

Magnetic topological insulators as a laboratory for nonstandard axion electrodynamics

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Outline

Quick recap on axion electrodynamics

- Phenomenology
- Realization through Dirac fermions

Nonstandard axion electrodynamics

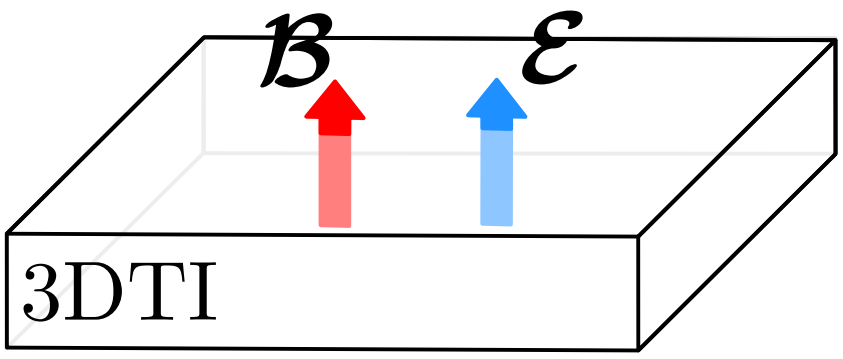
- Dirac fermions and P, T breaking
- Field-induced magnetoelectric coupling

Condensed matter platforms

- ME response of 3D topological insulators
- E and B induced axion-like coupling
- Outlook: dynamical axion mode

Engineering EM response

MH, Liang, Kotetes, arXiv:2610.25452



$\mathcal{T}$ TRS-breaking: magnetic field or magnetization

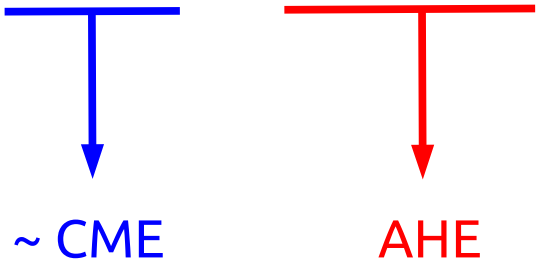
$\mathcal{P}$ parity breaking: electric field or gating

What electromagnetic responses can we engineer using Dirac electrons?

Axion electrodynamics

$$S_{\mathrm{AED}} = \int \mathrm{d}^4x \left(\frac{\varepsilon}{2}\boldsymbol{E}^2 - \frac{1}{2\mu}\boldsymbol{B}^2 + \frac{e^2 C}{2\pi h}\theta \boldsymbol{E}\cdot\boldsymbol{B} \right)$$

$$\boldsymbol{J} = \frac{e^2 C}{2\pi h}\left((\partial_t \theta)\boldsymbol{B} + (\nabla\theta)\times\boldsymbol{E} \right)$$



Realizing AED

$$H = \psi^\dagger \mathcal{H} \psi = \psi^\dagger \left(\gamma^0 \boldsymbol{\gamma} \cdot \boldsymbol{p} - \gamma^5 \partial_t \theta + \boldsymbol{\gamma} \gamma^5 \cdot \nabla \theta \right) \psi$$

$$S_{\mathrm{AED}} = \int \mathrm{d}^4 x \left(\frac{\varepsilon}{2} \boldsymbol{E}^2 - \frac{1}{2\mu} \boldsymbol{B}^2 + \frac{e^2 C_A}{2\pi h} \theta \boldsymbol{E} \cdot \boldsymbol{B} \right)$$

Realizing AED

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$\frac{1}{2\pi}$

pseudoscalar

Realizing AED

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$\frac{1}{2\pi}$

pseudoscalar

$$\partial^\mu \theta = A_5^\mu$$

axial imbalance: $A_{5,0} \sim \mu_5$

rotation: $\boldsymbol{A}_5 \sim \boldsymbol{\omega}$

dynamical axion field

$$\boldsymbol{J} = \frac{e^2}{2\pi h} C_A \left(A_{5,0} \boldsymbol{B} + \boldsymbol{A}_5 \times \boldsymbol{E} \right)$$

Effect of fermion mass

$$H = \psi^\dagger \mathcal{H} \psi = \psi^\dagger \left(\gamma^0 \boldsymbol{\gamma} \cdot \boldsymbol{p} + \gamma^0 m - \gamma^5 \partial_t \theta + \boldsymbol{\gamma} \gamma^5 \cdot \nabla \theta \right) \psi$$

$$S_{\mathrm{AED}} = \cdots + \frac{e^2}{2\pi h} \int \mathrm{d}^4 x \, \left(C_A \theta \boldsymbol{E} \cdot \boldsymbol{B} + C_V \partial_t \theta \nabla \theta \cdot (\boldsymbol{E} \times \boldsymbol{B}) \right)$$

pseudoscalar

polar vector

$\sim \frac{1}{m^2}$

C_A : suppressed for $m > 0$
(weak fields, adiabatic limit)

C_V : zero for $m = 0$

Nonstandard axion electrodynamics

particle physics

axion



condensed matter

ME-coupling

Nonstandard axion electrodynamics

particle physics  **condensed matter**

axion — ME-coupling

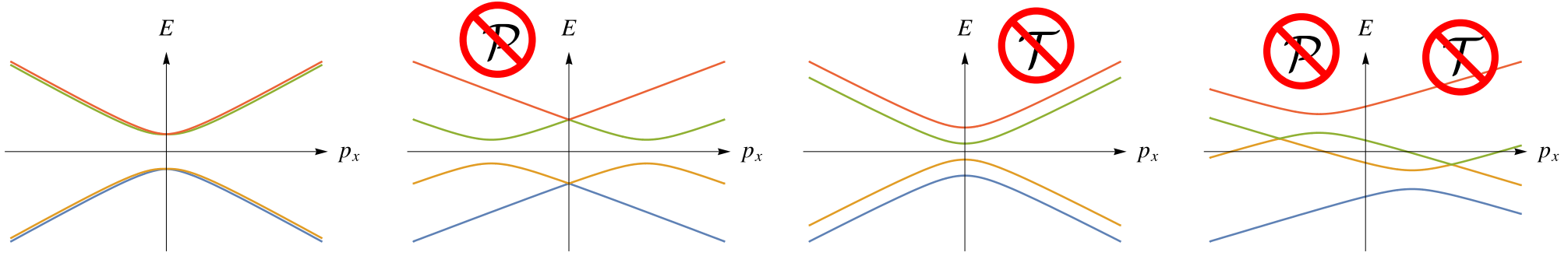
$$S_{\mathrm{AED}} = \int \mathrm{d}^4x \left(\frac{1}{2}\boldsymbol{E}\cdot(\check{\boldsymbol{\varepsilon}}\boldsymbol{E}) - \frac{1}{2}\boldsymbol{B}\cdot(\check{\boldsymbol{\mu}}^{-1}\boldsymbol{B}) + \frac{e^2}{2\pi h}\boldsymbol{E}\cdot(\check{\boldsymbol{\Theta}}\boldsymbol{B}) \right)$$

$$\boldsymbol{J} = \frac{e^2}{2\pi h}\left(\partial_t(\check{\boldsymbol{\Theta}}\boldsymbol{B}) + \nabla\times(\check{\boldsymbol{\Theta}}^{\top}\boldsymbol{E})\right)$$

Symmetry-breaking Dirac

$$\mathcal{H} = \gamma^0 \left(v_F \boldsymbol{\gamma} \cdot \boldsymbol{p} + m + i\gamma^5 m_5 + \mathcal{F}_{\mu\nu} \frac{i}{2} [\gamma^\mu, \gamma^\nu] \right)$$

$$\mathcal{F}^{0i} \sim \mathcal{E}^i \qquad \mathcal{F}^{ij} \sim \epsilon^{ijk} \mathcal{B}^k \qquad m_5 \sim \boldsymbol{\mathcal{E}} \cdot \boldsymbol{\mathcal{B}}$$



Symmetry-breaking Dirac

$$\mathcal{H} = \gamma^0 \left(\upsilon_F \boldsymbol{\gamma} \cdot \boldsymbol{p} + m + i\gamma^5 m_5 + \mathcal{F}_{\mu\nu} \frac{i}{2} [\gamma^\mu, \gamma^\nu] \right)$$

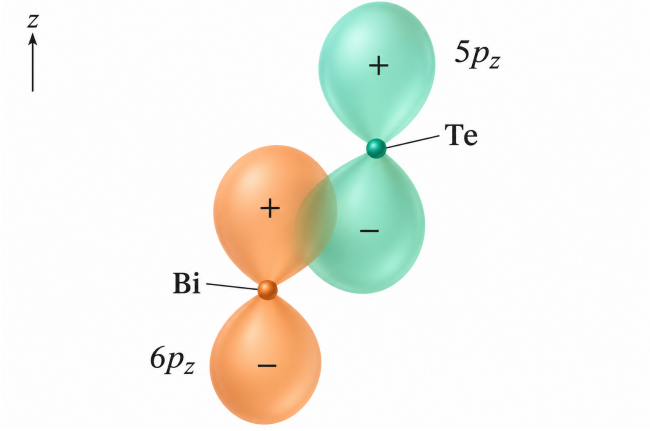
$$\mathcal{F}^{0i} \sim \mathcal{E}^i \qquad \mathcal{F}^{ij} \sim \epsilon^{ijk} \mathcal{B}^k \qquad m_5 \sim \boldsymbol{\mathcal{E}} \cdot \boldsymbol{\mathcal{B}}$$

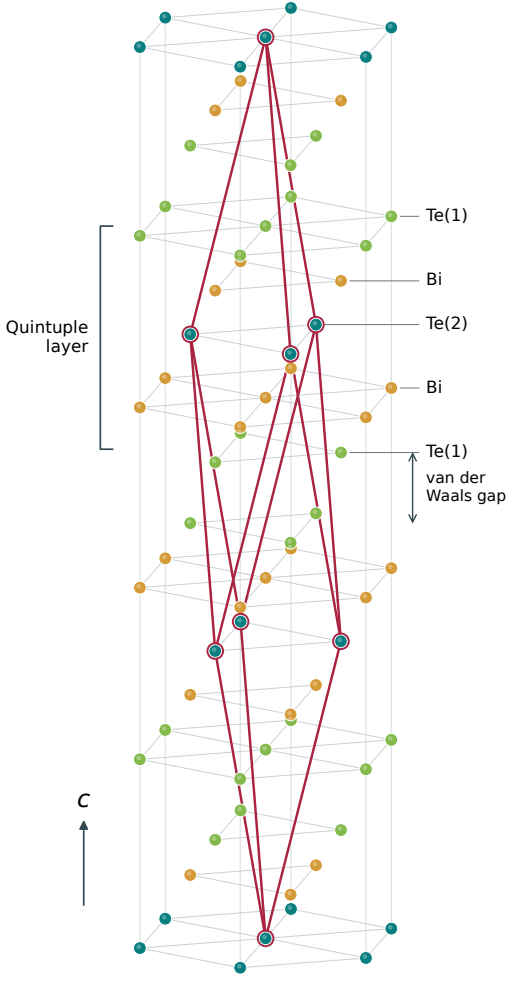
$$\check{\boldsymbol{\Theta}} = \vartheta_s \frac{\boldsymbol{\mathcal{E}} \circ \boldsymbol{\mathcal{B}} + \boldsymbol{\mathcal{B}} \circ \boldsymbol{\mathcal{E}}}{2} + \vartheta_a \frac{\boldsymbol{\mathcal{E}} \circ \boldsymbol{\mathcal{B}} - \boldsymbol{\mathcal{B}} \circ \boldsymbol{\mathcal{E}}}{2} + \vartheta_d \boldsymbol{\mathcal{E}} \cdot \boldsymbol{\mathcal{B}} \, \mathbb{1}$$

Platforms

- 3DTIs: versatile semiconducting materials; Examples: Bi_2Te_3, Bi_2Se_3, Sb_2Te_3

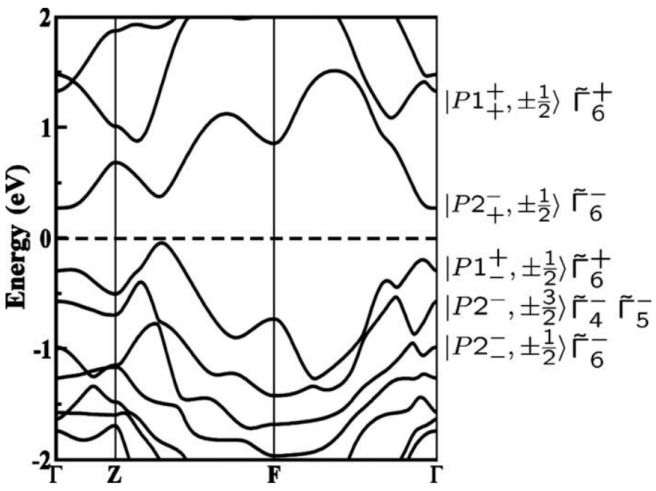
- Harboring low-lying Dirac electron bands





Platforms

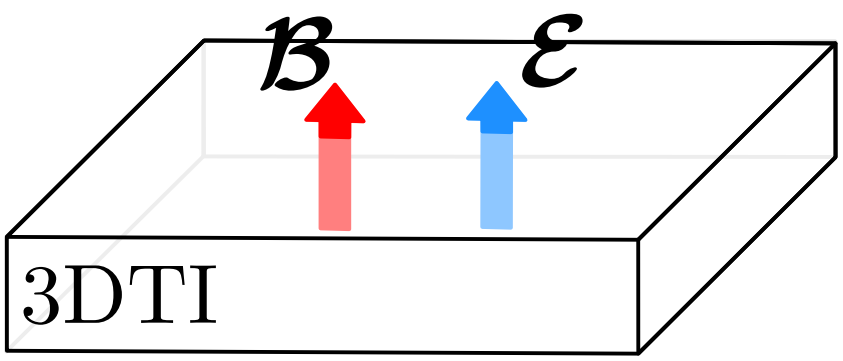
- 3DTIs: versatile semiconducting materials;
  Examples: Bi_2Te_3, Bi_2Se_3, Sb_2Te_3
- Harboring low-lying Dirac electron bands
- strong SOC & Zeeman coupling to magnetic field
- Magnetization by doping or proximity



Liu *et al.*, PRB **82**, 045122 (2010)

$$\mathcal{H}_{\mathrm{TI}}(\boldsymbol{p}) = v_\perp\left(-p_x\tau_x\sigma_y + p_y\tau_x\sigma_x\right) + v_z p_z\tau_y + \Delta(\boldsymbol{p})\tau_z - \frac{\mu_{\mathrm{B}}}{2}\boldsymbol{\mathcal{M}}\cdot\left(\check{\boldsymbol{g}}^+\boldsymbol{\sigma} + \tau_z\check{\boldsymbol{g}}^-\boldsymbol{\sigma}\right)$$

Platforms





TRS-breaking: magnetic field or magnetization

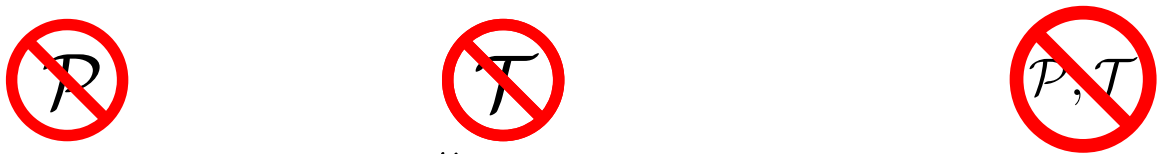


parity breaking: electric field or gating

Predictions

MH, Liang, Kotetes, arXiv:2610.25452

- First: effective Hamiltonian of TI with static E and B



$$\mathcal{H}(\boldsymbol{p}) = v_\perp \boldsymbol{p}\cdot\boldsymbol{\alpha} + \Delta(\boldsymbol{p})\tau_z + d_\perp \boldsymbol{\mathcal{E}}\cdot\boldsymbol{\beta} + m_{\mathcal{O}}\boldsymbol{\mathcal{B}}\cdot\boldsymbol{\gamma} - \frac{\mu_{\mathrm{B}}}{2}\boldsymbol{\mathcal{B}}\cdot\left(\check{\boldsymbol{g}}^{+}\boldsymbol{\sigma} + \tau_z\check{\boldsymbol{g}}^{-}\boldsymbol{\sigma}\right) + \boldsymbol{\mathcal{E}}\cdot(\check{\mathcal{X}}\boldsymbol{\mathcal{B}})\hat{\Gamma}$$

- Second: linear response for ME coefficients

$$\frac{e^2}{2\pi h}\Theta^{ij}_{\mathcal{O}} = \frac{i}{4}\varepsilon^{jk\ell}\partial_\omega\partial^k_{\boldsymbol{q}}\Pi^{i\ell}_{JJ}(\omega=0,\boldsymbol{q}=\boldsymbol{0})$$

$$\frac{e^2}{2\pi h}\Theta^{ij}_{\mathcal{S}} = \frac{i}{2}\partial_\omega\Pi^{ij}_{JS}(\omega=0,\boldsymbol{q}=\boldsymbol{0})$$

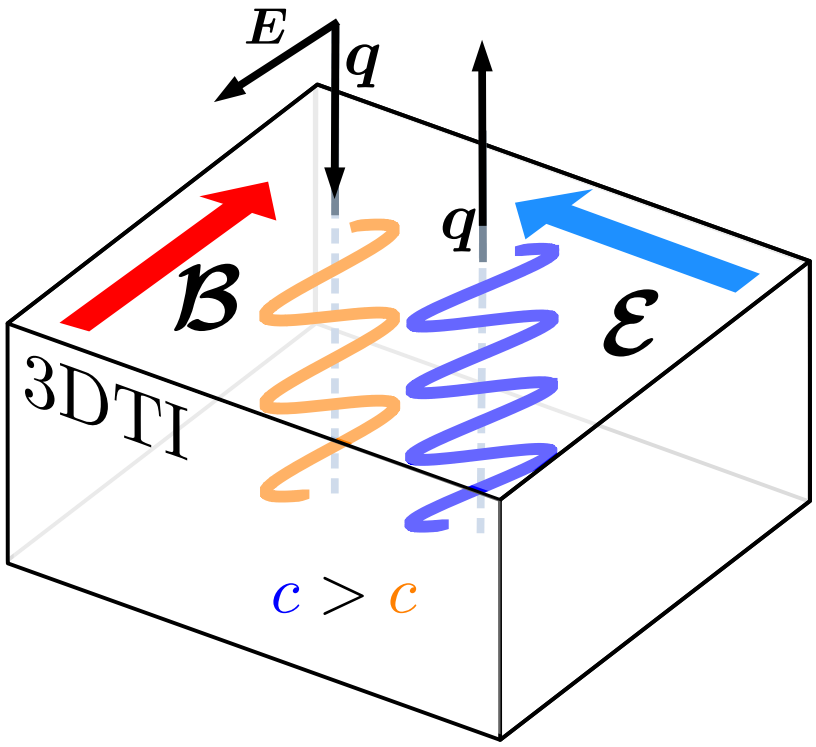
$$\Pi^{ij}_{JJ}(\omega,\boldsymbol{q}) = \left\langle \hat{J}^i\hat{J}^j\right\rangle$$

$$\Pi^{ij}_{JS} = \left\langle \hat{J}^i\hat{S}^j\right\rangle$$

$$\boldsymbol{S} = -(\mu_{\mathrm{B}}/2)(\check{\boldsymbol{g}}^{+} + \check{\boldsymbol{g}}^{-}\tau_z)\boldsymbol{\sigma}$$

Optical signatures

MH, Liang, Kotetes, arXiv:2610.25452



Maxwell's in medium

$$\nabla \cdot (\varepsilon \boldsymbol{E} + \boldsymbol{P}) = \nabla \cdot \boldsymbol{D} = 0$$

$$\nabla \times (\boldsymbol{B}/\mu - \boldsymbol{M}) = \nabla \times \boldsymbol{H} = \partial_t \boldsymbol{D}$$

Current induced by ME

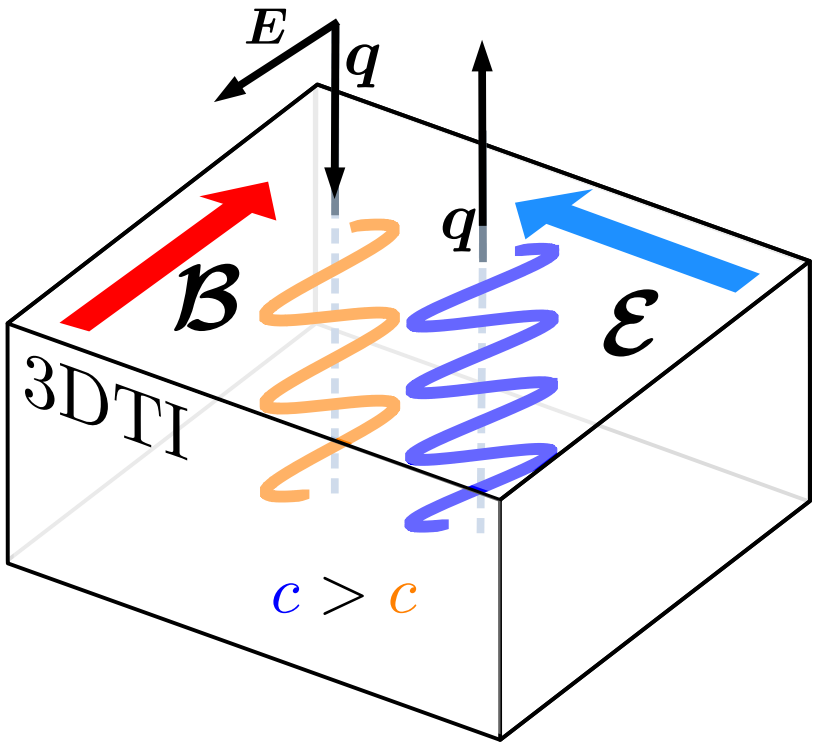
$$\boldsymbol{J} = \partial_t \boldsymbol{P} + \nabla \times \boldsymbol{M}$$

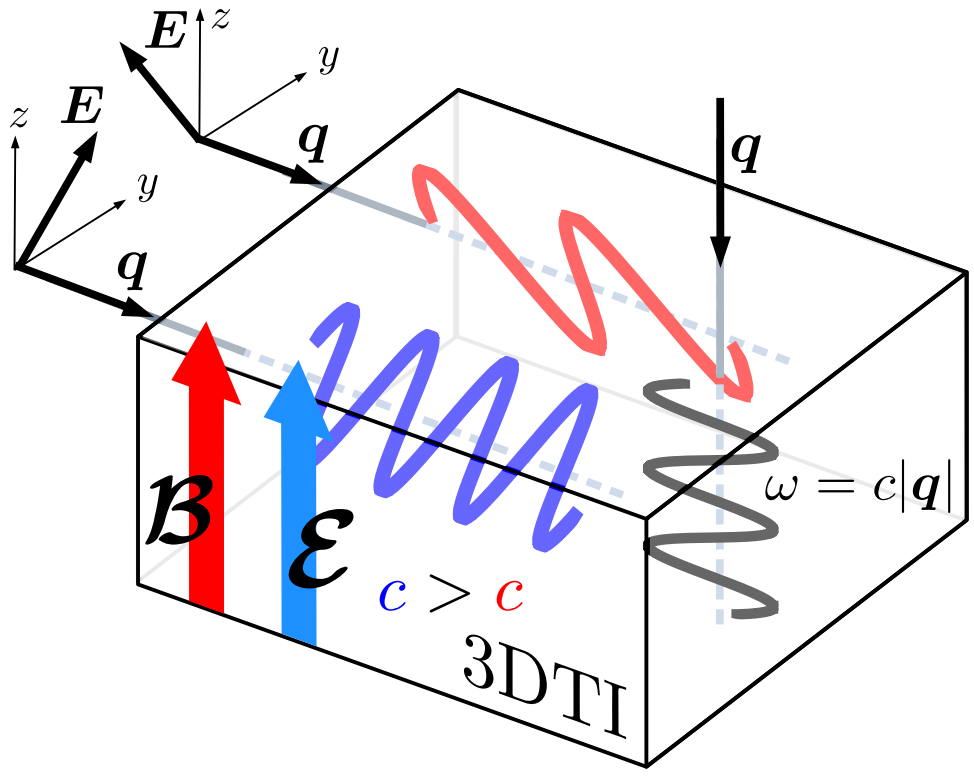
$$\boldsymbol{P} = \frac{e^2}{2\pi h} \check{\boldsymbol{\Theta}} \boldsymbol{B} \qquad \boldsymbol{M} = \frac{e^2}{2\pi h} \check{\boldsymbol{\Theta}}^\top \boldsymbol{E}$$

$$\frac{c_\pm}{c} = \sqrt{1 + \eta_\Theta^2} \pm \mathrm{sgn}(q)\eta_\Theta \qquad \eta_\Theta \sim |\boldsymbol{\mathcal{E}}||\boldsymbol{\mathcal{B}}|$$

Nonreciprocal birefringence

MH, Liang, Kotetes, arXiv:2610.25452

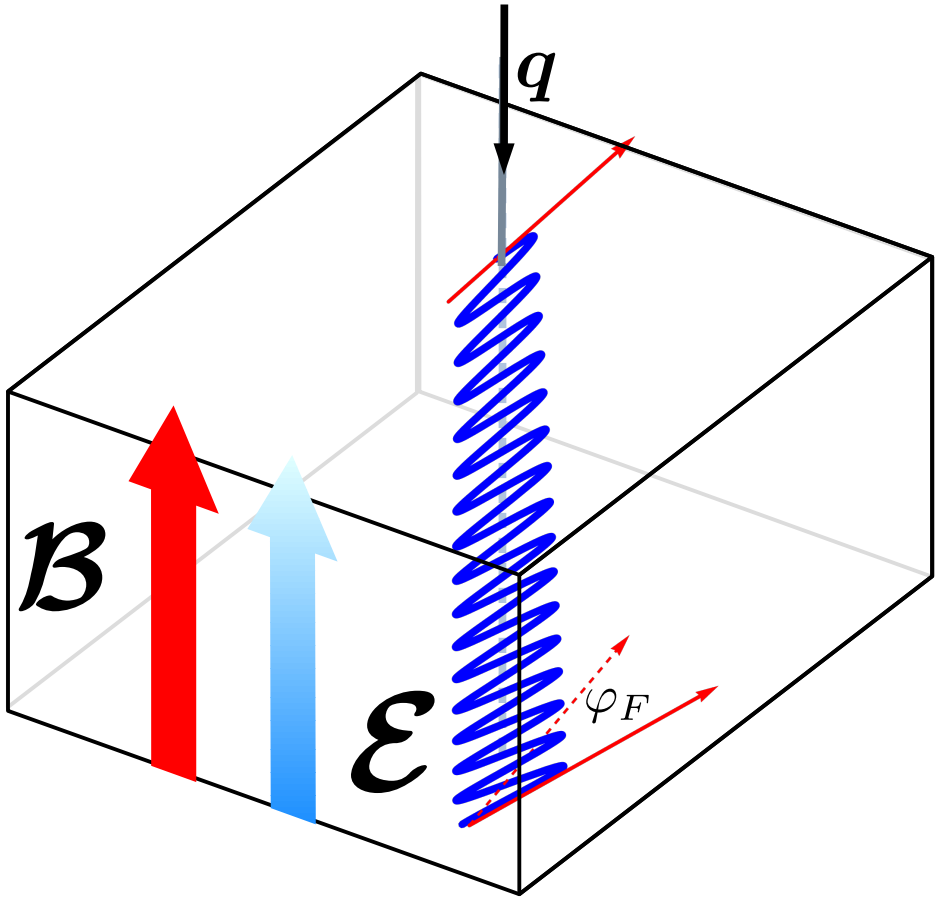




$$\frac{c_\pm}{c} = \sqrt{1+\eta_\Theta^2} \pm \mathrm{sgn}(q)\eta_\Theta \qquad \eta_\Theta = \frac{e^2}{2\pi h\varepsilon}\frac{\mu_\mathrm{B} e\hbar v_\perp |\boldsymbol{\mathcal{E}}||\boldsymbol{\mathcal{B}}|}{\Delta_0^3}\Delta\vartheta$$

Faraday rotation

MH, Kotetes (in preparation)



Activated ME coupling:

$$\Theta_{zz} \sim \mathcal{E}_z \mathcal{B}_z$$

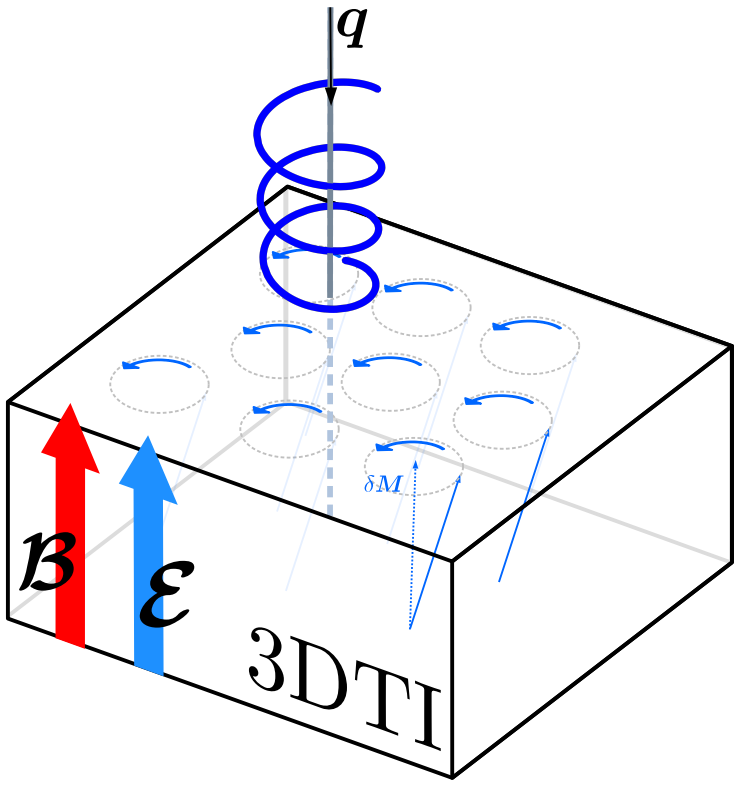
$$\frac{\mathrm{d}\varphi_F}{\mathrm{d}z} = \frac{\sqrt{\omega(\omega+\omega_0)} - \sqrt{\omega(\omega-\omega_0)}}{2c}$$

$$\approx \frac{e^2}{4\pi h c \varepsilon} \frac{\mu_{\mathrm{B}} e \hbar v_\perp |\partial_z \mathcal{E}_z \mathcal{B}_z|}{\Delta_0^3} \vartheta = \frac{\omega_0}{2c}$$

$\omega \gg \omega_0$

Outlook: dynamical axion

- Magnetization fluctuation allow axion mode to emerge in magnetic TIs
- Non-reciprocal circular dichroism
- Generation of higher harmonics
- Both controllable through magnon frequency



Summary

- **Axion electrodynamics in the context of magnetoelectricity**

  → coupling may have a topological part (e.g. for massless fermions)

  → symmetry breaking can induce it as well: way of control

- **Condensed matter platforms for 3D Dirac fermions**

  → controllable systems like WSMs, DSM & TIs

  → engineering “axion background”

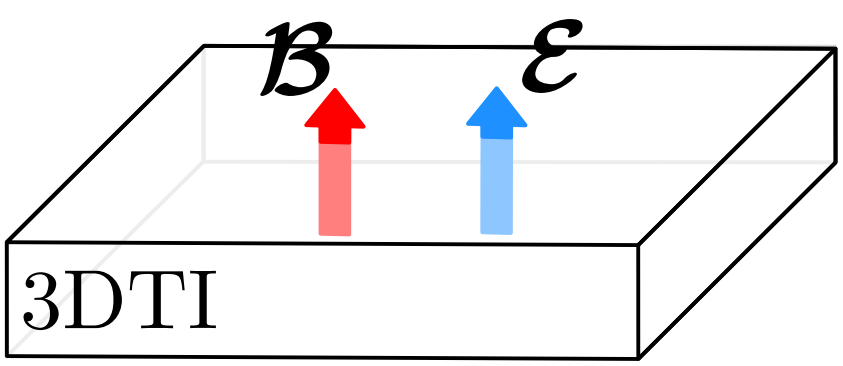
  → magnetic fluctuations can animate an axion mode

Thank you for listening!

Questions? Comments? 谢谢大家!

Backups

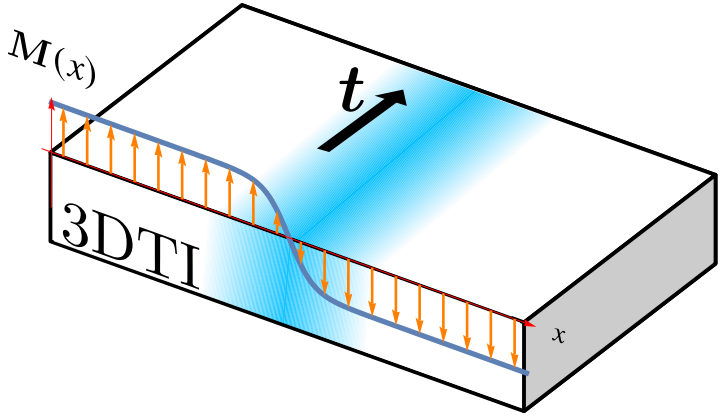
Platforms



$\mathcal{T}$ TRS-breaking: magnetic field or magnetization

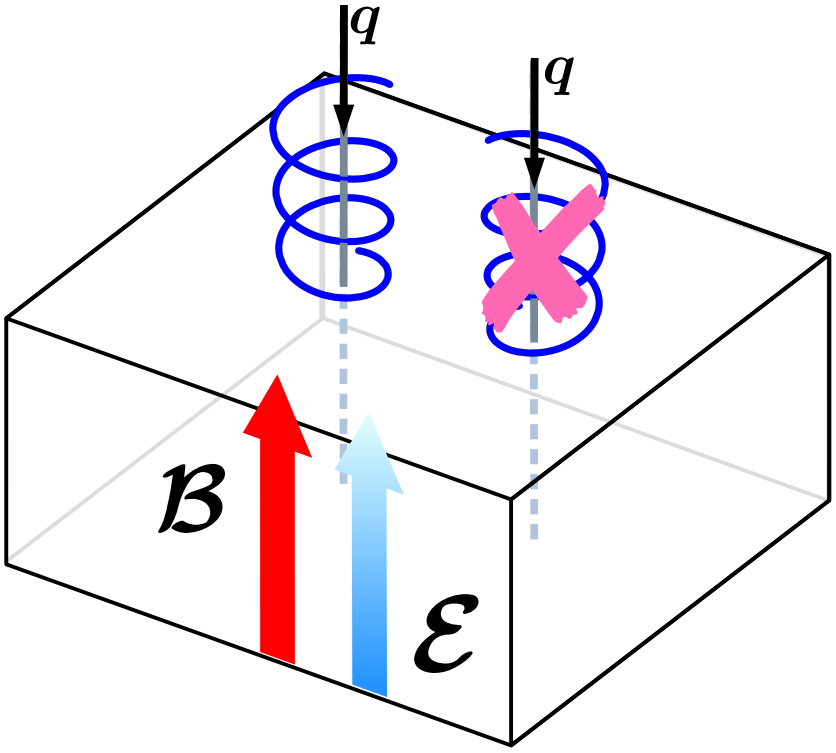
$\mathcal{P}$ parity breaking: electric field or gating

$\mathcal{P},\mathcal{T}$ Magnetization gradient (e.g. domain wall): can break both TRS and inversion



Circular dichroism

MH, Kotetes (in preparation)



$$\omega < \omega_0$$

$$\omega_0 = \frac{e^2}{2\pi h\varepsilon}\frac{\mu_{\mathrm{B}} e\hbar v_\perp |\partial_z \mathcal{E}_z \mathcal{B}_z|}{\Delta_0^3}\vartheta$$

Predictions

MH, Liang, Kotetes, arXiv:2610.25452

- ME coefficients induced by both spin and orbital effects

	Bi_2Te_3		Bi_2Se_3		Sb_2Te_3	
	coeff.	$\mathcal{S}/\mathcal{O}[\%/\%]$	coeff.	$\mathcal{S}/\mathcal{O}[\%/\%]$	coeff.	$\mathcal{S}/\mathcal{O}[\%/\%]$
$\vartheta_{\perp\perp}$	1.80	$\frac{4.56\%}{95.44\%}$	0.08	$\frac{40.18\%}{59.82\%}$	1.74	$\frac{20.94\%}{79.06\%}$
$\vartheta'_{\perp\perp}$	−5.53	$\frac{5.32\%}{94.68\%}$	1.71	$\frac{82.03\%}{17.97\%}$	−3.83	$\frac{14.90\%}{85.10\%}$
ϑ_{zz}	−3.90	$\frac{37.68\%}{62.32\%}$	15.50	$\frac{94.80\%}{5.20\%}$	5.84	$\frac{99.67\%}{0.33\%}$
$\vartheta_{\perp z}$	169.46	$\frac{6.50\%}{93.50\%}$	−1.14	$\frac{76.70\%}{23.30\%}$	−3.14	$\frac{97.03\%}{2.97\%}$
$\vartheta'_{\perp z}$	0.08	$\frac{2.99\%}{97.01\%}$	0.06	$\frac{25.26\%}{74.74\%}$	0.47	$\frac{11.36\%}{88.64\%}$
$\vartheta''_{\perp z}$	−6.44	$\frac{0.32\%}{99.68\%}$	3.88	$\frac{94.94\%}{5.06\%}$	−2.40	$\frac{88.11\%}{11.89\%}$

$$\frac{\mathcal{L}_\Theta}{\frac{e^2}{2\pi h}\cdot g_\Theta} = \vartheta_{\perp\perp}\boldsymbol{E}_\perp\cdot\boldsymbol{\mathcal{E}}_\perp\boldsymbol{B}_\perp\cdot\boldsymbol{\mathcal{B}}_\perp+\vartheta'_{\perp\perp}\big(\boldsymbol{E}_\perp\cdot\boldsymbol{\mathcal{B}}_\perp\boldsymbol{\mathcal{E}}_\perp\cdot\boldsymbol{B}_\perp+\boldsymbol{\mathcal{E}}_\perp\cdot\boldsymbol{\mathcal{B}}_\perp\boldsymbol{E}_\perp\cdot\boldsymbol{B}_\perp\big)+\vartheta_{zz}\mathcal{E}_z\mathcal{B}_zE_zB_z+\vartheta_{\perp z}\boldsymbol{\mathcal{E}}_\perp\cdot\boldsymbol{E}_\perp\mathcal{B}_zB_z$$
$$+\ \vartheta'_{\perp z}\boldsymbol{\mathcal{B}}_\perp\cdot\boldsymbol{B}_\perp\mathcal{E}_zE_z+\vartheta''_{\perp z}\big(\boldsymbol{\mathcal{E}}_\perp\cdot\boldsymbol{B}_\perp\mathcal{B}_zE_z+\boldsymbol{\mathcal{B}}_\perp\cdot\boldsymbol{E}_\perp\mathcal{E}_zB_z+\boldsymbol{E}_\perp\cdot\boldsymbol{B}_\perp\mathcal{E}_z\mathcal{B}_z+\boldsymbol{\mathcal{E}}_\perp\cdot\boldsymbol{\mathcal{B}}_\perp E_zB_z\big)$$

Order of magnitude

MH, Liang, Kotetes, arXiv:2610.25452

$$c_\pm \approx \left(1 \pm \frac{e^2}{2\pi h c\varepsilon}\frac{\mu_B|\boldsymbol{\mathcal{B}}|}{\Delta_0}\frac{e\hbar v_\perp|\boldsymbol{\mathcal{E}}|}{\Delta_0^2}\vartheta\right)c \sim 0.1-10$$

$\Delta_0 \sim 0.1\text{eV}$ ~ 0.01 $\sim 10^{-4}|\boldsymbol{\mathcal{B}}|[\text{T}]$ $\sim 10^{-2}|\boldsymbol{\mathcal{E}}|[\text{V}]/[\mu\text{m}]$

$$\frac{\Delta c}{c} \sim 10^{-9} - 10^{-7} \qquad \frac{\omega_0}{c}L \sim 10^{-6} - 10^{-2}\ \text{rad}$$

Birefringence setup

Roth & Rikken, PRL **88**, 063001 (2002)

