{\leftrightarrow} n'S(D), nS \overset{\omega}{\leftrightarrow} pP \overset{\omega'}{\leftrightarrow} jD$$

$\rho_{nS(D),n'S(D)}$ not small when $n \neq n'$

$\rho_{nS,jD} > \rho_{jS,jD}$ may happen when $n < j$

Recoilless limit $(\omega, \omega' \to 0)$ they give Real / Imaginary potential contributions

02 Mechanism-Orbital

Open quantum system

Lindblad equation

Find steady states $\dot{\rho}_{sys} = 0$

GAME form

(Geometric Arithmetic Master Equation)

$$0 = -i[H_{sys} + \delta H] + \mathcal{J}[\rho_{sys}] - \frac{1}{2}\{\mathcal{K}, \rho_{sys}\}$$

Recycling

$$\mathcal{J}[\rho_{sys}] = \sum_{\omega,\omega'} \sum_{\alpha,\beta=\pm} \left(\frac{\gamma_{ij}(\omega) + \gamma_{ij}(\omega')}{2} + i\left(S_{ij}(\omega) - S_{ij}(\omega')\right) \right) V_{\beta,j}(\omega) \rho_{sys} V^{\dagger}_{\alpha,i}(\omega')$$

$\rho_{ss} \leftrightarrow \rho_{oo}$

$$= \sum_{\omega,\omega'} \sum_{\alpha,\beta=\pm} \left(\sqrt{\gamma_{ij}(\omega)\,\gamma_{ij}(\omega')} + f(\omega,\omega') \right)$$

$\omega' \approx \omega, f(\omega,\omega') \to 0$

ω away from $\omega', f(\omega,\omega') \xrightarrow{average} 0$

Anti-Hermitian correction

$$-\frac{1}{2}\{\mathcal{K}, \rho_{sys}\} = \sum_{\omega,\omega'} \sum_{\alpha,\beta=\pm} \left(\frac{\gamma_{ij}(\omega) + \gamma_{ij}(\omega')}{2} + i\left(S_{ij}(\omega) - S_{ij}(\omega')\right) \right) \frac{-1}{2}\{V^{\dagger}_{\alpha,i}(\omega') V_{\beta,j}(\omega), \rho_{sys}\}$$

Recoilless limit $(\omega, \omega' \to 0)$ they give Real / Imag

D. Davidović Quantum 2020

02 Mechanism-Orbital

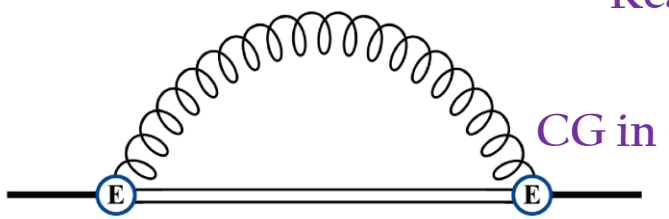
Steady states solution

$$0 = i\left[\rho_{sys}, H_{sys} + \delta H\right] + \mathcal{J}\left[\rho_{sys}\right] - \frac{1}{2}\left\{\mathcal{K}, \rho_{sys}\right\}$$

$\boldsymbol{n = (1, \vec{0}), u = \gamma_v(1, \vec{v})}$

Real thermal gluon $\mathfrak{Q}\sim\mathbf{T}$

HTL gluon $\mathfrak{Q}\sim\mathbf{gT}$

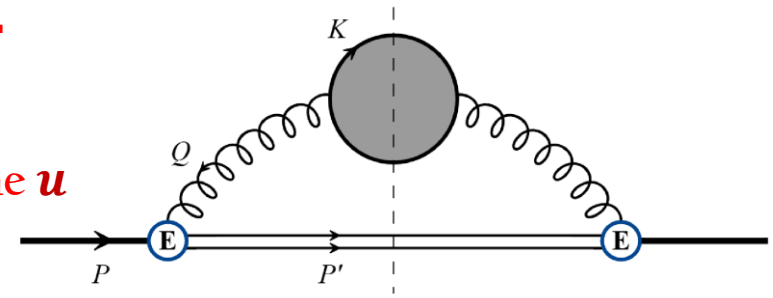


CG in Quarkonium rest frame $\boldsymbol{n}$

$(\boldsymbol{\nabla \cdot A})_{\boldsymbol{n}} = \mathbf{0}$

CG in QGP rest frame $\boldsymbol{u}$

$(\boldsymbol{\nabla \cdot A})_{\boldsymbol{u}} = \mathbf{0}$



02 Mechanism-Orbital

Steady states solution

$$0 = i[\rho_{sys}, H_{sys} + \delta H] + \mathcal{J}[\rho_{sys}] - \frac{1}{2}\{\mathcal{K}, \rho_{sys}\}$$

Setting $\boldsymbol{J/\psi(c\bar{c})}$ $\int^{T} dq, \int^{T} d\omega, |q_0| < |q|$ $T = 0.15\text{GeV}$

$\mathcal{H}_s = \{|nS\rangle, |jD^m\rangle\}, n = 1\sim10, j = 3\sim10, m = -2,-1,0,1,2$

$\mathcal{H}_o = \{|p_\nu P^m\rangle\}, \nu = 1\sim150, m = -1,0,1$

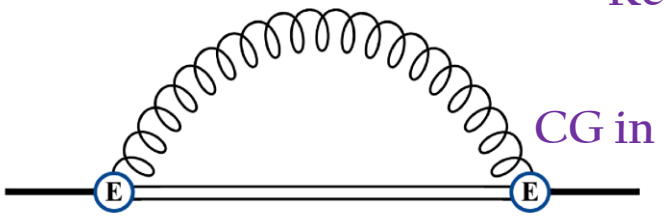
Box normalization

Focusing on relative number of S- and D-wave

$\boldsymbol{n = (1,\vec{0}), u = \gamma_v(1,\vec{v})}$

Real thermal gluon $\mathfrak{Q}\sim\mathbf{T}$

HTL gluon $\mathfrak{Q}\sim\mathbf{gT}$

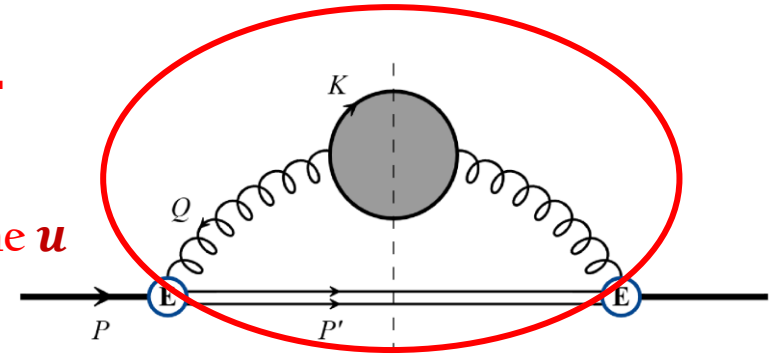


CG in Quarkonium rest frame $\boldsymbol{n}$

$(\nabla \cdot A)_n = 0$

CG in QGP rest frame $\boldsymbol{u}$

$(\nabla \cdot A)_u = 0$



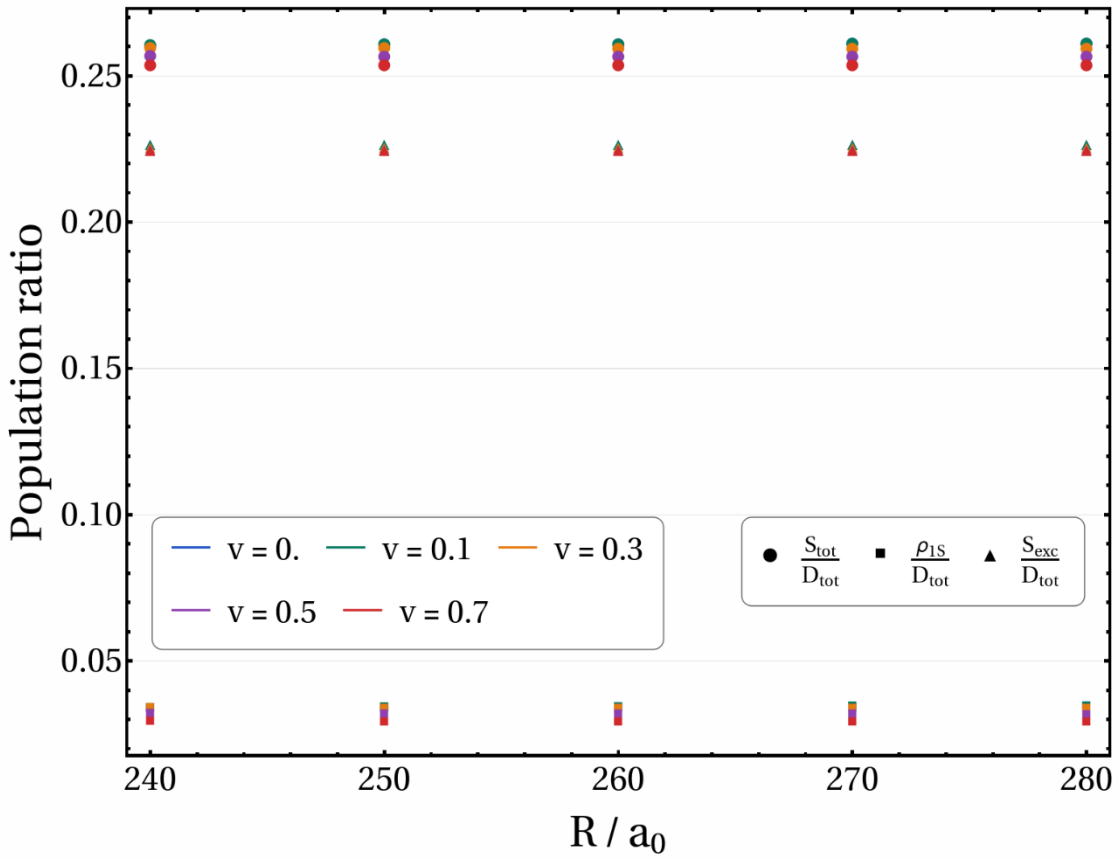
03 Results

Steady states solution

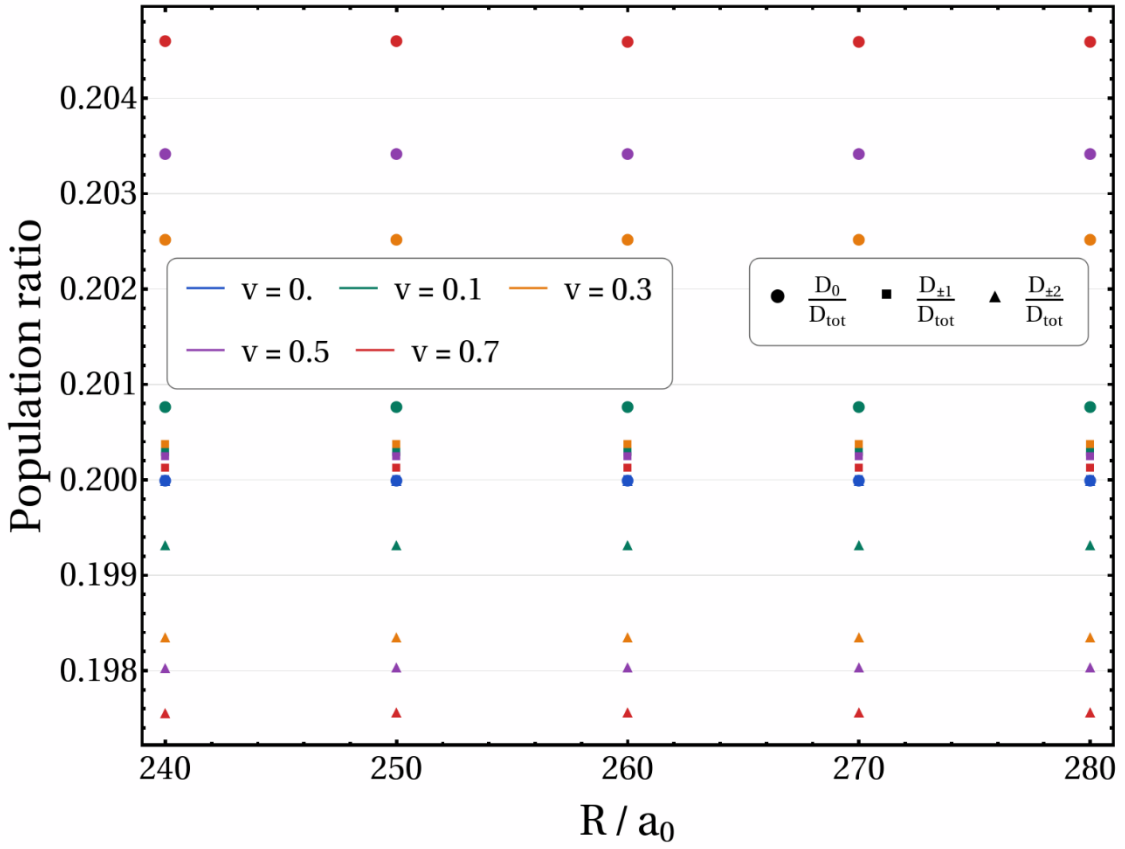
Red

Stationary of box size

$$\frac{\rho_{SS}^{tot,1S,nS}}{\rho_{DD}^{tot}}$$



$$\frac{\rho_{D^0D^0,D^{\pm1}D^{\pm1},D^{\pm2}D^{\pm2}}}{\rho_{DD}^{tot}}$$



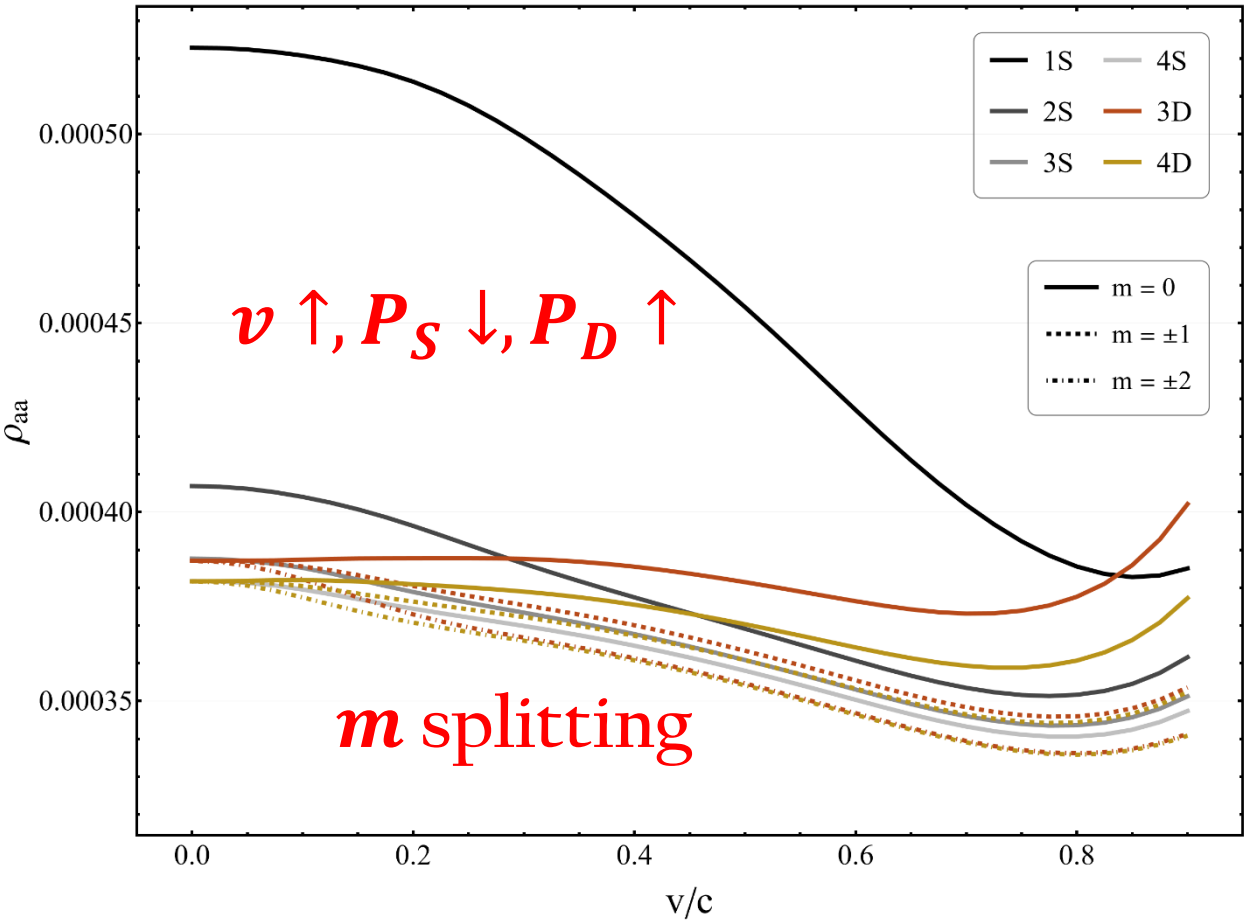
03 Results

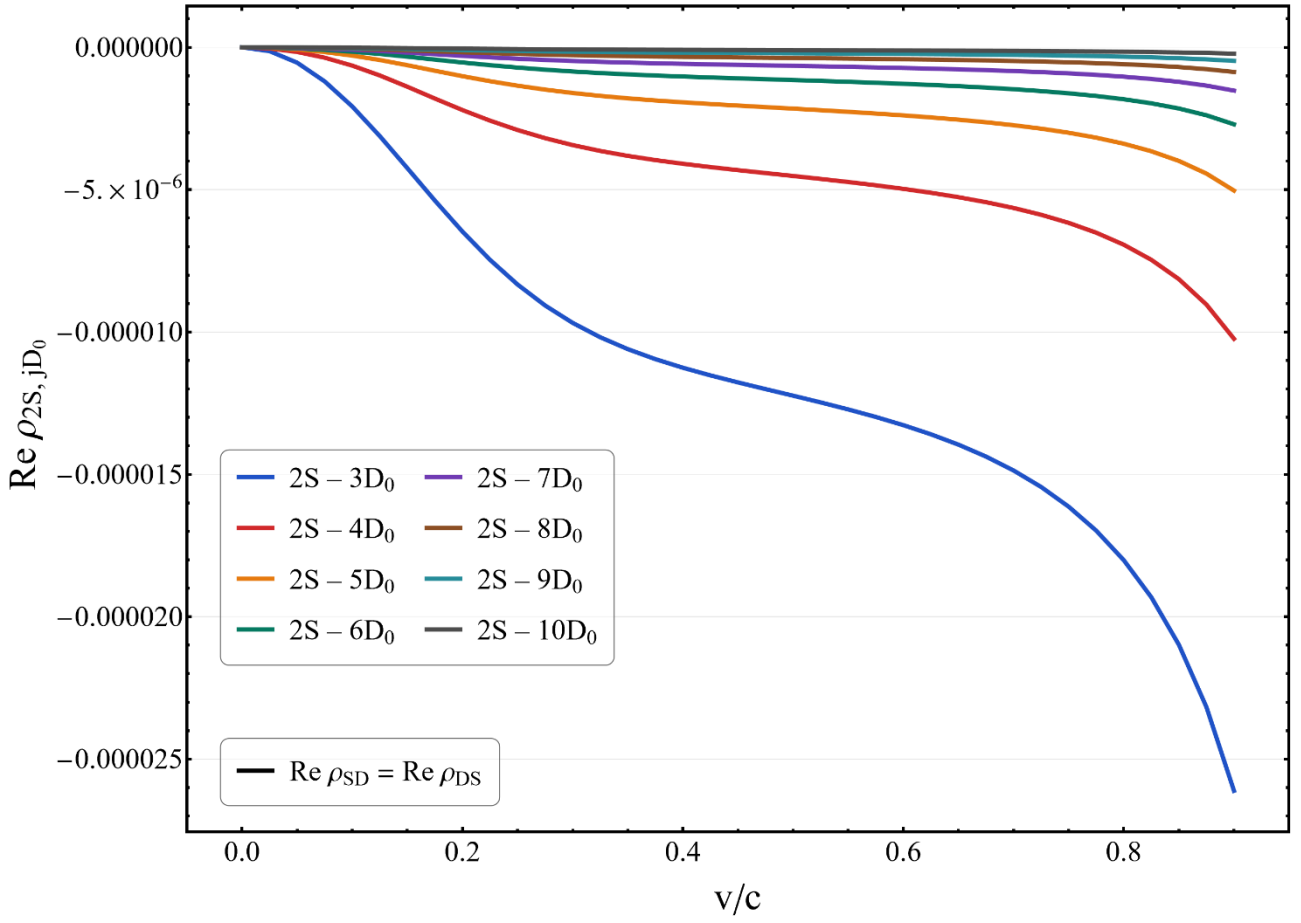
Steady states solution

Red

Splitting of D^m states

Non-zero coherence





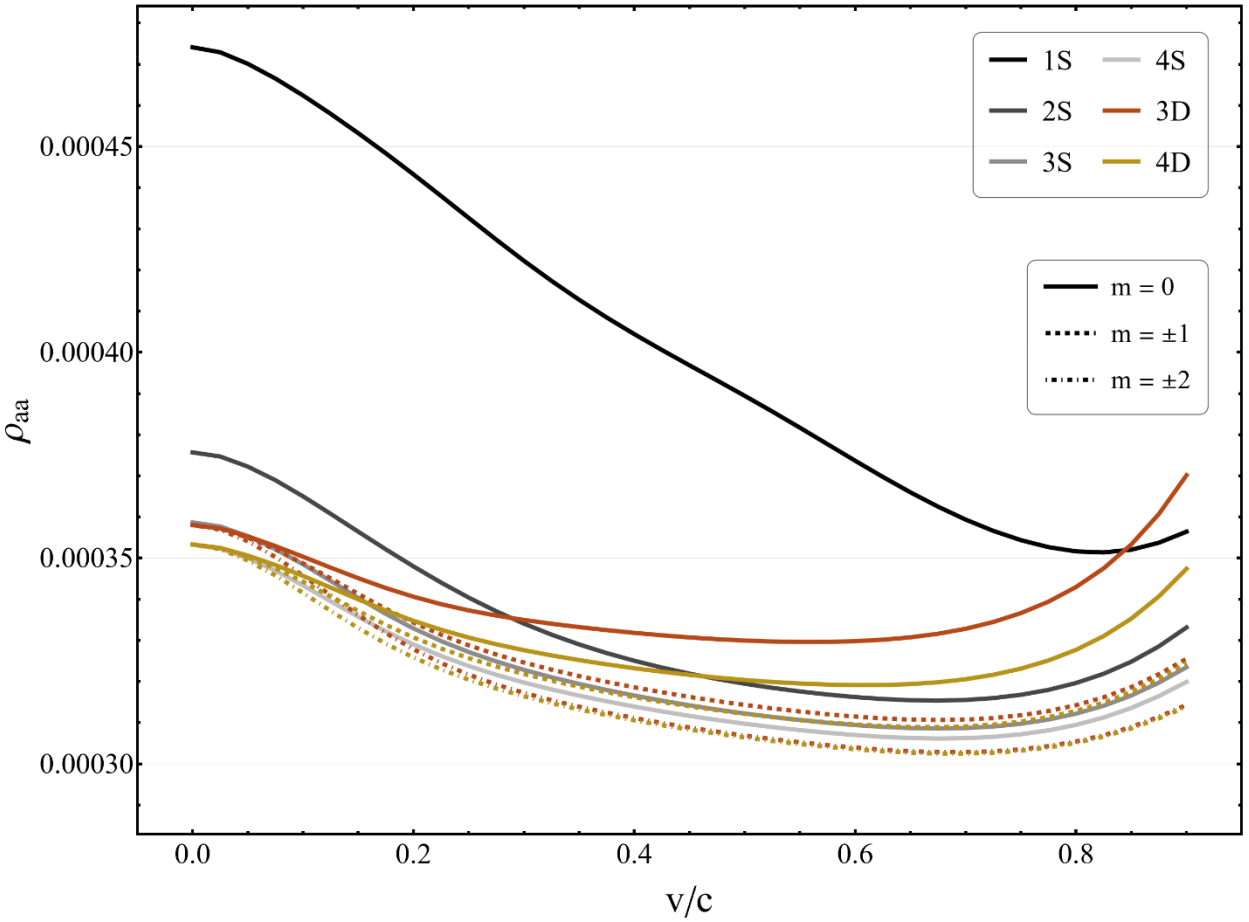
03 Results

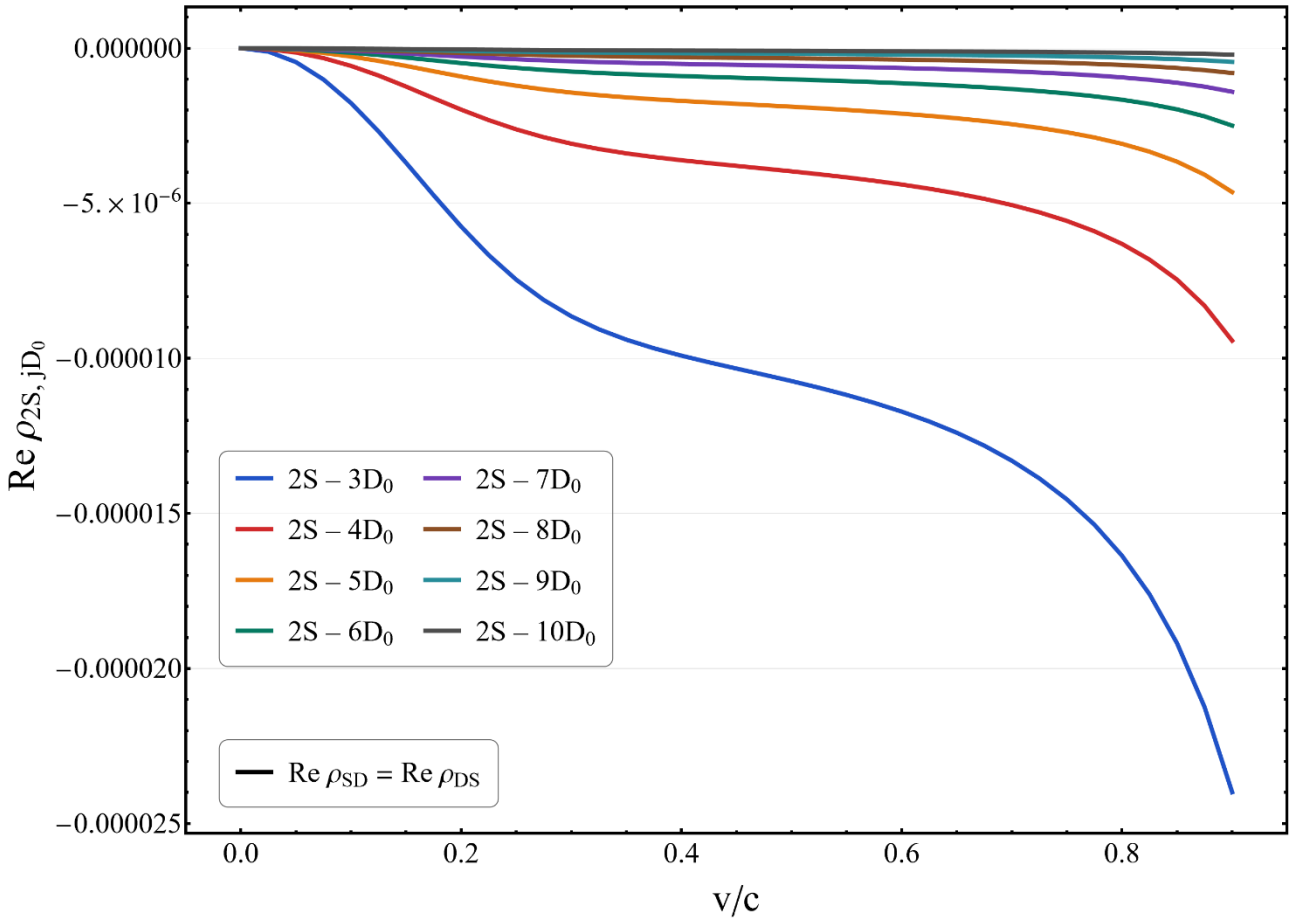
Steady states solution

GAME

Splitting of D^m states

Non-zero coherence





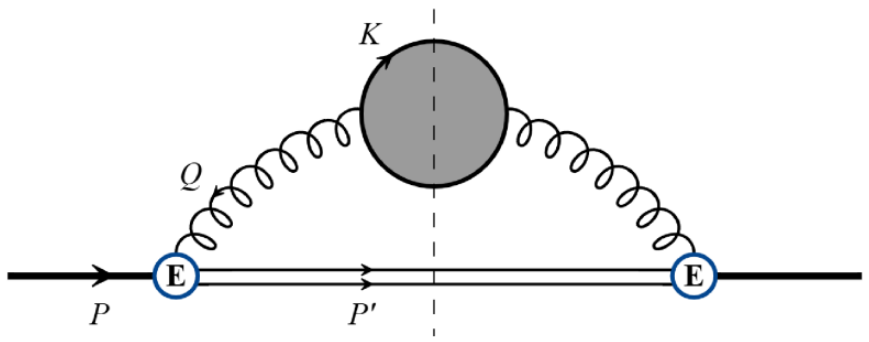
03 Results

Steady states solution

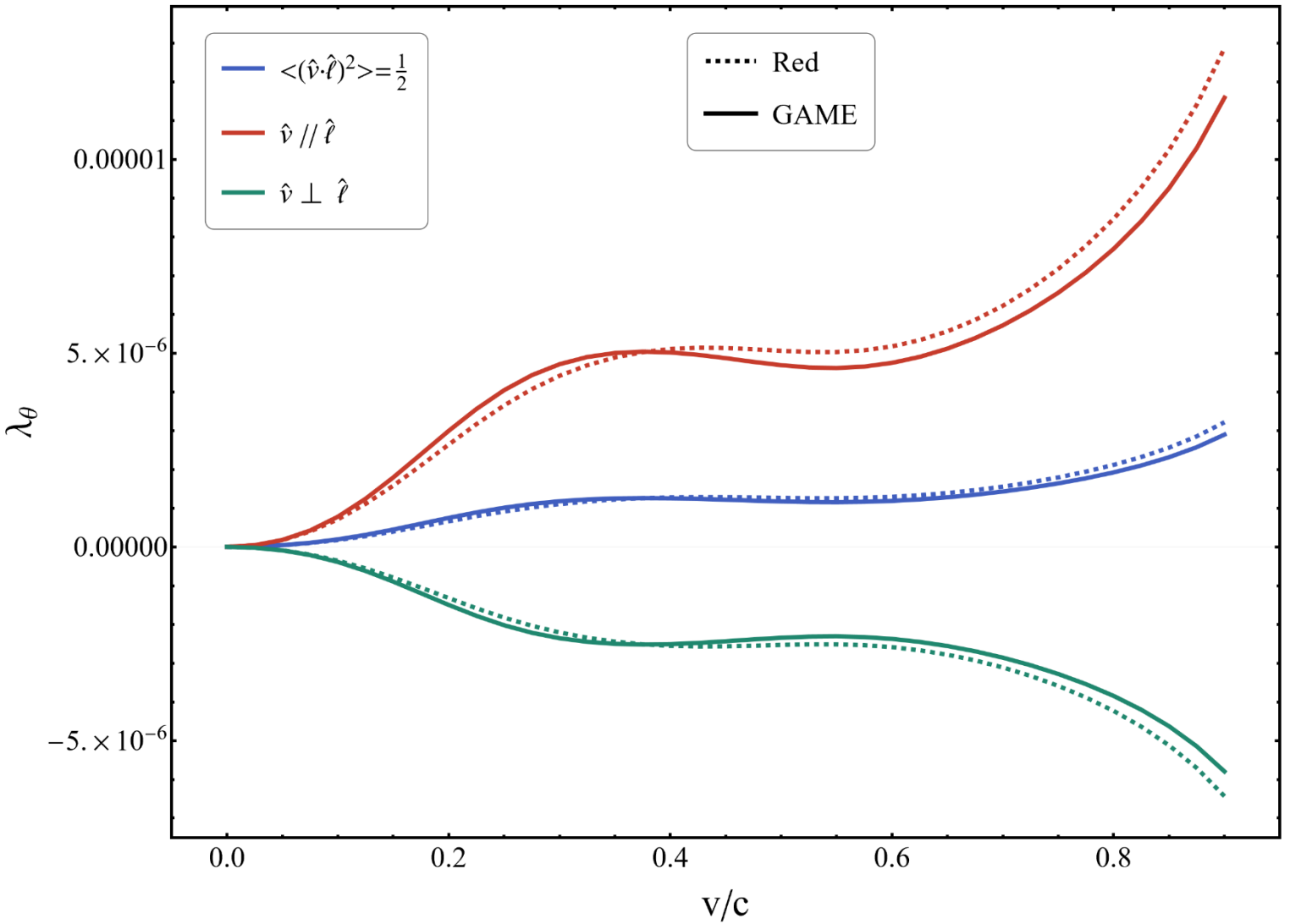
Spin alignment

$$\lambda_{\vartheta}^{\langle\theta_{vl}\rangle} > 0$$

$\sim 10^{-5}$ from HTL gluon $\mathfrak{Q} \sim gT$



$$\Pi_{T/L} = \Pi_{T/L}(\rho_{SS}, \rho_{D^m D^m}, Re[\rho_{SD^0}])$$



$$\lambda_\theta = \frac{\left(-\Pi^{(SD^0)} - \Pi^{(D^0)} - \frac{1}{2}\Pi^{(D^{\pm 1})} + \Pi^{(D^{\pm 2})}\right)(3\cos^2\theta_{vl} - 1)}{\left(4\Pi^{(SS)} - \frac{1}{3}\Pi^{(SD^0)} + \frac{7}{3}\Pi^{(D^0)} + \frac{5}{2}\Pi^{(D^{\pm 1})} + 3\Pi^{(D^{\pm 2})}\right) + \left(\Pi^{(SD^0)} + \Pi^{(D^0)} + \frac{1}{2}\Pi^{(D^{\pm 1})} - \Pi^{(D^{\pm 2})}\right)\cos^2\theta_{vl}}$$

04 Summary and outlook

summary

- pNRQCD and Redfield(GAME-Lindblad) equation.
- Relative motion induces spin alignment.
- Numerical simulation gives the right sign.

outlook

- Thermal gluon's contribution.
- More different waves (P_s, F_o …) join the game.
- Find better relations between v and l?

Thanks !

Back up

$\tau_{env} \sim \frac{1}{m_D}(soft)\ or\ \frac{1}{T}(hard)$: environment correlation time

$\tau_{sys} \sim \frac{1}{\epsilon_B}$: system intrinsic time scale

$\tau_R \sim \frac{m_Q}{g^4 T^2}$: system relaxation time

Redfield equation $\tau_R \gg \tau_{env}$ $\frac{\tau_R}{\tau_{env}} \sim \frac{m_Q(g)\,T}{g^4T^2} = \frac{m_Q}{g^{4(3)}T} \gg 1$

$$\dot{\rho}_{sys} = \sum_{\omega,\omega'} \sum_{\alpha,\beta=\pm} e^{i(\omega'-\omega)t} \Gamma_{ij}(\omega) \left(V_{\beta,j}(\omega) \rho_{sys} V^{\dagger}_{\alpha,i}(\omega') - V^{\dagger}_{\alpha,i}(\omega') V_{\beta,j}(\omega) \rho_{sys} \right) + (h.c.)$$

Interaction picture

Lindblad equation from RWA $(\omega = \omega')$ $\tau_R \gg \tau_{sys}$ $\frac{\tau_R}{\tau_{sys}} \sim \frac{m_Q \epsilon_B}{g^4T^2} \sim \frac{m_Q}{g^3 T} \gg 1$

$$\dot{\rho}_{sys} = i\left[\rho_{sys}, H_{sys} + \delta H\right] + \sum_{\omega} \sum_{\alpha,\beta=\pm} \gamma_{ij}(\omega) \left(V_{\beta,j}(\omega) \rho_{sys} V^{\dagger}_{\alpha,i}(\omega) - \frac{1}{2} \left\{ V^{\dagger}_{\alpha,i}(\omega) V_{\beta,j}(\omega), \rho_{sys} \right\} \right)$$

$\omega = E_s - E_o, \omega' = E_s - E'_o \quad \omega' \to \omega$

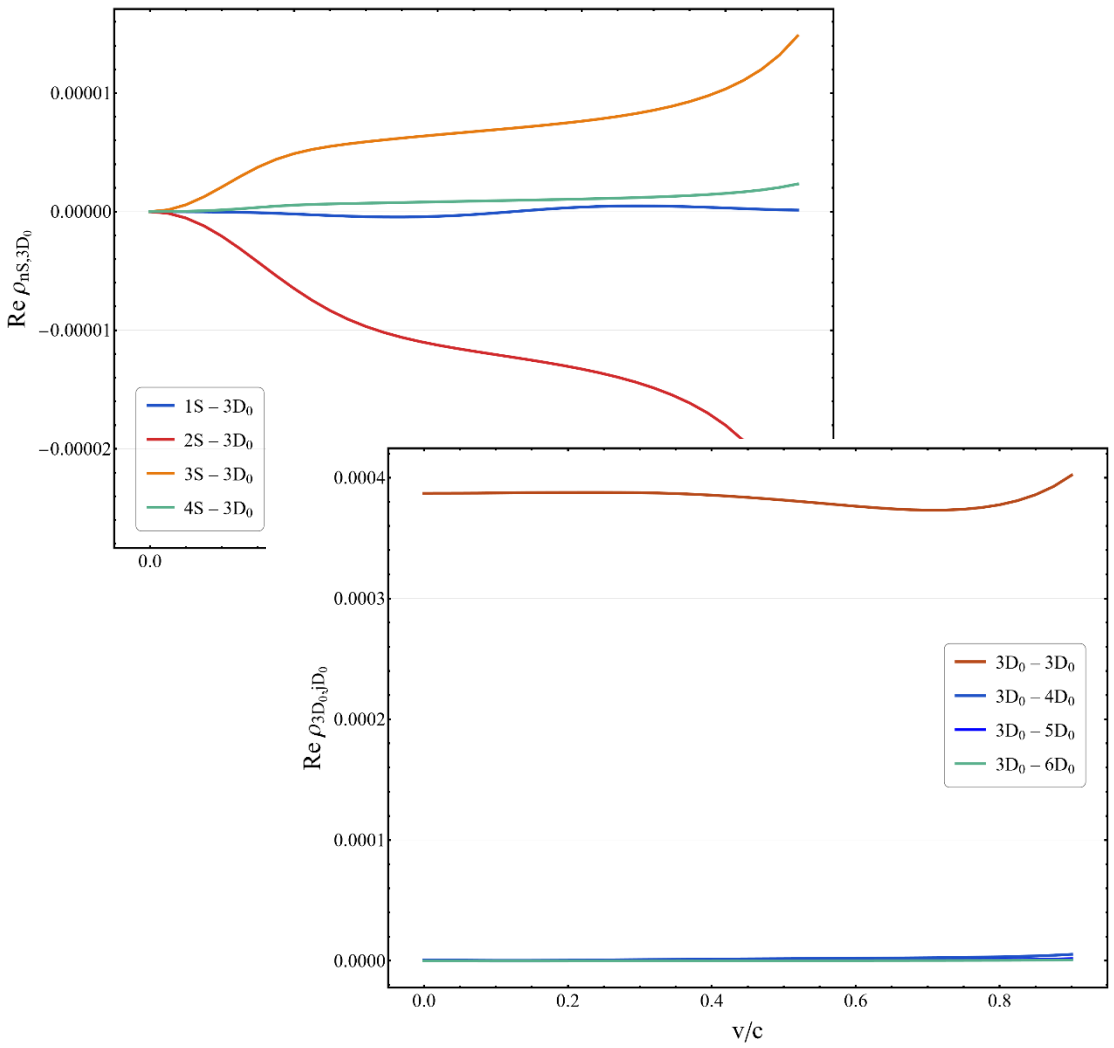
Lindblad equation from QBM $(\omega \sim 0)$

$\Gamma_{ij}(\omega) = \Gamma_{ij}(0) + \omega \Gamma'_{ij}(0) + \cdots$ $\tau_{sys} \gg \tau_{env}$ $hard\ \frac{\tau_{sys}}{\tau_{env}} \sim \frac{T}{\epsilon_B} \sim \frac{1}{g} \gg 1$ $soft\ \frac{\tau_{sys}}{\tau_{env}} \sim \frac{m_D}{\epsilon_B} \sim 1$

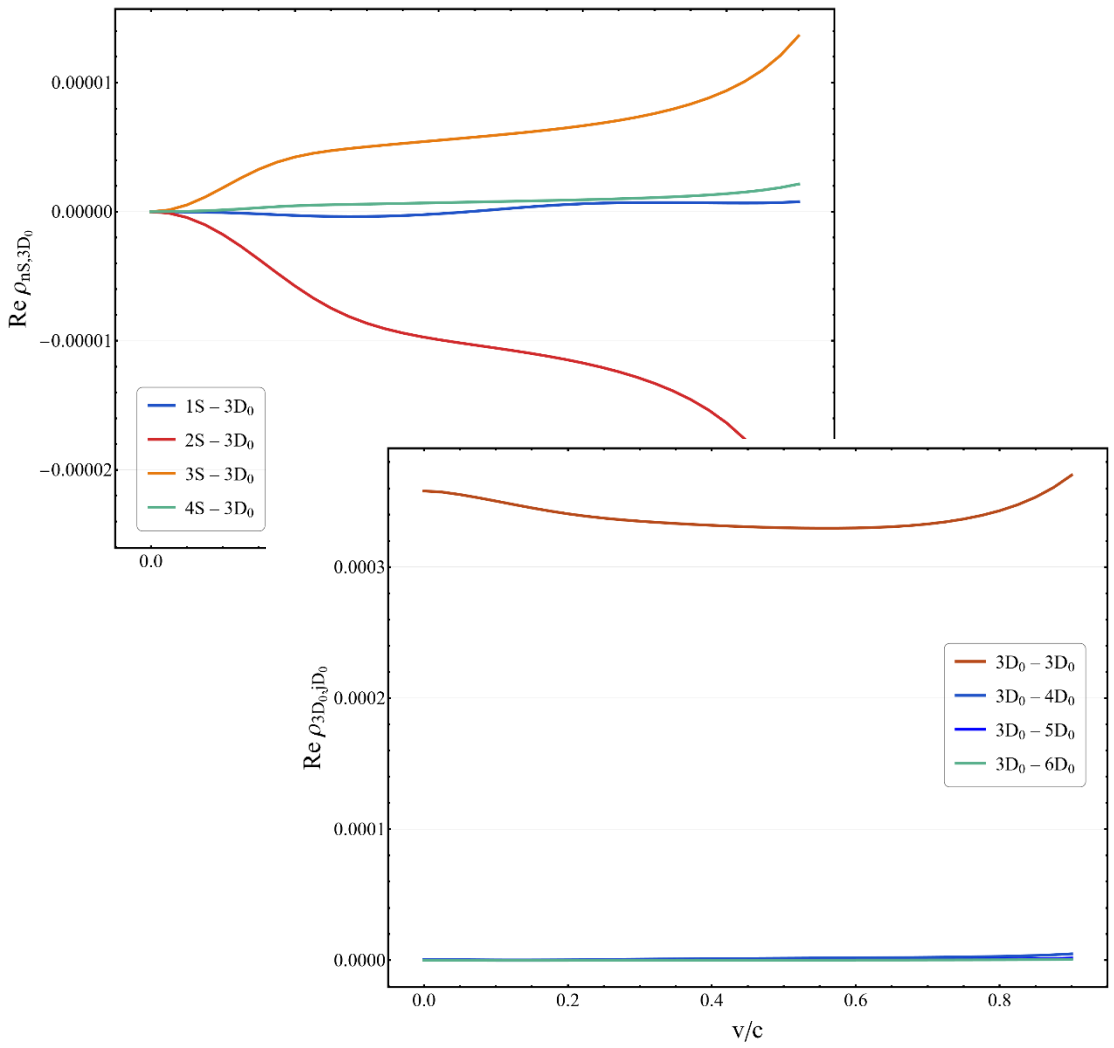
$$\dot{\rho}^{(0)}_{sys} = \sum_{\alpha,\beta=\pm} \Gamma_{ij}(0) \left(V_{\beta,j}(t) \rho_{sys} V^{\dagger}_{\alpha,i}(t) - V^{\dagger}_{\alpha,i}(t) V_{\beta,j}(t) \rho_{sys} \right) + (h.c.)$$

Back up

Red



GAME

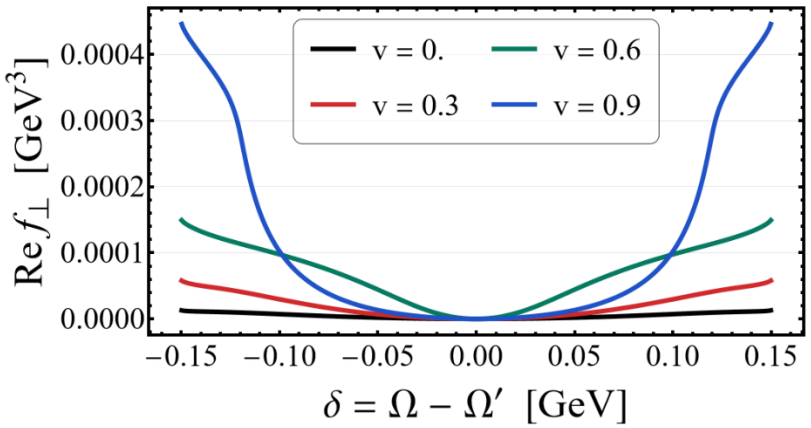


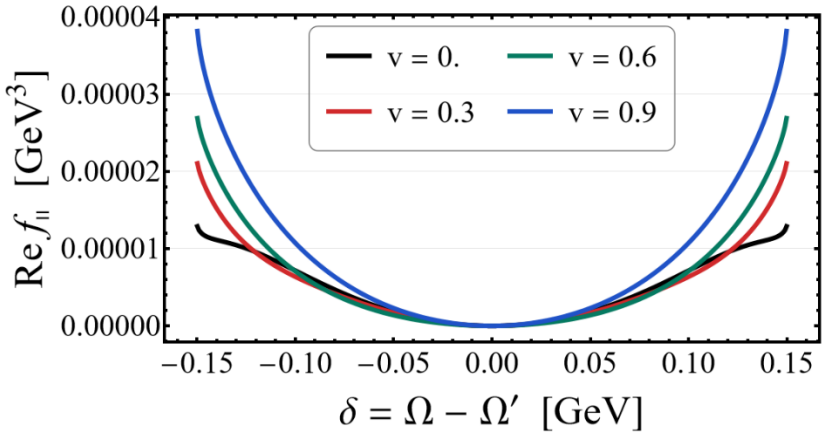
Back up

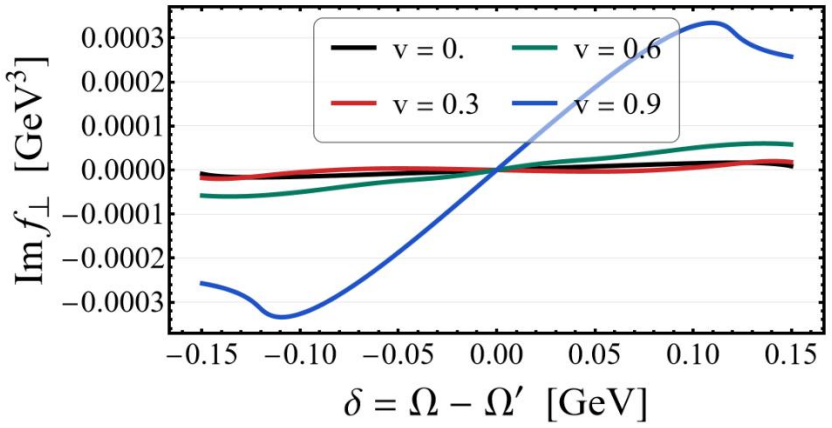
$\bar{\Omega} = -0.075$ GeV

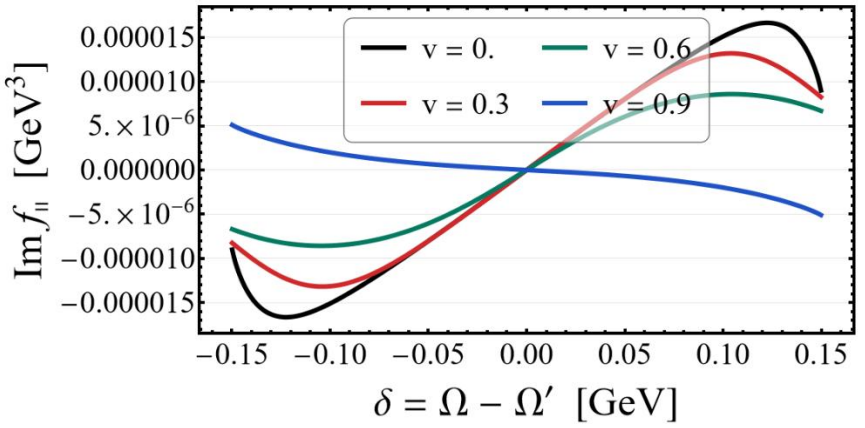
$f(\Omega, \Omega')$

$\bar{\Omega} = -0.075$ GeV

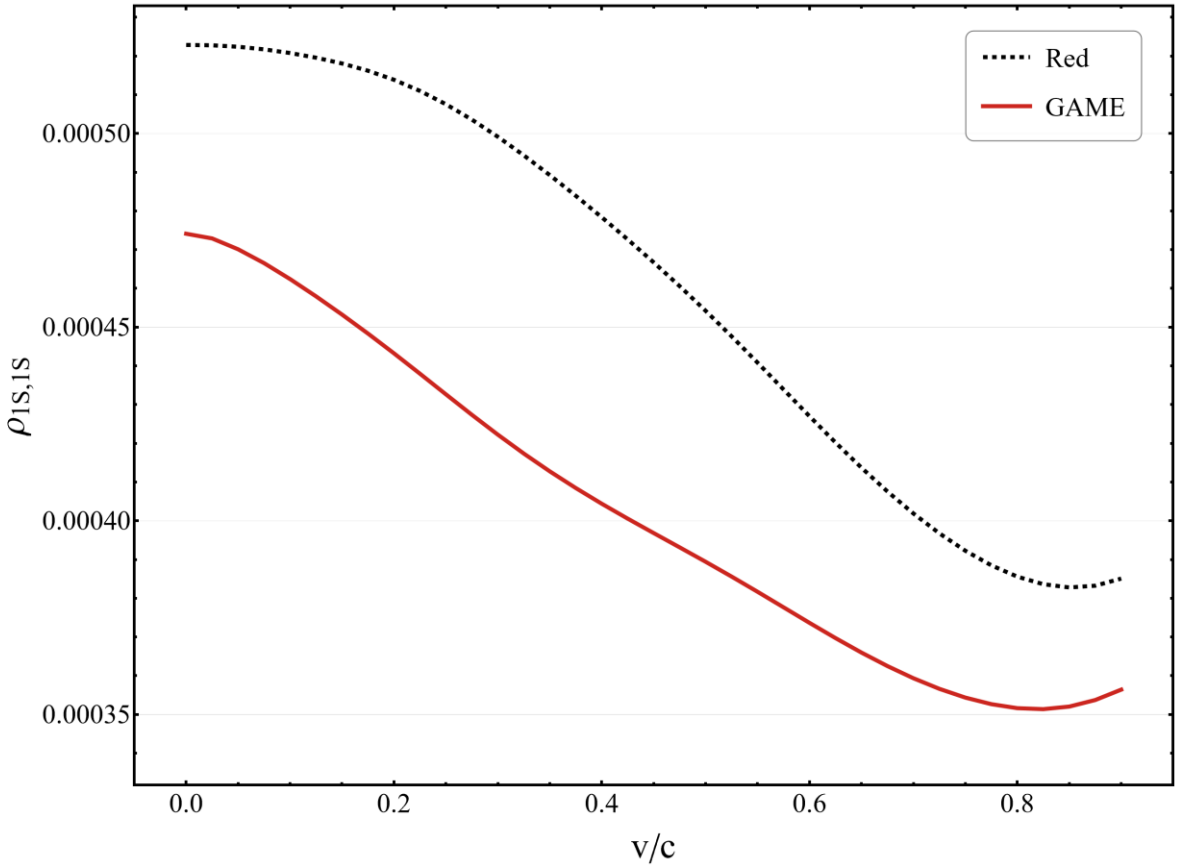


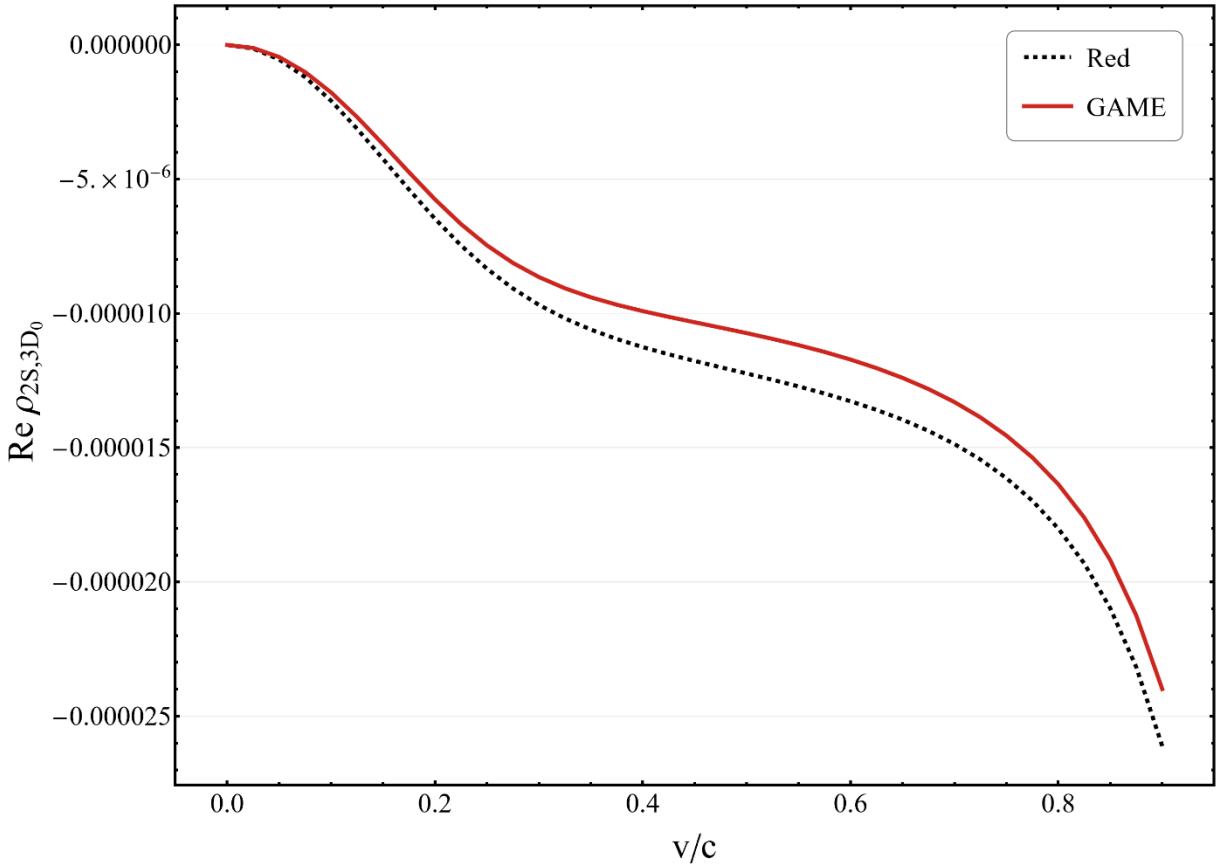






Back up


Red
GAME
0.00050
0.00045
0.00040
0.00035
$\rho_{1S,1S}$
0.0
0.2
0.4
0.6
0.8
v/c


Red
GAME
0.000000
$-5.\times10^{-6}$
−0.000010
−0.000015
−0.000020
−0.000025
Re $\rho_{2S,3D_0}$
0.0
0.2
0.4
0.6
0.8
v/c

Back up

$$0 = i\left[\rho_{sys}, H_{sys} + \delta H\right] + \sum_{\omega,\omega'} \sum_{\alpha,\beta=\pm} \mathrm{G}_{ij}(\omega,\omega')\left(V_{\beta,j}(\omega)\rho_{sys}V^{\dagger}_{\alpha,i}(\omega') - \frac{1}{2}\left\{V^{\dagger}_{\alpha,i}(\omega')V_{\beta,j}(\omega), \rho_{sys}\right\}\right)$$

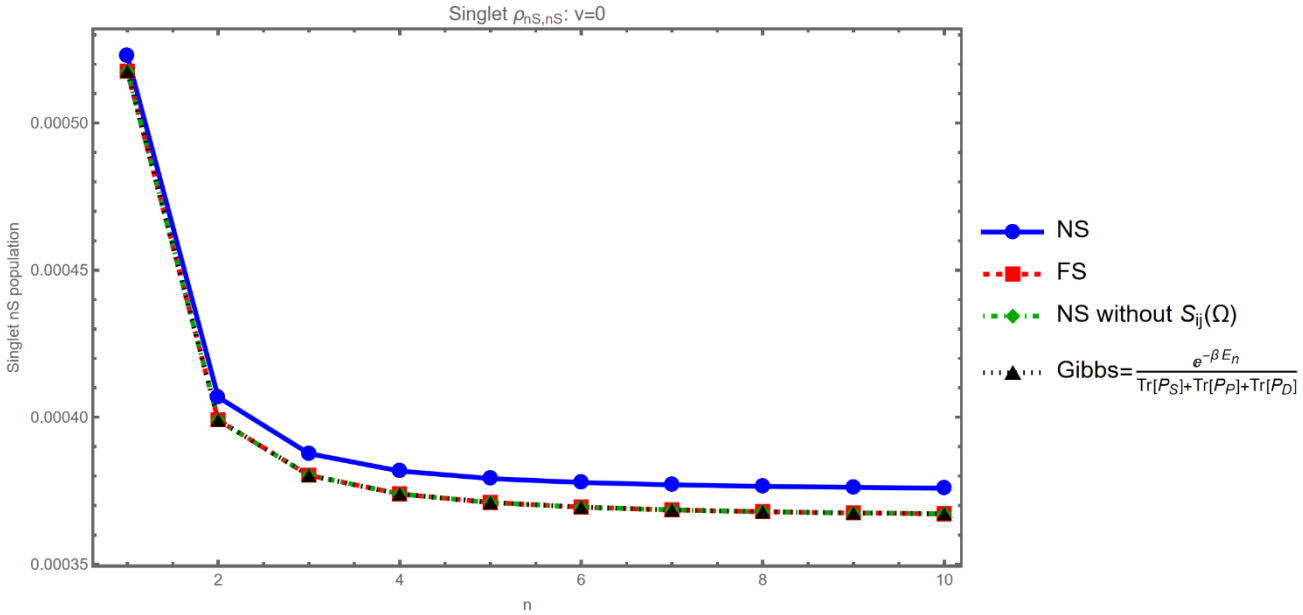
NS: non-secular (Redfield) equation

$FS(\omega = \omega')$: full-secular equation

$NS\ without\ S(S_{ij} = 0)$

Satisfy Gibbs distribution!

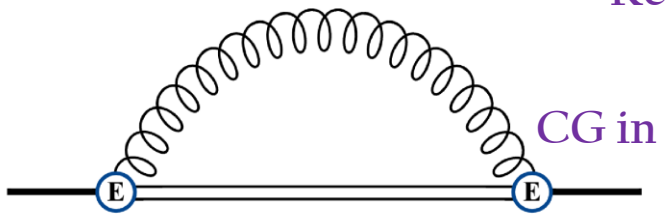
Proved by Lee, Yeo PRE 2022



$\boldsymbol{n = (1, \vec{0}), u = \gamma_v(1, \vec{v})}$

Real thermal gluon $\mathfrak{Q}$~T

HTL gluon $\mathfrak{Q}$~gT



CG in Quarkonium rest frame $\boldsymbol{n}$

$(\nabla \cdot A)_n = 0$

CG in QGP rest frame $\boldsymbol{u}$

$(\nabla \cdot A)_u = 0$

