# Topological charge density in the polarized nucleon

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# 1. Introduction ---- Topological charge



**Topological Charge density**

$$q_{\rm top}(x) = \frac{\alpha_s}{4\pi} {\rm Tr}\, F^{\mu\nu}(x) \widetilde{F}_{\mu\nu}(x).$$

- The spacetime integral gives the winding number of the gluon configuration:

$$\int d^4x\, q_{\rm top}(x) = n \in \mathbb{Z}.$$

- Topological charge density is P- and CP- odd.

- In HIC, topological charge can locally fluctuate event by event.

  → Chiral Magnetic Effect

[Kharzeev, McLerran, Warringa. *Phys.Rev.D*. (2008)]

[Fukushima, Kharzeev, Warringa. *Phys.Rev.D*. (2008)]

# 1. Introduction ---- Topological charge

**Chiral anomaly**

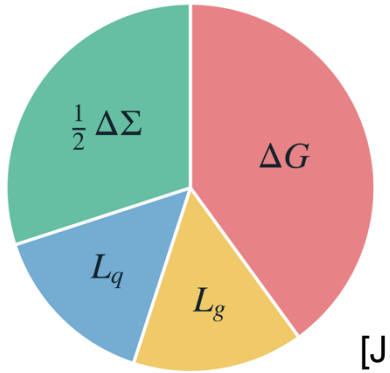
$$\partial_\mu J_5^\mu = 2N_f q_{\rm top} + 2i \sum_f m_f \bar{\psi}_f \gamma_5 \psi_f .$$

- Topological charge is related to the axial current of quarks, or quark spin.

$$\frac{1}{2} J_5^3 = \frac{1}{2} \psi^\dagger \gamma^0 \gamma^3 \gamma_5 \psi = \psi^\dagger \frac{\Sigma^3}{2} \psi = s_q .$$

Cf. Proton spin decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$




[Jaffe, Manohar. *Nucl.Phys.B*. (1990)]

# 1. Introduction ---- Topological charge

**In a nucleon,**

[Hatta. *Phys.Rev.D,* (2020)]

**Topological Form Factor**

$$\frac{1}{M_N}\langle p'|q_{\rm top}|p\rangle = \bar{u} i\gamma_5 u\, F_Q(t), \quad 2N_f F_Q(t) = 2G_A^{(0)}(t) + \frac{t^2}{2M_N} G_P^{(0)}(t),$$

**Due to the anomalous Ward identity in the chiral limit**

**Cf. Axial Form Factor**

$$\langle p'|J_5^\mu|p\rangle = \bar{u}\left[\gamma^\mu\gamma_5 G_A^{(0)}(t) + \frac{\Delta^\mu}{2M_N}\gamma_5 G_P^{(0)}(t)\right] u.$$

**Forward limit**

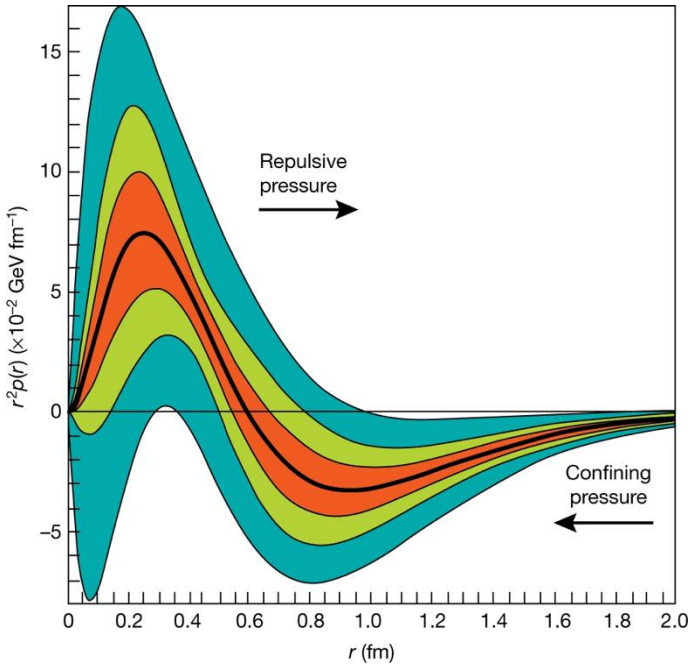
$$G_A^{(0)}(0) = \Delta\Sigma,$$
$$F_Q(0) = \Delta\Sigma/N_f.$$

# 1. Introduction ---- Topological charge

**Form Factors are Fourier conjugate of the spatial distributions**

| Form Factor (FF) | Spatial Distributions |
|---|---|
| Electromagnetic FF | Charge |
| Gravitational FF | Energy, pressure, etc… |
| Topological FF | Topological charge |

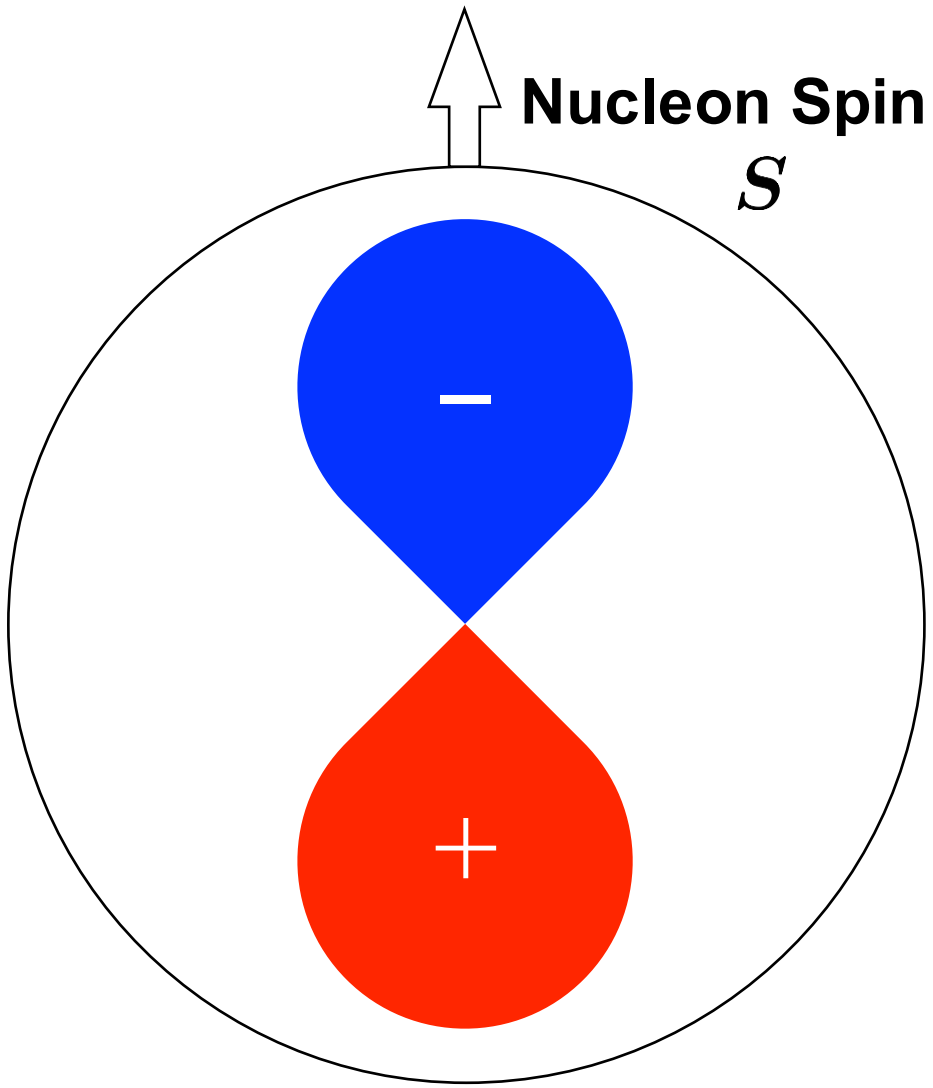
[Burkert, et al. *Nature*. (2018)]




**Question: How does the topological charge distribute in a nucleon?**

# 2. Results ---- Topological Dipole

**The answer is dipole!  Polarized nucleon is a "Topological Dipole".**




In the Breit frame, i.e., $\Delta^\mu = (0, \boldsymbol{\Delta})$

$$\bar{u}(p', s) i\gamma_5 u(p, s) = -i\boldsymbol{S} \cdot \boldsymbol{\Delta}.$$

The Fourier transform of

$$\frac{1}{M_N} \langle p' | q_{\rm top} | p \rangle = \bar{u} i\gamma_5 u \, F_Q(t),$$

reads

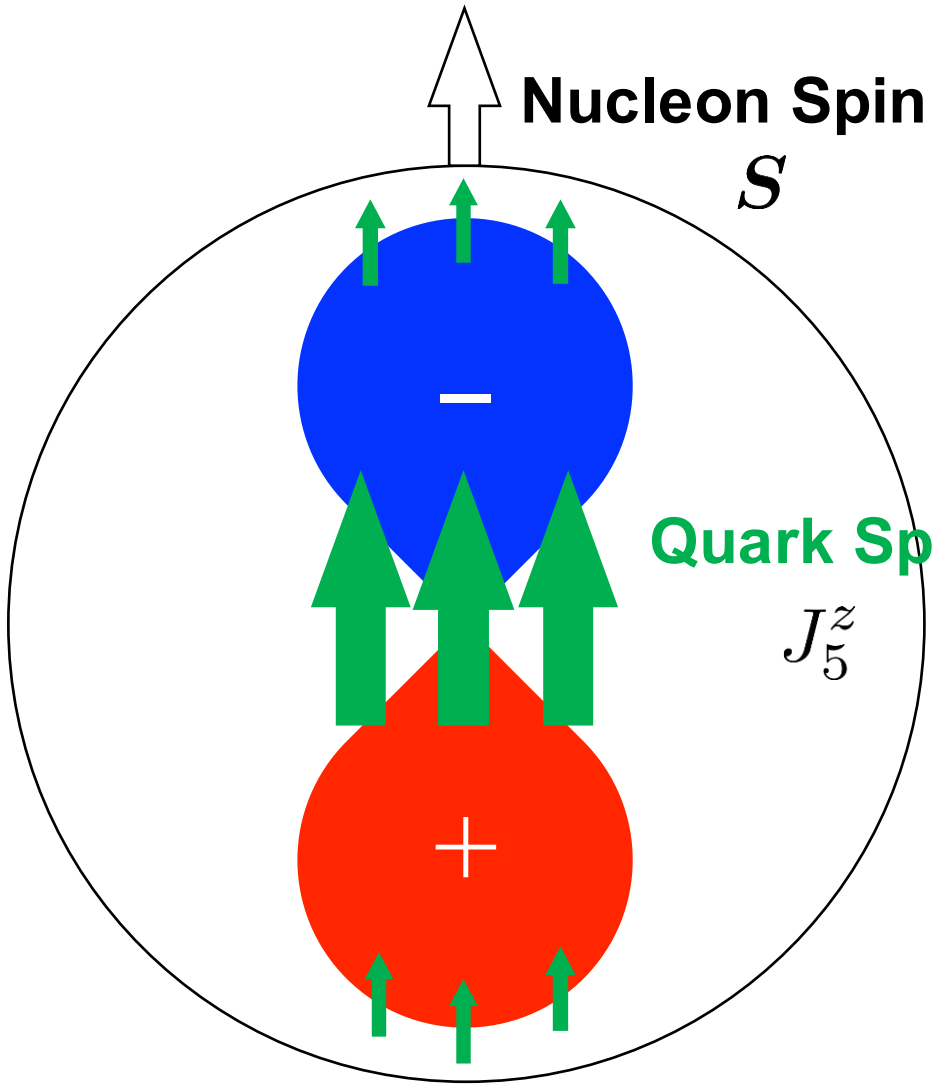
$$q_{\rm top}(\boldsymbol{r}) = \boldsymbol{S} \cdot \hat{\boldsymbol{r}} \, \widetilde{F}'_Q(r).$$

The dipole moment is negative:

$$d_Q = -F_Q(0) = -\Delta\Sigma/N_f < 0.$$

# 2. Results ---- Why negative?

**The reason why the upper part is negative**




In the static situation, $\nabla \cdot \boldsymbol{J}_5 = 2N_f q_{\mathrm{top}}$.

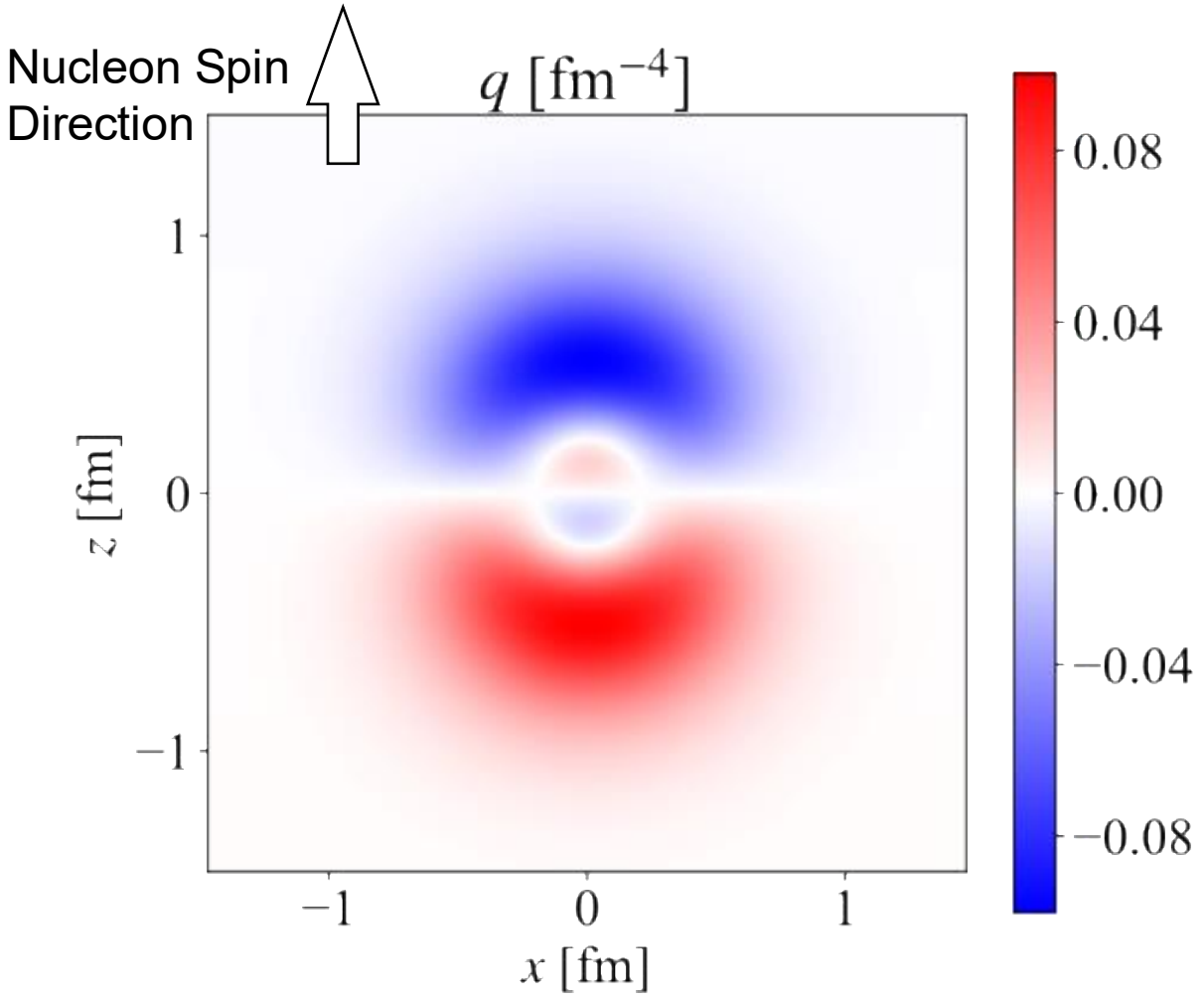
The axial current $J_5^z$ is quark spin.

⇓

Divergence of quark spin is $q_{\mathrm{top}}$.

# 3. Model Realization

**In the Skyrme model, topological dipole manifests as**




- The dipole moment is

$$d_Q = \int d^3\boldsymbol{r}\, \boldsymbol{S}\cdot\boldsymbol{r}\, q_{\rm top}(\boldsymbol{S},\boldsymbol{r})$$

- It is also related to

$$d_Q = -\frac{F_Q(0)}{2} = -\frac{\Delta\Sigma}{2N_f}$$

- In our model calculation,

$$d_Q = -0.077$$

# 4. Discussions ---- How to measure

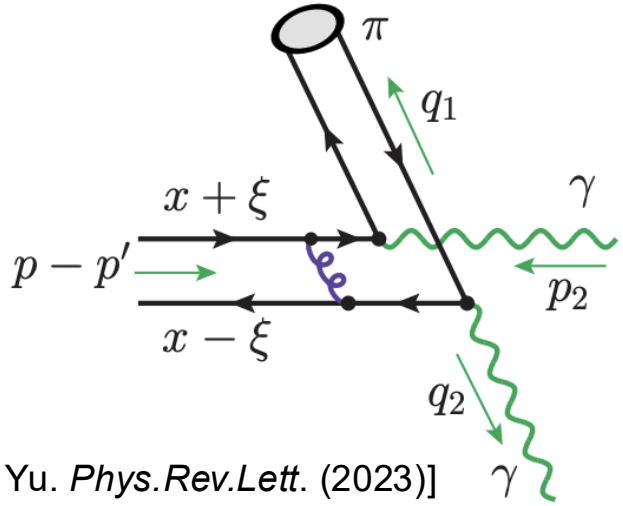
**Exclusive photoproduction in SDHEP** $N(p)+\gamma(p_2)\to N'(p')+\pi(q_1)+\gamma(q_2).$

- This process is sensitive to the twist-2 axial GPDs,

$$\widetilde{H}^{(0)}(x,\xi,t),\ \widetilde{E}^{(0)}(x,\xi,t).$$

- Their moments give $G_A^{(0)}(t),\ G_P^{(0)}(t),$ and

$$2N_f F_Q(t)=2G_A^{(0)}(t)+\frac{t^2}{2M_N}G_P^{(0)}(t).$$



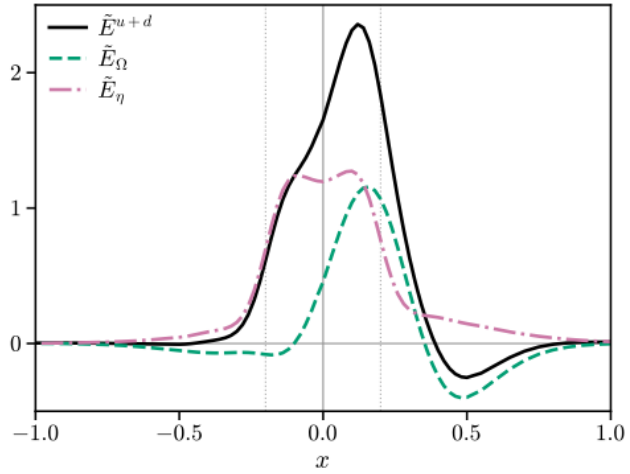

[Qiu, Yu. *Phys.Rev.Lett.* (2023)]

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- GPD $\widetilde{E}^{(0)}(x,\xi,t)$ contains chiral anomaly information.

[Bhattacharya, Hatta, Vogelsang. *Phys.Rev.D.* (2023)]

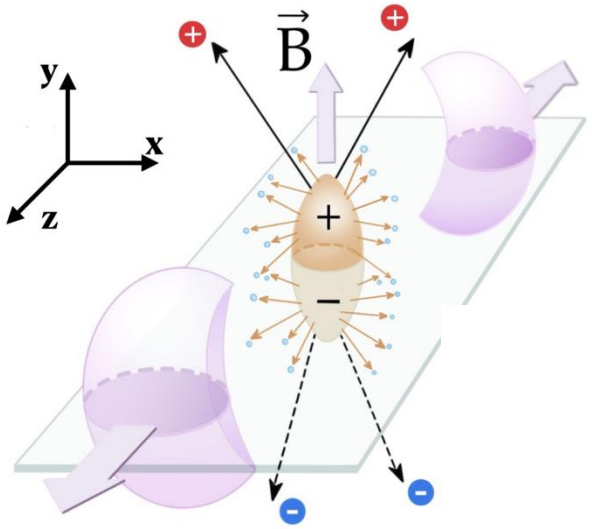
- Can we see chiral anomaly effect in EIC?



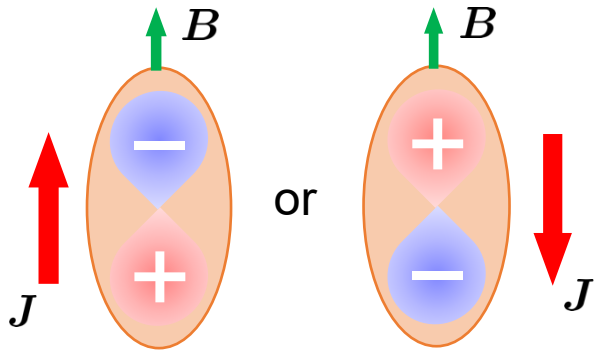

[Minamiguchi, Uji. arXiv:2611.07869]

# 4. Discussions ---- Heavy Ion Collision

**Local Parity Violation has been searched through the Chiral Magnetic Effect**




[Kharzeev, Liao, Tribedy. (2024)]




$$\boldsymbol{J} = \frac{e^2 \mu_5}{2\pi^2} \boldsymbol{B}.$$

- CME is the macroscopic manifestation of LPV.
- Electric current flows parallel or anti-parallel to the magnetic field.
- The direction of the current fluctuates event-by-event.

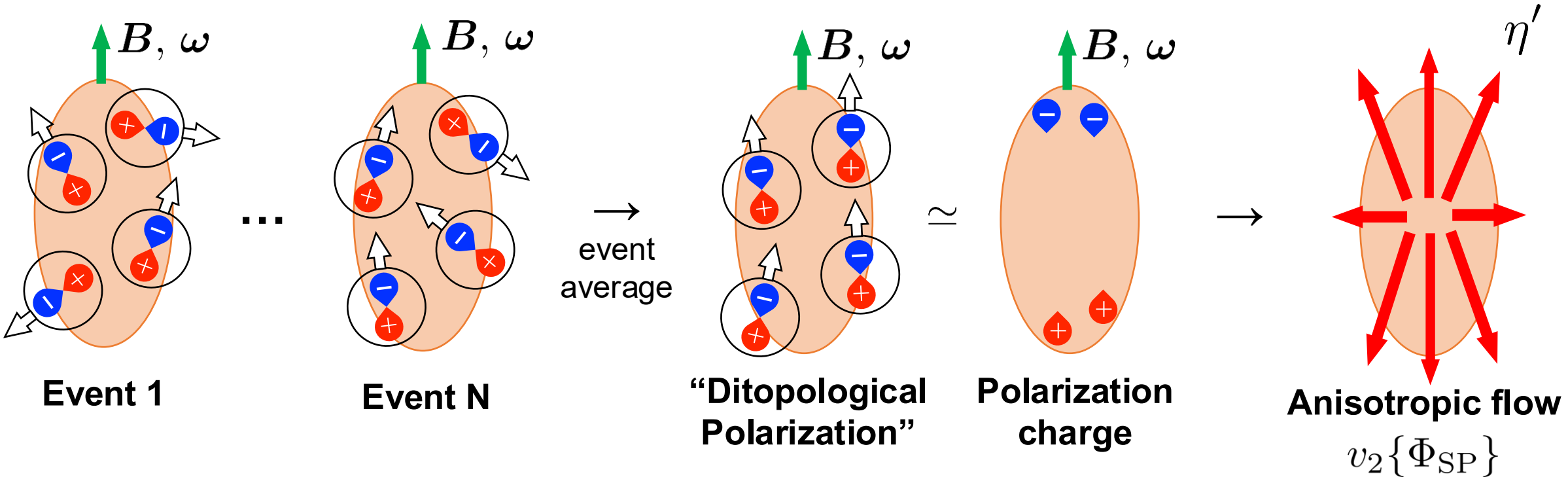
- The CME signal is the P-even flow of the charged hadrons.

$$\langle \Delta a_1 \rangle = 0, \quad \text{but} \quad \langle (\Delta a_1)^2 \rangle \neq 0. \quad \rightarrow \quad \Delta\gamma$$

# 4. Discussions ---- Heavy Ion Collision

**Can we see topological dipole in HIC?**

- If the magnetic field or vorticity induces global nucleon spin alignment, hadronic matter may become the “dielectric polarization” of the topological dipole.




$$\mathcal{M}_{\eta'} \propto \sin(\phi - \Psi_{\rm SP}) \qquad |\mathcal{M}_{\eta'}|^2 \propto 1 - \cos 2(\phi - \Psi_{\rm SP})$$

# 5. Summary

- Topological charge is distributed like dipole-shape in a nucleon.
- GPDs have information about the topological dipole.
- We can measure the GPDs through hard exclusive processes.
- Topological dipole may make the anisotropic flow $v_2^{\eta'}\{\Phi_{\mathrm{SP}}\}$ in HIC.

  Cf. Conventional elliptic flow is $v_2\{\Phi_{\mathrm{PP}}\}$