2026年8月29日，第八届粒子物理天问论坛，桂林市

超子弱辐射衰变的理论研究

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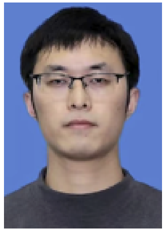

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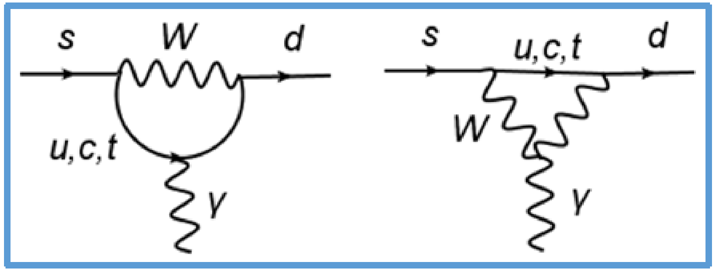
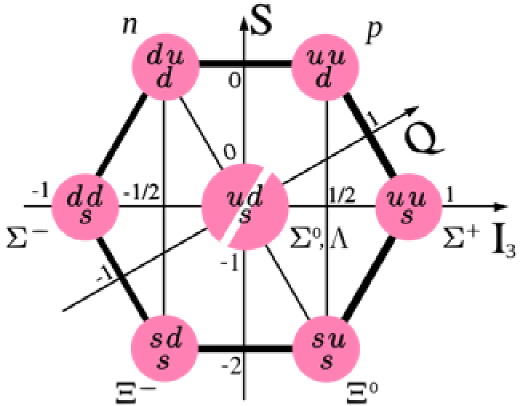
- ☞Summary and outlook

Contents

What are weak radiative hyperon decays

- **Weak radiative hyperon decays (WRHDs)** are interesting physical processes **involving the electromagnetic, weak, and strong interactions**
- $s \to d\,\gamma$ transitions at the quark level

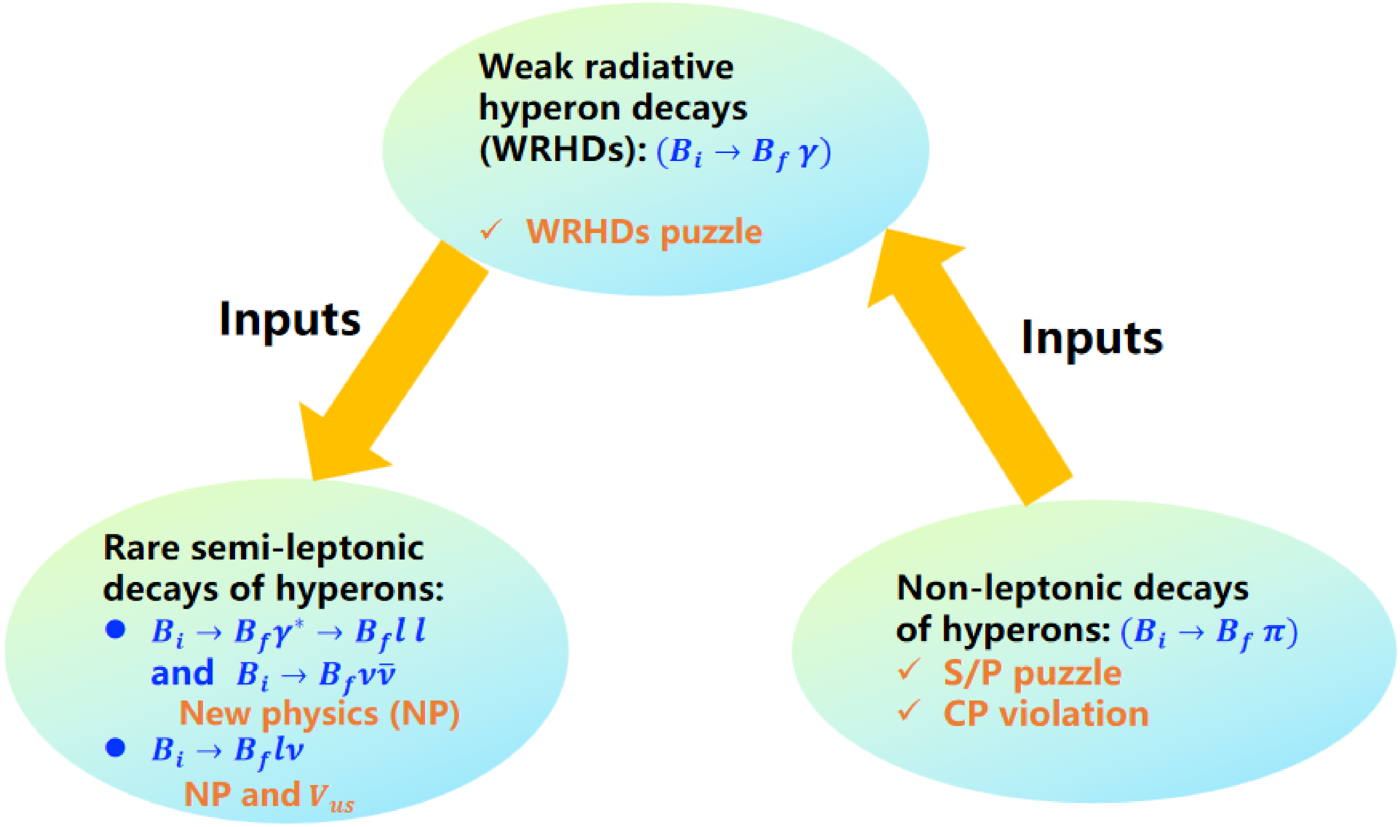




- **Six** WRHDs channels of the ground-state octet baryons

$$\Lambda \to n\gamma \qquad \Sigma^0 \to n\gamma \qquad \Xi^0 \to \Sigma^0\gamma$$
$$\Sigma^+ \to p\gamma \qquad \Xi^0 \to \Lambda\gamma \qquad \Xi^- \to \Sigma^-\gamma$$

Weak decays of hyperons: related to various processes

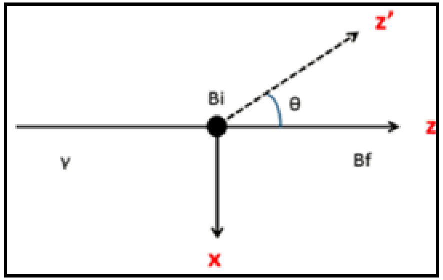


What are weak radiative hyperon decays

- The effective Lagrangian describing the $B_i \to B_f\,\gamma$ WRHDs

$$\mathcal{L} = \frac{eG_F}{2}\bar{B}_f(a + b\gamma_5)\sigma^{\mu\nu}B_i F_{\mu\nu},$$

a: partity-conserving amplitude b: partity-violating amplitude



- **Only two observables** for the WRHDs

$$\frac{d\Gamma}{d\cos\theta} = \frac{e^2G_F^2}{\pi}(|a|^2 + |b|^2)[1 + \frac{2\mathrm{Re}(ab^*)}{|a|^2 + |b|^2}\cos\theta]\cdot|\vec{k}|^3,$$

$$\boxed{\alpha_\gamma = \frac{2\mathrm{Re}(ab^*)}{|a|^2 + |b|^2}, \qquad \Gamma = \frac{e^2G_F^2}{\pi}(|a|^2 + |b|^2)\cdot|\vec{k}|^3,} \qquad |\vec{k}| = \frac{m_i^2 - m_f^2}{2m_i}$$

α_γ: **asymmetry parameter**
θ: **angle between spin of the initial baryon B_i and 3-momentum $\vec{k}$ of the final baryon B_f**

Why study WRHDs: the WRHDs puzzle

Hara's theorem *Y. Hara, PRL12, 378 (1964)*

- Based on gauge invariance, CP conservation, and U-spin symmetry

- Hara' s theorem dictates that the WRHDs $B \to B'\gamma$ and $B' \to B\gamma$ must be identical under the U-spin transformation $s \Leftrightarrow d$

$$\mathcal{L}_{\text{p.c.}} = a\left(\bar{p}\sigma^{\mu\nu}\Sigma^+ F_{\mu\nu} + \bar{\Sigma}^+\sigma^{\mu\nu} p F_{\mu\nu}\right)\frac{eG_F}{2},$$

$$\mathcal{L}_{\text{p.v.}} = b\left(\bar{p}\sigma^{\mu\nu}\gamma_5\Sigma^+ F_{\mu\nu} - \bar{\Sigma}^+\sigma^{\mu\nu}\gamma_5 p F_{\mu\nu}\right)\frac{eG_F}{2},$$

leads to

$$b = -b, \quad \text{i.e.}, \quad b = 0$$

$$\boxed{\alpha_\gamma = \frac{2\,\mathrm{Re}(ab^*)}{|a|^2 + |b|^2} = 0}$$

Why study WRHDs: the WRHDs puzzle

PHYSICAL REVIEW VOLUME 188, NUMBER 5 25 DECEMBER 1969

Asymmetry Parameter and Branching Ratio of $\Sigma^+ \to p\gamma$*

LAWRENCE K. GERSHWIN,† MARGARET ALSTON-GARNJOST, ROGER O. BANGERTER, ANGELA BARBARO-GALTIERI, TERRY S. MAST, FRANK T. SOLMITZ, AND ROBERT D. TRIPP

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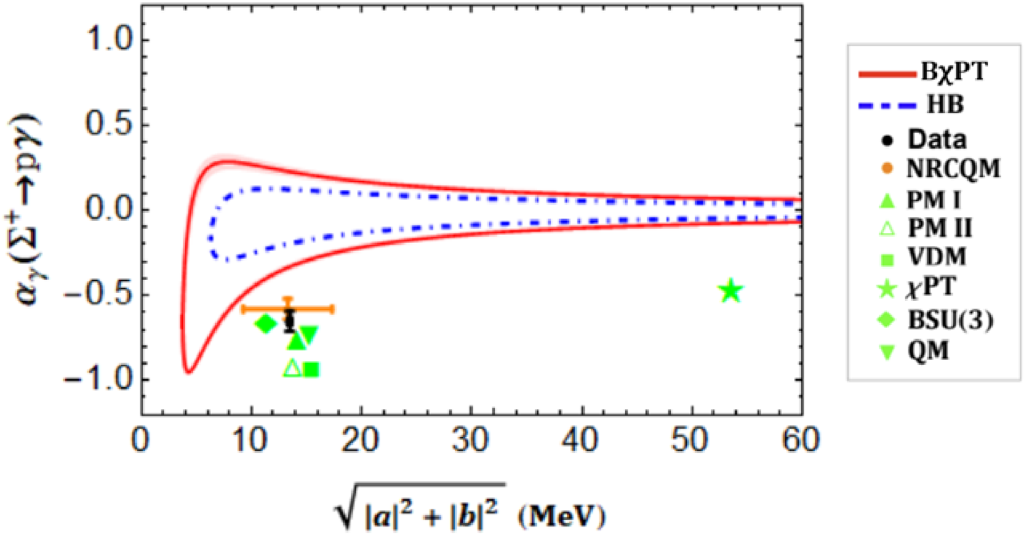
(Received 25 August 1969)

An experiment to study the decay $\Sigma^+ \to p\gamma$ was performed in the Berkeley 25-in. hydrogen bubble chamber. An analysis was made of 48 000 events of the type $K^-p \to \Sigma^+\pi^-$, $\Sigma^+ \to p+$neutral with K^- momenta near 400 MeV/c. The Σ's produced in this momentum region are polarized because of the interference of the Y_0^* (1520) amplitude with the background amplitudes. We have measured the proton asymmetry parameter α for 61 $\Sigma^+ \to p\gamma$ events with an average polarization of 0.4. We found $\alpha = -1.03_{-0.42}^{+0.52}$. $SU(3)$ predicts a value $\alpha = 0$. A more restricted sample of events was used to determine the $\Sigma^+ \to p\gamma$ branching ratio. From 31 $\Sigma^+ \to p\gamma$ events and 11 670 $\Sigma^+ \to p\pi^0$ events, we found $(\Sigma^+ \to p\gamma)/(\Sigma^+ \to p\pi^0) = (2.76 \pm 0.51) \times 10^{-3}$. The result is in agreement with the previous measurements.

Why study WRHDs: the WRHDs puzzle

- The $\Sigma^+ \to p\,\gamma$ asymmetry parameter remains **large and negative**:
  −0.652±0.056stat±0.020syst.



Data: BESIII, PRL130 (2023) 21, 211901

HB χPT: E. E. Jenkins et al, NPB 397, 84 (1993)

$B\chi PT$: H. Neufeld, Nucl. Phys. B 402, 166 (1993)

NRCQM: Qiang Zhao et al, CPC45, 013101 (2021)

PM1: M. B. Gavela et al, PLB 101, 417 (1981)

PM2: G. Nardulli, PLB 190, 187 (1987)

VDM: P. Zenczykowski, PRD 44, 1485 (1991)

χPT: B. Borasoy et al, PRD 59, 054019 (1999)

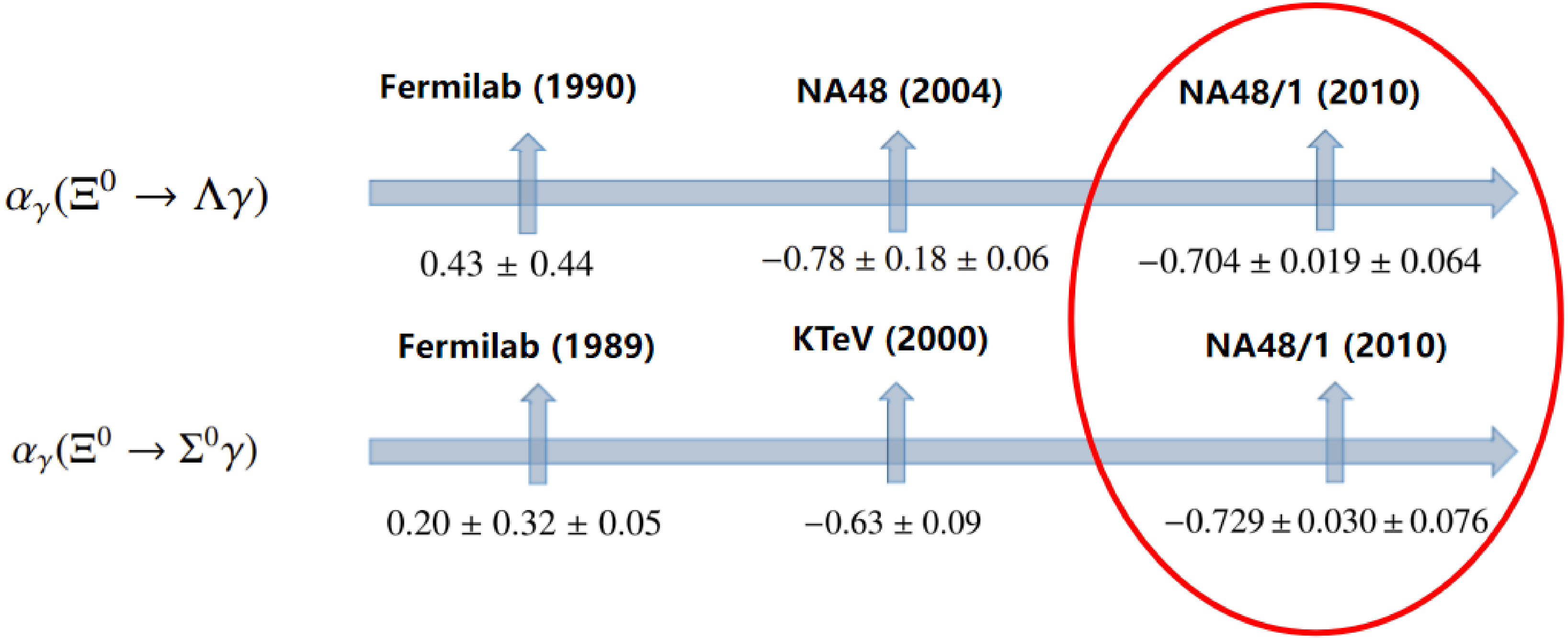
BSU(3): P. Zenczykowski, PRD 73, 076005 (2006)

QM: E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)

- Although some predictions agree with the measured large asymmetry of the $\Sigma^+ \to p\,\gamma$ decay, **they explain poorly the data of other WRHDs (as shown later)**

Why study WRHDs: experimentally challenging

- **Significant changes** in the asymmetry parameters of $\Xi^0 \to \Sigma^0 \gamma$ and $\Xi^0 \to \Lambda \gamma$



Why study WRHDs--$\Lambda \to n\gamma$

New BESIII measurement for the $\Lambda \to n\gamma$ decay (PRL129(2022)21,212002)

Decay Mode	$\Lambda \to n\gamma$	$\bar{\Lambda} \to \bar{n}\gamma$
N_{ST} $(\times 10^3)$	6853.2 ± 2.6	7036.2 ± 2.7
$\varepsilon_{\mathrm{ST}}$ (%)	51.13 ± 0.01	52.53 ± 0.01
N_{DT}	723 ± 40	498 ± 41
$\varepsilon_{\mathrm{DT}}$ (%)	6.58 ± 0.04	4.32 ± 0.03
BF $(\times 10^{-3})$	$0.820 \pm 0.045 \pm 0.066$	$0.862 \pm 0.071 \pm 0.084$
	$0.832 \pm 0.038 \pm 0.054$	
α_γ	$-0.13 \pm 0.13 \pm 0.03$	$0.21 \pm 0.15 \pm 0.06$
	$-0.16 \pm 0.10 \pm 0.05$	

PDG2022

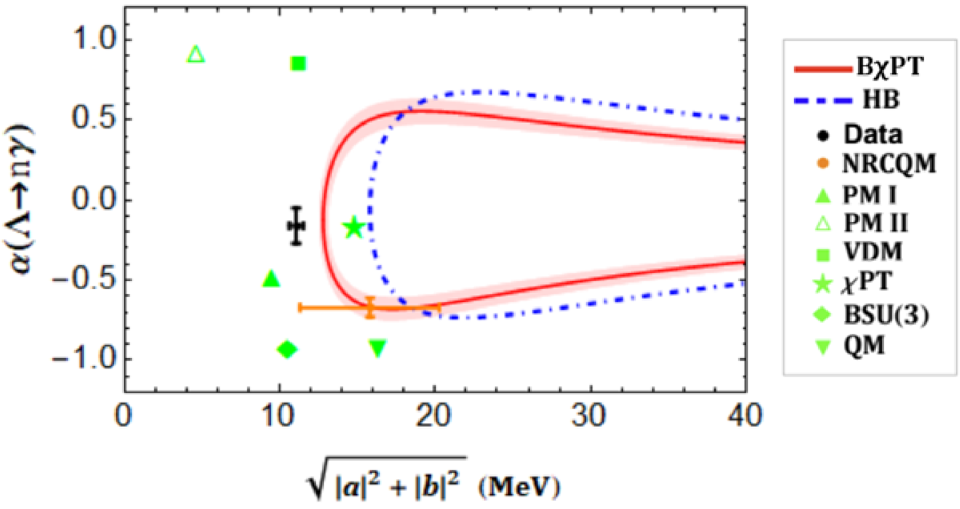
$\Gamma(n\gamma)/\Gamma_{\mathrm{total}}$ Γ_3/Γ

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
1.75±0.15 OUR FIT					
1.75±0.15	1816	LARSON	93	SPEC	$K^- p$ at rest
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$1.78 \pm 0.24^{+0.14}_{-0.16}$	287	NOBLE	92	SPEC	See LARSON 93

- The branching fraction is only **about one half** of the current PDG average
- The asymmetry parameter α_γ **is determined for the first time**

Why study WRHDs—$\Lambda \to n\gamma$

- **None of the existing predictions can describe** the new BESIII measurement for the $\Lambda \to n\gamma$ decay



Data: *BESIII, PRL129(2022)21,212002*

HB χPT: *E. E. Jenkins et al, NPB 397, 84 (1993)*

BχPT: *H. Neufeld, Nucl. Phys. B 402, 166 (1993)*

NRCQM: *Qiang Zhao et al, CPC45, 013101 (2021)*

PM1: *M. B. Gavela et al, PLB 101, 417 (1981)*

PM2: *G. Nardulli, PLB 190, 187 (1987)*

VDM: *P. Zenczykowski, PRD 44, 1485 (1991)*

χPT: *B. Borasoy et al, PRD 59, 054019 (1999)*

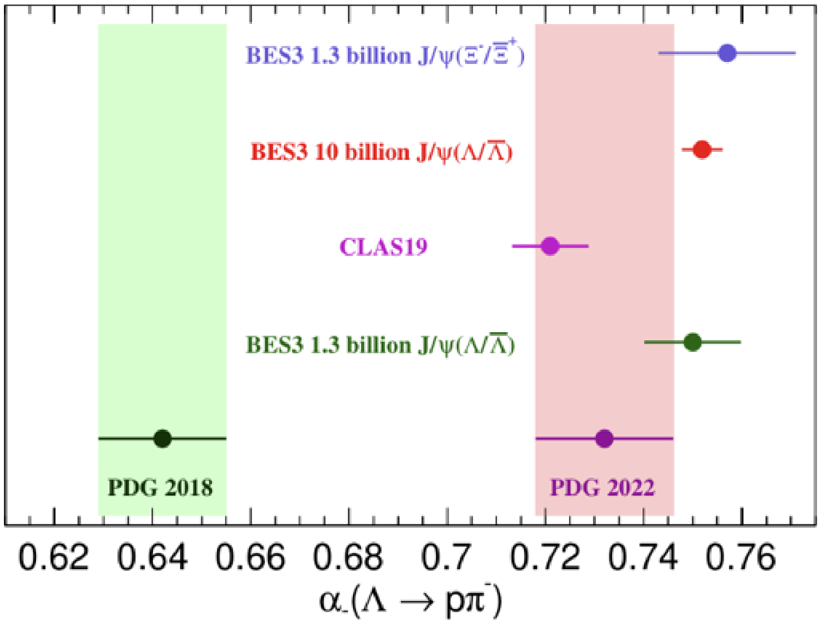
BSU(3): *P. Zenczykowski, PRD 73, 076005 (2006)*

QM: *E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)*

Why study WRHDs

□ New BESIII and CLAS data for hyperon non-leptonic decays

BESIII: Nature Phys. 15, 631 (2019)	CLAS: PRL123,182301 (2019)
BESIII: Nature 606, 64 (2022)	BESIII: PRL129,131801 (2022)



- Definition of decay parameter for the $\Lambda \to p\,\pi^-$ decay

$$\mathcal{M}(B_i \to B_f \pi) = iG_F m_\pi^2 \bar{B}_f (A_S - A_P \gamma_5) B_i$$

$$\boxed{\alpha_\pi = \frac{2\mathrm{Re}\,(s \cdot p)}{|s|^2 + |p|^2}} \qquad s = A_S \qquad p = A_P |\vec{q}|/(E_f + m_f)$$

- Featured by a **larger statistics** and a **small uncertainty** and very different from previous PDG average

- A significant change for the baryon decay parameter of $\Lambda \to p\,\pi^-$ may **greatly affect the values of LECs hD, hF and hyperon non-leptonic decay amplitudes as inputs to WRHDs**

Why study WRHDs—theoretical tools

- Theoretically, **two phenomenological models** are able to explain the current experimental data of WRHDs at least qualitatively **except for the $\Lambda \to n\,\gamma$ decay**

  - *E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)—QM*
  - *P. Zenczykowski, PRD 73, 076005 (2006)—BSU(3)*

- **Chiral perturbation theory (χPT)** studies on the WRHDs

  - *B. Borasoy et al, PRD 59, 054019 (1999)*
  - *E. E. Jenkins et al, NPB397, 84 (1993)*
  - *J. W. Bos et al, PRD 51, 6308 (1995)*
  - *J. W. Bos et al, PRD 54, 3321 (1996)*
  - *J. W. Bos, et al, PRD 57, 4101 (1998)*

  (Tree or loop level in the heavy baryon formulation)

  - *H. Neufeld, NPB 402, 166 (1993)* *(Loop level in the covariant formulation)*

Our purpose

Our goal is to study the WRHDs in covariant baryon chiral perturbation theory (BχPT) with the extended-on-mass-shell (EOMS) renormalization scheme

- **The work in the BχPT** *H. Neufeld, NPB 402, 166 (1993)*
  - ✓ The used low energy constants (LECs) and hyperon non-leptonic decay amplitudes are **out of date**
  - ✓ No efforts were taken to ensure a **consistent power counting**

Updating the relevant LECs and hyperon non-leptonic decay amplitudes → Calculating the branching fractions and asymmetry parameters, i.e., **amplitudes a and b**, of the WRHDs order by order → Comparing our predictions with those from other approaches/experimental data

Contents

- Brief introduction: motivation and purpose
- Theoretical framework: covariant ChEFT
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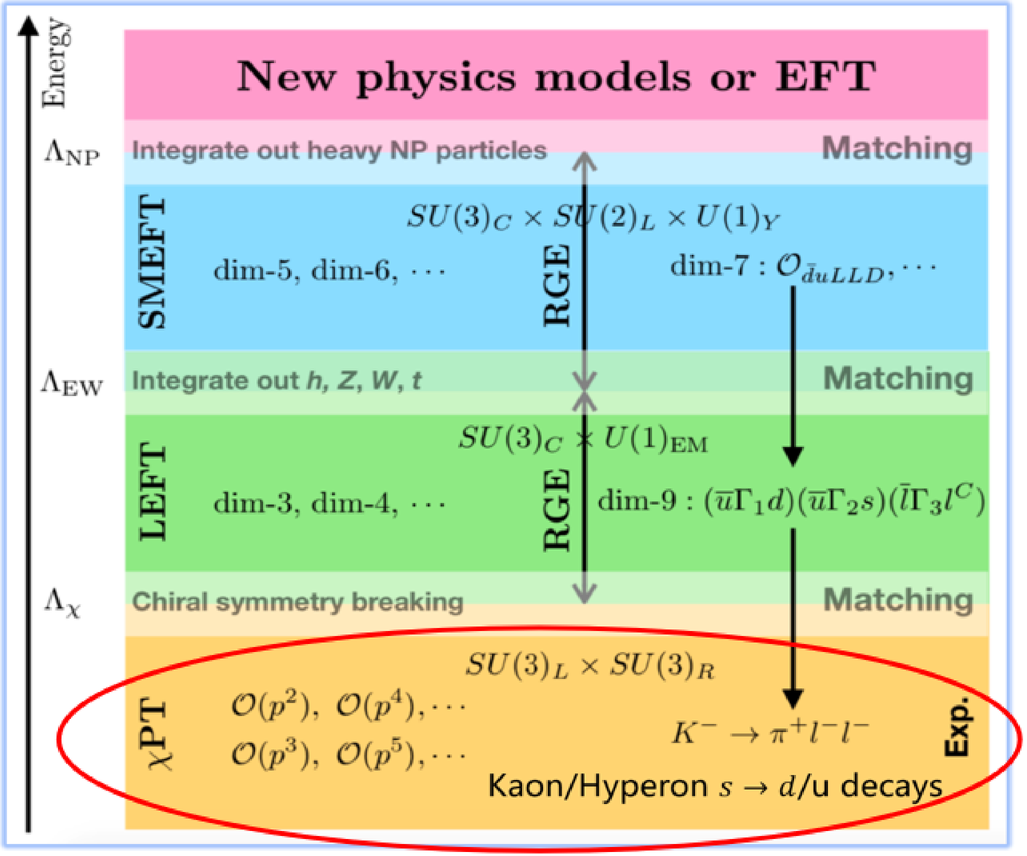
Chiral perturbation theory : a bottom-up EFT approach



- **Effective theory:** the physics in low energy regions does not depend on the details of the higher energy physics, which has been integrated out

- **Chiral perturbation theory is a powerful tool to study the WRHDs**

Chiral perturbation theory

Because of quark confinement and asymptotic freedom,
low energy QCD can not be solved perturbatively

□ Chiral perturbation theory—low energy EFT of QCD

- ✓ Maps quark (u, d, s) dof' s to those of the asymptotic states, hadrons
- ✓ Allows a perturbative formulation of low energy QCD in powers of external momenta and light quark masses, by utilizing chiral symmetry and its breaking pattern (the third feature of QCD)

Power counting:

⇩

$$O(Q/\Lambda)^n$$

Steven Weinberg
Nobel Prize in Physics in 1979

□ The effective Lagrangian of the general form

$$\mathcal{L} = \sum_i c_i\,(Q,\Lambda)\,O_i(\{\psi\})$$

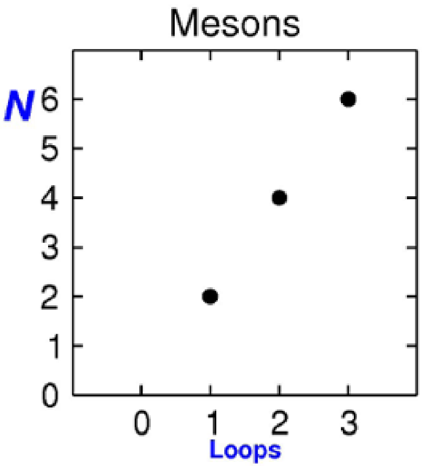
Q is the soft scale, Λ is the hard scale, c_i are LECs, O_i refer to operators containing field ψ.

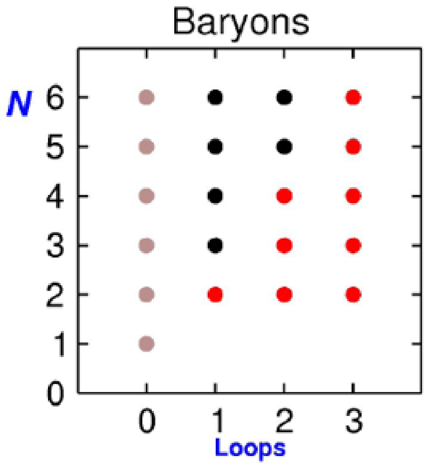
□ Power counting rule

Chiral order: $N = 4L - 2N_M - N_B + \sum k\,V_k$

Power-Counting-Breaking in the baryon sector

- **ChPT very successful in the study of Nanbu-Goldstone boson self-interactions, at least in SU(2)**
- **In the baryon sector, things become problematic because of the nonzero (large) baryon mass in the chiral limit, which leads to the fact that high-order loops contribute to lower-order results, i.e., a systematic power counting is lost!**





red dots denote possible PCB terms (pion-nucleon scattering)

J. Gasser et al., NPB 307, 779(1988)

HB vs. Infrared vs. EOMS

LSG, Front.Phys.(Beijing) 8 (2013) 328

Extended-on-mass-shell (EOMS) BChPT

- satisfies all symmetry and analyticity constraints
- converges relatively faster--an appealing feature

Heavy baryon (HB) ChPT

- non-relativistic
- breaks analyticity of loop amplitudes
- converges slowly (particularly in three-flavor sector)
- strict PC and simple nonanalytical results

Infrared BChPT

- relativistic
- breaks analyticity of loop amplitudes
- converges slowly (particularly in three-flavor sector)
- analytical terms the same as HB ChPT

Three applications of covariant ChPT

LSG, Front.Phys.(Beijing) 8 (2013) 328

PRL **101**, 222002 (2008) PHYSICAL REVIEW LETTERS week ending 28 NOVEMBER 2008

Leading SU(3)-Breaking Corrections to the Baryon Magnetic Moments in Chiral Perturbation Theory

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[1]*Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, E-46071 Valencia, Spain*
[2]*Departamento de Física, Universidad de Murcia, E-30071 Murcia, Spain*
(Received 9 May 2008; published 26 November 2008)

We calculate the baryon magnetic moments using covariant chiral perturbation theory (χPT) within the extended-on-mass-shell renormalization scheme. By fitting the two available low-energy constants, we improve the Coleman-Glashow description of the data when we include the leading SU(3)-breaking effects coming from the lowest-order loops. This success is in dramatic contrast with previous attempts at the same order using heavy-baryon χPT and covariant infrared χPT. We also analyze the source of this improvement with particular attention to the comparison between the covariant results.

PHYSICAL REVIEW LETTERS **130**, 071902 (2023)

Cross-Channel Constraints on Resonant Antikaon-Nucleon Scattering

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(Received 9 September 2022; revised 22 December 2022; accepted 24 January 2023; published 17 February 2023)

Chiral perturbation theory and its unitarized versions have played an important role in our understanding of the low-energy strong interaction. Yet, so far, such studies typically deal exclusively with perturbative or nonperturbative channels. In this Letter, we report on the first global study of meson-baryon scattering up to one-loop order. It is shown that covariant baryon chiral perturbation theory, including its unitarization for the negative strangeness sector, can describe meson-baryon scattering data remarkably well. This provides a highly nontrivial check on the validity of this important low-energy effective field theory of QCD. We show that the $\bar{K}N$ related quantities can be better described in comparison with those of lower-order studies, and with reduced uncertainties due to the stringent constraints from the πN and KN phase shifts. In particular, we find that the two-pole structure of $\Lambda(1405)$ persists up to one-loop order reinforcing the existence of two-pole structures in dynamically generated states.

PHYSICAL REVIEW LETTERS **128**, 142002 (2022)

Accurate Relativistic Chiral Nucleon-Nucleon Interaction up to Next-to-Next-to-Leading Order

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[3]*Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France*
[4]*Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 102206, China*
[5]*School of Physics and Microelectronics, Zhengzhou University, Zhengzhou, Henan 450001, China*
[6]*State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China*
[7]*Physik Department, Technische Universität München, D-85747 Garching, Germany*

(Received 21 November 2021; revised 25 February 2022; accepted 21 March 2022; published 6 April 2022)

We construct a relativistic chiral nucleon-nucleon interaction up to the next-to-next-to-leading order in covariant baryon chiral perturbation theory. We show that a good description of the np phase shifts up to $T_{\rm lab} = 200$ MeV and even higher can be achieved with a $\tilde{\chi}^2$/d.o.f. less than 1. Both the next-to-leading-order results and the next-to-next-to-leading-order results describe the phase shifts equally well up to $T_{\rm lab} = 200$ MeV, but for higher energies, the latter behaves better, showing satisfactory convergence. The relativistic chiral potential provides the most essential inputs for relativistic *ab initio* studies of nuclear structure and reactions, which has been in need for almost two decades.

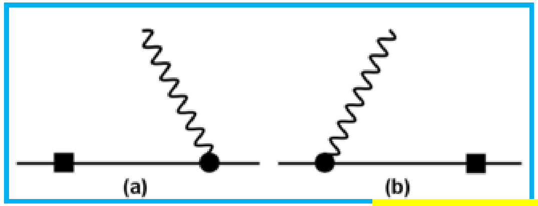
WRHDs in the EOMS BχPT

$$a_{B_iB_f} = a^{(1,\text{tree})}_{B_iB_f} + a^{(2,\text{tree})}_{B_iB_f} + a^{(2,\text{loop})}_{B_iB_f}$$
$$b_{B_iB_f} = b^{(2,\text{tree})}_{B_iB_f} + b^{(2,\text{loop})}_{B_iB_f}$$

Feynman diagrams

Lagrangians

Order contributions

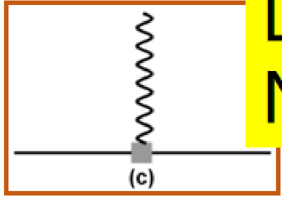


$$\mathcal{L}^{(0)}_{\Delta S=1} = \sqrt{2}G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle,$$
$$\mathcal{L}^{(2)}_{MB} = \frac{b_6^D}{8m_B}\langle \bar{B}\sigma^{\mu\nu}\{F^+_{\mu\nu}, B\}\rangle + \frac{b_6^F}{8m_B}\langle \bar{B}\sigma^{\mu\nu}[F^+_{\mu\nu}, B]\rangle,$$

$a^{(1,\text{tree})}_{B_iB_f}$

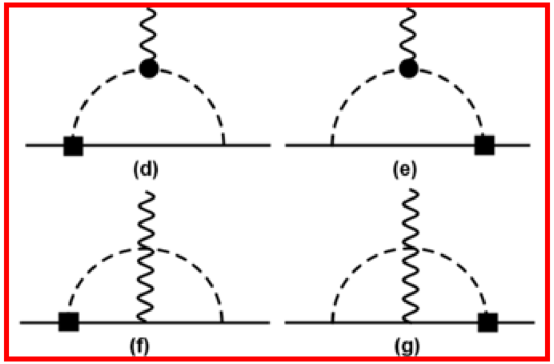
LECs b_6^D and b_6^F : <u>the experimental data of Octet baryon magnetic moment</u>

Leading order LECs hD & hF
NLO LECs: five C’ s



$$\mathcal{L}^{(2)}_{\sigma} = C_\sigma \langle \bar{B}\sigma^{\mu\nu} F_{\mu\nu} \lambda B Q\rangle,$$
$$\mathcal{L}^{(2)}_{\rho} = C_\rho \left(\langle \bar{B}\sigma^{\mu\nu}\gamma_5 F_{\mu\nu} Q\rangle\langle B\lambda\rangle - \langle \bar{B}\sigma^{\mu\nu}\gamma_5 F_{\mu\nu}\lambda\rangle\langle BQ\rangle\right)$$

$a^{(2,\text{tree})}_{B_iB_f}$ $\quad b^{(2,\text{tree})}_{B_iB_f}$



$$\mathcal{L}^{(0)}_{\Delta S=1} = \sqrt{2}G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle$$
$$\mathcal{L}^{(1)}_{B} = \langle \bar{B} i\gamma^\mu D_\mu B - m_0 \bar{B}B\rangle,$$
$$\mathcal{L}^{(2)}_{M} = \frac{F_\phi^2}{4}\langle u_\mu u^\mu + \chi^+\rangle,$$
$$\mathcal{L}^{(1)}_{MB} = \frac{D}{2}\langle \bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\rangle + \frac{F}{2}\langle \bar{B}\gamma^\mu\gamma_5[u_\mu, B]\rangle,$$

LECs D and F have been determined in Ref. <u>LSG et al, PRD 90, 054502 (2014)</u>

$a^{(2,\text{loop})}_{B_iB_f}$ $\quad b^{(2,\text{loop})}_{B_iB_f}$

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- Brief introduction: motivation and purpose
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LECs hD, hF and hyperon non-leptonic decay amplitudes

- The hyperon non-leptonic decay amplitudes for the octet-to-octet transitions have the following form

$$\mathcal{M}(B_i \to B_f \pi) = i G_F m_\pi^2 \bar{B}_f \left(A_S - A_P \gamma_5\right) B_i$$

Hyperon non-leptonic decay amplitudes: S-wave amplitude A_S and P-wave amplitude A_P

- **Decay width and baryon decay parameters α_π, β_π and γ_π** for $B_i \to B_f \pi$ decays

$$\Gamma(B_i \to B_f \pi) = \frac{\left(G_F m_\pi^2\right)^2}{8\pi m_i^2} |\vec{q}| \left\{ \left[(m_i + m_f)^2 - m_\pi^2\right] |s|^2 + \left[(m_i - m_f)^2 - m_\pi^2\right] \left| p \cdot \frac{(E_f + m_f)}{|\vec{q}|} \right|^2 \right\}$$

$$\alpha_\pi = \frac{2\,\mathrm{Re}(s \cdot p)}{|s|^2 + |p|^2}, \beta_\pi = \frac{2\,\mathrm{Im}(s \cdot p)}{|s|^2 + |p|^2}, \gamma_\pi = \frac{|s|^2 - |p|^2}{|s|^2 + |p|^2},$$

with

$$s = A_S \qquad p = A_P |\vec{q}| / (E_f + m_f) \qquad \alpha^2 + \beta^2 + \gamma^2 = 1$$

where E_f and $\vec{q}$ are the energy and 3-momentum of the final baryon

LECs hD, hF and hyperon non-leptonic decay amplitudes

Table: By means of isospin symmetry, the Lee-Sugawara relations and the criterion that $A_S(\Lambda \to p\pi^-)$ is conventionally positive, **S - and P-wave hyperon non-leptonic decay amplitudes are uniquely determined by fitting to the recent data** [3,51-53] of branching fraction $\mathcal{B}$, baryon decay parameters α_π and γ_π

Decay modes	$\mathcal{B}$ [3]	α_π [3, 51–53]	ϕ_π (°) [3, 52]	$s = A_S^{(\text{Expt})}$ This work	$s = A_S^{(\text{Expt})}$ [49]	$p = A_P^{(\text{Expt})}\lvert\vec{q}\rvert/(E_f + m_f)$ This work	$p = A_P^{(\text{Expt})}\lvert\vec{q}\rvert/(E_f + m_f)$ [49]
$\Sigma^+ \to n\pi^+$	0.4831(30)	0.068(13)	167(20)	0.06(1)	0.06(1)	1.81(1)	1.81(1)
$\Sigma^- \to n\pi^-$	0.99848(5)	−0.068(8)	10(15)	1.88(1)	1.88(1)	−0.06(1)	−0.06(1)
$\Lambda \to p\pi^-$	0.639(5)	0.7462(88)	−6.5(35)	1.38(1)	1.42(1)	0.62(1)	0.52(2)
$\Xi^- \to \Lambda\pi^-$	0.99887(35)	−0.376(8)	0.6(12)	−1.99(1)	−1.98(1)	0.39(1)	0.48(2)
$\Sigma^+ \to p\pi^0$	0.5157(30)	−0.982(14)	36(34)	−1.50(3)	−1.43(5)	1.29(4)	1.17(7)
$\Lambda \to n\pi^0$	0.358(5)	0.74(5)	⋯	−1.09(2)	−1.04(1)	−0.48(4)	−0.39(4)
$\Xi^0 \to \Lambda\pi^0$	0.99524(12)	−0.356(11)	21(12)	1.62(10)	1.52(2)	−0.30(10)	−0.33(2)

$$\gamma_\pi = \sqrt{1 - \alpha_\pi^2}\cos(\phi_\pi)$$

- Comparing our results with those of Ref. [49]:
  - ✓ P-wave amplitudes, especially for $A_P(\Lambda \to p\pi^-)$ and $A_P(\Xi^- \to \Lambda\pi^-)$, differ a lot, which would **affect the imaginary parts of the parity-conserving amplitude a**
  - ✓ Experimental S -wave amplitudes remain almost unchanged

[3] P. A. Zyla et al. PDG, PTEP 2020, 083C01(2020) [49] E. E. Jenkins, NPB 375, 561 (1992) [51] M. Ablikim et al., BESIII, 2204.11058 (2022)
[52] M. Ablikim et al. (BESIII), Nature 606, 64, 2105.11155 (2022) [53] D. G. Ireland et al, PRL 123,182301 (2019)

Non-leptonic decay amplitudes—S/P puzzle

□ Amplitudes of hyperon non-leptonic decays

$$\mathcal{M}(B_i \to B_f \pi) = i G_F m_\pi^2 \bar{B}_f \left(A_S - A_P \gamma_5\right) B_i$$

Here, both S-wave amplitude A_S and P-wave amplitude A_P are functions of LECs hD and hF

□ The so-called S/P puzzle: if the two LECs hD and hF can describe well the experimental S-wave amplitudes, they reproduce very poorly the P-wave amplitudes

As a result, we only updated the values of hD and hF by fitting to the experimental S -wave amplitudes for hyperon non-leptonic decays

LECs hD, hF and hyperon non-leptonic decay amplitudes

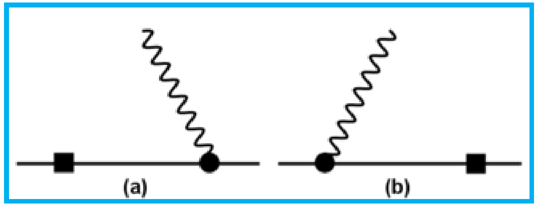
Table: LECs hD and hF determined by fitting to the S –wave hyperon non-leptonic decay amplitudes.

Decay modes	$A_S^{\rm th}$	$A_S^{\rm Expt}$
$\Sigma^+ \to n\pi^+$	0	0.06(1)
$\Sigma^- \to n\pi^-$	$-h_D + h_F$	1.88(1)
$\Lambda \to p\pi^-$	$\frac{1}{\sqrt{6}}(h_D + 3h_F)$	1.38(1)
$\Xi^- \to \Lambda\pi^-$	$\frac{1}{\sqrt{6}}(h_D - 3h_F)$	−1.99(1)
$\Sigma^+ \to p\pi^0$	$\frac{1}{\sqrt{2}}(h_D - h_F)$	−1.50(3)
$\Lambda \to n\pi^0$	$-\frac{1}{2\sqrt{3}}(h_D + 3h_F)$	−1.09(2)
$\Xi^0 \to \Lambda\pi^0$	$-\frac{1}{2\sqrt{3}}(h_D - 3h_F)$	1.62(10)
$\chi^2/\text{d.o.f.} = 0.24$	$h_D = -0.61(24)$	$h_F = 1.42(14)$

- In our least-squares fit, an absolute uncertainty of 0.3 is added to each S -wave amplitude in order to match the theoretical predictions with the experimental data at 1σ confidence level
- The tree-level formulae for the S -wave amplitudes derived from the following Lagrangian

$$\mathcal{L}^{(0)}_{\Delta S=1} = \sqrt{2}G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle$$

Real part of amplitude a at $O(p^1)$—tree



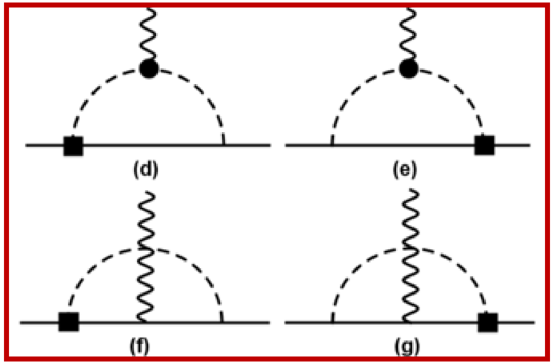
$$\mathcal{L}^{(0)}_{\Delta S=1} = \sqrt{2}G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle,$$
$$\mathcal{L}^{(2)}_{MB} = \frac{b_6^D}{8m_B}\langle \bar{B}\sigma^{\mu\nu}\{F^+_{\mu\nu}, B\}\rangle + \frac{b_6^F}{8m_B}\langle \bar{B}\sigma^{\mu\nu}[F^+_{\mu\nu}, B]\rangle,$$

$$a^{(1,\text{tree})}_{\Lambda n} = \frac{m_\pi^2 F_\phi}{2m_B}\left[\frac{1}{\sqrt{3}}(h_D+3h_F)\frac{\mu_n^{(2)}-\mu_\Lambda^{(2)}}{m_\Lambda - m_n} - (h_D-h_F)\frac{\mu^{(2)}_{\Lambda\Sigma^0}}{m_{\Sigma^0}-m_n}\right],$$
$$a^{(1,\text{tree})}_{\Sigma^+ p} = \frac{m_\pi^2 F_\phi}{2m_B}\left[-\sqrt{2}(h_D-h_F)\frac{\mu_p^{(2)}-\mu_{\Sigma^+}^{(2)}}{m_{\Sigma^+}-m_p}\right],$$
$$a^{(1,\text{tree})}_{\Sigma^0 n} = \frac{m_\pi^2 F_\phi}{2m_B}\left[(h_D-h_F)\frac{\mu_n^{(2)}-\mu_{\Sigma^0}^{(2)}}{m_{\Sigma^0}-m_n} - \frac{1}{\sqrt{3}}(h_D+3h_F)\frac{\mu^{(2)}_{\Sigma^0\Lambda}}{m_\Lambda - m_n}\right],$$
$$a^{(1,\text{tree})}_{\Xi^0 \Lambda} = \frac{m_\pi^2 F_\phi}{2m_B}\left[\frac{1}{\sqrt{3}}(h_D-3h_F)\frac{\mu_\Lambda^{(2)}-\mu_{\Xi^0}^{(2)}}{m_{\Xi^0}-m_\Lambda} + (h_D+h_F)\frac{\mu^{(2)}_{\Sigma^0\Lambda}}{m_{\Xi^0}-m_{\Sigma^0}}\right],$$
$$a^{(1,\text{tree})}_{\Xi^0 \Sigma^0} = \frac{m_\pi^2 F_\phi}{2m_B}\left[(h_D+h_F)\frac{\mu_{\Sigma^0}^{(2)}-\mu_{\Xi^0}^{(2)}}{m_{\Xi^0}-m_{\Sigma^0}} + \frac{1}{\sqrt{3}}(h_D-3h_F)\frac{\mu^{(2)}_{\Lambda\Sigma^0}}{m_{\Xi^0}-m_\Lambda}\right],$$
$$a^{(1,\text{tree})}_{\Xi^- \Sigma^-} = \frac{m_\pi^2 F_\phi}{2m_B}\left[\sqrt{2}(h_D+h_F)\frac{\mu_{\Xi^-}^{(2)}-\mu_{\Sigma^-}^{(2)}}{m_{\Xi^-}-m_{\Sigma^-}}\right],$$

- **hD and hF are LECs**
- **$\mu_B^{(2)}$ are the experimental baryon magnetic moments**

$$a_{B_iB_f} = \boxed{a^{(1,\text{tree})}_{B_iB_f}} + a^{(2,\text{tree})}_{B_iB_f} + a^{(2,\text{loop})}_{B_iB_f}$$
$$b_{B_iB_f} = b^{(2,\text{tree})}_{B_iB_f} + b^{(2,\text{loop})}_{B_iB_f}$$

Amplitude b and imaginary part of amplitude a at $O(p^2)$—loop



$$\mathcal{L}^{(0)}_{\Delta S=1} = \sqrt{2} G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle$$

$$\mathcal{L}^{(1)}_{B} = \langle \bar{B} i\gamma^\mu D_\mu B - m_0 \bar{B}B\rangle,$$

$$\mathcal{L}^{(2)}_{M} = \frac{F_\phi^2}{4}\langle u_\mu u^\mu + \chi^+\rangle,$$

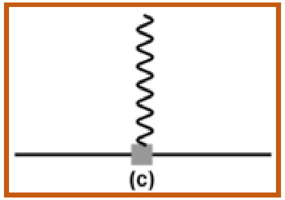
$$\mathcal{L}^{(1)}_{MB} = \frac{D}{2}\langle \bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\rangle + \frac{F}{2}\langle \bar{B}\gamma^\mu\gamma_5[u_\mu, B]\rangle,$$

Real part of amplitude a in the loop level cannot be reliably determined due to S/P puzzle in hyperon non-leptonic decays.

$$a_{B_iB_f} = a^{(1,\mathrm{tree})}_{B_iB_f} + a^{(2,\mathrm{tree})}_{B_iB_f} + a^{(2,\mathrm{loop})}_{B_iB_f}$$

$$b_{B_iB_f} = b^{(2,\mathrm{tree})}_{B_iB_f} + \boxed{b^{(2,\mathrm{loop})}_{B_iB_f}}$$

Real part of amplitude a and b at $O(p^2)$—tree



$$\mathcal{L}_\alpha^{(2)} = C_\alpha \langle \bar{B}\sigma^{\mu\nu}F_{\mu\nu}\lambda Q B\rangle,$$
$$\mathcal{L}_\beta^{(2)} = C_\beta \langle \sigma^{\mu\nu}F_{\mu\nu}\bar{B} Q B\lambda\rangle,$$
$$\mathcal{L}_\gamma^{(2)} = C_\gamma \langle \bar{B}\sigma^{\mu\nu}F_{\mu\nu}B\lambda Q\rangle,$$
$$\mathcal{L}_\sigma^{(2)} = C_\sigma \langle \bar{B}\sigma^{\mu\nu}F_{\mu\nu}\lambda B Q\rangle,$$
$$\mathcal{L}_\rho^{(2)} = C_\rho \left(\langle \bar{B}\sigma^{\mu\nu}\gamma_5 F_{\mu\nu} Q\rangle\langle B\lambda\rangle - \langle \bar{B}\sigma^{\mu\nu}\gamma_5 F_{\mu\nu}\lambda\rangle\langle BQ\rangle\right)$$

counter-terms

- CPS is CP followed by the SU(3) transformation of $u \to -u$, $d \to s$ and $s \to d$ which exchanges s and d quarks.
- CPS symmetry dictates the existence of five unknown LECs

Table: Contributions to the real parts of amplitudes a and b at tree-level $O(p)^2$. The normalization $2(eG_F)^{-1}$ has been factored out.

	$\Lambda \to n\gamma$	$\Sigma^+ \to p\gamma$	$\Sigma^0 \to n\gamma$	$\Xi^0 \to \Lambda\gamma$	$\Xi^0 \to \Sigma^0\gamma$	$\Xi^- \to \Sigma^-\gamma$
$a^{(2,\text{tree})}$	$\frac{2C_\alpha - C_\beta - C_\gamma + 2C_\sigma}{3\sqrt{6}}$	$\frac{2C_\beta - C_\gamma}{3}$	$\frac{C_\beta + C_\gamma}{3\sqrt{2}}$	$-\frac{C_\alpha - 2C_\beta - 2C_\gamma + C_\sigma}{3\sqrt{6}}$	$\frac{C_\alpha + C_\sigma}{3\sqrt{2}}$	$\frac{2C_\sigma - C_\alpha}{3}$
$b^{(2,\text{tree})}$	$-\frac{C_\rho}{\sqrt{6}}$	0	$-\frac{C_\rho}{\sqrt{2}}$	$\frac{C_\rho}{\sqrt{6}}$	$\frac{C_\rho}{\sqrt{2}}$	0

$$b^{(2,\text{tree})}_{\Xi^0\Sigma^0} = \sqrt{3}\, b^{(2,\text{tree})}_{\Xi^0\Lambda}, \quad b^{(2,\text{tree})}_{\Lambda n} = -b^{(2,\text{tree})}_{\Xi^0\Lambda},$$
$$b^{(2,\text{tree})}_{\Sigma^0 n} = -\sqrt{3}\, b^{(2,\text{tree})}_{\Xi^0\Lambda}, \quad b^{(2,\text{tree})}_{\Sigma^+ p} = 0, \quad b^{(2,\text{tree})}_{\Xi^-\Sigma^-} = 0.$$

Determining the contributions of counter-terms

- **Total amplitudes a and b are a sum of the tree and loop contributions and read:**

$$a_{B_iB_f} = \boxed{a^{(1,\text{tree})}_{B_iB_f}} + a^{(2,\text{tree})}_{B_iB_f} + \boxed{a^{(2,\text{loop})}_{B_iB_f}} = \text{Re}\, a_{B_iB_f} + \boxed{\text{Im}\, a^{(2,\text{loop})}_{B_iB_f}}$$
$$b_{B_iB_f} = b^{(2,\text{tree})}_{B_iB_f} + \boxed{b^{(2,\text{loop})}_{B_iB_f}}$$

- **Using $b^{(2,\text{tree})}_{\Xi^0\Sigma^0} = \sqrt{3}b^{(2,\text{tree})}_{\Xi^0\Lambda}$ and fitting to $\mathcal{B}$ and α_γ for $\Xi^0 \to \Sigma^0\gamma$ and $\Xi^0 \to \Lambda\gamma$ decays, we determine for the first time the contributions of counter-terms**

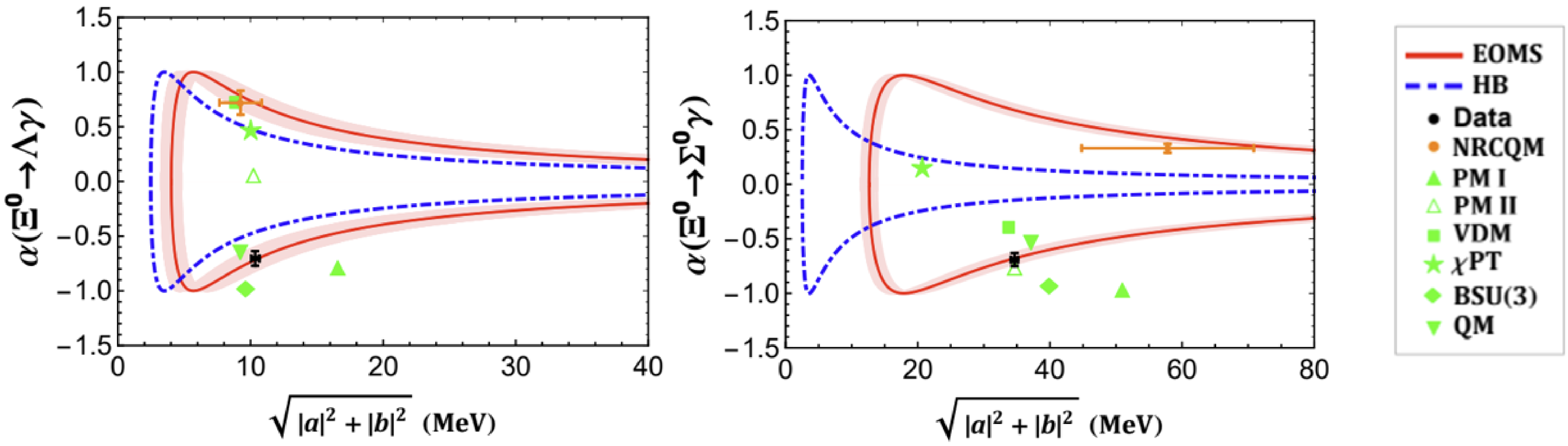
	Solution I	Solution II
$b^{(2,\text{tree})}_{\Xi^0\Lambda}$	5.62(53)	−8.34(48)
Re $a_{\Xi^0\Lambda}$	−9.56(34)	3.89(45)
Re $a_{\Xi^0\Sigma^0}$	−32.22(64)	32.50(61)
χ^2/d.o.f.	0.04	1.22

- The $\chi^2/d.o.f.$ of Solution I much smaller than that of Solution II.
- Contributions of counter-terms for other WRHDs obtained by the following relations $b^{(2,\text{tree})}_{\Lambda n} = -b^{(2,\text{tree})}_{\Xi^0\Lambda}$ $b^{(2,\text{tree})}_{\Sigma^0 n} = -\sqrt{3}b^{(2,\text{tree})}_{\Xi^0\Lambda}$, $b^{(2,\text{tree})}_{\Sigma^+ p} = 0$, $b^{(2,\text{tree})}_{\Xi^-\Sigma^-} = 0$

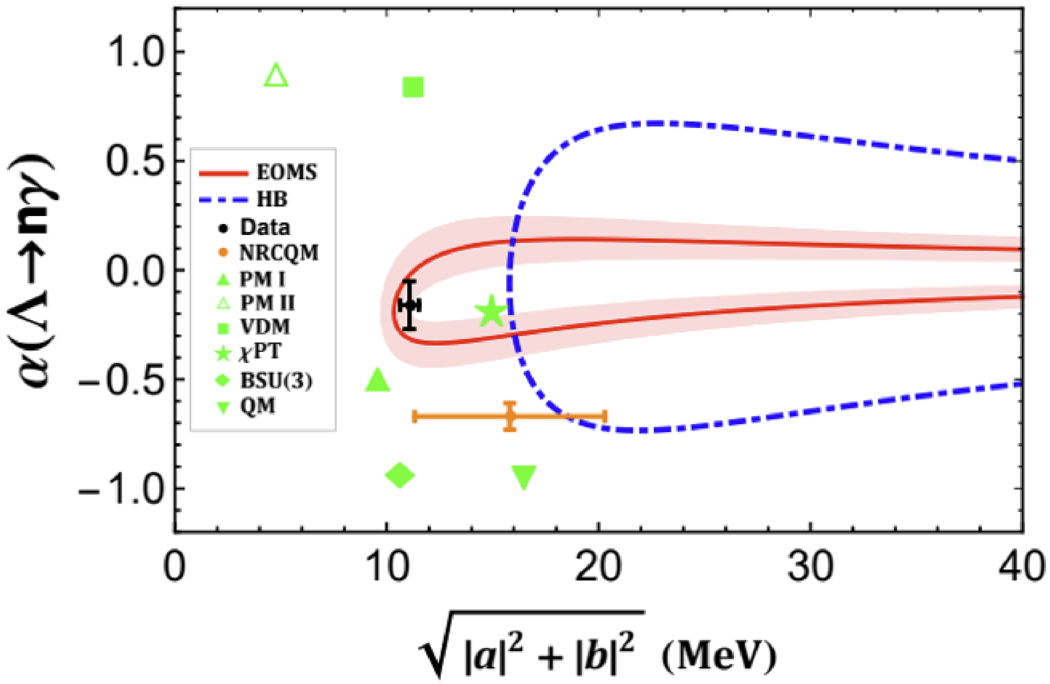
$$a_{B_iB_f} = \boxed{a^{(1,\text{tree})}_{B_iB_f}} + a^{(2,\text{tree})}_{B_iB_f} + \boxed{a^{(2,\text{loop})}_{B_iB_f}} = \text{Re}\, a_{B_iB_f} + \boxed{\text{Im}\, a^{(2,\text{loop})}_{B_iB_f}}$$
$$b_{B_iB_f} = \boxed{b^{(2,\text{tree})}_{B_iB_f}} + \boxed{b^{(2,\text{loop})}_{B_iB_f}}$$

Therefore, we take the Re a for each WRHD as a free parameter due to the unknown real parts of amplitudes a at $O(p^2)$ order

α_γ of $\Xi^0 \to \Sigma^0\,\gamma$ and $\Xi^0 \to \Lambda\,\gamma$ as a function of $\sqrt{|a|^2 + |b|^2}$



α_γ of the $\Lambda \to n\,\gamma$ decay as a function of $\sqrt{|a|^2+|b|^2}$



Data: *BESIII, PRL129(2022)21,212002*

HB χPT : *E. E. Jenkins et al, NPB 397, 84 (1993)*

NRCQM: *Qiang Zhao et al, CPC45, 013101 (2021)*

PM1: *M. B. Gavela et al, PLB 101, 417 (1981)*

PM2: *G. Nardulli, PLB 190, 187 (1987)*

VDM: *P. Zenczykowski, PRD 44, 1485 (1991)*

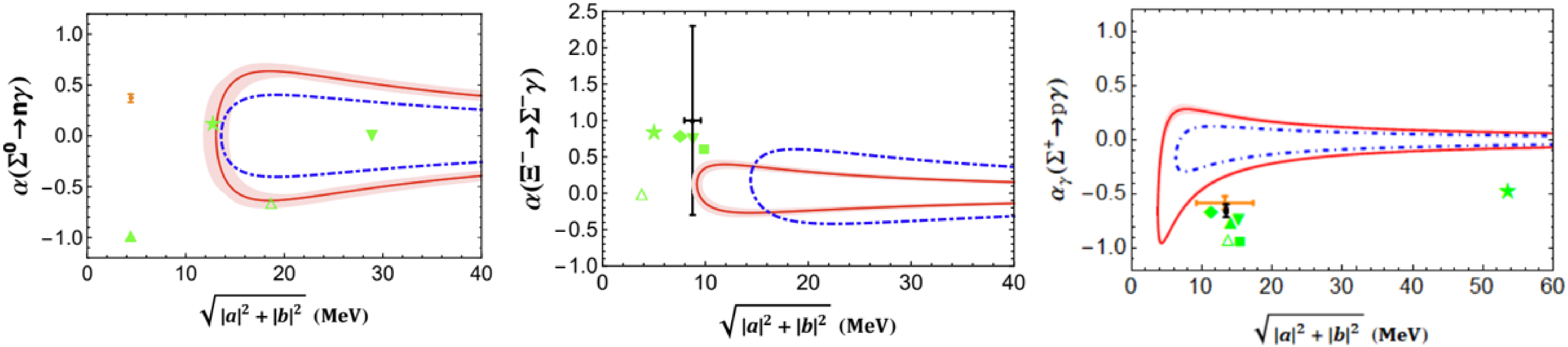
χPT: *B. Borasoy et al, PRD 59, 054019 (1999)*

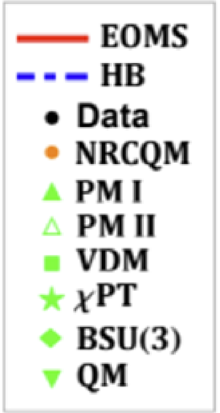
BSU(3): *P. Zenczykowski, PRD 73, 076005 (2006)*

QM: *E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)*

- Interestingly, only **EOMS BχPT agrees with** the latest BESIII measurement
- The prediction in the HB χPT **with counter-term contributions** is very close to the BESIII data
- The vector dominance model (VDM) and the pole model (PM II) **are disfavored** by the BESIII data

α_γ of the other WRHDs as a function of $\sqrt{|a|^2+|b|^2}$





- For the $\Sigma^0 \to n\,\gamma$ decay, not yet measured, **our result contradicts** the predictions of PM I and NRCQM
- For the $\Xi^- \to \Sigma^-\gamma$ decay, **our prediction agrees better** with the experimental measurement, and the current PDG data disfavor the results of PM II and tree-level χPT
- **For the $\Sigma^+ \to p\,\gamma$ decay, the results predicted in all the χPT deviate from the PDG average but our prediction is closer**

Hara' s theorem: α_γ for $\Xi^- \to \Sigma^-\gamma$ and $\Sigma^+ \to p\,\gamma$ should not be too large.

What happed to $\Sigma^+ \to p\,\gamma$? What is still missing?

- For the $\Sigma^+ \to p\,\gamma$ decay, the results predicted in all the χPT deviate from the PDG average but our prediction is closer
- Could this be somehow rescued?
  - How about contributions of heavier resonances? Have been tried previously, but the results do not look good, e.g., B. Borasoy et al, PRD 59, 054019(1999)

$$\alpha^{p\Sigma^+} = -0.49 \qquad \alpha^{\Sigma^-\Xi^-} = 0.84$$

$$\alpha^{n\Sigma^0} = 0.12 \qquad \alpha^{n\Lambda} = -0.19$$

$$\alpha^{\Sigma^0\Xi^0} = 0.15 \qquad \alpha^{\Lambda\Xi^0} = 0.46.$$

$\Lambda(1405)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\Sigma\pi$	100 %
Γ_2	$\Lambda\gamma$	
Γ_3	$\Sigma^0\gamma$	
Γ_4	$N\overline{K}$	

$N(1535)$ DECAY MODES

The following branching fractions are our estimates, not fits or averages.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	32–52 %
Γ_2	$N\eta$	30–55 %
Γ_3	$N\pi\pi$	4–31 %
Γ_4	$\Delta(1232)\pi$, D-wave	1–4 %
Γ_5	$N\rho$	2–17 %
Γ_6	$N\rho$, S=1/2 , S-wave	2–16 %
Γ_7	$N\rho$, S=3/2 , D-wave	<1 %
Γ_8	$N\sigma$	2–10 %
Γ_9	$N(1440)\pi$	5–12 %
Γ_{10}	$p\gamma$, helicity=1/2	0.15–0.30 %
Γ_{11}	$n\gamma$, helicity=1/2	0.01–0.25 %

Uncertainties of the relevant LECs are important but remain unstudied

Contents

- Brief introduction: motivation and purpose
- Theoretical framework: covariant ChEFT
- Results & discussions
  - Conventional ChPT results
  - Contribution of negative parity heavy resonances (preliminary)
- Summary and outlook

Contributions of heavier resonances

□ **Consider only additional contributions of heavier resonances at tree-level**

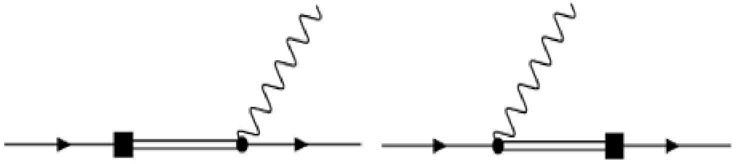
$$\mathcal{L}_W^{(0)} = G_F m_\pi^2 \sqrt{2} F_\phi \cdot \left(i w_d \left(\mathrm{tr}\left(\bar{R}\{h_+, B\}\right) - \mathrm{tr}\left(\bar{B}\{h_+, R\}\right)\right) + i w_f \left(\mathrm{tr}\left(\bar{R}[h_+, B]\right) - \mathrm{tr}\left(\bar{B}[h_+, R]\right)\right)\right)$$

$$\begin{aligned}\mathcal{L}_{RB}^{s} = & \, i r_d \left[\mathrm{tr}\left(\bar{R}\sigma_{\mu\nu}\gamma_5 \left\{f_+^{\mu\nu}, B\right\}\right) + \mathrm{tr}\left(\bar{B}\sigma_{\mu\nu}\gamma_5 \left\{f_+^{\mu\nu}, R\right\}\right)\right] \\ & + i r_f \left[\mathrm{tr}\left(\bar{R}\sigma_{\mu\nu}\gamma_5 \left[f_+^{\mu\nu}, B\right]\right) + \mathrm{tr}\left(\bar{B}\sigma_{\mu\nu}\gamma_5 \left[f_+^{\mu\nu}, R\right]\right)\right]\end{aligned}$$

Following B. Borasoy et al, PRD 59, 054019(1999)

$$a_{B_iB_f} = a_{B_iB_f}^{(1,\mathrm{tree})} + a_{B_iB_f}^{(2,\mathrm{tree})} + a_{B_iB_f}^{(2,\mathrm{loop})} = \mathrm{Re}\; a_{B_iB_f} + \mathrm{Im}\; a_{B_iB_f}^{(2,\mathrm{loop})}$$

$$b_{B_iB_f} = b_{B_iB_f}^{(\mathrm{res})} + b_{B_iB_f}^{(2,\mathrm{tree})} + b_{B_iB_f}^{(2,\mathrm{loop})}.$$



- At tree level, only $\frac{1}{2}^-$ **states** contributing to $\mathrm{Re}\,\mathrm{b}$ can affect the EOMS results because $\frac{1}{2}^+$ states contribute to $\mathrm{Re}\,a$ which are taken as free parameters
- Due to charge conservation, contributions of $\boldsymbol{b_{B_iB_f}^{(\mathrm{res})}}$ to $\Sigma^+ \to p\gamma$ and $\Xi^- \to \Sigma^-\gamma$ are dominated by $N(1535)$. For other channels, $\boldsymbol{b_{B_iB_f}^{(\mathrm{res})}}$ are mainly from $\Lambda(1405)$.

Contributions of heavier resonances

- $b_{B_iB_f}^{(\mathrm{res})}$ at leading order

$$b(\Lambda \to n) = \frac{8r_dF_\phi m_\phi^2(w_d+3w_f)}{3\sqrt{3}(m_\Lambda - m_R)} - \frac{8r_dF_\phi m_\phi^2(w_d-3w_f)}{3\sqrt{3}(m_R - m_n)}$$

$$b(\Sigma^+ \to p) = \frac{4\sqrt{2}F_\phi m_\phi^2(r_d+3r_f)(w_d-w_f)}{3(m_R - m_p)} + \frac{4\sqrt{2}F_\phi m_\phi^2(r_d+3r_f)(w_d-w_f)}{3(m_{\Sigma_+} - m_R)}$$

$$b(\Sigma^0 \to n) = \frac{8r_dF_\phi m_\phi^2(w_d-w_f)}{3(m_{\Sigma_0} - m_R)} - \frac{8r_dF_\phi m_\phi^2(w_d+w_f)}{3(m_R - m_n)}$$

$$b(\Xi^0 \to \Lambda) = \frac{8r_dF_\phi m_\phi^2(w_d-3w_f)}{3\sqrt{3}(m_R - m_\Lambda)} + \frac{8r_dF_\phi m_\phi^2(w_d+3w_f)}{3\sqrt{3}(m_R - m_{\Xi_0})}$$

$$b(\Xi^0 \to \Sigma^0) = \frac{8r_dF_\phi m_\phi^2(w_d-w_f)}{3(m_R - m_{\Xi_0})} + \frac{8r_dF_\phi m_\phi^2(w_d+w_f)}{3(m_R - m_{\Sigma_0})}$$

$$b(\Xi^- \to \Sigma^-) = \frac{4\sqrt{2}F_\phi m_\phi^2(r_d-3r_f)(w_d+w_f)}{3(m_{\Xi_-} - m_R)} + \frac{4\sqrt{2}F_\phi m_\phi^2(r_d-3r_f)(w_d+w_f)}{3(m_R - m_{\Sigma_-})}$$

$\Lambda(1405)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\Sigma\pi$	100 %
Γ_2	$\Lambda\gamma$	
Γ_3	$\Sigma^0\gamma$	
Γ_4	$N\overline{K}$	

$N(1535)$ DECAY MODES

The following branching fractions are our estimates, not fits or averages.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	32–52 %
Γ_2	$N\eta$	30–55 %
Γ_3	$N\pi\pi$	4–31 %
Γ_4	$\Delta(1232)\pi$, D-wave	1–4 %
Γ_5	$N\rho$	2–17 %
Γ_6	$N\rho$, S=1/2 , S-wave	2–16 %
Γ_7	$N\rho$, S=3/2 , D-wave	<1 %
Γ_8	$N\sigma$	2–10 %
Γ_9	$N(1440)\pi$	5–12 %
Γ_{10}	$p\gamma$, helicity=1/2	0.15–0.30 %
Γ_{11}	$n\gamma$, helicity=1/2	0.01–0.25 %

- r_d and r_f can be determined by fitting to **electromagnetic decays** of the resonances
- ω_d and ω_f can be determined by fitting to **the nonleptonic hyperon** decays

B. Borasoy et al, PRD 59, 054019(1999), PRD 59, 094025(1999)

Contributions of heavier resonances

☐ Determination of LECs r_d and r_f

$$\mathcal{L}^{s}_{RB} = i r_d \left[\mathrm{tr}\left(\bar{R}\sigma_{\mu\nu}\gamma_5\left\{f_+^{\mu\nu}, B\right\}\right) + \mathrm{tr}\left(\bar{B}\sigma_{\mu\nu}\gamma_5\left\{f_+^{\mu\nu}, R\right\}\right)\right]$$
$$+ i r_f \left[\mathrm{tr}\left(\bar{R}\sigma_{\mu\nu}\gamma_5\left[f_+^{\mu\nu}, B\right]\right) + \mathrm{tr}\left(\bar{B}\sigma_{\mu\nu}\gamma_5\left[f_+^{\mu\nu}, R\right]\right)\right]$$

$$\Gamma^{ji} = \frac{1}{8\pi M_R^2}|\mathbf{k}_\gamma||\mathcal{T}^{ji}|^2 \qquad (A4)$$

with

$$|\mathbf{k}_\gamma| = E_\gamma = \frac{1}{2M_R}(M_R^2 - M_B^2) \qquad (A5)$$

being the three-momentum of the photon in the rest frame of the resonance and M_R and M_B being the masses of the resonance and the ground state baryon, respectively. For the resonance mass M_R we use the mass of $N(1535)$. The mistake we make in the case of the other resonance octet states is of higher chiral order and, therefore, beyond the accuracy of our calculation. For the transition matrix, one obtains

$$|\mathcal{T}^{ji}|^2 = 128 e^2 (p_i \cdot k)^2 (C^{ji})^2 \qquad (A6)$$

with p_i the momentum of the decaying baryon and the coefficients

$$C^{p\, p(1535)} = \tfrac{1}{3} r_d + r_f, \quad C^{n\, n(1535)} = -\tfrac{2}{3} r_d. \qquad (A7)$$

使用$N(1535) \to p\,\gamma$ 和 $N(1535) \to n\,\gamma$衰变的实验数据来确定 r_d和 r_f

N(1535) BREIT-WIGNER WIDTH

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
125 to 175 (≈ 150) OUR ESTIMATE				
147 ± 5	[6] HUNT	19	DPWA	Multichannel
163 ± 25	KASHEVAROV	17	DPWA	$\gamma p \to \eta p$, $\eta' p$
120 ± 10	SOKHOYAN	15A	DPWA	Multichannel

https://pdg.lbl.gov Page 3 Created: 5/30/2025 07:46

新数据不同于1999年之前的结果，因此文献*PRD(1999) 59, 094025*中的r_d和r_f需要被更新，我们做了这个事情。

Citation: S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024) and 2025 update

131 ± 12	[6] SHKLYAR	13	DPWA	Multichannel
188.4 ± 3.8	[6] ARNDT	06	DPWA	$\pi N \to \pi N$, ηN
240 ± 80	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
120 ± 20	HOEHLER	79	IPWA	$\pi N \to \pi N$

Contributions of heavier resonances

☐ Determination of LECs ω_d and ω_f

$$\boxed{\mathcal{M}(B_i \to B_f \pi) = i G_F m_\pi^2 \bar{B}_f \left(A_S - A_P \gamma_5\right) B_i,}$$

$$As(\Sigma^+ \to n\pi^+) = \frac{1}{3}\sqrt{2}\left(\frac{s_d\,(w_d+3w_f)}{m_n - M_\Lambda} + \frac{3sf\,(-w_d+w_f)}{m_n - M_{\Sigma^0}} - \frac{3(s_d+sf)\,(w_d-w_f)}{m_{\Sigma^+} - M_{N^+}}\right)\cdot\frac{1}{F_\phi}\cdot(m_f - m_i)$$

$$As(\Sigma^- \to n\pi^-) = \frac{-(d-f)}{\sqrt{2}}\cdot\frac{1}{F_\phi} + \frac{1}{3}\sqrt{2}\left(\frac{s_d\,(w_d+3w_f)}{m_n - M_\Lambda} + \frac{3s_f\,(w_d-w_f)}{m_n - M_{\Sigma^0}}\right)\cdot\frac{1}{F_\phi}\cdot(m_f - m_i)$$

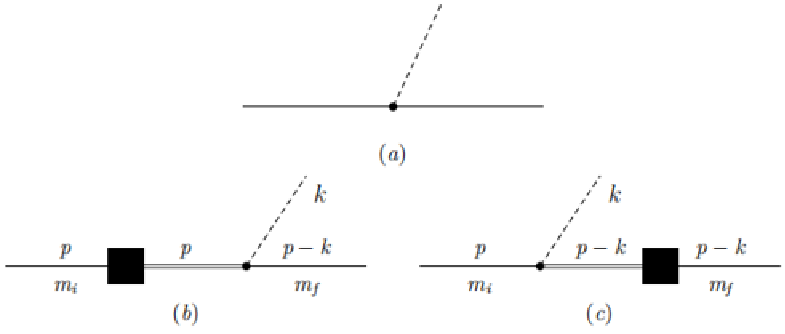
$$As(\Sigma^+ \to p\pi^0) = \frac{(d-f)}{2}\cdot\frac{1}{F_\phi} - (w_d - w_f)\left(\frac{2s_f}{m_p - M_{\Sigma^+}} + \frac{s_d+s_f}{m_{\Sigma^+} - M_{N^+}}\right)\cdot\frac{1}{F_\phi}\cdot(m_f - m_i)$$

$$As(\Lambda \to p\pi^-) = \frac{d+3f}{2\sqrt{3}}\cdot\frac{1}{F_\phi} + \frac{1}{\sqrt{3}}\left(\frac{2\,sd\,(-w_d+w_f)}{m_p - M_{\Sigma^+}} + \frac{(s_d+s_f)\,(w_d+3w_f)}{m_\Lambda - M_{N^0}}\right)\cdot\frac{1}{F_\phi}\cdot(m_f - m_i)$$

$$As(\Lambda \to n\pi^0) = -\frac{d+3f}{2\sqrt{6}}\cdot\frac{1}{F_\phi} + \frac{1}{\sqrt{6}}\left(-\frac{(s_d+s_f)\,(w_d+3w_f)}{m_\Lambda - M_{N^0}} + \frac{2\,s_d\,(w_d-w_f)}{m_n - M_{\Sigma^0}}\right)\cdot\frac{1}{F_\phi}\cdot(m_f - m_i)$$

$$As(\Xi^- \to \Lambda\pi^-) = \frac{d-3f}{2\sqrt{3}}\cdot\frac{1}{F_\phi} + \frac{1}{\sqrt{3}}\left(-\frac{2\,s_d\,(w_d+w_f)}{m_{\Xi^-} - M_{\Sigma^-}} + \frac{(s_d-s_f)\,(w_d-3w_f)}{m_\Lambda - M_{\Sigma^0}}\right)\cdot\frac{1}{F_\phi}\cdot((m_f - m_i))$$

$$As(\Xi^0 \to \Lambda\pi^0) = -\frac{d-3f}{2\sqrt{6}}\cdot\frac{1}{F_\phi} + \frac{1}{\sqrt{6}}\left(-\frac{(s_d-s_f)\,(w_d-3w_f)}{m_\Lambda - M_{\Xi^0}} + \frac{2\,s_d\,(w_d+w_f)}{m_{\Xi_0} - M_{\Sigma^0}}\right)\cdot\frac{1}{F_\phi}\cdot(m_f - m_i)$$



$M_i = M_R$ 是手征极限质量

$$\boxed{m_n \approx m_p,\quad M_{\Sigma^0} = M_{\Sigma^-} = M_{\Sigma^+},\quad m_{\Xi^0} = m_{\Xi^-},\quad m_{\Sigma^0} = m_{\Sigma^-} = m_{\Sigma^+}}$$

满足同位旋关系：

$$As(\Lambda \to p\pi^-) + \sqrt{2}As(\Lambda \to n\pi^0) = 0$$

$$As(\Xi^- \to \Lambda\pi^-) + \sqrt{2}As(\Xi^0 \to \Lambda\pi^0) = 0$$

$$\sqrt{2}As(\Sigma^+ \to p\pi^0) + As(\Sigma^- \to n\pi^-) - As(\Sigma^+ \to n\pi^+) = 0$$

Contributions of heavier resonances

☐ Determination of LECs s_d and s_f

$$\mathcal{L}_S^{(1)} = i s_d \left[\mathrm{tr}\left(\bar{R}\gamma_\mu\{u^\mu, B\}\right) - \mathrm{tr}\left(\bar{B}\gamma_\mu[u^\mu, R]\right)\right] + i s_f \left[\mathrm{tr}\left(\bar{R}\gamma_\mu[u^\mu, B]\right) - \mathrm{tr}\left(\bar{B}\gamma_\mu[u^\mu, R]\right)\right]$$

除了前面ppt提到的N(1535)新数据不同于1999年之前的结果，Λ(1405)数据相较1999年之前也发生了变化。因此文献*PRD(1999) 59, 094025*中的s_d和s_f需要被更新，我们做了这个事情。

ACKNOWLEDGMENTS

This work was supported in part by the Deutsche Forschungsgemeinschaft and by the National Science Foundation.

APPENDIX: DETERMINATION OF THE $\frac{1}{2}^-$-RESONANCE COUPLINGS s_d AND s_f

The decays listed in the particle data book, which determine the coupling constants s_d and s_f, are $N(1535)\to N\pi$, $N(1535)\to N\eta$ and $\Lambda(1405)\to\Sigma\pi$. The width follows via

$$\Gamma = \frac{1}{8\pi M_R^2}|\mathbf{k}_\phi||\mathcal{T}|^2 \tag{A1}$$

with

$$|\mathbf{k}_\phi| = \frac{1}{2M_R}[(M_R^2-(M_B+m_\phi)^2)(M_R^2-(M_B-m_\phi)^2)]^{1/2} \tag{A2}$$

$$|\mathcal{T}|^2 = \frac{2}{F_\pi^2}(M_R-M_B)^2[(M_R+M_B)^2-m_\phi^2]A_{R\phi} \tag{A3}$$

with the coefficients

$$A_{N(1535)\pi} = \frac{3}{2}(s_d+s_f)^2,$$

$$A_{N(1535)\eta} = \frac{1}{6}(s_d-3s_f)^2,\quad A_{\Lambda(1405)\pi} = 2s_d^2. \tag{A4}$$

Using the experimental values for the decay widths we arrive at the central values

$$s_d = 0.17,\quad s_f = -0.12 \tag{A5}$$

$\Lambda(1405)$ WIDTH

PRODUCTION EXPERIMENTS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
50.5± 2.0 OUR AVERAGE					
62 ±10		HASSANVAND	13	SPEC	$pp \to p\Lambda(1405)K^+$
50 ± 2		[1] DALITZ	91		M-matrix fit
• • • We do not use the following data for averages, fits, limits, etc. • • •					
24 $^{+4}_{-3}$		ESMAILI	10	RVUE	$^4\mathrm{He}\,K^- \to \Sigma^\pm\pi^\mp X$ at rest
32 ± 1	700	[1] HEMINGWAY	85	HBC	K^-p 4.2 GeV/c
45 to 55	400	[2] THOMAS	73	HBC	π^-p 1.69 GeV/c
35	120	BARBARO-...	68B	DBC	K^-d 2.1–2.7 GeV/c
50 ±10	67	BIRMINGHAM	66	HBC	K^-p 3.5 GeV/c
89 ±20		ENGLER	65	HDBC	
60 ±20		MUSGRAVE	65	HBC	
35 ± 5		ALEXANDER	62	HBC	
50		ALSTON	62	HBC	
20		ALSTON	61B	HBC	

Contributions of heavier resonances

☐ Determination of LECs ω_d and ω_f

```
MIGRAD MINIMIZATION HAS CONVERGED.

MIGRAD WILL VERIFY CONVERGENCE AND ERROR MATRIX.
COVARIANCE MATRIX CALCULATED SUCCESSFULLY

FCN=   0.3033017     FROM HESSE     STATUS=OK            52 CALLS    200260 TOTAL
                     EDM=   0.77E-03     STRATEGY= 1      ERROR MATRIX ACCURATE

 EXT PARAMETER                                         STEP         FIRST
 NO.   NAME       VALUE          ERROR              SIZE       DERIVATIVE
  1      wd      -0.11597      0.38385E-01   0.37513E-06   -4.8885
  2      wf       0.11603      0.40858E-01   0.37398E-06   -3.3510
  3      hd      -0.58620      0.36891       0.19484E-03  -0.20482E-03
  4      hf        1.3173      0.21756       0.10394E-03  -0.34094E-02
  5      sd       0.34302      0.13601E-02   0.75576E-06  -0.53674
  6      sf      -0.23805      0.33415E-02   0.18135E-05   0.11039E-01
```

$M_R \in [1.26\,, 1.36\,]\mathrm{GeV}$

Contributions of heavier resonances

☐ Predictions of amplitudes $b_{B_iB_f}$

TABLE IX. Decomposition of the contributions to the parity-violating amplitudes b (in units of MeV).

Decay modes	This work				EOMS BχPT (2022)		
	$b^{(2,\mathrm{tree})}$	$b^{(\mathrm{res})}$	$b^{(2,\mathrm{loop})}$	$b^{(2,\mathrm{tot})}$	$b^{(2,\mathrm{tree})}$	$b^{(2,\mathrm{loop})}$	$b^{(2,\mathrm{tot})}$
$\Lambda \to n\gamma$	−6.77(45)	−0.03(17)	7.24(112) + 9.26(122)i	0.44 + 9.26i	−5.62(53)	7.87(73) + 10.04(81)i	2.25(90) + 10.04(81)i
$\Sigma^+ \to p\gamma$	0	−2.18	−1.82(17) − 1.67(18)i	−4.0 − 1.67i	0	−1.96(11) − 1.75(12)i	−1.96(11) − 1.75(12)i
$\Sigma^0 \to n\gamma$	−11.73(78)	1.51(36)	1.31(17) + 9.46(117)i	−8.91 + 9.46i	−9.73(92)	1.41(11) + 10.09(78)i	−8.32(93) + 10.09(78)i
$\Xi^0 \to \Lambda\gamma$	6.77(45)	−2.38(114)	−1.43(76)	2.96	5.62(53)	−1.60(48)	4.02(72)
$\Xi^0 \to \Sigma^0\gamma$	11.73(78)	2.61(82)	2.62(99)	16.96	9.73(92)	2.91(67)	12.64(114)
$\Xi^- \to \Sigma^-\gamma$	0	0.0(32)	−2.75(47) − 8.05(79)i	−2.75 − 8.05i	0	−3.00(29) − 8.64(54)i	−3.00(29) − 8.64(54)i

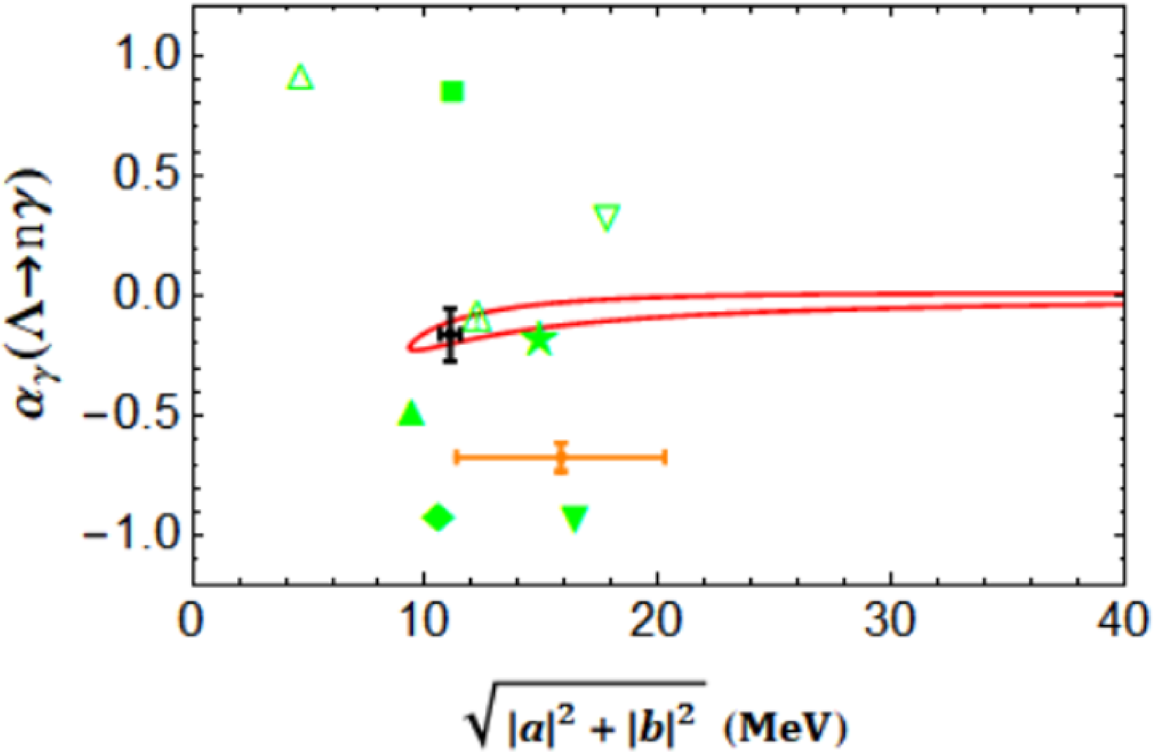
☐ Using $b^{(2,\mathrm{tree})}_{\Xi^0\Sigma^0} = \sqrt{3}\, b^{(2,\mathrm{tree})}_{\Xi^0\Lambda}$ and fitting to $\mathcal{B}$ and α_γ for $\Xi^0 \to \Sigma^0\gamma$ and $\Xi^0 \to \Lambda\gamma$ decays, we determine the contributions of counter-terms

$$a_{B_iB_f} = a^{(1,\mathrm{tree})}_{B_iB_f} + a^{(2,\mathrm{tree})}_{B_iB_f} + a^{(2,\mathrm{loop})}_{B_iB_f} = \mathrm{Re}\, a_{B_iB_f} + \mathrm{Im}\, a^{(2,\mathrm{loop})}_{B_iB_f}$$

$$b_{B_iB_f} = b^{(\mathrm{res})}_{B_iB_f} + b^{(2,\mathrm{tree})}_{B_iB_f} + b^{(2,\mathrm{loop})}_{B_iB_f}.$$

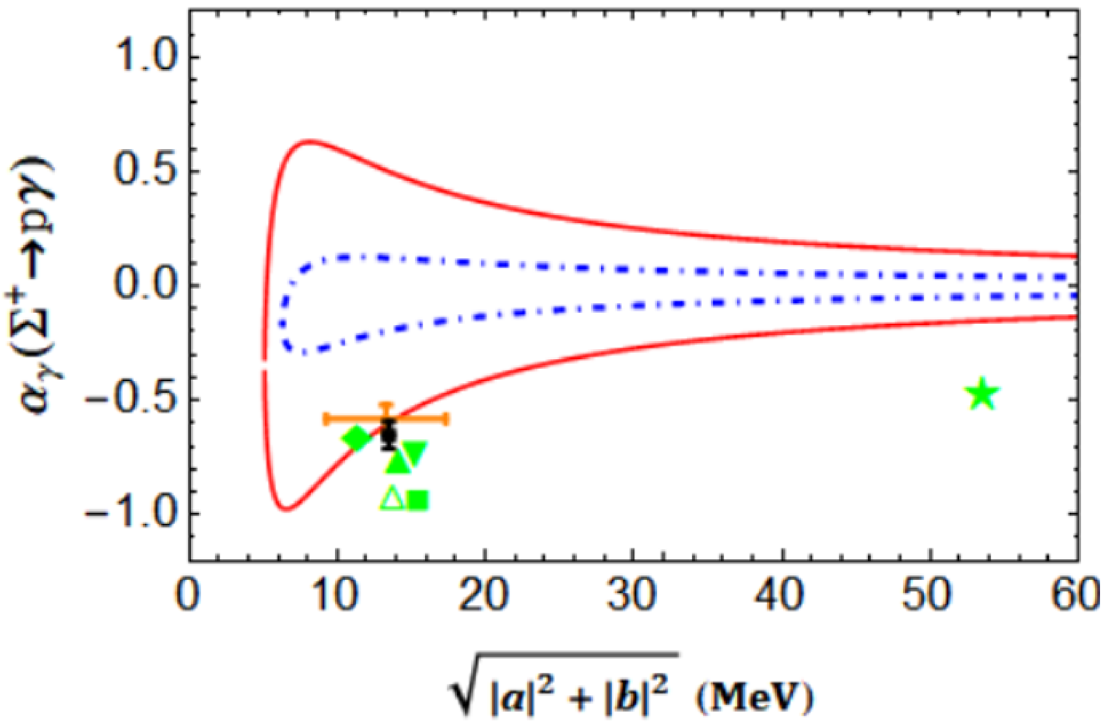
```
MIGRAD MINIMIZATION HAS CONVERGED.

MIGRAD WILL VERIFY CONVERGENCE AND ERROR MATRIX.
COVARIANCE MATRIX CALCULATED SUCCESSFULLY

FCN=  0.2250674     FROM HESSE     STATUS=OK           61 CALLS    462331 TOTAL
                    EDM=  0.71E-03    STRATEGY= 1      ERROR MATRIX ACCURATE

 EXT PARAMETER                                    STEP         FIRST
 NO.   NAME        VALUE          ERROR           SIZE         DERIVATIVE
  1    Rea1       -10.133         0.38774       0.19915E-03  -0.58650E-02
  2    Reb1         6.7724        0.49304       0.24555E-03   0.16039E-02
  3    Rea2       -32.964         1.3257        0.69679E-03  -0.25160E-03
```

Contributions of heavier resonances



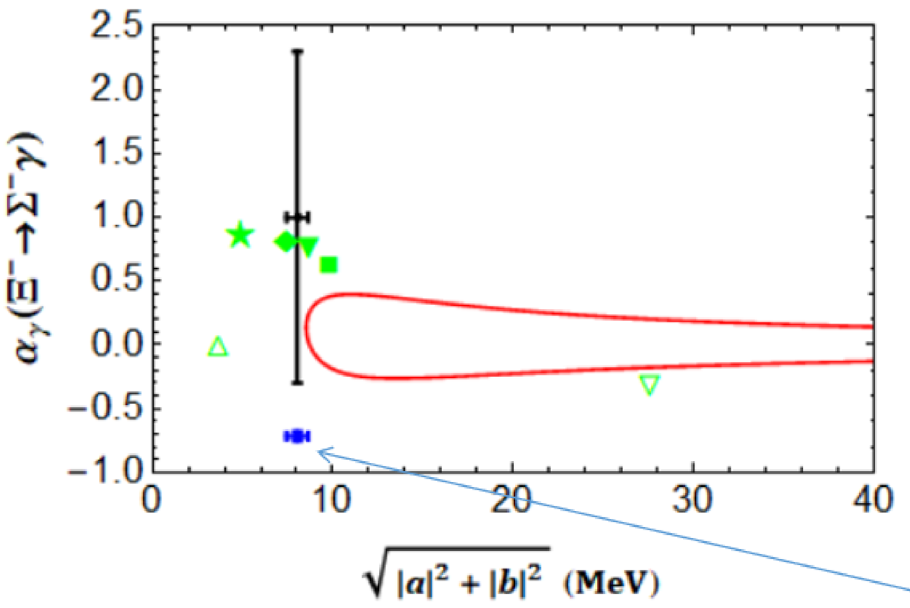
BESIII data (black)

Contributions of heavier resonances



BESIII data (black)

Contributions of heavier resonances



Zhi-Peng Xing, Ruilin Zhu et al, arXiv:2608.03248

Observable	Exp	Our work
$Br(\Sigma^+\to p\gamma)$	0.996(28)[16]	0.996(28)
$\alpha_\gamma(\Sigma^+\to p\gamma)$	−0.69(5)[2]	−0.685(35)
$Br(\Lambda\to n\gamma)$	0.832(66)[15]	0.831(66)
$\alpha_\gamma(\Lambda\to n\gamma)$	−0.16(11)[15]	−0.17(11)
$Br(\Sigma^0\to n\gamma)$	-	$3.7(3.0)\times 10^{-7}$
$\alpha_\gamma(\Sigma^0\to n\gamma)$	-	−0.685(35)
$Br(\Xi^0\to\Lambda\gamma)$	1.24(7)[2]	1.241(70)
$\alpha_\gamma(\Xi^0\to\Lambda\gamma)$	−0.72(5)[2]	−0.746(36)
$Br(\Xi^0\to\Sigma^0\gamma)$	3.69(24)[18]	3.68(24)
$\alpha_\gamma(\Xi^0\to\Sigma^0\gamma)$	−0.807(96)[18]	−0.716(33)
$Br(\Xi^-\to\Sigma^-\gamma)$	0.127(23)[2]	0.127(23)
$\alpha_\gamma(\Xi^-\to\Sigma^-\gamma)$	1.0(1.3)[30]	−0.716(33)

Contents

- ☞Brief introduction: motivation and purpose
- ☞Theoretical framework: covariant ChEFT
- ☞Results & discussions
  - ➢ Conventional ChPT results
  - ➢ Contribution of negative parity heavy resonances (preliminary)
- ☞Summary and outlook

Summary and outlook

- Motivated by the latest BESIII results and the success of the covariant baryon chiral perturbation theory, we revisited the long-standing WRHDs.
  - LECs hD, hF and hyperon non-leptonic decay amplitudes are determined by fitting to the latest experimental data on the $B_i \to B_f\,\pi$ decays
  - $O(p^2)$ counter-term contributions are determined by fitting to $\Xi^0 \to \Sigma^0\gamma$ and $\Xi^0 \to \Lambda\gamma$ for the first time
- We showed that the latest measurement of $\Lambda \to n\,\gamma$ by the BESIII Collaboration can be well explained. The contributions of heavier $\frac{1}{2}^-$ states are important to explain the $\Sigma^+ \to p\,\gamma$ asymetry, and finally bring an overall solution to the WRHDs puzzle.
- This work provides essential SM inputs for studying new physics in the rare hyperon semi-leptonic decay $B_i \to B_f\gamma^* \to B_f l\,l$

LHCb: JHEP 05 (2019) 048 and CERN Yellow Rep.Monogr. 7 (2019) 867-1158

Summary and outlook

- A more precise measurement of $\alpha_\gamma(\Xi^- \to \Sigma^-\gamma)$ is highly desirable in order to test Hara's theorem and confirm the present experimental result.

Super tau-charm factory:

Zhou XR, PoSCHARM2020(2021)007

A.Y.Barnyakov, JPhysConfSer1561(1)(2020)012004

Summary and outlook

Revisiting the S/P puzzle of hyperon non-leptonic decays $(B_i \to B_f\, \pi)$

- ✓ Latest BESIII study shows that $\Delta I = 1/2$ rule may be violated

$$\langle \alpha_{\Lambda 0} \rangle / \langle \alpha_{\Lambda -} \rangle = 0.870 \pm 0.012^{+0.011}_{-0.010}$$

BESIII: PRL 132 (2024) 10, 101801

- ✓ **Previous theoretical studies in HBχPT neglected the contributions of either the counterterms or intermediate decuplet-baryons**

HB χPT: Borasoy B et al, EPJC 6 (1999) 85-107
Abd El-Hady A, PRD 61 (2000) 114014

Inputs

Revisiting CP violation (CPV) of hyperon non-leptonic decays $(B_i \to B_f\, \pi)$

CPV observables	SM predictions	BESIII data
A_{CP}^{Λ}	$(-3\sim3) \times 10^{-5}$	$(-2.5 \pm 4.6 \pm 1.2) \times 10^{-3}$
A_{CP}^{Ξ}	$(0.5\sim6) \times 10^{-5}$	$(6 \pm 13.4 \pm 5.6) \times 10^{-3}$
B_{CP}^{Ξ}	$(-3.8 \sim -0.3) \times 10^{-4}$	$(1.2 \pm 3.4 \pm 0.8) \times 10^{-2}$

- Jusak Tandean et al , PRD 67 (2003) 056001
- Salone N et al , PRD 105 (2022) 11, 116022
- Xiao-Gang He et al, Sci.Bull. 67 (2022) 1840-1843
- Wang XF,arXiv:2312.17486

- ✓ The large uncertainties predicted in SM are related to the S/P puzzle

$$\alpha = \frac{2\mathrm{Re}(S^*P)}{|S|^2+|P|^2}, \quad \beta = \frac{2\mathrm{Im}(S^*P)}{|S|^2+|P|^2}$$
$$\gamma = \frac{|S|^2-|P|^2}{|S|^2+|P|^2}$$

$$\Rightarrow \quad A_{CP} = \frac{\alpha+\bar{\alpha}}{\alpha-\bar{\alpha}}, \quad B_{CP} = \frac{\beta+\bar{\beta}}{\alpha-\bar{\alpha}}$$

Summary and outlook

$|\Delta I| = 3/2$ non-leptonic hyperon decays in covariant baryon chiral perturbation theory #1

Jie Xu (Beihang U.), Jun-Xu Lu (Beihang U.), Rui-Xiang Shi (Guangxi Normal U. and Guangxi Key Lab. Nucl. Phys. Technol.), Li-Sheng Geng (Beihang U. and KAIST, Taejon and Beijing, CSRC and Hui Zhou, Inst. Modern Phys.) (Aug 11, 2026)

e-Print: 2608.11176 [hep-ph]

pdf cite claim reference search 0 citations

Results for $|\Delta I = 1/2|$ non-leptonic hyperon decays is coming soon !


Thanks for your attention!