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A singlet framework for the μ problem and type-II seesaw in a generalized NMSSM

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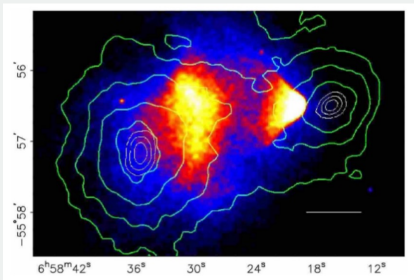
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Based on work with Yusi Pan, Fei Wang and Yang Zhang

1. Motivation and background

Dark matter and neutrino masses

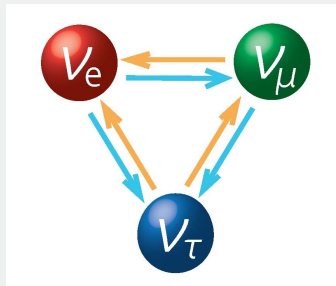
Dark matter



Cluster X-rays and lensing

Many strong evidence for dark matter.
The Standard Model has no established dark-matter particle.

Neutrino masses



Oscillations imply massive neutrinos. The Standard Model has massless neutrinos.

Supersymmetry is one of the most widely studied New Physics model.

Original defence images; cluster lensing: Clowe et al., arXiv:astro-ph/0608407.

1. Motivation and background

The μ problem and the NMSSM

MSSM: an independent μ

$$W_{\text{MSSM}} \supset \mu H_u H_d$$

The MSSM is the minimal supersymmetric standard model with two Higgs doublets.

EWSB requires μ near the soft scale. Why should an independent parameter have this size? This is the " μ problem".

NMSSM: add a singlet S

$$W_{Z_3} \supset \lambda S H_u H_d + \frac{\kappa}{3} S^3$$

$\mu_{\text{eff}} = \lambda \langle S \rangle$ in the scale-invariant model

The NMSSM adds a gauge-singlet field S. When S gets a vacuum expectation value, it generates an effective μ in the Z_3 model. This links μ to the singlet sector.

Still need to add a seesaw mechanism to explain neutrino masses.

Ellwanger et al., arXiv:0910.1785; generalized NMSSM: Ross et al., arXiv:1205.1509.

1. Motivation and background

NMSSM + Seesaw

Type-I

Cerdeño & Seto (2009) and Cao et al. (2019, 2020) studied sneutrino DM, Higgsino coannihilation and Higgs physics.

[arXiv:0903.4677](#), [1807.03762](#), [1908.07206](#)

Inverse

Cao and collaborators (2017–21) studied sneutrino DM, unitarity, direct detection and lepton $g-2$.

[arXiv:1707.09626](#), [1903.01124](#), [1912.10225](#), [2102.11355](#); comparison: [1910.14317](#)

Type-II

Li & Wang (2020) studied triplet mediation. Ouahid & Ahl Laamara (2022) studied neutrino mixing and Higgs physics.

[arXiv:2001.04155](#); Nucl. Phys. B 974 (2022) 115640

This work is inspired by the Li & Wang (2020).

1. Motivation and background

From Z_3 NMSSM to the generalized NMSSM

$$W_{Z_3} = W_{\text{Yukawa}} + \lambda S H_u H_d + \frac{\kappa}{3} S^3$$

$$W_{\text{GNMSSM}} = W_{Z_3} + \mu_0 H_u H_d + L_1 S + \frac{M_S}{2} S^2$$

The generalized model adds an explicit μ term, a linear S term and a quadratic S term. These terms break the usual Z_3 symmetry.

$$\mu_{\text{eff}} = \mu_0 + \lambda s, \quad (\mathcal{M}_{\chi^0})_{SS} = M_S + 2\kappa s, \quad s = v_S / \sqrt{2}$$

The equations give the effective μ and the singlino mass entry.

Need a large singlet VEV to bring the μ_{eff} to EW scale for μ_0 being at SUSY invariant scale.

Add the triplet fields needed for type-II seesaw.

Fixed singlet basis; constant omitted. Ellwanger et al., arXiv:0910.1785; Ross et al., arXiv:1205.1509.

2. Model and matching

Our framework: GNMSSM + type-II seesaw

Superfield	$(SU(3)_c, SU(2)_L, Y)$	Role
S	$(1, 1, 0)$	Higgs and triplet mass terms
$T, \bar{T}$	$(1, 3, +1), (1, 3, -1)$	Type-II seesaw pair

We add two electroweak triplet superfields, T and $\bar{T}$, with opposite hypercharges.

$$W \supset (\mu_0 + \lambda S) H_u H_d + (M_T^{(0)} + \Lambda_T S) \bar{T} T$$

The triplets couple to Higgs doublets, the singlet S and leptons.

$$+ \chi_u H_u \bar{T} H_u + \chi_d H_d T H_d + (\chi_l)_{ij} L_i T L_j$$

The χ_u and χ_l couplings give neutrino masses through type-II seesaw.

$$\langle S \rangle = v_S / \sqrt{2}, \quad Q = T_3 + Y$$

How the heavy triplets give small neutrino masses.

Type-II NMSSM: Li and Wang, arXiv:2001.04155. Common singlet and neutralino dark matter: this work.

Type-II seesaw matching

$M_T \gg m_{\text{light}}$: integrate out T and $\bar{T}$

$$W_{\text{EFT}} \supset \frac{1}{2} C_V^{ij} (L_i H_u) (L_j H_u), \quad C_V^{ij} = -\frac{2(\chi_l)_{ij} \chi_u}{M_T}$$

$$m_\nu = \frac{v_u^2}{2} C_V = -\frac{\chi_u v_u^2}{M_T} \chi_l$$

$$\chi_l = -\frac{M_T}{\chi_u v_u^2} U_\nu^* \text{diag}(m_1, m_2, m_3) U_\nu^\dagger$$

$$v_u = v \sin \beta, \quad v = 246.22 \text{ GeV}.$$

The triplets to be much heavier than the electroweak scale.

Integrating them out gives the Weinberg operator, which generates neutrino masses after electroweak symmetry breaking.

For sizable χ_l and χ_u , small neutrino masses favour a high triplet scale.

How can the singlet generate this large triplet mass?

Du et al., leading type-II matching; $C_H = -\gamma \frac{d\gamma}{dM}$, $T=0$ for $\gamma_d=0$.

One singlet and two mass combinations

$$\mu_{\text{eff}} = \mu_0 + \lambda s, \quad M_T = M_T^{(0)} + \Lambda_T s$$

$$s = v_S/\sqrt{2}, \quad M_T^{(0)}(M_T) = 0$$

Singlet vev $\rightarrow$ supersymmetric triplet mass; $V_F \supset |M_T|^2 (|T|^2 + |\bar{T}|^2)$.

For $|\lambda s| \gg m_{\text{SUSY}}$, keeping μ_{eff} near the soft scale requires

$$\mu_0 = -\lambda s + \mathcal{O}(m_{\text{SUSY}})$$

A large singlet expectation value can make the triplet mass large.

It also gives a large λs contribution to effective μ . We impose a cancellation with μ_0 to keep effective μ near the soft scale.

The UV origin remains open. The singlet and soft sectors must also satisfy their own conditions.

The heavy triplets may also help transmit supersymmetry breaking.

Du et al., common-scale singlet mass relations. The cancellation is an imposed boundary condition.

A possible UV picture: triplet messengers

SUSY-breaking sector $\longrightarrow$ triplet messengers $\longrightarrow$ soft masses

$$F_{\text{mess}} = \Lambda_T F_S + k_X F_X,$$

$$\Lambda_{\text{GM}} = F_{\text{mess}}/M_T$$

F_S, F_X : SUSY-breaking F -terms; k_X : messenger coupling.

$|F_{\text{mess}}| \ll |M_T|^2$: approximately supersymmetric triplet threshold

The heavy triplets may transmit supersymmetry breaking as gauge messengers.

Gluino mass needs extra coloured messengers.

We use independent soft inputs. A complete messenger model and the UV origin of the μ correlation remain open.

The low-energy neutralino sector

$$\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S})$$

The triplet multiplets have been integrated out at M_T .

Below the triplet scale, we have a bino, a neutral wino, two Higgsinos and a singlino.

We set $\kappa = 0$. The singlino diagonal mass is then controlled by M_S , while physical masses also depend on mixing.

$$(\mathcal{M}_{\chi^0})_{SS} = M_S + \sqrt{2} \kappa v_S \xrightarrow{\kappa=0} M_S$$

Bino-like

$$f_B = |N_{11}|^2 \geq 0.50$$

Singlino-like

$$f_S = |N_{15}|^2 \geq 0.50$$

We focus on bino-like and singlino-like dark matter because direct-detection limits are strong.

Physical assumptions and adopted constraints

Boundary conditions

$$10^7 \leq M_T/\text{GeV} \leq 10^{15}$$

Universal sfermion masses at M_T .

We use independent gaugino and soft inputs at the triplet matching scale.

Physical constraints

$$122 \leq m_h/\text{GeV} \leq 128$$

We apply Higgs and approximate collider bounds, flavour and leading-EFT CLFV limits, EWSB and perturbativity tests.

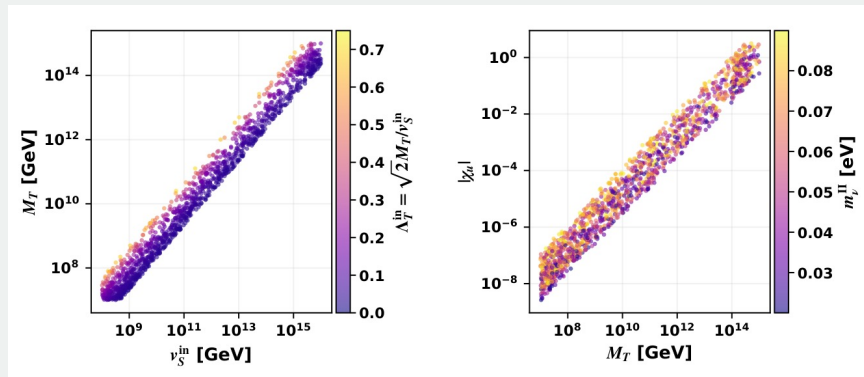
$$0 < \Omega_\chi h^2 \leq 0.144, \quad 0.096 \leq \Omega_\chi h^2 \leq 0.144 \text{ highlighted}$$

The neutralino may provide only part of dark matter.
Our abundance bound includes a theory tolerance.

Du et al., Secs. 4.1–4.2. SI/SD limits apply separately; directional tests do not give a global vacuum lifetime.

3. Neutralino phenomenology

Type-II threshold and singlet input coordinates



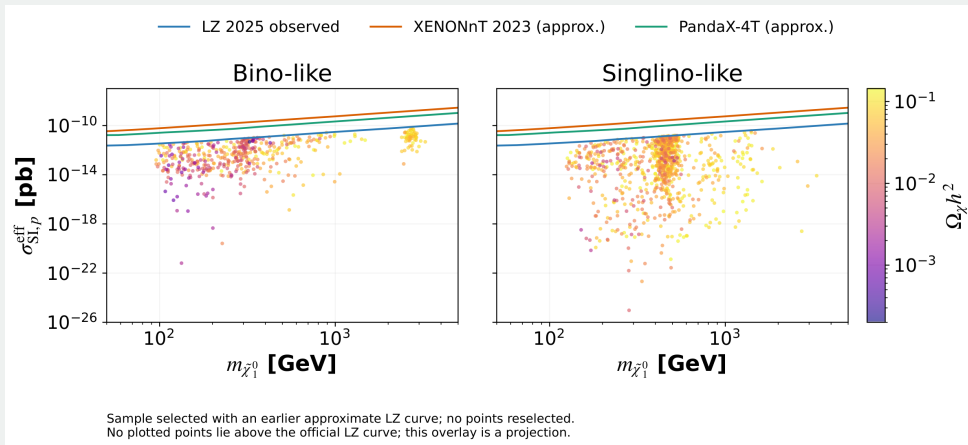
Left: the input singlet VEV against the triplet mass. Colour is the triplet-coupling input coordinate.

We evolve parameters to the triplet scale for matching.

Right: the Higgs-triplet coupling. Colour is a leading neutrino-mass diagnostic, rather than a full three-flavour spectrum.

3. Neutralino phenomenology

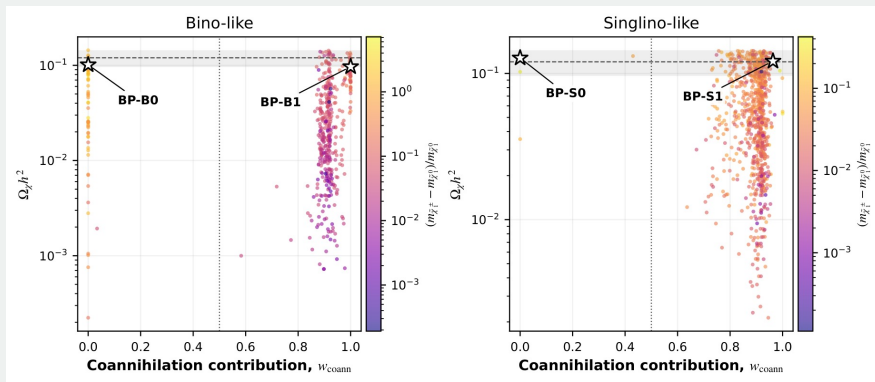
Direct detection of bino and singlino dark matter



The left panel is bino-like; the right panel is singlino-like. Colour gives Ωh^2 . The vertical axis is the proton SI cross section weighted by the relic fraction. All plotted points lie below the official observed LZ curve in this projection.

Single-nucleon projection, not a full likelihood. Original selection used an approximate LZ curve.

Coannihilation contribution and the relic abundance



BP-B0: h_2/a_1 Higgs funnel.

BP-B1: gluino-pair coannihilation.

BP-S0: thermally accessible h_3/a_2 funnel.

BP-S1: nearly degenerate wino-like partners.

The stars mark the four benchmarks. These examples lead to our conclusions.

Weakly interacting dark matter can need extra reactions to reduce its relic abundance.

The horizontal axis gives the listed coannihilation weight. Colour gives the chargino gap, with separate scales.

A zero weight is a resonance. Coannihilation assumes chemical equilibrium after check.

Conclusions and the ultraviolet question

- 1 In the selected scan, a triplet scale near 10^{15} GeV can coexist with light neutralino dark matter under our boundary conditions.
- 2 The four benchmarks illustrate different thermal mechanisms. Higgs funnels and partner reactions can reduce the relic abundance. The coannihilation interpretation assumes chemical equilibrium.
- 3 The UV origin of the μ correlation remains open. The singlet and soft sectors also need a consistent UV explanation.

Thank you for your attention.

Generalized singlet sector and field-basis freedom

$$W_S = L_1 S + \frac{M_S}{2} S^2 + \frac{\kappa}{3} S^3$$

$$\begin{aligned} S = S' + \delta : \quad \mu'_0 &= \mu_0 + \lambda \delta, \quad M_T^{(0)'} = M_T^{(0)} + \Lambda_T \delta \\ s' = s - \delta, \quad M'_S &= M_S + 2\kappa \delta, \quad L'_1 = L_1 + M_S \delta + \kappa \delta^2 \end{aligned}$$

$$I_{T\mu} = M_T^{(0)} - \frac{\Lambda_T}{\lambda} \mu_0 = M_T - \frac{\Lambda_T}{\lambda} \mu_{\text{eff}}$$

μ_{eff} and M_T are invariant. The choice $M_T^{(0)} = 0$ fixes a convenient basis.

$$B_{\mu, \text{eff}} = B_{\mu_0} + T_\lambda s + \lambda(L_1 + M_S s + \kappa s^2)$$

A singlet shift changes the separate parameters, but preserves effective μ and M_T .

Setting the explicit triplet mass to zero is a choice of basis.

The μ correlation, effective B_μ and singlet scalar sector still need a consistent UV origin.

A possible triplet-messenger interpretation

$$W \supset (M_T^{(0)} + \Lambda_T S + k_X X) \bar{T} T, \quad X = \theta^2 F_X$$

$$M_{\text{mess}} = M_T, \quad F_{\text{mess}} = \Lambda_T F_S + k_X F_X$$

$$x = |F_{\text{mess}}|/|M_T|^2 < 1, \quad x \ll 1 \text{ for SUSY threshold matching}$$

$n_a = (18/5, 4, 0)$: additional coloured messengers are required for a leading gluino mass.

Triplets can transmit SUSY breaking if their splitting is small enough for matching. They give bino and wino masses. A leading gluino mass needs coloured messengers. We use independent soft inputs. A full model must also keep the neutralino lighter than other R-odd states, including the gravitino.

Bino benchmark inputs and spectra

	BP-B0	BP-B1
Triplet scale M_t [GeV]	1.472×10^{13}	1.775×10^9
$\tan \beta$	3.164	12.06
λ ; μ effective [GeV]	0.01727 ; 2217	0.08064 ; 4342
Higgs h_1, h_2, h_3 [GeV]	124.3, 1270.6, 1564.5	125.4, 4645.4, 9402.4
Higgs $a_1, a_2, H^\pm$ [GeV]	1269.4, 3729.2, 1271.7	9402.4, 10076.4, 9402.8
Neutralinos χ_1^0, χ_2^0 [GeV]	634.5, 1569.0	3064.7, 4358.8
Light chargino [GeV]	2264.0	4358.6
Gluino [GeV]	14340	3094
Bino fraction	0.9994	0.9993
Higgsino fraction	6.076×10^{-4}	6.632×10^{-4}
Ωh^2 ; effective SI [pb]	0.1013 ; 6.591×10^{-12}	0.09674 ; 5.460×10^{-12}
LSP-pair ; coann. weight	0.994239 ; 0	0 ; 0.999970

BP-B0 is near the h_2/a_1 poles. Their individual contributions were not separated.
 BP-B1 has a nearly degenerate gluino. Its listed gluino-pair reactions dominate, assuming fast conversion. A full conversion-rate test remains open.

Higgs and sparticle masses are pole masses. Effective SI includes the neutralino relic fraction.

Singlino benchmark inputs and spectra

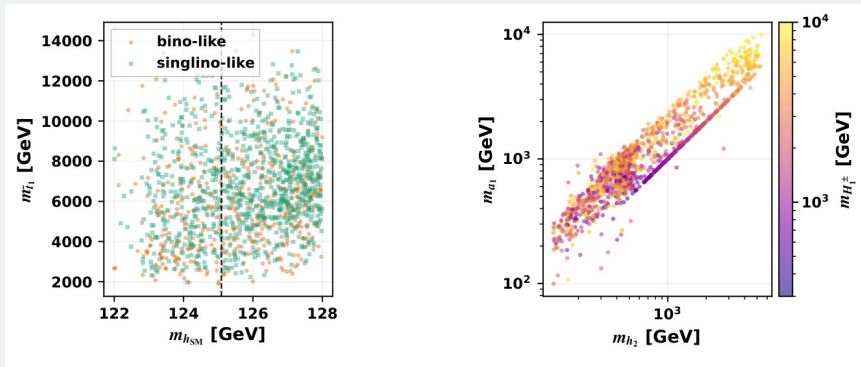
	BP-S0	BP-S1
Triplet scale M_t [GeV]	3.615×10^9	3.095×10^9
$\tan \beta$	6.072	14.44
λ ; μ effective [GeV]	0.05649 ; 484.2	0.01894 ; 4568
Higgs h_1, h_2, h_3 [GeV]	126.2, 374.4, 776.5	127.4, 1437.9, 4584.1
Higgs $a_1, a_2, H^\pm$ [GeV]	330.5, 775.7, 780.0	3275.5, 4584.1, 4584.9
Neutralinos χ_1^0, χ_2^0 [GeV]	385.1, 457.2	1436.5, 1441.7
Light chargino [GeV]	459.9	1440.7
Gluino [GeV]	10520	9934
Singlino fraction	0.9951	0.9998
Higgsino fraction	0.004241	9.542×10^{-7}
Ωh^2 ; effective SI [pb]	0.1275 ; 2.213×10^{-12}	0.1213 ; 7.704×10^{-17}
LSP-pair ; coann. weight	0.992883 ; 0	0 ; 0.963331

BP-S0 can access the h_3/a_2 poles thermally. Their individual weights are unknown.

BP-S1 has nearly degenerate wino-like partners. Their listed reactions dominate under chemical equilibrium. Zero weight means no leading listed channel.

Higgs and sparticle masses are pole masses. Effective SI includes the neutralino relic fraction.

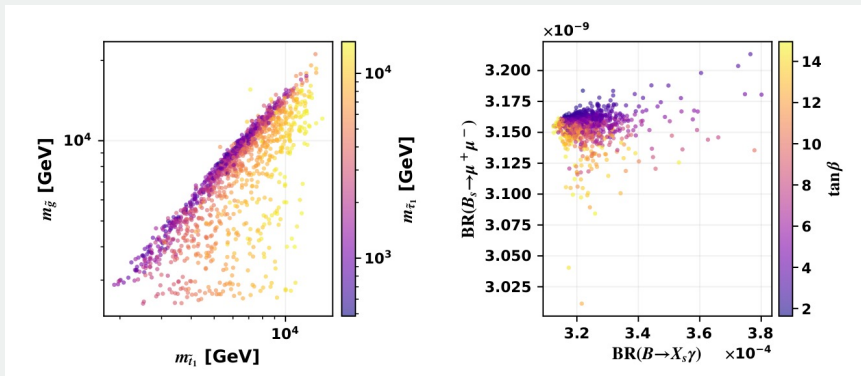
Higgs and stop spectra



Left: the SM-like Higgs mass and lighter-stop mass.
Right: extra CP-even and CP-odd Higgs masses.
Colour gives the charged-Higgs mass.

These solutions satisfy our chosen Higgs conditions. The directional tests do not give a complete vacuum lifetime.

Coloured-superpartner and flavour observables



Left: lighter-stop and gluino masses. Colour gives the lighter-stau mass.

Right: two rare B-meson decay rates. Colour gives $\tan \beta$.

The points pass our adopted flavour and approximate collider bounds. A detailed collider interpretation depends on production and decays.

Related work and the present framework

NMSSM background

Ellwanger et al. (2010) review singlet Higgs physics, effective μ and neutralino phenomenology.

[arXiv:0910.1785](#)

SUSY type-II

Rossi (2002) studied type-II neutrino mass and lepton-flavour violation.

[arXiv:hep-ph/0207006](#)

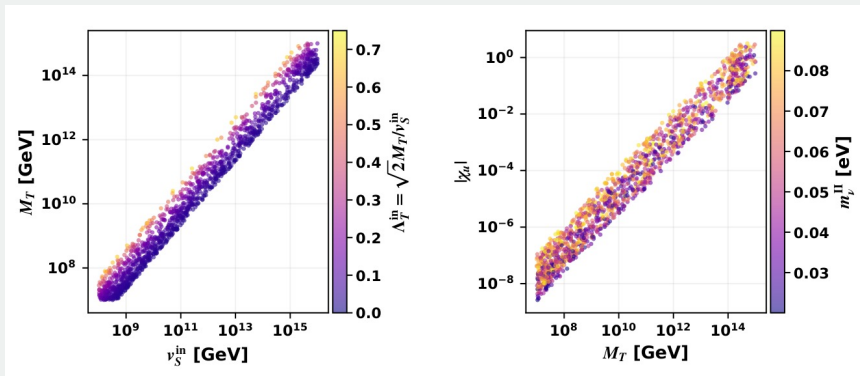
NMSSM + type-II

Li and Wang (2020) linked the extension to SUSY-breaking mediation. Joaquim and Rossi studied triplet messengers.

[arXiv:2001.04155](#); [hep-ph/0604083](#)

Here we study neutralino dark matter below a high triplet threshold, with a correlated Higgs bilinear and independent soft inputs.

Type-II threshold and singlet input coordinates

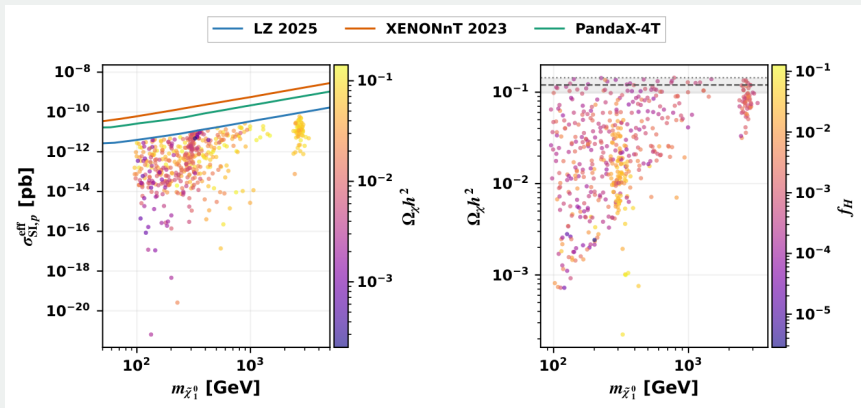


Left: the input singlet VEV against the triplet mass. Colour is the triplet-coupling input coordinate.

We evolve parameters to the triplet scale for matching.

Right: the Higgs-triplet coupling. Colour is a leading neutrino-mass diagnostic, rather than a full three-flavour spectrum.

Bino dark matter: scattering and relic abundance

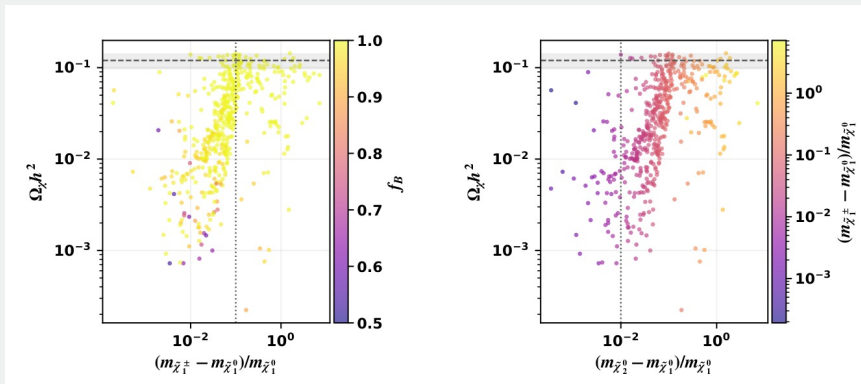


Left: the proton SI cross section weighted by the relic fraction.

Right: relic abundance. Colour gives the Higgsino fraction. The shaded band includes our theory tolerance.

These are the earlier approximate reference curves. The main slide uses official observed LZ data.

Bino dark matter: electroweakino mass gaps



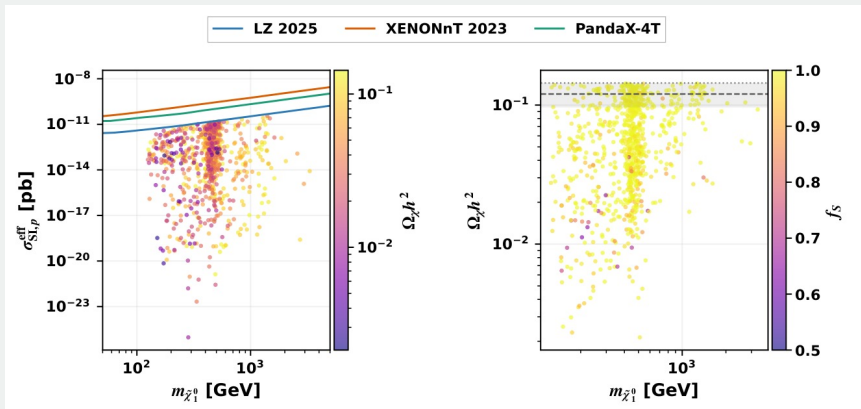
The panels show the gaps to the lightest chargino and second neutralino.

A small gap suggests a possible thermal partner. Even if these gaps are large, a gluino or sfermion may contribute.

We must also check the reaction channels to identify the thermal mechanism.

Du et al., Fig. 3, lower row. Left colour: bino fraction; right colour: chargino relative gap.

Singlino dark matter: scattering and relic abundance

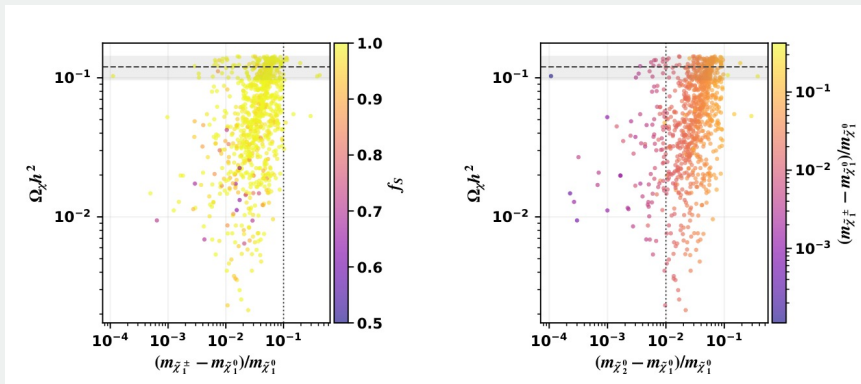


Left: the abundance-weighted proton SI cross section.

Right: relic abundance.
Colour gives the singlino fraction.

The adopted abundance range allows very small signals.
Mixing and couplings also matter. This backup uses the earlier approximate LZ curve.

Singlino dark matter: electroweakino mass gaps



Left and right show the chargino and second-neutralino gaps separately.

The nearest gap is the smaller of the two. In the adopted abundance band, its median is 4.7% for singlino and 12.6% for bino.

A small gap alone does not prove chemical equilibrium or fix reaction weights.

Du et al., Fig. 4, lower row. Left colour: singlino fraction; right colour: chargino relative gap.