speaker: **G. Yu. Prokhorov** [1,2]

in collaboration with:

R.V. Khakimov [1,2,3], **D. D. Lapygin** [4,1],
O. V. Teryaev [1,2,4], **V. I. Zakharov** [2,1]

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Causality, trace anomaly and entanglement viscosity in accelerated frame

based on works:

[GP, PRD letter 114 (2026) 4, L041703]

[GP, O. Teryaev, PRD letter 114 (2026) 8, L081701]

[D. Lapygin, GP, O. Teryaev, V. Zakharov, PRD 112 (2025) 6, 065012]

[R. Khakimov, GP, O. Teryaev, 2605.22409 (2026)]

Chirality 2026,
Zhuhai, China,
November 16-20, 2026

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Spoiler: main results

Introduction

1. **Entanglement viscosity**: *a) universal formulas*
   b) unitarity vs irreversibility
2. **Novel sum rule** from the **isotropy** of **radiation** in the **Unruh** effect: *direct verification*
3. **Shear viscosity** to **entropy density ratio**: *KSS-bound and causality*
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Preliminary results, conclusion and outlook

Spoiler: main results

- **Viscosity** from **acceleration**? Related with gravitational **conformal anomaly**.

$$T = \frac{a}{2\pi} \Rightarrow \frac{\eta}{s} = ? \Rightarrow \boxed{\frac{\eta}{s} = \frac{1}{4\pi c_s^2}} \Rightarrow \boxed{\eta = -8\lambda a^3 \quad \text{related to anomaly} \quad T^\mu_\mu = \lambda C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} + \dots}$$

Unruh temperature

"Entanglement viscosity"

Causality: $c_s^2 < 1 \Rightarrow \frac{\eta}{s} > \frac{1}{4\pi}$ famous **KSS-bound**

speed of sound

[Kovtun, Son, Starinets, PRL (2005)]
[Policastro, Son, Starinets. PRL (2001)]

- Condition of **isotropy** (**"Pascal law"**) of **Unruh radiation**

Novel sum rule for fundamental spectral functions $c^{(0)}(\mu)$ and $c^{(2)}(\mu)$.

-- Directly **confirmed** for **CFT** and **massive Dirac** field in any dimensions.

- **Similarity** between **thermal irreversibility** and **irreversibility** of **RG flows**.

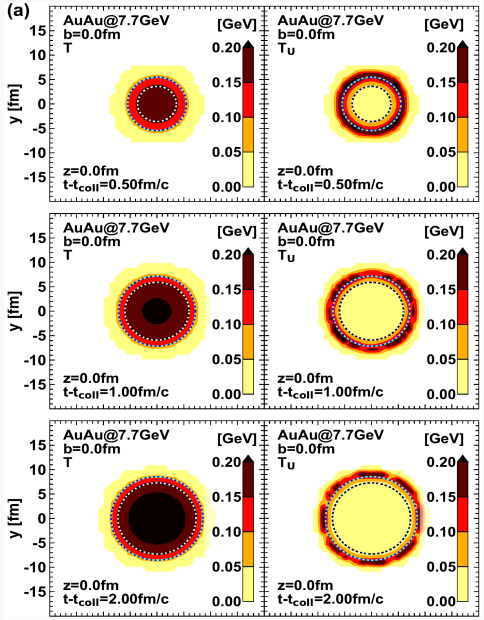
[A. B. Zamolodchikov. JETP Lett., 43:730–732, 1986]

Introduction

Motivation: acceleration

- Plenty of novel effects of **magnetic field** and **vorticity** (CME, CVE, **KVE**, vortical polarization...)
- **Acceleration** is not so well studied, see (in particular) talks of Xu-Guang Huang (16/11), Artem Roenko (18/11)

Phenomenological motivation:



Huge acceleration in heavy-ion collisions is expected:

$$a \sim 1\,GeV \sim 10^{32}\,m/sec^2$$

[D. Kharzeev and K. Tuchin, Nucl. Phys. A 753, 316 (2005)]

[I. Karpenko and F. Becattini, Nucl. Phys. A 982, 519 (2019), arXiv:1811.00322 [nucl-th].]

[GP, D. A. Shohonov, O. V. Teryaev, N. S. Tsegelnik, and V. I. Zakharov. Phys. Rev. C, 112(6):064907, 2025.]

Theoretical interest:

- Flat space laboratory to investigate black holes: "**black hole in HICs**"
- "Purely" quantum **entanglement** effect
- **Unruh effect** $T = \frac{a}{2\pi}$

Minimal viscosity bounds

Hydrodynamics in linear gradients - corrections to EMT with dissipation:

$$T^{diss}_{\mu\nu} = -\eta(\nabla_\mu u_\nu + \nabla_\nu u_\mu - u_\mu u^\alpha \nabla_\alpha u_\nu - u_\nu u^\alpha \nabla_\alpha u_\mu) - \left(\zeta - \frac{2}{3}\eta\right)\nabla^\alpha u_\alpha (g_{\mu\nu} - u_\mu u_\nu) + \mathcal{O}(\nabla^2 u)$$

Bound inspired by **string theory:**

- **There are no completely ideal fluids!**
- It is believed that **QGP** near this limit

[Kovtun, Son, Starinets, PRL (2005)]

"KSS bound"

$$\frac{\eta}{s} \geqslant \frac{1}{4\pi}$$

- Another well-known bound for **bulk viscosity** also from holography:

[Buchel, PLB (2008), arXiv:0708.3459]

$$\frac{\zeta}{\eta} \geqslant 2\left(\frac{1}{p} - c_s^2\right)$$

number of space dimensions

Statement of the problem

- Does **radiation** in the **Unruh effect** have **viscosity**?
- How is it related to the **KSS bound**?

$$T = \frac{a}{2\pi} \quad \Rightarrow \quad \frac{\eta}{s} = ?$$

- Direct calculation for **quantum fluid** -- **unlike** the **membrane paradigm**, where the viscosity is **classical**.

First step (known):

[G. Chirco, C. Eling, and S. Liberati. Phys. Rev. D, 82:024010, 2010]

For massless scalar field - **finite shear viscosity**, saturates **KSS bound**

This work:

We generalize to **“any” theory**:

-- demonstrate the role of **Unitarity** and **Causality**

-- demonstrate the **relationship** with **irreversibility** of **renormalization group flows**

-- something more ...

[A. B. Zamolodchikov. JETP Lett., 43:730–732, 1986]

Part 1

Entanglement viscosity: universal formulas

Kubo formulas in curved spacetime

- Linear response theory allows us to write **Kubo formula** in **curved spacetime**:

  [Dam T. Son. Acta Phys. Polon. B, 39:3173–3182, 2008]

Shear viscosity:

$$\eta(x') = \mathrm{Im} \lim_{\omega \to 0^+} \frac{i}{\sqrt{g^{00}(x')}\omega} \int_{-\infty}^{\infty} dt\, \theta(t-t') \cdot e^{i\omega(t-t')} \int_V d^{d-1}x \sqrt{-g(x)}\, \langle [\hat{T}_{12}(x), \hat{T}_{12}(x')] \rangle$$

Bulk viscosity: similar formula with trace of EMT

- Apply Kubo formula to the ***d*-dimensional Rindler space:**

$$ds^2 = \rho^2 d\tau^2 - (dx^1)^2 - \ldots - (dx^{d-2})^2 - d\rho^2$$

- Consider **Minkowski vacuum state**:

  Thermal correlator corresponds to the **Minkowski vacuum** one (with a change of coordinates)

  [W. G. Unruh and N. Weiss. Phys. Rev., D29:1656, 1984]

Universal spectral representation

- **General representation** for **<TT>:**

[A. Cappelli, D. Friedan, and J. I. Latorre. Nucl. Phys. B, 352:616–670, 1991]
[M. Smolkin and S. N. Solodukhin, Phys. Rev. D 91, no.4, 044008 (2015)]

$$\Pi^{(0)}_{\alpha\beta,\rho\sigma}(\partial') = \frac{1}{\Gamma(d)} S_{\alpha\beta} S_{\rho\sigma}$$
$$S_{\alpha\beta}(\partial') = \partial'_\alpha \partial'_\beta - \eta_{\alpha\beta}\partial'^2$$

$$\langle 0|\hat{T}_{\alpha\beta}(x)\hat{T}_{\rho\sigma}(x')|0\rangle_M = \frac{A_d}{(d-1)^2}\int_0^\infty d\mu\, c^{(0)}(\mu)\Pi^{(0)}_{\alpha\beta,\rho\sigma}(\partial')G_d(x-x',\mu)$$
$$+\frac{A_d}{(d-1)^2}\int_0^\infty d\mu\, c^{(2)}(\mu)\Pi^{(2)}_{\alpha\beta,\rho\sigma}(\partial')G_d(x-x',\mu)$$

spectral function

Scalar massive propagator

- **Valid** for **“any”** theory
- Unitarity implies: $c^{(0)}(\mu) \geq 0 \quad c^{(2)}(\mu) \geq 0$
- Only $c^{(0)}(\mu)$ for 2d theory: positivity $c^{(0)}(\mu) \geq 0$ ⇒ irreversibility of RG flows

[A. B. Zamolodchikov. JETP Lett., 43:730–732, 1986]

Unitarity provides irreversibility of renormalization group flows

Key integral

- Apply Kubo formula to the ***d*-dimensional Rindler space:**

$$ds^2 = \rho^2 d\tau^2 - (dx^1)^2 - ... - (dx^{d-2})^2 - d\rho^2$$

- Use universal **spectral representaion**
- Use equation for (Whightman) scalar Green function: $\Box G_d(x-x') = -\mu^2 G_d(x-x')$
- Use **descent method:** $\int dx^{d-2} G^d(x) \to G^2(x)$

$$k_d = \frac{\pi^{d/2}}{(d+1)2^{d-1}\Gamma(d)\Gamma(d/2)}$$

Main integral
to be found:

$$\eta(\rho') = k_d \rho' \int d\mu\, \mu^4 c^{(2)}(\mu)\, \mathrm{Im} \lim_{\omega\to 0^+} \frac{i}{\omega} \int_0^\infty d\tau\; e^{i\omega\tau}$$
$$\cdot \int_0^\infty d\rho\, \frac{\rho}{2\pi} \left\{ K_0\left(\mu\sqrt{\rho^2+\rho'^2-2\rho\rho'\cosh(\tau-i\varepsilon)}\right) - (\tau \to -\tau) \right\}$$

Result: final universal formulas

- **Entanglement** (local) **shear viscosity** of radiation in **Unruh effect**:

$$\eta(\rho) = k_d \rho \int_0^\infty d\mu \, c^{(2)}(\mu) \mu^2 K_0(\mu\rho)$$

Defined by acceleration:
$\rho = 1/a$

The calculation for **bulk viscosity** is very similar:

$$\zeta(\rho) = \frac{2k_d\rho}{(d-1)^2} \int_0^\infty d\mu \, c^{(0)}(\mu) \mu^2 K_0(\mu\rho)$$

- Valid in general case (conformal, nonconformal, interacting…).
- $c^{(2)}(\mu)$ defines **shear viscosity**.
- $c^{(0)}(\mu)$ defines **bulk viscosity**.
- In full accordance with particular direct QFT calculations (scalar, Dirac, photon):

[G. Chirco, C. Eling, and S. Liberati. Phys. Rev. D, 82:024010, 2010]
[D. Lapygin, G. Prokhorov, O. Teryaev, V. Zakharov, Phys.Rev.D 112 (2025) 6, 065012]

Discussion: Unitarity *vs* Irreversibility

$$\eta(\rho) = k_d \rho \int_0^\infty d\mu\, c^{(2)}(\mu) \mu^2 K_0(\mu\rho)$$

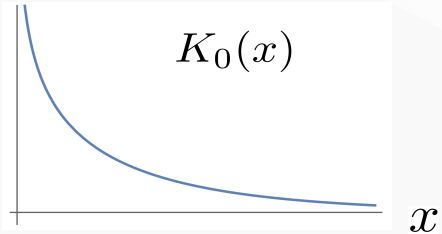
$$\zeta(\rho) = \frac{2k_d\rho}{(d-1)^2} \int_0^\infty d\mu\, c^{(0)}(\mu) \mu^2 K_0(\mu\rho)$$

- **Unitarity:**

$$c^{(0)}(\mu) \geq 0 \qquad c^{(2)}(\mu) \geq 0$$

- Include positive-defined Bessel function:

$$K_0(x) > 0$$



$$\eta, \zeta \geq 0$$

$$\partial_\mu s^\mu \geq 0$$

- **Unitarity** underlies **thermal irreversibility** for a subsystem separated by a horizon
- **Full analogy** with the **irreversibility** of **RG flows** in 2d.

[Andrea Cappelli, Daniel Friedan, and Jose I. Latorre. Nucl. Phys. B, 352:616–670, 1991]

Part 2

Novel sum rule for spectral densities

Anisotropy of pressure: modular Hamiltonian

- Using the results of **modular Hamiltonian** perturbative technique, one can write the derivatives of pressure with respect to temperature:

[M. Smolkin and S. N. Solodukhin, Phys. Rev. D 91, no.4, 044008 (2015)]
[V. Rosenhaus and M. Smolkin. JHEP, 12:179, 2014.]

$$\left.\frac{\partial p_\perp}{\partial T}\right|_a = \frac{-2\pi\rho A_d}{(d-1)^2\Gamma(d)}\int_0^\infty d\mu\,\mu^2 K_0(\mu\rho) \cdot\left\{c^{(0)}(\mu)-(d-1)c^{(2)}(\mu)\right\},$$

$$\left.\frac{\partial p_{||}}{\partial T}\right|_a = \frac{2\pi A_d}{(d-1)^2\Gamma(d)}\int_0^\infty d\mu\,\mu K_1(\mu\rho) \cdot\left\{c^{(0)}(\mu)+(d-1)(d-2)c^{(2)}(\mu)\right\}$$

- The key point – **different** formulas for **longitudinal** and **transverse pressures:**

In general, **pressure** can be **anisotropic!**

Compare with talk of Koichi Hattori 16/11

Pascal's law

- Let us require (or introduce a criterion) **isotropy** of **Unruh radiation**:

$$\frac{\partial p_{||}}{\partial T} = \frac{\partial p_{\perp}}{\partial T}$$

- **Novel sum rule**, relating $c^{(0)}(\mu)$ and $c^{(2)}(\mu)$ spectral functions:

$$\int_0^\infty d\mu \left\{ c^{(0)}(\mu)\mathcal{A}^{(0)}(\mu,\rho) + c^{(2)}(\mu)\mathcal{A}^{(2)}(\mu,\rho) \right\} = 0$$

where:

$$\mathcal{A}^{(2)}(\mu,\rho) = -(d-1)\mu^2 K_0(\mu\rho) + (d-1)(d-2)\frac{\mu}{\rho}K_1(\mu\rho)$$
$$\mathcal{A}^{(0)}(\mu,\rho) = \mu^2 K_0(\mu\rho) + \frac{\mu}{\rho}K_1(\mu\rho)$$

- Postulation of **Pascal law** for Unruh radiation
- It is a criterion of **fluidity** (anisotropic pressure in solid body)
- Can be compared with the **Burkhardt-Cottingham sum rule**

[Hugh Burkhardt and W. N. Cottingham. Annals Phys., 56:453–463, 1970]

Direct verifiaction

- Can be **verified** in **conformal** case in **any dimensions**.
- Can be **verified** in **nonconformal** case: free massive Dirac field in any dimensions.

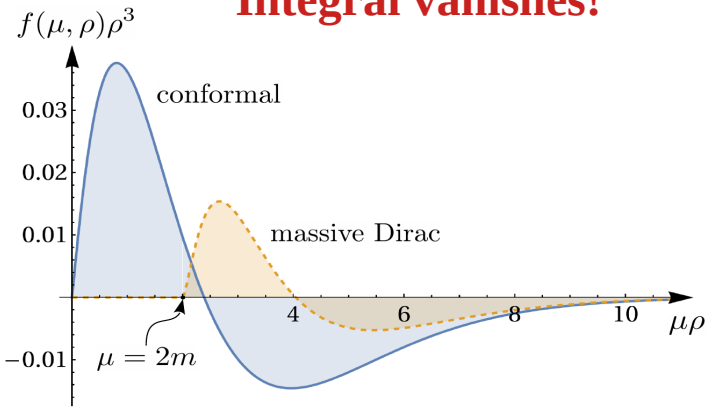
Spectral functions of **massive fermions**:

$$c^{(2)}(\mu,m) = \frac{2^{\lfloor d/2\rfloor}(d-1)\Gamma\left(\frac{d}{2}\right)^2}{8\pi^d}\mu^{d-3}\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{d-1}{2}} \cdot\left(1+\frac{8}{d-1}\cdot\frac{m^2}{\mu^2}\right)\theta(\mu-2m)\,.$$

$$c^{(0)}(\mu,m) = \frac{2^{\lfloor d/2\rfloor}(d^2-1)\Gamma\left(\frac{d}{2}\right)^2}{2\pi^d}m^2\mu^{d-5}\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{d-1}{2}} \cdot\theta(\mu-2m)\,,$$

[Andrea Cappelli, Daniel Friedan, and Jose I. Latorre. Nucl. Phys. B, 352:616–670, 1991]

Integral vanishes!



Scalar field (problem):

- Satisfied for conformal scalar field
- Violated for massless nonconformal scalar field
- Violated for massive scalar field

- Anysotropic contribution $T_{\mu\nu} \sim a_\mu a_\nu$

[Frolov, V., & Serebryanyi, E. (1987), Phys. Rev. D, 35, 3779–3782]

- Surface terms are important, but **do not solve** the problem

[Herzog, C., & Nishioka, T. (2016). JHEP, 12, 138.]

Part 3

Viscosity to entropy density ratio

Viscosity to heat capacity ratio

- What is about **KSS bound** for **entanglement viscosity**?
- Central universal expression for entanglement shear viscosity:

$$\eta(\rho) = k_d \rho \int_0^\infty d\mu \, c^{(2)}(\mu) \mu^2 K_0(\mu\rho)$$

- **Heat capacity** is the **temperature derivative** of **energy density**:

  - can be obtained via modular Hamiltonian perturbative approach

[M. Smolkin and S. N. Solodukhin, Phys. Rev. D 91, no.4, 044008 (2015)]

$$\left.\frac{\partial \varepsilon}{\partial T}\right|_a = \frac{2\pi\rho A_d}{(d-1)^2\Gamma(d)} \int_0^\infty d\mu \left[\mu^2 K_0(\mu\rho) + \frac{\mu}{\rho} K_1(\mu\rho)\right] \left\{c^{(0)}(\mu) + (d-1)(d-2)c^{(2)}(\mu)\right\}$$

⇩ use **isotropy sum rule**

$$c_V(\rho) = \frac{2\pi\rho A_d}{\Gamma(d)} \int_0^\infty d\mu \, c^{(2)}(\mu) \mu^2 K_0(\mu\rho)$$

only via $c^{(2)}(\mu)$

- **One can easily see:**

$$\frac{\eta}{c_V} = \frac{1}{4\pi}$$

- Valid in general case (conformal, nonconformal, interacting...)
- Obtained without holography

Viscosity to entropy density ratio

- Can be transformed to the form with entropy density
- Use relation:

$$c_s^2(a,T) = \frac{\partial p}{\partial \varepsilon}\Big|_a = \frac{\partial p(a,T)/\partial T}{\partial \varepsilon(a,T)/\partial T} = \frac{s}{c_V}$$

we obtain:

$$\frac{\eta}{c_V} = \frac{1}{4\pi} \quad \Rightarrow \quad \boxed{\frac{\eta}{s} = \frac{1}{4\pi c_s^2}}$$

- For example, in conformal limit $c_s^2 = \frac{1}{3}$ then $\frac{\eta}{s} = \frac{3}{4\pi}$

- Relationship between **KSS bound** and **causality**:

Causality: $c_s^2 < 1 \quad \Rightarrow \quad \frac{\eta}{s} > \frac{1}{4\pi}$

Compare with:

[Mauro Brigante, Hong Liu, Robert C. Myers, Stephen Shenker, and Sho Yaida. Phys. Rev. Lett., 100:191601, 2008]

$$\frac{\eta}{s} > \frac{16}{25 \cdot 4\pi}$$

Part 4

Bulk viscosity

Bulk viscosity

- What is about **entanglement bulk viscosity**?

$$\zeta(\rho) = \frac{2k_d\rho}{(d-1)^2}\int_0^\infty d\mu\, c^{(0)}(\mu)\mu^2 K_0(\mu\rho)$$

- Use **isotropy sum rule:**

-- entropy density
-- heat capacity

Speed of sound

$$c_s^2(a,T) = \frac{s}{c_V}$$

- Use similar “spectral” formula for shear viscosity
- Everything is expressed through integrals with $c^{(0)}(\mu)$ and $c^{(2)}(\mu)$

Simple algebra

- **Bulk viscosity** saturates **well-known bound**:

$$\frac{\zeta}{\eta} = 2\left(\frac{1}{d-1} - c_s^2\right)$$

[Buchel, PLB (2008), arXiv:0708.3459]

$$\frac{\zeta}{s} = \frac{1}{2\pi c_s^2}\left(\frac{1}{3} - c_s^2\right)$$

Part 5

Entanglement viscosity and conformal anomaly

Entanglement viscosity and conformal anomaly

-- Various effects of Chiral anomaly

-- **Conformal anomaly** effects are **not so well studied**.

[M. N. Chernodub, PRL (2016)]
[C.-S. Chu, R.-X. Miao, PRL (2018)]

[Yang, Gao, Liang, Prokhorov, Shi Pu, Teryaev, Zakharov, 2604.23849]

Conformal gravitational anomaly:

$$\langle \hat{T}^{\mu}_{\mu} \rangle = a(-H + \frac{2}{3}\Box R) - bE_4 + c\Box R$$

$$\eta = 8a\alpha^3$$

In the case of CFT conformal **central charge**

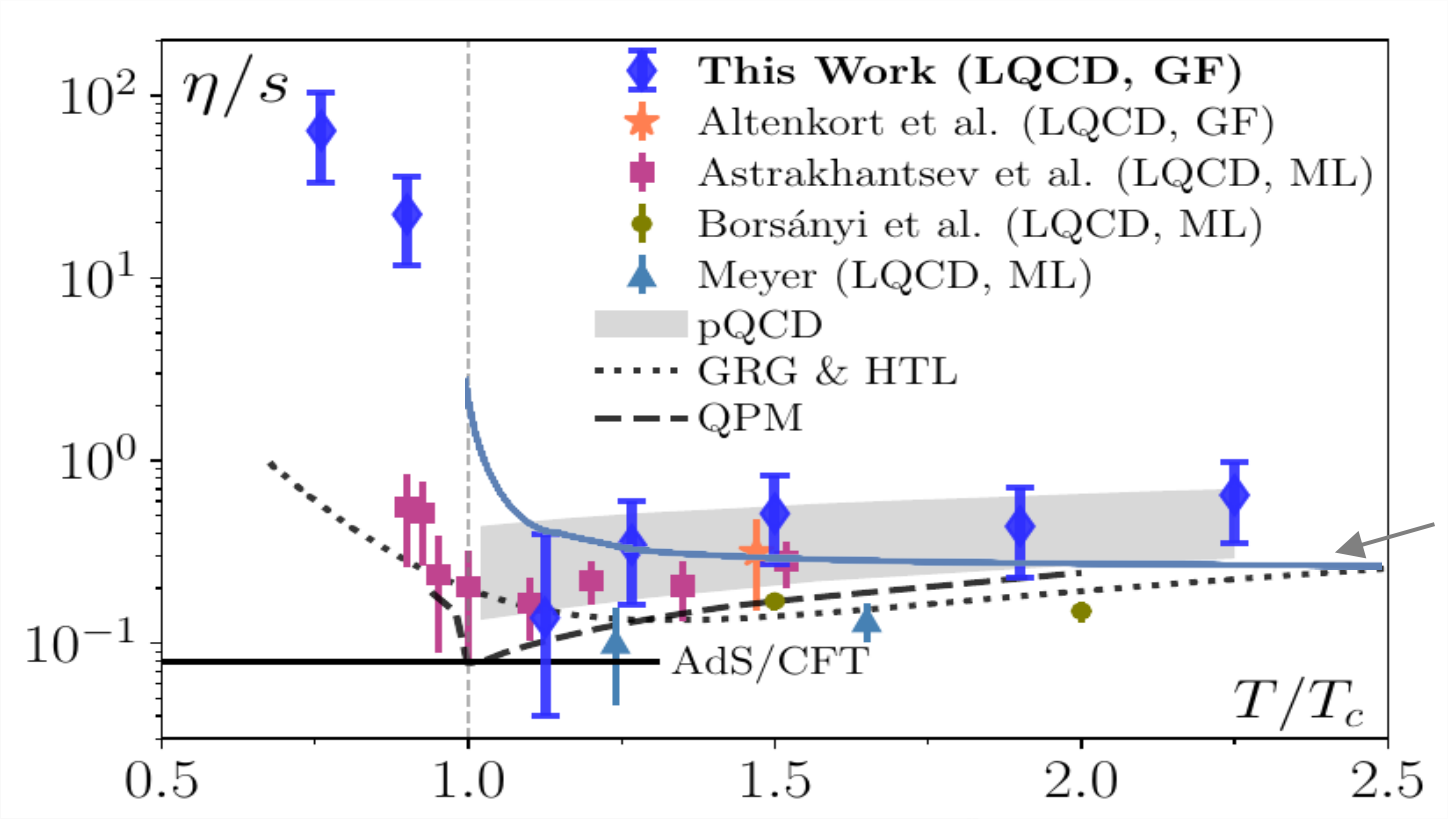
Novel anomalous effect in systems with **extreme acceleration** α

- Entanglement viscosity is defined by the conformal central charge – compare with “maximal entanglement principle”: [D. E. Kharzeev, 2601.00405 [quant-ph]]
- *“Cheshire cat’s grin”* of conformal gravitational anomaly in flat spacetime (similar to CVE and KVE), or manifestation of the **equivalence principle**



Preliminary results, conclusion and outlook

Preliminary results: comparison with lattice

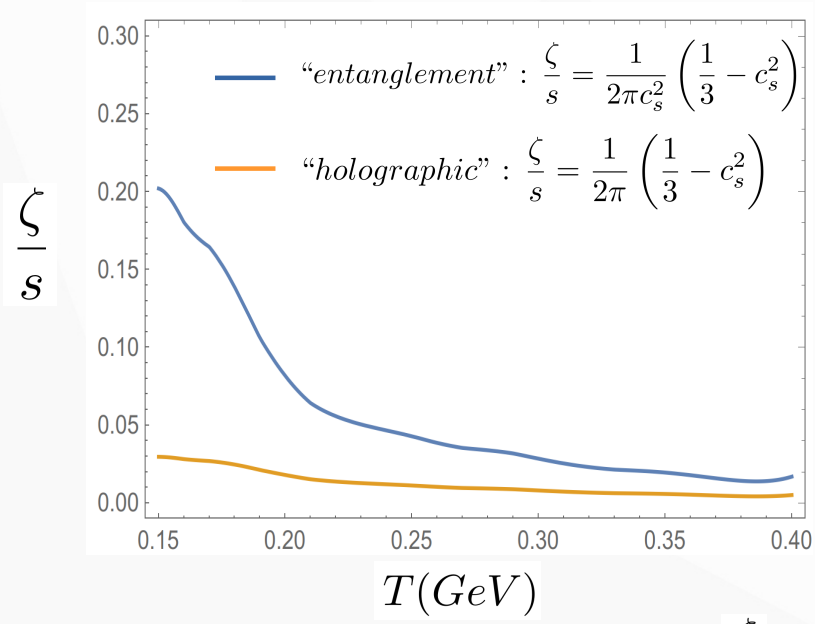


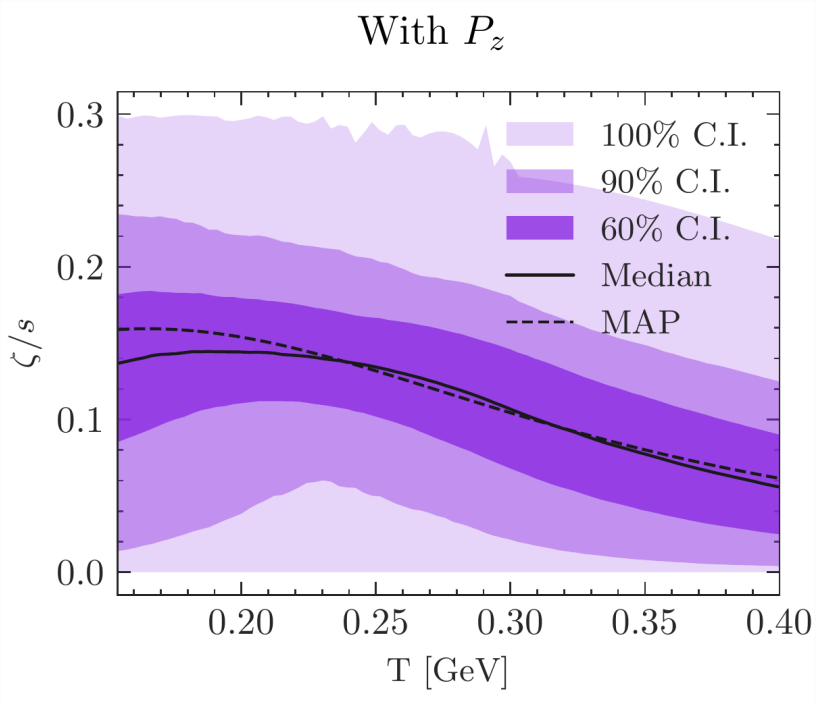
[Heng-Tong Ding, Hai-Tao Shu, Cheng Zhang, 2601.14967, Phys.Rev.D 113 (2026) 7, 074503]

- Naive comparison: explains the **excess** at **high temperatures**
- **Violated** near T_c - **non-hydrodynamic** behavior of the QGP near the transition?

Preliminary: bulk viscosity and polarization

Naive comparison with bayesian constraints for longitudinal spin polarization:





- "Entanglement" predicts higher ratio $\frac{\zeta}{s}$ as favored by the bayesian constraints.
- Cannot be considered as "serious" prediction:
  (problem with peak of shear viscosity near critical temperature...)

[Sushant K. Singh, Eduardo Grossi and Francesco Becattini, e-print: 2605.29383 [nucl-th]]

See talk of F. Becattini 16/11

Summary & Conclusion

- **Universal formulas** for entanglement shear and bulk viscosities in **accelerated** frame in terms of fundamental **spectral functions** are obtained:

  -- Analogy **thermal irreversibility** ⟺ **RG irreversibility**

- **Novel sum rule** for different spectral functions is obtained from **Pascal law (isotropy condition)** for radiation in the **Unruh effect**:

  -- Verified: **CFT** and **nonconformal** case (massive fermions)

- It is shown (**without holography**) that the entanglement viscosities **saturate bounds**:

  -- **Novel** formula for **local ratio** with speed of sound

  -- Connection of the **KSS bound** with **causality** is shown

  -- The known bound for **bulk viscosity** is **saturated**

  -- New expression for **bulk viscosity** to **entropy** density ratio ⇒

$$\frac{\eta}{s} = \frac{1}{4\pi c_s^2}$$

or in more general case:

$$\frac{\eta}{c_V} = \frac{1}{4\pi}$$

$$\frac{\zeta}{s} = \frac{1}{2\pi c_s^2}\left(\frac{1}{3} - c_s^2\right)$$

Outlook: other sum rules?

Rotating medium of fermions



Also **isotropic pressure**!

Term $T_{\mu\nu} \sim \omega_\mu \omega_\nu$ vanishes

[Buzzegoli, M., Grossi, E., & Becattini, F. (2017), JHEP, 10, 091]



Other **new sum rules**?



Questions:

-- Is there an analog of “modular Hamiltonian” for rotating medium?

-- What are the properties of the Minkowski vacuum from the point of view of a rotating observer?

Thank you for your attention!

Additional slides

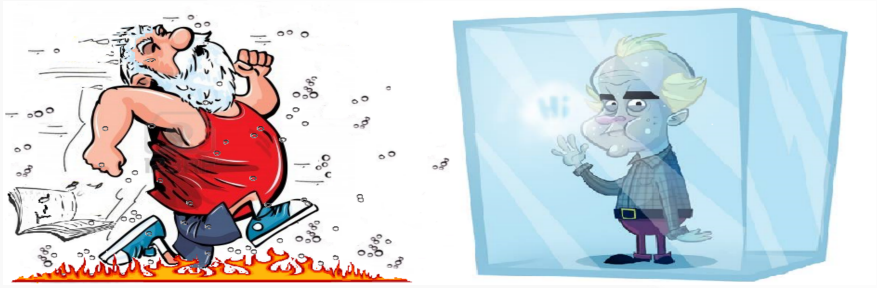
Unruh effect

Formulation:

The **Minkowski vacuum** is perceived by an **accelerated** observer as a medium with a finite **Unruh temperature**

$$T_U = \frac{a}{2\pi}$$

[W. G. Unruh. Notes on black hole evaporation. Phys. Rev., D14:870, 1976.]



[Blasone, (2018), e-Print: 1911.06002]

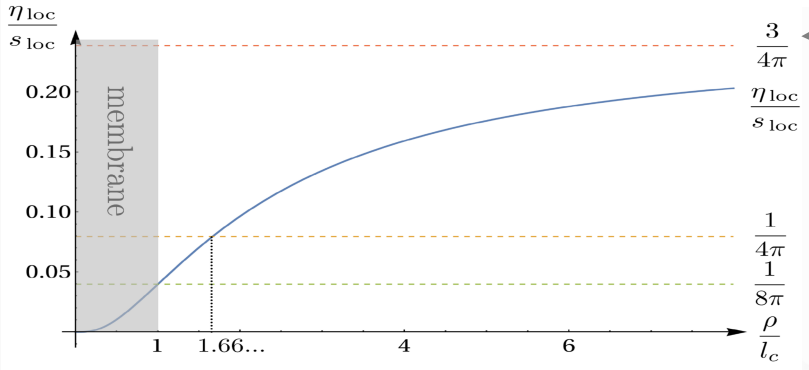
[The equation of state in (2+1)-flavor QCD, A. Bazavov, et al, 1407.6387v2]

Viscosity to entropy density ratio: local *vs* global

- There are **local** and **global** quantities:

Local

- At certain distance frρ the horizon
- We can now regularize integrals by stretched horizon $\rho > l_c$
- Any CFT in 4d: $\dfrac{\eta}{s} = \dfrac{1}{4\pi c_s^2}$



$$\frac{\eta_{\rm loc}}{s_{\rm loc}}(\rho) = f(\rho/l_c) = \frac{3\rho^4\left[\rho^4 + 4\rho^2 {l_c}^2 - 5{l_c}^4 - 4{l_c}^2(2\rho^2 + l_c^2)\ln\left(\frac{\rho}{l_c}\right)\right]}{4\pi(\rho^2 - l_c^2)^4}$$

- **Universal function** for any CFT in 4d

Global

$$\eta_{glob} = \int_{l_c}^{\infty} \eta(\rho)d\rho = \frac{C_T\pi^2}{480 l_c^2}$$

$$s_{glob} = \int_{l_c}^{\infty} s(\rho)d\rho = \frac{C_T\pi^3}{120 l_c^2}$$

$$\boxed{\frac{\eta_{glob}}{s_{glob}} = \frac{1}{4\pi}}$$

- **KSS bound** is **saturated globally** for any CFT in 4d

(all the previous discussion – for $l_c \to 0$*)*

New area: anomalous transport effects

New **(non-dissipative)** effects are predicted at the intersection of quantum field theory and gravity *(only some of them)*:

- Chiral Magnetic Effect **(CME)**:
  [K. Fukushima, D. E. Kharzeev, H. J. Warringa, PRD 78, 074033 (2008), 0808.3382]

$$\langle \hat{j}^\mu \rangle = Ce^2 \mu_A B^\mu$$

Associated with **axial anomaly** in the electromagnetic field:

$$\langle \partial_\mu \hat{j}^\mu_A \rangle = -\frac{Ce^2}{8} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

- Axial Vortical Effect **(AVE)**:
  [D. T. Son, P. Surowka, PRL 103, 191601 (2009), 0906.5044]

$$\langle \hat{j}^\mu_A \rangle = C(\mu^2 + \mu_A^2)\omega^\mu$$

- Thermal part of AVE:
  [K. Landsteiner, E. Megias, F. Pena-Benitez, PRL 107, 021601 (2011), 1103.5006]

$$\langle \hat{j}^\mu_A \rangle \sim \mathcal{N} T^2 \omega^\mu$$

Associated with an **axial anomaly** in the **gravitational field:**

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\rho\sigma} R_{\alpha\beta}{}^{\rho\sigma}$$

- **Kinematic Vortical Effect (KVE)**
  [G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, PRL 129, 151601 (2022), 2207.04449]

$$\langle \hat{j}^A_\mu \rangle = (\lambda_1 \omega^2 + \lambda_2 a^2)\omega_\mu$$

Connection with **anomaly:**

$$\lambda_1 - \lambda_2 = 32\mathcal{N}$$

Modular Hamiltonian perturbation theory

- There is technique to find perturbatively effects of small angular deficit using **modular Hamiltonian** (boost operator in our case):

[M. Smolkin and S. N. Solodukhin, Phys. Rev. D 91, no.4, 044008 (2015)]
[V. Rosenhaus and M. Smolkin. JHEP, 12:179, 2014.]

$$\lim_{\nu\to 1}\frac{\partial}{\partial\nu}\langle\hat{T}_{ij}\rangle = \langle\hat{T}_{ij}\hat{K}_0\rangle$$

- Euclidean Rindler space - describes an accelerated system at a finite temperature

$$ds^2 = \rho^2 d\tau^2 + (dx^1)^2 + \ldots + (dx^{d-2})^2 + d\rho^2$$

*It is also a **conical space***

- The derivative with respect to the angular deficit corresponds to the derivative with respect to temperature:

$$\nu = \frac{2\pi T}{a} \quad \Rightarrow \quad \lim_{\nu\to 1}\frac{\partial}{\partial\nu} \quad \leftrightarrows \quad \lim_{T\to T_U}\frac{\partial}{\partial T}$$

Higher spins

- **Negative viscosity** for **higher spins** (without supergravity)?
  [R. Khakimov, G. Prokhorov, O. Teryaev, 2605.22409 [hep-th]]

- **Rarita-Schwinger-Adler** theory

$$\mathcal{L} = -\varepsilon^{\alpha\beta\mu\nu}\bar{\psi}_\alpha\gamma_5\gamma_\mu\partial_\nu\psi_\beta + i\bar{\lambda}\gamma^\mu\partial_\mu\lambda - im\bar{\lambda}\gamma^\mu\psi_\mu + im\bar{\psi}_\mu\gamma^\mu\lambda$$

  [Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018]

- We show that it has features of **conformal theory** on the one-loop level in the limit: $m \to \infty$ (unlike usual Rarita-Schwinger field)

- **Negative viscosity**

  Negative conformal **central charge**

  Entropy density is also **negative**

General ratio is **confirmed**

$$\frac{\eta}{s} = \frac{1}{4\pi c_s^2}$$

Outlook

1. Signatures of $\frac{\eta}{s} = \frac{1}{4\pi c_s^2}$ or $\frac{\eta}{c_V} = \frac{1}{4\pi}$ in other systems?

2. Is there any connection with **DSW decoherence**?

-- both are related with absorption of soft particles (photons, gravitons) by horizon.

3. Why is the **isotropy** of Unruh radiation and the corresponding **sum rule** for **massive scalar** fields violated?

4. Other new sum rules?

-- isotropy in rotating systems, ...

Key integral: solution

- However, everything **becomes easy** in the **"momentum" representation**!



$$\frac{1}{2\pi} K_0\left(\mu\sqrt{\rho^2+\rho'^2-2\rho\rho'\cosh(\tau-i\varepsilon)}\right) =$$

$$= \frac{1}{\pi^2}\int_0^\infty d\lambda\, K_{i\lambda}(\mu\rho)K_{i\lambda}(\mu\rho')\cosh(\pi\lambda - i\lambda(\tau-i\varepsilon))$$

Dependence on $\boldsymbol{\rho}$ and $\boldsymbol{\tau}$ is **factorized**

[A.P. Prudnikov, Yu.A. Brychkov, and O.I. Marichev. Integrals and series: Special functions vol 2.]

- Now integral over the distance from the horizon can be easily found:

$$\int_0^\infty d\rho\,\rho K_{i\lambda}(\mu\rho) = \frac{\pi\lambda}{2\mu^2\sinh(\pi\lambda/2)}$$

- Integral over τ leads to frequency parts of delta function – principal value part are real and drops out:

$$\eta = \frac{k_d\rho'}{4}\lim_{\omega\to 0^+}\frac{1}{\omega}\int_0^\infty d\mu\, c^{(2)}(\mu)\mu^2\int_0^\infty d\lambda\,\frac{\lambda K_{i\lambda}(\mu\rho')}{\sinh(\pi\lambda/2)}$$

$$\cdot\left\{e^{\pi\lambda}\delta(\omega-\lambda) + e^{-\pi\lambda}\delta(\omega+\lambda) - e^{\pi\lambda}\delta(\omega+\lambda) - e^{-\pi\lambda}\delta(\omega-\lambda)\right\}$$

Phenomenological applications

- **Local ratio** with speed of sound:

expressed in terms of general hydrodynamic quantities ⇒ can be more general ⇒ **compare with QGP**

QGP viscosity from the experiments with HICs: $0.05 < \eta/s < 0.2$

[John W. Harris and Berndt Müller. ”QGP Signatures” Revisited. Eur. Phys. J. C, 84(3):247, 2024]

Naive prediction of our formula:

$$\frac{\eta}{s} = \frac{1}{4\pi c_s^2} \Rightarrow c_s^2 \sim 1/3 \Rightarrow \eta/s \sim 0.24$$

“Wandering” Planck constant

- In the original holographic derivation (or within membrane paradigm) of KSS bound the **viscosity** is **“classical”** – Planck constant comes from the **“quantum”** Bekenstein-Hawking **entropy**:

$$\left.\begin{aligned} \eta &\sim \mathcal{O}(\hbar^0) \\ s &\sim \frac{A}{\hbar G} \sim \mathcal{O}(\hbar^{-1}) \end{aligned}\right\} \quad \frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

- In our case, the **viscosity** is **quantum:** determined by a **one-loop** diagram, contains Planck constant:

$$\left.\begin{aligned} \eta_{\mathrm{loc}} &\sim \frac{T_U^3}{\hbar^2} \\ s_{\mathrm{loc}} &\sim \frac{T_U^3}{\hbar^3} \end{aligned}\right\} \quad \left.\frac{\eta}{s}\right|_{loc} = \frac{\hbar c^2}{4\pi k_B c_s^2}$$

However, the result is the same:
“Wandering” Planck constant