



復旦大學
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Direct CP violation measurement of decay $B^0 \rightarrow K_S^0 \pi^0$

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- 2 Common Setup
- 3 Belle II Analysis
 - Continuum Suppression
 - Shape Extraction
 - A_{CP} Measurement with 16rdMC
- 4 Belle Analysis
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Motivation

- The decay $B^0 \rightarrow K_S^0 \pi^0$ is a penguin-dominated $b \rightarrow s$ transition and a sensitive probe of CP-violating effects.
- The long-standing $K\pi$ puzzle, i.e. the tension pattern among direct CP asymmetries in $B \rightarrow K\pi$ modes, motivates improved neutral-mode measurements.
- The $K\pi$ isospin sum rule:

$$I_{K\pi} \equiv \mathcal{A}_{K^+\pi^-} + \mathcal{A}_{K^0\pi^+} \frac{\mathcal{B}(K^0\pi^+)\tau_{B^0}}{\mathcal{B}(K^+\pi^-)\tau_{B^+}} - 2\mathcal{A}_{K^+\pi^0} \frac{\mathcal{B}(K^+\pi^0)\tau_{B^0}}{\mathcal{B}(K^+\pi^-)\tau_{B^+}} - 2\mathcal{A}_{K^0\pi^0} \frac{\mathcal{B}(K^0\pi^0)}{\mathcal{B}(K^+\pi^-)} \approx 0.$$

- $B^0 \rightarrow K_S^0 \pi^0$ provides the essential neutral-mode input; a significant deviation from the expectation would be of strong physics interest.
- This analysis targets:
 - Branching fraction $\mathcal{B}(B^0 \rightarrow K_S^0 \pi^0)$,
 - Direct CP asymmetry \mathcal{A}_{CP} .

Previous Measurements and Analysis Strategy

- Previous measurements of $B^0 \rightarrow K_S^0 \pi^0$:

Publications	Data statistic ($\times 10^6$)	\mathcal{B} [$\times 10^{-6}$]	\mathcal{A}_{CP}
Phys.Rev.D 109 (2024) 1, 012001	387	$10.40 \pm 0.66 \pm 0.60$	$-0.06 \pm 0.15 \pm 0.04$
Phys.Rev.Lett. 131 (2023) 11, 111803	387	$11.15 \pm 0.68 \pm 0.62$	$0.04 \pm 0.15 \pm 0.05$
Phys.Rev.D 81 (2010) 011101	657	$8.7 \pm 0.5 \pm 0.6$	$0.14 \pm 0.13 \pm 0.06$

- Analysis strategy:
 - Use the combined **Belle + Belle II** program to improve the precision of the \mathcal{A}_{CP} measurement.
 - Belle integrated luminosity: $\sim 711 \text{ fb}^{-1}$.
 - Apply the Belle II reconstruction strategy within the B2BII framework.
 - Belle II integrated luminosity: $\sim 500 \text{ fb}^{-1}$.
 - The Belle II dataset is expected to increase further with ongoing data taking.

Software Version and MC Samples

- Basf2: light-2503-ceres.
- Run-dependent MC_16rd samples are used for selection studies and PDF modeling.
- Official signal MC is used for signal-shape determination and efficiency evaluation.
- Inclusive MC samples include $e^+e^- \rightarrow q\bar{q}$ and generic $B\bar{B}$ components.

Sample	Name	Data size
Signal	MC_16rd_signal_1110022103	2M
continuum	MC_16rd_prompt(proc16)	1946.34 fb ⁻¹
BB	MC_16rd_prompt(proc16)	1946.34 fb ⁻¹

- Physics target: direct CP-violation measurement in $B^0 \rightarrow K_S^0 \pi^0$.
- **Belle II** – nominal fit with MC_16rd sample ($4\times$ data statistics):
 - $\mathcal{A}_{CP} = -0.030 \pm 0.060$.
 - Scaled to Belle II data statistics: $\sigma(\mathcal{A}_{CP}) \approx 0.120$.
- Experiment-specific details, validations, and fit performances are presented separately in the Belle II and Belle sections.

Nominal Observables and Fit Strategy

- Nominal observables:

$$M'_{bc}, \quad \Delta E, \quad \text{CSO}_{\text{mu.}}$$

- Signal categories in the fit:
 - signal,
 - continuum $q\bar{q}$,
 - generic $B\bar{B}$.
- The ACP extraction is performed with a simultaneous fit in flavor-tag bins.

- Common analysis flow:

- ① event reconstruction and preselection,
 - ② selection optimization,
 - ③ validation of input variables and classifier outputs,
 - ④ PDF modeling,
 - ⑤ nominal fit and ACP extraction.
- The following sections show how this workflow is implemented first in Belle II and then in Belle.

Reconstruction Chain

- K_S^0 :
stdKshorts + ksSelector (v0mva/lammva kept).
- $\pi^0 \rightarrow \gamma\gamma$:
photon candidates are taken from gamma:cdc
 π^0 candidates are reconstructed from two photons
a mass-constrained kinematic fit is applied to the π^0 .
- $B^0 \rightarrow K_S^0\pi^0$:
candidates are reconstructed in a broad kinematic region.

Category	Belle II selection conditions
Event level	$nTracks \geq 2$
γ reconstruction	$E_\gamma > 0.10$ GeV (Forward), $E_\gamma > 0.05$ GeV (Backward/Barrel)
π^0 selection	$0.110 < \text{InvM}_{\pi^0} < 0.160$ GeV/ c^2 <code>pi0_mva</code> > 0.50
K_S^0 selection	$0.479 < \text{InvM}_{K_S^0} < 0.517$ GeV/ c^2 <code>K_S0_lammva</code> > 0.10, <code>K_S0_v0mva</code> > 0.90
B^0 window	$5.24 < M_{bc} < 5.30$ GeV/ c^2 , $ \Delta E < 0.30$ GeV

Ks MVA Selection: FOM Optimization

- FOM definition:

$$\text{FOM} = \frac{N_S}{\sqrt{N_S + N_B}}$$

- Optimized selection:

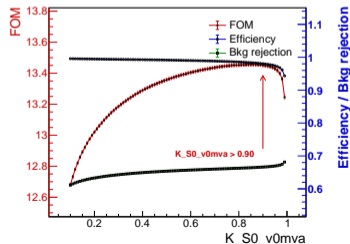
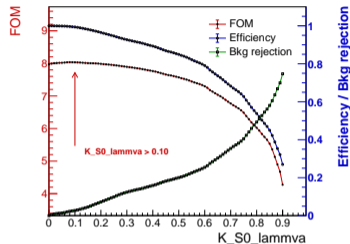
- $K_{S0_lammva} > 0.10$
- $K_{S0_v0mva} > 0.90$

- lammva:

- eff. at cut 0.10: $(99.38 \pm 0.01)\%$
- bkg rejection at cut 0.10: 2.72%

- v0mva:

- eff. at cut 0.90: $(98.04 \pm 0.01)\%$
- bkg rejection at cut 0.90: 66.62%



π^0 MVA Training

- Classifier: FastBDT

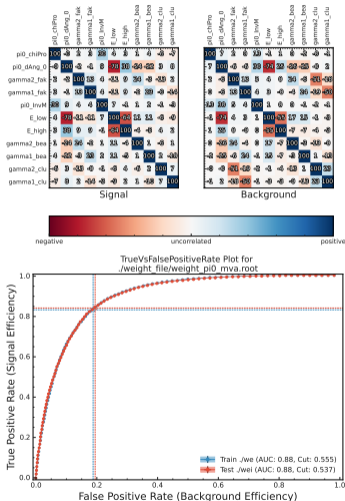
- Input variables:

- π^0 variables:

InvM, daughterAngle, chiProb, pt

- photon variables:

fakePhotonSuppression,
beamBackgroundSupreesion,
clusterSecondMoment, E_low,
E_high

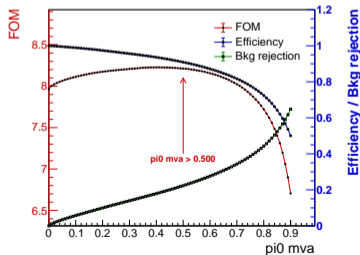
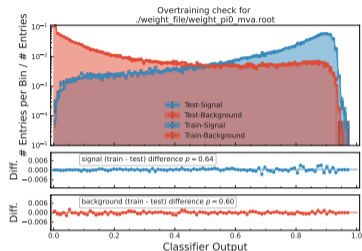


π^0 MVA Performance and FOM Optimization

- FOM definition:

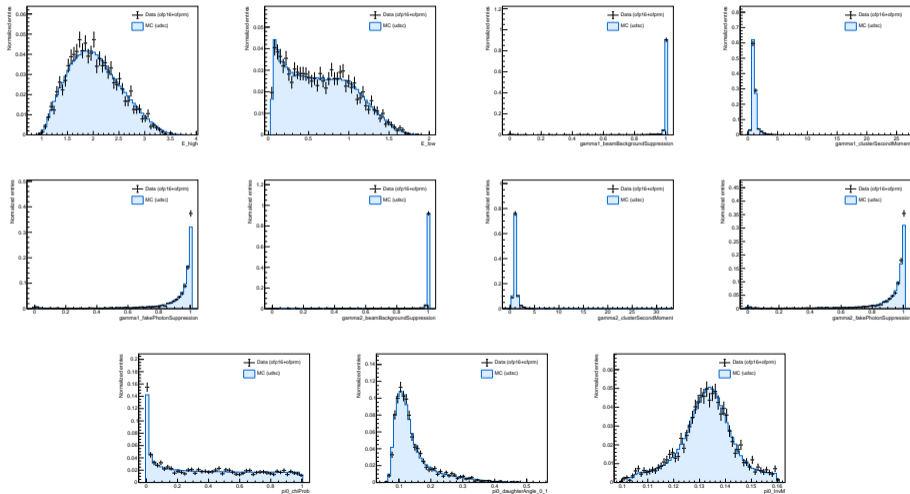
$$\text{FOM} = \frac{N_S}{\sqrt{N_S + N_B}}$$

- Optimized cut: $\text{pi0_mva} > 0.40$.
- Signal efficiency at cut 0.50: $(90.92 \pm 0.03)\%$.
- Background rejection at cut 0.50: $(22.32 \pm 0.03)\%$.

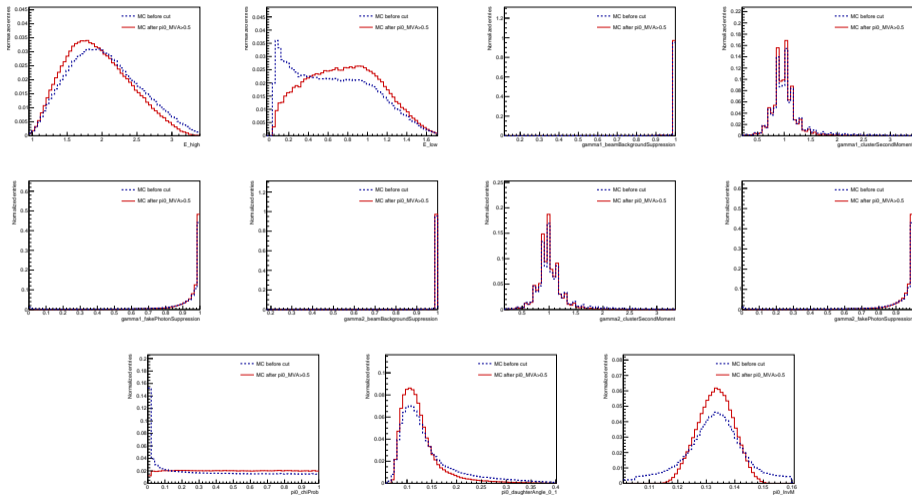


Data-MC Comparison of π^0 -MVA Inputs

Data-MC comparison of the π^0 -MVA input variables

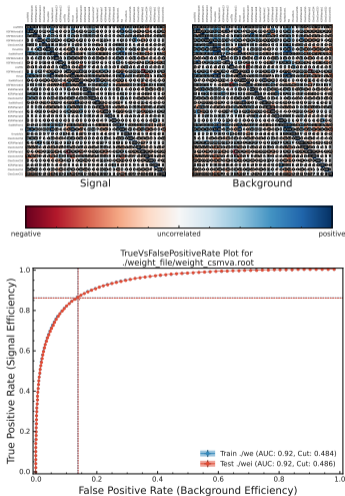


Effect of the π^0 -MVA Requirement

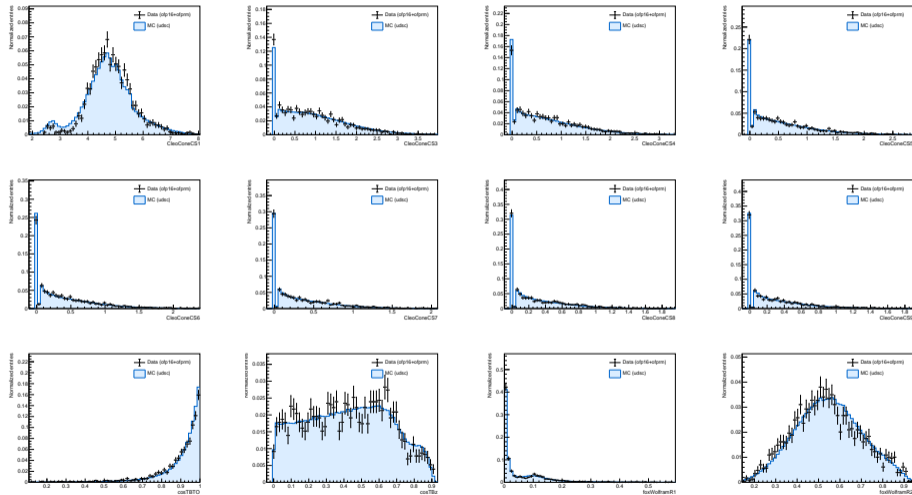


Continuum Suppression MVA Training

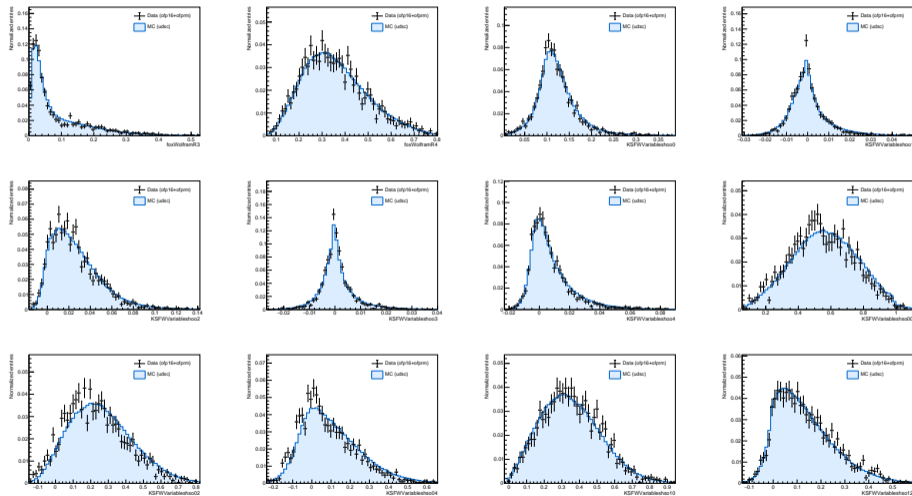
- Classifier: FastFBDT
- Input variables:
 - Modified Fox-Wolfram / KSFW moments
 - KSFWVariables(mm2) and KSFWVariables(et)
 - CLEO-cone variables
 - Event-thrust variables:
 - thrust0m, thrustBm
 - cosTBT0, cosTBz
 - thrustAxisCosTheta
 - R2 (normalized 2nd Fox-Wolfram moment)



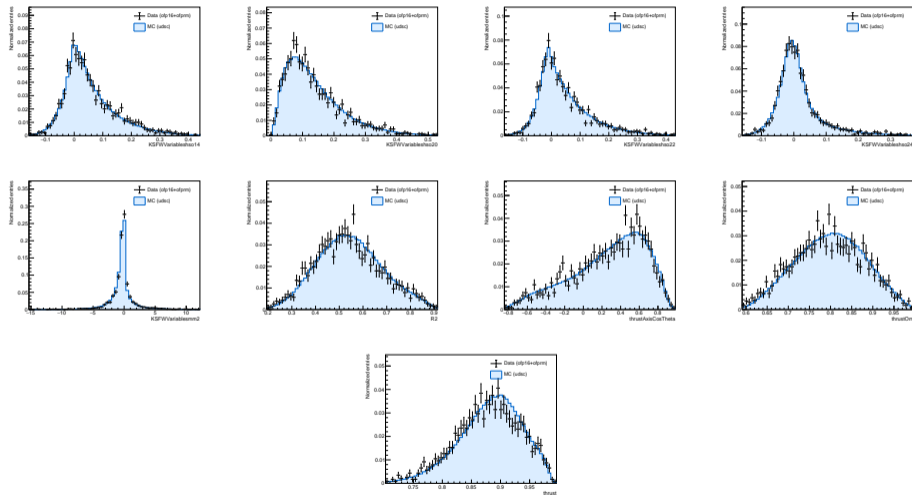
Data-MC Comparisons for CSMVA Event-Shape Inputs



Data-MC Comparisons for CSMVA KSFW Inputs I

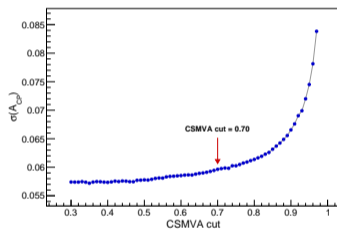


Data-MC Comparisons for CSMVA KSFW Inputs II and Thrust

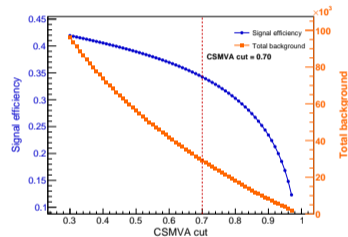


Continuum Suppression: Optimization

- The CSMVA requirement is optimized directly with the Belle II expected fit uncertainty on A_{CP} .
- We adopt CSMVA > 0.70 for the Belle II nominal selection:
 - $\sigma(A_{CP}) = 0.0596$
 - $\epsilon = 34.2\%$
 - $N_{\text{cont}} = 26292$,
 $N_{BB} = 2858$

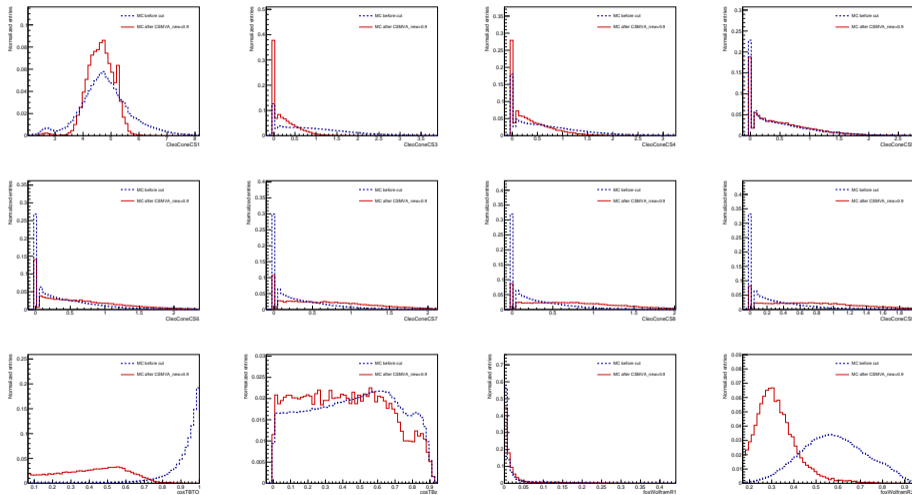


Expected $\sigma(A_{CP})$ vs. CSMVA cut

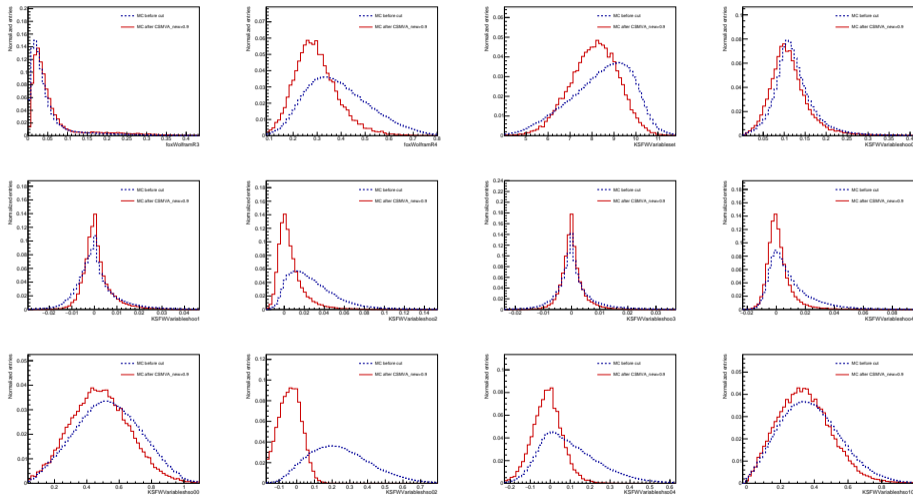


Signal efficiency and background vs. CSMVA cut

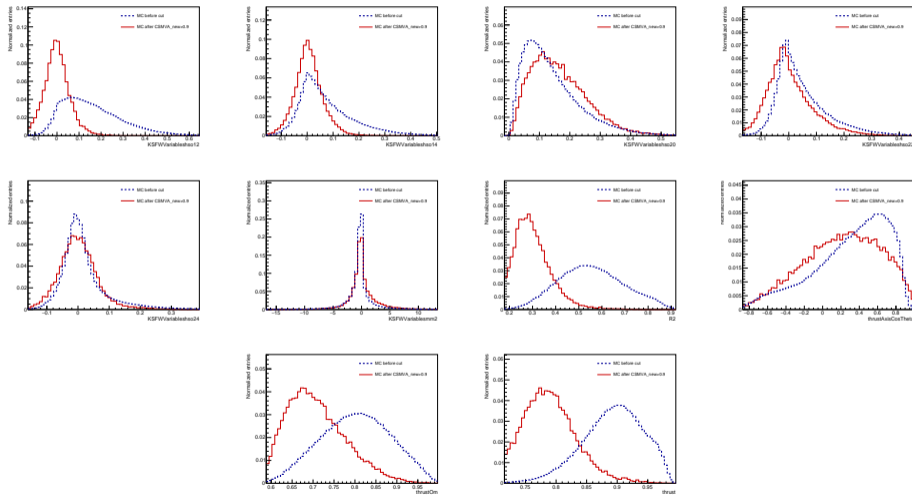
Effect of the CSMVA Requirement on Event-Shape Inputs



Effect of the CSMVA Requirement on KSFW Inputs I



Effect of the CSMVA Requirement on KSFW Inputs II and Thrust



- s is the output of continuum suppression results and define

$$\text{CSOMu}(s) = \int_0^s \mathcal{P}_{CS}(t) dt.$$

- where $\mathcal{P}_{CS}(t)$ is the distribution of the continuum-suppression classifier.
- The modified M_{bc} , CSOMu and ΔE will be used as fitting variables.

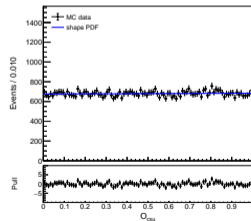
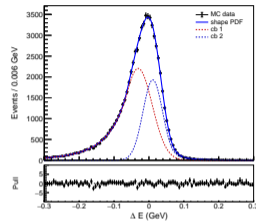
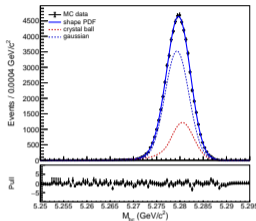
Signal PDF Model

- Sample: signal MC.
- M'_{bc} : Crystal Ball + Gaussian.
- ΔE : sum of two Crystal Ball functions.
- CSOmu: first-order polynomial

$$\mathcal{P}_{\text{sig}}^{\text{CSOmu}} = 1 + p_1^{\text{sig}} \text{CSOmu}.$$

- Total signal PDF:

$$\mathcal{P}_{\text{sig}}(\vec{x}) = \mathcal{P}_{\text{sig}}^{M'_{bc}}(M'_{bc}) \mathcal{P}_{\text{sig}}^{\Delta E}(\Delta E) \mathcal{P}_{\text{sig}}^{\text{CSOmu}}(\text{CSOmu}).$$



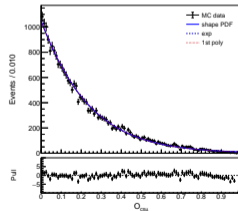
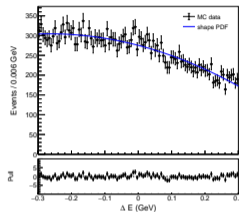
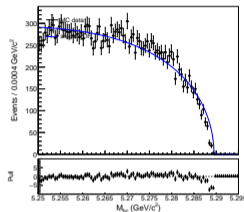
Continuum PDF Model

- Sample: generic $q\bar{q}$ MC.
- M'_{bc} : double ARGUS function.
- ΔE : second-order Bernstein polynomial.
- CSOmu: exponential \times first-order polynomial

$$e^{\lambda \text{CSOmu}} (1 + p_1 \text{CSOmu}).$$

- Total continuum PDF:

$$\mathcal{P}_{q\bar{q}}(\vec{x}) = \mathcal{P}_{q\bar{q}}^{M'_{bc}}(M'_{bc}) \mathcal{P}_{q\bar{q}}^{\Delta E}(\Delta E) \mathcal{P}_{q\bar{q}}^{\text{CSOmu}}(\text{CSOmu}).$$



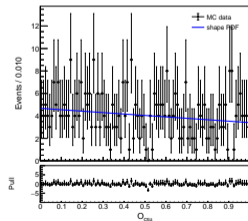
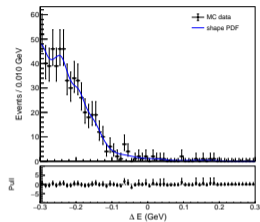
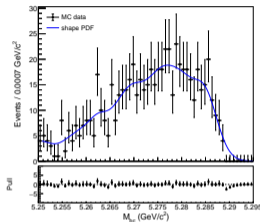
Generic $B\bar{B}$ PDF Model

- Sample: generic $B\bar{B}$ MC.
- $(M'_{bc}, \Delta E)$: non-parametric 2D KDE.
- CSOmu: first-order polynomial

$$\mathcal{P}_{BB}^{\text{CSOmu}} = 1 + p_1^{\text{bb}} \text{CSOmu}.$$

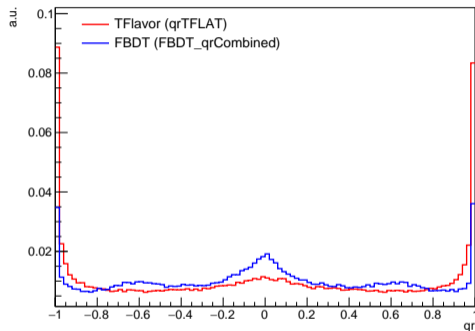
- Combined form:

$$\mathcal{P}_{BB}(\vec{x}) = \mathcal{P}_{BB}^{\text{2D KDE}}(M'_{bc}, \Delta E) \mathcal{P}_{BB}^{\text{CSOmu}}(\text{CSOmu}).$$



14-category simultaneous fit in $q \cdot r$ bins

- Nominal ACP extraction uses 14 categories: 7 bins in $|qr|$ and $q = \pm 1$ in each bin.
- Bin boundaries:
 $|qr| \in [0.0, 0.1), [0.1, 0.25), [0.25, 0.5), [0.5, 0.625), [0.625, 0.75), [0.75, 0.875), [0.875, 1.0].$
- Fit ranges:
 - $5.24 < M'_{bc} < 5.295 \text{ GeV}/c^2$
 - $-0.30 < \Delta E < 0.30 \text{ GeV}$
 - $0.0 < \text{CSOmu} < 1.0$
- Correlation handling in nominal model: continuum uses $2D(M'_{bc}, \text{CSOmu})$, while generic- $B\bar{B}$ uses $2D(M'_{bc}, \Delta E)$.
- Per-category yield construction is shown on the next slide.



Updated qr distribution used for the flavor-tag binning reference.

Per-category yield construction in $q \cdot r$ bins

$$N_{\text{sig}}^{i,q} = \frac{1}{2} N_{\text{sig}}^{\text{tot}} \epsilon_i \left[1 - q\Delta\omega_i + q\mu_i(1 - 2\omega_i) \right. \\ \left. + (q(1 - 2\omega_i) + \mu_i(1 - q\Delta\omega_i))(1 - 2\chi_d)\mathcal{A}_{CP} \right].$$

$$N_{\text{cont}}^{i,q} = \frac{1}{2} N_{\text{cont},i}^{\text{tot}}, \quad N_{BB}^{i,q} = \frac{1}{2} N_{BB,i}^{\text{tot}}.$$

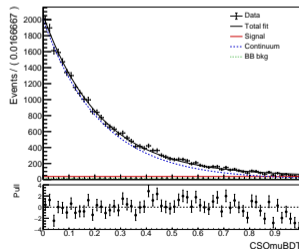
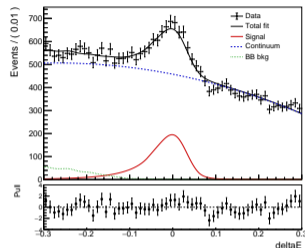
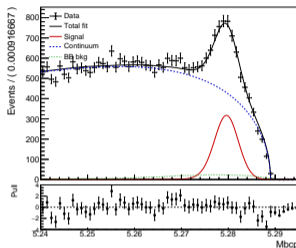
- $i = 0, \dots, 6$ labels the $|qr|$ bin; $q = \pm 1$ is the tag-sign category.
- Signal yield is controlled by $(\epsilon_i, \omega_i, \Delta\omega_i, \mu_i, \chi_d, \mathcal{A}_{CP})$.
- Continuum and generic- $B\bar{B}$ yields float per r bin and are split symmetrically in q .

Projections of the Nominal $q \cdot r$ -Binned Fit

- Fitted with 16rd MC (CSMVA > 0.70 , $4 \times$ data statistics):

$$\mathcal{A}_{CP} = -0.030 \pm 0.060$$

- Scaled to Belle II data statistics:
 $\sigma(\mathcal{A}_{CP}) \approx 0.120$.



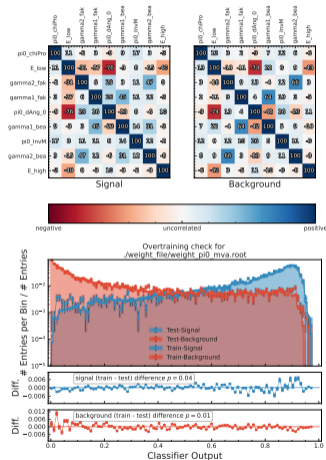
Category	Belle selection conditions
γ reconstruction	$E_\gamma > 0.100$ GeV (Forward), $E_\gamma > 0.050$ GeV (Barrel/Backward)
π^0 selection	$0.100 < \text{InvM}_{\pi^0} < 0.160$ GeV/ c^2
K_S^0 selection	$0.479 < \text{InvM}_{K_S^0} < 0.517$ GeV/ c^2 <code>goodBelleKshort=1</code>
B^0 window	$5.24 < M_{bc} < 5.30$ GeV/ c^2 , $ \Delta E < 0.30$ GeV

Belle π^0 MVA Training

- π^0 -MVA training inputs:

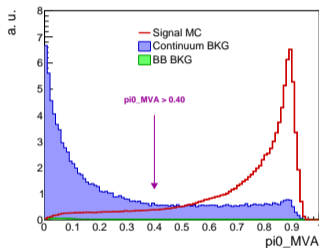
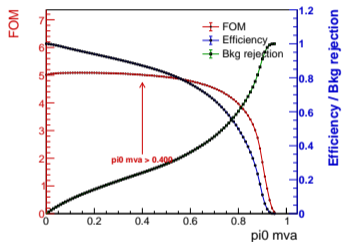
- $E_{\text{high}}, E_{\text{low}}$
- $\text{gamma1/2_beamBackgroundSuppression}$
- $\text{gamma1/2_fakePhotonSuppression}$
- $\text{pi0_chiProb},$
 $\text{pi0_daughterAngle_0_1}$
- $\text{pi0_InvM}, \text{pi0_pt}$

- $\text{gamma1/2_clusterSecondMoment}$ is excluded in b2bii because of nan values.



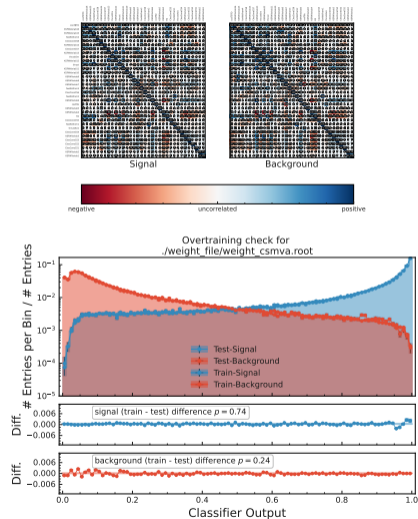
Belle π^0 MVA Optimization

- π^0 MVA cut: > 0.4 .
- Signal efficiency at cut 0.4: $(88.33 \pm 0.03)\%$.
- Background rejection at cut 0.4: $(28.49 \pm 0.03)\%$.
- The signal-background separation and the FOM scan are shown on the right.



Belle Continuum Suppression

- Present the Belle continuum-suppression strategy and the corresponding classifier inputs.
- The default b2bii CSMVA training uses continuum background only.
- The Belle-side monitoring is shown with evaluation and overtraining plots.



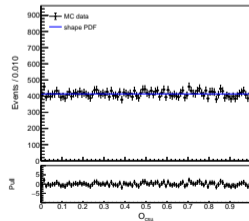
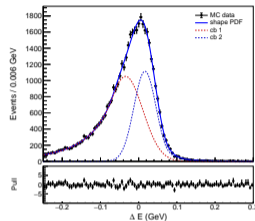
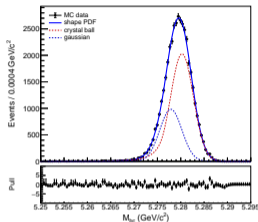
Belle Signal PDF Model

- Sample: signal MC (Belle).
- M'_{bc} : Crystal Ball + Gaussian.
- ΔE : sum of two Crystal Ball functions.
- CSOmu: first-order polynomial

$$\mathcal{P}_{\text{sig}}^{\text{CSOmu}} = 1 + p_1^{\text{sig}} \text{CSOmu}.$$

- Total signal PDF:

$$\mathcal{P}_{\text{sig}}(\vec{x}) = \mathcal{P}_{\text{sig}}^{M'_{bc}}(M'_{bc}) \mathcal{P}_{\text{sig}}^{\Delta E}(\Delta E) \mathcal{P}_{\text{sig}}^{\text{CSOmu}}(\text{CSOmu}).$$



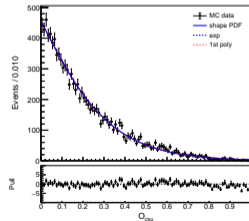
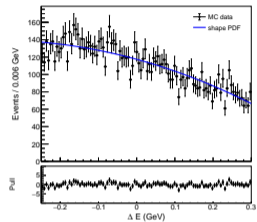
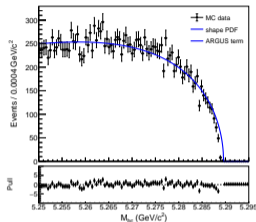
Belle Continuum PDF Model

- Sample: continuum MC (Belle).
- M'_{bc} : single ARGUS function.
- ΔE : second-order Bernstein polynomial.
- CSOmu: exponential \times first-order polynomial

$$e^{\lambda \text{CSOmu}} (1 + p_1 \text{CSOmu}).$$

- Total continuum PDF:

$$\mathcal{P}_{q\bar{q}}(\vec{x}) = \mathcal{P}_{q\bar{q}}^{M'_{bc}}(M'_{bc}) \mathcal{P}_{q\bar{q}}^{\Delta E}(\Delta E) \mathcal{P}_{q\bar{q}}^{\text{CSOmu}}(\text{CSOmu}).$$



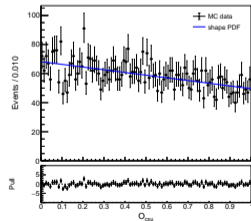
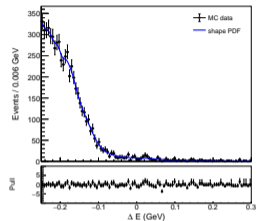
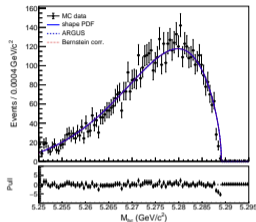
Belle Generic $B\bar{B}$ PDF Model

- Sample: generic $B\bar{B}$ MC (Belle).
- M'_{bc} : ARGUS \times quadratic Bernstein correction.
- ΔE : RooKeysPdf (adaptive kernel density).
- CSOmu: first-order polynomial

$$\mathcal{P}_{BB}^{\text{CSOmu}} = 1 + p_1^{\text{bb}} \text{CSOmu}.$$

- Total BB PDF:

$$\mathcal{P}_{BB}(\vec{x}) = \mathcal{P}_{BB}^{M'_{bc}}(M'_{bc}) \mathcal{P}_{BB}^{\Delta E}(\Delta E) \mathcal{P}_{BB}^{\text{CSOmu}}(\text{CSOmu}).$$



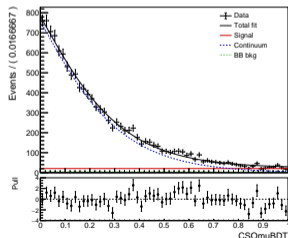
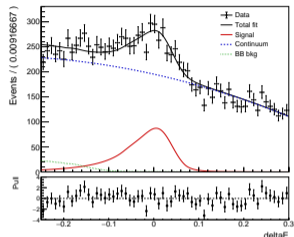
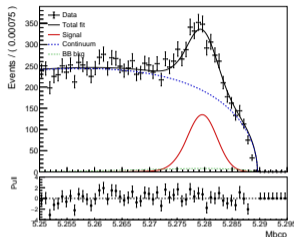
Projections of the Belle B2BII Closure Fit

- Pseudo-data: twice Belle data statistics.

- Closure result:

$$\mathcal{A}_{CP} = 0.081 \pm 0.104$$

- Scaled to Belle data statistics: $\sigma(\mathcal{A}_{CP}) \approx 0.147$.



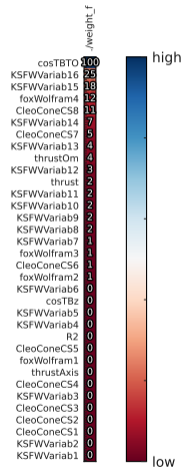
- A common analysis strategy for $B^0 \rightarrow K_S^0 \pi^0$ has been established across Belle II and Belle (B2BII).
- **Belle II** – nominal fit with MC_16rd sample ($4\times$ data statistics):
 - $\mathcal{A}_{CP} = -0.030 \pm 0.060$.
 - Scaled to Belle II data statistics: $\sigma(\mathcal{A}_{CP}) \approx 0.120$.
- **Belle** – B2BII closure fit with $2\times$ data statistics:
 - $\mathcal{A}_{CP} = 0.081 \pm 0.104$ (closure test).
 - Expected sensitivity: $\sigma(\mathcal{A}_{CP}) \approx 0.147$ at Belle data scale.

Thank you for your attention!

Backup

Belle Continuum Suppression: Auxiliary Monitoring

- The input-importance plot is shown separately for readability.
- We do not emphasize the Belle-side FOM scan here, since the final selection discussion will use the A_{CP} -sensitivity scan.



Preselection

- K_S^0 mass window:

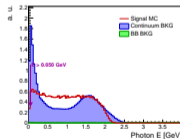
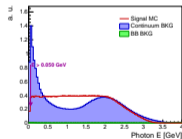
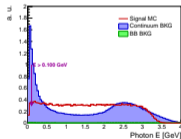
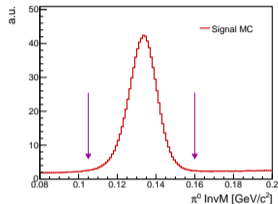
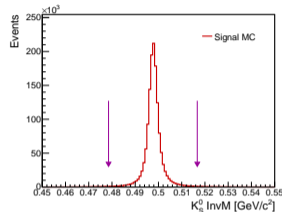
$$0.479 < \text{Inv}M_{K_S^0} < 0.517 \text{ GeV}/c^2$$

- π^0 preselection:

- $0.110 < \text{Inv}M_{\pi^0} < 0.160 \text{ GeV}/c^2$
- $E_\gamma > 0.10 \text{ GeV}$ (Forward)
- $E_\gamma > 0.05 \text{ GeV}$ (Backward/Barrel)

- B^0 window:

- $5.24 < M_{bc} < 5.30 \text{ GeV}/c^2$
- $|\Delta E| < 0.30 \text{ GeV}$

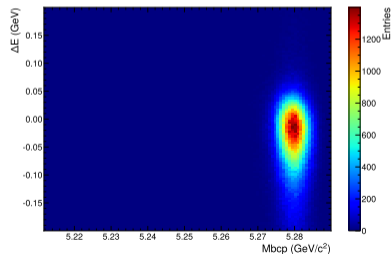
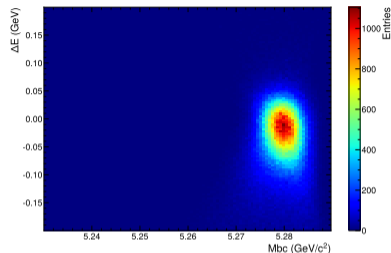


Modified M_{bc} for Nominal Fit

- The π^0 energy leakage induces a sizable ΔE - M_{bc} correlation.
- We therefore define a modified variable:

$$M'_{bc} = \sqrt{(E_{\text{beam}}^*)^2 - \left(\vec{p}_{K_S^0}^* + \frac{\vec{p}_{\pi^0}^*}{|\vec{p}_{\pi^0}^*|} \sqrt{(E_{\text{beam}}^* - E_{K_S^0}^*)^2 - m_{\pi^0}^2} \right)^2}.$$

- In signal MC, $\rho(\Delta E, M_{bc}) \approx 37.85\%$ is reduced to $\rho(\Delta E, M'_{bc}) \approx 0.38\%$.

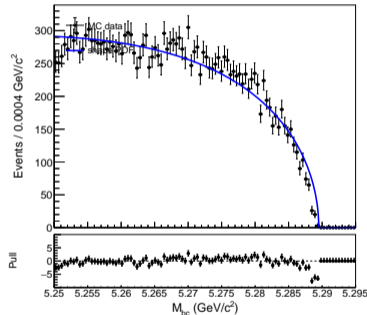


Continuum PDF: Conditional M'_{bc} Model (Detail)

Model form used in Slide 16

$$\frac{\exp\left(a_1(M'_{bc} - x_0) + a_2(M'_{bc} - x_0)^2 + a_3(M'_{bc} - x_0)^3\right)}{1 + \exp\left[(M'_{bc} - x_c)/w\right]}$$

- x_0 : pivot for stable polynomial expansion.
- a_1, a_2, a_3 : slope, curvature, and higher-order shape control.
- x_c : center of the fast-drop region.
- w : width parameter controlling the drop steepness.
- Used as the conditional M'_{bc} term in $\mathcal{P}_{q\bar{q}}(M'_{bc}, \text{CSOmu})$.



pi0 MVA: Training Variables

- pi0_InvM
- pi0_daughterAngle_0_1
- pi0_chiProb
- pi0_pt
- gamma1_clusterSecondMoment
- gamma1_beamBackgroundSuppression
- gamma1_fakePhotonSuppression
- gamma2_clusterSecondMoment
- gamma2_beamBackgroundSuppression
- gamma2_fakePhotonSuppression
- E_low (The lower energy of two photons)
- E_high (The higher energy of two photons)

Event-shape core

- R2
- thrustBm
- thrust0m
- cosTBT0
- cosTBz
- thrust
- thrustAxisCosTheta

KSFW low-order

- KSFWVariables(et)
- KSFWVariables(mm2)

CleoCone

- CleoConeCS(1), CleoConeCS(2), CleoConeCS(3)
- CleoConeCS(4), CleoConeCS(5), CleoConeCS(6)
- CleoConeCS(7), CleoConeCS(8), CleoConeCS(9)

Continuum Suppression: Training Variables

KSFW hso

- KSFWVariables(hso00)
- KSFWVariables(hso02)
- KSFWVariables(hso04)
- KSFWVariables(hso10)
- KSFWVariables(hso12)
- KSFWVariables(hso14)
- KSFWVariables(hso20)
- KSFWVariables(hso22)
- KSFWVariables(hso24)

KSFW hoo

- KSFWVariables(hoo0)
- KSFWVariables(hoo1)
- KSFWVariables(hoo2)
- KSFWVariables(hoo3)
- KSFWVariables(hoo4)

Fox-Wolfram

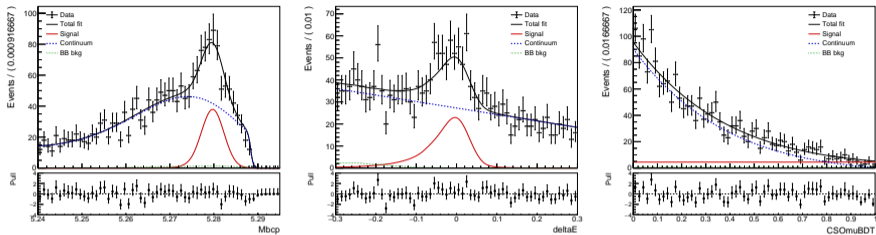
- foxWolframR1, foxWolframR2
- foxWolframR3, foxWolframR4

Flavor-tag calibration inputs in $q \cdot r$ bins

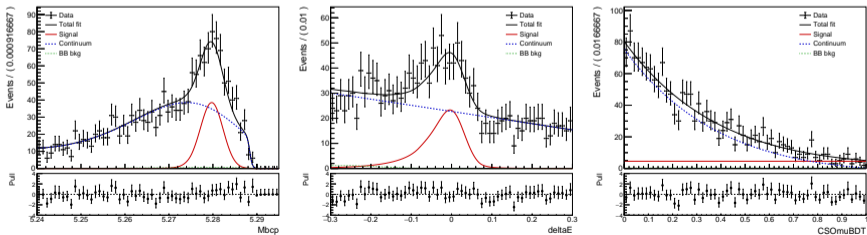
- The flavor-tag calibration parameters are taken from signal MC.

Bin	$ qr $ range	ϵ_i	ω_i	$\Delta\omega_i$	μ_i
0	[0.000, 0.100)	0.1559 ± 0.0005	0.4777 ± 0.0018	-0.0015 ± 0.0035	0.0013 ± 0.0035
1	[0.100, 0.250)	0.1569 ± 0.0005	0.4213 ± 0.0017	0.0171 ± 0.0035	-0.0008 ± 0.0035
2	[0.250, 0.500)	0.2049 ± 0.0006	0.3206 ± 0.0014	-0.0071 ± 0.0029	-0.0078 ± 0.0031
3	[0.500, 0.625)	0.1155 ± 0.0004	0.2240 ± 0.0017	0.0096 ± 0.0034	-0.0047 ± 0.0041
4	[0.625, 0.750)	0.1094 ± 0.0004	0.1645 ± 0.0016	-0.0002 ± 0.0031	-0.0018 ± 0.0042
5	[0.750, 0.875)	0.0887 ± 0.0004	0.1058 ± 0.0014	-0.0062 ± 0.0029	-0.0012 ± 0.0047
6	[0.875, 1.000]	0.1687 ± 0.0005	0.0331 ± 0.0006	-0.0035 ± 0.0012	0.0027 ± 0.0034

$q \cdot r$ -binned projections: rbin0 and rbin1

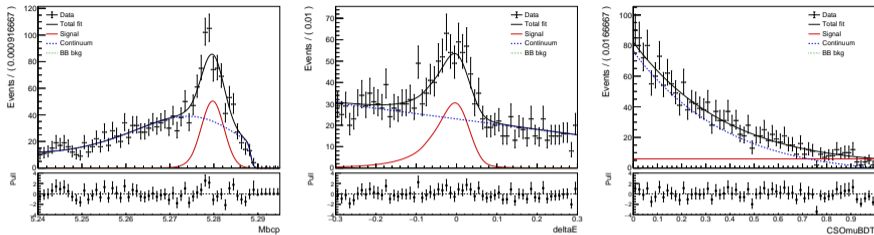


rbin0: M_{bc} / ΔE / $CS_{0\mu}BDT$

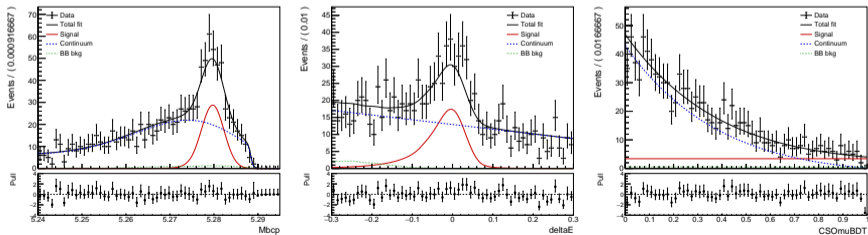


rbin1: M_{bc} / ΔE / $CS_{0\mu}BDT$

$q \cdot r$ -binned projections: rbin2 and rbin3

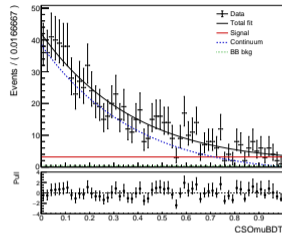
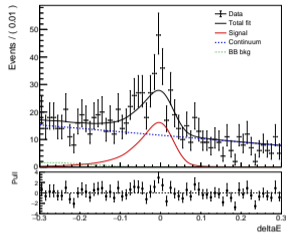
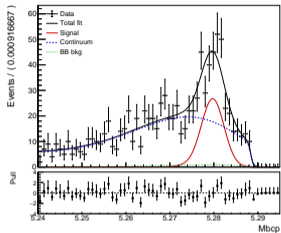


rbin2: Mbc / deltaE / CS0muBDT

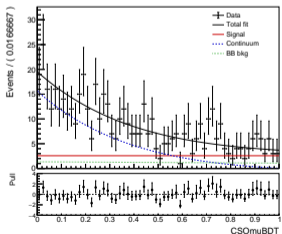
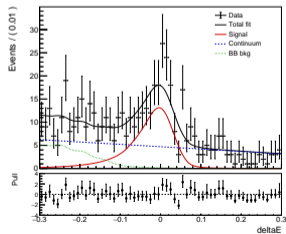
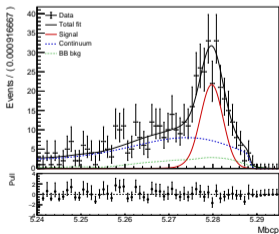


rbin3: Mbc / deltaE / CS0muBDT

$q \cdot r$ -binned projections: rbin4 and rbin5

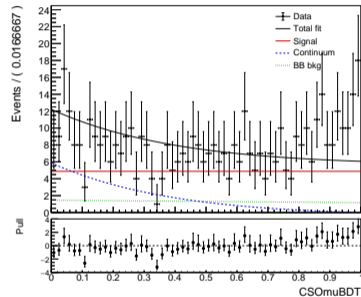
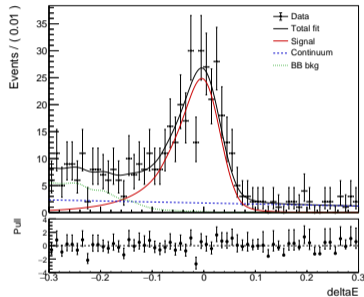
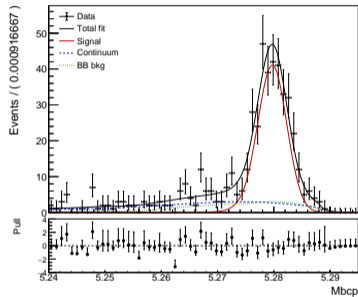


rbin4: Mbc / deltaE / CS0muBDT



rbin5: Mbc / deltaE / CS0muBDT

$q \cdot r$ -binned projections: rbin6



rbin6: Mbc π / ΔE / CS0muBDT (no paired bin)

Belle Preselection

- Summarize the Belle-side event reconstruction and the corresponding B2BII processing chain.
- Present the Belle preselection for the same decay mode and the corresponding baseline yields.
- The K_S^0 selection follows the established requirement and is not re-optimized in the Belle study.

