Physics-Informed Neural Networks for 3D Magnetic Field Reconstruction in Precision Muon Experiments

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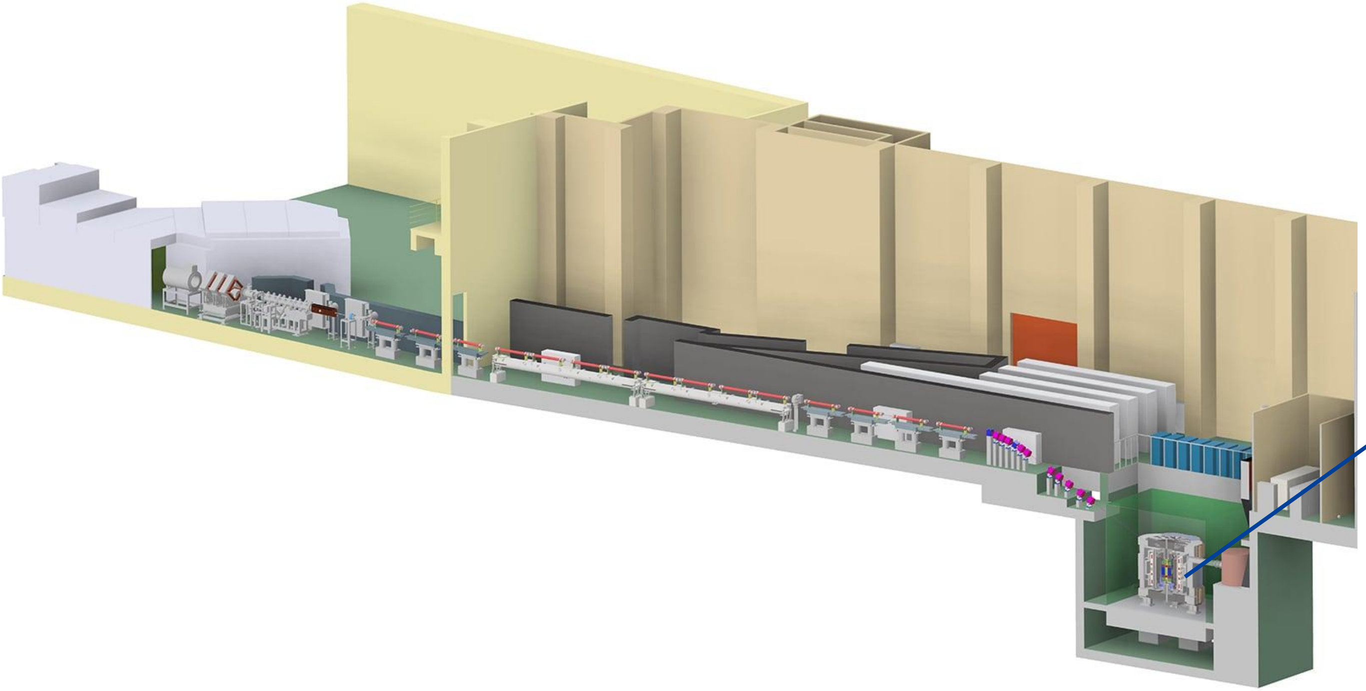
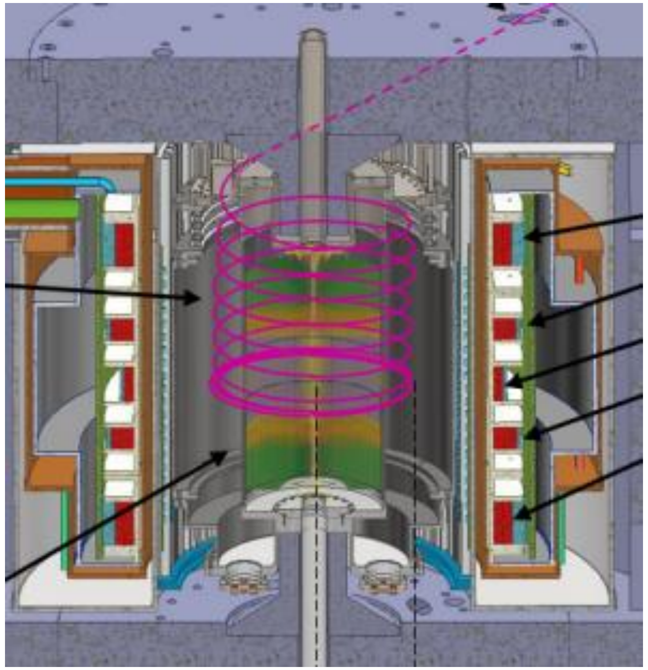
arXiv:2605.25640

J-PARC Muon g−2/EDM Experiment



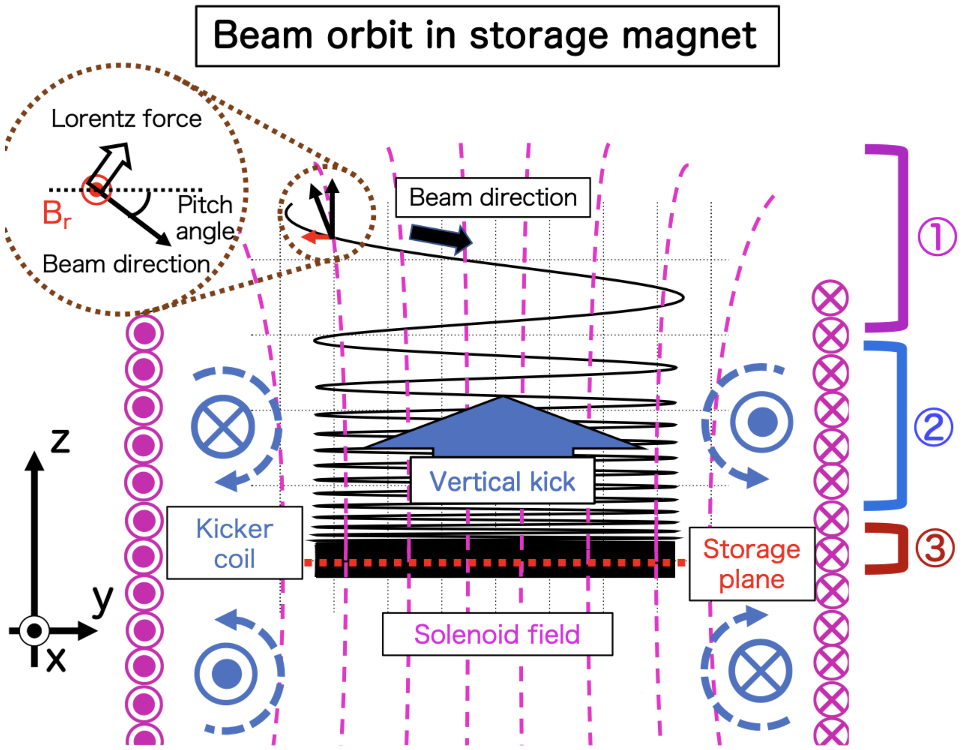
- Storage region



- Scientific Goals
  - Precision measurement of muon g−2
  - Search for muon EDM
  - Probe physics beyond the Standard Model
- Accurate magnetic field knowledge is essential throughout the experiment

Magnetic Field Reconstruction Challenge

- Challenges
  - Vacuum chambers restrict probe placement
  - Sparse boundary measurements
  - Complex 3D magnetic field
  - High precision required



- Existing Solution:
  - Laplace Multipole Expansion
  - Truncation error
  - Sensitive to measurement noise

- Goal: Reconstruct inaccessible magnetic fields from sparse measurements.

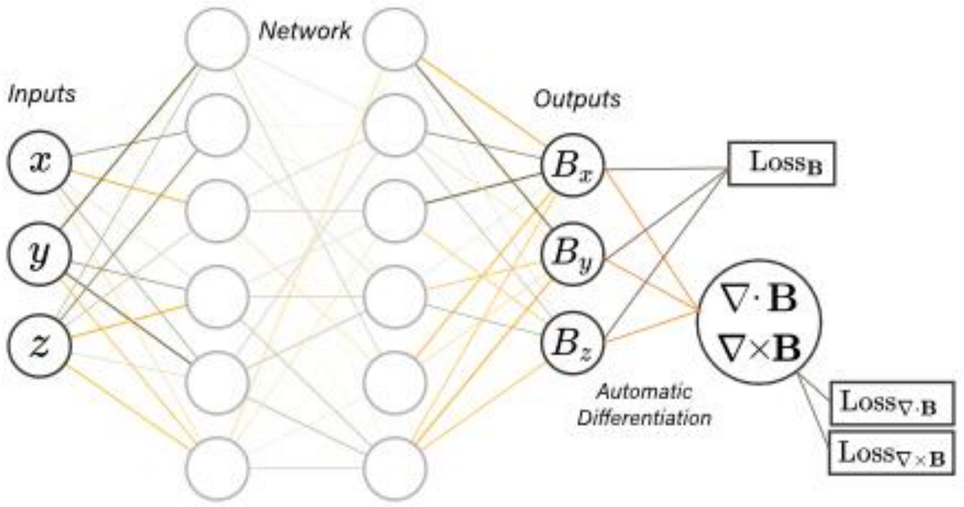
Laplace Multipole Expansion vs Physics-Informed Neural Networks

- Laplace Multipole Expansion :
- Advantages
  - Mature mathematical framework
  - Widely used in accelerator physics
- Limitations
  - Truncation error
  - Sensitive to noise
  - Manual order selection

- Physics-Informed Neural Networks
- Advantages
  - Continuous field representation
  - No truncation error
  - Better performance with sparse measurements
  - Physical constraints naturally embedded

PINN Framework

- Input / Output
  - Inputs: spatial coordinates (x,y,z)
  - Outputs: magnetic field components (Bx ,By ,Bz)
- Network
  - A fully-connected neural network maps inputs to field components
  - Continuous and differentiable field representation
- Loss Function
  - Data loss ($Loss_B$): fits measured field values
  - Physics losses ($Loss_{\nabla\cdot B}$, $Loss_{\nabla\times B}$): enforce Maxwell equations
  - Automatic differentiation enables exact computation of derivatives

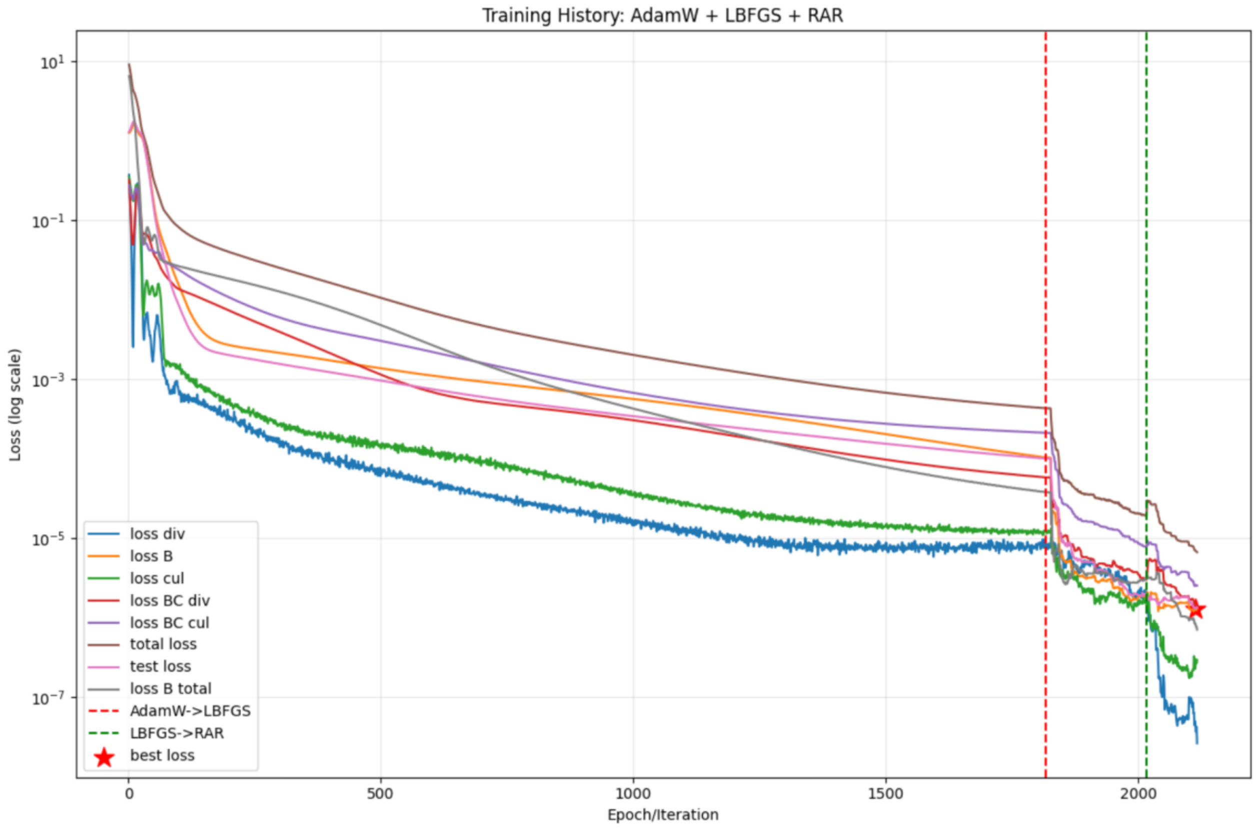


- Source-Free Maxwell Equations
  - $\nabla\cdot B=0$ (Divergence-free magnetic field)
  - $\nabla\times B=0$ (Curl-free in current-free regions)
- By combining measurement data with Maxwell's equations, PINNs can produce continuous, physically consistent 3D magnetic field reconstructions

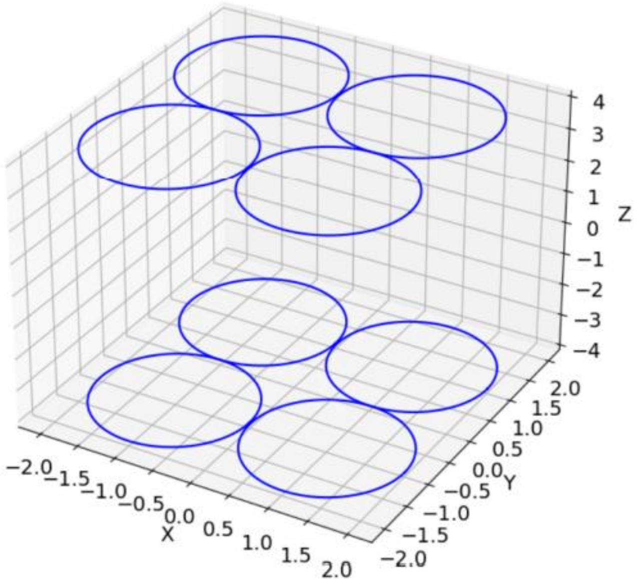
Key Features

arXiv:2605.25640

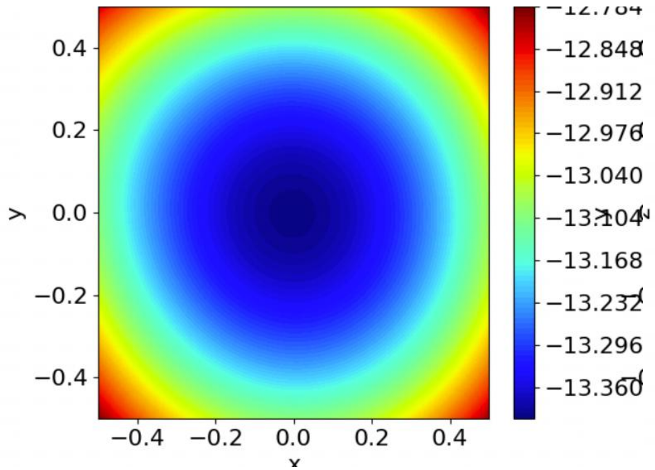
- Physics constraints
  - Domain collocation points
  - Measurement locations
- Three-stage optimization
  - AdamW -> LBFGS -> RAR
- SIREN representation
  - Periodic activation functions for high-frequency magnetic fields.



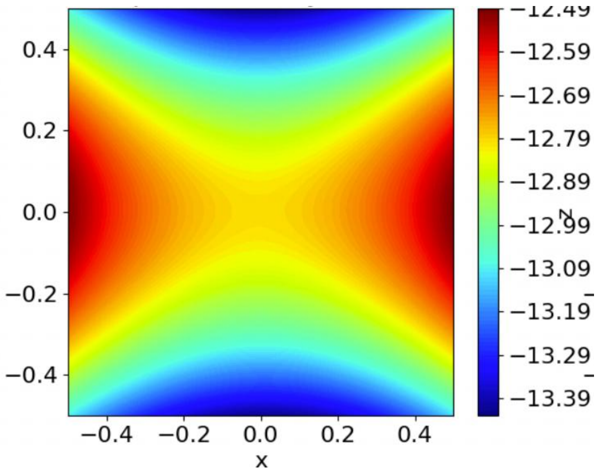
Simulation Validation



Vertical Field Bz at z=0

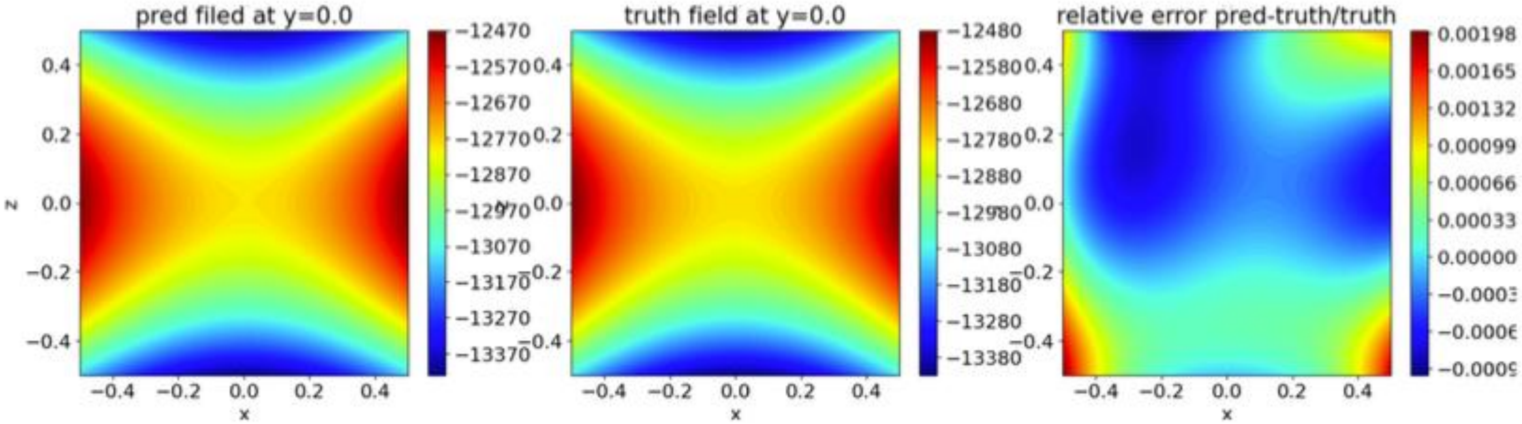


Vertical Field Bz at y=0



- Simulation Setup
  - Biot–Savart magnetic field
  - Multiple circular current loops
  - Cubic reconstruction domain
  - Sparse boundary sampling

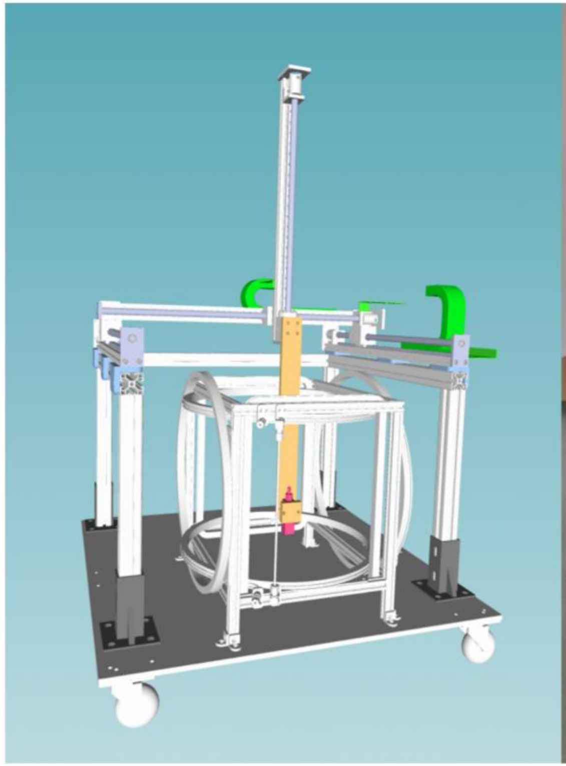
Spatial Reconstruction with Sparse Boundary Measurements



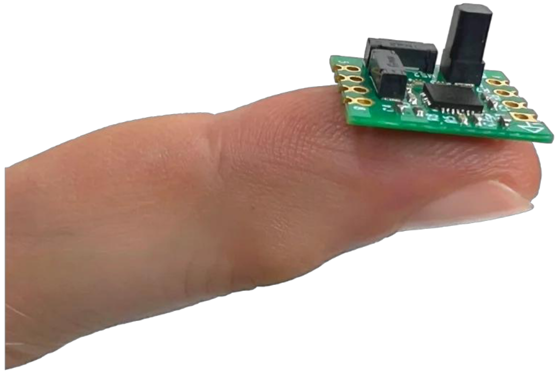
Spatial reconstruction quality at the y=0 plane.
From left to right: PINN prediction, Biot–Savart ground truth, and relative error distribution.
The proposed method achieves approximately 10^{-4} accuracy despite using only sparse boundary measurements.

- Key Results
  - Excellent agreement with analytical solution
  - Accurate reconstruction using only 18 boundary measurements
  - Reconstruction accuracy reaches 10^{-4}

Experimental Platform: 3D Magnetic Field Mapping System

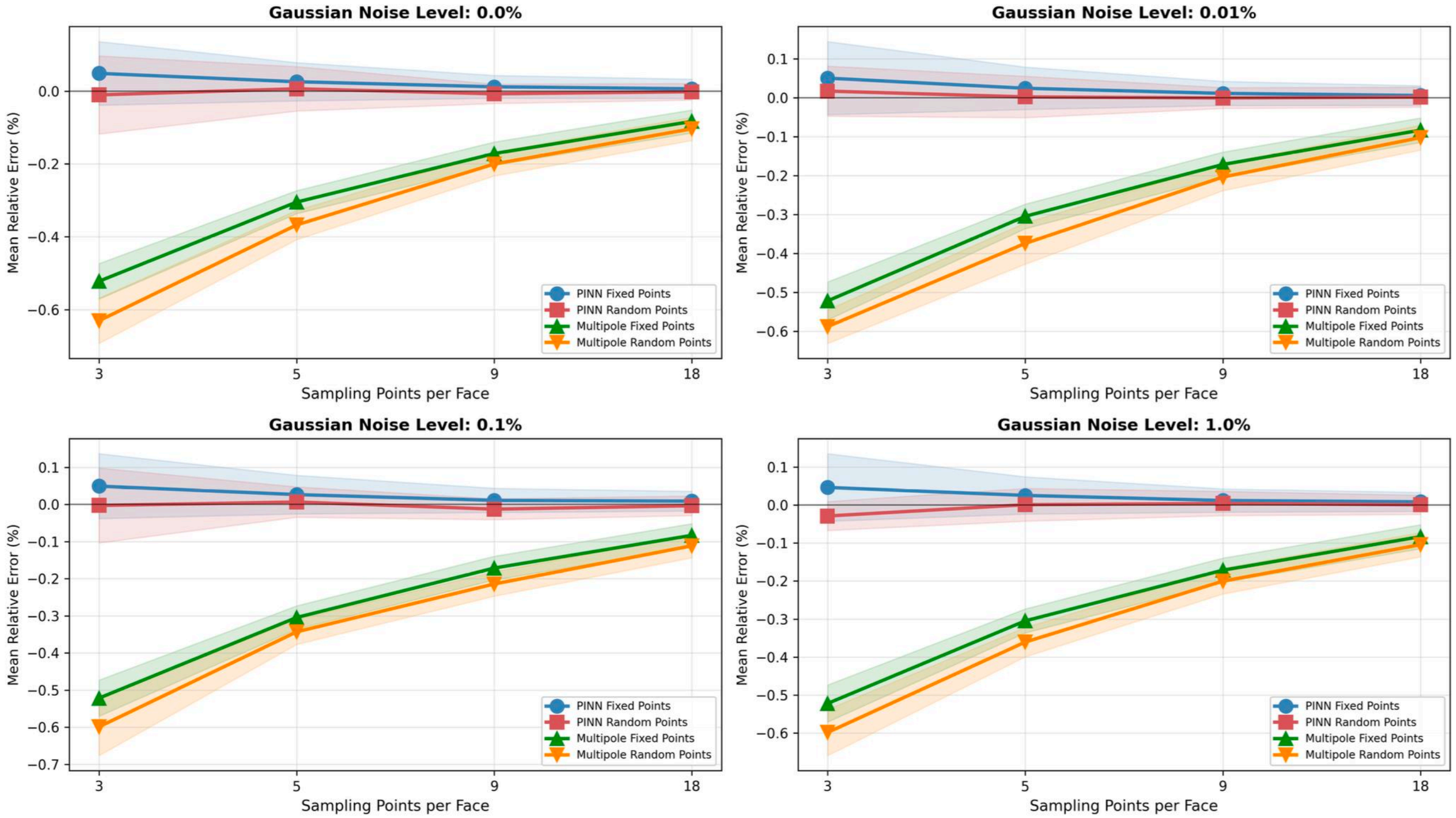


- Experimental system includes
  - Orthogonal coil system
  - Three-axis translation stage
  - RM3100 magneto-inductive sensor
- Measurement range: 0–400 μT
- Validation performed under ambient laboratory conditions.



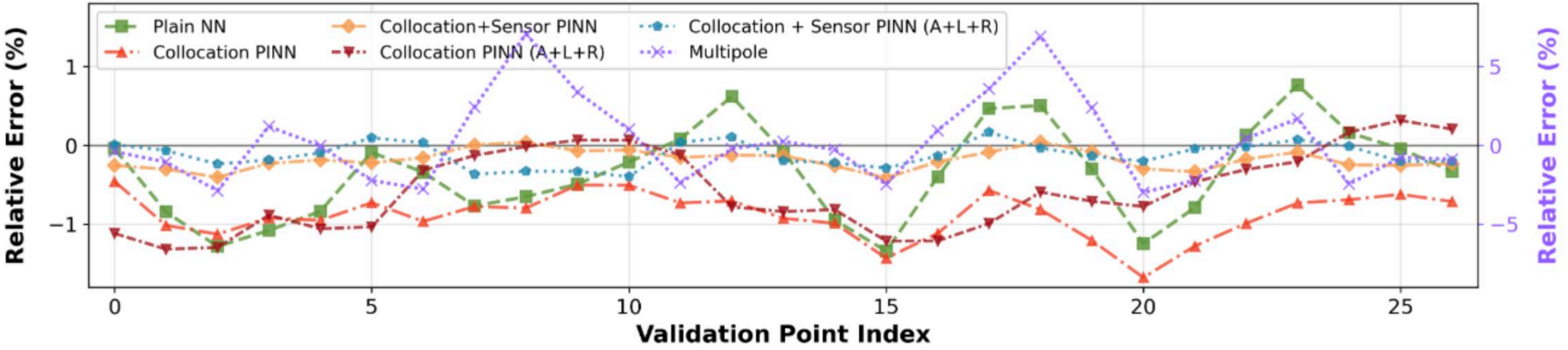
- Resolution: 13 nT
- Noise floor: 15 nT
- Three-axis translation stage
- Orthogonal coil system

Experimental Results: 10^{-3} Accuracy with High Robustness



- PINN consistently outperforms multipole expansion.
- Performance
  - Sub-percent relative accuracy
  - Best performance reaches 10^{-3}
  - Robust under sparse measurements
- Key observations
  - Better local consistency
  - Lower prediction variance
  - Higher robustness to experimental noise

Ablation Study

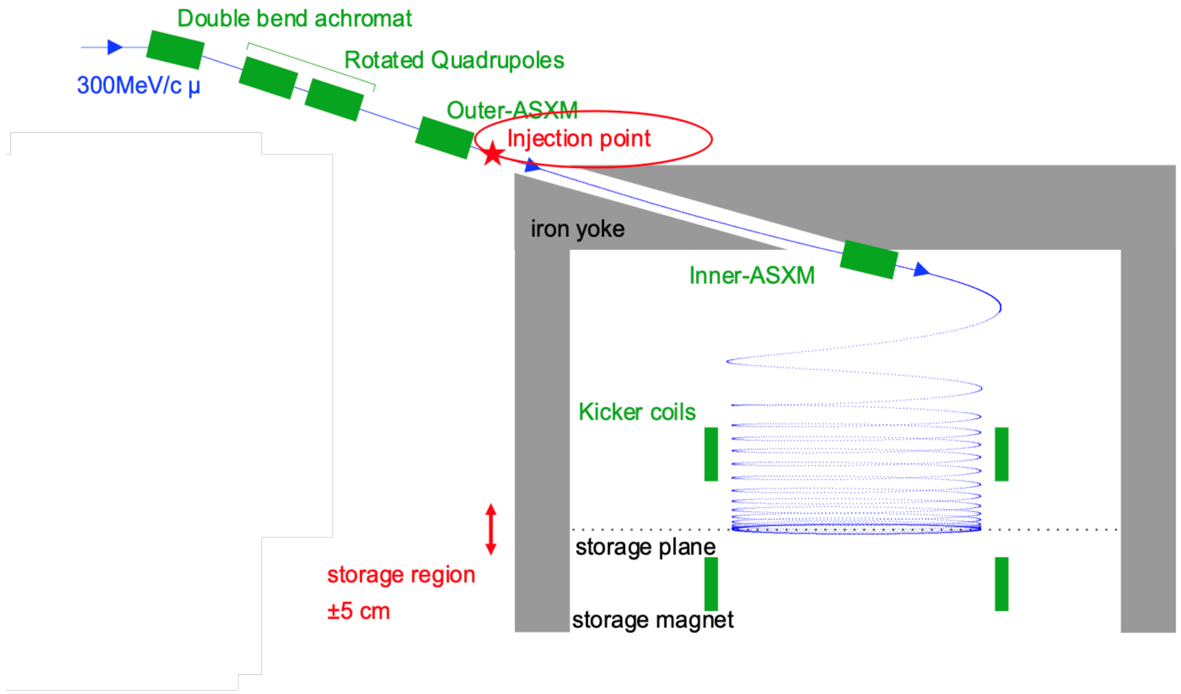


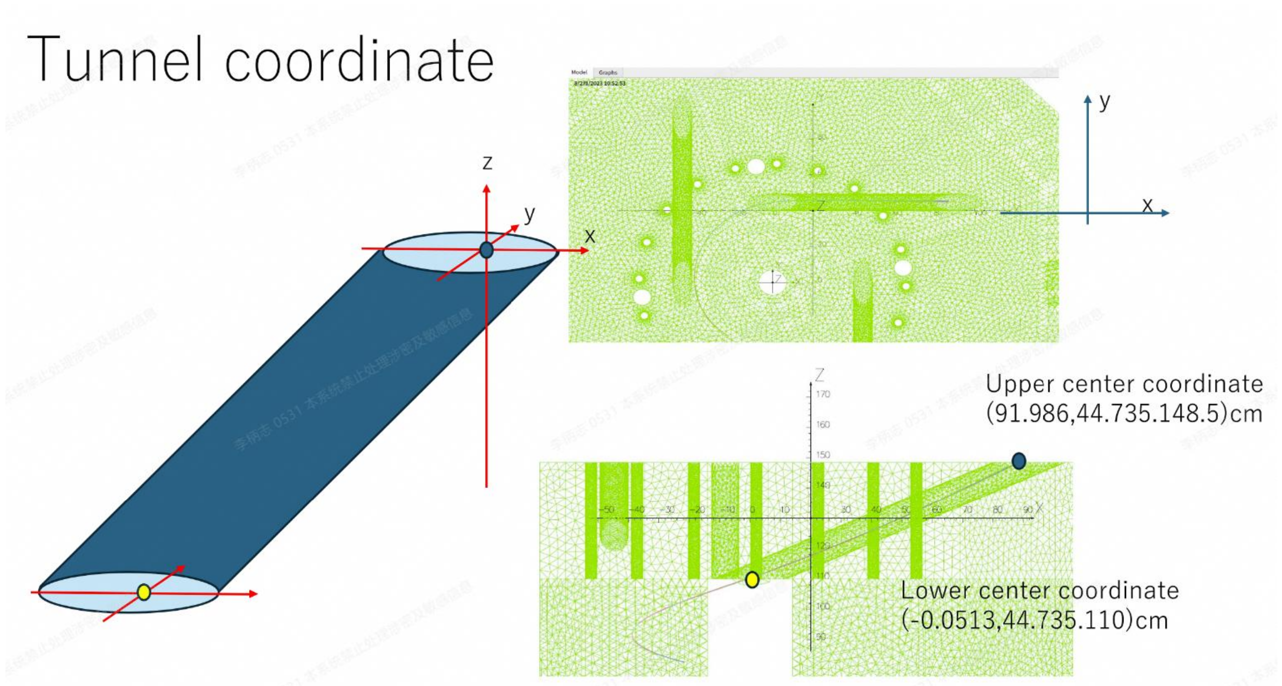
Comparison of relative reconstruction errors for the Multipole, Plain NN, PINN and Full PINN

- To understand where the performance gain comes from, we evaluate each component of the proposed framework independently.
- Key Findings
  - Physics constraints significantly improves local reconstruction accuracy by enforcing physical consistency at measurement locations.
  - **RAR** further reduces residual errors by concentrating training on challenging regions.
  - Combining all proposed components yields the best overall performance.

Application to the J-PARC Injection Line

- Apply the PINN framework to the magnetic field reconstruction of the J-PARC Muon g−2/EDM injection line.
- First apply on 3D-injection dataset that comes from simulation of the injection line



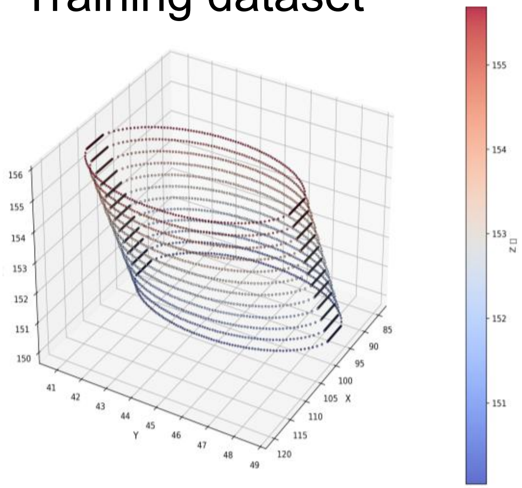


Z range : (94.60~155.69)cm

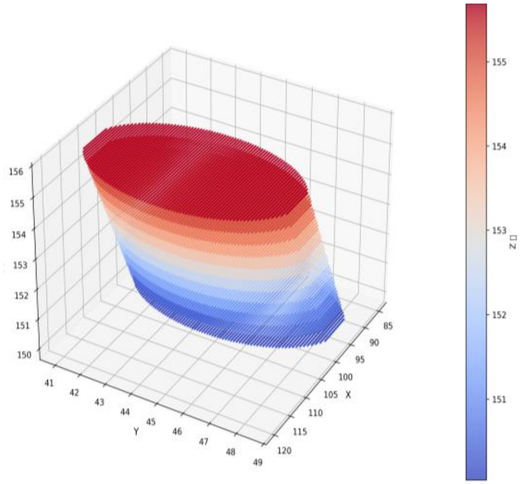
Initial Validation on Injection-Line Simulation

- Test Design
  - Data are distributed across many Z-planes.
  - We first selected 12 Z-planes from one end of the dataset.
  - This subset corresponds to roughly 10% of the full simulation data.
- Training / Test Split
  - Outer boundary layers → used for training.
  - Central interior volume → used as the test region.
- Result
  - Initial PINN verification successful
  - Accuracy ≈ 1%
- Even with sparse boundary information, the PINN successfully reconstructs the interior magnetic field structure of the injection-line simulation.

Training dataset



Test dataset



Conclusions & Outlook

- Conclusions
  - **PINN provide a physics-consistent framework** for 3D magnetic field reconstruction.
  - **Physics and data constrained optimization** significantly improves reconstruction accuracy.
  - Achieved **$\sim 10^{-4}$** accuracy in simulation and **$\sim 10^{-3}$** precision in experimental validation.
  - Demonstrated the feasibility of applying PINNs to the **J-PARC Muon g−2/EDM injection line**.
- Outlook
  - More application and integration with J-PARC Muon g−2/EDM experiment
  - Deployment in future precision experiments

Call for Collaboration

- We welcome collaborations with experiments requiring high-precision magnetic-field reconstruction.
- Additional simulation and experimental magnetic-field datasets are highly appreciated for further validation and development.
- My email address: hhzxlsjtu@sjtu.edu.cn

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THANKS FOR WATCHING

SJTU