From spin 1/2 to spin 1: foundations of perfect spin hydrodynamics

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talk based on: arXiv:2511.09580, arXiv:2606.15364

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Spin 1/2 vs 1

Experiment:

- **Spin 1/2** measurements of local and global spin polarization of hadrons
- **Spin 1** measurements of tensor polarizabilities of vector mesons

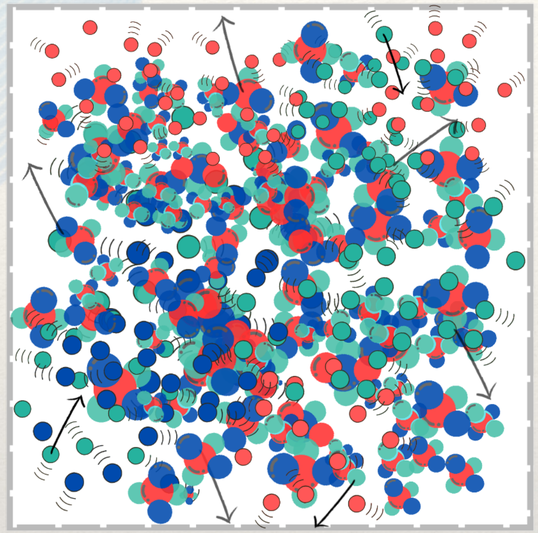
Theory:

- Relativistic spin hydrodynamics — searching for “missing” terms to describe the data
- Clarifying relations between different spin observables, e.g. pseudogauge ambiguity, local vs global equilibrium, …

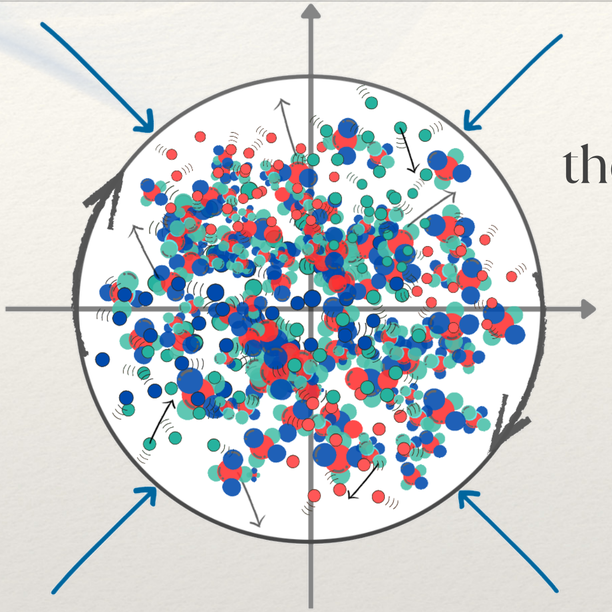
Various pictures with spin

spin polarization tensor $\omega^{\mu\nu}$

ω components — Lagrange multipliers for spin conservation



$\omega \rightarrow \varpi$



thermal vorticity $\varpi^{\mu\nu}$

Local equilibrium
no rotation, no boundary

Global equilibrium
with rotation and boundary

Our approach: spin conserved separately from the angular momentum;
spin-orbit coupling encoded in the dissipation

[Drogosz, Florkowski, Witkowski, 2611.06945]

Spin 1/2 vs 1

Boltzmann (spin 1/2 or 1) → Fermi-Dirac — Kar, V.M., arXiv:2511.09580

Boltzmann (spin 1/2 or 1) → Bose-Einstein — Kar, V.M., arXiv:2606.15364

- Local-equilibrium Wigner function with various statistics
- Obtaining relevant tensors $T^{\mu\nu}$, $S^{\lambda,\mu\nu}$, S^{μ}, N^{μ}
- Checking thermodynamic relations
- Divergence-type theory, nonlinear causality and stability

For low T and high μ, where polarization is the most pronounced

Spin 1/2: Boltzmann

Local equilibrium Wigner function

De Groot, Relativistic Kinetic Theory. Principles and Applications

Semiclassical approximation

$$W^{+}(x,k) = \frac{1}{2}\sum_{r,s=1}^{2}\int dP\,\delta^{(4)}(k-p)\,u_r(p)\,f^{+}_{rs}(x,p)\,\bar{u}_s(p)$$

Spin density matrix

Hermitian 2x2

$$\left[f^{+}(x,p)\right]_{rs} \equiv f^{+}_{rs}(x,p) = \frac{1}{2m}\,\bar{u}_r(p)\,X^{+}\,u_s(p)$$

$$\left[f^{-}(x,p)\right]_{rs} \equiv f^{-}_{rs}(x,p) = -\frac{1}{2m}\,\bar{v}_s(p)\,X^{-}\,v_r(p)$$

4x4 spinor density matrix

$$W^{\pm}(x,k) = \frac{1}{4m}\int dP\,\delta^{(4)}(k\mp p)(p_\mu\gamma^\mu \pm m)X^{\pm}(x,p)(p_\mu\gamma^\mu \pm m)$$

The key ingredient — Local equilibrium Wigner function

$$W^{\pm}(x,k) = \frac{1}{4m}\int dP\,\delta^{(4)}(k \mp p)(p_\mu\gamma^\mu \pm m)X^{\pm}(x,p)(p_\mu\gamma^\mu \pm m)$$

$$\rho^{\pm}(p) = \pm\frac{1}{4m}\Big(p^\mu\gamma_\mu \pm m\Big)\underbrace{\rho'^{\pm}(p)}_{\parallel}\Big(p^\mu\gamma_\mu \pm m\Big)$$

auxiliary spin density matrix

$$1 + \gamma_5\gamma^\mu\zeta^{\pm}_\mu(p)$$

[Berestetskii, Lifshitz, Pitaevskii, Quantum Electrodynamics]

NEW

Spinor density matrix

$$X^{\pm}(x,p) = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu + \gamma_5\gamma^\mu a_\mu\right]$$

[Bhadury, Drogosz, Florkowski, Kar, V.M., 2505.02657]

$$a_\mu(x,p) = -\frac{1}{4m}\epsilon^{\mu\nu\alpha\beta}\omega_{\alpha\beta}(x)p^\nu$$

spin polarization tensor

polarization vector

Differs from earlier version by Becattini et al., Ann. Phys. 338 (2013):

$$X^{\pm} = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu \pm \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]$$

$$\Sigma^{\mu\nu} = \sigma^{\mu\nu}/2 = (i/4)[\gamma^\mu,\gamma^\nu]$$

Spin-extended Boltzmann statistics

$$X^{\pm}(x,p) = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu + \gamma_5 a_\mu \gamma^\mu\right]$$

$$a^{\pm}_\mu(x,p) = -\frac{1}{4m}\epsilon^{\mu\nu\alpha\beta}\omega_{\alpha\beta}(x)p^\nu$$

Polarization vector normalization and direction

$$\zeta^\mu_\pm = \frac{a^\mu_\pm}{\sqrt{-a^2_\pm}}\tanh\sqrt{-a^2_\pm}$$

similar to Palermo, Becattini, Eur. Phys. J. Plus 138 (2023)

$$-\zeta^2_\pm = (\vec{\zeta}_{\pm *})^2 = \tanh^2\sqrt{-a^2_\pm} \leq 1$$

$$\vec{\zeta}^{\pm}_*(x,\vec{p}) = -\frac{1}{2}\vec{b}_*(x,\vec{p})$$

agrees with Becattini et al., Ann. Phys. 338 (2013)

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & e^1 & e^2 & e^3 \\ -e^1 & 0 & -b^3 & b^2 \\ -e^2 & b^3 & 0 & -b^1 \\ -e^3 & -b^2 & b^1 & 0 \end{bmatrix}$$

Spin 1/2: Fermi-Dirac

From Boltzmann to Fermi-Dirac

Towards quantum-statistical effects

$$X^{\pm}(x,p) = \exp\left[\pm\xi(x) - \beta(x)\cdot p + \gamma_5\,\gamma_\mu a^\mu(x,p)\right] = f_0(x,p)\,\exp\left(\gamma_5\,\gamma_\mu a^\mu(x,p)\right)$$

Using $\exp\left(\gamma_5\,\gamma_\mu a^\mu\right) = \cosh\sqrt{-a^2} + \dfrac{\gamma_5\,\gamma_\mu a^\mu}{\sqrt{-a^2}}\sinh\sqrt{-a^2}$ we get

$$X^{\pm}_{\alpha\beta} = \frac{1}{2}\left[\left(e^{\pm\xi-\beta\cdot p+\sqrt{-a^2}} + e^{\pm\xi-\beta\cdot p-\sqrt{-a^2}}\right)\,\delta_{\alpha\beta} + \frac{(\gamma_5\,\gamma_\mu a^\mu)_{\alpha\beta}}{\sqrt{-a^2}}\,\left(e^{\pm\xi-\beta\cdot p+\sqrt{-a^2}} - e^{\pm\xi-\beta\cdot p-\sqrt{-a^2}}\right)\right]$$

$$X^{\pm}_{\alpha\beta} = \frac{1}{2}\left[\left(g_{\rm B}^{\pm+} + g_{\rm B}^{\pm-}\right)\,\delta_{\alpha\beta} + \frac{(\gamma_5\,a^\mu\gamma_\mu)_{\alpha\beta}}{\sqrt{-a^2}}\,\left(g_{\rm B}^{\pm+} - g_{\rm B}^{\pm-}\right)\right]$$

spin-extended Boltzmann distributions

$$g_{\rm B}^{\pm\pm} \overset{?}{\to} g_{\rm FD}^{\pm\pm}$$

From Boltzmann to Fermi-Dirac statistics

$$X^{\pm}_{\alpha\beta} = \frac{1}{2}\left[\left(g^{\pm+}_{\mathrm{FD}} + g^{\pm-}_{\mathrm{FD}}\right)\,\delta_{\alpha\beta} + \frac{(\gamma_5 a^{\mu}\gamma_{\mu})_{\alpha\beta}}{\sqrt{-a^2}}\,\left(g^{\pm+}_{\mathrm{FD}} - g^{\pm-}_{\mathrm{FD}}\right)\right]$$

Boltzmann

$$g^{\pm\pm}_{\mathrm{B}}(x,p) = \exp\left[\pm\xi(x) - \beta(x)\cdot p \pm \sqrt{-a^2(x,p)}\right] \quad\Longrightarrow\quad g^{\pm\pm}_{\mathrm{FD}}(x,p) = \left[\exp\left(\mp\xi(x) + \beta(x)\cdot p \mp \sqrt{-a^2(x,p)}\right) + 1\right]^{-1}$$

Fermi-Dirac

From spinor density matrices $X^{\pm}$ to spin density matrices $f^{\pm}$ → polarization vector

$$\left[f^{+}(x,p)\right]_{rs} \equiv f^{+}_{rs}(x,p) = \frac{1}{2m}\bar{u}_r(p)X^{+}u_s(p)$$

$$\left[f^{-}(x,p)\right]_{rs} \equiv f^{-}_{rs}(x,p) = -\frac{1}{2m}\bar{v}_s(p)X^{-}v_r(p)$$

$$f^{\pm}_{rs}(x,p) = \frac{1}{2}\left[\left(g^{\pm+}_{\mathrm{FD}} + g^{\pm-}_{\mathrm{FD}}\right)\,\delta_{rs} + \frac{\vec{a}_*\cdot\vec{\sigma}_{rs}}{\sqrt{a^2_*}}\,\left(g^{\pm+}_{\mathrm{FD}} - g^{\pm-}_{\mathrm{FD}}\right)\right]$$

Pauli matrices

$$\vec{P} = \frac{1}{2}\frac{\mathrm{tr}\left[(f^{+}+f^{-})\vec{\sigma}\right]}{\mathrm{tr}\left(f^{+}+f^{-}\right)} = \frac{\vec{a}_*}{2\sqrt{a^2_*}}\frac{\sum_{i,j=\pm} j\, g^{ij}_{\mathrm{FD}}}{\sum_{i,j=\pm} g^{ij}_{\mathrm{FD}}}$$

properly bounded, mean polarization vector along the OAM, agrees with Palermo, Becattini, Eur. Phys. J. Plus 138 (2023)

Thermodynamic currents for spin 1/2

$$W^{\pm}(x,k) = \pm\frac{1}{2}\int \mathrm{d}P\,\delta^{(4)}(k \mp p)(p_\mu\gamma^\mu \pm m)X^{\pm}(x,p) \implies \int dP\,\mathrm{tr}_4\left[\cdots X^{\pm}(x,p)\cdots\right]$$

traces over the spinor space

Boltzmann

$$T^{\mu\nu} = \sum_{i,j=\pm}\int dP\,p^\mu p^\nu g^{ij}_{\mathrm{B}} = 2\int dP\,p^\mu p^\nu\left[f_0^+ + f_0^-\right]\cosh\sqrt{-a^2}$$

$$S^{\lambda,\mu\nu} = \frac{1}{m}\int dP\,p^\lambda\,\frac{\sinh\sqrt{-a^2}}{\sqrt{-a^2}}\epsilon^{\mu\nu}{}_{\rho\sigma}\,a^\rho p^\sigma\left[f_0^+ + f_0^-\right]$$

$$f_0^{\pm}(x) = \mathrm{e}^{\pm\xi(x) - p^\mu\beta_\mu(x)}$$

$$\mathcal{N}^\mu(x) = 2\int dP\,p^\mu\left[f_0^+ + f_0^-\right]\cosh\sqrt{-a^2}$$

$$S^\mu = -2\int dP\,p^\mu\left[\left(\left[f_0^+(\xi - p_\mu\beta^\mu) - f_0^+\right] - \left[f_0^-(\xi + p_\mu\,\beta^\mu) + f_0^-\right]\right)\cosh\sqrt{-a^2}\right.$$
$$\left. -\left(f_0^+ + f_0^-\right)\frac{\sinh\sqrt{-a^2}}{\sqrt{-a^2}}\,a^2\right]$$

GLW pseudogauge

$$T^{\mu\nu}(x) = \sum_{i,j=\pm}\int dP\,p^\mu p^\nu g^{ij}_{\mathrm{FD}}$$

Fermi-Dirac

$$S^{\lambda,\mu\nu}(x) = \frac{1}{2m}\sum_{i,j=\pm} j\int dP\,p^\lambda\frac{1}{\sqrt{-a^2}}\epsilon^{\mu\nu}{}_{\alpha\beta}\,a^\alpha p^\beta g^{ij}_{\mathrm{FD}}$$

$$\mathcal{N}^\mu = -\sum_{i,j=\pm}\int dP\,p^\mu\ln\left(1 - g^{ij}_{\mathrm{FD}}\right)$$

$$S^\mu = -\sum_{i,j=\pm}\int dP\,p^\mu\left[g^{ij}_{\mathrm{FD}}\ln g^{ij}_{\mathrm{FD}} + \left(1 - g^{ij}_{\mathrm{FD}}\right)\ln\left(1 - g^{ij}_{\mathrm{FD}}\right)\right]$$

$$g^{\pm\pm}_{\mathrm{FD}}(x,p) = \left[\exp\left(\mp\xi(x) + \beta(x)\cdot p \mp\sqrt{-a^2(x,p)}\right) + 1\right]^{-1}$$

Thermodynamic currents for spin 1/2

satisfy a generalized Gibbs-Duhem relation: $$d\mathcal{N}^\mu = N^\mu d\xi - T^{\mu\lambda} d\beta_\lambda + \frac{1}{2} S^{\mu,\alpha\beta} d\omega_{\alpha\beta}$$

generalized first law of spin thermodynamics: $$dS^\mu = -\xi dN^\mu + \beta_\lambda dT^{\lambda\mu} - \frac{1}{2}\omega_{\alpha\beta} dS^{\mu,\alpha\beta}$$

[Florkowski, Hontarenko, PRL 124 (2025)]

Entropy conservation is a direct consequence of the conservation of other tensors:

[Bhadury, Drogosz, Florkowski, Kar, V.M., 2505.02657]

$$S^\mu = -\xi N^\mu + \beta_\lambda T^{\lambda\mu} - \frac{1}{2}\omega_{\alpha\beta} S^{\mu,\alpha\beta} + \mathcal{N}^\mu$$

Spin 1:
from Boltzmann to Bose-Einstein

Spin density matrix

The most general form of the covariant spin-1 density matrix [Xia, Li, Huang, PLB 817 (2021)]

$$\rho_{\mu\nu}(x,p) = -\frac{1}{3}\left[\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}\right) + \frac{3i\epsilon_{\mu\nu\lambda\rho}\mathscr{P}^\lambda(x,p)\,p^\rho}{2m} + \sqrt{6}\,\mathscr{T}_{\mu\nu}(x,p)\right]$$

$(S^i)_{jk} = -i\epsilon_{ijk}$
Adjoint representation

$a_\mu(x,p) = -\frac{1}{4m}\epsilon^{\mu\nu\alpha\beta}\omega_{\alpha\beta}(x)p^\nu$

$$\chi^{\mathrm{B}}_{rs}(x,p) \sim \mathrm{e}^{\pm\xi(x) - p^\mu\beta_\mu(x)}\,\rho^S_{rs}(x,p)$$ spin-dependent matrix distribution function

★ $$\chi^{\mathrm{B}}_{rs*} = \exp(-\beta\cdot p + 2\vec{a}_*\cdot\vec{S})_{rs}$$

$$= g_0^{\mathrm{B}}\delta_{rs} + \frac{(\hat{a}_*\cdot\vec{S})_{rs}}{2}\left(g_+^{\mathrm{B}} - g_-^{\mathrm{B}}\right) + \frac{[(\hat{a}_*\cdot\vec{S})^2]_{rs}}{2}\left(g_-^{\mathrm{B}} - 2g_0^{\mathrm{B}} + g_+^{\mathrm{B}}\right)$$

★ $$\chi^{\mathrm{BE}}_{rs*} = \left(\exp(\beta\cdot p - 2\vec{a}_*\cdot\vec{S}) - 1\right)^{-1}_{rs}$$

$$= g_0^{\mathrm{BE}}\delta_{rs} + \frac{(\hat{a}_*\cdot\vec{S})_{rs}}{2}\left(g_+^{\mathrm{BE}} - g_-^{\mathrm{BE}}\right) + \frac{[(\hat{a}_*\cdot\vec{S})^2]_{rs}}{2}\left(g_-^{\mathrm{BE}} - 2g_0^{\mathrm{BE}} + g_+^{\mathrm{BE}}\right)$$

+ tensor polarizabilities $\mathscr{T}_{\mu\nu}$
+ spin alignment
  $\mathscr{A} = \rho_{00} - \frac{1}{3} = -\sqrt{\frac{2}{3}}\mathscr{T}^{22}_*$
+ Pauli-Lubański vector

[Florkowski, Kar, V.M., 2602.00819, Acta Phys.Polon.B 57 (2026)]

Spin-1 thermodynamics

$$\mathscr{W}_{\mu\nu}(x,k) = -\int dP\,\delta^{(4)}(k-p)\chi_{\nu\mu}(x,p)$$

[Weickgenannt, Wagner, Speranza, PRD 105 (2022)]

$$T^{\mu\nu}(x) = \int d^4p\, p^\mu p^\nu \,\mathrm{tr}[\mathscr{W}(\mathrm{x},\mathrm{p})] \qquad S^{\lambda,\mu\nu}(x) = 2i\int d^4p\, p^\lambda \mathscr{W}^{[\mu\nu]}(x,p)$$

$$T^{\mu\nu}_{\mathrm{B}}(x) = \int dP\, p^\mu p^\nu f_0\left[2\cosh(2a_*)+1\right]$$

$$S^{\lambda,\mu\nu}_{\mathrm{B}}(x) = 2\int dP\, p^\lambda f_0 \frac{\sinh\left(2\sqrt{-a^2}\right)}{m\sqrt{-a^2}}\epsilon^{\mu\nu\alpha\beta}a_\alpha p_\beta$$

2nd order expansion

$$\omega_{\mu\nu} = k_\mu u_\nu - k_\nu u_\mu + \epsilon_{\mu\nu\alpha\beta}u^\alpha\omega^\beta$$

$$a^2 = -\frac{\omega_{\mu\nu}\omega^{\mu\nu}}{8} - \frac{p_\alpha\omega^{\alpha\beta}\omega_{\beta\gamma}p^\gamma}{4m^2}$$

★ $$T^{\mu\nu}_{\mathrm{B}}(x) = 3\int dP\, p^\mu p^\nu g^{\mathrm{B}}_0 + (k^2-\omega^2)\int dP\, p^\mu p^\nu g^{\mathrm{B}}_0 + \frac{\omega_{\alpha\beta}\omega^\beta_\gamma}{m^2}\int dP\, p^\alpha p^\gamma p^\mu p^\nu g^{\mathrm{B}}_0 + \mathcal{O}(\omega^4)$$

$$Z^{\mu\nu\cdots} = \int dP p^\mu p^\nu \cdots g^{\mathrm{B}}_0 \qquad Z^{\alpha\gamma\mu\nu}$$

★ $$S^{\lambda,\mu\nu}_{\mathrm{B}}(x) = 2\omega^{\mu\nu}Z^\lambda + \frac{2}{m^2}\omega^\nu{}_\alpha Z^{\lambda\alpha\mu} - \frac{2}{m^2}\omega^\mu{}_\alpha Z^{\lambda\alpha\nu}$$

Spin-1 thermodynamics

Expanded:

Boltzmann

$$Z^{\mu\nu\cdots} = \int dP p^\mu p^\nu \cdots g_0^{\mathrm{B}}$$

$$T^{\mu\nu}_{\mathrm{B}}(x) = 3Z^{\mu\nu} + (k^2 - \omega^2)Z^{\mu\nu} + \frac{\omega_{\alpha\beta}\omega^\beta_{\ \gamma}}{m^2} Z^{\alpha\gamma\mu\nu}$$

even in ω

$$S^{\lambda,\mu\nu}_{\mathrm{B}}(x) = 2\omega^{\mu\nu}Z^\lambda + \frac{2}{m^2}\omega^\nu_{\ \alpha}Z^{\lambda\alpha\mu} - \frac{2}{m^2}\omega^\mu_{\ \alpha}Z^{\lambda\alpha\nu}$$

odd in ω

Bose-Einstein

$$Z_2^{\mu\nu\cdots} = \int dP\, p^\mu p^\nu \cdots g_0^{\mathrm{BE}}(1 + g_0^{\mathrm{BE}})(1 + 2g_0^{\mathrm{BE}})$$

$Z_1^{\mu\nu\cdots}$

$$T^{\mu\nu}_{\mathrm{BE}}(x) = 3Z^{\mu\nu} + (k^2 - \omega^2)Z_2^{\mu\nu} + \frac{\omega_{\alpha\beta}\omega^\beta_{\ \gamma}}{m^2} Z_2^{\alpha\gamma\mu\nu}$$

$$S^{\lambda,\mu\nu}_{\mathrm{BE}}(x) = 2\omega^{\mu\nu}Z_1^\lambda + \frac{2}{m^2}\omega^\nu_{\ \alpha}Z_1^{\lambda\mu\alpha} - \frac{2}{m^2}\omega^\mu_{\ \alpha}Z_1^{\lambda\nu\alpha}$$

Full expressions are also available: spin generalization of the pressure current $P\beta^\mu$

$$\mathcal{N}^\mu_{\mathrm{B}} = \int dP\, p^\mu (g_-^{\mathrm{B}} + g_0^{\mathrm{B}} + g_+^{\mathrm{B}})$$

$$\mathcal{N}^\mu_{\mathrm{BE}} = \int dP\, p^\mu \left[\ln(1 + g_-^{\mathrm{BE}}) + \ln(1 + g_0^{\mathrm{BE}}) + \ln(1 + g_+^{\mathrm{BE}})\right]$$

Nonlinear causality and stability

[Abboud, Gavassino, Singh, Speranza, PRD 112 (2025)]

[Bhadury, Drogosz, Florkowski, Kar, V.M., PRD 114 (2026)]

$$M^\lambda = \left(Z\frac{\partial}{\partial\xi} - Z_\mu\frac{\partial}{\partial\beta_\mu} + \frac{1}{2}Z_{\mu\nu}\frac{\partial}{\partial\omega_{\mu\nu}} \right)^2 \mathcal{N}^\lambda$$

If M^λ is future-oriented and timelike for any non-vanishing real elements Z, Z_μ, $Z_{\mu\nu}$, the theory is nonlinearly causal and stable

spin 1/2

$$\mathcal{N}^\mu_{\rm B} = \sum_{i,j=\pm}\int dP\, p^\mu\, g^{ij}_{\rm B}$$

$$\mathcal{N}^\lambda_{\rm FD} = -\sum_{i,j=\pm}\int dP\, p^\lambda \ln(1-g^{ij}_{\rm FD})$$

spin 1

$$\mathcal{N}^\mu_{\rm B} = \int dP\, p^\mu (g^{\rm B}_- + g^{\rm B}_0 + g^{\rm B}_+)$$

$$\mathcal{N}^\mu_{\rm BE} = \int dP\, p^\mu \left[\ln(1+g^{\rm BE}_-) + \ln(1+g^{\rm BE}_0) + \ln(1+g^{\rm BE}_+)\right]$$

Valid only if all orders of ω are considered!

Divergence-type structure of the theory:

$$\mathcal{N}^\lambda = -\frac{\partial\chi}{\partial\beta_\lambda}, \qquad N^\lambda = -\frac{\partial^2\chi}{\partial\xi\partial\beta_\lambda}, \qquad T^{\lambda\mu} = \frac{\partial^2\chi}{\partial\beta_\lambda\partial\beta_\mu}, \qquad S^{\lambda\mu\nu} = -\frac{\partial^2\chi}{\partial\beta_\lambda\partial\omega_{\mu\nu}}$$

Generating functions χ for spin 1/2:

$$\chi_{\rm B} = \sum_{i,j=\pm}\int dP\, g^{ij}_{\rm B}(x,p),$$

$$\chi_{\rm FD} = \sum_{i,j=\pm}\int dP \left[\frac{1}{2}\left(\ln\frac{1-g^{ij}_{\rm FD}}{g^{ij}_{\rm FD}}\right)^2 + {\rm Li}_2\left(\frac{g^{ij}_{\rm FD}-1}{g^{ij}_{\rm FD}}\right) + \frac{\pi^2}{6}\right]$$

[Kar, V.M., 2511.09580; 2606.15364]

Summary

★ Local-equilibrium Wigner function constructed for spin-1/2 and 1

★ Generalized thermodynamic relations satisfied

★ Proper normalization (and direction) of the (mean) polarization vector

★ Divergence-type theory with nonlinearly causal and stable equations

★ **Spin-extended kinetic theory for massive particles in local equilibrium**