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Identification of $D_{s1}(2933)^+$ as the $2P_1$ State in the Quark Model

J. Wang et al., arXiv: 2609.32586 (hep-ph)

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- 3 Numerical Results and Discussion
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1 Introduction



Charm-strange meson spectroscopy

Experimental developments

Positive-parity states $D_{s0}^*(2317)$ and $D_{s1}(2460)$ were observed in 2003.

B. Aubert et al., *Phys. Rev. Lett.* 90, 242001 (2003), D. Besson et al., *Phys. Rev. D* 68, 032002 (2003)

Higher excited states include $D_{sJ}(2700)$ and $D_{sJ}(2860)$.

J. Brodzicka et al., *Phys. Rev. Lett.* 100, 092001 (2008), R. Aaij et al., *Phys. Rev. Lett.* 113, 162001 (2014)

Structure and dynamics

- Masses, widths, and decay properties show discrepancies with conventional $c\bar{s}$ quark-model predictions.
- Coupled-channel effects, hadronic molecules, and tetraquark interpretations have been discussed.
- New signals provide clues to the internal structure of excited charm-strange mesons.

S. Godfrey et al., *Phys. Rev. D* 93, 034035 (2016), S. Godfrey et al., *Phys. Rev. D* 32, 189–231 (1985),

F.-K. Guo et al., *Rev. Mod. Phys.* 90, 015004 (2018)



LHCb observation of $D_{s1}(2933)^+$

Full phase-space amplitude analysis

$$B^0 \rightarrow D^+ D^- K^+ \pi^-.$$

The new charm-strange meson is observed in two quasi-two-body channels:

$$\begin{aligned} B^0 &\rightarrow D^- D_{s1}(2933)^+, \\ D_{s1}(2933)^+ &\rightarrow D^+ K^{*0} (\rightarrow K^+ \pi^-), \\ D_{s1}(2933)^+ &\rightarrow K^+ D_2^{*0} (\rightarrow D^+ \pi^-). \end{aligned}$$

Mass and quantum numbers

$$m \simeq 2933 \text{ MeV}, \quad J^P = 1^+.$$

R. Aaij et al., arXiv: 2604.21257 (hep-ex)



The mass discrepancy and nearby channels

Comparison with the conventional quark model

- The lowest $GI\ 2P_1$ state is expected at approximately 3.02 GeV.
- The observed mass is about 80–100 MeV lower.
- A similar discrepancy occurs for $D_{s0}^*(2317)$ and $D_{s1}(2460)$.

S. Godfrey et al., *Phys. Rev. D* 93, 034035 (2016), S. Godfrey et al., *Phys. Rev. D* 32, 189–231 (1985),
S. Godfrey et al., *Phys. Rev. D* 43, 1679–1687 (1991)

Motivation for coupled-channel effects

- $D_{s1}(2933)$ lies near the D^*K^* and $D_s^{(*)}\phi$ thresholds.
- Coupled-channel effects or near-threshold molecular components may play an important role.

F.-K. Guo et al., *Rev. Mod. Phys.* 90, 015004 (2018), W. Hao et al., *Phys. Rev. D* 106, 074014 (2022),
E. van Beveren et al., *Phys. Rev. Lett.* 91, 012003 (2003), T. Barnes et al., *Phys. Rev. D* 68, 054006 (2003)



Structural interpretations

A radially excited $c\bar{s}$ state

- $J^P = 1^+$ and a mass above the $1P$ multiplet motivate a $2P_1$ assignment.
- Meson loops can shift masses, change $^1P_1$ - $^3P_1$ mixing, and affect partial widths.

S. Godfrey et al., *Phys. Rev. D* 93, 034035 (2016), W. Hao et al., *Phys. Rev. D* 106, 074014 (2022),
T. Barnes et al., *Phys. Rev. C* 77, 055206 (2008)

Lessons from the lower axial-vector states

- Coupled channels affect the $^1P_1$ - $^3P_1$ description of $D_{s1}(2460)$ and $D_{s1}(2536)$.
- The nearby D^*K threshold also motivates a molecular interpretation of $D_{s1}(2460)$.

X.-G. Wu et al., *Phys. Rev. D* 85, 034040 (2012), R. Molina et al., *Phys. Rev. D* 82, 014010 (2010)

The mass and decay branching ratios must be considered together.



Aim and strategy of this work

Quark-model starting point

Treat $D_{s1}(2933)$ as a $2P_1$ excitation of the $c\bar{s}$ system. The GI spin-orbit interaction supplies the bare mixing; the $^3P_0$ model supplies the strong couplings.

S. Godfrey et al., *Phys. Rev. D* 93, 034035 (2016), L. Micu, *Nucl. Phys. B* 10, 521–526 (1969),
E. S. Ackleh et al., *Phys. Rev. D* 54, 6811–6829 (1996)

Mass and decay tests

1. Evaluate two-point meson-loop corrections to the 2^1P_1 – 2^3P_1 mass matrix.
2. Use the observed mass to determine the corrected matrix and mixing angle.
3. Calculate partial and total widths from the physical-state wave functions.
4. Compare the relative three-body branching ratio through DK^* and D_2^*K with experiment.



2 Formalism



Bare states and mixing convention

The GI model predicts $M(2P_1) = 3.018$ GeV and $M(2P'_1) = 3.038$ GeV.

Physical mixtures of the $2P$ basis states

$$\begin{aligned} |P\rangle &= \cos \theta_P |^1P_1\rangle + \sin \theta_P |^3P_1\rangle, \\ |P'\rangle &= -\sin \theta_P |^1P_1\rangle + \cos \theta_P |^3P_1\rangle. \end{aligned}$$

$$\mathcal{P} = \begin{pmatrix} \cos \theta_P & -\sin \theta_P \\ \sin \theta_P & \cos \theta_P \end{pmatrix}, \quad M^0 = \mathcal{P} M_{\text{diag}} \mathcal{P}^{-1}.$$

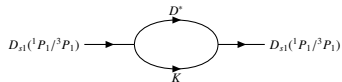
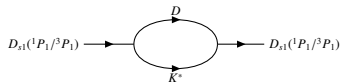
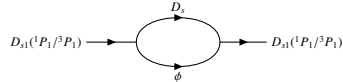
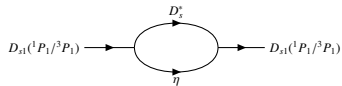
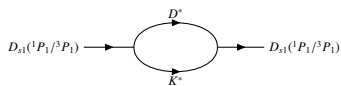
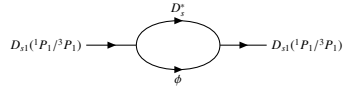
- M_{diag} contains the mixed-state masses predicted by the GI model.
- The bare $^1P_1$ – $^3P_1$ mixing is induced by the spin-orbit interaction.
- Coupled-channel corrections are added to obtain the physical masses and mixing without spin-orbit interaction.

S. Godfrey et al., *Phys. Rev. D* 93, 034035 (2016), S. Godfrey et al., *Phys. Rev. D* 32, 189–231 (1985),
S. Godfrey et al., *Phys. Rev. D* 43, 1679–1687 (1991)



Six intermediate meson channels

$$VP : D^*K, DK^*, D_s\phi, D_s^*\eta; \quad VV : D^*K^*, D_s^*\phi.$$



Two-point self-energy diagrams. Only the S -wave contributions are used for the mass corrections.



Effective Lagrangians for the loop vertices

VP channels

$$\mathcal{L}_{D_{s1}D^*K} = g_{D_{s1}D^*K} D_{s1}^\mu D_\mu^* K,$$

$$\mathcal{L}_{D_{s1}DK^*} = g_{D_{s1}DK^*} D_{s1}^\mu K_\mu^* D,$$

$$\mathcal{L}_{D_{s1}D_s\phi} = g_{D_{s1}D_s\phi} D_{s1}^\mu D_s \phi_\mu,$$

$$\mathcal{L}_{D_{s1}D_s^*\eta} = g_{D_{s1}D_s^*\eta} D_{s1}^\mu D_{s\mu}^* \eta.$$

VV channels

$$\mathcal{L}_{D_{s1}D^*K^*} = ig_{D_{s1}D^*K^*} \epsilon_{\mu\nu\alpha\beta} \partial^\mu D_{s1}^\nu D^{*\alpha} K^{*\beta},$$

$$\mathcal{L}_{D_{s1}D_s^*\phi} = ig_{D_{s1}D_s^*\phi} \epsilon_{\mu\nu\alpha\beta} \partial^\mu D_{s1}^\nu D_s^{*\alpha} \phi^\beta.$$

The couplings are determined at the bare-state masses using the S -wave $^3P_0$ amplitudes.



VP self-energy and regularization

For $D_{s1}(p) \rightarrow V(q) + P(p-q) \rightarrow D_{s1}(p)$:

$$\Sigma_{VP}^{\mu\nu}(p) = ig_{VP}g'_{VP} \int \frac{d^4q}{(2\pi)^4} \frac{-g^{\mu\nu} + q^\mu q^\nu / m_V^2}{(q^2 - m_V^2 + i\epsilon)[(p-q)^2 - m_P^2 + i\epsilon]} \mathcal{F}(q^2).$$

Form factor for the ultraviolet divergence

$$\mathcal{F}(q^2) = \frac{\Lambda^2 - m_V^2}{\Lambda^2 - q^2} \frac{\Lambda^2 - m_P^2}{\Lambda^2 - (q-p)^2}.$$

$$\Lambda = m_V(m_P) + \alpha\Lambda_{\text{QCD}}, \quad \Lambda_{\text{QCD}} = 220 \text{ MeV}.$$

Replace the couplings and meson masses for each of the D^*K , DK^* , $D_s\phi$, and $D_s^*\eta$ channels.



VV self-energy

For $D_{s1}(p) \rightarrow V_1(q) + V_2(p - q) \rightarrow D_{s1}(p)$:

Two-vector loop amplitude

$$\begin{aligned} \Sigma_{VV}^{\mu\nu}(p) = & ig_{V_1 V_2} g'_{V_1 V_2} g^{\mu\alpha} g^{\delta\nu} \int \frac{d^4 q}{(2\pi)^4} \varepsilon_{\alpha\beta\rho\sigma} \varepsilon_{\delta\zeta\tau\kappa} p^\beta p^\zeta \\ & \times \frac{\left(-g^{\rho\tau} + \frac{q^\rho q^\tau}{m_{V_1}^2}\right) \left(-g^{\sigma\kappa} + \frac{(p-q)^\sigma (p-q)^\kappa}{m_{V_2}^2}\right)}{(q^2 - m_{V_1}^2 + i\epsilon)((p-q)^2 - m_{V_2}^2 + i\epsilon)} \mathcal{F}(q^2). \end{aligned}$$

- The two VV channels are $D^* K^*$ and $D_s^* \phi$.
- The channel-dependent couplings and masses enter the same expression.
- The form factor regularizes the loop integral.



Transverse self-energy and effective mass matrix

For a spin-1 particle, the self-energy is decomposed as

$$\Sigma^{\mu\nu}(p) = \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2} \right) \Sigma_T(p^2) + \frac{p^\mu p^\nu}{p^2} \Sigma_L(p^2).$$

Projection onto the transverse part

$$\Sigma_T(p^2) = -\frac{1}{3} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Sigma^{\mu\nu}(p).$$

The physically relevant mass correction comes from Σ_T .

Coupled-channel correction to the mass matrix

$$M_{ij}(m) = M_{ij}^0 \delta_{ij} + \frac{\text{Re } \Sigma_{ij}^T(m^2)}{2m}.$$

T. Barnes et al., *Phys. Rev. C* 77, 055206 (2008)



Fixed-reference-mass prescription

Evaluate the self-energies at one reference mass

$$m = 2933 \text{ MeV}, \quad \det(M_{ij}(m) - m\delta_{ij}) = 0.$$

1. Choose α so that the **lower eigenvalue** of the effective matrix equals the reference mass.
2. Diagonalize this same matrix to obtain the partner mass and mixing angle.
3. The self-energy is **not re-evaluated at the partner mass**.

Alternative assignment

Keep all other model inputs unchanged and require the higher eigenvalue to equal the same reference mass.



The ${}^3P_0$ transition operator

Quark-pair creation

$$T = -3\gamma \sum_m \langle 1, m; 1, -m | 0, 0 \rangle \int d^3\mathbf{p}_3 d^3\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \\ \times \mathcal{Y}_1^m\left(\frac{\mathbf{p}_3 - \mathbf{p}_4}{2}\right) \chi_{1,-m}^{34} \phi_0^{34} \omega_0^{34} b_{3i}^\dagger(\mathbf{p}_3) d_{4j}^\dagger(\mathbf{p}_4).$$

- γ is the pair-creation coupling constant; i, j are color indices.
- $\chi_{1,-m}^{34}$ is the spin wave function; ϕ_0^{34} is the flavor singlet.
- The color wave function and solid harmonic are

$$\omega_0^{34} = \frac{\delta_{ij}}{\sqrt{3}}, \quad \mathcal{Y}_1^m(\mathbf{p}) = |\mathbf{p}| Y_1^m(\theta, \phi).$$

E. S. Ackleh et al., *Phys. Rev. D* 54, 6811–6829 (1996), H. G. Blundell, arXiv: hep-ph/9608473,
A. Le Yaouanc et al., *Phys. Rev. D* 8, 2223–2234 (1973), L. Micu, *Nucl. Phys. B* 10, 521–526 (1969)



Meson wave functions

Quark-model state

$$\begin{aligned}
 |M(\mathbf{P}, J, J_z)\rangle = & \sum_{S_z, L_z, c_i} \langle L, L_z; S, S_z | J, J_z \rangle \int d^3\mathbf{p}_1 d^3\mathbf{p}_2 \\
 & \times \delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{P}) \psi_{N, L, L_z}(\mathbf{p}_1, \mathbf{p}_2) \\
 & \times \frac{\delta_{c_1 c_2}}{\sqrt{3}} \phi_{f_1, f_2} \chi_{s_1, s_2}^{S, S_z} b_{c_1, f_1, s_1, \mathbf{p}_1}^\dagger d_{c_2, f_2, s_2, \mathbf{p}_2}^\dagger |0\rangle.
 \end{aligned}$$

- c_j , s_j , and f_j label color, spin, and flavor.
- $b^\dagger$ and $d^\dagger$ create the quark and antiquark.
- ψ_{N, L, L_z} is the spatial wave function in the harmonic-oscillator basis.

C. Hayne et al., *Phys. Rev. D* 25, 1944 (1982)



Amplitude matching and partial decay widths

From quark-model amplitudes to hadronic couplings

$$\mathcal{M}_q = \langle BC|T|A\rangle, \quad \mathcal{M}_h = 8\pi^{3/2} \sqrt{m_A m_B m_C} \mathcal{M}_q.$$

Substitute the meson wave functions into $\mathcal{M}_q$ and match to the effective Lagrangians to determine the vertex couplings.

Decay widths of the physical states

- Determine the physical-state masses and mixing angle from the corrected mass matrix.
- Use these states in the ${}^3P_0$ calculation of the partial widths.
- The dominant channels of $D_{s1}(2933)$ are

$$DK^*, \quad D^*K, \quad D^*K^*, \quad D_s^*\eta.$$



Relative three-body branching ratios

Experimental ratio from the two decay chains

$$R_{\text{exp}} = \frac{\text{BR}(B^0 \rightarrow D^- D_{s1}(2933) \rightarrow D^+ K^{*0}(\rightarrow K^+ \pi^-))}{\text{BR}(B^0 \rightarrow D^- D_{s1}(2933) \rightarrow K^+ D_2^{*0}(\rightarrow D^+ \pi^-))}.$$

The common $B^0 \rightarrow D^- D_{s1}(2933)$ production process is identical.

Theory: two intermediate states, the same $D^+ K^+ \pi^-$ final state

$$R_{\text{th}} = \frac{\Gamma(D_{s1}(2933) \rightarrow D^+ K^{*0}(\rightarrow K^+ \pi^-))}{\Gamma(D_{s1}(2933) \rightarrow K^+ D_2^{*0}(\rightarrow D^+ \pi^-))}.$$

Both secondary decays are retained. Comparing the relative three-body ratios tests the structural assignment.



Three-body decay vertices

For $D_{s1}(2933) \rightarrow D^+ K^+ \pi^-$ through DK^* and $D_2^* K$:

Effective Lagrangians

$$\mathcal{L}_{D_{s1}DK^*} = g_{D_{s1}DK^*} D_{s1}^\mu K_\mu^* D,$$

$$\mathcal{L}_{K^*K\pi} = ig_{K^*K\pi} K_\mu^* (\partial^\mu K \pi - K \partial^\mu \pi),$$

$$\mathcal{L}_{D_{s1}D_2^*K} = ig_{D_{s1}D_2^*K} D_{s1}^\mu D_{2\mu\nu}^* \partial^\nu K,$$

$$\mathcal{L}_{D_2^*D\pi} = g_{D_2^*D\pi} D_{2\mu\nu}^* \partial^\mu D \partial^\nu \pi.$$

- The primary couplings follow from the $^3P_0$ model.
- Since $D_{s1}(2933)$ is below the $D_2^* K$ threshold, the on-shell approximation is used to determine that coupling.
- The secondary-decay couplings follow from experimental decay information.



3 Results



Input parameters

Harmonic-oscillator strengths and constituent quark masses, in GeV:

HO strength	Value	Quark mass	Value
$\beta_{D_s(2^1P_1)} (\beta_{D_s(2^3P_1)})$	0.433 (0.434)	m_c	1.628
$\beta_D (\beta_{D^*})$	0.601 (0.516)	m_s	0.419
$\beta_{D_s} (\beta_{D_s^*})$	0.651 (0.562)	$m_{u/d}$	0.22
$\beta_{D_2^*}$	0.437		

Baseline calculation

The pair-creation coupling is $\gamma = 0.4$; the remaining parameters follow the reference quark-model calculation.

S. Godfrey et al., *Phys. Rev. D* 93, 034035 (2016), A. Le Yaouanc et al., *Phys. Rev. D* 8, 2223–2234 (1973),
 A. Le Yaouanc et al., *Phys. Rev. D* 9, 1415–1419 (1974),
 J. Segovia et al., *Phys. Lett. B* 715, 322–327 (2012)



Bare-state coupling constants

VP couplings are in GeV; VV couplings are dimensionless.

Channel	$g(^1P_1)$	$g(^3P_1)$
$D^0 K^{*+}$	-0.411	0.683
$D^+ K^{*0}$	-0.340	0.586
$D^{*0} K^+$	-1.44	2.09
$D^{*+} K^0$	-1.42	2.06
$D_s \phi$	-2.74	3.72
$D_s^* \eta$	0.074	-0.053
$D^{*+} K^{*0}$	0.534	—
$D^{*0} K^{*+}$	0.579	—
$D_s^* \phi$	1.41	—

The couplings are calculated with the $^3P_0$ model at the bare-state masses.



S -wave selection rule in the $^3P_0$ model

The $^3P_1 \rightarrow VV$ amplitude contains the Wigner $9j$ symbol

$$\left\{ \begin{matrix} 1/2 & 1/2 & 1 \\ 1/2 & 1/2 & 1 \\ 1 & 1 & 1 \end{matrix} \right\} = 0.$$

Consequences for the loop corrections

- The VV contribution to the $^3P_1$ diagonal mass shift vanishes in the S -wave treatment.
- The same VV channels can contribute to the $^1P_1$ diagonal element.
- The off-diagonal loop correction is generated solely by the VP channels.

The VV channels can therefore shift diagonal masses without inducing $^1P_1$ – $^3P_1$ mixing.

T. Barnes et al., *Phys. Rev. D* 68, 054014 (2003), H. G. Blundell, arXiv: hep-ph/9608473,

A. Le Yaouanc et al., *Phys. Rev. D* 8, 2223–2234 (1973),

E. S. Ackleh et al., *Phys. Rev. D* 54, 6811–6829 (1996)



Bare mass matrix for the numerical calculation

GI-model input in the $^1P_1$ - $^3P_1$ basis

$$M^0 = \begin{pmatrix} 3023 & 8.729 \\ 8.729 & 3032 \end{pmatrix} \text{ MeV.}$$

The matrix is obtained from the GI masses and mixing angle through the similarity transformation.

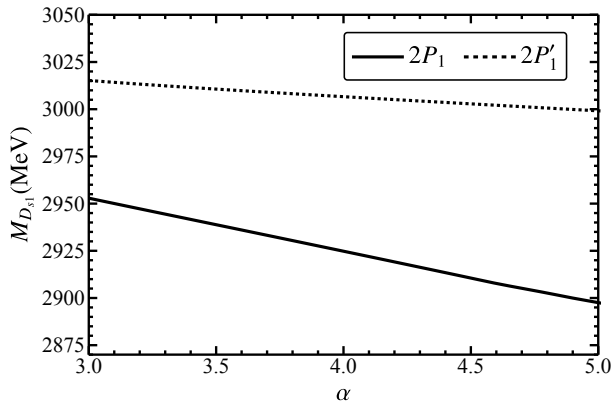
S. Godfrey et al., *Phys. Rev. D* 93, 034035 (2016)

From couplings to physical eigenvalues

1. Use the calculated channel couplings to evaluate $\Sigma_{ij}^T(m^2, \alpha)$.
2. Construct the effective mass matrix at $m = 2933$ MeV.
3. Examine the dependence of its two eigenvalues on α .



Masses as functions of the cutoff parameter



The two mass eigenvalues versus α .

Lower-eigenvalue solution

Fix the lower eigenvalue to the reference mass 2933 MeV.

$$\alpha = 3.7, \quad \gamma = 0.4.$$

$$\alpha \Lambda_{\text{QCD}} = 0.82 \text{ GeV}.$$

The measured mass is accommodated with a cutoff scale in the reasonable $\sim 1 \text{ GeV}$ range.



Effective mass matrix and partner state

At $m = 2933$ MeV, with $\gamma = 0.4$ and $\alpha = 3.7$:

$$M = \begin{pmatrix} 2936 & 17.23 \\ 17.23 & 3008 \end{pmatrix} \text{ MeV.}$$

Diagonalizing the same effective matrix

$$M_{2P_1} = 2933 \text{ MeV (input),} \quad M_{2P'_1} = 3012 \text{ MeV.}$$

$$\theta_P = -12.9^\circ.$$

- The partner mass and angle follow from this same matrix, without re-evaluating the self-energy at the partner mass.
- The angle differs significantly from the heavy-quark ideal value -35.3° .



Alternative higher-state assignment

Keep the bare matrix, intermediate channels, S -wave approximation, and $\gamma = 0.4$ unchanged.

Require the higher eigenvalue to equal 2933 MeV

$$\alpha = 13.9, \quad \alpha \Lambda_{\text{QCD}} = 3.1 \text{ GeV}.$$

$$M_{2P_1} = 2497 \text{ MeV}.$$

Why this assignment is disfavored

- The required cutoff scale is physically unreasonable.
- The lower state falls in the region of the $1P$ partners $D_{s1}(2460)$ and $D_{s1}(2536)$.
- Overlap of the $2P$ and $1P$ multiplets without affecting their properties is unlikely.



Mixing pattern and dominant loop contributions

Origin of the relatively small mixing angle

- VV loops mainly shift the $^1P_1$ diagonal element downward; their $^3P_1$ contribution vanishes.
- The off-diagonal correction comes from VP channels and remains small relative to the difference of the diagonal elements.

Primary contributions to the mass shifts

$$D^*K^*, \quad D_s\phi, \quad D_s^*\phi.$$

- These thresholds are closest to the bare-state mass.
- In the S -wave treatment, there is no momentum-factor suppression near threshold.



Decay widths: baseline $\gamma = 0.4$

Widths use the full $^3P_0$ amplitudes; the S -wave restriction applies to the self-energies.

Decay channel	$\Gamma(2P_1)$ [MeV]	$\Gamma(2P'_1)$ [MeV]
DK^*	11.7	20.1
D^*K	51.2	40.3
D^*K^*	46.5	27.2
$D_0(1^3P_0)K$	0.71	0.07
$D_1(1P_1)K$	—	3.37
$D_1(1P'_1)K$	—	0.89
$D_2(1^3P_2)K$	—	32.21
$D_s\phi$	—	9.25
$D_s^*\eta$	4.82	1.39
Total	115	135

$$\Gamma_{\text{exp}} = 72^{+18}_{-12}(\text{stat})^{+7}_{-10}(\text{syst}) \text{ MeV.}$$

R. Aaij et al., arXiv: 2604.21257 (hep-ex)



Sensitivity to the pair-creation coupling

Determine γ from the measured total width

Requiring the calculated width to be 72 MeV gives $\gamma = 0.32$. This is **constrained by the measured width**, not an independent prediction.

$$M = \begin{pmatrix} 2936 & 17.17 \\ 17.17 & 3008 \end{pmatrix} \text{ MeV.}$$

$$\alpha = 5.2, \quad \alpha\Lambda_{\text{QCD}} = 1.14 \text{ GeV.}$$

- The matrix is nearly identical to the $\gamma = 0.4$ result.
- The mixing angle remains -12.9° .



Decay widths: width-constrained $\gamma = 0.32$

Decay channel	$\Gamma(2P_1)$ [MeV]	$\Gamma(2P'_1)$ [MeV]
DK^*	7.31	12.5
D^*K	31.9	25.2
D^*K^*	29.1	17.0
$D_0(1^3P_0)K$	0.44	0.04
$D_1(1P_1)K$	—	2.10
$D_1(1P'_1)K$	—	0.56
$D_2(1^3P_2)K$	—	20.1
$D_s\phi$	—	5.80
$D_s^*\eta$	3.01	0.87
Total	71.8	84.1

This sensitivity check uses γ constrained by the measured total width.



Inputs for the relative three-body ratio

Primary couplings from the $^3P_0$ model

$$g_{D_{s1}DK^*}^{\text{phys}} = 1.01 \text{ GeV}, \quad g_{D_{s1}D_2^*K}^{\text{phys}} = 2.66.$$

The D_2^*K coupling uses the on-shell approximation.

Secondary-decay couplings

Assume $\Gamma(D_2^*) = \Gamma(D_2^* \rightarrow D\pi) + \Gamma(D_2^* \rightarrow D^*\pi)$.

$$\frac{\Gamma(D_2^* \rightarrow D\pi)}{\Gamma(D_2^* \rightarrow D^*\pi)} = 1.52 \pm 0.14.$$

With isospin symmetry, $\Gamma(D_2^{*0} \rightarrow D^+\pi^-) = 19 \text{ MeV}$, giving

$$g_{K^*K\pi} = 4.40, \quad g_{D_2^*D\pi} = 25.7 \text{ GeV}^{-1}.$$

S. Navas et al., *Phys. Rev. D* 110, 030001 (2024), A. F. Falk et al., *Phys. Lett. B* 292, 119–127 (1992)



Relative three-body branching ratio

Theory: the two chains into $D^+ K^+ \pi^-$

$$R_{\text{th}} = \frac{\Gamma(D_{s1}(2933) \rightarrow D^+(K^{*0} \rightarrow K^+ \pi^-))}{\Gamma(D_{s1}(2933) \rightarrow K^+(D_2^{*0}(2460) \rightarrow D^+ \pi^-))} = 1.98.$$

Experiment: the same two chains in B^0 decay

$$R_{\text{exp}} = \frac{\text{BR}(B^0 \rightarrow D^- D_{s1}(2933) \rightarrow D^+ K^{*0}(\rightarrow K^+ \pi^-))}{\text{BR}(B^0 \rightarrow D^- D_{s1}(2933) \rightarrow K^+ D_2^{*0}(\rightarrow D^+ \pi^-))} = 1.27_{-0.95}^{+1.29}.$$

- The common $^3P_0$ coupling factor γ cancels: the result is the same for $\gamma = 0.4$ and 0.32 .
- Theory and experiment are consistent within the sizable experimental uncertainties.

R. Aaij et al., arXiv: 2604.21257 (hep-ex)



4 Summary



Summary

Main results

- We treat the newly observed $D_{s1}(2933)^+$ as the **lower $2P_1$ excited state** of the $c\bar{s}$ system.
- Coupled-channel self-energy corrections from DK^* , D^*K , D^*K^* , $D_s\phi$, $D_s^*\phi$, $D_s^*\eta$ loops shift the bare GI masses downward; the lower eigenvalue is fixed to the measured mass.
- Diagonalizing the corrected mass matrix yields:
 - ▶ $2P_1'$ partner at **3012 MeV**
 - ▶ Mixing angle $\theta_P = -12.9^\circ$ (deviates from HQ ideal-mixing -35.3°)
- With $\gamma = 0.32$ (fitted to Γ_{exp}), the total width 71.8 MeV agrees with $72^{+18}_{-12}(\text{stat})^{+7}_{-10}(\text{syst})$ MeV.
- The three-body relative branching ratio $R_{\text{th}} = 1.98$ is consistent with $R_{\text{exp}} = 1.27^{+1.29}_{-0.95}$ within the large experimental uncertainties.

R. Aaij et al., arXiv: 2604.21257 (hep-ex)



Outlook

Interpretation

The $D_{s1}(2933)$ assignment is strongly affected by the nearby S -wave coupling channels.

Partial branching fractions

More precise measurements at LHCb and BESIII, particularly of

$$\frac{\Gamma(D_{s1}(2933) \rightarrow D_2^* K)}{\Gamma(D_{s1}(2933) \rightarrow DK^*)},$$

would provide a more stringent test of the assignment and facilitate the search for the $2P_1'$ partner state.



Thank You