重子的半轻衰变和形状因子

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preparing

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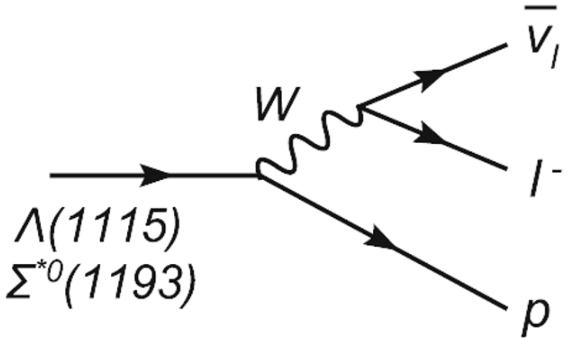

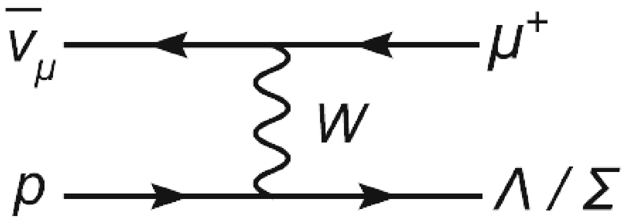

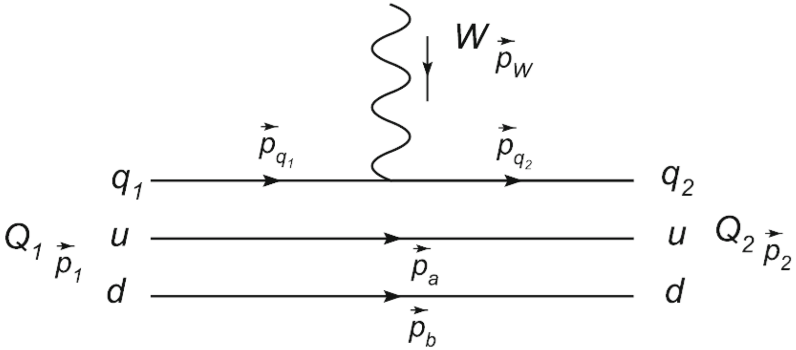

- 如何利用该顶点研究强子的性质？
- 如何利用该顶点寻找超出传统夸克模型的强子成分？

背景：研究超子物理的反应

$\gamma p \to KY^*, \pi p \to KY^*, e^+e^- \to Y\bar{Y}^*, ep \to eKY^*, pp \to KNY^*$

问题：多强子末态带来的困扰！

$K^-p \to Y^*$

问题：束流能量选择有限制！

背景：研究超子物理的反应

$$\gamma p \to KY^*, \pi p \to KY^*, e^+e^- \to Y\bar{Y}^*, ep \to eKY^*, pp \to KNY^*$$

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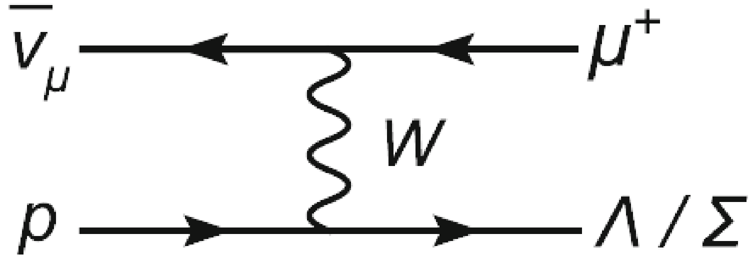
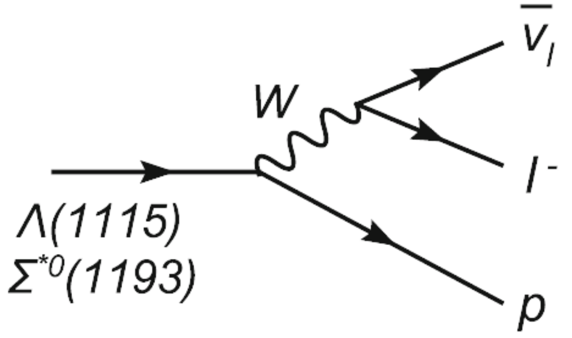
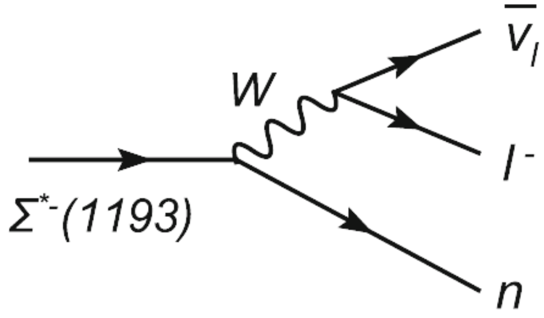
$$\bar{\nu}p \to l^+Y^*, \quad \Lambda_c \to \bar{\nu}l^-Y^*$$

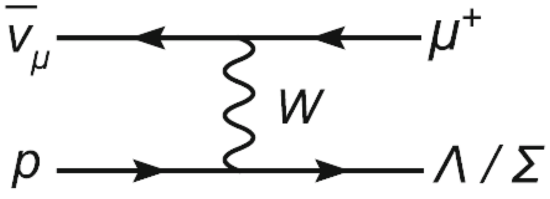


如何利用该顶点研究强子的性质？

超子反应振幅计算







$$d\sigma = \frac{(2\pi)^4}{2E_\nu}\frac{1}{2}\sum_{s_z^\nu,\, s_z^{N_1}}\sum_{s_z^l,\, s_z^{N_2}}|\mathcal{M}|^2\delta^{(4)}_{(p_\nu+p_{N_1}-p_l-p_{N_2})}\frac{d^3\mathbf{p}_{N_1}m_{N_1}}{(2\pi)^3E_{N_1}}\frac{d^3\mathbf{p}_l m_l}{(2\pi)^3E_l}$$

$$d\Gamma = \frac{(2\pi)^4}{2M_\Lambda}\frac{1}{2}\sum_{s_z^\Lambda}\sum_{s_z^\nu, s_z^l,\, s_z^p}|\mathcal{M}|^2\delta^{(4)}_{(p_\nu+p_l+p_p)}\frac{d^3\mathbf{p}_p m_p}{(2\pi)^3E_p}\frac{d^3\mathbf{p}_\nu}{(2\pi)^3 2E_\nu}\frac{d^3\mathbf{p}_l m_l}{(2\pi)^3E_l}$$

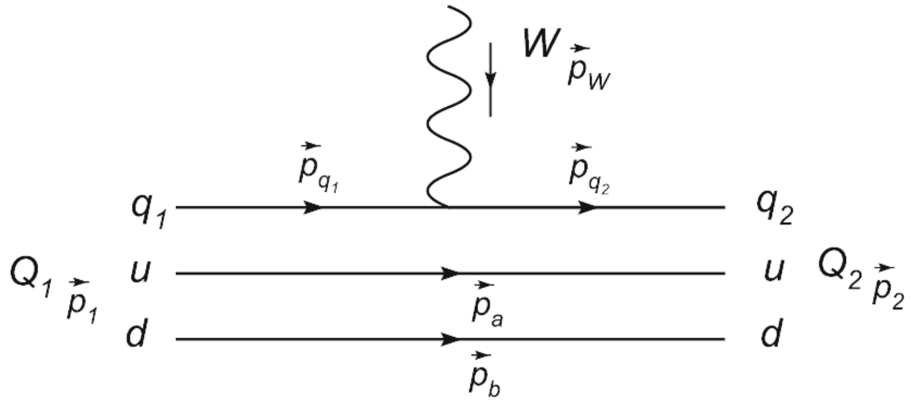
$$G_W^{\mu\nu} = \frac{-g^{\mu\nu}+p_W^\mu p_W^\nu/m_W^2}{p_W^2-m_W^2}$$

$$\mathcal{M}(s_\ell, s_{\bar\nu};\lambda_f,\lambda_i) = \frac{G_F}{\sqrt{2}}V_{\text{CKM}}\,L_-^\mu(s_\ell,s_{\bar\nu})\,g_{\mu\nu}\,H^\nu(\lambda_f,\lambda_i;P_i,P_f),$$

$$L_-^\mu(s_\ell,s_{\bar\nu}) = \bar{u}_\ell(p_\ell,s_\ell)\,\gamma^\mu(1-\gamma^5)\,v_{\bar\nu}(p_\nu,s_{\bar\nu})$$

$$H^\nu(\lambda_f,\lambda_i;P_i,P_f) \equiv \sum_{j=1}^{3}\langle B_f(P_f,\lambda_f)\,|J_j^\nu(0)|\,B_i(P_i,\lambda_i)\rangle$$

W-强子对的顶点计算



$$H^\nu(\lambda_f,\lambda_i;P_i,P_f) \equiv \sum_{j=1}^{3}\langle B_f(P_f,\lambda_f)\,|J_j^\nu(0)\,|\,B_i(P_i,\lambda_i)\rangle$$

$$H^\nu = H_V^\nu - H_A^\nu$$

基于强子层次的计算

$$H_V^\nu(\lambda_f,\lambda_i;P_i,P_f) = \bar{u}_f(P_f,\lambda_f)\left[f_1(q^2)\gamma^\nu - i\frac{f_2(q^2)}{M}\sigma^{\nu\alpha}q_\alpha + \frac{f_3(q^2)}{M}q^\nu\right]u_i(P_i,\lambda_i),$$

$$H_A^\nu(\lambda_f,\lambda_i;P_i,P_f) = \bar{u}_f(P_f,\lambda_f)\left[g_1(q^2)\gamma^\nu\gamma^5 - i\frac{g_2(q^2)}{M}\sigma^{\nu\alpha}q_\alpha\gamma^5 + \frac{g_3(q^2)}{M}q^\nu\gamma^5\right]u_i(P_i,\lambda_i),$$

基于夸克层次的计算

$$H^\nu(\lambda_f,\lambda_i;P_i,P_f) = \sum_{\kappa_f=1}^{3}\sum_{\kappa_i=1}^{3}(c_{\kappa_f}^{B_f,1/2})^* c_{\kappa_i}^{B_i,1/2}\, H_{\kappa_f\kappa_i}^\nu(\lambda_f,\lambda_i;P_i,P_f),$$

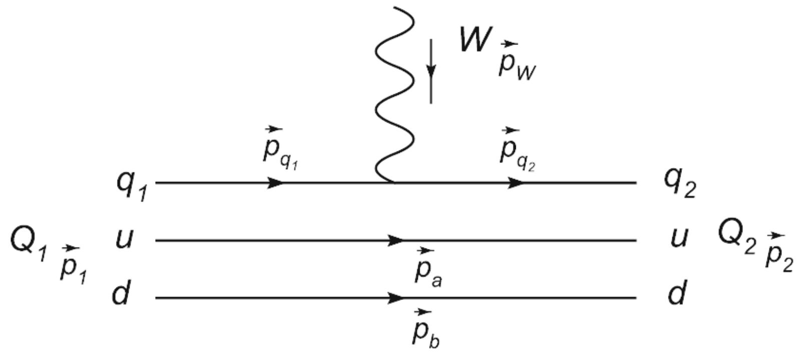
$$H_{\kappa_f\kappa_i}^\nu(\lambda_f,\lambda_i;P_i,P_f) \equiv \sum_{j=1}^{3}\left\langle B_{f,\kappa_f}(P_f,\lambda_f)\left|J_j^\nu(0)\right| B_{i,\kappa_i}(P_i,\lambda_i)\right\rangle$$

$$\left\langle \Psi_f\Phi_f(\lambda_f)\left|J_j^\nu(0)\right|\Psi_i\Phi_i(\lambda_i)\right\rangle = \sqrt{2E_i\,2E_f}\int\frac{d^3\boldsymbol{k}_\rho}{(2\pi)^3}\frac{d^3\boldsymbol{k}_\lambda}{(2\pi)^3}\frac{\sqrt{\mathcal{J}_f(\{\boldsymbol{p}_1',\boldsymbol{p}_2',\boldsymbol{p}_3'\},\boldsymbol{P}_f)}}{\sqrt{4e_je_j'}}\left(\prod_{r\neq j}\mathcal{S}_r\right)$$

$$\times\,\widetilde{\Psi}_f^*(\boldsymbol{k}_\rho',\boldsymbol{k}_\lambda')\,\widetilde{\Psi}_i(\boldsymbol{k}_\rho,\boldsymbol{k}_\lambda)\sum_{\alpha,\beta}\left\langle\Phi_f(\lambda_f)\left|\hat{F}_j^{q_fq_i}\hat{S}_j^{\alpha\beta}\right|\Phi_i(\lambda_i)\right\rangle[O_{W,j}^\nu]^{\alpha\beta}$$

W-强子对的顶点计算

基于夸克层次的计算



$$H^{\nu}(\lambda_f,\lambda_i;P_i,P_f)=\sum_{\kappa_f=1}^{3}\sum_{\kappa_i=1}^{3}(c_{\kappa_f}^{B_f,1/2})^{*}c_{\kappa_i}^{B_i,1/2}\,H^{\nu}_{\kappa_f\kappa_i}(\lambda_f,\lambda_i;P_i,P_f),$$

$$H^{\nu}_{\kappa_f\kappa_i}(\lambda_f,\lambda_i;P_i,P_f)\equiv\sum_{j=1}^{3}\left\langle B_{f,\kappa_f}(P_f,\lambda_f)\left|J^{\nu}_j(0)\right|B_{i,\kappa_i}(P_i,\lambda_i)\right\rangle$$

$$\left\langle\Psi_f\Phi_f(\lambda_f)\left|J^{\nu}_j(0)\right|\Psi_i\Phi_i(\lambda_i)\right\rangle=\underbrace{\sqrt{2E_i\,2E_f}}_{\text{Normalization Factor}}\int\underbrace{\frac{d^3\boldsymbol{k}_\rho}{(2\pi)^3}\frac{d^3\boldsymbol{k}_\lambda}{(2\pi)^3}}_{\text{Momentum Integration}}\underbrace{\frac{\sqrt{\mathcal{J}_f(\{\boldsymbol{p}'_1,\boldsymbol{p}'_2,\boldsymbol{p}'_3\},\boldsymbol{P}_f)}}{\sqrt{4e_je'_j}}\left(\prod_{r\neq j}\mathcal{S}_r\right)}_{\text{Relativistic Factor}}\underbrace{\widetilde{\Psi}^{*}_f(\boldsymbol{k}'_\rho,\boldsymbol{k}'_\lambda)\widetilde{\Psi}_i(\boldsymbol{k}_\rho,\boldsymbol{k}_\lambda)}_{\text{Spatial wave functions}}\underbrace{\sum_{\alpha,\beta}\left\langle\Phi_f(\lambda_f)\left|\hat{F}^{q_fq_i}_j\hat{S}^{\alpha\beta}_j\right|\Phi_i(\lambda_i)\right\rangle}_{\text{Spin–flavor transition matrices}}\underbrace{[O^{\nu}_{W,j}]^{\alpha\beta}}_{\text{Wqq vertex}}$$

Normalization Factor

Relativistic Factor

Spatial wave functions

Spin–flavor transition matrices

Momentum Integration

$$\boldsymbol{k}_\rho=\frac{1}{\sqrt{2}}(\boldsymbol{k}_1-\boldsymbol{k}_2),$$

$$\boldsymbol{k}_\lambda=\frac{1}{\sqrt{6}}(\boldsymbol{k}_1+\boldsymbol{k}_2-2\boldsymbol{k}_3),$$

$$\boldsymbol{P}_B=\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3,$$

Kinematic part， Jacobi factor

$$\psi_f^{(\boldsymbol{P}_f)}(\{\boldsymbol{p}'_a\})=\sqrt{\mathcal{J}_f(\{\boldsymbol{p}'_a\},\boldsymbol{P}_f)}\,\psi_f^{(\boldsymbol{0})}(\{\boldsymbol{k}'_a\})\quad\mathcal{J}_f(\{\boldsymbol{p}'_a\},\boldsymbol{P}_f)=\frac{E_f}{M_f}\prod_{a=1}^{3}\frac{\epsilon'_a}{e'_a}$$

Spin part， Wigner Rotation

$$\mathcal{S}_r=\frac{1}{2}\,\mathrm{Tr}\left[D^{\dagger}(\boldsymbol{k}'_r,\boldsymbol{P}_f;m_r,M_f)\,D(\boldsymbol{k}_r,\boldsymbol{P}_i;m_r,M_i)\right]$$

$$D(\boldsymbol{k},\boldsymbol{P};m,M)\equiv S_m^{-1}(\boldsymbol{p})\,S_M(\boldsymbol{P})\,S_m(\boldsymbol{k})$$

$$S_m(\boldsymbol{p})=\frac{(e+m)\mathbb{I}_2+\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\sqrt{2m(e+m)}},\quad S_M(\boldsymbol{P})=\frac{(E+M)\mathbb{I}_2+\boldsymbol{\sigma}\cdot\boldsymbol{P}}{\sqrt{2M(E+M)}}$$

$$I^{fi,j}_{\zeta_f\zeta_i}\equiv\left\langle\phi^{\zeta_f}_{\mathbf{8},B_f}\left|\hat{F}^{q_fq_i}_j\right|\phi^{\zeta_i}_{\mathbf{8},B_i}\right\rangle,$$

$$(C^{\sigma_f\sigma_i,j}_{\lambda_f\lambda_i})_{\alpha\beta}\equiv\left\langle\chi^{\sigma_f}_{1/2,\lambda_f}\left|\hat{S}^{\alpha\beta}_j\right|\chi^{\sigma_i}_{1/2,\lambda_i}\right\rangle,$$

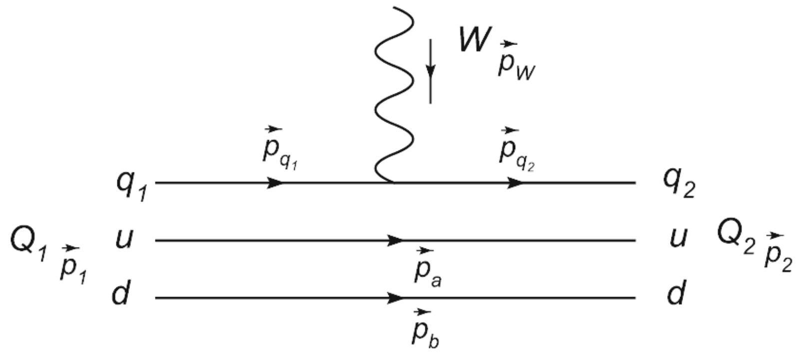
Wqq vertex

$$[O^{\nu}_{W,j}]^{s'_js_j}=\left[D^{\dagger}(\boldsymbol{k}'_j,\boldsymbol{P}_f;m'_j,M_f)\,O^{\nu}(\boldsymbol{p}'_j,\boldsymbol{p}_j)D(\boldsymbol{k}_j,\boldsymbol{P}_i;m_j,M_i)\right]^{s'_js_j}$$

$$[O^{\nu}(\boldsymbol{p}'_j,\boldsymbol{p}_j)]^{s'_js_j}=\bar{u}_{q_f}(p'_j,s'_j)\,\gamma^{\nu}(1-\gamma^5)\,u_{q_i}(p_j,s_j)$$

W-强子对的顶点计算

基于夸克层次的计算



$$H^{\nu}(\lambda_f, \lambda_i; P_i, P_f) = \sum_{\kappa_f=1}^{3} \sum_{\kappa_i=1}^{3} (c_{\kappa_f}^{B_f,1/2})^* c_{\kappa_i}^{B_i,1/2} H^{\nu}_{\kappa_f \kappa_i}(\lambda_f, \lambda_i; P_i, P_f),$$

$$H^{\nu}_{\kappa_f \kappa_i}(\lambda_f, \lambda_i; P_i, P_f) \equiv \sum_{j=1}^{3} \left\langle B_{f,\kappa_f}(P_f, \lambda_f) \middle| J_j^{\nu}(0) \middle| B_{i,\kappa_i}(P_i, \lambda_i) \right\rangle$$

$$\left\langle \Psi_f \Phi_f(\lambda_f) \middle| J_j^{\nu}(0) \middle| \Psi_i \Phi_i(\lambda_i) \right\rangle = \underbrace{\sqrt{2E_i\, 2E_f}}_{\text{Normalization Factor}} \int \underbrace{\frac{d^3 \boldsymbol{k}_\rho}{(2\pi)^3} \frac{d^3 \boldsymbol{k}_\lambda}{(2\pi)^3}}_{\text{Momentum Integration}} \underbrace{\frac{\sqrt{\mathcal{J}_f(\{\boldsymbol{p}_1', \boldsymbol{p}_2', \boldsymbol{p}_3'\}, \boldsymbol{P}_f)}}{\sqrt{4 e_j e_j'}}}_{\text{Relativistic Factor}} \left(\prod_{r \neq j} \mathcal{S}_r \right) \underbrace{\widetilde{\Psi}_f^*(\boldsymbol{k}_\rho', \boldsymbol{k}_\lambda') \widetilde{\Psi}_i(\boldsymbol{k}_\rho, \boldsymbol{k}_\lambda)}_{\text{Spatial wave functions}} \underbrace{\sum_{\alpha,\beta} \left\langle \Phi_f(\lambda_f) \middle| \hat{F}_j^{q_f q_i} \hat{S}_j^{\alpha\beta} \middle| \Phi_i(\lambda_i) \right\rangle}_{\text{Spin–flavor transition matrices}} \underbrace{[O^{\nu}_{W,j}]^{\alpha\beta}}_{\text{Wqq vertex}}$$

Normalization Factor

Momentum Integration

Relativistic Factor

Spatial wave functions

Spin–flavor transition matrices

Wqq vertex

$$|N(939)\rangle = 0.90|N_8^2 S_S\rangle + 0.34|N_8^2 S_S'\rangle - 0.27|N_8^2 S_M\rangle,$$

$$|\Lambda(1115)\rangle = 0.93|\Lambda_8^2 S_S\rangle + 0.30|\Lambda_8^2 S_S'\rangle - 0.20|\Lambda_8^2 S_M\rangle,$$

$$\left| B_8^2 S_S, \frac{1}{2}^+ \right\rangle = \frac{1}{\sqrt{2}} \left(|B\rangle_\lambda \left| \frac{1}{2}, s_z \right\rangle_\lambda + |B\rangle_\rho \left| \frac{1}{2}, s_z \right\rangle_\rho \right) \Phi_{000}(\mathbf{q}_\lambda, \mathbf{q}_\rho)$$

Kinematic part，Jacobi factor

$$\psi_f^{(\boldsymbol{P}_f)}(\{\boldsymbol{p}_a'\}) = \sqrt{\mathcal{J}_f(\{\boldsymbol{p}_a'\}, \boldsymbol{P}_f)}\, \psi_f^{(\mathbf{0})}(\{\boldsymbol{k}_a'\})$$

Spin part， Wigner Rotation

$$\mathcal{S}_r = \frac{1}{2} \operatorname{Tr} \left[D^\dagger(\boldsymbol{k}_r', \boldsymbol{P}_f; m_r, M_f)\, D(\boldsymbol{k}_r, \boldsymbol{P}_i; m_r, M_i) \right]$$

$$D(\boldsymbol{k}, \boldsymbol{P}; m, M) \equiv S_m^{-1}(\boldsymbol{p})\, S_M(\boldsymbol{P})\, S_m(\boldsymbol{k})$$

$$S_m(\boldsymbol{p}) = \frac{(e+m)\mathbb{I}_2 + \boldsymbol{\sigma}\cdot\boldsymbol{p}}{\sqrt{2m(e+m)}}, \quad S_M(\boldsymbol{P}) = \frac{(E+M)\mathbb{I}_2 + \boldsymbol{\sigma}\cdot\boldsymbol{P}}{\sqrt{2M(E+M)}}$$

$$_\rho\left\langle \frac{1}{2}, s_z^\Lambda | \hat{O}^\mu | \frac{1}{2}, s_z^p \right\rangle_\rho = O^\mu(s_z^\Lambda, s_z^p), \qquad _\lambda\left\langle \frac{1}{2}, s_z^\Lambda = \frac{1}{2} | \hat{O}^\mu | \frac{1}{2}, s_z^p = \frac{1}{2} \right\rangle_\lambda$$

$$_\rho\left\langle \frac{1}{2}, s_z^\Lambda | \hat{O}^\mu | \frac{1}{2}, s_z^p \right\rangle_\lambda = 0, \qquad = \frac{1}{3}\left(O^\mu\left(\frac{1}{2}, \frac{1}{2}\right) + 2O^\mu\left(-\frac{1}{2}, -\frac{1}{2}\right) \right)$$

$$_\lambda\left\langle \frac{1}{2}, s_z^\Lambda | \hat{O}^\mu | \frac{1}{2}, s_z^p \right\rangle_\rho = 0, \qquad _\lambda\left\langle \frac{1}{2}, s_z^\Lambda = -\frac{1}{2} | \hat{O}^\mu | \frac{1}{2}, s_z^p = \frac{1}{2} \right\rangle_\lambda = \frac{1}{3} O^\mu\left(-\frac{1}{2}, \frac{1}{2}\right)$$

$$_\lambda\langle \Lambda | \chi_s^+ \chi_u | p \rangle_\lambda = {_\lambda\langle \Lambda | \chi_s^+ \chi_u | p \rangle_\rho} = {_\rho\langle \Lambda | \chi_s^+ \chi_u | p \rangle_\lambda} = 0,$$

$$_\rho\langle \Lambda | \chi_s^+ \chi_u | p \rangle_\rho = \frac{\sqrt{6}}{3}.$$

$$O^\mu\left(s_z^s, s_z^u\right) = \bar{u}_s\left(\mathbf{q}_s,\ s_z^s\right) \gamma^\nu \left(1 - \gamma^5\right) u_u\left(\mathbf{q}_u,\ s_z^u\right)$$

夸克模型：重子波函数的计算

$$H=\sum_{i=1}^{3}\sqrt{\boldsymbol{p}_i^2+m_i^2}+\sum_{i<j}V(r_{ij})+C_0,$$

$$V(r_{ij})=V_{\rm corn}(r_{ij})+V_{\rm hyp}(r_{ij})$$

$$V_{\rm corn}(r_{ij})=-\frac{2}{3}\frac{\alpha_s^{ij}}{r_{ij}}+\frac{b}{2}r_{ij}$$

$$V_{\rm hyp}(r_{ij})=\frac{16\pi\alpha_s^{ij}}{9\,m_im_j}\frac{e^{-r_{ij}^2/r_0^2(ij)}}{\pi^{3/2}r_0^3(ij)}(\boldsymbol{S}_i\cdot\boldsymbol{S}_j)$$

$$\sum_{k'=1}^{\mathcal{N}}(H_{kk'}-E\,N_{kk'})c_{k'}=0,$$

$$\Psi_{JM_J}=\sum_{k=1}^{\mathcal{N}}c_k\left[\psi_{\rm space}^{(k)}\otimes\chi_{\rm spin}\otimes\phi_{\rm flavor}\otimes\phi_{\rm color}\right]_{JM_J}$$

重子波函数

$$\left|B^{\mathbf{8}},1/2^+\right\rangle=\sum_{\kappa=1}^{3}c_\kappa^{\mathbf{8},1/2}\left|B_\kappa^{\mathbf{8}},1/2^+\right\rangle$$

$$\left|B^{\mathbf{10}},3/2^+\right\rangle=\sum_{\kappa=1}^{2}c_\kappa^{\mathbf{10},3/2}\left|B_\kappa^{\mathbf{10}},3/2^+\right\rangle$$

$$\left|B_1^{\mathbf{8}},1/2^+\right\rangle=\Phi_{\mathbf{56,8}}^s\Psi_{000}^s$$

$$\left|B_2^{\mathbf{8}},1/2^+\right\rangle=\Psi_{200}^s\Phi_{\mathbf{56,8}}^s,$$

$$\left|B_3^{\mathbf{8}},1/2^+\right\rangle=\frac{1}{\sqrt{2}}\left(\Psi_{200}^\rho\Phi_{\mathbf{70}}^\rho+\Psi_{200}^\lambda\Phi_{\mathbf{70}}^\lambda\right)$$

$$\left|B_1^{\mathbf{10}},3/2^+\right\rangle=\Psi_{000}^s\Phi_{\mathbf{56,10}}^s$$

$$\left|B_2^{\mathbf{10}},3/2^+\right\rangle=\Psi_{200}^s\Phi_{\mathbf{56,10}}^s$$

TABLE I: Parameters of the semirelativistic potential model.

$m_{u/d}$ (MeV)	m_s (MeV)	m_c (MeV)	m_b (MeV)
368.5	460.7	1550.8	5005.7
$g_{u/d}^{\rm OGE}$	$g_s^{\rm OGE}$	$g_c^{\rm OGE}$	$g_b^{\rm OGE}$
0.9445	0.7592	0.3888	0.4245
$A_{\rm sm}$ (GeV$^{B_{\rm sm}-1}$)	$B_{\rm sm}$	b (GeV^2)	C_0 (MeV)
0.9425	0.8869	0.1472	−654.7

$$\Phi_{\mathbf{56,8}}^s=\frac{1}{\sqrt{2}}\left(\phi_{\mathbf{8}}^\rho\chi_{1/2}^\rho+\phi_{\mathbf{8}}^\lambda\chi_{1/2}^\lambda\right)$$

$$\Phi_{\mathbf{70}}^\rho=\frac{1}{\sqrt{2}}\left(\phi_{\mathbf{8}}^\rho\chi_{1/2}^\lambda+\phi_{\mathbf{8}}^\lambda\chi_{1/2}^\rho\right)$$

$$\Phi_{\mathbf{70}}^\lambda=\frac{1}{\sqrt{2}}\left(\phi_{\mathbf{8}}^\rho\chi_{1/2}^\rho-\phi_{\mathbf{8}}^\lambda\chi_{1/2}^\lambda\right)$$

$$\Phi_{\mathbf{56,10}}^s=\phi_{\mathbf{10}}^s\chi_{3/2}^s,$$

$$\Phi_{\mathbf{56,10}}^s=\phi_{\mathbf{10}}^s\chi_{3/2}^s.$$

夸克模型：重子波函数的计算

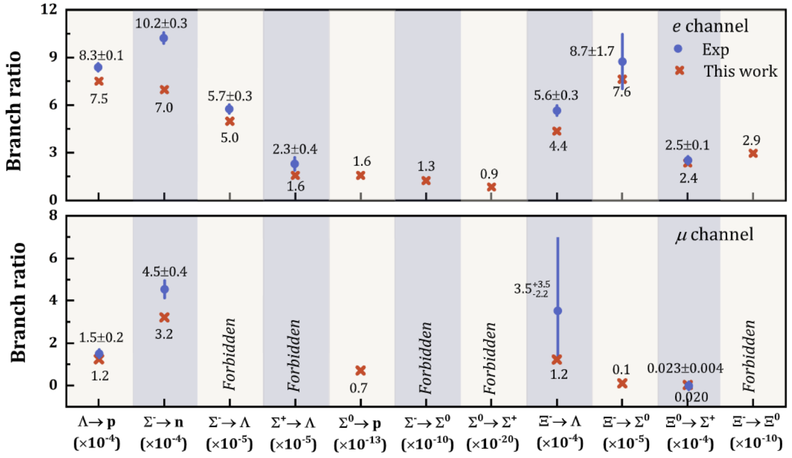
Light baryons			Charmed baryons			Bottom baryons		
States	$M_{\text{thy.}}$	$M_{\text{ref.}}$	States	$M_{\text{thy.}}$	$M_{\text{ref.}}$	States	$M_{\text{thy.}}$	$M_{\text{ref.}}$
p	947	938	Λ_c	2281	2286	Λ_b	5611	5620
Δ	1251	1235	Σ_c	2448	2453	Σ_b	5803	5811
Λ	1123	1116	Σ_c^*	2498	2519	Σ_b^*	5827	5830
Σ	1180	1193	Ξ_c	2465	2469	Ξ_b	5799	5794
Σ^*	1397	1385	Ξ_c'	2580	2579	Ξ_b'	5936	5935
Ξ	1319	1318	Ξ_c^*	2624	2646	Ξ_b^*	5956	5952
Ξ^*	1535	1533	Ω_c	2706	2695	Ω_b	6062	6046
Ω	1666	1672	Ω_c^*	2743	2766	Ω_b^*	6079	6085
			Ξ_{cc}	3633	3622			
			Ω_{cc}	3753	3726			

State	Coefficients
Light octet: $\left(\left\|B^{\mathbf{8}}_{1,2,3}, 1/2^+\right\rangle\right)$	
p	(0.9908, 0.0906, −0.1002)
Λ	(0.9940, 0.0417, −0.1010)
Σ	(0.9959, 0.0202, −0.0880)
Ξ	(0.9955, −0.0186, −0.0934)
Light decuplet: $\left(\left\|B^{\mathbf{10}}_{1,2}, 3/2^+\right\rangle\right)$	
Δ	(0.9942, −0.1075)
Σ^*	(0.9889, −0.1486)
Ξ^*	(0.9825, −0.1863)
Ω	(0.9754, −0.2202)
A-type $(1/2^+)$: $\left(\left\|B^{A}_{1,2,3,4}, 1/2^+\right\rangle\right)$	
Λ_c	(0.9854, 0.0963, −0.1388, 0.0211)
Λ_b	(0.9909, 0.1259, −0.0458, 0.0102)
S-type $(1/2^+)$: $\left(\left\|B^{S}_{1,2,3,4}, 1/2^+\right\rangle\right)$	
Σ_c	(0.9906, −0.0555, −0.1209, 0.0314)
Ω_c	(0.9783, −0.1285, −0.1602, 0.0260)
Σ_b	(0.9979, −0.0337, −0.0533, 0.0168)
Ω_b	(0.9881, −0.1107, −0.1055, 0.0143)
Ξ_{cc}	(0.9704, −0.2331, −0.0593, 0.0226)
Ω_{cc}	(0.9658, −0.2342, −0.1094, 0.0204)
Mixed Ξ_Q–Ξ_Q' $(1/2^+)$: $A:\left(\left\|B^{A}_{1,2,3,4}, 1/2^+\right\rangle\right)$; $S:\left(\left\|B^{S}_{1,2,3,4}, 1/2^+\right\rangle\right)$	
Ξ_c	A : (0.9849, 0.0361, −0.1639, 0.0217) S : (0.0353, −0.0017, −0.0027, 0.0004)
Ξ_c'	A : (−0.0343, −0.0007, 0.0094, 0.0027) S : (0.9844, −0.0930, −0.1425, 0.0286)
Ξ_b	A : (0.9946, 0.0657, −0.0790, 0.0108) S : (0.0135, −0.0002, 0.0001, 0.0003)
Ξ_b'	A : (−0.0132, −0.0006, 0.0029, 0.0030) S : (0.9938, −0.0728, −0.0816, 0.0155)
S-type $(3/2^+)$: $\left(\left\|B^{S}_{1,2,3}, 3/2^+\right\rangle\right)$	
Σ_c^*	(0.9806, −0.0820, −0.1782)
Ω_c^*	(0.9673, −0.1481, −0.2059)
Σ_b^*	(0.9953, −0.0485, −0.0840)
Ω_b^*	(0.9837, −0.1227, −0.1312)
Ξ_Q^* $(3/2^+)$: $\left(\left\|B^{S}_{1,2,3}, 3/2^+\right\rangle; \left\|B^{A}_{4}, 3/2^+\right\rangle\right)$	
Ξ_c^*	(0.9741, −0.1161, −0.1938, −0.0007)
Ξ_b^*	(0.9902, −0.0864, −0.1098, 0.0012)

超子半轻衰变计算

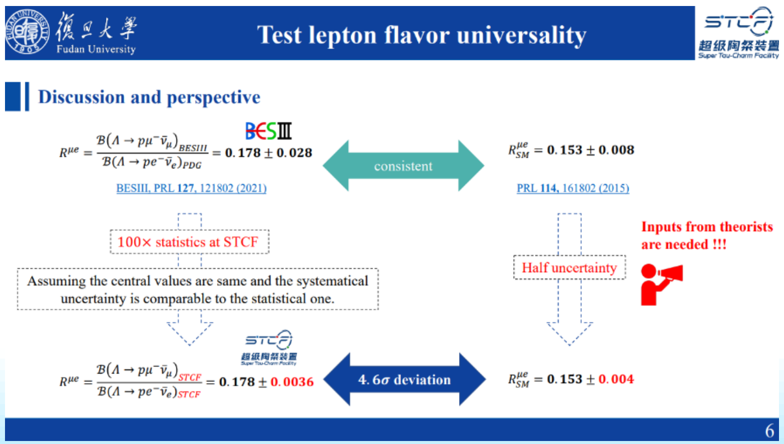
$\Lambda \to p$

Method	Γ (10^6 s^{-1}) $\ell = e$	Γ (10^6 s^{-1}) $\ell = \mu$	$\mathcal{B}$ (10^{-4}) $\ell = e$	$\mathcal{B}$ (10^{-4}) $\ell = \mu$	$R_{\mu e}$
SU(3) IRA [11]	3.18(5)	0.53(1)	8.32(14)	1.40(2)	0.168(4)
LFQM [44]	3.51	0.58	9.19	1.52	0.165
CCQM [50]	3.28	0.57	8.58	1.49	0.174
NR3QM [67]	4.09	0.70	10.70	1.82	0.170
	4.51	0.75	11.8	1.95	0.165
QCDSR [39]	2.95(24)	0.52(4)	7.72(64)	1.35(11)	0.175(31)
	2.72(27)	0.44(4)	7.12(70)	1.15(11)	0.162(35)
LFD [102]	$3.18(^{+12}_{-21})$	$0.53(^{+2}_{-3})$	$8.32(^{+32}_{-54})$	$1.38(^{+5}_{-9})$	$0.166(^{+9}_{-15})$
	$3.18(^{+12}_{-21})$	$0.50(^{+2}_{-3})$	$8.32(^{+32}_{-54})$	$1.31(^{+4}_{-8})$	$0.158(^{+7}_{-14})$
LQCD [36]	2.93(18)	0.51(6)	7.68(48)	1.33(16)	0.1735(98)
BESIII [65]	—	—	8.16(22)(15)	—	—
BESIII [63]	—	—	—	1.48(21)(8)	0.178(28)
LHCb [64]	—	—	—	1.462(16)(100)(11)	0.175(12)
RPP [66]	—	—	8.34(14)	1.51(19)	0.181(23)
This work	**2.85**	**0.47**	**7.46**	**1.24**	**0.166**



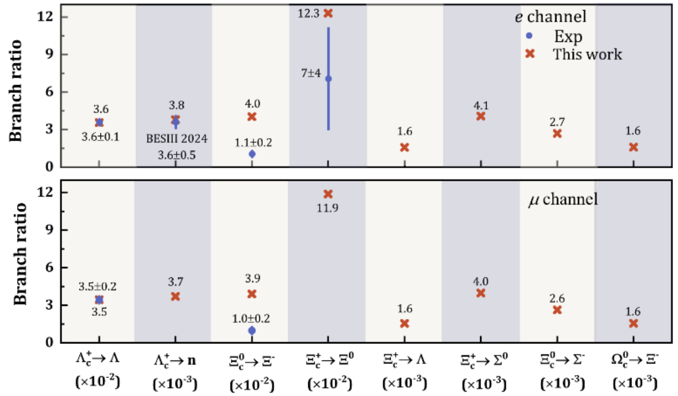
Method	Γ (10^6 s^{-1}) $\ell = e$	Γ (10^6 s^{-1}) $\ell = \mu$	$\mathcal{B}$ $\ell = e$	$\mathcal{B}$ $\ell = \mu$	$R_{\mu e}$
$\Sigma^- \to n\ell^-\bar{\nu}_\ell$ [$\mathcal{B}$ in units of 10^{-4}]					
SU(3) IRA [11]	6.88(23)	3.06(10)	10.17(34)	4.53(15)	0.445
LFQM [44]	5.74	2.54	8.49	3.76	0.443
CCQM [50]	6.50	3.15	9.61	4.66	0.485
NR3QM [67]	5.52	2.67	8.16	3.95	0.484
	7.00	3.28	10.35	4.85	0.469
QCDSR [39]	6.76(122)	3.20(59)	10.00(180)	4.73(88)	0.473
	5.88(95)	2.53(42)	8.70(140)	3.74(62)	0.430
RPP [66]	—	—	10.17(34)	4.5(4)	0.442(56)
This work	**4.73**	**2.17**	**6.99**	**3.22**	**0.460**
$\Sigma^- \to \Lambda e^-\bar{\nu}_e$ [$\mathcal{B}$ in units of 10^{-5}]					
SU(3) IRA [11]	0.39(2)	—	5.73(27)	—	—
LFQM [44]	0.39	—	5.75	—	—
CCQM [50]	0.43	—	6.36	—	—
RPP [66]	—	—	5.73(27)	—	—
This work	**0.34**	**Forbidden**	**4.98**	**Forbidden**	—
$\Sigma^+ \to \Lambda e^+\nu_e$ [$\mathcal{B}$ in units of 10^{-5}]					
SU(3) IRA [11]	0.23(1)	—	1.88(11)	—	—
LFQM [44]	0.24	—	1.92	—	—
CCQM [50]	0.26	—	2.08	—	—
RPP [66]	—	—	2.3(4)	—	—
This work	**0.20**	**Forbidden**	**1.62**	**Forbidden**	—
$\Sigma^0 \to p\ell^-\bar{\nu}_\ell$ [$\mathcal{B}$ in units of 10^{-13}]					
SU(3) IRA [11]	3.26(43)	1.42(19)	2.41(32)	1.05(14)	0.436
LFQM [44]	2.72	1.18	2.01	0.87	0.434
NR3QM [67]	2.54	1.19	1.88	0.88	0.469
	3.22	1.46	2.38	1.08	0.453
QCDSR [39]	3.38(95)	1.59(46)	2.50(70)	1.18(34)	0.472
	2.95(78)	1.27(34)	2.18(58)	0.94(25)	0.431
This work	**2.22**	**1.00**	**1.64**	**0.74**	**0.449**
$\Sigma^- \to \Sigma^0 e^-\bar{\nu}_e$ [$\mathcal{B}$ in units of 10^{-10}]					
SU(3) IRA [11]	$2.95(271) \times 10^{-6}$	—	4.36(401)	—	—
LFQM [44]	1.47×10^{-6}	—	2.17	—	—
This work	$\mathbf{8.49 \times 10^{-7}}$	**Forbidden**	**1.26**	**Forbidden**	—
$\Sigma^0 \to \Sigma^+ e^-\bar{\nu}_e$ [$\mathcal{B}$ in units of 10^{-20}]					
SU(3) IRA [11]	$4.61(432) \times 10^{-7}$	—	3.41(320)	—	—
LFQM [44]	3.65×10^{-8}	—	0.27	—	—
This work	$\mathbf{1.17 \times 10^{-7}}$	**Forbidden**	**0.86**	**Forbidden**	—

Method	Γ (10^6 s^{-1}) $\ell = e$	Γ (10^6 s^{-1}) $\ell = \mu$	$\mathcal{B}$ $\ell = e$	$\mathcal{B}$ $\ell = \mu$	$R_{\mu e}$
$\Xi^- \to \Lambda\ell^-\bar{\nu}_\ell$ [$\mathcal{B}$ in units of 10^{-4}]					
SU(3) IRA [11]	3.34(9)	0.96(2)	5.47(15)	1.58(4)	0.289
LFQM [44]	2.96	0.80	4.85	1.31	0.270
CCQM [50]	3.35	0.96	5.49	1.57	0.287
QCDSR [39]	3.15(45)	0.94(14)	5.17(73)	1.54(23)	0.298
	2.85(40)	0.76(11)	4.67(66)	1.24(18)	0.266
BESIII [109]	—	—	3.60(40)(10)	—	—
RPP [66]	—	—	5.63(31)	$3.5(^{+35}_{-22})$	—
This work	**2.69**	**0.76**	**4.41**	**1.24**	**0.281**
$\Xi^- \to \Sigma^0\ell^-\bar{\nu}_\ell$ [$\mathcal{B}$ in units of 10^{-5}]					
SU(3) IRA [11]	0.50(4)	$6.59(55) \times 10^{-3}$	8.12(60)	0.11(1)	0.013
LFQM [44]	0.55	7.47×10^{-3}	9.00	0.12	0.014
CCQM [50]	0.52	6.70×10^{-3}	8.52	0.11	0.013
QCDSR [39]	0.48(5)	$6.47(73) \times 10^{-3}$	7.80(77)	0.11(1)	0.014
	0.47(5)	$5.92(67) \times 10^{-3}$	7.72(86)	0.10(1)	0.013
RPP [66]	—	—	8.7(17)	< 80	—
This work	**0.47**	$\mathbf{6.29 \times 10^{-3}}$	**7.62**	**0.103**	**0.014**
$\Xi^0 \to \Sigma^+\ell^-\bar{\nu}_\ell$ [$\mathcal{B}$ in units of 10^{-4}]					
SU(3) IRA [11]	0.87(3)	$7.52(34) \times 10^{-3}$	2.52(8)	0.022(1)	0.009
LFQM [44]	0.94	7.74×10^{-3}	2.73	0.022	0.008
CCQM [50]	0.93	8.10×10^{-3}	2.70	0.024	0.009
QCDSR [39]	0.83(10)	$7.14(93) \times 10^{-3}$	2.41(28)	0.021(3)	0.009
	0.82(11)	$6.59(86) \times 10^{-3}$	2.39(32)	0.019(3)	0.008
RPP [66]	—	—	2.52(8)	0.023(4)	0.0092(15)
This work	**0.81**	$\mathbf{6.98 \times 10^{-3}}$	**2.35**	**0.020**	**0.009**
$\Xi^- \to \Xi^0 e^-\bar{\nu}_e$ [$\mathcal{B}$ in units of 10^{-10}]					
SU(3) IRA [11]	$2.56(146) \times 10^{-6}$	—	4.2(24)	—	—
RPP [66]	—	—	$< 2.59 \times 10^6$	—	—
This work	$\mathbf{1.77 \times 10^{-6}}$	**Forbidden**	**2.90**	**Forbidden**	—



单粲重子半轻衰变计算

$\Lambda_c^+ \to \Lambda \ell^+ \nu_\ell$	$\mathcal{B}$ (10^{-2}) $\ell = e$	$\ell = \mu$	$R_{\mu e}$
CCQM [48]	2.78	2.69	0.968
RQM [49]	3.25	3.14	0.966
LFQM [53]	4.04(75)	3.90(73)	0.965
HBM [54]	3.78(25)	3.67(23)	0.971
QCDSR [36]	3.49(65)	3.37(54)	0.966
LQCD [7]	3.80(22)	3.69(22)	0.969
RPP [12]	3.56(13)	3.48(17)	0.98(6)
This work	**3.56**	**3.45**	**0.969**



$\Lambda_c^+ \to n \ell^+ \nu_\ell$	$\mathcal{B}$ (10^{-3}) $\ell = e$	$\ell = \mu$	$R_{\mu e}$
CQM [44]	2.74	—	—
CCQM [47]	2.07	2.02	0.976
SU(3) [31]	2.93(34)	—	—
RQM [49]	2.68	2.62	0.978
LFQM [51]	2.01	—	—
SU(3) [14]	5.1(4)	—	—
MBM [52]	2.79	2.73	0.978
LFCQM [52]	3.6(15)	3.4(14)	0.944
QCDSR [36]	2.81(56)	2.75(55)	0.979
LQCD [8]	4.10(26)	4.00(26)	0.977
BESIII [6]	3.57(34)(14)	—	—
This work	**3.79**	**3.71**	**0.979**

Method	$\Xi_c^+ \to \Xi^0 \ell^+ \nu_\ell$ $\mathcal{B}$ (10^{-2}) $\ell = e$	$\ell = \mu$	$R_{\mu e}$
RQM [15]	9.40	9.11	0.969
SU(3) [14]	11.9(13)	11.6(17)	0.975
	9.8(11)	9.8(11)	1.000
	10.7(9)	10.8(9)	1.009
LFQM [51]	5.39	—	—
LFCQM [16]	11.3(34)	10.8(33)	0.956
LCSR [17]	5.51(165)	5.34(161)	0.969
LCSR [18]	$8.43(^{+52}_{-45})$	$8.16(^{+50}_{-43})$	0.968
LCSR [20]	11.20(325)	10.8(31)	0.964
LQCD [22]	7.18(133)	6.91(127)	0.962
LQCD [23]	10.94(34)	10.61(33)	0.969
RPP [12]	7(4)	—	—
This work	**12.3**	**11.9**	**0.969**

Method	$\Xi_c^0 \to \Xi^- \ell^+ \nu_\ell$ $\mathcal{B}$ (10^{-2}) $\ell = e$	$\ell = \mu$	$R_{\mu e}$
RQM [15]	2.38	2.31	0.969
SU(3) [14]	3.0(3)	2.9(4)	0.967
	2.4(3)	2.4(3)	1.000
	2.7(2)	2.7(2)	1.000
LFQM [51]	1.35	—	—
LFCQM [16]	3.49(95)	3.34(94)	0.957
LCSR [17]	1.85(56)	1.79(54)	0.968
LCSR [18]	$2.81(^{+17}_{-15})$	$2.72(^{+17}_{-15})$	0.968
LCSR [20]	3.73(104)	3.59(101)	0.963
LQCD [22]	2.38(44)	2.29(42)	0.962
LQCD [23]	3.58(12)	3.47(12)	0.969
RPP [12]	1.06(21)	1.01(21)	—
This work	**4.03**	**3.91**	**0.968**

Method	$\Xi_c^+ \to \Lambda \ell^+ \nu_\ell$ $\mathcal{B}$ (10^{-3}) $\ell = e$	$\ell = \mu$	$R_{\mu e}$
NRQM [41]	0.884	—	—
LFQM [51]	0.82	—	—
SU(3) [14]	1.03(15)	—	—
	1.66(18)	—	—
	2.18(18)	—	—
RQM [15]	1.27	1.24	0.980
LFCQM [16]	1.2(4)	1.1(5)	0.92
LCSR [17]	0.92(28)	0.89(27)	0.967
SU(3) [13]	0.67(13)	0.69(21)	1.03
SU(3) [21]	1.17(14)	1.15(13)	0.983
This work	**1.58**	**1.55**	**0.980**

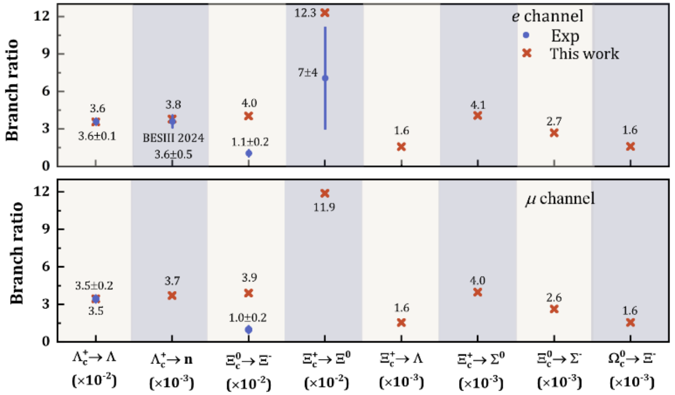
$\Xi_c^+ \to \Sigma^0 \ell^+ \nu_\ell$	$\mathcal{B}$ (10^{-3}) $\ell = e$	$\ell = \mu$	$R_{\mu e}$
NRQM [41]	4.42	—	—
LFQM [51]	1.87	—	—
SU(3) [14]	3.1(4)	—	—
	3.8(4)	—	—
	4.6(4)	—	—
LFCQM [16]	3.3(9)	3.1(9)	0.94
SU(3) [13]	4.96(46)	4.81(44)	0.970
SU(3) [21]	2.82(30)	2.75(29)	0.975
This work	**4.08**	**3.98**	**0.976**

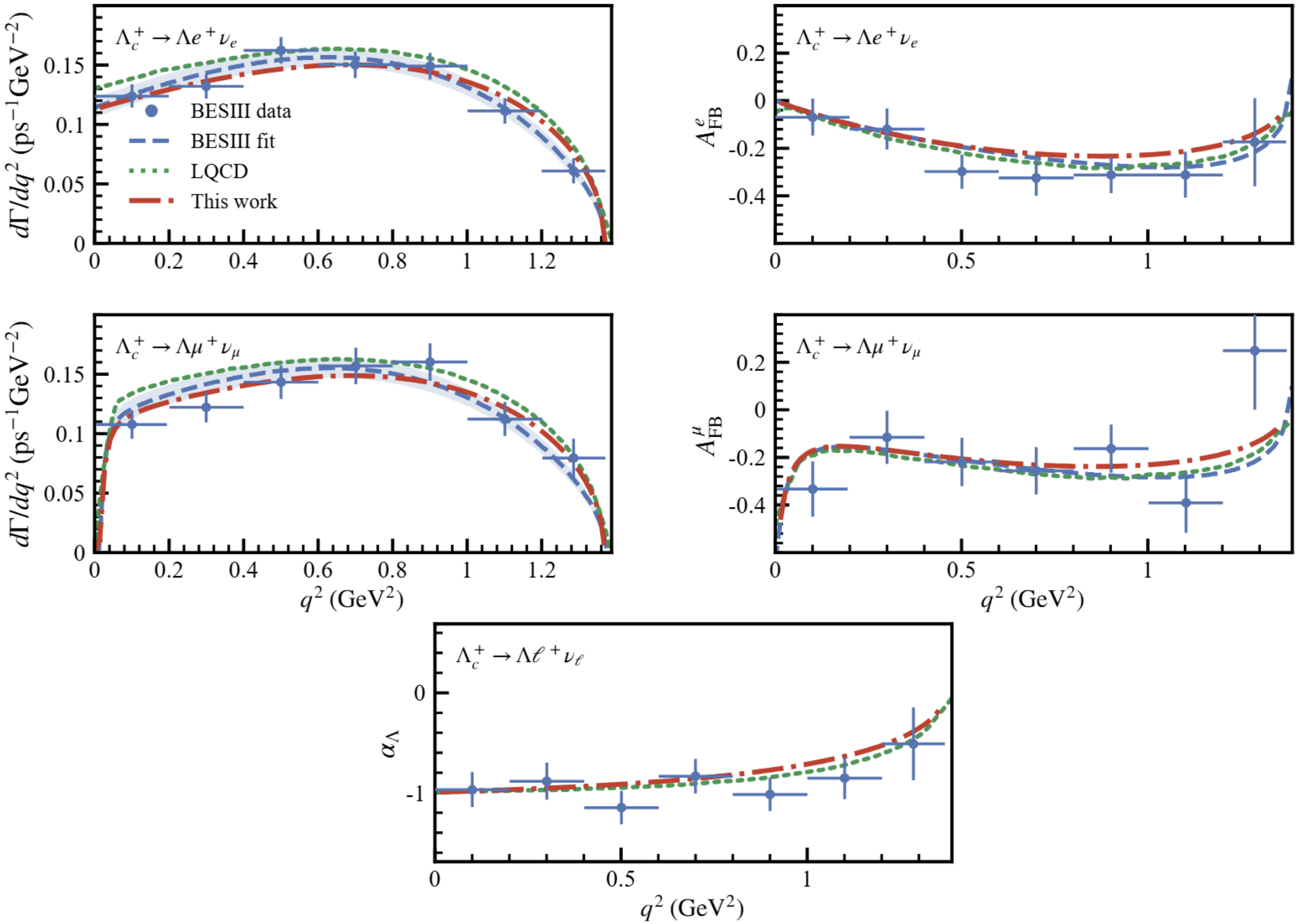
$\Xi_c^0 \to \Sigma^- \ell^+ \nu_\ell$	$\mathcal{B}$ (10^{-3}) $\ell = e$	$\ell = \mu$	$R_{\mu e}$
NRQM [41]	1.12	—	—
LFQM [51]	0.95	—	—
SU(3) [14]	1.57(22)	—	—
	1.92(21)	—	—
	2.32(19)	—	—
LFCQM [16]	2.2(6)	2.1(6)	0.95
SU(3) [13]	3.33(31)	3.23(29)	0.970
SU(3) [21]	1.86(20)	1.82(19)	0.978
This work	**2.69**	**2.63**	**0.976**

$\Omega_c^0 \to \Xi^- \ell^+ \nu_\ell$	$\mathcal{B}$ (10^{-3}) $\ell = e$	$\ell = \mu$	$R_{\mu e}$
CQM [45]	0.93–1.78	—	—
LFQM [51]	0.22	—	—
LCSR [39]	3.06(15)	—	—
This work	**1.59**	**1.56**	**0.983**

单粲重子半轻衰变计算

$\Lambda_c^+ \to \Lambda \ell^+ \nu_\ell$	$\mathcal{B}$ (10^{-2})		$R_{\mu e}$
	$\ell = e$	$\ell = \mu$	
CCQM [48]	2.78	2.69	0.968
RQM [49]	3.25	3.14	0.966
LFQM [53]	4.04(75)	3.90(73)	0.965
HBM [54]	3.78(25)	3.67(23)	0.971
QCDSR [36]	3.49(65)	3.37(54)	0.966
LQCD [7]	3.80(22)	3.69(22)	0.969
RPP [12]	3.56(13)	3.48(17)	0.98(6)
This work	**3.56**	**3.45**	**0.969**





$= e$	$\ell = \mu$	
.12	—	—
.95	—	—
7(22)	—	—
2(21)	—	—
2(19)	—	—
2(6)	2.1(6)	0.95
3(31)	3.23(29)	0.970
6(20)	1.82(19)	0.978
.69	**2.63**	**0.976**

$\mathcal{B}$ (10^{-3})		$R_{\mu e}$
$= e$	$\ell = \mu$	
3–1.78	—	—
.22	—	—
6(15)	—	—
.59	**1.56**	**0.983**

形状因子计算

基于强子层次的计算

$$H_V^\nu(\lambda_f,\lambda_i;P_i,P_f)=\bar{u}_f(P_f,\lambda_f)\left[f_1(q^2)\gamma^\nu-i\frac{f_2(q^2)}{M}\sigma^{\nu\alpha}q_\alpha+\frac{f_3(q^2)}{M}q^\nu\right]u_i(P_i,\lambda_i),$$

$$H_A^\nu(\lambda_f,\lambda_i;P_i,P_f)=\bar{u}_f(P_f,\lambda_f)\left[g_1(q^2)\gamma^\nu\gamma^5-i\frac{g_2(q^2)}{M}\sigma^{\nu\alpha}q_\alpha\gamma^5+\frac{g_3(q^2)}{M}q^\nu\gamma^5\right]u_i(P_i,\lambda_i),$$

g_2 is zero at exact SU(3) limit

f_3, g_3 terms are suppressed by $(m_l/M_B)^2$

基于夸克层次的计算

$$\langle\Psi_f\Phi_f(\lambda_f)|J_j^\nu(0)|\Psi_i\Phi_i(\lambda_i)\rangle=\sqrt{2E_i2E_f}\int\frac{d^3\boldsymbol{k}_\rho}{(2\pi)^3}\frac{d^3\boldsymbol{k}_\lambda}{(2\pi)^3}\frac{\sqrt{\mathcal{J}_f(\{\boldsymbol{p}_1',\boldsymbol{p}_2',\boldsymbol{p}_3'\},\boldsymbol{P}_f)}}{\sqrt{4e_je_j'}}\left(\prod_{r\neq j}\mathcal{S}_r\right)\times\widetilde{\Psi}_f^*(\boldsymbol{k}_\rho',\boldsymbol{k}_\lambda')\widetilde{\Psi}_i(\boldsymbol{k}_\rho,\boldsymbol{k}_\lambda)\sum_{\alpha,\beta}\left\langle\Phi_f(\lambda_f)\left|\hat{F}_j^{q_fq_i}\hat{S}_j^{\alpha\beta}\right|\Phi_i(\lambda_i)\right\rangle[O_{W,j}^\nu]^{\alpha\beta}$$

$$H^\nu(\lambda_f,\lambda_i;P_i,P_f)=\sum_{\kappa_f=1}^{3}\sum_{\kappa_i=1}^{3}(c_{\kappa_f}^{B_f,1/2})^*c_{\kappa_i}^{B_i,1/2}H_{\kappa_f\kappa_i}^\nu(\lambda_f,\lambda_i;P_i,P_f),$$

$$H_{\kappa_f\kappa_i}^\nu(\lambda_f,\lambda_i;P_i,P_f)\equiv\sum_{j=1}^{3}\left\langle B_{f,\kappa_f}(P_f,\lambda_f)\left|J_j^\nu(0)\right|B_{i,\kappa_i}(P_i,\lambda_i)\right\rangle$$

矢量流 (Vector Current, H_V^μ, 弱磁项衰变取 $-i\sigma q$，散射取 $+i\sigma q$)

0 (时间)	↑↑	$\bar{u}_f\gamma^0u_i$	$\mathcal{N}\left[f_1-\frac{E_f-M_f}{M}f_2+\frac{M_i-E_f}{M}f_3\right]$	$\mathcal{N}\left[f_1-\frac{E_f-M_f}{M}f_2+\frac{E_f-M_i}{M}f_3\right]$
1 (横向)	↑↓	$\bar{u}_f\gamma^1u_i$	$-\mathcal{N}R\left[f_1+\frac{M_i+M_f}{M}f_2\right]$	$-\mathcal{N}R\left[f_1+\frac{M_i+M_f}{M}f_2\right]$
2 (横向)	↑↓	$\bar{u}_f\gamma^2u_i$	$-i\mathcal{N}R\left[f_1+\frac{M_i+M_f}{M}f_2\right]$	$-i\mathcal{N}R\left[f_1+\frac{M_i+M_f}{M}f_2\right]$
3 (纵向)	↑↑	$\bar{u}_f\gamma^3u_i$	$\mathcal{N}\left[Rf_1+R\frac{M_i-E_f}{M}f_2-\frac{\lvert\mathbf{p}_f\rvert}{M}f_3\right]$	$\mathcal{N}\left[Rf_1+R\frac{M_i-E_f}{M}f_2+\frac{\lvert\mathbf{p}_f\rvert}{M}f_3\right]$

轴矢量流 (Axial Current, H_A^μ, 感生张量项衰变取 $-i\sigma q\gamma^5$，散射取 $+i\sigma q\gamma^5$)

0 (时间)	↑↑	$\bar{u}_f\gamma^0\gamma^5u_i$	$\mathcal{N}\left[Rg_1+\frac{\lvert\mathbf{p}_f\rvert}{M}g_2-R\frac{M_i-E_f}{M}g_3\right]$	$\mathcal{N}\left[Rg_1+\frac{\lvert\mathbf{p}_f\rvert}{M}g_2-R\frac{E_f-M_i}{M}g_3\right]$
1 (横向)	↑↓	$\bar{u}_f\gamma^1\gamma^5u_i$	$\mathcal{N}\left[g_1-\frac{M_i-M_f}{M}g_2\right]$	$\mathcal{N}\left[g_1-\frac{M_i-M_f}{M}g_2\right]$
2 (横向)	↑↓	$\bar{u}_f\gamma^2\gamma^5u_i$	$i\mathcal{N}\left[g_1-\frac{M_i-M_f}{M}g_2\right]$	$i\mathcal{N}\left[g_1-\frac{M_i-M_f}{M}g_2\right]$
3 (纵向)	↑↑	$\bar{u}_f\gamma^3\gamma^5u_i$	$\mathcal{N}\left[g_1-\frac{M_i-E_f}{M}g_2+R\frac{\lvert\mathbf{p}_f\rvert}{M}g_3\right]$	$\mathcal{N}\left[g_1-\frac{M_i-E_f}{M}g_2-R\frac{\lvert\mathbf{p}_f\rvert}{M}g_3\right]$

By introducing helicity matrix elements, $H^0_{V/A}(\uparrow\uparrow)$, $H^3_{V/A}(\uparrow\uparrow)$ and $H^1_{V/A}(\uparrow\downarrow)$ to obtain $f_{1\text{-}3}$，$g_{1\text{-}3}$.

$$f_2(q^2)=-\frac{M}{2M_i\mathcal{N}}\left[H_V^0(\uparrow\uparrow)+\frac{M_i-E_f}{|\mathbf{p}_f|}H_V^3(\uparrow\uparrow)+\left(\frac{1}{R}+\frac{M_i-E_f}{|\mathbf{p}_f|}\right)H_V^1(\uparrow\downarrow)\right],$$

$$f_1(q^2)=-\frac{M_i+M_f}{M}f_2(q^2)-\frac{H_V^1(\uparrow\downarrow)}{\mathcal{N}R},$$

$$f_3(q^2)=\frac{M}{M_i-E_f}\left[\frac{H_V^0(\uparrow\uparrow)}{\mathcal{N}}-f_1(q^2)+\frac{E_f-M_f}{M}f_2(q^2)\right].$$

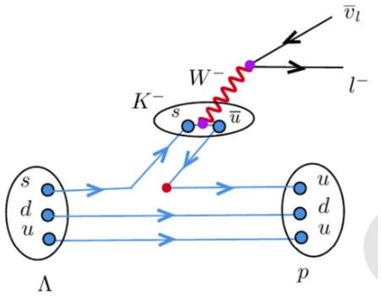
$$g_2(q^2)=\frac{M}{2M_i\mathcal{N}}\left[\frac{1}{R}H_A^0(\uparrow\uparrow)+\frac{M_i-E_f}{R|\mathbf{p}_f|}H_A^3(\uparrow\uparrow)-\left(1+\frac{M_i-E_f}{R|\mathbf{p}_f|}\right)H_A^1(\uparrow\downarrow)\right],$$

$$g_1(q^2)=\frac{H_A^1(\uparrow\downarrow)}{\mathcal{N}}+\frac{M_i-M_f}{M}g_2(q^2),$$

$$g_3^{\text{bare}}(q^2)=\frac{M}{R|\mathbf{p}_f|}\left[\frac{H_A^3(\uparrow\uparrow)}{\mathcal{N}}-g_1(q^2)+\frac{M_i-E_f}{M}g_2(q^2)\right].$$

$$\mathcal{N}=\sqrt{2M_i(E_f+M_f)}$$
$$R=|\mathbf{p}_f|/(E_f+M_f).$$

形状因子计算



$\Lambda \xrightarrow{W^-} \bar{v}_l + l^- + p$

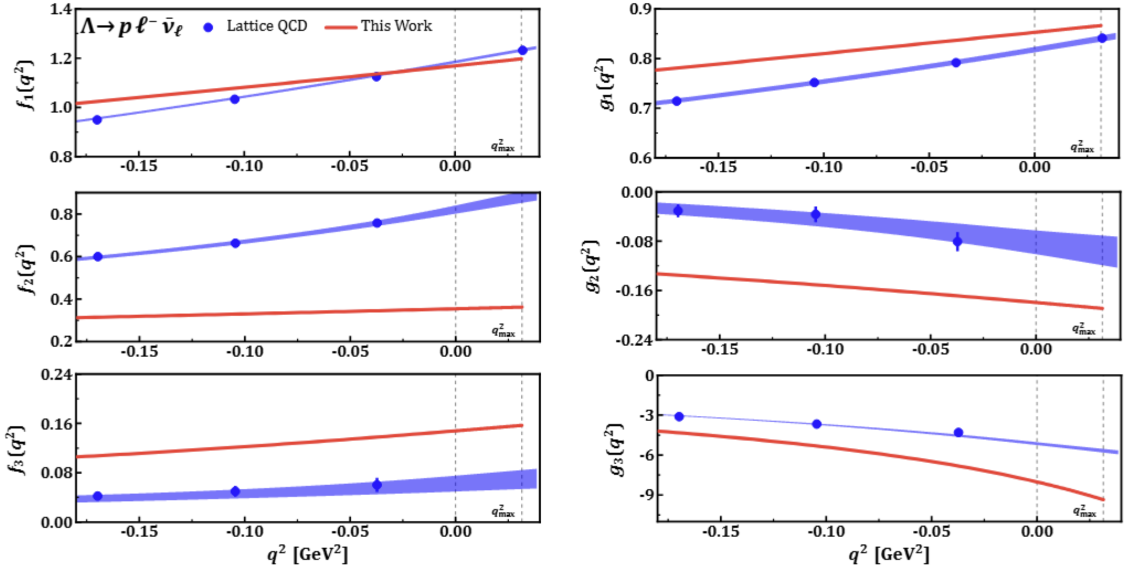
基于强子层次的计算

$$H_V^\nu(\lambda_f,\lambda_i;P_i,P_f)=\bar{u}_f(P_f,\lambda_f)\left[f_1(q^2)\gamma^\nu - i\frac{f_2(q^2)}{M}\sigma^{\nu\alpha}q_\alpha + \frac{f_3(q^2)}{M}q^\nu\right]u_i(P_i,\lambda_i),$$

$$H_A^\nu(\lambda_f,\lambda_i;P_i,P_f)=\bar{u}_f(P_f,\lambda_f)\left[g_1(q^2)\gamma^\nu\gamma^5 - i\frac{g_2(q^2)}{M}\sigma^{\nu\alpha}q_\alpha\gamma^5 + \frac{g_3(q^2)}{M}q^\nu\gamma^5\right]u_i(P_i,\lambda_i),$$

g_2 is zero at exact SU(3) limit

f_3, g_3 terms are suppressed by $(m_l/M_B)^2$

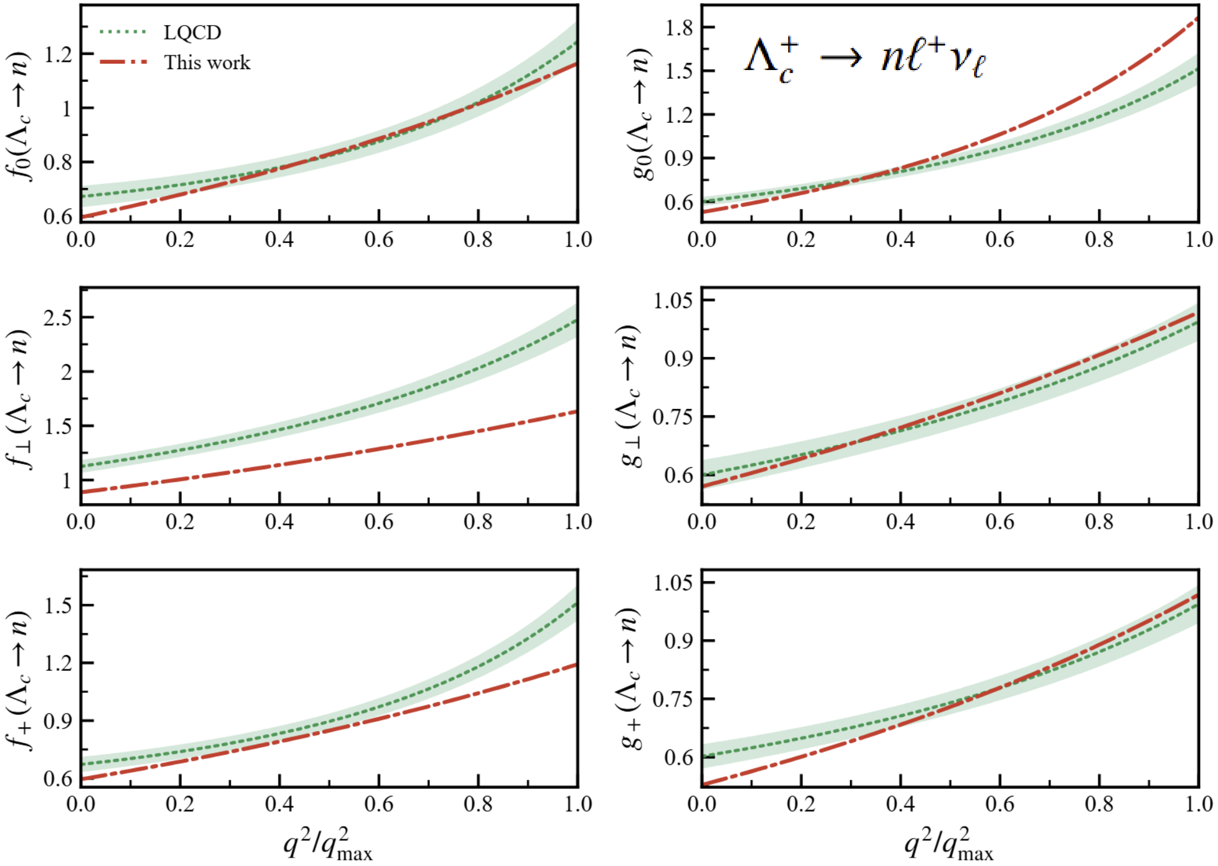


Method	$f_1(0)$	$f_2(0)$	$f_3(0)$	$g_1(0)$	$g_2(0)$	$g_3(0)$
LFQM [44]	1.190	0.950	—	0.990	0.028	—
χQM [45]	1.220	0.913	−0.337	0.820	0.054	−11.407
CCQM [50]	1.226	1.226	−0.067	0.888	0.072	−6.760
χQSM [47]	1.225	0.870	—	0.830	—	—
χCQM [48]	1.225	0.563	−0.225	0.909	0.092	−11.244
QCDSR [39]	1.179(75)	0.888(74)	—	0.843(10)	—	—
LFD [102]	$1.305(^{+15}_{-38})$	$1.139(^{+32}_{-13})$	$-0.917(^{+144}_{-182})$	$0.924(^{+12}_{-24})$	$0.269(^{+101}_{-115})$	$-7.487(^{+1377}_{-1169})$
LQCD [36]	1.185(6)	0.821(19)	0.062(13)	0.818(6)	−0.082(19)	−5.14(11)
This work	**1.169**	**0.354**	**0.148**	**0.853**	**−0.180**	**−8.017**

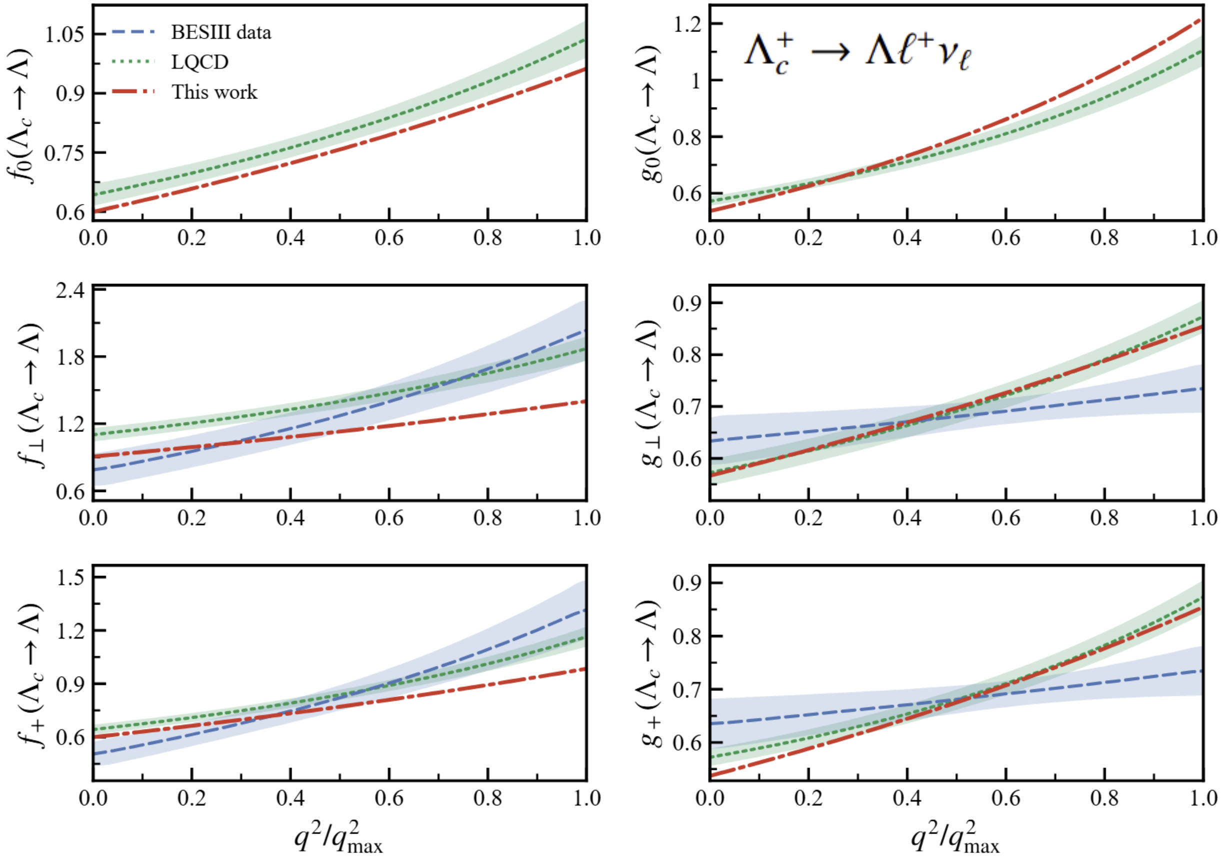
Method	$f_2(0)/f_1(0)$	$g_1(0)/f_1(0)$	$g_2(0)/f_1(0)$	$\lvert f_1(0)\rvert/\lvert f_1^{\text{stat.}}\rvert$	$\lvert g_1(0)\rvert/\lvert g_1^{\text{stat.}}\rvert$
LFQM [44]	0.798	0.826	0.023	0.972	0.808
χQM [45]	0.748	0.672	0.044	0.996	0.670
CCQM [50]	1.000	0.724	0.059	1.001	0.725
χQSM [47]	0.710	0.678	—	1.000	0.678
χCQM [48]	0.460	0.742	0.075	1.000	0.742
QCDSR [39]	0.752(74)	0.708(47)	—	0.963(61)	0.688(8)
LFD [102]	$0.872(^{+35}_{-14})$	$0.708(^{+12}_{-27})$	$0.206(^{+77}_{-89})$	$1.065(^{+12}_{-27})$	$0.755(^{+10}_{-20})$
LQCD [36]	0.693(17)	0.690(4)	−0.069(16)	0.967(5)	0.668(4)
BESIII [65]	$0.77(^{+53}_{-49})$	$0.706(^{+89}_{-86})$	$-0.19(^{+65}_{-63})$	—	—
RPP [66]	—	0.718(15)	—	—	—
This work	**0.303**	**0.730**	**−0.154**	**0.954**	**0.696**

Simone Bacchio, Andreas Konstantinou Cyprus group PRL 135 231901

形状因子计算

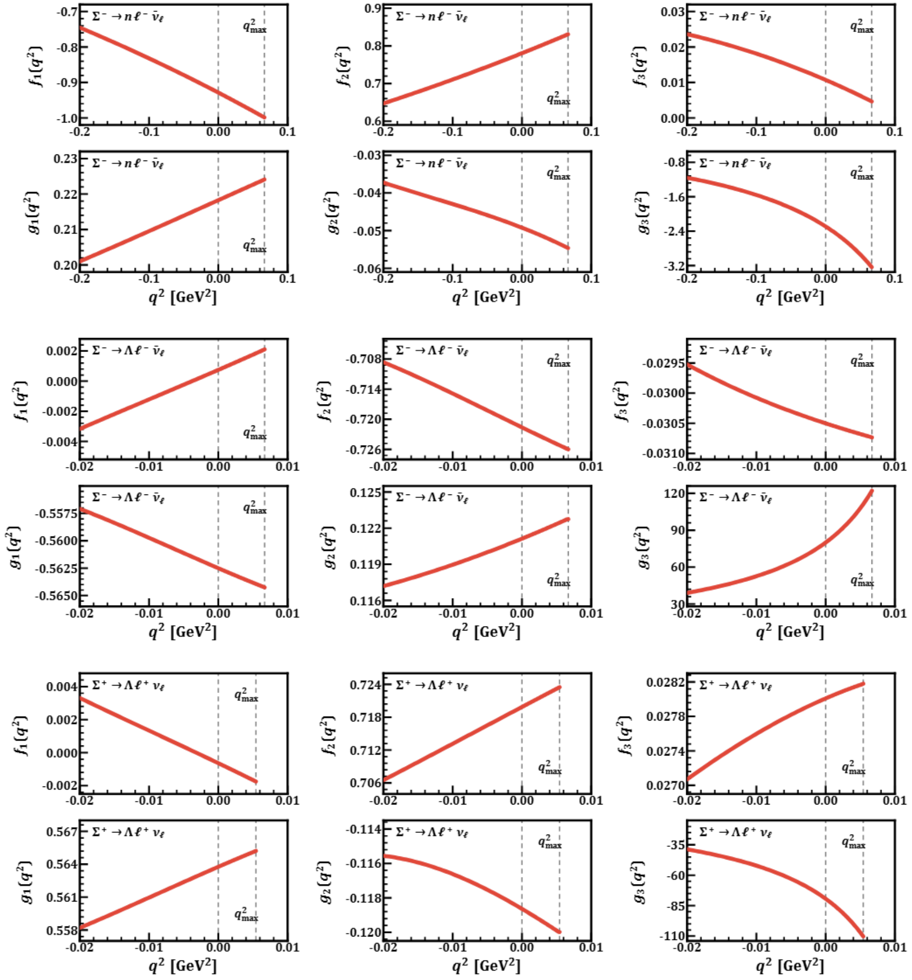


S. Meinel, Phys. Rev. D 97, 034511 (2018)



BESIII, Phys. Rev. D 108, L031105 (2023),
S. Meinel, Phys. Rev. Lett. 118, 082001 (2017)

形状因子计算



Method	$f_1(0)$	$f_2(0)$	$f_3(0)$	$g_1(0)$	$g_2(0)$	$g_3(0)$
$\Sigma^- \to n\ell^-\bar{\nu}_\ell$						
LFQM [44]	−0.970	0.742	—	0.270	0.007	—
χQM [45]	−1.000	1.020	0.471	0.220	−0.017	−4.034
CCQM [50]	−1.009	0.971	0.055	0.262	−0.078	−2.199
χQSM [47]	−1.000	0.960	—	0.270	—	—
χCQM [48]	−1.000	1.016	0.345	0.314	0.010	−5.155
QCDSR [39]	−0.993	1.025	—	0.324	—	—
LQCD [35]	−0.957(10)	—	—	—	—	—
LQCD [29]	−0.988(49)	0.841(450)	0.232(122)	0.284(53)	−0.349(145)	−3.376(1834)
This work	**−0.929**	**0.780**	**0.011**	**0.218**	**−0.050**	**−2.292**
$\Sigma^0 \to p\ell^-\bar{\nu}_\ell$						
LFQM [44]	−0.690	0.524	—	0.190	0.005	—
QCDSR [39]	−0.702	0.725	—	0.229	—	—
This work	**−0.658**	**0.552**	**0.007**	**0.155**	**−0.035**	**−1.612**

Method	$f_1(0)$	$f_2(0)$	$f_3(0)$	$g_1(0)$	$g_2(0)$	$g_3(0)$
$\Sigma^- \to \Lambda e^-\bar{\nu}_e$						
LFQM [44]	0	−1.245	—	−0.600	0	—
χQM [45]	0	−1.413	−0.052	−0.550	0.129	124.200
CCQM [50]	0.002	−1.206	0.016	−0.633	−0.013	84.446
χQSM [47]	0	−1.240	—	−0.600	—	—
χCQM [48]	0	−1.173	−0.041	−0.646	0.079	140.500
RPP [66]	—	—	—	—	—	—
This work	**0.001**	**−0.722**	**−0.030**	**−0.562**	**0.121**	**79.981**
$\Sigma^+ \to \Lambda e^+\nu_e$						
LFQM [44]	0	1.237	—	0.600	0	—
χQM [45]	0	1.404	0.046	0.550	−0.114	−108.400
χCQM [48]	0	1.165	0.037	0.646	−0.070	−126.900
This work	**−0.001**	**0.720**	**0.028**	**0.564**	**−0.119**	**−79.342**

小结和展望

利用夸克模型计算了一系列重子半轻衰变的分支比，大体上和实验符合，这给予了我们信心利用弱作用去更加细致的研究强子的性质。

期待相关实验的检验，并且我们相信现在的理论估计是非常粗糙的，只是在量级上估计，亟待实验数据的输入能够鉴别参数和约束更多的模型参数。

尤其是对于f2这类形状因子究竟反应了什么内部机制？
什么样的实验观测量能够对f2形状因子非常敏感？

谢谢