Spin hydrodynamics and an extension of the Bargmann-Michel-Telegdi equation

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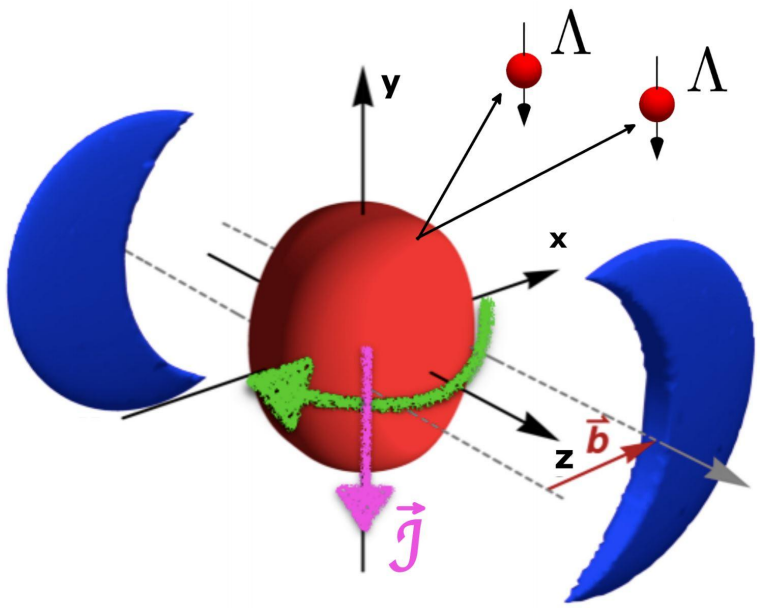
The 10th International Conference on Chirality, Vorticity and Magnetic Fields in Quantum Matter
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Outline

- Introduction
- Classical equations for spin
- Relativistic spin hydrodynamics
- Summary

Introduction

Rotation and polarization



Picture from Florkowski, Kumar and Ryblewski, PPNP (2019)

- **Large angular momentum in non-central heavy ion collisions**

$$J \sim \frac{A\sqrt{s}}{2} b \sim 10^5 \hbar$$

Vorticity of quark gluon plasma

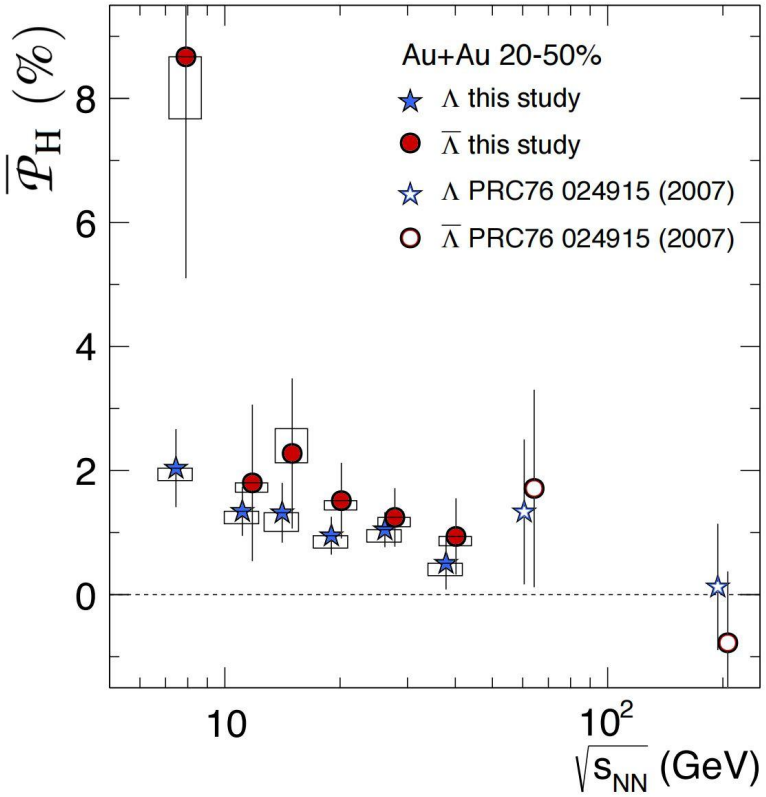
⇩ (Total angular momentum conservation)

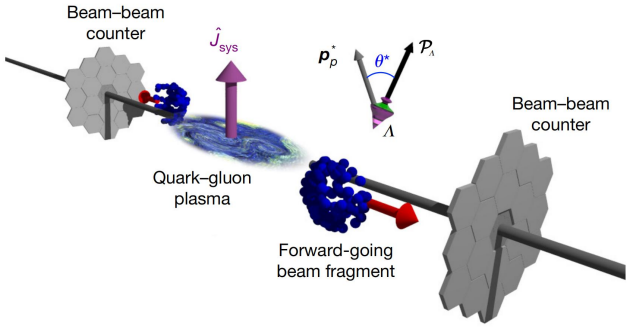
Polarization of particles along vorticity

Liang and Wang, PRL (2005); PLB (2005)
Gao, Chen, Deng, Liang, Wang and Wang, PRC (2008)

Most vortical fluid and spin polarization







- **Weak decay:** $\Lambda \to p + \pi^-$
- **Proton distribution:**

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}(1 + \alpha_H |\vec{\mathcal{P}}_H| \cos\theta^*)$$

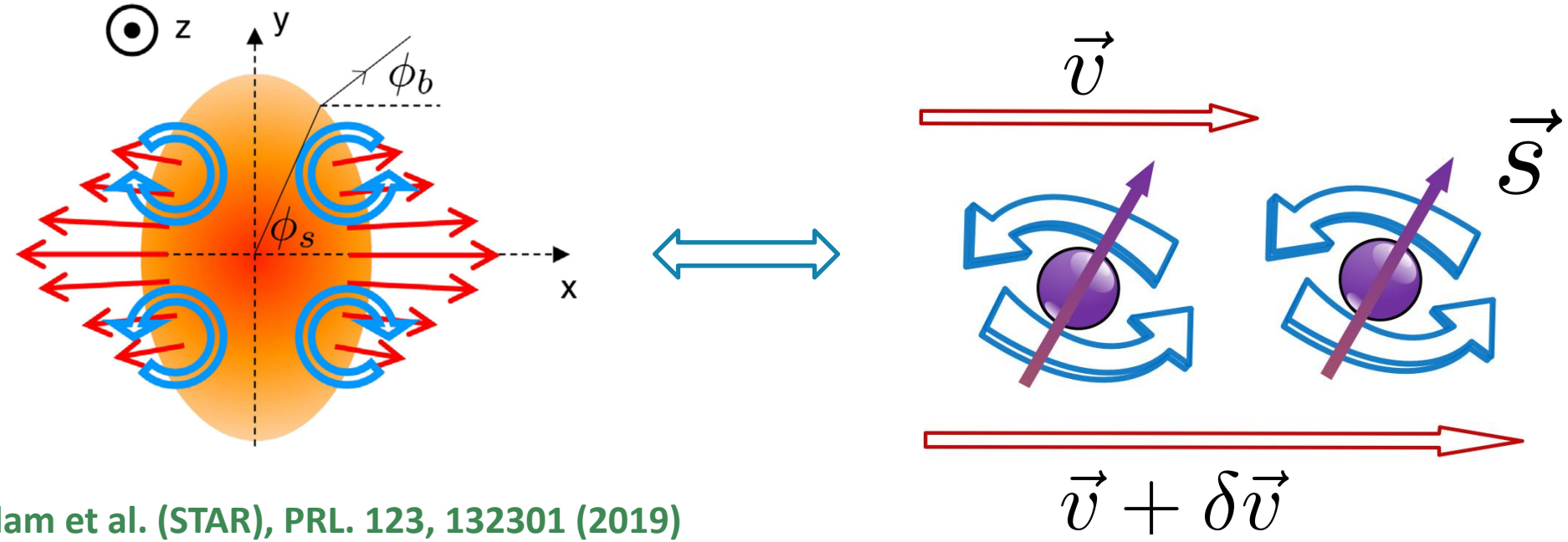
- **Large vorticity:**

$$\omega \approx (9 \pm 1) \times 10^{21} \mathrm{s}^{-1}$$

Most vortical fluid!

Adamczyk et al. (STAR), Nature 548, 62 (2017)

Spin-orbit angular momentum transfer



Adam et al. (STAR), PRL. 123, 132301 (2019)

How to describe the spin evolution?

Classical Equations for Spin

Bargmann-Michel-Telegdi (BMT) equation

- **The relativistic spin evolution of a charged particle in an electromagnetic field:**

$$s^\mu u_\mu = 0 \quad \dot{s}^\mu = \frac{ds^\mu}{d\tau}$$
$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Thomas precession
(caused by relativistic acceleration)

$$\dot{s}^\mu = \boxed{-u^\mu s^\nu \dot{u}_\nu} + \boxed{\gamma \Delta^{\mu\nu} F_{\nu\lambda} s^\lambda} + \text{damping}$$

Larmor precession
(caused by external electromagnetic fields)

Bargmann, Michel and Telegdi (1959)

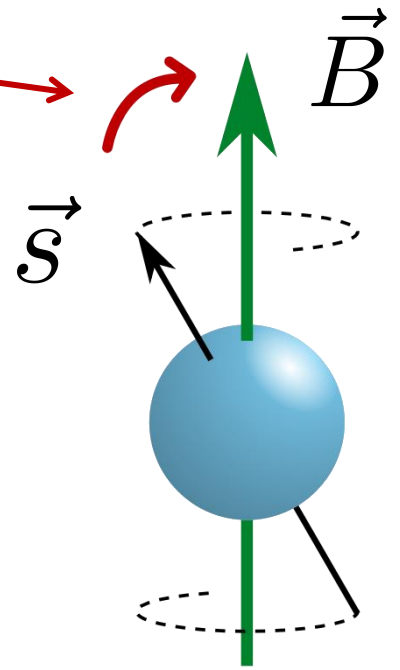
Landau-Lifshitz-Gilbert (LLG) equation

- **In non-relativistic limit, the Thomas precession term vanishes, and the spin evolution is governed by**

Precession **Damping**

$$\frac{d}{dt}\vec{s} = -\gamma\vec{B}\times\vec{s} + \lambda\gamma\vec{s}\times(\vec{s}\times\vec{B})$$

$$\approx -\gamma\vec{B}\times\vec{s} + \lambda\vec{s}\times\frac{d}{dt}\vec{s}$$



- **Finally, spin is parallel to the magnetic field.**

Landau and Lifshitz (1935); Gilbert (1955)

What is the equation for spin in fluids?

- **We expect the corresponding equation to include terms for Thomas precession, spin-vorticity precession, ...**

$$\dot{s}^{\mu} = \text{Thomas precession} + \text{spin-vorticity} + \text{spin-dissipation} + \ldots$$

$\Rightarrow$ **similar to** $\vec{B} \times \vec{s}$

Let's see if this guess is correct...

Relativistic Spin Hydrodynamics

Canonical spin hydrodynamics

- **Total angular momentum is decomposed into the orbital part and spin part**

spin current

$$J^{\lambda\mu\nu} = x^{\mu}\Theta^{\lambda\nu} - x^{\nu}\Theta^{\lambda\mu} + \boxed{\Sigma^{\lambda\mu\nu}}$$

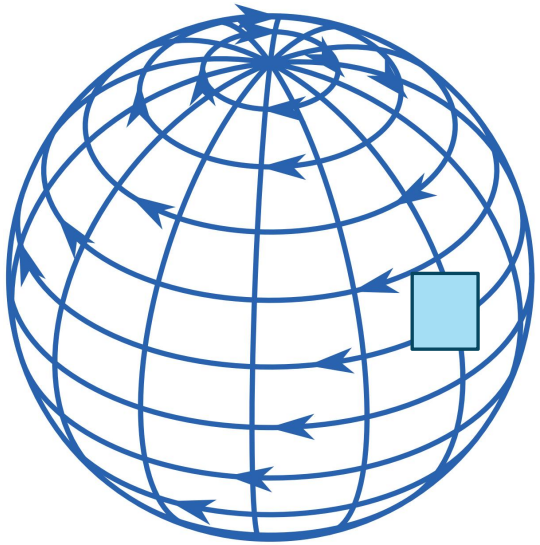
- **Conserved laws give the spin evolution equation**

$$\partial_\lambda J^{\lambda\mu\nu} = \partial_\mu \Theta^{\mu\nu} = 0 \Longrightarrow \boxed{\partial_\lambda \Sigma^{\lambda\mu\nu} = -2\Theta^{[\mu\nu]}}$$

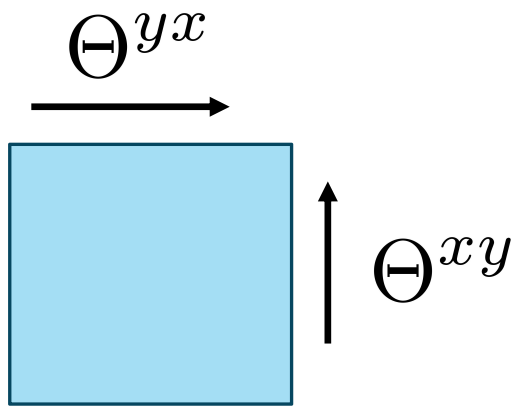
spin evolution equation

Becattini, PRL (2012)
Florkowski, Kumar and Ryblewski, PPNP (2019)
Hattori, Hongo, Huang, Matsuo and Taya, PLB (2019)

The role of the anti-symmetric part of EMT







$$\Theta^{xy} \neq \Theta^{yx}$$

There exists nonzero net torque acting on the fluid cell

$$\partial_\lambda \Sigma^{\lambda\mu\nu} = -2\Theta^{[\mu\nu]}$$

Thermodynamic relations

- **To include spin effects, the thermodynamic relations are assumed to be**

$$e + p = Ts + \mu n + \omega_{\mu\nu} S^{\mu\nu}$$

$$de = Tds + \mu dn + \omega_{\mu\nu} dS^{\mu\nu}$$

$$S^{\mu\nu} = u_\lambda \Sigma^{\lambda\mu\nu}$$

spin density: $S^{\mu\nu} \sim \mathcal{O}(1)$ **spin chemical potential:** $\omega_{\mu\nu} \sim \mathcal{O}(\partial)$

- **See also recent developments:**

Becattini, Daher, and Sheng (2024); Florkowski and Hontarenko (2025); Becattini and Singh (2025); Ambrus and Gecic (2026); Armas and Jain (2026); Palermo and Roselli (2026); Daher, Wagner, Shokri, and Rischke (2026).

Two different choices for the spin current

➢ **Choice-1**

$$\Sigma_1^{\lambda\mu\nu} = \frac{i}{4}\overline{\psi}\gamma^\lambda[\gamma^\mu,\gamma^\nu]\psi$$

1. **Non-Hermitian.**
2. **Anti-symmetric with respect to $\mu\nu$.**
3. **Spin tensor has 6 DoFs.**

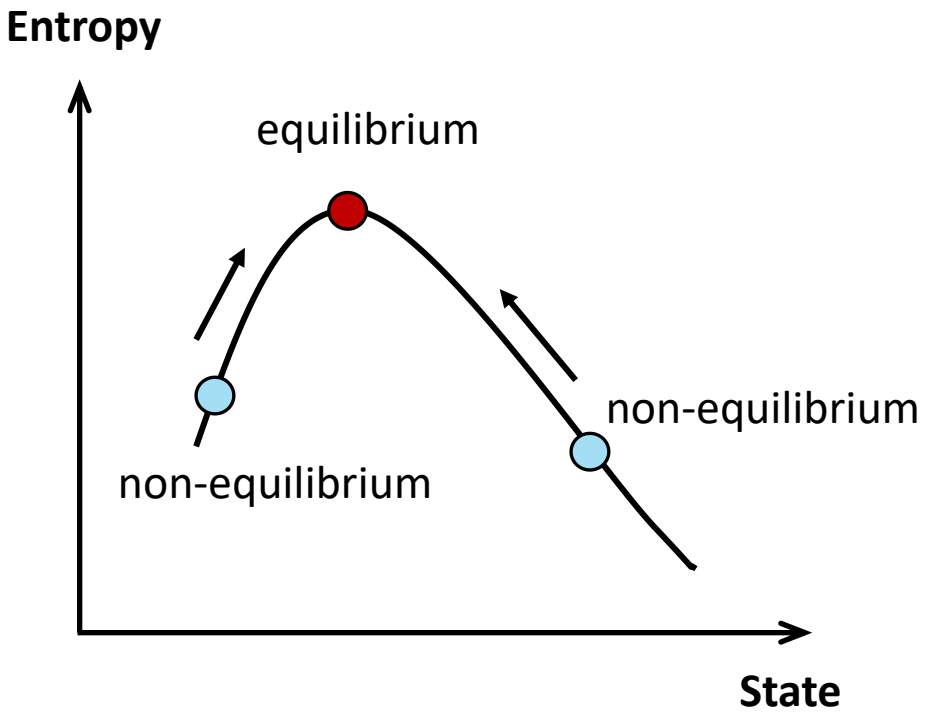
➢ **Choice-2**

$$\Sigma_2^{\lambda\mu\nu} = \frac{i}{8}\overline{\psi}\{\gamma^\lambda,[\gamma^\mu,\gamma^\nu]\}\psi$$
$$= \frac{1}{2}(\Sigma_1^{\lambda\mu\nu} + \Sigma_1^{\dagger\lambda\mu\nu})$$

1. **Hermitian.**
2. **Totally anti-symmetric.**
3. **Spin tensor has 3 DoFs.**

Becattini et al., Int.J.Mod.Phys.E 33 (2024)

Spin hydrodynamics based on Choice-1



$$\partial_\mu \mathcal{S}^\mu \geq 0$$

$$\Sigma_1^{\lambda\mu\nu} = u^\lambda S^{\mu\nu} + O(\partial)$$

$$\Theta^{[\mu\nu]} = 2q^{[\mu}u^{\nu]} + \phi^{\mu\nu}$$

$$q^\mu = \lambda(\beta\dot{u}^\mu - \Delta^{\mu\nu}\partial_\nu\beta - 4\beta\omega^{\mu\nu}u_\nu)$$

$$\phi^{\mu\nu} = \gamma(\Delta^{\mu\alpha}\Delta^{\nu\beta}\partial_{[\alpha}u_{\beta]} + 2\omega^{\mu\nu})$$

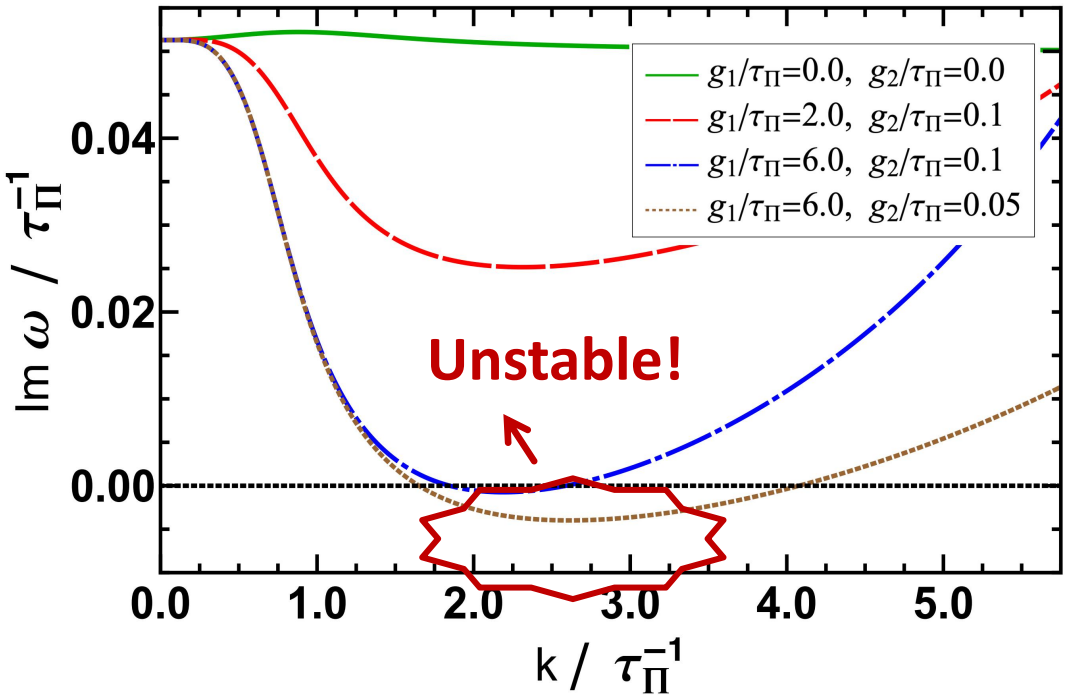
Hattori, Hongo, Huang, Matsuo and Taya, PLB (2019)

Two possible issues of Choice-1

1. **Stability conditions impose strong constraints on the equation of state, limiting the scope of application.**

Xie, DLW, Yang and Pu, PRD (2023)
Ren, Yang, DLW and Pu, PRD (2024)

See also:
Sarwar et al., PRD (2023)
Daher, Das and Ryblewski, PRD (2023)



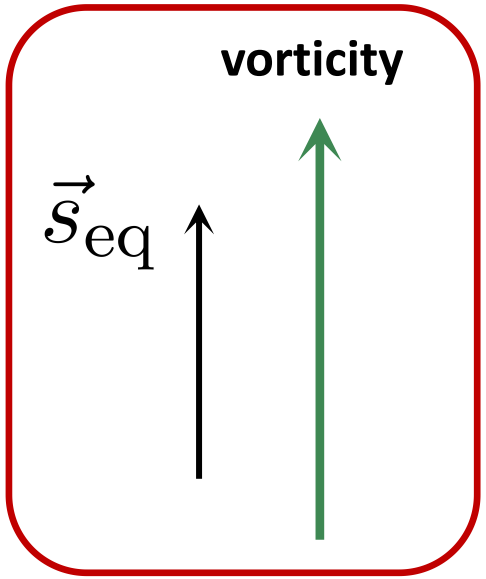
Two possible issues of Choice-1

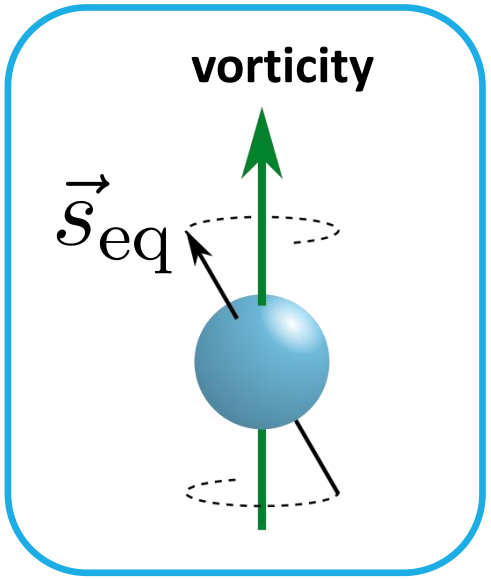
2. **In the equilibrium state, the spin vector should "look the same" when observed along the flow generated by the killing vector.**

$$\mathcal{L}_\beta s^\mu_{\rm eq} = 0$$

(Vanishing Lie derivative)

For dissipative systems





However, the spin hydrodynamics based on Choice-1 fails to satisfy the condition!

Choice-2: Totally anti-symmetric spin current

- **Adopting Choice-2, the spin current tensor is totally anti-symmetric and can be decomposed as:**

  Hongo, Huang, Kaminski, Stephanov and Yee, JHEP (2021)
  Cao, Hattori, Hongo, Huang and Taya, PRD (2022)

$$\Sigma_2^{\lambda\mu\nu} = u^\lambda S^{\mu\nu} + u^\mu S^{\nu\lambda} + u^\nu S^{\lambda\mu} + O(\partial)$$

- **Additional terms emerge in the entropy production rate:**

$$\partial_\mu \mathcal{S}^\mu = q^\mu(\partial_\mu\beta - \beta\dot{u}_\mu) + 2\beta\omega_{\mu\nu}S^{\lambda\mu}\partial_\lambda u^\nu + \ldots$$

$$q^\mu = -\frac{1}{2}u_\nu\partial_\lambda\Sigma^{\lambda\mu\nu}$$

It seems impossible to ensure $\partial_\mu \mathcal{S}^\mu \geq 0$ **?**

Constitutive relations with spin corrections

- **These additional terms can be absorbed into the terms related to heat flow h^μ, viscous tensor $\pi^{\mu\nu}$, and $\phi^{\mu\nu}$:**

$$\begin{aligned}\partial_\mu \mathcal{S}'^\mu &= (h^\mu - \mathcal{H}\nu^\mu + h_s^\mu)(\partial_\mu\beta + \beta\dot{u}_\mu) \\ &+\beta(\phi^{\mu\nu} + \phi_s^{\mu\nu})(\partial_{[\mu}u_{\nu]} + 2\omega_{\mu\nu}) \\ &+\beta(\pi^{\mu\nu} + \pi_s^{\mu\nu})\partial_{(\mu}u_{\nu)} + O(\partial^3)\end{aligned}$$

$$\mathcal{S}'^\mu = \mathcal{S}^\mu + S^{\mu\nu}\partial_\nu\beta$$

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Constitutive relations with spin corrections

- **These additional terms can be absorbed into the terms related to heat flow h^μ, viscous tensor $\pi^{\mu\nu}$, and $\phi^{\mu\nu}$:**

spin corrections

$$
\begin{aligned}
h^\mu - \mathcal{H}\nu^\mu + h_s^\mu &= -\sigma\Delta^{\mu\nu}(\partial_\nu\beta + \beta\dot{u}_\nu) \\
\pi^{\mu\nu} + \pi_s^{\mu\nu} &= \eta\partial^{<\mu}u^{\nu>} + \zeta(\partial\cdot u)\Delta^{\mu\nu} \\
\phi^{\mu\nu} + \phi_s^{\mu\nu} &= \gamma(\Delta^{\mu\alpha}\Delta^{\nu\beta}\partial_{[\alpha}u_{\beta]} + 2\omega^{\mu\nu})
\end{aligned}
$$

$$
\phi_s^{\mu\nu} = -2S^{[\mu\lambda}\omega^{\nu]}_{\ \lambda} - v_n^2(\partial\cdot u)S^{\mu\nu}
$$

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Extended BMT equation

- **Spin corrections modify the spin evolution equation:**

$\phi_s^{\mu\nu}$

$$\Delta^\mu_\rho \Delta^\nu_\sigma \partial_\lambda \Sigma_2^{\lambda\rho\sigma} = 2[v_n^2(\partial\cdot u)S^{\mu\nu} - 2\omega_\alpha^{[\nu} S^{\mu]\alpha}] + \text{original terms}$$

- **Extended BMT equation from the spin evolution equation:**

$$\dot{s}^\mu = -u^\mu s^\nu \dot{u}_\nu + (\epsilon^{\mu\nu\rho\sigma} s_\nu u_\rho - 2\gamma g^{\mu\sigma})(2\omega_\sigma - \mathfrak{m}_\sigma) - s_\nu \partial^{<\mu} u^{\nu>} - (2v_n^2 + 1/3) s^\mu (\partial\cdot u)$$

$$s^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} u_\nu S_{\rho\sigma}$$

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Extended BMT equation

➢ **Extended BMT equation**

Thomas precession

$$\dot{s}^\mu = \boxed{-u^\mu s^\nu \dot{u}_\nu} + (\epsilon^{\mu\nu\rho\sigma} s_\nu u_\rho - 2\gamma g^{\mu\sigma})(2\omega_\sigma - \mathfrak{m}_\sigma)$$
$$- s_\nu \partial^{<\mu} u^{\nu>} - (2v_n^2 + 1/3) s^\mu (\partial \cdot u)$$

➢ **Classical BMT equation:**

$$\dot{s}^\mu = \boxed{-u^\mu s^\nu \dot{u}_\nu} + \gamma \Delta^{\mu\nu} F_{\nu\lambda} s^\lambda + \text{damping}$$

Thomas precession

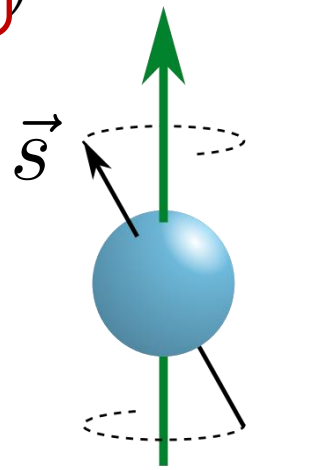
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Extended BMT equation

- **Extended BMT equation**

spin precession in hydro.

$$\dot{s}^\mu = -u^\mu s^\nu \dot{u}_\nu + (\epsilon^{\mu\nu\rho\sigma} s_\nu u_\rho - 2\gamma g^{\mu\sigma})(2\omega_\sigma - \mathfrak{m}_\sigma)$$
$$- s_\nu \partial^{<\mu} u^{\nu>} - (2v_n^2 + 1/3) s^\mu (\partial \cdot u)$$



precession

- **Classical BMT equation:**

$$\dot{s}^\mu = -u^\mu s^\nu \dot{u}_\nu + \gamma \Delta^{\mu\nu} F_{\nu\lambda} s^\lambda + \text{damping}$$

spin precession in EM fields

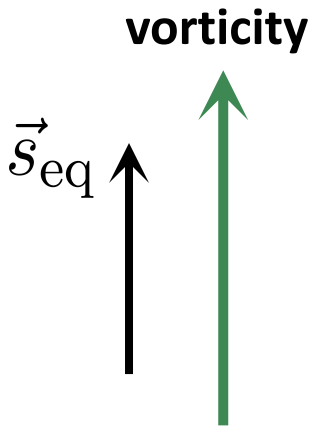
Phys. Rev. Lett. 136, 182301 (2026)

Extended BMT equation

➢ **Extended BMT equation in the equilibrium state**

$$\dot{s}^\mu = -u^\mu s^\nu \dot{u}_\nu + (\epsilon^{\mu\nu\rho\sigma} s_\nu u_\rho - 2\gamma g^{\mu\sigma})(2\omega_\sigma - \mathfrak{m}_\sigma)$$
$$- s_\nu \partial^{<\mu} u^{\nu>} - (2v_n^2 + 1/3) s^\mu (\partial \cdot u)$$

$$\dot{s}^\mu_{\rm eq} = -u^\mu s^\nu_{\rm eq} \dot{u}_\nu$$



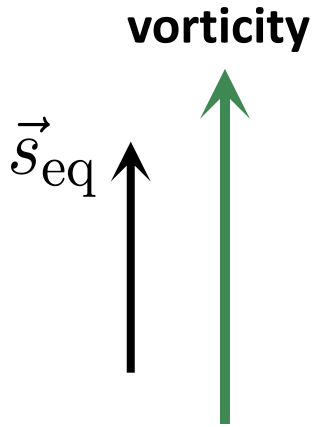
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Issues disappear

1. **Stable modes in linear mode analysis** $\chi_s = \delta\omega^{\mu\nu}/\delta S^{\mu\nu}$

$$\omega = \pm 4i\gamma\chi_s$$ **Choice-1** $\Longrightarrow$ $$\omega = -4i\gamma\chi_s$$ **Choice-2**

2. **As expected, the spin vector is parallel to the vorticity vector in equilibrium.**



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Summary

Summary

- **Spin hydrodynamics provides an effective tool for describing the spin evolution in quark-gluon-plasma.**
- **There are two different choices for the canonical spin current. Our results suggest that the totally anti-symmetric one is better.**
- **The extended BMT equation includes terms for Thomas precession and spin-hydrodynamic field couplings.**