Probing the imaginary parts and their q^2 dependences for the tauon $g-2$ and EDM

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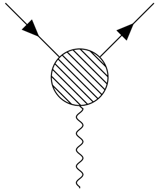
May 21, 2026

The Electromagnetic Current Vertex

To investigate the interactions between the τ lepton and the EM field, we start from the most general Lorentz-invariant effective vertex:



The vertex Γ^μ for $\gamma \to \tau^+\tau^-$ is parameterized as:

$$\begin{aligned}\Gamma^\mu = {} & F_1(q^2)\gamma^\mu \\ & + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_\tau} \quad \text{(MDM)} \\ & + F_3(q^2)\sigma^{\mu\nu}q_\nu\gamma_5 \quad \text{(EDM)} \\ & + F_4(q^2)(q^2\gamma^\mu - q^\mu \not{q})\gamma_5\end{aligned}$$

Physical Insight: QFT Interpretation

The scattering amplitude is given by $\mathcal{M} = -e\,\bar{u}(p')\,\Gamma^{\mu}\,u(p)\,A_{\mu}(q)$.

Components of the Amplitude

- **Asymptotic States ($\bar{u}, u$):** Represent the free particle states at $t \to \pm\infty$ in the interaction picture.
- **The Electromagnetic Field (A_{μ}):**
  1. $\vec{A}_{ext}$: Classical macroscopic background field (e.g., static $\vec{B}$ field).
  2. ϵ_{μ}: On-shell polarization of a quantized external photon.
  3. $-D_{\mu\nu}J^{\nu}$: Internal propagator contracted with another current (e.g., in $e^{+}e^{-}$ annihilation).

The Static Limit ($q^\mu \to 0$)

How do F_2 and F_3 map to classical dipole moments? We take the **Static Limit** ($q^\mu \to 0$):

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Bridge to Classical Electrodynamics

As $q^\mu \to 0$, the QFT amplitude reduces to the Born approximation of the interaction potential $V(\vec{x})$:

$$\mathcal{M} \xrightarrow{\text{Non-rel. Limit}} \mathcal{H}_{\text{int}} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$$

- **Magnetic Dipole Moment (MDM):** $a_\tau = F_2(0)$.
  The F_2 term couples to $\vec{B}$, describing the *anomalous* part.
- **Electric Dipole Moment (EDM):** $d_\tau = -eF_3(0)$.
  The γ_5 structure couples spin to $\vec{E}$, indicating *CP-violation* (P-odd, T-odd).

The Experimental Bottleneck: Lifetime Barrier

Measuring $a_{e,\mu}$ relies on **stable/quasi-stable** states interacting with macroscopic static fields ($q^\mu \to 0$).

Lepton	**Lifetime (τ)**	**Measurement Technique**
Electron (e)	*Stable*	Penning Traps (Quantum jumps)
Muon (μ)	$\sim 2.2 \times 10^{-6}$ s	Storage Rings (Spin precession)
Tauon (τ)	$\boldsymbol{\sim 2.9 \times 10^{-13}}$ **s**	**? ? ?**

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The Failure of the Static Limit

With a proper decay length of $c\tau \approx 87\,\mu$m, the τ decays almost instantaneously at the production vertex. It is fundamentally impossible to prepare a stable τ state to precess in macroscopic fields!

The Paradigm Shift

"If we cannot trap the tauon,
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Solution: Transition to the Time-like Region

We must abandon the space-like static limit ($q^\mu \to 0$) and transition to the **Time-like region** ($q^2 = s > 4m_\tau^2$).

We can extract the dipole information **dynamically** from the production and sequential decay spin-correlations at high-energy colliders (e.g., STCF, Belle II).

Dynamical Form Factors in the Timelike Region

The traditional matching ($q^\mu \to 0$) fails for the τ lepton because:

- **Quantum State Instability:** τ cannot serve as a stable asymptotic state in QFT.
- **Kinematic Threshold:** Collider production requires $q^2 = s > 4m_\tau^2$.

Static limit ($q^2 \to 0$) is unreachable in experiment.

Defining Dynamical $a_\tau(s)$ and $d_\tau(s)$

We promote the dipole constants to **dynamical form factors**:

$$\Gamma^\mu = F_1(s)\gamma^\mu + F_2(s)\frac{i\sigma^{\mu\nu}q_\nu}{2m_\tau} + F_3(s)\sigma^{\mu\nu}q_\nu\gamma_5 + \dots \quad (1)$$

- $a_\tau(s) \equiv F_2(s)$ and $d_\tau(s) \equiv -eF_3(s)$.
- **Analytic Continuation:** Recovers static values as $s \to 0$.
- **Unitarity:** Above threshold ($s > 4m_\tau^2$), form factors acquire **absorptive imaginary parts** via final-state rescattering.

QED Baseline: Unitarity and CP Properties

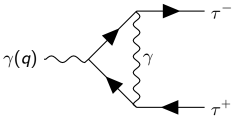

Figure: Final-state QED rescattering

In the timelike region, the QED 1-loop results are :

QED Results ($d_\tau = 0$)

$$\mathrm{Re}[a_\tau^{\mathrm{QED}}] = \frac{\alpha_{\mathrm{em}} m_\tau^2}{\pi s \beta_\tau} \ln\left(\frac{1-\beta_\tau}{1+\beta_\tau}\right) \quad (2)$$

$$\mathrm{Im}[a_\tau^{\mathrm{QED}}] = \frac{\alpha_{\mathrm{em}} m_\tau^2}{s \beta_\tau} \Theta(s - 4m_\tau^2) \quad (3)$$

Physics Insight: CP-Violation Requirement

- QED preserves CP symmetry $\Longrightarrow$ $d_\tau = 0$ at this order.
- **CP Criterion:** Generating d_τ requires an **odd number of CP-odd operators** from New Physics (NP).

Framework: Low-Energy EFT (LEFT)

At energy scales well below the electroweak scale, the relevant NP interactions are parameterized by LEFT operators:

Leading Four-Fermion and Dipole Operators

$$\mathcal{L}_{\rm eff} \supset \frac{C_{SS}^{\tau\tau}}{\Lambda^2}(\bar{\tau}\tau)^2 + \frac{C_{PP}^{\tau\tau}}{\Lambda^2}(\bar{\tau}i\gamma_5\tau)^2 + \frac{C_{SP}^{\tau\tau}}{\Lambda^2}(\bar{\tau}i\gamma_5\tau)(\bar{\tau}\tau) - \frac{ea_\tau^0}{4m_\tau}F^{\mu\nu}\bar{\tau}\sigma_{\mu\nu}\tau - d_\tau^0\frac{i}{2}F^{\mu\nu}\bar{\tau}\sigma_{\mu\nu}\gamma_5\tau$$

- **Hermiticity:** All parameters ($C_{SS}^{\tau\tau}$, $C_{PP}^{\tau\tau}$, $C_{SP}^{\tau\tau}$, a_τ^0, d_τ^0) must be **strictly real**. At tree level, they generate *no* imaginary parts.
- **CP Properties:**
  - **CP-even** ($C_{SS}^{\tau\tau}$, $C_{PP}^{\tau\tau}$, a_τ^0) contribute to the anomalous MDM.
  - **CP-odd** ($C_{SP}^{\tau\tau}$, d_τ^0) contribute to the EDM.

Framework: Matching LEFT to SMEFT

If the NP scale $\Lambda \gg v$, LEFT operators match onto SMEFT operators invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$\mathcal{L}_{\mathrm{SMEFT}} \supset \frac{C^{33}_{(eH)^2}}{\Lambda^4}\left[(\bar{L}_\tau H)\tau_R\right]^2 + \frac{C^{33}_{eB}}{\Lambda^2}(\bar{L}_\tau \sigma^{\mu\nu}\tau_R)HB_{\mu\nu} + \frac{C^{33}_{eW}}{\Lambda^2}(\bar{L}_\tau \sigma_{\mu\nu}\tau_R)\sigma_a H W_a^{\mu\nu} + \text{h.c.}$$

Matching after EWSB

$$C_{SS}^{\tau\tau} = -C_{PP}^{\tau\tau} = \frac{v^2}{4\Lambda^4}\mathrm{Re}[C^{33}_{(eH)^2}], \qquad C_{SP}^{\tau\tau} = \frac{v^2}{2\Lambda^4}\mathrm{Im}\left[C^{33}_{(eH)^2}\right],$$

$$a_\tau^0 = \frac{-2\sqrt{2}\, v\, m_\tau}{e\Lambda^2}\mathrm{Re}\left[c_W C^{33}_{eB} - s_W C^{33}_{eW}\right], \qquad d_\tau^0 = -\frac{\sqrt{2}v}{\Lambda^2}\mathrm{Im}\left[c_W\, C^{33}_{eB} - s_W\, C^{33}_{eW}\right]$$

Physical Insight

The anomalous MDM (a_τ) and EDM (d_τ) share a **common origin**. The Real parts of the Wilson coefficients govern the MDM, while the **Imaginary (CP-violating) parts** dictate the EDM.

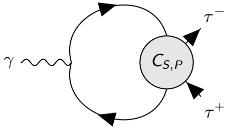

Figure: Closed τ loop

Kinematic Evolution :

$$F'_{2,3} \propto \frac{m_\tau^2 C_{SS,SP}}{\Lambda^2} g(s) \tag{4}$$

Loop Function $g(s)$

$$g(s) = 2 + \beta_\tau \ln\left(1 - \frac{s(1-\beta_\tau)}{2m_\tau^2}\right) \tag{5}$$

Loop Function $g(s)$

$$\mathrm{Im}(g(s)) = \pi\beta_\tau\Theta(s - 4m_\tau^2) \tag{6}$$

The Current Experimental Landscape: a_τ

Recent Experimental Limit (ATLAS at LHC)

Extracted via photon-photon collisions ($\gamma\gamma \to \tau^+\tau^-$) in Pb+Pb ultra-peripheral collisions:

$$-0.057 < a_\tau < 0.024 \quad (95\% \text{ C.L.})$$

The Current Experimental Landscape: a_τ

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Physical Insight: The Precision Deficit

- The current experimental uncertainty is $\mathcal{O}(10^{-2})$.
- The Standard Model QED 1-loop prediction (Schwinger term) is:

$$a_\tau^{\text{SM}} \simeq \frac{\alpha_{\text{em}}}{2\pi} \approx 1.177 \times 10^{-3}$$

- **Conclusion:** The experimental error is **more than** $10\times$ **larger** than the leading theoretical effect. We cannot even verify the QED baseline for the τ lepton yet!

The Current Experimental Landscape: d_τ

Current Best Limit (Belle at $\Upsilon(4S)$)

$$\mathrm{Re}(d_\tau) = (-6.2 \pm 6.3) \times 10^{-18}\ e \cdot \mathrm{cm}$$
$$\mathrm{Im}(d_\tau) = (-4.0 \pm 3.2) \times 10^{-18}\ e \cdot \mathrm{cm}$$

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Physical Insight: The Massive Discovery Window

- In the SM, the EDM requires 4-loop interactions and heavily suppressed CP-phases.
- The theoretical expectation is extraordinarily small:

$$d_\tau^{\mathrm{SM}} \sim \mathcal{O}(10^{-37})\ e \cdot \mathrm{cm}$$

- **Conclusion:** There is a ~ 19 **orders of magnitude gap**! This makes the τ EDM an exceptionally clean, background-free probe for New Physics.

Motivation: The Need for Novel Methodologies

How do we bridge these massive precision gaps?

- Traditional static field trapping is **impossible** (femtosecond lifetime).

Our Strategy

We must systematically exploit the **kinematics, angular distributions, and spin correlations** of $\tau^+\tau^-$ pairs at high-luminosity facilities:

Super Tau-Charm Facility (STCF) & **Belle II**

Experimental Methodology: Kinematic Basis

To extract the dipole form factors $a_\tau(s)$ and $d_\tau(s)$ from $e^+e^- \to \tau^+\tau^-$, we must exploit the full kinematic information.

Defining the Momentum Vectors

We define three fundamental unit momentum directions:

- $\hat{\boldsymbol{p}}$: The electron beam direction in the CM frame.
- $\hat{\boldsymbol{k}}$: The τ^- flight direction in the CM frame.
- $\hat{\boldsymbol{l}}_\pm$: The momentum direction of the charged hadron $h^\pm$ (e.g., $\pi^\pm, \rho^\pm$) in the respective $\tau^\pm$ rest frame.

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The Challenge

Dipole moments couple to the τ **spin**. But the τ decays too fast to measure its spin directly! How do we access the polarization?

The Hadronic Decay as a Spin Analyzer

We use the hadronic decay products as **Spin Analyzers**. The angular distribution of the secondary hadron is intimately correlated with the parent $\tau^{\pm}$ spin:

$$\frac{1}{\Gamma}\frac{d\Gamma(\tau^{\mp} \to h^{\mp}\nu)}{d\cos\theta_{\mp}} = \frac{1}{2}\left(1 \pm \alpha_h \cos\theta_{\mp}\right) \quad \text{where} \;\; \cos\theta_{\pm} \equiv \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{l}}_{\pm}$$

- α_h: Helicity sensitivity parameter ($\alpha_\pi = 1, \alpha_\rho = 0.45$).

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Theoretical Bridge

1. **Vertex:** Interaction is governed by the effective current $\Gamma^\mu(F_1, F_2, F_3)$.
2. **Amplitude:** $|\mathcal{M}|^2 \propto L_{\mu\nu} T^{\mu\nu}$ (Leptonic tensor $\times$ Hadronic/τ tensor).
3. **Integration:** Yields the multidimensional angular distribution of $\hat{\boldsymbol{p}}, \hat{\boldsymbol{k}}, \hat{\boldsymbol{l}}_\pm$.

Constructing CP-Even Observables for a_τ

By analyzing the differential cross-section, we construct specialized operators to isolate the real and imaginary parts of the anomalous MDM, $a_\tau(s)$:

Optimal Operators

$$\mathcal{O}_5 = \left(\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{k}}\right)^2, \qquad \mathcal{O}_6 = \frac{(\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{k}})\left[\hat{\boldsymbol{l}}_{\mp} \cdot (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}})\right]}{|\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}|}$$

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Evaluating the expectation values $\langle \mathcal{O}_i \rangle$ successfully decouples the parameters:

$$\mathrm{Re}[a_\tau] = \frac{2(s+2m_\tau^2)(s+m_\tau^2)}{s(s-4m_\tau^2)} - \frac{5(s+2m_\tau^2)^2}{s(s-4m_\tau^2)} \langle \mathcal{O}_5 \rangle$$

$$\mathrm{Im}[a_\tau] = -\frac{64 m_\tau (2m_\tau^2 + s)}{\pi \alpha_{h^\pm} \sqrt{s}(s-4m_\tau^2)} \langle \mathcal{O}_6 \rangle$$

Statistical Sensitivity & A Physics Milestone

The precision is dictated by event statistics ($\Delta\hat{\mathcal{O}}_i \sim 1/\sqrt{N_{\text{eff}}}$). Applying this to future high-luminosity facilities yields:

Facility ($\sqrt{s}$)	$\lvert\mathrm{Re}(a_\tau^{QED})\rvert$	$\delta(\mathrm{Re}\,[a_\tau])$	$\lvert\mathrm{Im}(a_\tau^{QED})\rvert$	$\delta(\mathrm{Im}\,[a_\tau])$
STCF ($m_{\psi(2S)}$)	1.11×10^{-3}	2.17×10^{-3}	6.39×10^{-3}	4.90×10^{-3}
Belle II ($m_{\Upsilon(4S)}$)	2.44×10^{-4}	6.29×10^{-5}	2.18×10^{-4}	5.02×10^{-5}

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Verifying the Schwinger Term

Breakthrough: The projected uncertainty at Belle II ($\sim 6 \times 10^{-5}$) is **smaller** than the 1-loop QED prediction ($a_\tau^{\mathrm{SM}} \sim 1.177 \times 10^{-3}$).

$\Longrightarrow$ We can **experimentally verify** the τ QED baseline for the first time, and explicitly measure the dynamically generated **Imaginary Parts**!

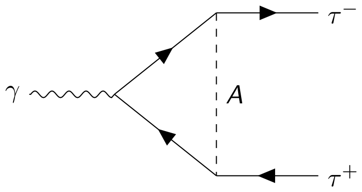
Testing New Physics: The UV Complete Model

How does our enhanced experimental precision translate into model-testing power?

Simplified Interaction (Eq. 23)

We begin with a generic CP-violating interaction between a neutral scalar mediator A and the τ lepton:

$$\mathcal{L}_A = A\bar{\tau}(a - ib\gamma_5)\tau$$



The NP contribution to $a_\tau(s)$ is:

$$a_\tau^{\text{NP}}(s) = a^2 f_1(s, m_A) + b^2 f_2(s, m_A)$$

Where f_1, f_2 correspond to the scalar and pseudoscalar couplings.

Physics Insights: Low-Mass Enhancement & Interference

Key observations from Fig. 3:

- **Low-Mass Enhancement**: Effect grows significantly at $m_A \sim$ GeV. In a 2HDM, we can focus exclusively on the light scalar A.
- **Destructive Interference**: $\mathrm{Re}(f_1)$ and $\mathrm{Re}(f_2)$ have strictly opposite signs.

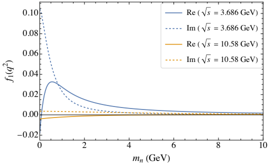


Figure: $f_1(s, m_A)$ (Scalar contribution)

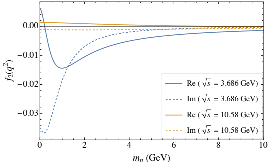


Figure: $f_2(s, m_A)$ (Pseudoscalar contribution)

Modeling Strategy

To generate an a_τ large enough to reach our experimental sensitivity, we must evade the **cancellation region**. This requires one coupling (typically a) to dominate.

The General 2HDM and the Basis Choice

Starting Point: The General Basis
Consider two complex scalar doublets Φ_1 and Φ_2 with hypercharge $Y = 1/2$. The most general $SU(2)_L \times U(1)_Y$ invariant potential is:

$$V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + h.c.) + \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + (\frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + h.c.)$$

Both doublets acquire Vacuum Expectation Values (VEVs):

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2/\sqrt{2} \end{pmatrix}, \quad \tan\beta = v_2/v_1, \quad v = \sqrt{v_1^2 + v_2^2}$$

Problem: In this general basis, the "physical" Goldstone bosons and the Higgs particle are linear combinations of both Φ_1 and Φ_2. This complicates the identification of the SM-like Higgs.
$\Rightarrow$ We need a more transparent basis: The Higgs Basis.

Defining the Higgs Basis

The Rotation: We perform a basis rotation by the angle β to isolate the VEV:

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

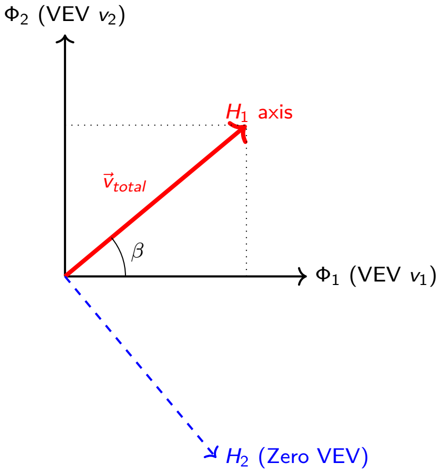
where $c_\beta = \cos\beta, s_\beta = \sin\beta$.

Key Property: In this new basis, only H_1 carries the VEV:

$$\langle H_1 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$\langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

H_1: The "SM-like" doublet (contains Goldstones $G^\pm, G^0$).
H_2: The "New Physics" doublet (contains $H^\pm, H, A$).



Visualizing the Higgs Basis Rotation

The Scalar Potential in Higgs Basis

The potential parameters transform ($\lambda_i \to \Lambda_i$). The relevant mass terms for the neutral scalars can be extracted rigorously. In the basis $\{h, H, A\}$ where $h = \mathrm{Re}(H_1^0), H = \mathrm{Re}(H_2^0), A = \mathrm{Im}(H_2^0)$:

Mass Matrix Structure (Neutral Sector)

$$\mathcal{M}^2_{neutral} = \begin{pmatrix} \Lambda_1 v^2 & \Lambda_6 v^2 & 0 \\ \Lambda_6 v^2 & M_{22}^2 + \frac{1}{2}(\Lambda_3 + \Lambda_4 + \Lambda_5)v^2 & -\frac{1}{2}\mathrm{Im}(\Lambda_5)v^2 \\ 0 & -\frac{1}{2}\mathrm{Im}(\Lambda_5)v^2 & M_{22}^2 + \frac{1}{2}(\Lambda_3 + \Lambda_4 - \Lambda_5)v^2 \end{pmatrix}$$

- $\Lambda_1 v^2$: Corresponds to the SM Higgs mass (m_h^2).
- $\Lambda_6 v^2$: Describes the mixing between the SM-like Higgs (h) and the new scalar (H).
- $\mathrm{Im}(\Lambda_5)$: Describes the mixing between CP-even H and CP-odd A.

The Aligned Limit and Its Consequences

1. Definition of Alignment Limit

We require the scalar h (from H_1) to be **exactly** the observed 125 GeV Higgs boson.

$$\text{Alignment} \implies \text{Mixing}(h, H) = 0 \implies \Lambda_6 = 0$$

(Or equivalently $\cos(\beta - \alpha) = 0$ in the standard notation).

2. Consequences in this Limit

(1) Decoupling of h: The mass matrix becomes block diagonal. h is physically isolated.

(2) Heavy Sector Mixing: The remaining 2×2 block for H and A:

$$\mathcal{M}^2_{H/A} = \begin{pmatrix} M_H^2 & -\frac{1}{2}v^2\text{Im}(\Lambda_5) \\ -\frac{1}{2}v^2\text{Im}(\Lambda_5) & M_A^2 \end{pmatrix}$$

(3) CP Violation Origin:

- If $\text{Im}(\Lambda_5) \neq 0$: H and A mix $\to$ Mass eigenstates $h_{2,3}$ are CP-mixed.
- Even if $\text{Im}(\Lambda_5) = 0$: CPV can still enter via complex Yukawa couplings.

Equivalence of CP Sources in the "Invisible" Sector

Motivation: Intuitively, CP violation (CPV) from the scalar potential (mixing) and from the Yukawa sector seem distinct. However, in the Higgs Basis, for "invisible" (off-shell) processes involving only fermions, they are mathematically equivalent via a basis transformation.

Source 1: Potential Mixing (Λ_5)

CPV arises from the mass mixing between CP-even H and CP-odd A:

$$\mathcal{M}^2 = \begin{pmatrix} M_H^2 & -\frac{v^2}{2}\mathrm{Im}\Lambda_5 \\ -\frac{v^2}{2}\mathrm{Im}\Lambda_5 & M_A^2 \end{pmatrix}$$

$\Rightarrow$ Mass Eigenstates are mixed.

Source 2: Yukawa Phase (Y_2)

CPV arises from the complex coupling to fermions (e.g., τ):

$$-\mathcal{L}_Y \supset \bar{\tau}(\mathrm{Re}Y_2 + i\mathrm{Im}Y_2\gamma_5)\tau H + \ldots$$

$\Rightarrow$ Interaction vertices are complex.

2HDM Realization

We embed this mechanism into a 2HDM framework using the **Alignment Limit**:

- **Scalar Potential**: We assume no explicit CP violation.
- **Yukawa Sector**: CP violation is introduced exclusively via complex Yukawa couplings.

Matching to Effective Parameters

$$a = -\frac{\mathrm{Im}(Y_2)_{33}}{\sqrt{2}}, \qquad b = -\frac{\mathrm{Re}(Y_2)_{33}}{\sqrt{2}}$$

Physical Insight

Since the mass spectrum naturally allows $m_H \gg m_A$, the heavy CP-even state H effectively decouples and can be safely neglected.

Navigating Experimental Constraints

The chosen parameters must survive stringent experimental limits:

1. $\gamma^* \to A\gamma$ **(BESIII)**

- The τ triangle loop restricts the absolute magnitude ($|ab|$).

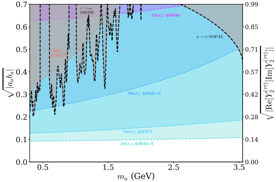


Figure: Limit on a vs b

2. $h \to \tau^+\tau^-$ **(Higgs Decay)**

- 1-loop radiative corrections stringently limit the ratio (b/a).

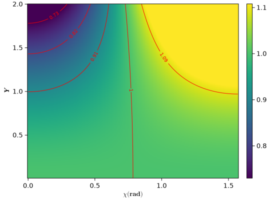


Figure: Fig. 7: Signal strength limit

Numerical Results: Benchmark Point (BP)

We select an optimal Benchmark Point: $m_A = 2$ GeV, $b/a = -1/6$, $a^2 + b^2 = 1 (a = 0.9864, b = -0.1644)$.

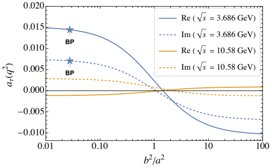


Figure: Fig. 4: Predicted a_τ at BP

NP predictions at collider energies:

- **STCF** ($\psi(2S)$):

$$\simeq 0.0014 + 0.0071i$$

- **Belle II** ($\Upsilon(4S)$):

$$\simeq -0.0011 + 0.0028i$$

Conclusion

The predicted values lie well within the reach of our proposed methodology. This explicitly demonstrates that our precision observables possess the capability to test realistic UV-complete theories.

Numerical Results: Anomalous MDM at Benchmark Point

We select the scalar-dominated Benchmark Point: $m_A = 2$ GeV, $b/a = -1/6$, $a^2 + b^2 = 1(a = 0.9864, b = -0.1644)$.

NP vs. QED Predictions (a_τ)

Energy ($\sqrt{s}$)	NP Contribution (a_τ^{NP})	QED Baseline (a_τ^{QED})
STCF	$\simeq 0.0014 + 0.0071i$	$-0.0011 + 0.0064i$
Belle II	$\simeq -0.0011 + 0.0028i$	$0.0002 + 0.0002i$
Static ($s = 0$)	$\simeq 0.0055$	0.0012

Interference Pattern & Conclusion

- **Collider Energies:** The combined effect of NP and QED exhibits a **cancellation in the real part** but an **enhancement in the imaginary part**. This highlights the crucial role of probing the imaginary part!
- **Static Limit:** NP and QED contributions are both positive, and the combined prediction safely satisfies the recent ATLAS limit.

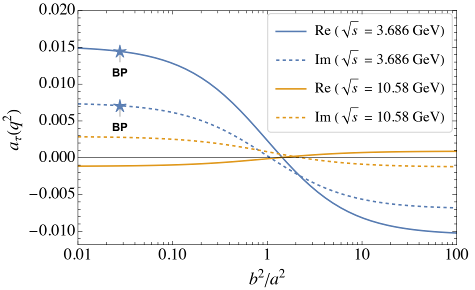


Figure: Predicted $a_\tau(s)$ versus b^2/a^2.

Numerical Results: Tauon EDM at Benchmark Point

Constrained by the required hierarchical coupling $b/a = -1/6$, the corresponding EDM form factor reaches and robustly maintains a magnitude at the $\mathcal{O}(10^{-17})$ $e \cdot \mathrm{cm}$ level.

EDM predictions at Collider Energies (d_τ^{NP})

- **STCF** ($\sqrt{s} = 3.686$ GeV):

$$d_\tau^{\mathrm{NP}} \simeq (2.3046 + 1.2959i) \times 10^{-17} \; e \cdot \mathrm{cm}$$

- **Belle II** ($\sqrt{s} = 10.58$ GeV):

$$d_\tau^{\mathrm{NP}} \simeq (-1.8482 + 3.7232i) \times 10^{-18} \; e \cdot \mathrm{cm}$$

- **Static Limit** ($s = 0$):

$$d_\tau^{\mathrm{NP}}(0) \simeq 8.2992 \times 10^{-18} \; e \cdot \mathrm{cm}$$

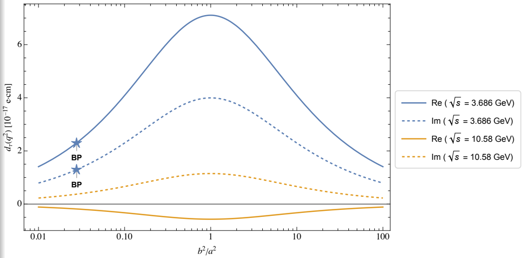


Figure: Predicted τ EDM form factor $d_\tau(s)$ versus b^2/a^2.

Phenomenological Consistency

The analytical continuation to the static limit $s = 0$ and the collider predictions are strictly consistent with the current best upper bounds set by the **Belle Collaboration**:

Summary & Conclusion

- **Theoretical Paradigm Shift**:
  Transitioning to the time-like region ($q^2 = s > 4m_\tau^2$) reveals rich dynamics.

- **Novel Experimental Methodology**:
  By exploiting the τ hadronic decays as **spin analyzers**, we constructed tailored kinematic observables to independently extract the Real and Imaginary parts of the dipole form factors at e^+e^- colliders.

- **Physics Milestone at STCF & Belle II**:
  Our proposed method improves the expected sensitivity of a_τ by **more than one order of magnitude**, enabling the first experimental verification of the τ QED Schwinger term. It also provides the necessary precision to rigorously test UV-complete theories like the 2HDM.

Thank you for your attention!