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Lattice QCD calculation of $\Xi^- \rightarrow \Lambda \ell^- \bar{\nu}_\ell$ semileptonic decay

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Wuhan · The 12th China LHC Physics Conference (CLHCP2026)

Outline

1. Motivation

2. Theoretical framework

3. Numerical results

4. Summary

CKM unitarity

-3-

PDG 2024:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

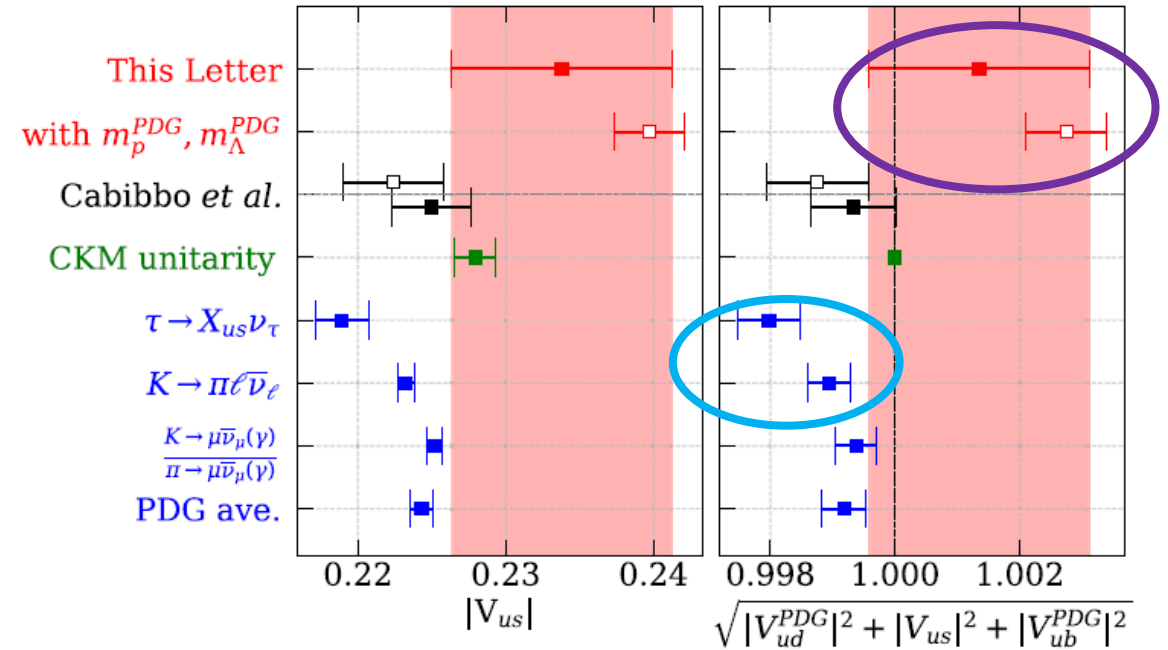
First row: 10^{-4} accuracy

$$|V_{ud}| = 0.97367 \pm 0.00032 \quad 0.03\%$$

$$|V_{us}| = 0.22431 \pm 0.00085 \quad 0.4\%$$

$$|V_{ub}| = 0.00382 \pm 0.00020 \quad 5\%$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0007$$

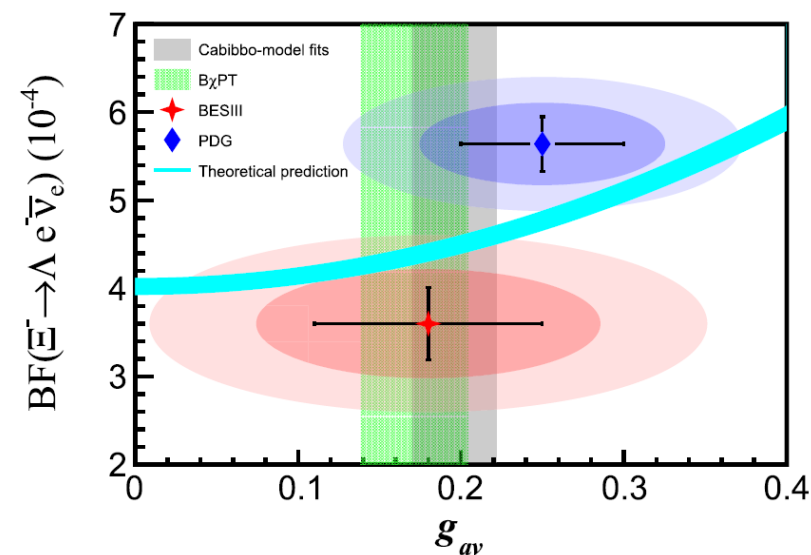


$\Lambda \rightarrow p \ell^- \bar{\nu}_\ell$: Lattice: S. Bacchio, *et.al.* Phys.Rev.Lett. 135 (2025) 231901
BESIII: M.Ablikim, *et.al.* Nature 657 (2026) 8130, 92-97

$|V_{us}|$ of τ and K decay : differ by more than 2σ .

Hyperon decays provide an independent tests of **CKM unitarity**

Experiment measurement



BESIII: M.Ablikim, *et.al.* Phys.Rev.Lett. 137, 091801

Branching ratio of $\Xi^- \rightarrow \Lambda e \bar{\nu}_e$:

BESIII: $(3.60 \pm 0.41) \times 10^{-4}$

PDG2024: $(5.63 \pm 0.51) \times 10^{-4}$

3.9 σ

Phenomenological calculation -4-

SU(3) symmetry, Quark model, QCD sum rule ...

F. Schlumpf, Phys. Rev. D 51, 2262 (1995)
A. Faessler, et.al. Phys. Rev. D 78, 094005 (2008)
R.M. Wang, *et.al.* Phys. Rev. D 100, 076008 (2019)
S.Q. Zhang, *et.al.* JHEP 06 (2024) 122
R.H. Ni, *et.al.* Phys. Rev. D 114 (2026) 074005

$Br \times 10^{-4}$	$\Xi^- \rightarrow \Lambda e \bar{\nu}_e$	$\Xi^- \rightarrow \Lambda \mu \bar{\nu}_\mu$
SU(3)	5.47(15)	1.58(4)
LFQM	4.85	1.31
CCQM	5.49	1.57
QCDSR	5.17(73)	1.54(23)
R3QM	4.41	1.24

First-principles calculations are necessary.

Outline

1. Motivation

2. Theoretical framework

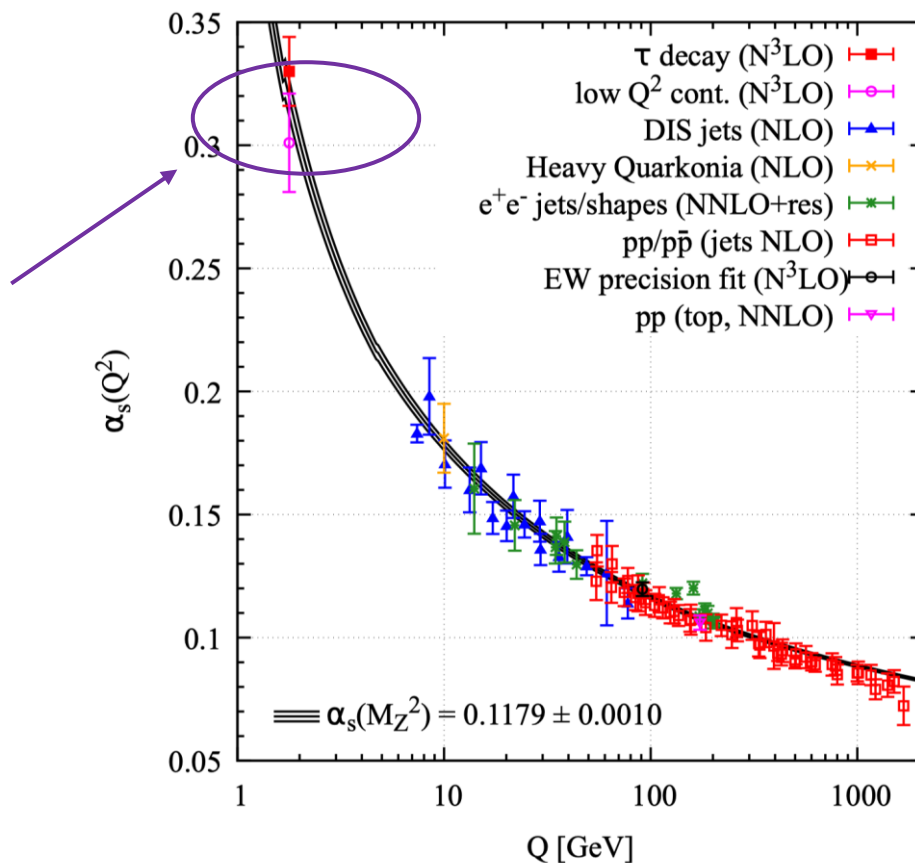
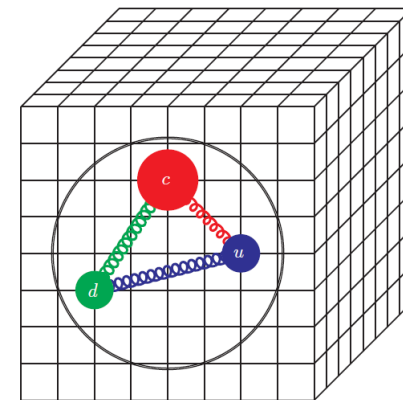
3. Numerical results

4. Summary and outlook

Hyperon decay: non-perturbative QCD

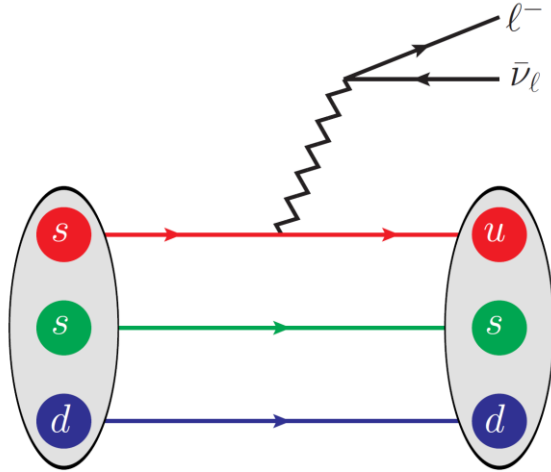
6

- Around the proton radius scale (~ 1 GeV), QCD cannot be solved analytically, and the theoretical calculation at this scale is one of the seven Millennium Prize Problems.
- Hyperon decay is just on the region where the transition from perturbative to nonperturbative occurs.



Siyuan @ TDLi

Hyperon decay is suitable for lattice QCD simulations.



Differential decay width:

$$\frac{d\Gamma(B_1 \rightarrow B_2 l \nu_l)}{dq^2} = \frac{G_F^2 |V_{us}|^2 q^2 \sqrt{Q_+ Q_-}}{384 \pi^3 M_\Xi^3} L^{\mu\nu} \underline{H_{\mu\nu}}$$

Hadronic part

$$H_{\mu\nu} = M_\mu^{V-A} M_\nu^{V-A}$$

$$\underline{M_\mu^{V-A}} = \langle \Lambda(p_f, s_f) | \bar{u} \gamma_\mu (1 - \gamma_5) s | \Xi(p_i, s_i) \rangle$$

Matrix element

Weinberg form factors:

$$\langle \Lambda(p_f, s_f) | \bar{u} \gamma_\mu s | \Xi^-(p_i, s_i) \rangle$$

$$= \bar{u}(p_f, s_f) \left(\gamma_\mu \textcolor{red}{f}_1(q^2) + i \sigma_{\mu\nu} q^\nu \frac{\textcolor{red}{f}_2(q^2)}{m_i} + q_\mu \frac{\textcolor{red}{f}_3(q^2)}{m_i} \right) u(p_i, s_i),$$

$$\langle \Lambda(p_f, s_f) | \bar{u} \gamma_\mu \gamma_5 s | \Xi^-(p_i, s_i) \rangle$$

$$= \bar{u}(p_f, s_f) \left(\gamma_\mu \textcolor{red}{g}_1(q^2) + i \sigma_{\mu\nu} q^\nu \frac{\textcolor{red}{g}_2(q^2)}{m_i} + q_\mu \frac{\textcolor{red}{g}_3(q^2)}{m_i} \right) \gamma_5 u(p_i, s_i).$$

Axial charge $\textcolor{green}{g}_A^H$: $\textcolor{green}{g}_A^H = f_1^{SU(3)} * \textcolor{blue}{g}_{av}$

Axial-vector coupling: $\textcolor{blue}{g}_{av} = g_1(0)/f_1(0)$

Vector form factor in SU(3) limit: $f_1^{SU(3)} = -\sqrt{3/2}$



Helicity form factor

$$\begin{aligned}
& \langle \Lambda(p_f, s_f) | \bar{u} \gamma_\mu s | \Xi^-(p_i, s_i) \rangle \\
&= \bar{u}(p_f, s_f) \left[(m_i - m_f) \frac{q^\mu}{q^2} f_0(q^2) \right. \\
&+ \frac{m_i + m_f}{s_+} \left(p_i^\mu + p_f^\mu - \frac{q^\mu}{q^2} (m_i^2 - m_f^2) \right) f_+(q^2) \\
&+ \left. \left(\gamma^\mu - \frac{2m_f}{s_+} p_i^\mu - \frac{2m_i}{s_+} p_f^\mu \right) f_\perp(q^2) \right] u(p_i, s_i)
\end{aligned}$$

$$\begin{aligned}
& \langle \Lambda(p_f, s_f) | \bar{u} \gamma_\mu \gamma_5 s | \Xi^-(p_i, s_i) \rangle \\
&= -\bar{u}(p_f, s_f) \left[(m_i + m_f) \frac{q^\mu}{q^2} g_0(q^2) \right. \\
&+ \frac{m_i - m_f}{s_-} \left(p_i^\mu + p_f^\mu - \frac{q^\mu}{q^2} (m_i^2 - m_f^2) \right) g_+(q^2) \\
&+ \left. \left(\gamma^\mu + \frac{2m_f}{s_-} p_1^\mu - \frac{2m_i}{s_-} p_2^\mu \right) g_\perp(q^2) \right] u(p_i, s_i)
\end{aligned}$$

Differential decay width:

$$\begin{aligned}
\frac{d\Gamma}{dq^2} &= \frac{G_F^2 |V_{us}|^2 \sqrt{s_+ s_-}}{384 \pi^3 m_\Xi^3} \left(1 - \frac{m_\ell^2}{q^2} \right)^2 \\
&\times \left\{ 2(m_\ell^2 + 2q^2)(s_+ g_\perp^2 + s_- f_\perp^2) \right. \\
&+ \frac{m_\ell^2 + 2q^2}{q^2} (s_+ [(m_\Xi - m_\Lambda) g_+]^2 \\
&+ s_- [(m_\Xi + m_\Lambda) f_+]^2) \\
&+ \left. \frac{3m_\ell^2}{q^2} (s_+ [(m_\Xi - m_\Lambda) f_0]^2 + s_- [(m_\Xi + m_\Lambda) g_0]^2) \right\}.
\end{aligned}$$

Differential decay width expressed as helicity form factor.

Correlation function in Lattice QCD

Interpolator current of the baryons:

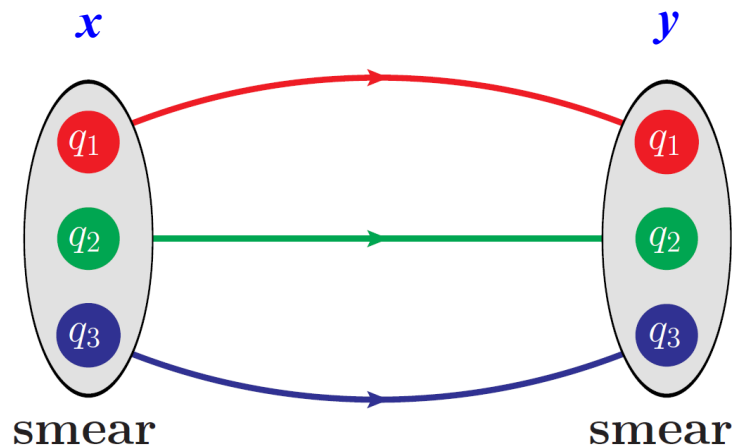
$$O_{\Xi^-} = \epsilon_{ijk} (d_i^T C \gamma_5 s_j) P_+ s_k,$$

$$O_{\Lambda} = \epsilon_{ijk} (u_i^T C \gamma_5 d_j) s_k,$$

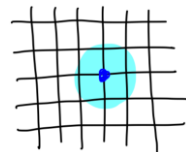
Weak current:

$$J_{\Gamma} = \bar{u} \gamma_{\mu} (1 - \gamma_5) s$$

Two point correlation functions:



$$C_{2pt}(t) = \sum_x e^{-ip \cdot x} \text{Tr} \{ \langle 0 | O_B(x), \bar{O}_B(0) | 0 \rangle T \}$$

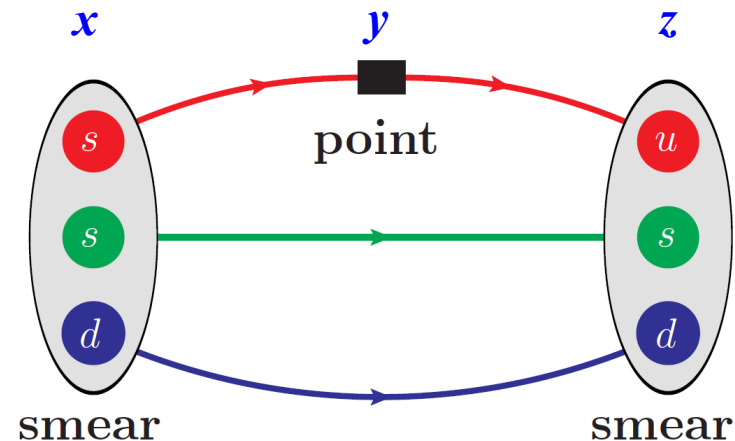


smear

point

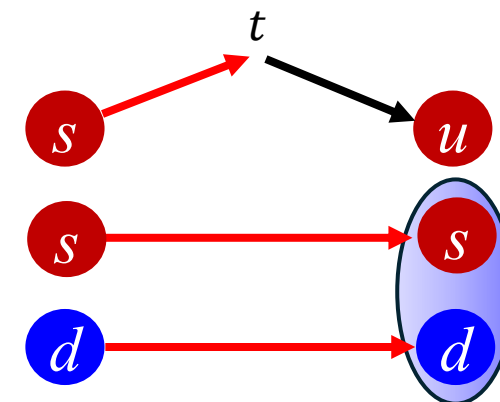
Three point correlation functions:

-9-



$$C^{3pt}(t, t_{seq}, \vec{p}_f, \Gamma, T) = \sum_{\vec{x}, \vec{y}} e^{-i\vec{p}_f \cdot \vec{x}} e^{-i\vec{p}_i \cdot \vec{y}} \times \text{Tr} \{ \langle O_{\Lambda}(t_{seq}, \vec{y}), J_{\Gamma}(t, \vec{x}), O_{\Xi}(0) \rangle T \}$$

Sequential source method:



Sequential time: t_{seq}

Lattice set-up

-10-

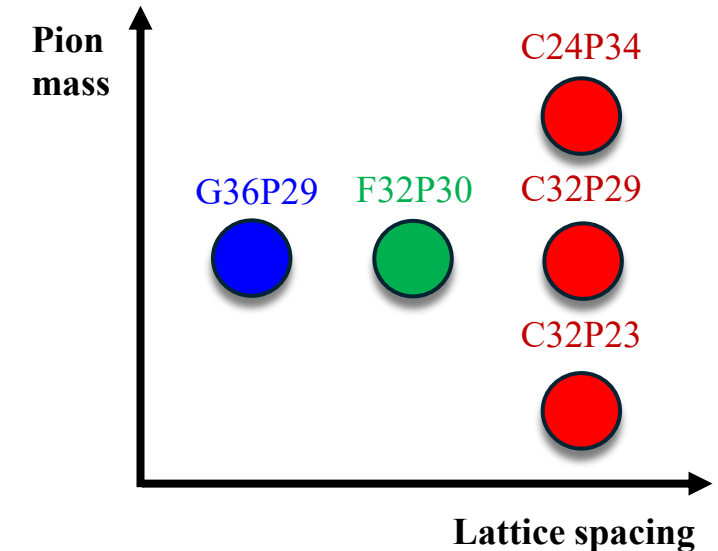
Ensembles from CLQCD Collaboration:

Lattice spacing: $[0.069, 0.1052]$ fm

Pion mass : $[227.9, 340.2]$ MeV

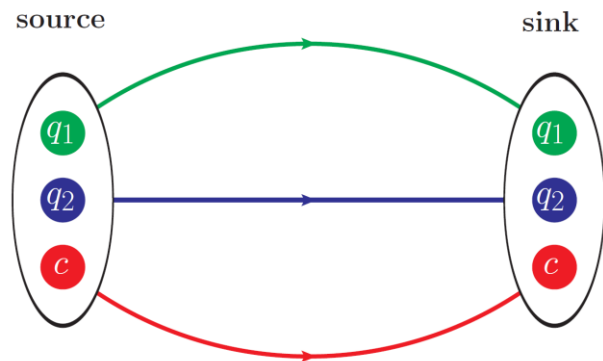


Ensemble	$L^3 \times T$	a (fm)	smear size	π (MeV)	Nconf	Nsrc
C24P34	$24^3 \times 64$	0.105	4 30	340	300	34
C32P29	$32^3 \times 64$	0.105	4 30	290	984	4
C32P23	$32^3 \times 64$	0.105	4 30	230	954	11
F32P30	$32^3 \times 96$	0.077	5 50	300	777	6
G36P29	$36^3 \times 108$	0.069	5 50	290	656	4

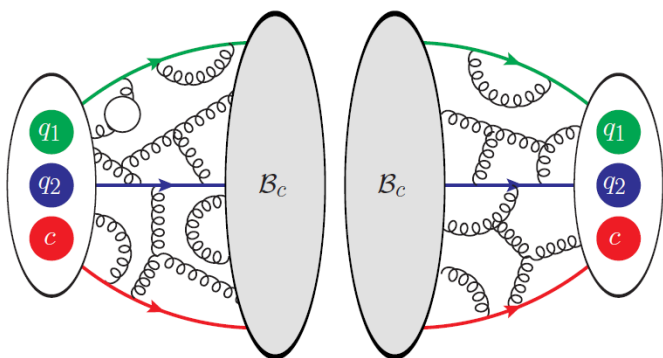


Extrapolation: Continuum limit and physical pion mass.

Two point correlation function



$$C_{2pt}(t) = \sum_x e^{-ip \cdot x} \text{Tr}\{\langle 0 | O_B(x), \bar{O}_B(0) | 0 \rangle T\}$$



$$C_{2pt}(t) = \underbrace{Z_B^{(0)2}}_{\text{Decay constant}} e^{-\underbrace{m_B t}_{\text{Effective mass}}} (1 + \Delta C \cdot e^{\Delta m \cdot t})$$

Decay constant Effective mass

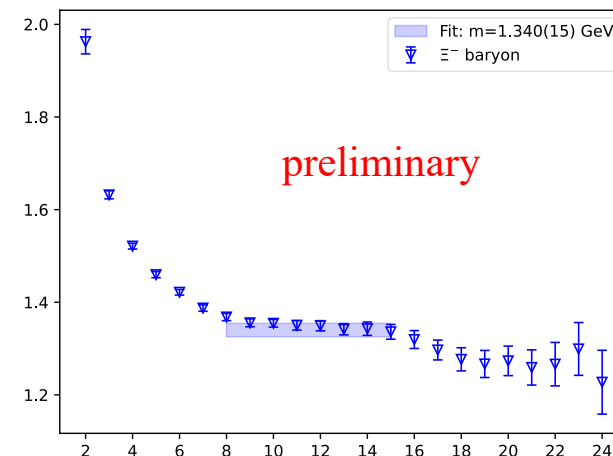
L.Y. Li, *et al.* Phys. Rev. D 113 054509

$$\mathbb{1} = \frac{1}{V} \sum_n \frac{1}{2E_n} |n\rangle \langle n|$$

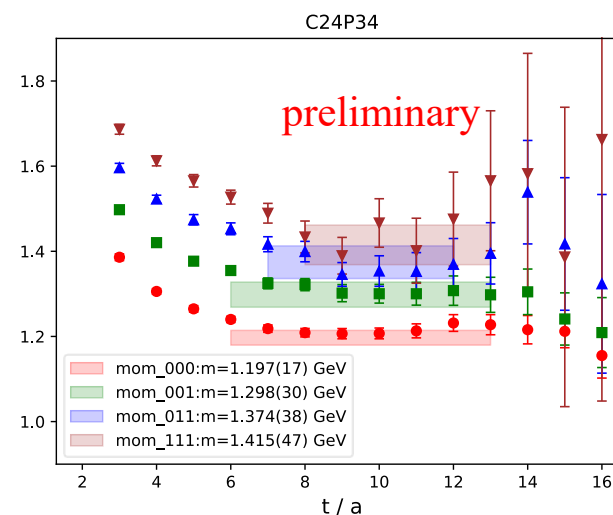
Complete set of hadronic states

Ensembles: C24P34

-11-



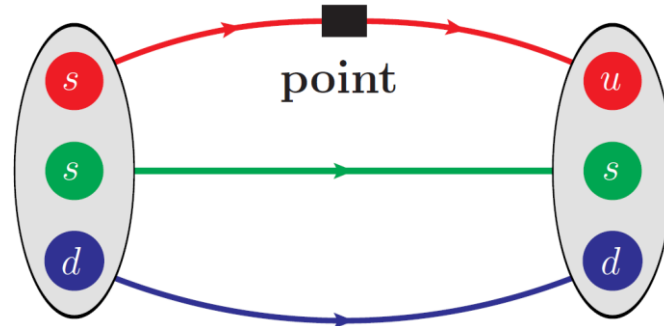
Ξ^- - baryon effective mass



Λ - baryon effective energy

Three point correlation functions

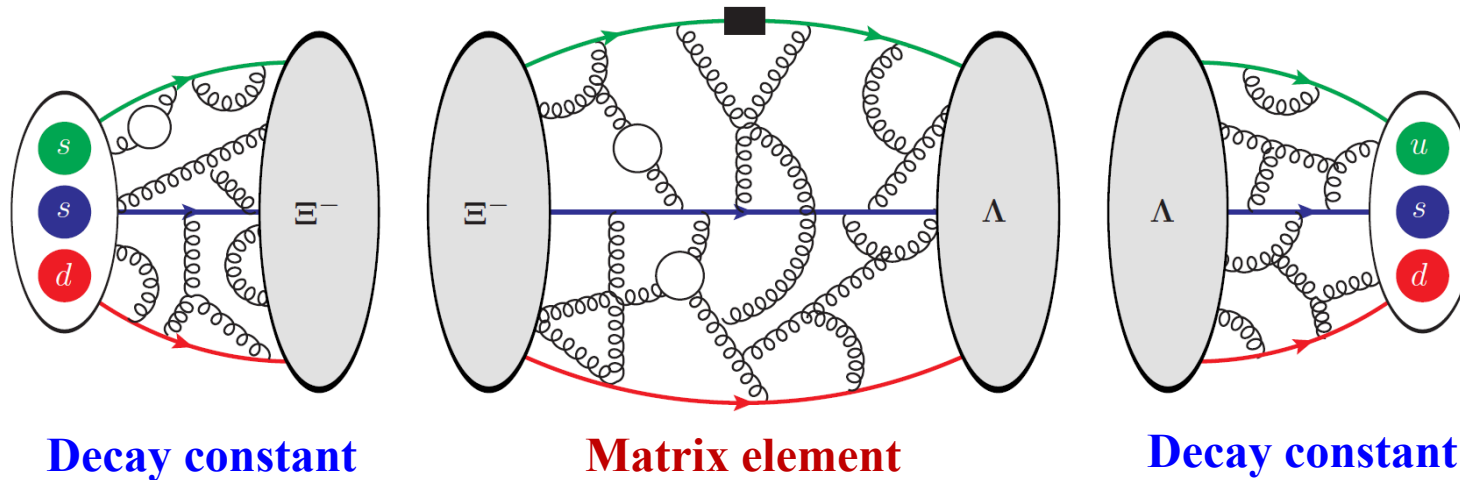
-12-



$$C^{3\text{pt}}(t, t_{\text{seq}}, \vec{p}_f, \Gamma, T) = \sum_{\vec{x}, \vec{y}} e^{-i\vec{p}_f \cdot \vec{x}} e^{-i\vec{p}_i \cdot \vec{y}} \text{Tr} \{ \langle O_\Lambda(t_{\text{seq}}, \vec{y}), J_\Gamma(t, \vec{x}), O_\Xi(0) \rangle T \}$$

$$1 = \frac{1}{V} \sum_n \frac{1}{2E_n} |n\rangle \langle n|$$

Complete set of
hadronic states



Ratio of 3pt/2pt:

$$R(\Gamma, T, \vec{p}_f, t, t_{\text{seq}}) = \sqrt{\frac{C^{3\text{pt}}(t, t_{\text{seq}}, \vec{p}_f, \Gamma, T) C^{3\text{pt}}(t, t_{\text{seq}} - t, \vec{p}_f, \Gamma, T)}{C_\Xi^{2\text{pt}}(t_{\text{seq}}, \vec{p}_i) C_\Lambda^{2\text{pt}}(t_{\text{seq}}, \vec{p}_f)}}$$

Reduction formula

Reduction formula:

ground state + first excited state

$$R(\Gamma, T, \vec{p}_f, t, t_{seq}) = \frac{F(\Gamma, T, \vec{p}_f)}{8\sqrt{E_i E_f m_i m_f}} \sqrt{\frac{1 + D(e^{-\Delta E t} + e^{\Delta E(t-t_{seq})})}{1 + A_f e^{-\Delta E t_{seq}}}}$$

where $F(\Gamma, T, \vec{p}_f) = V_i$ or A_i

mom=(0, 0, 1)	$\Gamma = \gamma_x$	$\Gamma = \gamma_y$	$\Gamma = \gamma_z$	$\Gamma = \gamma_t$
$T = 1$	0	0	V_1	V_2
$T = \gamma^x$	V_3	0	0	0
$T = \gamma^y$	0	V_3	0	0
$T = \gamma^x \gamma^y$	0	0	0	0
$T = \gamma^z$	0	0	V_4	V_5
$T = \gamma^x \gamma^z$	V_6	0	0	0
$T = \gamma^y \gamma^z$	0	V_6	0	0
$T = \gamma^5 \gamma^t$	0	0	0	0

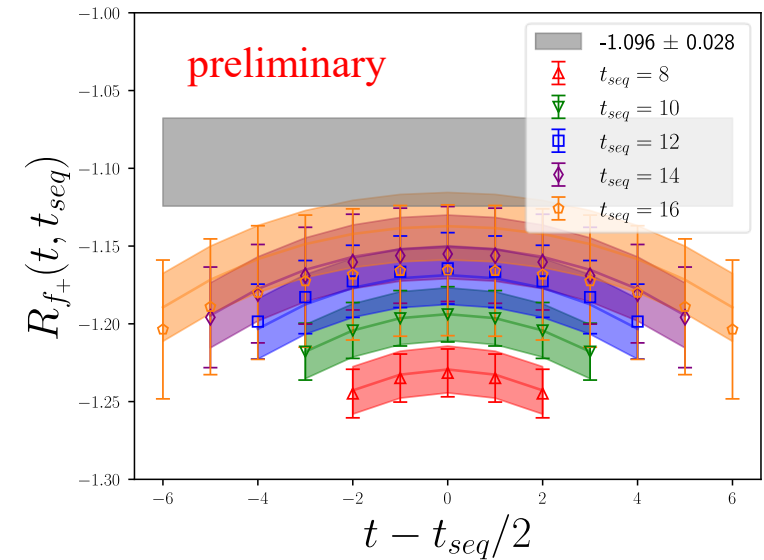
-13-

Linear combination of helicity form factors

$$V_i = af_+(q^2) + bf_\perp(q^2) + cf_0(q^2), \quad i = 1 \sim 6$$

$$A_i = ag_+(q^2) + bg_\perp(q^2) + cg_0(q^2), \quad i = 1 \sim 6$$

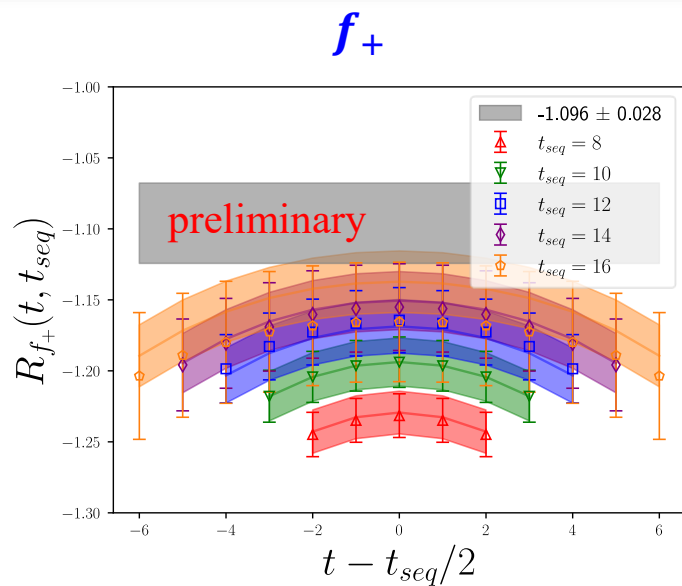
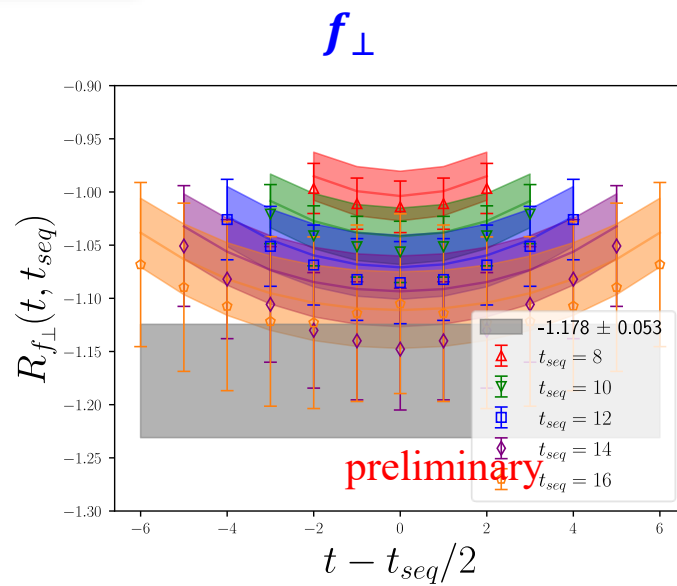
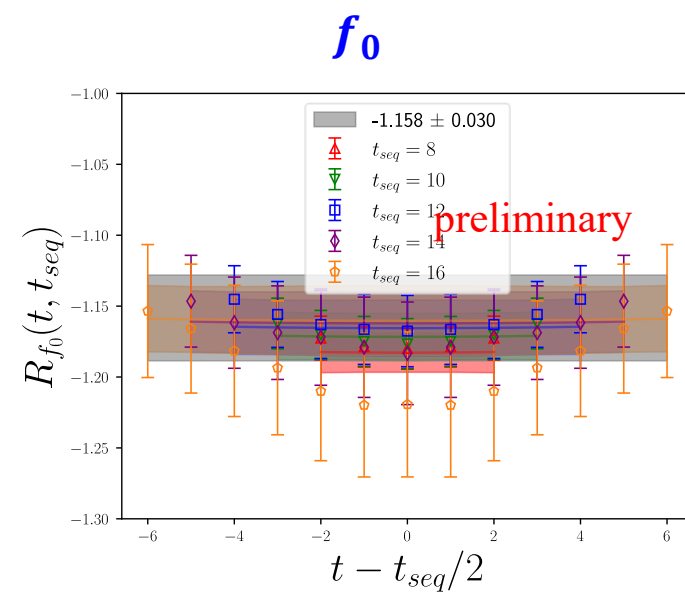
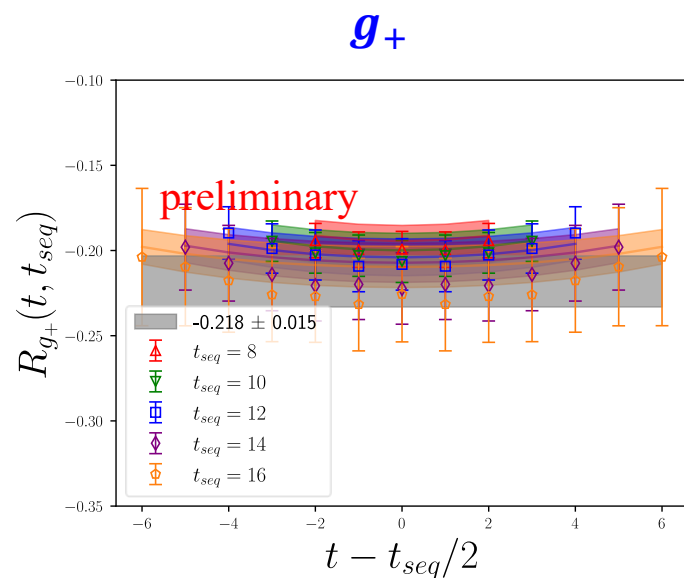
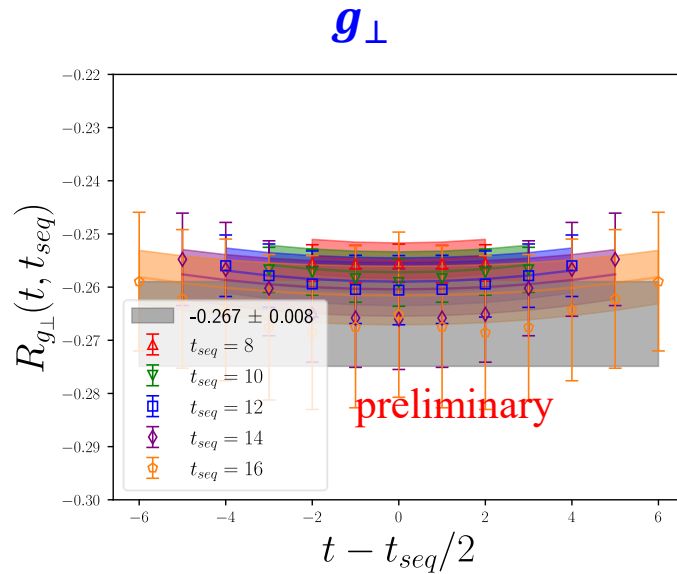
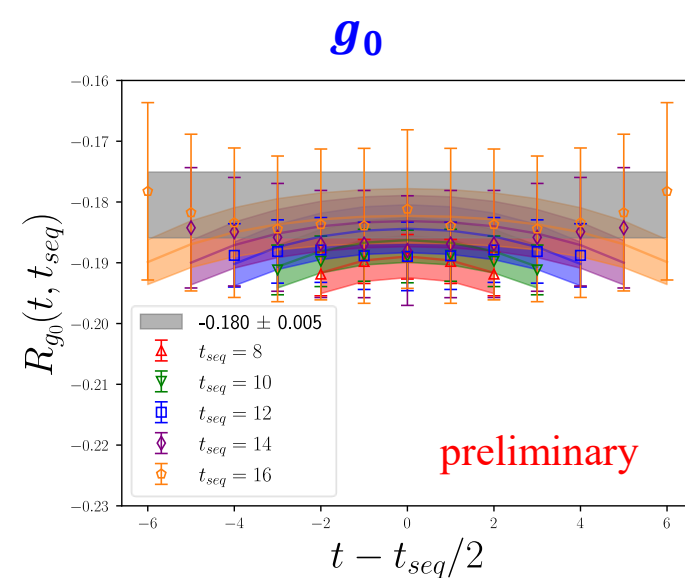
f_+



Extrapolation to sequential time

C32P23: mom=(0, 0, 1)

-14-



Bare lattice currents

$$V_\mu^{\text{lat}} = \bar{u}\gamma_\mu s, \quad A_\mu^{\text{lat}} = \bar{u}\gamma_\mu\gamma_5 s$$

$$V_\mu^R = Z_V^{us} V_\mu^{\text{lat}}, \quad A_\mu^R = Z_A^{us} A_\mu^{\text{lat}}$$

Renormalization of form factors

$$f_i^R(q^2) = Z_V^{us} f_i^{\text{bare}}(q^2), \quad g_i^R(q^2) = Z_A^{us} g_i^{\text{bare}}(q^2)$$

For the flavor-changing current

$J = \bar{u}\Gamma_{V/A}s$, we use

$$Z_V^{us} = \sqrt{Z_V Z_V^s}, \quad Z_A^{us} = \sqrt{Z_V Z_V^s} \frac{Z_A}{Z_V}$$

Renormalization constants of CLQCD ensembles:

TABLE VI. The renormalization constants on each ensemble.

a (fm)	Symbol	Z_V	Z_V^s (optional)	Z_V^c	Z_A/Z_V
0.10524(05)(62)	C24P34	0.79676(32)	0.85034(10)	1.57130(20)	1.07677(72)
	C24P29	0.79814(23)	0.85184(06)	1.57353(18)	1.07244(70)
	C32P29	0.79810(13)	0.85167(04)	1.57163(14)	1.07648(63)
	C32P23	0.79957(13)	0.85350(04)	1.57644(12)	1.07375(40)
	C48P23	0.79954(05)	0.85339(02)	1.57326(08)	1.07317(70)
	C48P14	0.79957(06)	0.85359(02)	1.57415(08)	1.07320(68)
0.08973(20)(53)	E28P35	0.81768(04)	0.85877(02)	1.40361(05)	1.06029(53)
0.07753(03)(45)	F32P30	0.83548(12)	0.86900(03)	1.30566(07)	1.05549(54)
	F48P30	0.83511(04)	0.86880(02)	1.30451(06)	1.05687(66)
	F32P21	0.83579(09)	0.87031(03)	1.30782(08)	1.05416(50)
	F48P21	0.83567(05)	0.86880(02)	1.30673(04)	1.05434(88)
0.06887(12)(41)	G36P29	0.84636(09)	0.87473(05)	1.23990(13)	1.04500(22)
0.05199(08)(31)	H48P32	0.86855(04)	0.88780(01)	1.12882(11)	1.03802(28)

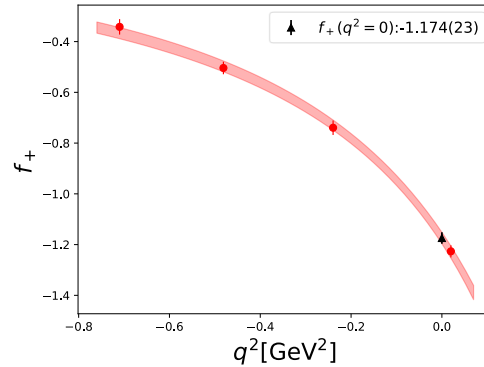
CLQCD Collaboration: H.Y. Du, *et.al.* Phys. Rev. D 111, 054504 (2025)

q^2 dependenc : z-series expansion

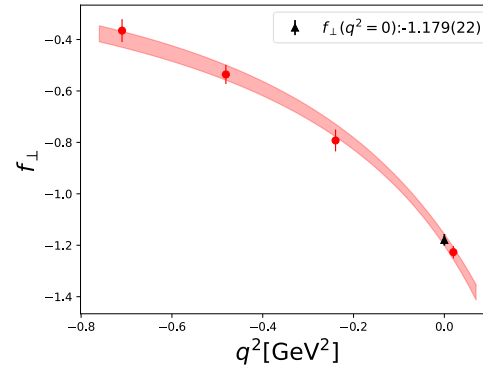
-16-

C24P34

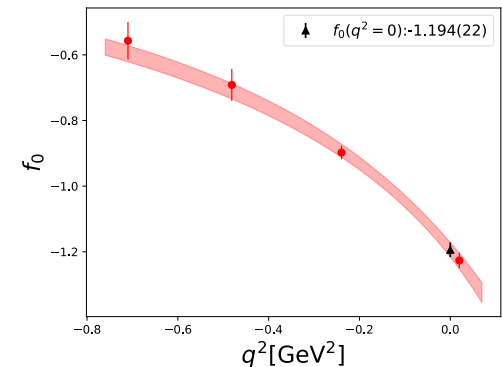
f_+ $m_{pole} = m_{K^*} = 0.892$ GeV



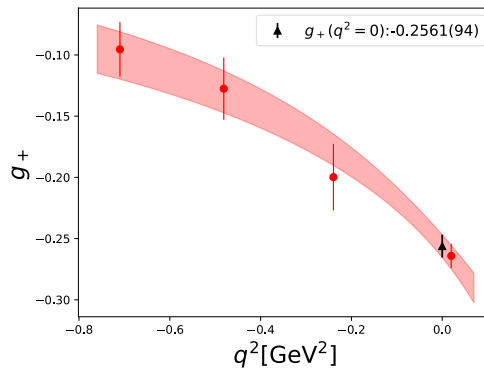
$f_\perp$ $m_{pole} = m_{K^*} = 0.892$ GeV



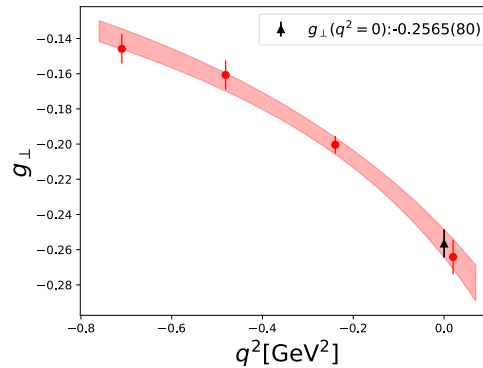
f_0 $m_{pole} = m_{K_0^*} = 0.838$ GeV



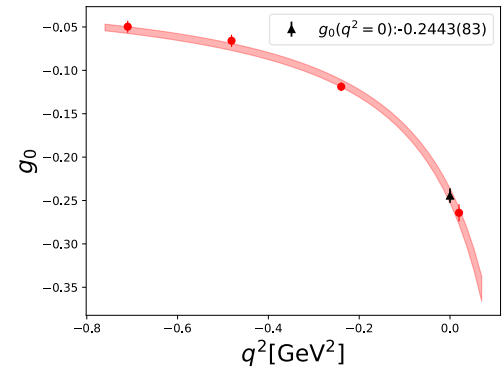
g_+ $m_{pole} = m_{K_1} = 1.27$ GeV



$g_\perp$ $m_{pole} = m_{K_1} = 1.27$ GeV



g_0 $m_{pole} = m_{K^\pm} = 0.494$ GeV



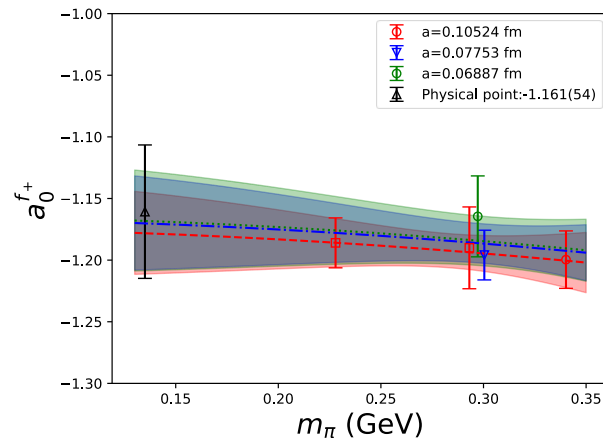
z-series expansion:
$$f(q^2) = \frac{1}{1 - q^2/(m_{pole})^2} \sum_{n=0}^{n_{max}} \boxed{a_n^f} [z(q^2)]^n$$

S. Meinel, Phys. Rev. Lett. 118, 082001 (2017).

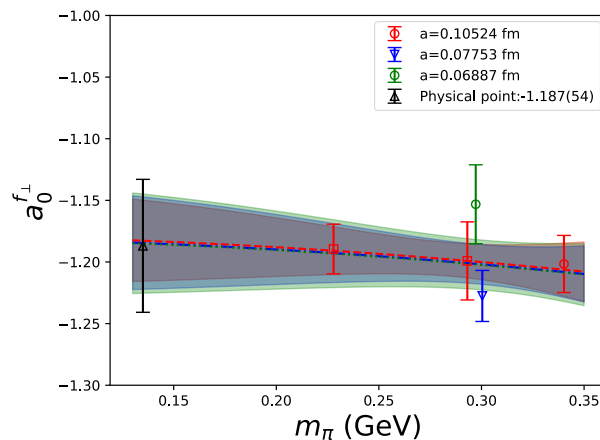
Chiral and continuum limit

-17-

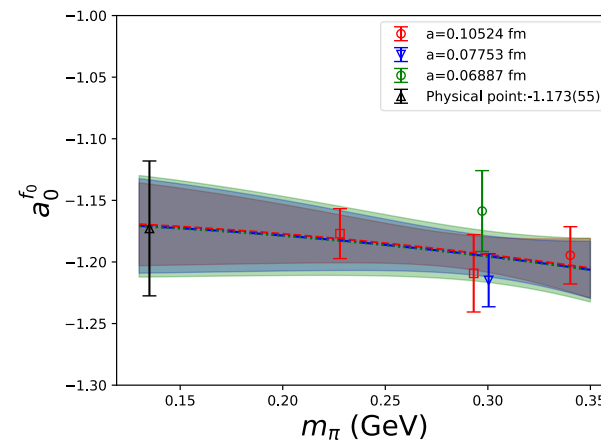
$f_+ : a_0$



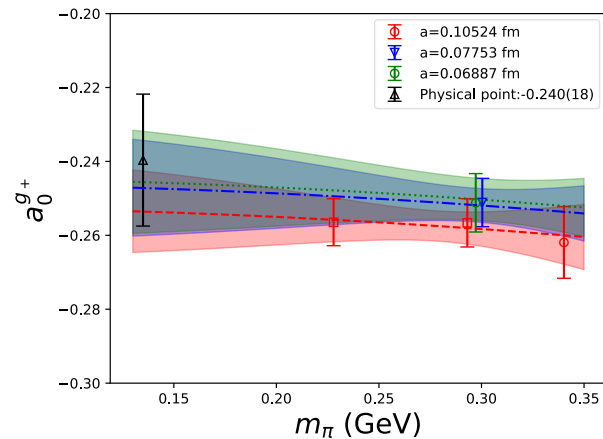
$f_\perp : a_0$



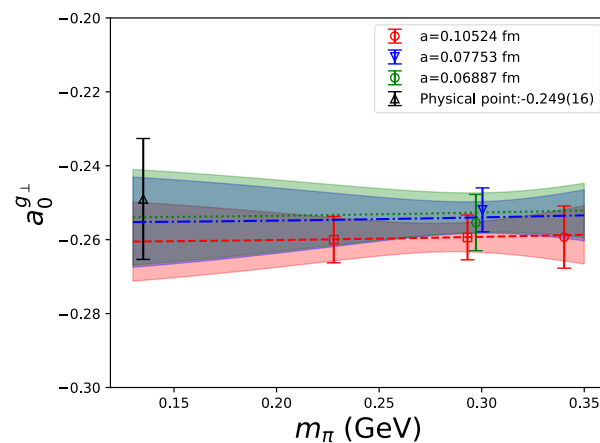
$f_0 : a_0$



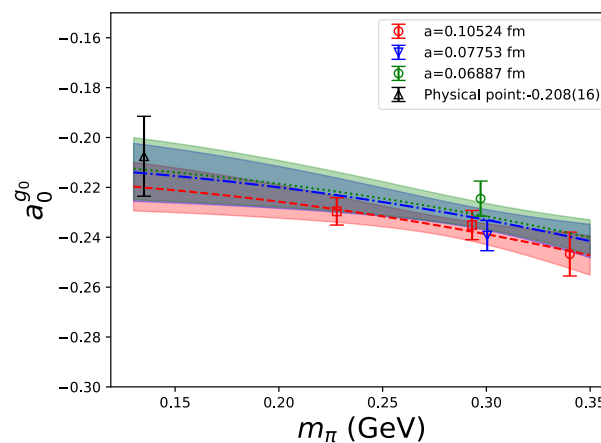
$g_+ : a_0$



$g_\perp : a_0$



$g_0 : a_0$



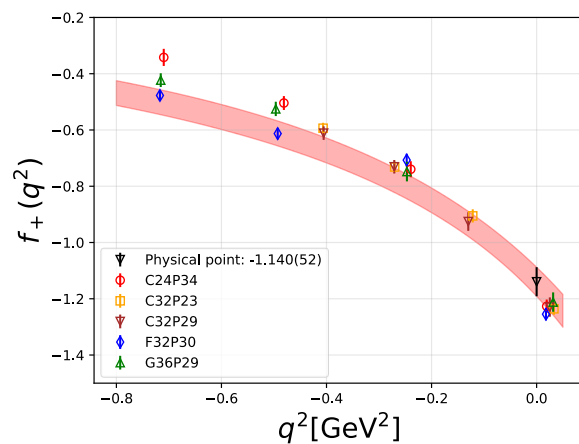
Extrapolation formula

$$a_i = a_{i,phy} + c_1(m_\pi^2 - m_{\pi,phy}^2) + c_2 a^2, \quad i = 0, 1$$

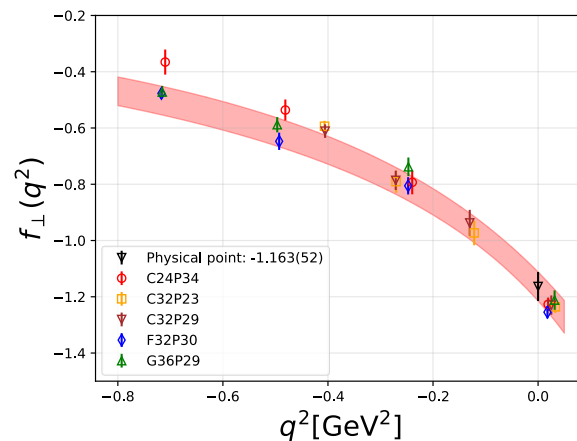
Form factors at the physical point

-18-

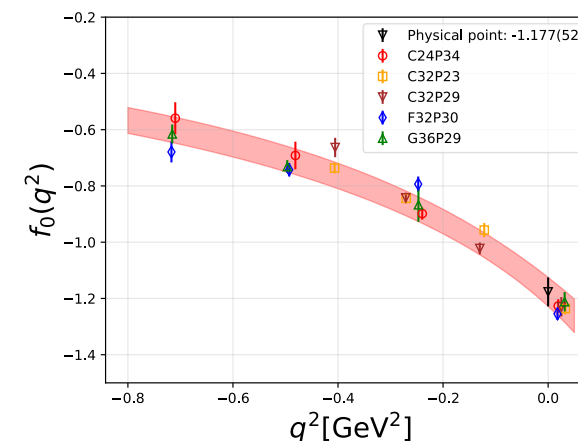
f_+



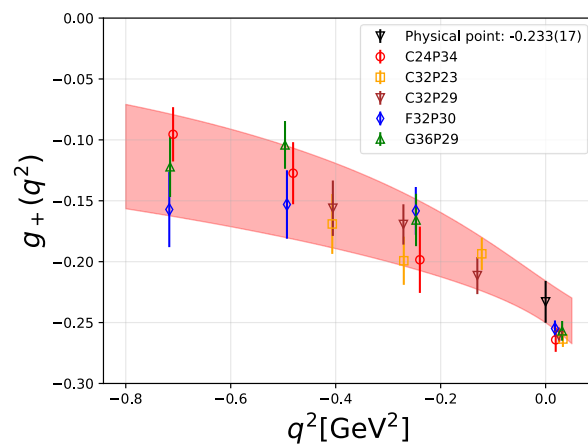
$f_\perp$



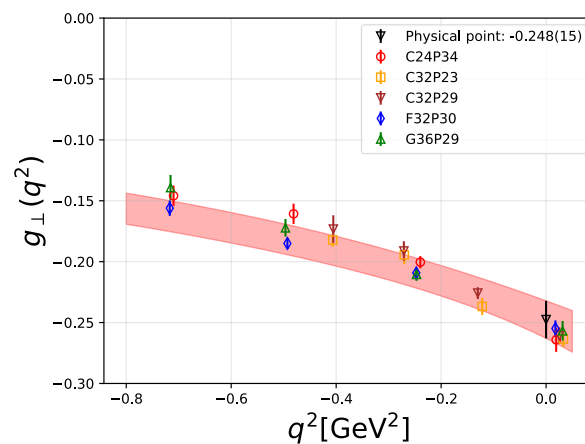
f_0



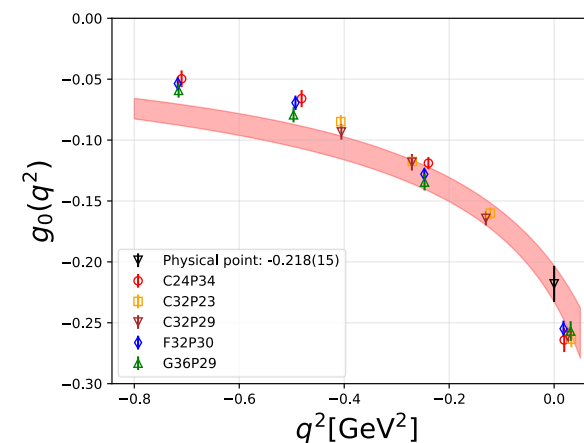
g_+



$g_\perp$



g_0



Outline

1. Motivation

2. Theoretical framework

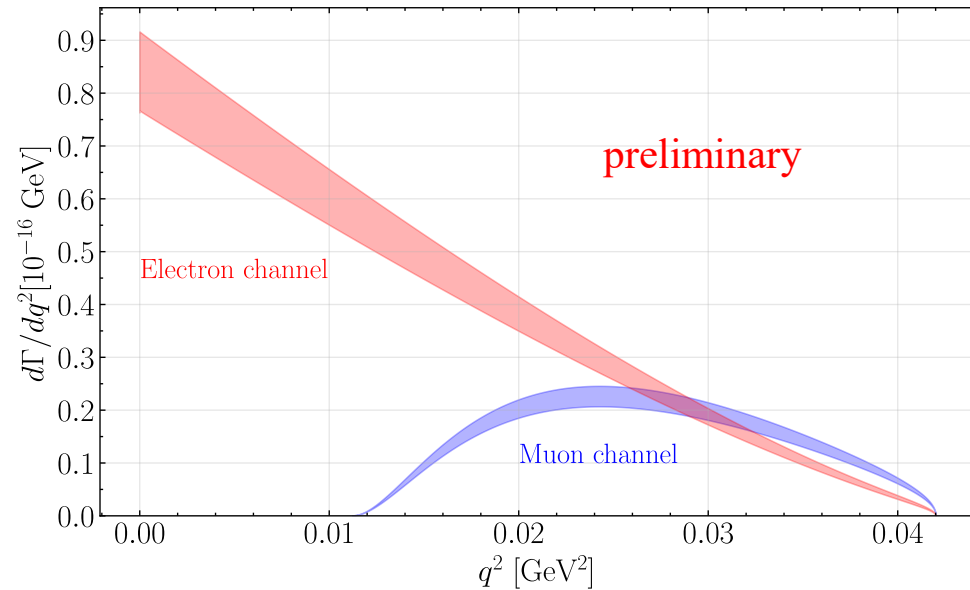
3. Numerical results

4. Summary

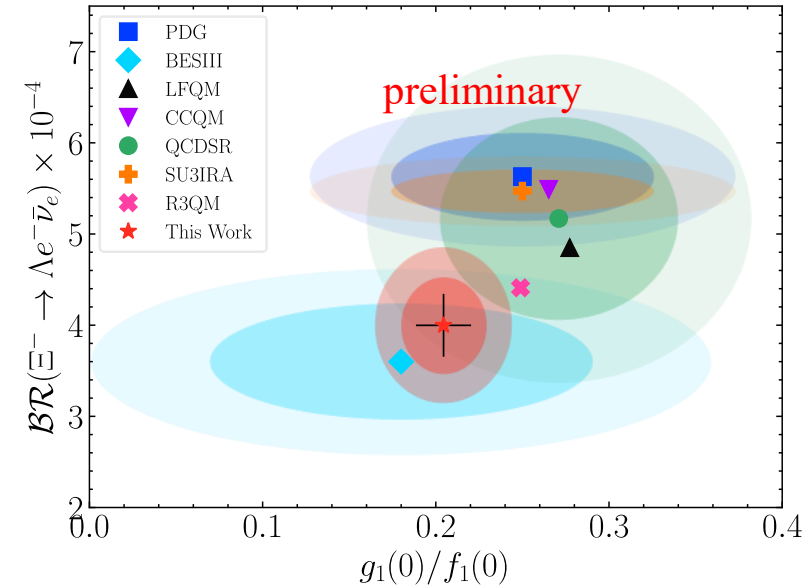
Numerical results

-20-

Differential decay width



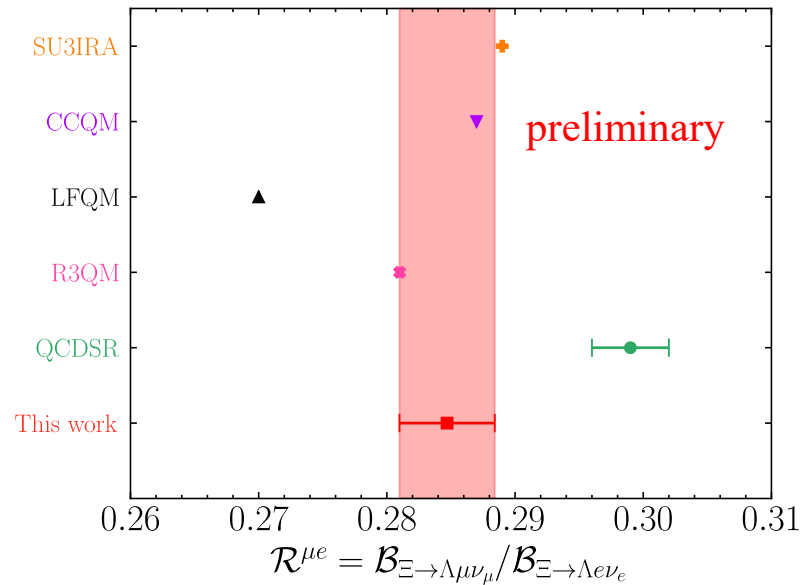
Branching ratio



	This work	BESIII	PDG
$\text{Br}(\Xi \rightarrow \Lambda e^- \bar{\nu}_e) \times 10^4$	$4.00(34)_{\text{stat}}$	$3.60(40)_{\text{stat}}(10)_{\text{syst}}$	$5.63(31)_{\text{total}}$

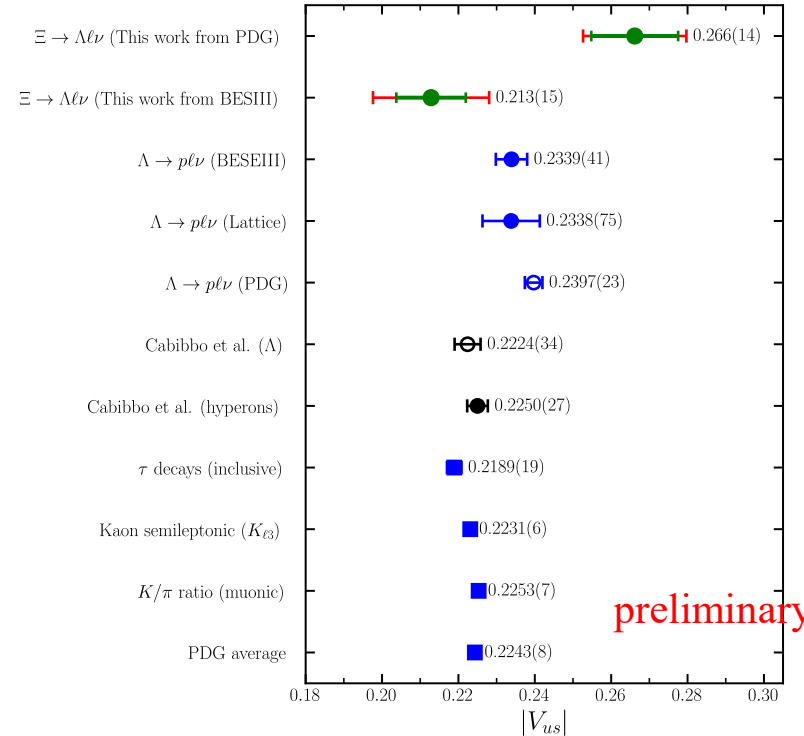
Electron-channel branching fraction: agree with BESIII, but more than 2σ with PDG.

Lepton-flavor ratio

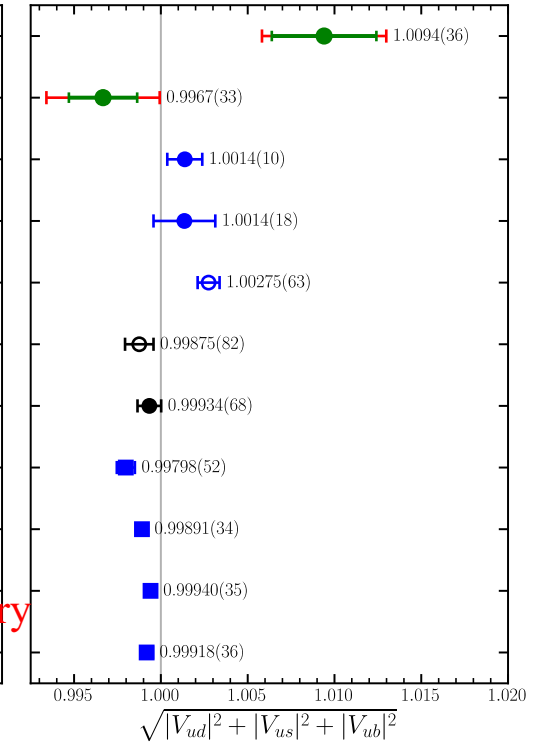


$$\mathcal{R} = \frac{\text{Br}(\Xi^- \rightarrow \Lambda \mu^- \bar{\nu}_\mu)}{\text{Br}(\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e)} = 0.2847(37)$$

CKM matrix element $|V_{us}|$



Unitarity



CKM matrix unitarity: within 1σ of BESIII, but more than 2σ from PDG.

Summary

-22-

1. We calculated helicity form factors of $\Xi \rightarrow \Lambda \ell \bar{\nu}_\ell$ in Lattice QCD
2. Performed chiral and continuum limit extrapolations
3. Obtained the preliminary results of branching ratio and g_{av}

Outlook

- Other processes like $\Xi_c \rightarrow \Xi \ell \bar{\nu}_\ell \dots$
- Apply 3pt to other baryon observables

Thanks for your attention!