Late-time attractors in relativistic spin hydrodynamics in Gubser flow

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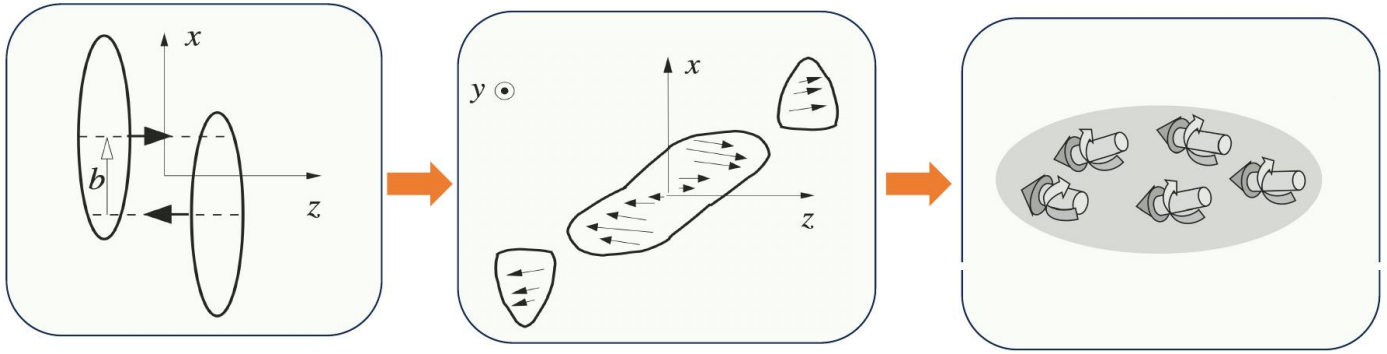
2026.11.19

Based on Li, X. Ren, Wang, Pu, Phys. Rev. D 114, 074029

Outline

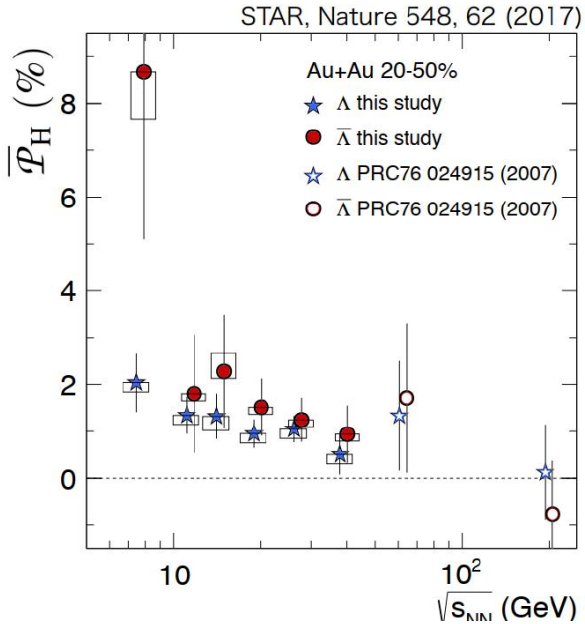
- **Background**
- **Two problems** in spin hydrodynamics
- **Attractors** in Gubser flow
- **Summary**

Spin polarization in relativistic heavy-ion collisions



- Non-central collisions: huge global orbital angular momentum, $L \sim 10^5\ \hbar$
- Spin–orbit coupling ⇒ polarization of Λ hyperons and spin alignment of vector mesons

  Liang, Wang, PRL (2005); PLB (2005);
  Gao et al., PRC (2008)
- Global polarization of Λ, $\bar{\Lambda}$ measured by STAR ⇒ described by thermal vorticity



STAR, Nature 548, 62 (2017)

Spin is a new degree of freedom in the QGP!

Relativistic spin hydrodynamics

▶ Conventional hydrodynamics

Conservation equations

$$\partial_\mu \Theta^{\mu\nu} = 0, \quad \partial_\lambda J^{\lambda\mu\nu} = 0,$$

Total angular momentum contains only the orbital part:

$$J^{\lambda\mu\nu} = x^\mu \Theta^{\lambda\nu} - x^\nu \Theta^{\lambda\mu}$$

▶ Relativistic spin hydrodynamics ⟶ A macroscopic theory describing spin

Total angular momentum contains both contributions:

$$J^{\lambda\mu\nu} = \underbrace{x^\mu \Theta^{\lambda\nu} - x^\nu \Theta^{\lambda\mu}}_{\text{orbital part}} + \underbrace{\Sigma^{\lambda\mu\nu}}_{\text{spin part}}$$

Conservation of J then implies

$$\nabla_\lambda \Sigma^{\lambda\mu\nu} = -2\Theta^{[\mu\nu]} \neq 0$$

spin and orbital angular momentum are continuously interconverted.

Two problems of spin hydrodynamics

▶ Problem I: fast decay of spin density

- its imprint on the freeze-out hypersurface may be negligible.
- one may even question whether spin density S is a meaningful hydrodynamic variable.

Expected decay with proper time

Charge density n	power-law decay
spin density S	exponential decay

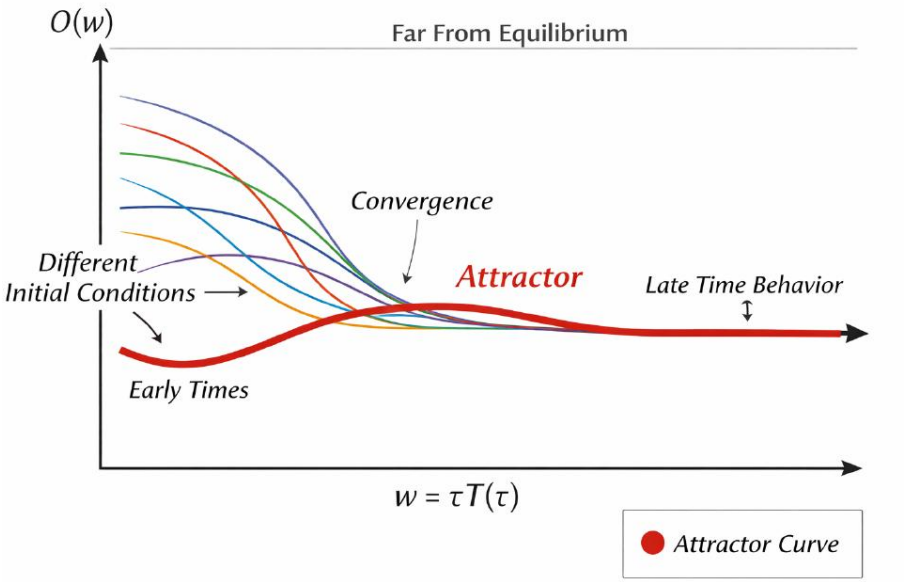
Wang, Fang, Pu, PRD(2021)

▶ Problem II: unknown initial density

- There is no established framework to fix the initial condition of the spin density S.
- Even its order of magnitude in the initial state is essentially unknown.

… unless the spin sector possesses a late-time attractor!

Hydrodynamic attractors

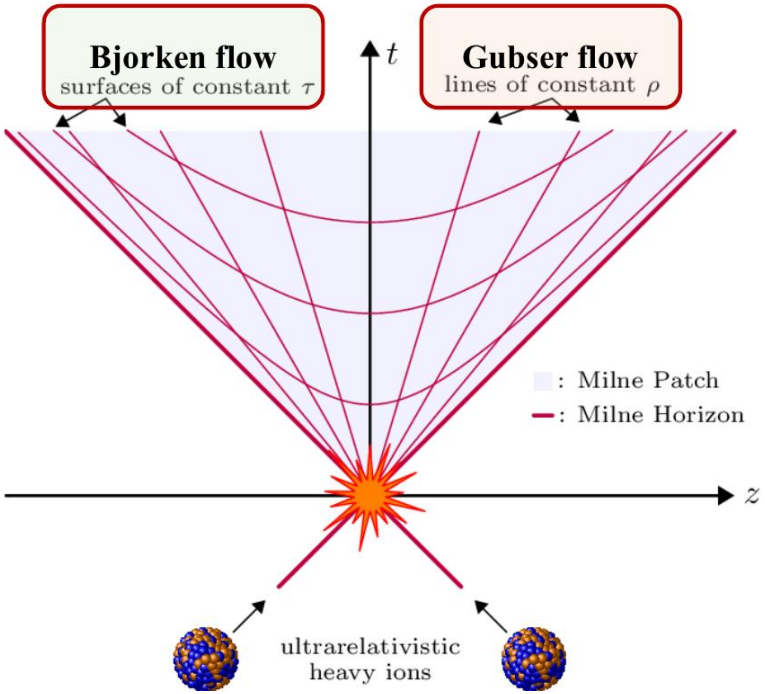


- Solutions with different initial conditions converge to one universal attracting curve
- Non-hydrodynamic modes damp out quickly
- Hydrodynamics becomes applicable well before local equilibrium is reached.
- Memory of the microscopic initial state is washed out

Attractor ⇒ late-time evolution becomes universal,
largely insensitive to the unknown initial state

Heller, Spalinski, PRL (2015); Romatschke, PRL (2018); Denicol, Noronha, PRD (2018);
Strickland, JHEP (2018); Almaalol, Kurkela, Strickland, PRL (2020); …

Theoretical framework -- Gubser flow



- Bjorken flow: boost invariant, but no transverse expansion
- Gubser flow: analytic 1+1d flow with radial (transverse) expansion
- Key — Weyl transformation to $dS_3 \times \mathbb{R}$

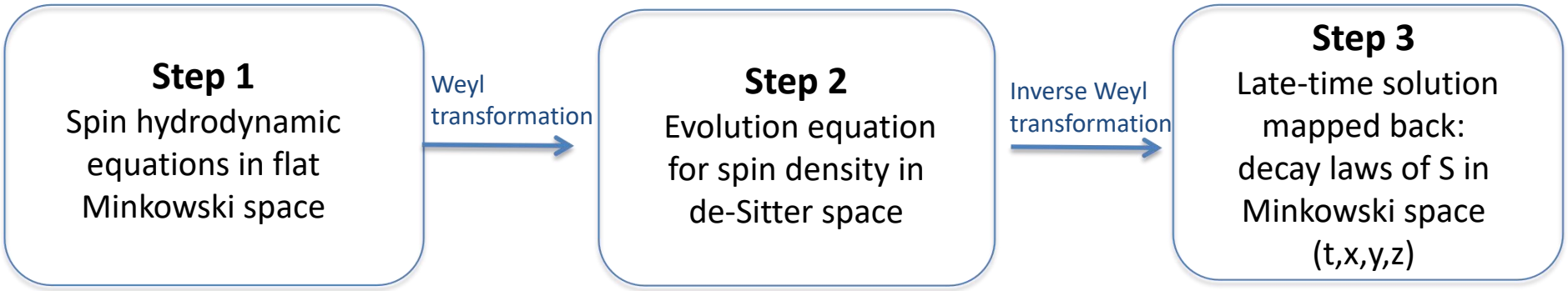
$$d\hat{s}^2 = d\rho^2 - \cosh^2\rho\,(d\theta^2 + \sin^2\theta\, d\phi^2) - d\eta^2$$

where the fluid is at rest, $\hat{u} = (1,0,0,0)$:

Gubser, PRD (2010);

Gubser, Yarom, NPB (2011)

Our strategy



Does the spin density S decay as a power law (like baryon number density $n \sim \tau^{-1}$) or exponentially (negligible at chemical freeze-out)?

Master equation in canonical form

▶ Conservation equation (angular momentum conservation for $\hat{S}$):

$$\partial_\rho \hat{S}^{\phi\eta} + 3\tanh\rho\,\hat{S}^{\phi\eta} + 2\hat{\varphi}^{\phi\eta} = 0$$

▶ Constitutive equation (minimal causal theory):

$$0 = \hat{\partial}_\rho \hat{\phi}^{\varphi\eta} + \left(\frac{11}{3}\tanh\rho + \frac{\tau}{\tau_\phi}\right)\hat{\phi}^{\varphi\eta} - \frac{4\tau^4\gamma_s}{\hat{\chi}\tau_\phi}\hat{S}^{\varphi\eta}.$$

χ, χ' are two spin susceptibilities

$$\hat{S}^{\mu\nu} = \hat{\chi}\hat{\omega}^{\mu\nu} + \hat{\chi}'\hat{u}_\lambda(\hat{\omega}^{\lambda\mu}\hat{u}^\nu - \hat{\omega}^{\lambda\nu}\hat{u}^\mu).$$

▶ The master equation for $\hat{S}$

$$0 = \frac{d^2\hat{S}}{d\rho^2} + \left(\frac{20}{3}\tanh\rho + \hat{\tau}_\phi^{-1}\right)\frac{d\hat{S}}{d\rho} + \left(3 + 8\tanh^2\rho + 3\hat{\tau}_\phi^{-1}\tanh\rho + 8\hat{\gamma}_s\hat{\tau}_\phi^{-1}\hat{\chi}^{-1}\right)\hat{S}$$

Late-time analysis

Late time $\rho \to +\infty$ ⇒ $w = 1/(1-\tanh\rho) \to \infty$ (near $r \to 0$, midrapidity)

▶ Auxiliary function — logarithmic derivative of $\hat{S}$:

$$f(w) = \frac{2w-1}{\hat{S}}\frac{d\hat{S}}{dw}$$

- $f \sim$ const ⇒ power-law decay
- $f \sim -w^{\mathrm{m}}$ ⇒ exponential decay

▶ Power-law parametrization of transport coefficients:

$$\hat{\tau}_\phi^{-1} = \alpha\, w^{\Delta_1}, \qquad 8\hat{\gamma}_s\hat{\tau}_\phi^{-1}\hat{\chi}^{-1} = \beta\, w^{\Delta_2}, \qquad \alpha, \beta > 0,\ \Delta_1 > 0$$

$\widehat{\tau_\phi}^{-1}$: effective Knudsen number Kn

D.-L. Wang, L. Yan, and S. Pu, PRD(2025)
J.-P. Blaizot and L. Yan, PRC(2021)

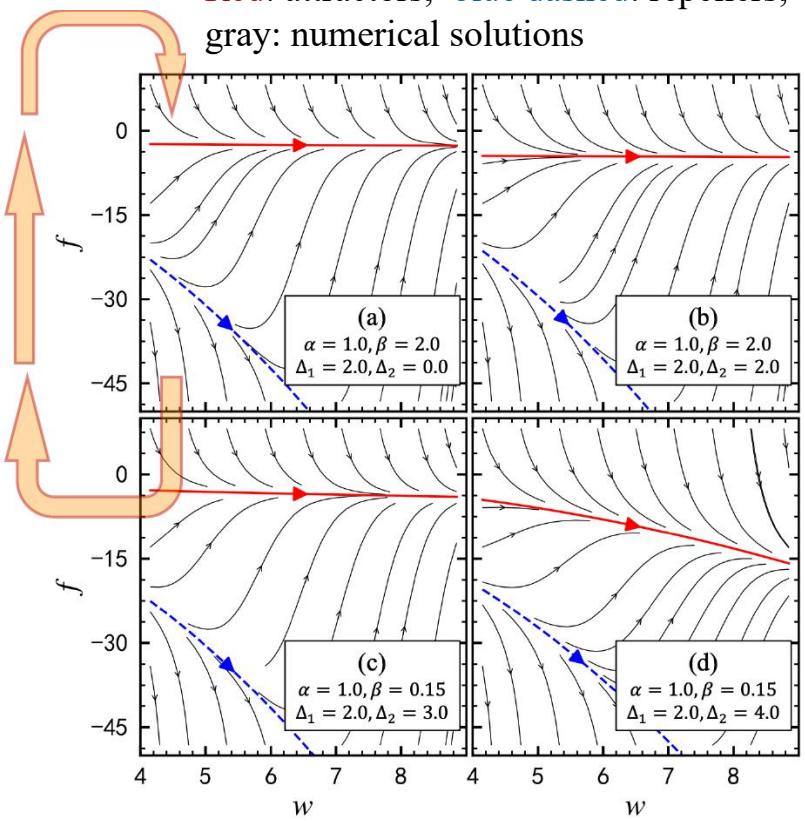
▶ The equation for $f(w)$ is of Riccati type: $v' + v^2 = I(w)$

⇒ slow-roll expansion gives systematic large-w asymptotic solutions

M. Strickland, J. Noronha, and G. Denicol, PRD(2018)
G. S. Denicol and J. Noronha, PRD(2019)

Results: attractors and repellers

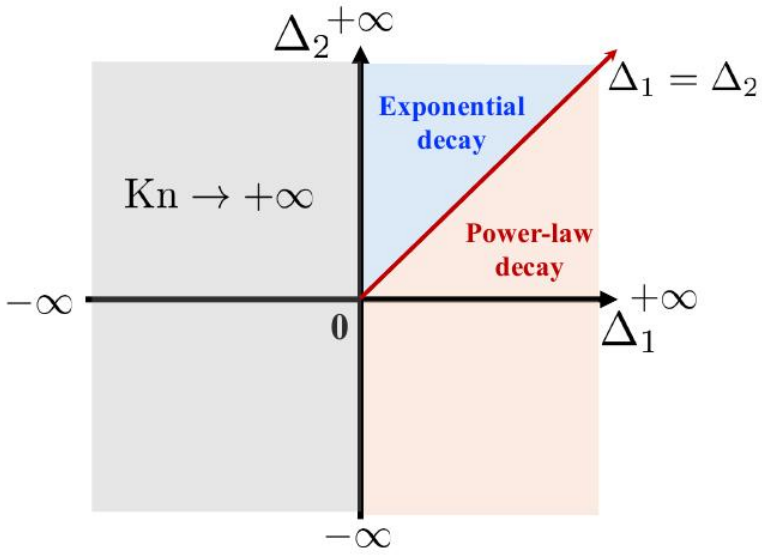
Red: attractors; blue dashed: repellers;
gray: numerical solutions



$$f(w) = \frac{2w-1}{\hat{S}}\frac{d\hat{S}}{dw}$$

- $f \sim$ const ⇒ power-law decay;
- $f \sim -w^{\mathrm{m}}$ ⇒ exponential decay
- Generic initial conditions converge to a unique attractor
- $f \to -\infty$ is not a singularity: $\hat{S}$ crosses zero, f re-enters from $+\infty$ and rejoins the attractor

Results: power-law vs exponential decay



- Only two parameter regions yield power-law decay:
  - (i) $\Delta_1 > \Delta_2$: $\hat{S} \propto w^{-3/2}$
  - (ii) $\Delta_1 = \Delta_2$: $\hat{S} \propto w^{-(3\alpha+\beta)/(2\alpha)}$
- Otherwise $\hat{S} \sim e^{-w^{\wedge}m}$: exponential decay

parameter region	asymptotic branches $f_\pm(w)$
$\Delta_1 > \Delta_2$	$f_+ \to -3, \quad f_- = -\alpha w^{\Delta_1}$
$\Delta_1 = \Delta_2$	$f_+ \to -\left(3 + \frac{\beta}{\alpha}\right), \quad f_- = -\alpha w^{\Delta_1}$
$\Delta_1 < \Delta_2 < 2\Delta_1$	$f_+ = -\frac{\beta}{\alpha} w^{\Delta_2 - \Delta_1}, \quad f_- = -\alpha w^{\Delta_1}$
$\Delta_1 < \Delta_2 = 2\Delta_1$	$f_\pm = \frac{1}{2}\left(-\alpha \pm \sqrt{\alpha^2 - 4\beta}\right) w^{\Delta_1}$
$\Delta_1 < 2\Delta_1 < \Delta_2$	$f_\pm = \pm i\sqrt{\beta} w^{\Delta_2/2} - \frac{\alpha}{2} w^{\Delta_1}$

Power-law decay ⇒ spin density may survive at chemical freeze-out!

Results: power-law decay in Minkowski space

Mapping back to Minkowski space (t, x, y, z): nonzero components S^{tx}, S^{ty}, S^{zx}, S^{zy}

▶ Case (i) $\Delta_1 > \Delta_2$:

$$L \gg \tau: \quad S^{tx}, S^{ty}, S^{zx}, S^{zy} \sim \frac{1}{\tau L^2}; \qquad \tau \gg L: \quad \sim \frac{L^4}{\tau^7}$$

For $L \gg \tau$ the decay in τ is as slow as the charge density $n \sim \tau^{-1}$ in Bjorken flow!

▶ Case (ii) $\Delta_1 = \Delta_2$:

$$L \gg \tau: \quad S^{tx}, \cdots \sim \frac{\tau^{(\beta-\alpha)/\alpha}}{L^{(\beta+2\alpha)/\alpha}}$$

$\beta = \alpha$: spin density independent of τ;

$\beta > \alpha$: spin density can even grow with τ

Physical picture — spin–orbit exchange vs expansion

Evolution of $\hat{S}$ = local balance of total angular momentum:

$$\partial_\rho \hat{S} \quad + \underbrace{3\tanh\rho\,\hat{S}}_{\text{expansion dilution}} \quad = \underbrace{-2\hat{\varphi}}_{\text{spin–orbit transfer}} \quad \sim \frac{\beta}{\alpha} w^{\Delta_2-\Delta_1}\hat{S}$$

- $\Delta_1 > \Delta_2$: exchange parametrically fast;

late-time evolution governed by expansion term ⇒ power-law dilution

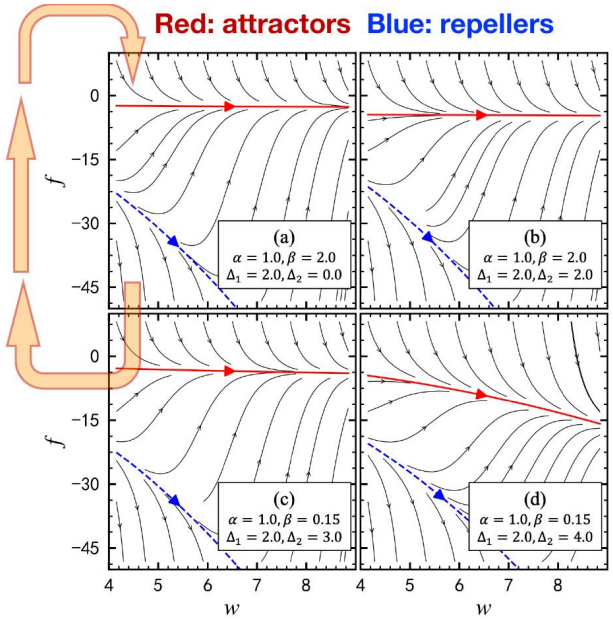
- $\Delta_1 = \Delta_2$: exchange remains comparable to expansion;

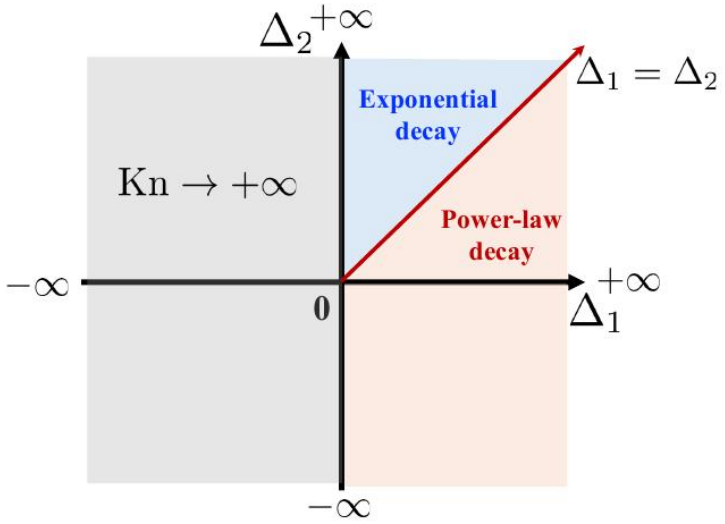
  $\gamma_s \sim \beta/\alpha \times \chi/8 \times \tau^{-1}$

  - $\beta > \alpha$: source decays slower than τ^{-1} ⇒ can counteract dilution
  - $\beta < \alpha$: source suppressed ⇒ usual power-law decay

Summary

- Two obstacles of spin hydrodynamics — spin density decays too fast and unknown initial conditions — could both be circumvented if the system has late-time attractors.
- We studied minimal causal spin hydrodynamics under Gubser flow (transverse expansion + boost invariance).
- Power-law decay regions identified: $\Delta_1 \geq \Delta_2$. For $L \gg \tau$ the spin density decays as slowly as $1/\tau$ — comparable to ordinary hydrodynamic variables.





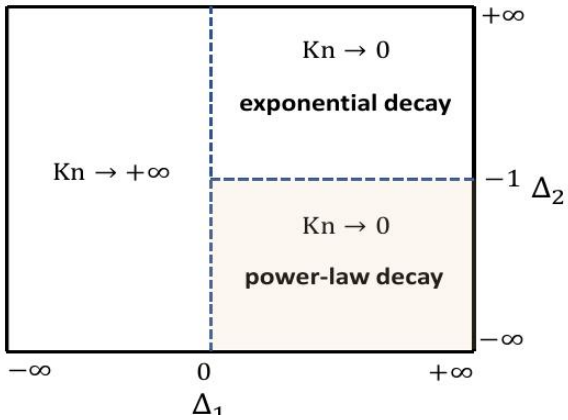
The spin density can behave as a conventional hydrodynamic quantity governed by the late-time scaling — and survive at chemical freeze-out.

Thank you!

Backup: previous test in Bjorken flow

- Late-time attractors observed in minimal causal spin hydrodynamics in Bjorken flow
- Power-law decay of the spin density found for suitable parameters
- But: Bjorken flow is boost invariant with no transverse expansion

⇒ motivates the present Gubser-flow study



D.-L. Wang, L. Yan, S. Pu, PRD 111, 034033 (2025)

Backup: full list of asymptotic solutions

parameter region	asymptotic branches $f_\pm(w)$
$\Delta_1 > \Delta_2$	$f_+ \to -3, \quad f_- = -\alpha w^{\Delta_1}$
$\Delta_1 = \Delta_2$	$f_+ \to -\left(3 + \frac{\beta}{\alpha}\right), \quad f_- = -\alpha w^{\Delta_1}$
$\Delta_1 < \Delta_2 < 2\Delta_1$	$f_+ = -\frac{\beta}{\alpha} w^{\Delta_2 - \Delta_1}, \quad f_- = -\alpha w^{\Delta_1}$
$\Delta_1 < \Delta_2 = 2\Delta_1$	$f_\pm = \frac{1}{2}\left(-\alpha \pm \sqrt{\alpha^2 - 4\beta}\right) w^{\Delta_1}$
$\Delta_1 < 2\Delta_1 < \Delta_2$	$f_\pm = \pm i\sqrt{\beta} w^{\Delta_2/2} - \frac{\alpha}{2} w^{\Delta_1}$

Complex (imaginary) branches ⇒ oscillatory behavior of $\hat{S}$.

Uniqueness of the attractor: $v' + v^2 = I \;\Rightarrow\; V'' - IV = 0$, $V = c_1\, e^{\wedge} I_0 + c_2\, e^{\wedge} {-I_0}$; any $c_1 \neq 0$ flows to $v \to +\sqrt{I_0}$.

Backup: setup and Weyl rescaling

- Weyl rescaling rules:

$$\hat{g}_{\mu\nu} = \tau^2 g_{\mu\nu}, \quad \hat{\Theta}^{\mu\nu} = \tau^6 \Theta^{\mu\nu}, \quad \hat{S}^{\mu\nu} = \tau^5 S^{\mu\nu}$$

- Angular momentum conservation in $dS_3 \times \mathbb{R}$:

$$\hat{\nabla}_\lambda(\hat{u}^\lambda \hat{S}^{\mu\nu}) + 2\hat{\varphi}^{\mu\nu} - \tau^{-1}\hat{\nabla}_\sigma\tau\left(\hat{g}^{\mu\sigma}\hat{u}_\alpha\hat{S}^{\alpha\nu} + \hat{g}^{\nu\sigma}\hat{u}_\alpha\hat{S}^{\mu\alpha} + \hat{u}^\mu\hat{S}^{\nu\sigma} - \hat{u}^\nu\hat{S}^{\mu\sigma}\right) = 0$$

- Evolution of the surviving component:

$$\partial_\rho \hat{S}^{\phi\eta} + 3\tanh\rho\,\hat{S}^{\phi\eta} + 2\hat{\varphi}^{\phi\eta} = 0$$

- Constitutive equation:

$$\partial_\rho \hat{\varphi}^{\phi\eta} + \left(\tfrac{11}{3}\tanh\rho + \tfrac{\tau}{\tau_\phi}\right)\hat{\varphi}^{\phi\eta} - \tfrac{4\tau^4\gamma_s}{\hat{\chi}\tau_\phi}\hat{S}^{\phi\eta} = 0$$

- Other components: $\hat{S}$, $\hat{S}$, $\hat{S}$, $\hat{S}$ mutually coupled (inconsistent with the Gubser profile); $\hat{S}$ decouples from dissipation (no relaxation information)