Scattering-Amplitude Methods in Effective Field Theory

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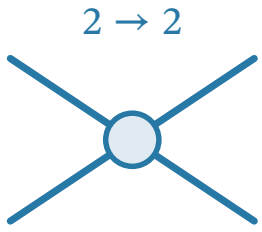
Selected works with Teng Ma

[1] *SM EFT from On-shell Amplitudes*, Chin. Phys. C 47 (2023) 023105.
[2] *On-shell operator basis for all masses and spins*, Phys. Rev. D 107 (2023) L111901.
[3] *Generic effective field theory for all masses and spins*, Phys. Rev. D 106 (2022) 116010.
[4] *The new formulation of Higgs effective field theory*, JHEP 09 (2023) 101.
[5] *An EFT hunter's guide to two-to-two scattering*, JHEP 05 (2023) 241.
[6] *An Efficient On-shell Framework for EFT Matching*, arXiv:2507.17829.
[7] *Static–Recoil Factorization in Heavy-Baryon Chiral EFTs*, arXiv:2607.12037.
[8] *Dark photons and high spin particles: complete EFT operator basis*, JHEP 07 (2025) 104.
[9] *Quantifying EFT Uncertainties in LHC Searches*, JHEP 01 (2026) 094.

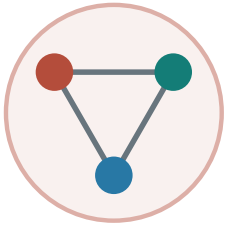
Collaborators Spencer Chang, Zi-yu Dong, HongKai Liu, Markus A. Luty, Yael Shadmi, Jing Shu, Michael Waterbury, Andrea Wulzer, Zi-Zheng Zhou.

Why effective field theory?

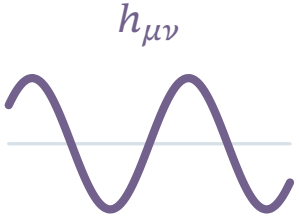
- One framework for physics at a chosen energy scale



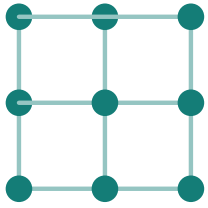
High-energy scattering



Low-energy QCD



Gravity and cosmology



Condensed matter

- Heavy new physics can leave measurable low-energy effects

Electroweak precision tests | **Dark matter searches** | **Flavor physics**

Resolve low-energy effects, constrain short-distance physics.

What defines an EFT basis?

- At fixed fields, symmetries and operator dimension, list all allowed local interactions.
- Different expressions can describe the same interaction.

Equations of motion | **Integration by parts** | **Index identities and statistics**

- IBP example: move a derivative between two factors.

$$\int d^4x\,(\partial_\mu\phi)J^\mu = -\int d^4x\,\phi\,\partial_\mu J^\mu.$$

J^μ: any local vector operator; boundary terms vanish.

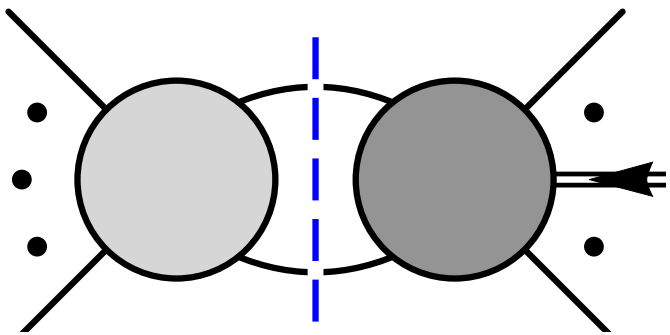
The two operators are related by IBP.

- Hilbert series count independent interactions.
  On-shell amplitudes construct explicit representatives.

B. Henning, X. Lu, T. Melia and H. Murayama, JHEP 10 (2017) 199, arXiv:1706.08520.

What on-shell amplitudes reveal

Unitarity cuts determine RG mixing



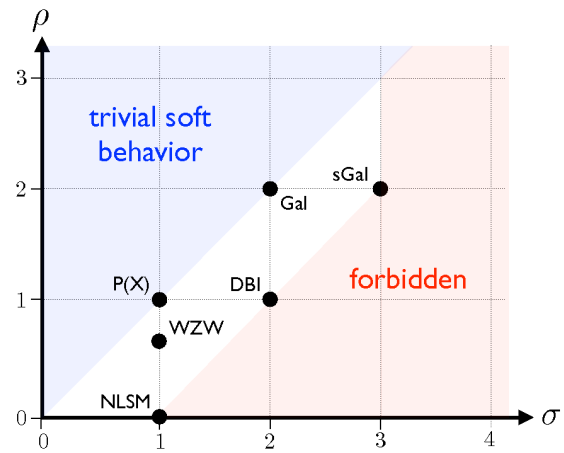
$$\frac{dc_i}{d\log\mu} = \frac{1}{16\pi^2}\sum_j \gamma_{ij} c_j$$

On-shell selection rules expose zeros in γ_{ij}.

Z. Bern, J. Parra-Martinez and E. Sawyer, JHEP 10 (2020) 211, arXiv:2005.12917.

Soft behavior classifies scalar EFTs

$$A_n \sim p^\sigma \qquad (p \to 0)$$



Consistency separates allowed and forbidden EFTs.

C. Cheung, K. Kampf, J. Novotny, C.-H. Shen and J. Trnka, JHEP 02 (2017) 020, arXiv:1611.03137.

Constructing EFTs from on-shell amplitudes

Shadmi–Weiss: scalar/vector couplings to gluons.

Ma–Shu–Xiao: massless amplitude bases ↔ independent SMEFT operators.

Y. Shadmi and Y. Weiss, JHEP 02 (2019) 165, arXiv:1809.09644.

T. Ma, J. Shu and M.-L. Xiao, Chin. Phys. C 47 (2023) 023105, arXiv:1902.06752.

On-shell kinematics and little groups

- Massive spinor and its little group (LG) $SU(2)_i$

$$(p_i)_{\alpha\dot{\alpha}} = (p_i)_\mu (\sigma^\mu)_{\alpha\dot{\alpha}} = |i^I]_{\dot{\alpha}} \langle i_I|_\alpha$$

EOM: $p_i|i^I] = m_i|i^I\rangle$

Lorentz × little group

$SU(2)_l \otimes SU(2)_r \otimes SU(2)_i$

$|i^I] \sim (1,2,2)$

$|i^I\rangle \sim (2,1,2)$

- For massless spinor, its little group is $U(1)_j$

$$|j] \to e^{-i\theta_j}|j], \qquad |j\rangle \to e^{i\theta_j}|j\rangle$$

- Minimal Lorentz scalar: spinor product

$$[ij]^{IJ} = \epsilon^{\dot{\alpha}\dot{\beta}}|i^I]_{\dot{\alpha}}|j^J]_{\dot{\beta}}, \qquad \langle ij\rangle^{IJ} = \epsilon^{\alpha\beta}|i^I\rangle_\alpha |j^J\rangle_\beta$$

- On-shell scattering amplitudes are the functions of spinor products

$$\mathcal{M}_n = \mathcal{M}_n([ij], \langle ij\rangle)$$

N. Arkani-Hamed, T.-C. Huang and Y.-t. Huang, JHEP 11 (2021) 070, arXiv:1709.04891.

Amplitudes under the little group

- A massless leg of helicity h_j fixes the amplitude's $U(1)_j$ weight:

$$\mathcal{M}(\{e^{-i\theta_j}|j],e^{i\theta_j}|j\rangle,h_j\}) = e^{-2ih_j\theta_j}\mathcal{M}(\{|j],|j\rangle,h_j\}).$$

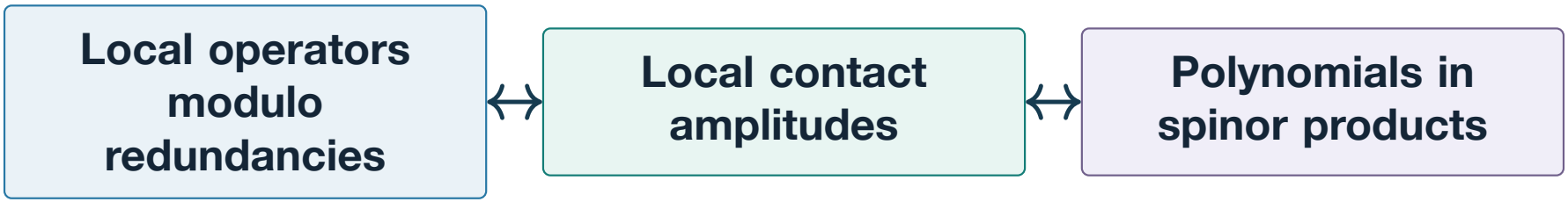
- A massive spin-s_i leg carries $2s_i$ symmetric $SU(2)_i$ indices, giving $2s_i+1$ spin states:

$$\mathcal{M}^{\{I_1\cdots I_{2s_i}\}} \longmapsto w^{I_1}{}_{J_1}\cdots w^{I_{2s_i}}{}_{J_{2s_i}}\mathcal{M}^{\{J_1\cdots J_{2s_i}\}}.$$

Little-group covariance fixes the spin structure of every allowed local amplitude.

H. Elvang and Y.-t. Huang, *Scattering Amplitudes*, arXiv:1308.1697.
N. Arkani-Hamed, T.-C. Huang and Y.-t. Huang, JHEP 11 (2021) 070, arXiv:1709.04891.

From operators to local amplitudes



- **EOM:** massless external spinors are already on shell.

$$p_i|i] = 0, \qquad p_i|i\rangle = 0.$$

- **IBP:** total derivatives become total momentum.

$$\partial_\mu \mathcal{O} \quad \longleftrightarrow \quad i\Big(\sum_i p_i\Big)_\mu \mathcal{M} = 0.$$

- **Next step:** use $U(N)$ representations to select independent spinor polynomials.

T. Ma, J. Shu and M.-L. Xiao, Chin. Phys. C 47 (2023) 023105; B. Henning and T. Melia, Phys. Rev. D 100 (2019) 016015; Y. Shadmi and Y. Weiss, arXiv:1809.09644.

Spinors and Young tableaux

- Treat the particle label $i = 1, \ldots, N$ as an auxiliary $U(N)$ index

  Representations under $SU(2)_l \times SU(2)_r \times U(N)$

Square spinor
$\tilde{\lambda}^i_{\dot{\alpha}} \equiv |i]$
$(\mathbf{1}, \mathbf{2}, \mathbf{N})$

Angle spinor
$\lambda_{i\alpha} \equiv |i\rangle$
$(\mathbf{2}, \mathbf{1}, \overline{\mathbf{N}})$

- Antisymmetric spinor products correspond to columns.

$$\begin{array}{|c|}\hline i \\ \hline j \\ \hline \end{array} = \epsilon^{\dot{\alpha}\dot{\beta}} \tilde{\lambda}^i_{\dot{\alpha}} \tilde{\lambda}^j_{\dot{\beta}} = [ij]$$

$$\begin{array}{|c|}\hline k_1 \\ \hline \cdot \\ \hline \cdot \\ \hline k_{N-2} \\ \hline \end{array} = \frac{\epsilon^{ijk_1\cdots k_{N-2}}}{(N-2)!} \langle ij \rangle$$

Two-box column: square product $[ij]$

Dual column ($N-2$ boxes): $\langle ij \rangle$

B. Henning and T. Melia, Phys. Rev. D 100 (2019) 016015, arXiv:1902.06754.

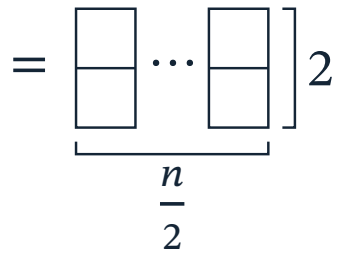
Single-chirality sectors and IBP

- IBP redundancy can be removed by $U(N)$ symmetry

Square-spinor sector

Only n right-handed spinors

$$f^{(n)}_{\text{hol}}(\{\tilde{\lambda}\}) = g_{SU(2)_r} \otimes g_{U(N)}$$

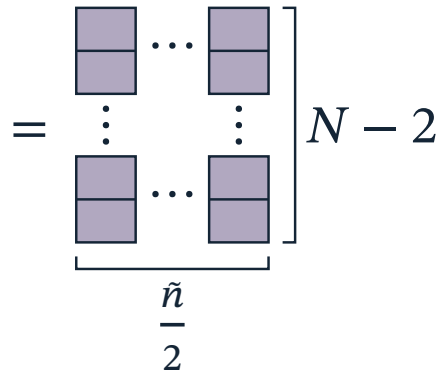


White column: square product $[ij]$

Angle-spinor sector

Only $\tilde{n}$ left-handed spinors

$$f^{(\tilde{n})}_{\text{anti-hol}}(\{\lambda\}) = g_{SU(2)_l} \otimes \tilde{g}_{U(N)}$$



Blue column: angle product $\langle ij \rangle$

- An IBP descendant contains total momentum $P = \sum_i |i]\langle i|$: it needs both chiralities. A single-chirality polynomial cannot contain this factor.

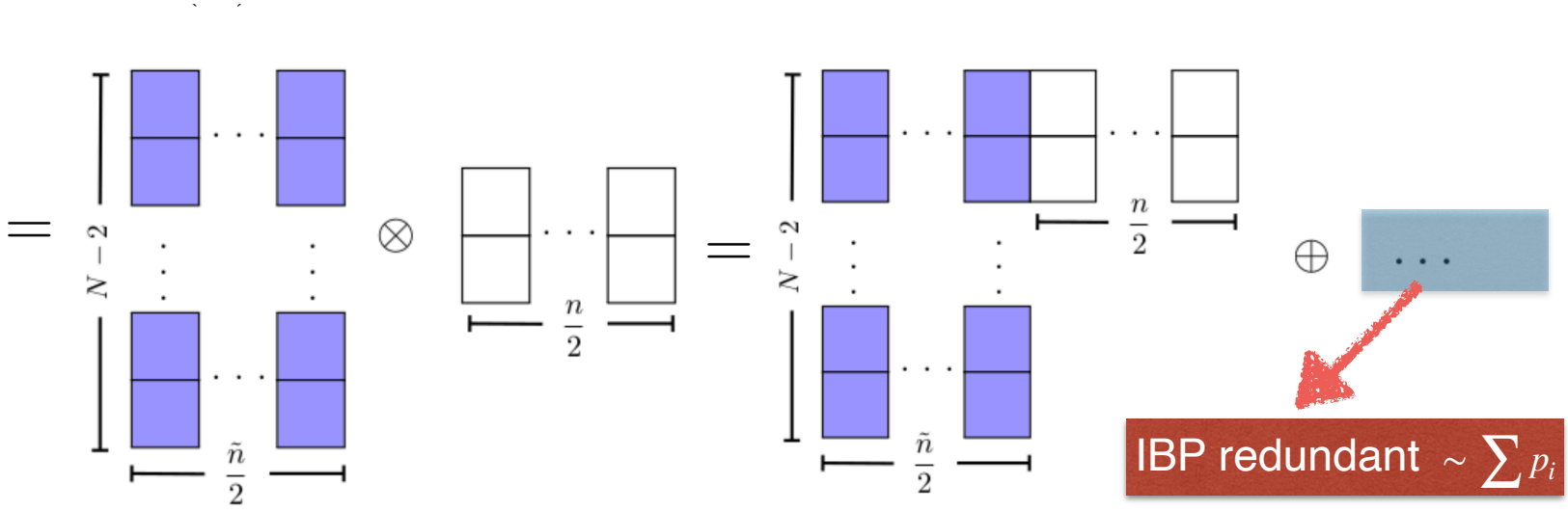
B. Henning and T. Melia, Phys. Rev. D 100 (2019) 016015, arXiv:1902.06754.

Mixed chirality: combine two sectors

- IBP redundancy can be removed by $U(N)$ symmetry

Combine n square spinors with $\tilde{n}$ angle spinors

$$f^{(n,\tilde{n})}(\{\lambda,\tilde{\lambda}\}) = f^{(n)}(\{\tilde{\lambda}\})f^{(\tilde{n})}(\{\lambda\}).$$



- Keep the first Young diagram shown above. The other terms contain total momentum and are IBP descendants.

B. Henning and T. Melia, Phys. Rev. D 100 (2019) 016015, arXiv:1902.06754.

Four-point examples: both chiralities

$N = 4$ examples

n right-handed spinors
White columns: $n/2$; height 2.

$\tilde{n}$ left-handed spinors
Blue columns: $\tilde{n}/2$; height $N - 2$.

$\psi_{1R}\psi_{2R}\psi_{3R}\psi_{4R} : (n, \tilde{n}) = (4,0)$ White column number is: $n/2 = 2$

1	3
2	4

1	2
3	4

$= [13][24] + [14][23]$

$\psi_{1L}\psi_{2L}\psi_{3L}\psi_{4L} : (n, \tilde{n}) = (0,4)$ Blue column number is: $\tilde{n}/2 = 2$

Blue column length: $N - 2 = 2$

1	3
2	4

1	2
3	4

$= \langle 13\rangle\langle 24\rangle + \langle 14\rangle\langle 23\rangle$

$\psi_{1R}\psi_{2R}\psi_{3L}\psi_{4L} : (\tilde{n}, n) = (2,2)$ Blue and white column: $n/2 = 1; \tilde{n}/2 = 1$

1	1
2	2

$= \langle 34\rangle[12]$

Massive amplitude factorization

Construct the massive spin structures and momentum dependence separately.

$$\mathcal{M}^{I}_{m,n} = \sum_{\{\dot{\alpha}\}} \underbrace{\mathcal{A}^{I}_{\{\dot{\alpha}\}}(\{\epsilon_{s_i}\})}_{\text{spin tensor}} \underbrace{G^{\{\dot{\alpha}\}}(|j], |j\rangle, p_i)}_{\text{kinematic tensor}}$$

I: physical massive-spin indices; $\dot{\alpha}$: contracted Lorentz indices.

$\mathcal{A}$: spin — Carries all massive little-group indices. Linear in each external polarization tensor.

G: kinematics — Contains momenta and massless spinors. Carries no massive little-group indices.

Construct the $\mathcal{A}$ basis and G basis separately; combining them gives the amplitude basis.

Z.-Y. Dong, **T. Ma**, and J. Shu, Phys. Rev. D 107 (2023) L111901, arXiv:2103.15837.

Constructing the spin basis

Constructing $\mathcal{A}$ reduces to decomposing the tensor product of the external polarizations' Lorentz $SU(2)_r$ representations.

$$\epsilon_{s_i} \equiv |i]^{\{I_1}_{\dot{\alpha}_1}, \ldots, |i]^{I_{2s_i}\}}_{\dot{\alpha}_{2s_i}} \sim \underbrace{\boxed{i}\cdots\boxed{i}_r}_{(2s_i)}$$

The polarization tensor is in the spin-s_i representation of the **Lorentz subgroup** $SU(2)_r$.

Lorentz $SU(2)_r$ tensor product **Irreducible representations**

$$\mathcal{A}^I \in \bigotimes_{i=1}^{m} \underbrace{\boxed{i}\boxed{\cdots}\boxed{i}}_{2s_i} \xrightarrow{\text{decompose}} \begin{array}{|c|c|c|}\hline & \cdots & \\ \hline & \cdots & \\ \hline \end{array} \cdots \oplus \cdots$$

Each irreducible representation gives an $\mathcal{A}$-basis tensor.

Z.-Y. Dong, **T. Ma**, and J. Shu, Phys. Rev. D 107 (2023) L111901, arXiv:2103.15837.

Example: $\psi\psi' Zh$ spin structures

- Labels $1, 2, 3$ denote ψ, ψ', Z; the scalar h carries no spin index.

$$\psi \sim \begin{array}{|c|}\hline 1 \\ \hline \end{array} \quad \psi' \sim \begin{array}{|c|}\hline 2 \\ \hline \end{array} \quad Z \sim \begin{array}{|c|c|}\hline 3 & 3 \\ \hline \end{array} \quad h \sim \bullet$$

$$\begin{array}{|c|}\hline 1 \\ \hline \end{array} \times \begin{array}{|c|}\hline 2 \\ \hline \end{array} \times \begin{array}{|c|c|}\hline 3 & 3 \\ \hline \end{array} \times \bullet$$

$$= \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 3 \\ \hline \end{array} \oplus \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 3 \\ \cline{1-1} \end{array} \oplus \begin{array}{|c|c|c|}\hline 1 & 3 & 3 \\ \hline 2 \\ \cline{1-1} \end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 3 \\ \hline \end{array}$$

- The first SSYT gives the following massive little-group tensor structure

$$\mathcal{A}^{I}_{[2,2]} = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 3 \\ \hline \end{array} \quad \longrightarrow \quad [1^I 3^{\{K_1}][2^J 3^{K_2\}}].$$

- The two columns give $[13]$ and $[23]$. Symmetrize the two Z indices K_1, K_2.

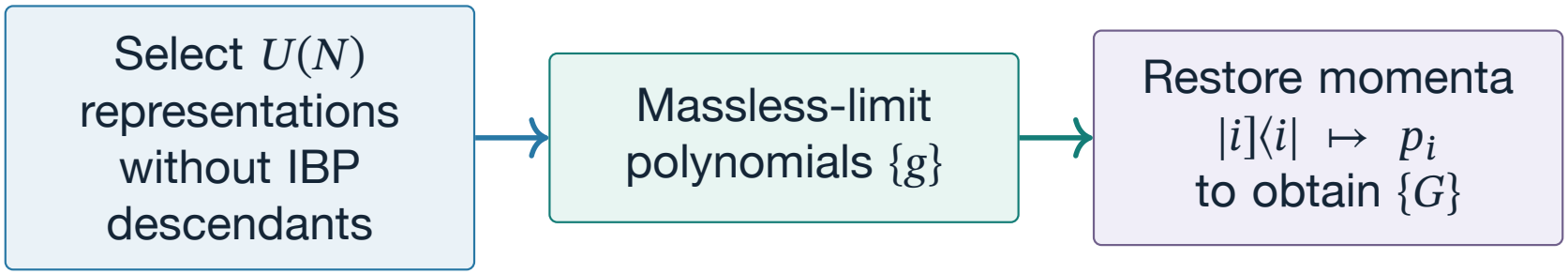
Removing kinematic redundancies

- $G(|j], |j\rangle, p_i)$ bases suffer from EOM and IBP redundancy
- EOM redundancy can be removed by first constructing $G(|j], |j\rangle, p_i)$ massless-limit bases

$$p_{i,\dot{\alpha}\alpha} \longrightarrow |i]_{\dot{\alpha}}\langle i|_{\alpha}$$

$$G(|j], |j\rangle, p_i) \longrightarrow g \equiv G(|j], |j\rangle, |i]\langle i|).$$

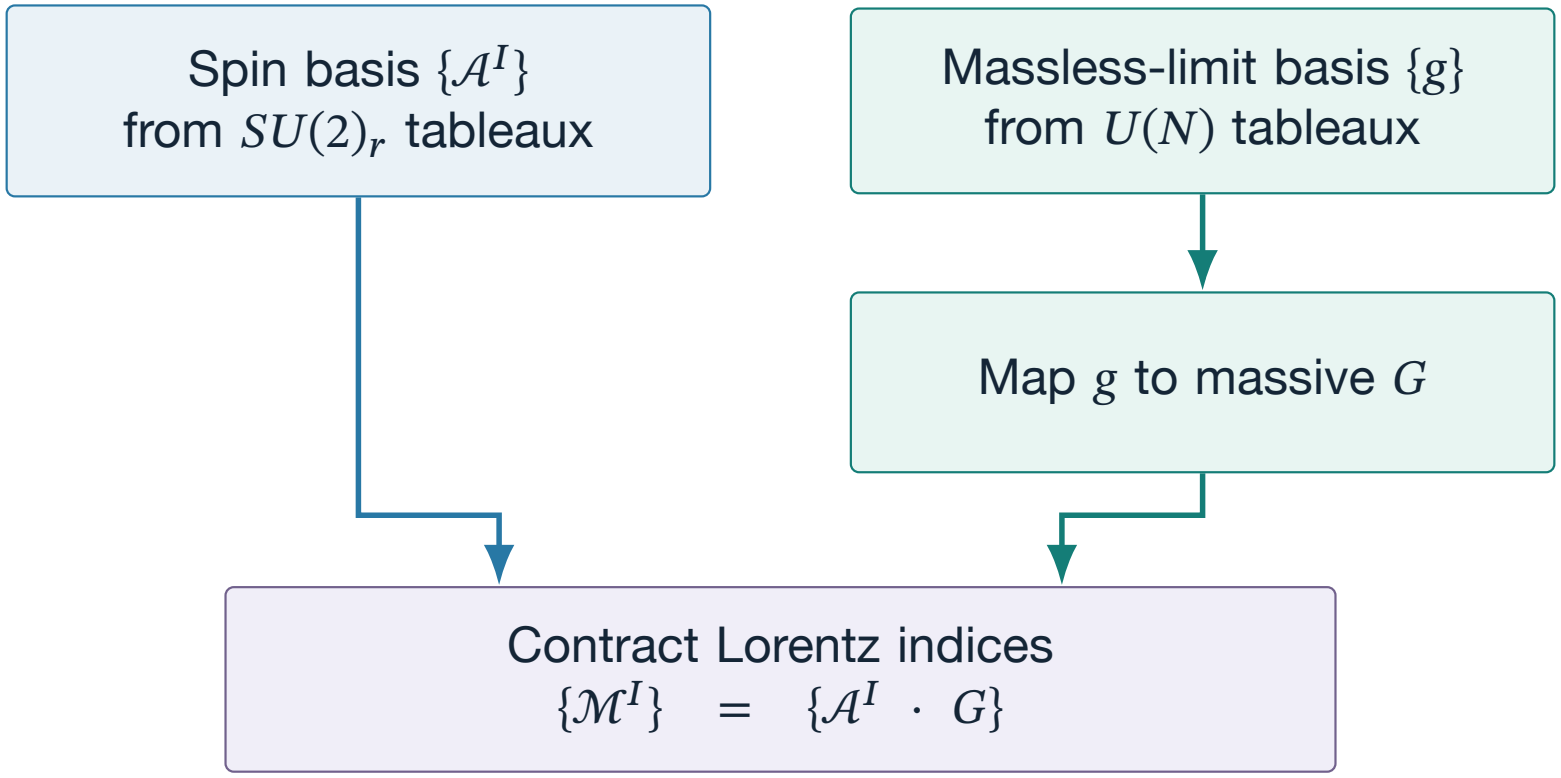
- IBP redundancy can be removed by $U(N)$ symmetry



Z.-Y. Dong, **T. Ma** and J. Shu, Phys. Rev. D 107 (2023) L111901.

Assembling independent amplitudes

- The spin and kinematic factors must carry matching Lorentz representations.



Z.-Y. Dong, **T. Ma** and J. Shu, Phys. Rev. D 107 (2023) L111901; Z.-Y. Dong et al., Phys. Rev. D 106 (2022) 116010.

Example: W^+W^-Z amplitude bases

- The Young-tableau construction gives the candidate W^+W^-Z bases

$$\mathcal{A}^I_{\{\dot\alpha\}}\left(\{\epsilon_{s_i}\}\right)\subset \begin{array}{|c|c|}\hline 1&1\\\hline\end{array}\times\begin{array}{|c|c|}\hline 2&2\\\hline\end{array}\times\begin{array}{|c|c|}\hline 3&3\\\hline\end{array}$$

$$= \begin{array}{|c|c|c|}\hline 1&1&2\\\hline 2&3&3\\\hline\end{array}\oplus\begin{array}{|c|c|c|c|}\hline 1&1&2&2\\\hline 3&3\\\cline{1-2}\end{array}\oplus\begin{array}{|c|c|c|c|}\hline 1&1&2&3\\\hline 2&3\\\cline{1-2}\end{array}\oplus\begin{array}{|c|c|c|c|}\hline 1&1&3&3\\\hline 2&2\\\cline{1-2}\end{array}$$

$$\oplus\begin{array}{|c|c|c|c|c|}\hline 1&1&2&2&3\\\hline 3\\\cline{1-1}\end{array}\oplus\begin{array}{|c|c|c|c|c|}\hline 1&1&2&3&3\\\hline 2\\\cline{1-1}\end{array}\oplus\begin{array}{|c|c|c|c|c|c|}\hline 1&1&2&2&3&3\\\hline\end{array}$$

$$\equiv \mathcal{A}^I_{[3,3]}\oplus\mathcal{A}^I_{[(4,2)^1]}\oplus\mathcal{A}^I_{[(4,2)^2]}\oplus\mathcal{A}^I_{[(4,2)^3]}$$
$$\oplus\mathcal{A}^I_{[(5,1)^1]}\oplus\mathcal{A}^I_{[(5,1)^2]}\oplus\mathcal{A}^I_{[6]}$$

- After kinematic constraints, seven independent bases remain.

$$\mathcal{A}^I_{[3,3]}\ :\ G^{\bullet}_{d=0}=1$$

$$\mathcal{A}^I_{[(4,2)^{1,2,3}]}\ :\ G^{[3]}_{d=2}=\begin{array}{|c|c|c|}\hline 1&2&3\\\hline\end{array}$$

$$\mathcal{A}^I_{[(5,1)^{1,2}]}\ :\ G^{[6]}_{d=4}=\begin{array}{|c|c|c|c|c|c|}\hline 1&1&2&2&3&3\\\hline\end{array}$$

$$\mathcal{A}^I_{[6]}\ :\ G^{[9]}_{d=6}=\begin{array}{|c|c|c|c|c|c|c|c|c|}\hline 1&1&1&2&2&2&3&3&3\\\hline\end{array}$$

Three-point kinematics

No independent momentum polynomial.

The seven survivors are spin amplitudes.

The seven three-point structures agree with Durieux et al., JHEP 12 (2020) 175 [arXiv:2008.09652].

EFT programs: HEFT + dark photons

Automated massive EFT bases

github.com/hamiguazzz/Massive

HEFT

Independent operator bases

Dark-photon EFT

Complete basis through $d = 8$

Representative operator output

A.91 Type: $ZZZZ$

A.91.1 Dimension = 4, $\mathcal{O}_4^1$

Type: $ZZZZ$ $d=4$ $\mathcal{O}_4^1$
$Z_\mu Z_\nu Z_\rho Z_\sigma \operatorname{Tr}(\sigma^\mu \bar{\sigma}^\rho) \operatorname{Tr}(\sigma^\nu \bar{\sigma}^\sigma)$

A.91.2 Dimension = 6, $\mathcal{O}_6^{1\sim4}$

Type: $ZZZZ$ $d=6$ $\mathcal{O}_6^{1\sim4}$
$Z_\mu Z_\nu Z^+_{\rho\sigma} Z^+_{\xi\tau} \operatorname{Tr}\left(\sigma^\mu \sigma^\nu \bar{\sigma}^{\xi\tau} \bar{\sigma}^{\rho\sigma}\right)$
$Z^-_{\mu\nu} Z_\rho Z_\sigma Z^+_{\xi\tau} \operatorname{Tr}\left(\sigma^\rho \sigma^{\mu\nu} \sigma^\sigma \bar{\sigma}^{\xi\tau}\right)$
$Z^-_{\mu\nu} Z^-_{\rho\sigma} Z_\xi Z_\tau \operatorname{Tr}\left(\sigma^\xi \sigma^{\mu\nu} \sigma^{\rho\sigma} \bar{\sigma}^\tau\right)$
$(D_\mu Z_\rho) Z_\sigma (D_\nu Z_\xi) Z_\tau \operatorname{Tr}\left(\sigma^\sigma \bar{\sigma}^\mu \sigma^\xi \bar{\sigma}^\tau\right) \operatorname{Tr}(\bar{\sigma}^\nu \sigma^\rho)$

A.86 Type: $ZZ\gamma^+\gamma^-$

A.86.1 Dimension = 6, $\mathcal{O}_6^1$

Type: $ZZ\gamma^+\gamma^-$ $d=6$ $\mathcal{O}_6^1$
$Z_\mu Z_\nu \gamma^+_{\rho\sigma} \gamma^-_{\xi\tau} \operatorname{Tr}\left(\sigma^\mu \sigma^{\xi\tau} \sigma^\nu \bar{\sigma}^{\rho\sigma}\right)$

A.86.2 Dimension = 8, $\mathcal{O}_8^{1\sim4}$

Type: $ZZ\gamma^+\gamma^-$ $d=8$ $\mathcal{O}_8^{1\sim4}$
$Z^-_{\mu\nu} Z^+_{\rho\sigma} \gamma^+_{\xi\tau} \gamma^-_{\zeta\eta} \operatorname{Tr}\left(\bar{\sigma}^{\rho\sigma} \bar{\sigma}^{\xi\tau}\right) \operatorname{Tr}\left(\sigma^{\mu\nu} \sigma^{\zeta\eta}\right)$
$Z^-_{\nu\rho} Z_\sigma \gamma^+_{\xi\tau} \left(D_\mu \gamma^-_{\zeta\eta}\right) \operatorname{Tr}\left(\sigma^\sigma \sigma^{\zeta\eta} \sigma^{\nu\rho} \bar{\sigma}^\mu \bar{\sigma}^{\xi\tau}\right)$
$Z_\nu Z^+_{\rho\sigma} \left(D_\mu \gamma^+_{\xi\tau}\right) \gamma^-_{\zeta\eta} \operatorname{Tr}\left(\bar{\sigma}^\mu \sigma^{\zeta\eta} \sigma^\nu\right) \operatorname{Tr}\left(\bar{\sigma}^{\rho\sigma} \bar{\sigma}^{\xi\tau}\right)$
$Z_\rho (D_\nu Z_\sigma) \gamma^+_{\xi\tau} \left(D_\mu \gamma^-_{\zeta\eta}\right) \operatorname{Tr}\left(\sigma^\rho \sigma^\mu \bar{\sigma}^\nu \sigma^{\zeta\eta} \sigma^\sigma \bar{\sigma}^{\xi\tau}\right)$

Z.-Y. Dong, **T. Ma**, C. Yang, and Z. Zhou, JHEP 07 (2025) 104, arXiv:2412.20096;
Z.-Y. Dong et al., JHEP 09 (2023) 101; Z. Dong et al., arXiv:2507.17829.

Application: SMEFT to HEFT matching

- On-shell matching maps SMEFT bases to HEFT bases and makes coefficient relations explicit

SMEFT bases $$\mathcal{A}(H^i H_k^\dagger H^j H_l^\dagger) \supset c^+_{HHHH}\frac{s_{13}}{\Lambda^2}T^{+\,ij}_{\;\;kl} + c^-_{HHHH}\frac{s_{12}-s_{14}}{\Lambda^2}T^{-\,ij}_{\;\;kl}$$

HEFT bases $$\boldsymbol{\mathcal{M}}(W^+W^-hh) = C^{00}_{6,WWhh}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$$

Just bold the spinors

$$\mathcal{A}(G^+G^-hh) = \frac{1}{2}\left(\mathcal{A}(H^1H_1^\dagger H^2 H_2^\dagger) + \mathcal{A}(H^1H_1^\dagger H_2^\dagger H^2)\right) = -\frac{c^+_{(H^\dagger H)^2} - 3c^-_{(H^\dagger H)^2}}{2}\frac{s_{12}}{2\Lambda^2}$$

$$-\frac{c^+_{(H^\dagger H)^2} - 3c^-_{(H^\dagger H)^2}}{2}\frac{s_{12}}{2\Lambda^2} \longrightarrow \frac{c^+_{(H^\dagger H)^2} - 3c^-_{(H^\dagger H)^2}}{2}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$$

An EFT hunter's guide to two-to-two scattering: HEFT and SMEFT on-shell amplitudes, H. Liu, **T. Ma**, Y. Shadmi, M. Waterbury, arXiv:2301.11349.

Nonrelativistic EFT

Heavy-baryon chiral EFT

Goal: construct the baryon EFT through $O(k^\nu)$.

$$P^\mu = Mv^\mu + k^\mu$$

Hard mass

$Mv^\mu, \quad M \sim \Lambda_\chi$

No soft power

Soft recoil

$k^\mu \sim k, \quad k \ll M$

Count powers of k

More derivatives do not always mean higher chiral order.

Separate hard mass from soft recoil *before* counting.

Z. Dong, **T. Ma**, C. Yang, and Z. Zhou, arXiv:2607.12037, Secs. 2–4.

Resolve static and recoil terms

Isolate the hard mass dependence in the heavy-pair tensor.

Group the heavy pair into channels of total angular momentum J_{12}.
In each channel, separate the heavy pair from the light sector:

$$\mathcal{A} = \sum_{J_{12},\ldots} \mathcal{H}_{J_{12},\ldots} \odot \mathcal{S}_{J_{12},\ldots}.$$

Heavy pair $\mathcal{H}$
Contains all hard M dependence.
Separate its static and recoil terms.

Light sector $\mathcal{S}$
Contains light fields and soft momenta.
Count its soft powers directly.

Count recoil in $\mathcal{H}$ together with the chiral order of $\mathcal{S}$.

$\odot$: contract the matching current indices; other channel labels are suppressed.

Z. Dong, **T. Ma**, C. Yang, and Z. Zhou, arXiv:2607.12037, Sec. 4.3.

Truncate by the actual soft order

Expand the actual current after resolving static cancellations.

$$\mathcal{H} = \underbrace{\mathcal{H}^{(0)}}_{\text{static}} + \underbrace{\mathcal{H}^{(1)} + \cdots}_{\text{recoil}}, \qquad \mathcal{H}^{(r)} = O((k/M)^r)\,.$$

Hard mass factors are factored out; $M \sim O(k^0)$.

Example: a nonzero light tensor starts at k^3.

$$\mathcal{H}^{(0)} \odot \mathcal{S}^{(3)} \sim k^3 \longrightarrow \text{Keep through } O(k^3)$$

$$\mathcal{H}^{(1)} \odot \mathcal{S}^{(3)} \sim k^4 \longrightarrow \text{Beyond } O(k^3)$$

If the static term cancels, the first nonzero recoil term sets the onset. Count only nonzero, symmetry-allowed contractions.

Keep all independent contributions with $d_\chi \le \nu$.

Z. Dong, **T. Ma**, C. Yang, and Z. Zhou, arXiv:2607.12037, Secs. 4–5.

On-shell matching: three routes

$$\sum_i C_i^{(1)} B_i = \left[\mathcal{A}_{\text{UV}}^{(1)} - \mathcal{A}_{\text{EFT}}^{(1)}[C^{(0)}]\right]_{\text{low energy, local}}$$

Direct matching [1] Expand UV amplitudes and subtract EFT terms. Project analytically or solve at on-shell points.

Unitarity cuts [2] Trees → loop masters → hard expansion. D-dimensional sewing retains rational contributions.

Dispersion [3] Scale $s_I \to z s_I$; extract a chosen low-energy order from heavy poles, cuts, and any boundary term.

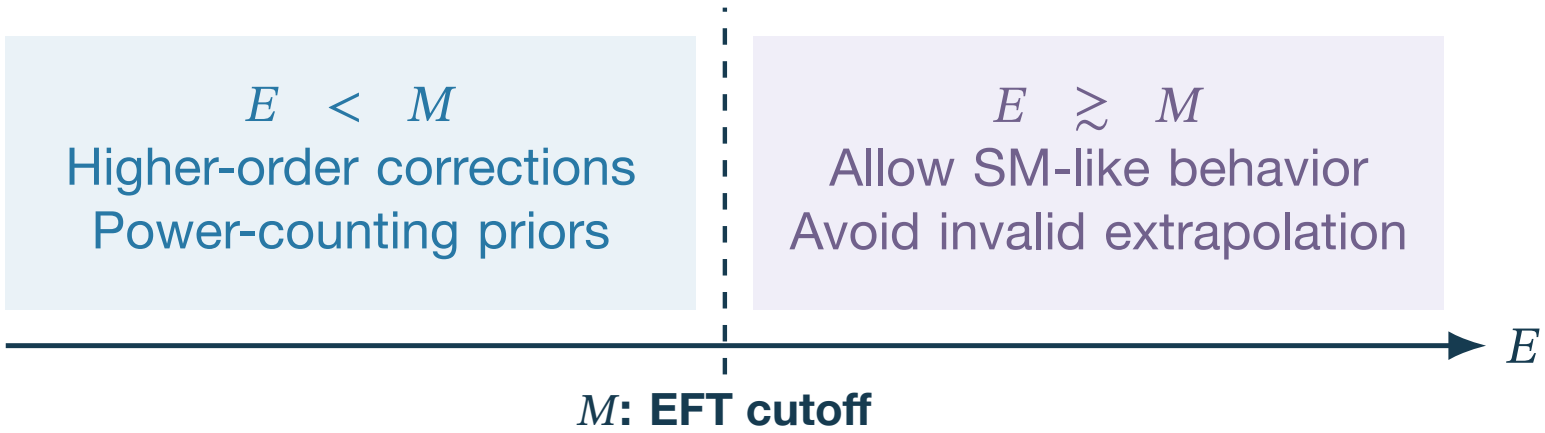
UV on-shell data → **Local hard remainder** → **Physical basis:** C_i

Same inputs, residues and IR scheme; retain rational/evanescent terms. Cuts alone do not fix all local terms.

[1] M. Chala et al., SciPost Phys. 18 (2025) 185. [2] Z. Dong, C. Li, **T. Ma** et al., arXiv:2507.17829v3 (2026). [3] S. De Angelis and G. Durieux, SciPost Phys. 16 (2024) 071.

Quantifying EFT uncertainties

High-energy EFT extrapolation can give overly strong exclusions.
Fit the Wilson coefficient together with unknown corrections.
Above the cutoff, allow the prediction to return to the SM.



Below M, parameterize the omitted terms:

$$G \longrightarrow G\,\mathcal{R}_\eta, \qquad \mathcal{R}_\eta = 1 + \eta_s \frac{\hat{s}}{M^2} + \eta_t \frac{\hat{t}}{M^2} + \cdots .$$

Fit the unknown η with power-counting priors.

Report limits on G versus M, including EFT uncertainty.

S. Chang, M. A. Luty, **T. Ma**, F. Montagno and A. Wulzer, *Quantifying EFT Uncertainties in LHC Searches*, JHEP 01 (2026) 094, arXiv:2507.15954.

Bootstrap constrains EFT coefficients

Which EFT coefficients admit a consistent UV amplitude?

$$\mathcal{A}_{\text{loc}} = \sum_{\alpha} C_\alpha B_\alpha \quad \longrightarrow \quad \text{dispersion constraints on } C_\alpha.$$

Identical stable scalars of mass m; subtract isolated light poles:

$$a_2 \equiv \frac{1}{2}\partial_s^2 \mathcal{A}(s,0)\Big|_{s=2m^2} = \frac{2}{\pi}\int_{4m^2}^{\infty} \frac{ds'\ \text{Im}\,\mathcal{A}(s',0)}{(s'-2m^2)^3} \geq 0.$$

Analyticity and crossing → **Unitarity: positive spectrum** → **Bounds on coefficients**

Assumptions: mass gap/controlled IR regulator, unitarity and sufficient high-energy boundedness. Beyond tree level, include calculable light-loop contributions.

A. Adams et al., JHEP 10 (2006) 014.

Photon EFT: why amplitude bases?

Symmetry fixes the structures; positivity constrains their coefficients.

Parity even; all momenta incoming; two independent structures at order p^4.

$$\mathcal{A}(++--) = g_2\,[12]^2\langle 34\rangle^2,$$

$$\mathcal{A}(++++) = f_2\big([12]^2[34]^2 + [13]^2[24]^2 + [14]^2[23]^2\big).$$

Operator dictionary:

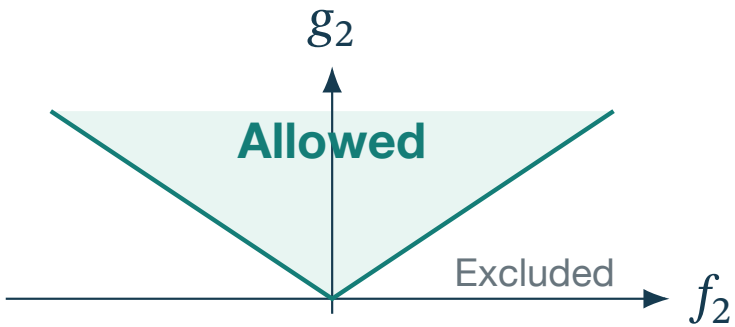
$$\mathcal{L}_8 = c_1(F_{\mu\nu}F^{\nu\mu})^2 + c_2 F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}, \qquad \begin{aligned} g_2 &= 2(4c_1 + 3c_2), \\ f_2 &= 2(4c_1 + c_2). \end{aligned}$$

Polarization positivity:

$$\boxed{g_2 \geq |f_2|}.$$

Polarizations probe g_2 and f_2 directly.

Necessary region; stronger tests can shrink it.



K. Haring et al., arXiv:2211.05795; N. Arkani-Hamed, T.-C. Huang and Y.-t. Huang, JHEP 05 (2021) 259.

Why interference matters for EFT limits

Small new-physics effects are usually probed through interference.

$$\sigma = \sigma_{\mathrm{SM}} + \underbrace{\frac{C_6}{\Lambda^2}\sigma_{\mathrm{int}}}_{\text{linear in } C_6} + \frac{1}{\Lambda^4}\left(C_6^2\sigma_{66} + C_8\sigma_{\mathrm{int},8}\right) + \cdots$$

- If interference is present, the leading correction is linear in C_6 at order $1/\Lambda^2$.
- If it vanishes, dim-6 squared and SM–dim-8 interference both enter at order $1/\Lambda^4$.

Check interference before interpreting the bound or choosing the EFT truncation.

A. Azatov, R. Contino, C. S. Machado and F. Riva, Phys. Rev. D 95 (2017) 065014, arXiv:1607.05236, Sec. III.

Helicity makes interference transparent

Only matching external-helicity components can interfere.

Local three-gluon basis for the dimension-six F^3 interaction

$$B_3(1^+,2^+,3^+) = [12][23][31], \qquad B_3(1^-,2^-,3^-) = \langle 12\rangle\langle 23\rangle\langle 31\rangle.$$

Four-gluon scattering $gg \to gg$

SM tree amplitude	$++--$ and permutations
One F^3 insertion	$++++$, $+++-$ and conjugates

All momenta outgoing; include permutations in both rows.

No shared helicity component ⇒ zero tree-level interference.

The helicity basis exposes the zero before squaring the full amplitude.

Loop corrections and additional hard radiation require separate checks.

A. Azatov, R. Contino, C. S. Machado and F. Riva, Phys. Rev. D 95 (2017) 065014, arXiv:1607.05236, Sec. II and Table I.

Summary

- **Construct independent interactions.**
  On-shell bases handle massless and massive fields systematically.
  Applications: HEFT, dark-photon, DM and CCWZ EFTs.
- **Separate scales before counting.**
  Heavy-baryon χEFT: remove hard M, then count soft recoil.
- **Match UV physics onto the basis.**
  Matching fixes coefficients; on-shell cuts determine their running.
- **Connect amplitudes to collider measurements.**
  Helicity structures expose interference; include EFT uncertainty.
- **Constrain UV consistency.**
  Dispersion and positivity restrict the physical coefficient region.

Thank you