Institute of High Energy Physics
Chinese Academy of Sciences

Discovering Hybrid Mesons

EXOHAD
EXOTIC HADRONS TOPICAL COLLABORATION

Spectroscopy: Light Revealing the Universe

From light to knowledge: spectroscopy has unlocked the structure of atoms, tested the foundations of quantum theory, and revealed a new world of particles.

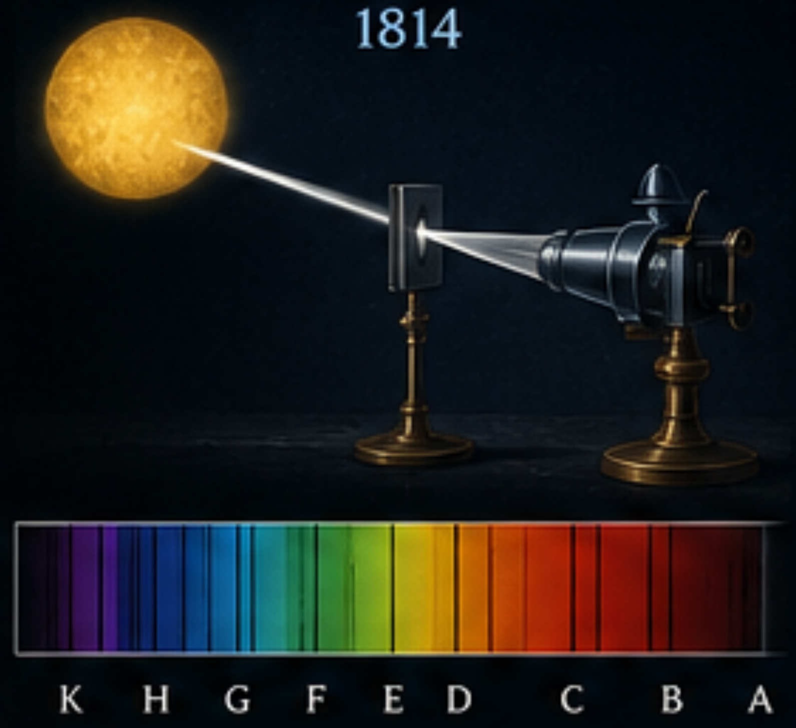
1. Newton's Prism Experiment

1666

Newton showed that white light is composed of colors. This was the birth of optical spectroscopy.

2. Fraunhofer's Solar Analysis

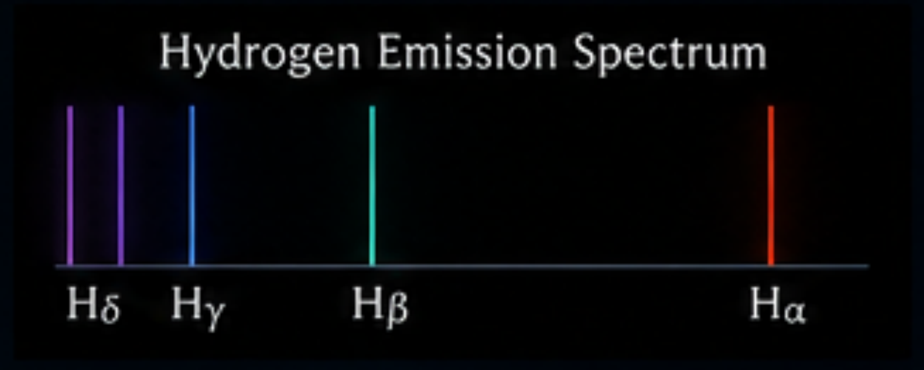
1814

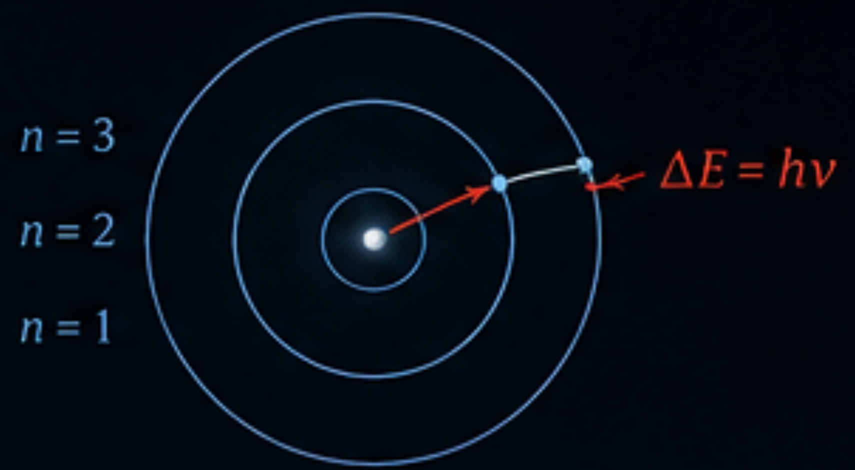


Fraunhofer discovered dark absorption lines in the solar spectrum. These lines revealed that atoms leave unique fingerprints in light.

3. Hydrogen Spectrum & Bohr Model

1885–1913

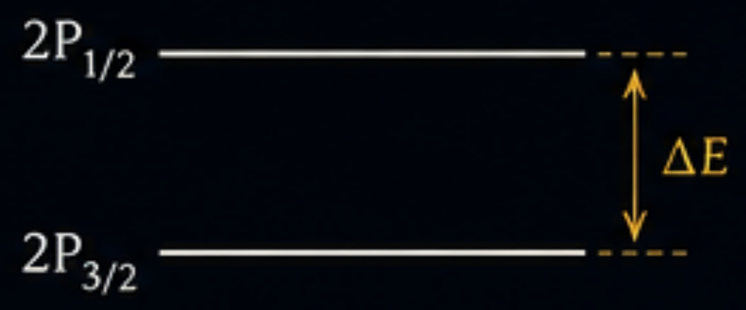


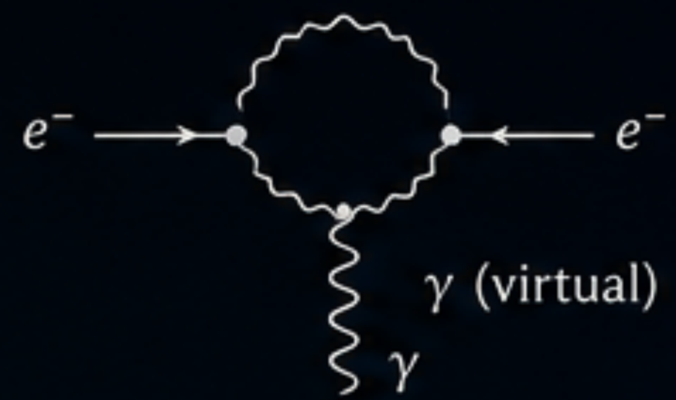


The discrete hydrogen spectrum led Bohr to propose quantized energy levels in atoms—launching modern atomic physics.

4. Lamb Shift & QED

1947–195 Lamb, 1947

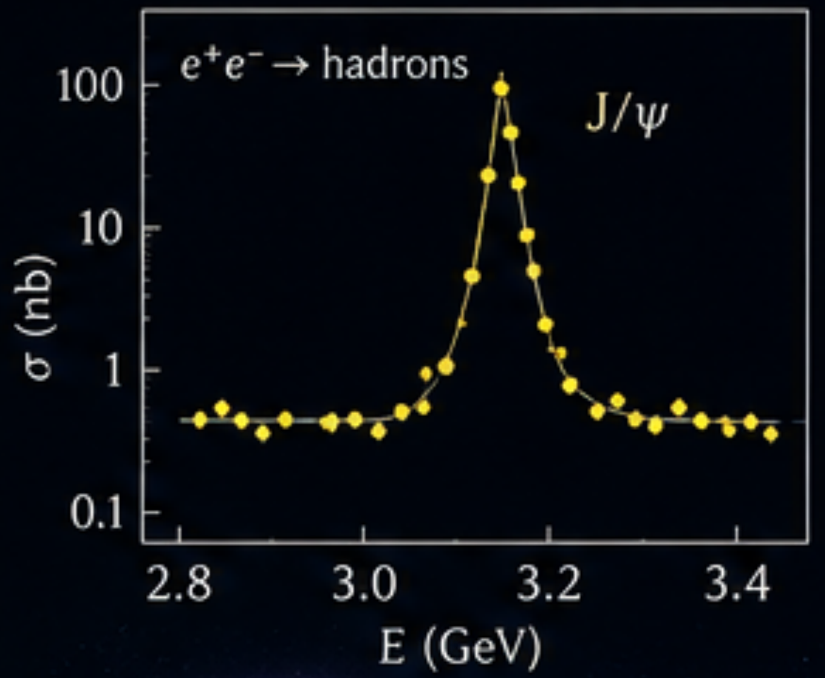


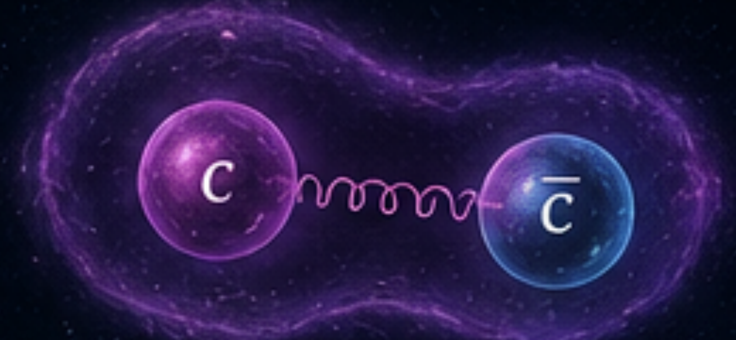


A tiny splitting (Lamb shift) in hydrogen revealed the effects of quantum vacuum fluctuations—confirmed by Quantum Electrodynamics, the most precise theory in physics.

5. Discovery of J/ψ

1974

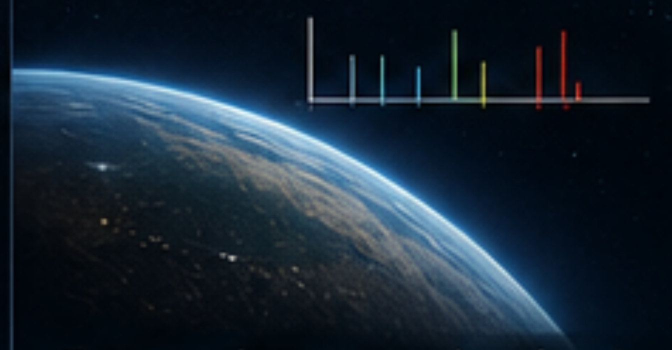




Spectroscopy of e^+e^- collisions revealed a new particle, the J/ψ, a bound state of charm quarks. This discovery opened the door to the rich field of the quark model and particle physics.

Why Spectroscopy Matters

Reveals the composition, temperature, density, and motion of stars, galaxies, and the cosmos.



Detects elements and molecules in planetary atmospheres, interstellar clouds, and exoplanets.

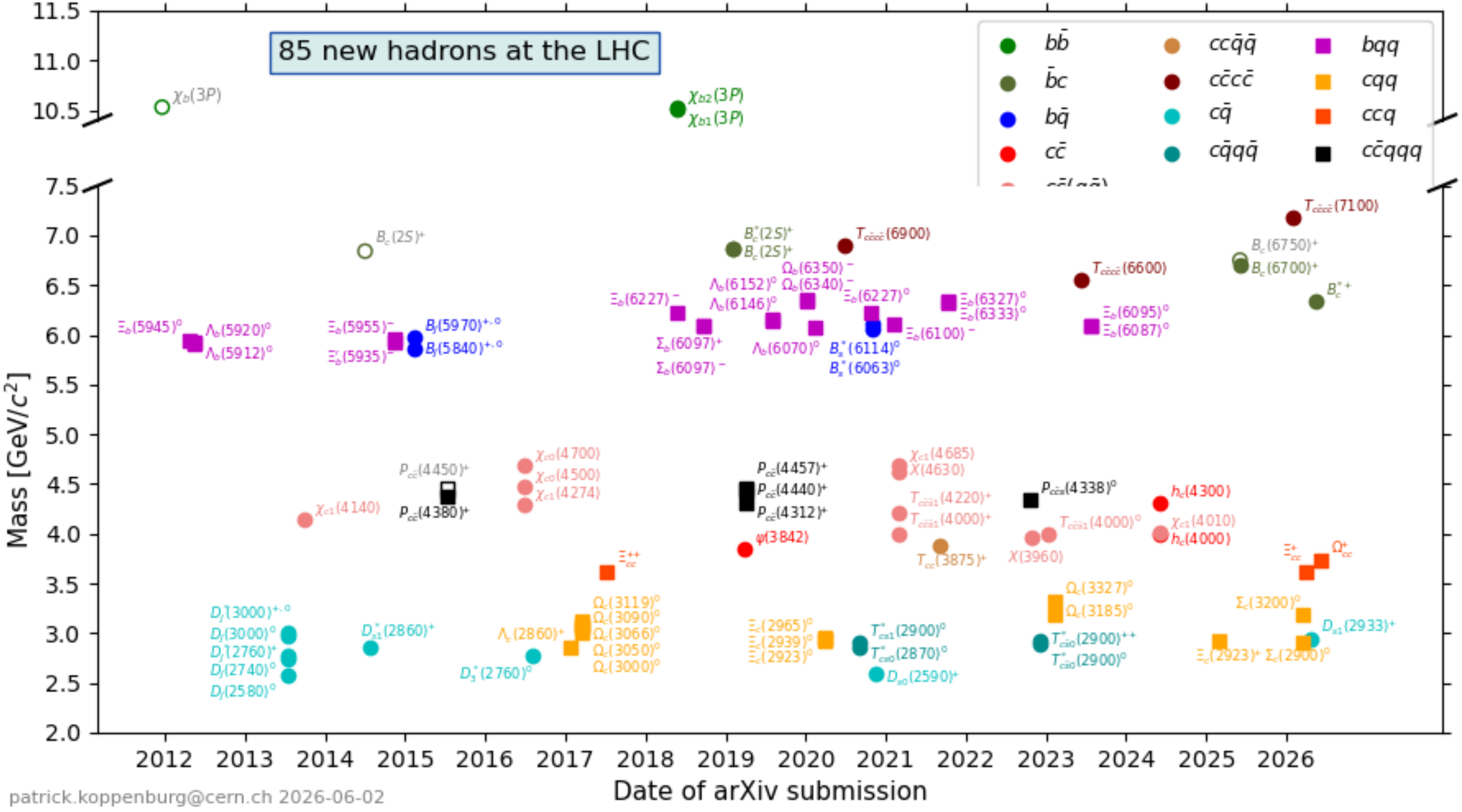
Essential tool in chemistry, materials science, and biology for identifying and quantifying matter.

Enables technologies from lasers and semiconductors to medical imaging and environmental monitoring.

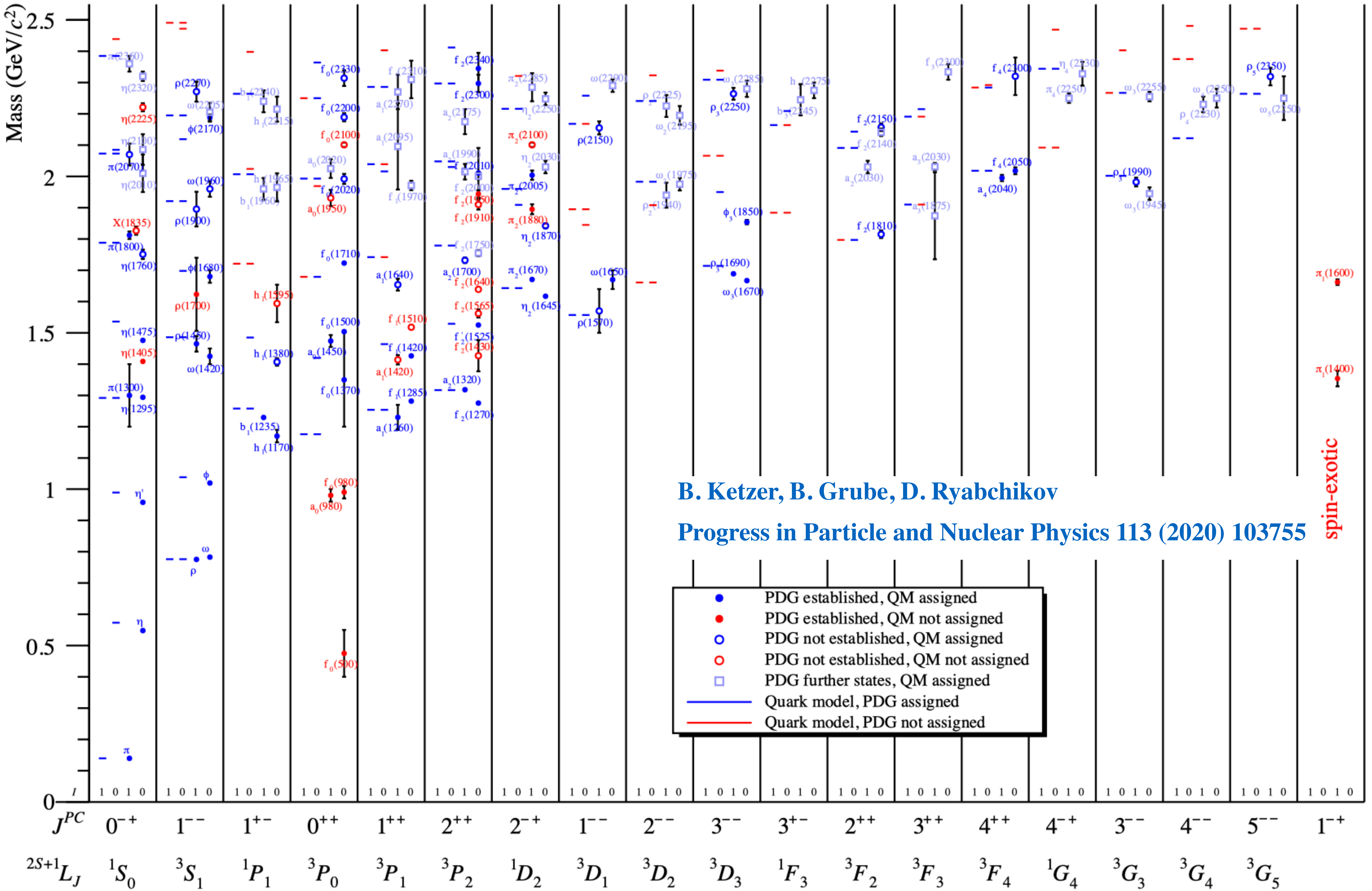
Tests fundamental physics and drives the development of new theories and technologies.

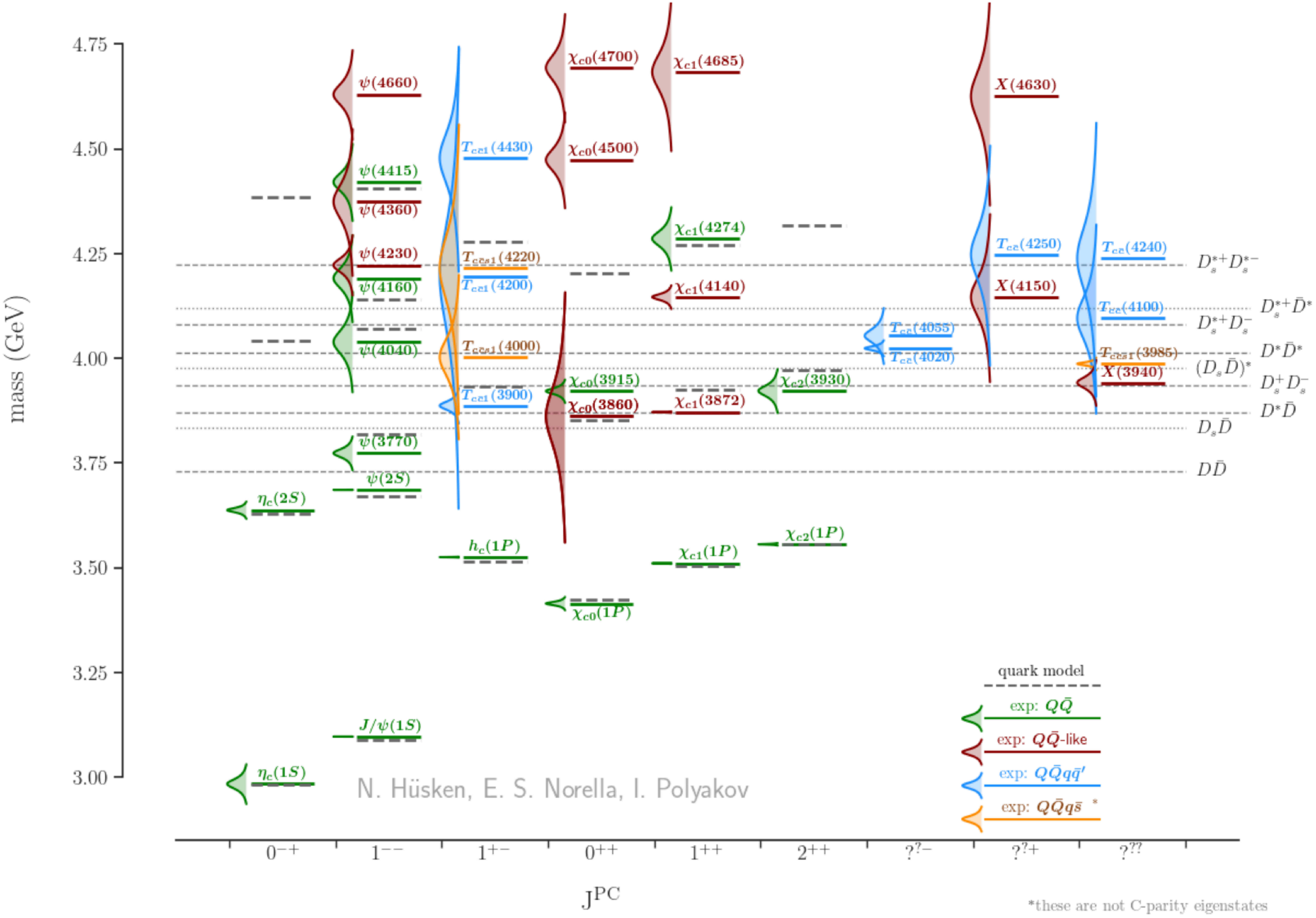
From prisms to particles, spectroscopy turns light into understanding—across all scales of nature.

Spectroscopy connects the very small to the very large—and continues to reveal the unknown.



85 new hadrons at the LHC
Mass [GeV/c2]
Date of arXiv submission
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025
2026
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
10.5
11.0
11.5
bb̄
b̄c
bq̄
cc̄
ccq̄q̄
cc̄cc̄
cq̄
cq̄qq̄
bqq
cqq
ccq
cc̄qqq
χb(3P)
χb2(3P)
χb1(3P)
Bc(2S)+
B*c(2S)+
Bc(2S)+
Tcc̄cc̄(6900)
Tcc̄cc̄(6600)
Tcc̄cc̄(7100)
Bc(6750)+
Bc(6700)+
B*c+
Ξb(5945)0
Λb(5920)0
Λb(5912)0
Ξb(5955)−
Ξ'b(5935)−
BJ(5970)+,0
BJ(5840)+,0
Ξb(6227)−
Σb(6097)+
Σb(6097)−
Λb(6152)0
Λb(6146)0
Ωb(6350)−
Ωb(6340)−
Λb(6070)0
Ξb(6227)0
B*s(6114)0
B*s(6063)0
Ξb(6100)−
Ξb(6327)0
Ξb(6333)0
Ξb(6095)0
Ξb(6087)0
χc1(4140)
Pcc̄(4450)+
Pcc̄(4380)+
χc0(4700)
χc0(4500)
χc1(4274)
Pcc̄(4457)+
Pcc̄(4440)+
Pcc̄(4312)+
ψ3(3842)
χc1(4685)
X(4630)
Tcc̄s1(4220)+
Tcc̄s1(4000)+
Tcc(3875)+
Pcc̄s(4338)0
Tcc̄s1(4000)0
X(3960)
hc(4300)
χc1(4010)
hc(4000)
Ξcc++
Ξcc+
Ωcc+
DJ*(3000)+,0
DJ(3000)0
DJ*(2760)+
DJ(2740)0
DJ(2580)0
D*s1(2860)+
D*s(2760)0
Λc(2860)+
Ωc(3119)0
Ωc(3090)0
Ωc(3066)0
Ωc(3050)0
Ωc(3000)0
Ξc(2965)0
Ξc(2939)0
Ξc(2923)0
T*cs1(2900)0
T*cs0(2870)0
Ds0(2590)+
Ωc(3327)0
Ωc(3185)0
T*cs̄0(2900)++
T*cs̄0(2900)0
Σc(3200)0
Ξc(2923)+
Σc(2900)0
Ds1(2933)+
patrick.koppenburg@cern.ch 2026-06-02


Mass (GeV/c^2)
B. Ketzer, B. Grube, D. Ryabchikov
Progress in Particle and Nuclear Physics 113 (2020) 103755
spin-exotic
PDG established, QM assigned
PDG established, QM not assigned
PDG not established, QM assigned
PDG not established, QM not assigned
PDG further states, QM assigned
Quark model, PDG assigned
Quark model, PDG not assigned
J^{PC} 0^{-+} 1^{--} 1^{+-} 0^{++} 1^{++} 2^{++} 2^{-+} 1^{--} 2^{--} 3^{--} 3^{+-} 2^{++} 3^{++} 4^{++} 4^{-+} 3^{--} 4^{--} 5^{--} 1^{-+}
$^{2S+1}L_J$ $^1S_0$ $^3S_1$ $^1P_1$ $^3P_0$ $^3P_1$ $^3P_2$ $^1D_2$ $^3D_1$ $^3D_2$ $^3D_3$ $^1F_3$ $^3F_2$ $^3F_3$ $^3F_4$ $^1G_4$ $^3G_3$ $^3G_4$ $^3G_5$


mass (GeV)
4.75
4.50
4.25
4.00
3.75
3.50
3.25
3.00
J^PC
0−+
1−−
1+−
0++
1++
2++
?^?−
?^?+
?^??
η_c(1S)
η_c(2S)
J/ψ(1S)
ψ(2S)
ψ(3770)
ψ(4040)
ψ(4160)
ψ(4230)
ψ(4360)
ψ(4415)
ψ(4660)
h_c(1P)
T_cc̄1(3900)
T_cc̄s1(4000)
T_cc̄1(4200)
T_cc̄s1(4220)
T_cc̄1(4430)
χ_c0(1P)
χ_c0(3860)
χ_c0(3915)
χ_c0(4500)
χ_c0(4700)
χ_c1(1P)
χ_c1(3872)
χ_c1(4140)
χ_c1(4274)
χ_c1(4685)
χ_c2(1P)
χ_c2(3930)
T_cc̄(4020)
T_cc̄(4055)
X(4150)
T_cc̄(4250)
X(4630)
X(3940)
T_cc̄s1(3985)
T_cc̄(4100)
T_cc̄(4240)
D_s^{*+}D_s^{*−}
D_s^{*+}D̄^*
D_s^{*+}D_s^−
D^*D̄^*
(D_sD̄)^*
D_s^+D_s^−
D^*D̄
D_sD̄
DD̄
N. Hüsken, E. S. Norella, I. Polyakov
quark model
exp: QQ̄
exp: QQ̄-like
exp: QQ̄qq̄′
exp: QQ̄qs̄ *
*these are not C-parity eigenstates

hybrid hadrons and gluodynamics

C. Meyer & E.S. Swanson, arXiv:1502.07276

Past ideas for hybrid mesons

Volume 60B, number 2 PHYSICS LETTERS 5 January 1976

UNCONVENTIONAL STATES OF CONFINED QUARKS AND GLUONS☆

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Received 27 October 1975

VOLUME 37, NUMBER 18 PHYSICAL REVIEW LETTERS 1 NOVEMBER 1976

ψ Spectroscopy of a Charm String*

R. C. Giles and S.-H. H. Tye
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 13 August 1976)

COLOURED QUARK AND GLUON CONSTITUENTS IN THE MIT BAG MODEL: A MODEL OF MESONS

Ted BARNES
Department of Physics, University of Southampton, Southampton SO9 5NH, England

Received 24 October 1977
(Revised 7 May 1979)

PHYSICAL REVIEW D VOLUME 17, NUMBER 3 1 FEBRUARY 1978

Model of mesons with constituent gluons*

D. Horn†
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J. Mandula‡
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 28 January 1977)

Volume 124B, number 3,4 PHYSICS LETTERS 28 April 1983

A FLUX TUBE MODEL FOR HADRONS

Nathan ISGUR [1,2] and Jack PATON
Department of Theoretical Physics, University of Oxford, 1 Keble Road, Oxford, OX1, 3NP, England

Received 20 December 1982

Lattice Hybrid Computations

Volume 129B, number 5 PHYSICS LETTERS 29 September 1983

MESONS WITH EXCITED GLUE

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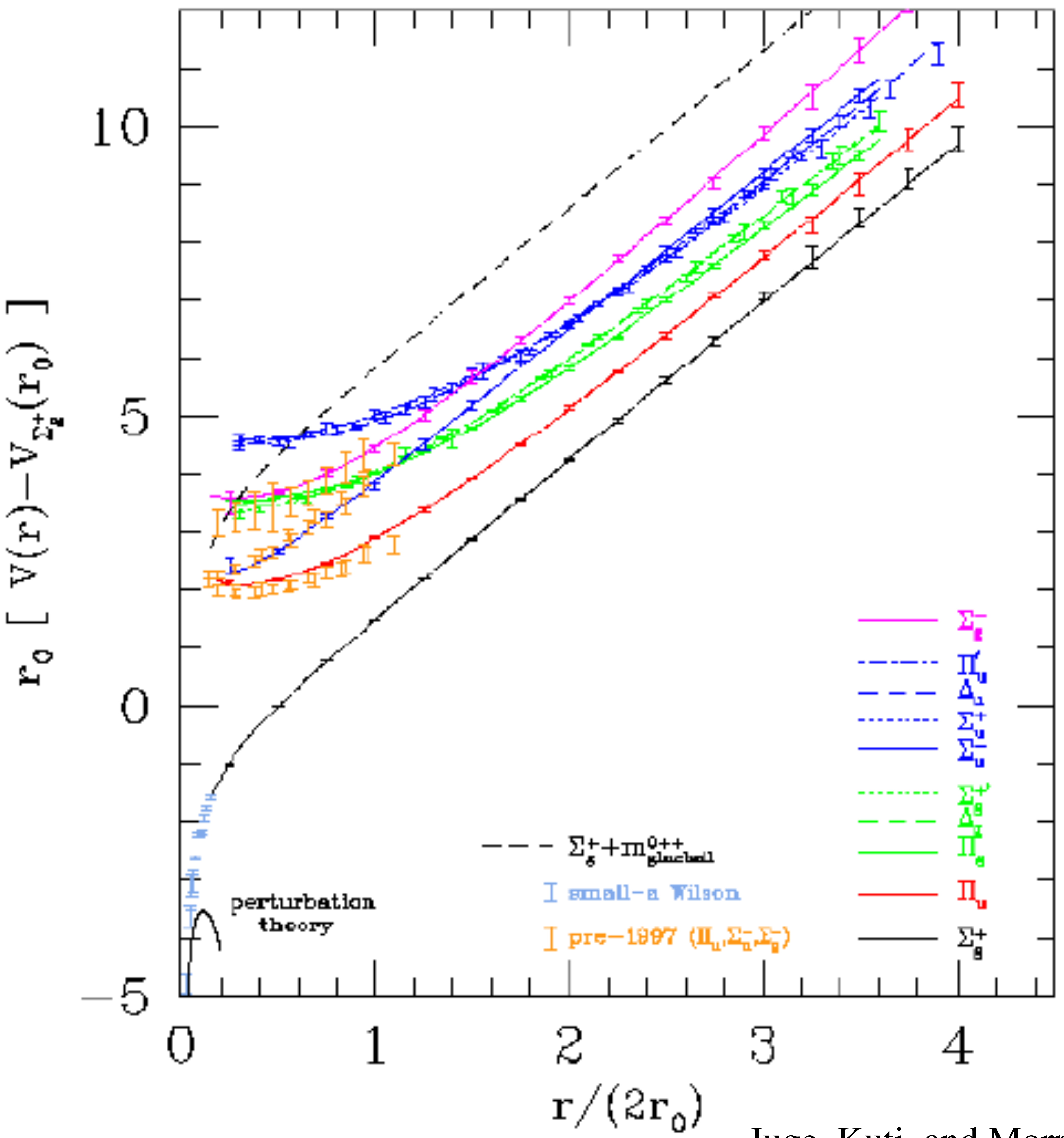
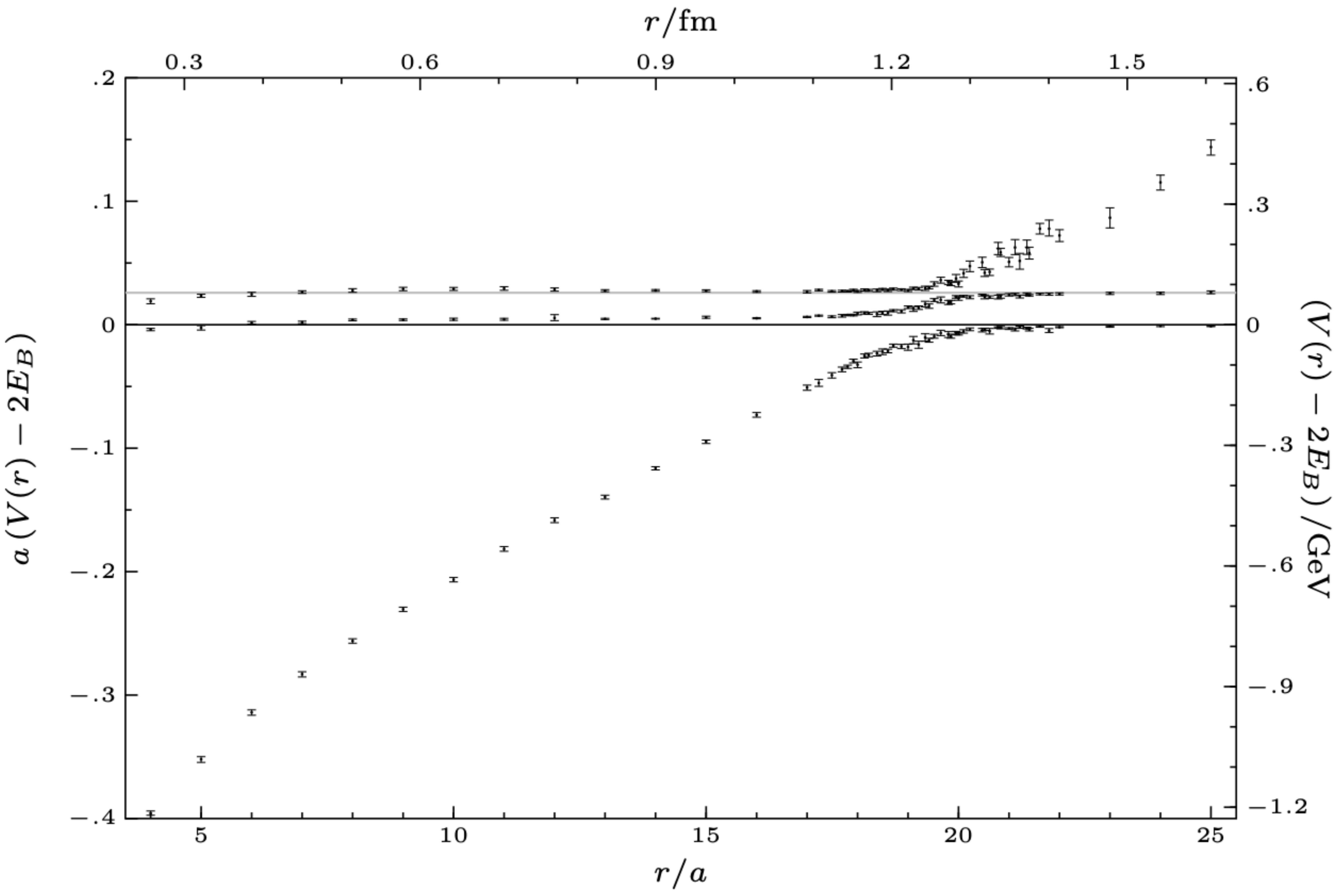
Lattice Hybrid Computations



Juge, Kuti, and Morningstar arXiv:hep-ph/9902336

Lattice Hybrid Computations



arXiv:1902.04006

Lattice Hybrid Computations

The 'gluelump' spectrum (static octet source + glue)

J^{PC}	mass (GeV)
1^{+-}	0.87(15)
1^{--}	1.25(16)
2^{--}	1.45(17)
2^{+-}	1.86(19)
3^{+-}	1.86(18)
0^{++}	1.98(18)
4^{--}	2.13(18)
1^{-+}	2.15(20)

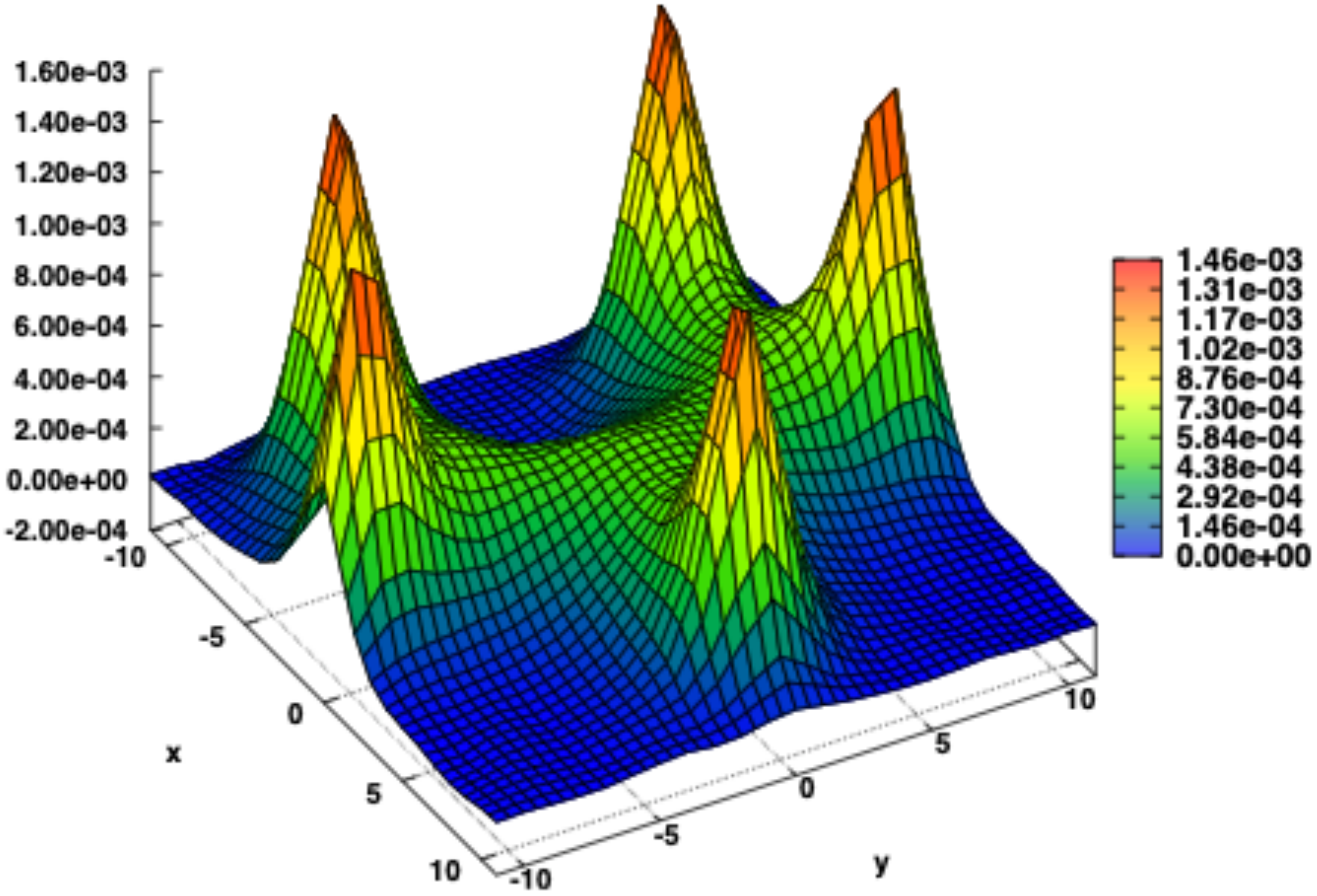
[only mass differences are well-defined]

M. Foster and C. Michael [UKQCD Collaboration], Phys. Rev. D 59, 094509 (1999).

G. S. Bali and A. Pineda, Phys. Rev. D 69, 094001 (2004)

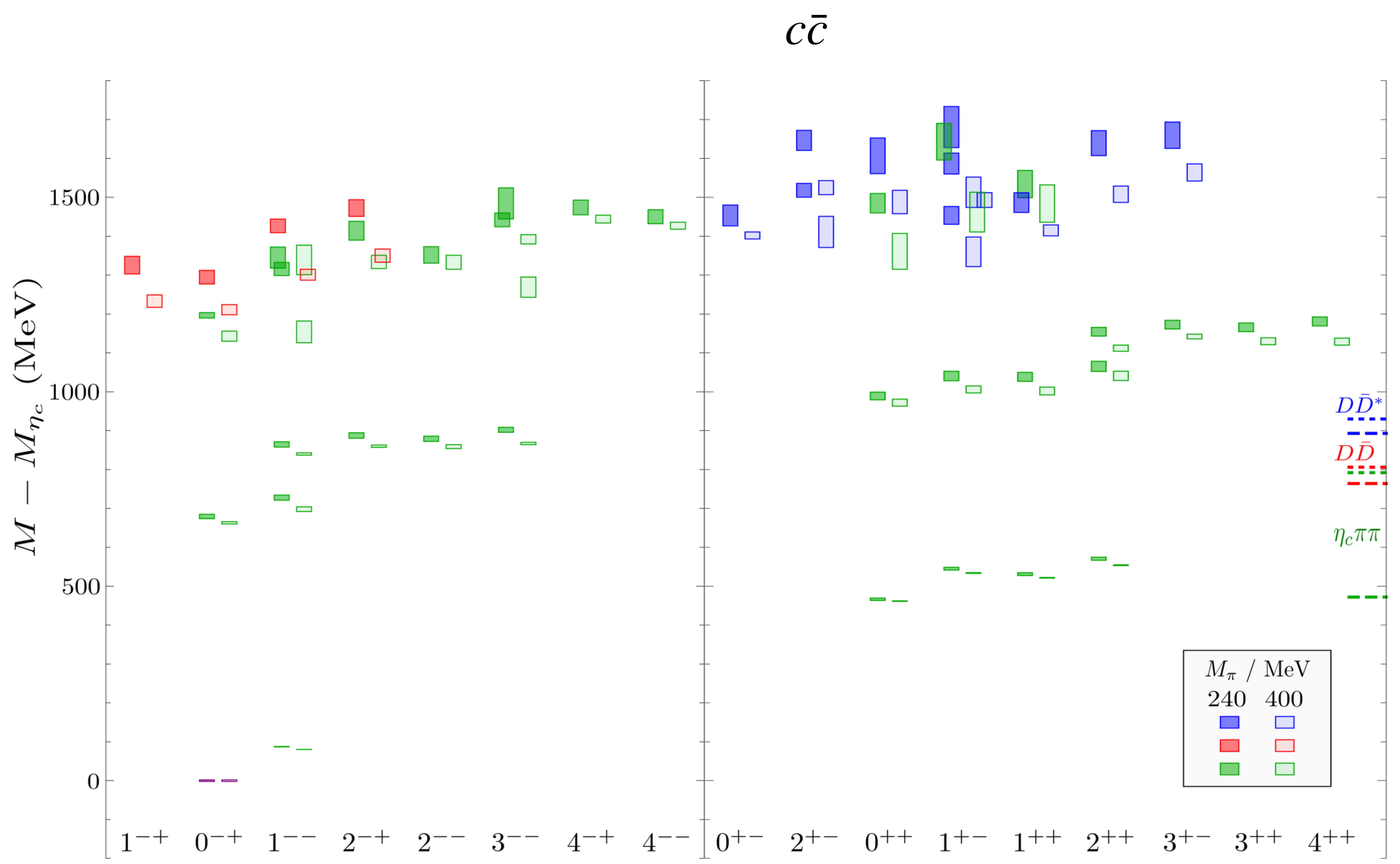
K. Marsh and R. Lewis, Phys. Rev. D 89, 014502 (2014)

Lattice Hybrid Computations



N Cardoso and P. Bicudo, arXiv:1209.1532

Lattice Hybrid Computations



Cheung et al. [HadSpec], JHEP 1612, 089 (2016).

Effective Field Theory

Obtain Schrödinger-type equations for heavy quark hybrids with pNRQCD.

$$\mathcal{L} = \mathrm{tr}\left(H^{i\dagger}\left(\delta_{ij} i\partial_0 - h_{H\,ij}\right) H_j\right)$$

$$h_{H\,ij} = \left(-\frac{\nabla^2}{m_Q} + V_{\Sigma_u^-}(r)\right)\delta_{ij} + (\delta_{ij} - \hat{r}_i\hat{r}_j)\left[V_{\Pi_u}(r) - V_{\Sigma_u^-}(r)\right]$$

$$\left[-\frac{1}{mr^2}\,\partial_r r^2 \partial_r + \frac{1}{mr^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)}\end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma^{(N)} \\ \psi_{-\Pi}^{(N)}\end{pmatrix} = \mathcal{E}_N \begin{pmatrix}\psi_\Sigma^{(N)} \\ \psi_{-\Pi}^{(N)}\end{pmatrix}$$

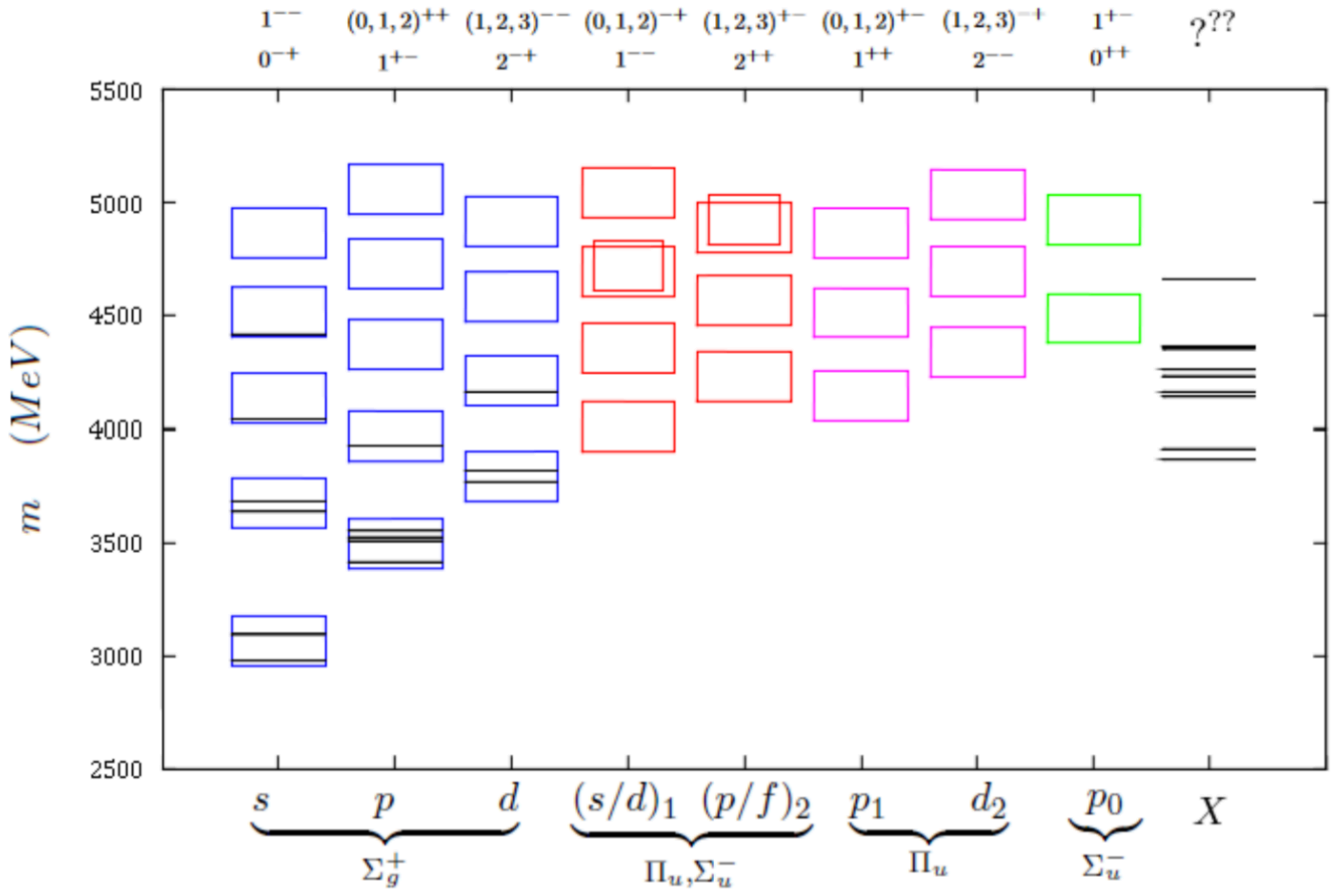
$$\left[-\frac{1}{mr^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{mr^2} + E_\Pi^{(0)}\right]\psi_{+\Pi}^{(N)} = \mathcal{E}_N\, \psi_{+\Pi}^{(N)}$$

M. Berwein, N. Brambilla, J. Castella, A. Vairo, arXiv:1510.04299

R. Oncala and J. Soto, arXiv: 1702.03900

N. Brambilla, W.-K. Lai, J. Segovia, J. Castella, A. Vairo, arXiv:1805.07713

Effective Field Theory



R. Oncala and J. Soto, arXix: 1702.03900

Born-Oppenheimer Approximation

all other models fail to reproduce this pattern

		multiplet	$\vec{\sigma}$
H_1	$\psi^\dagger \vec{B} \chi$	1^{--}	$(0,1,2)^{-+}$
H_2	$\psi^\dagger \nabla \times \vec{B} \chi$	1^{++}	$(0,1,2)^{+-}$
H_3	$\psi^\dagger \nabla \cdot \vec{B} \chi$	0^{++}	1^{+-}
H_4	$\psi^\dagger [\nabla \vec{B}]_2 \chi$	2^{++}	$(1,2,3)^{+-}$

constituent gluon with $(J^{PC})_g = 1^{+-}$

J. Dudek, arXiv:1106.5515

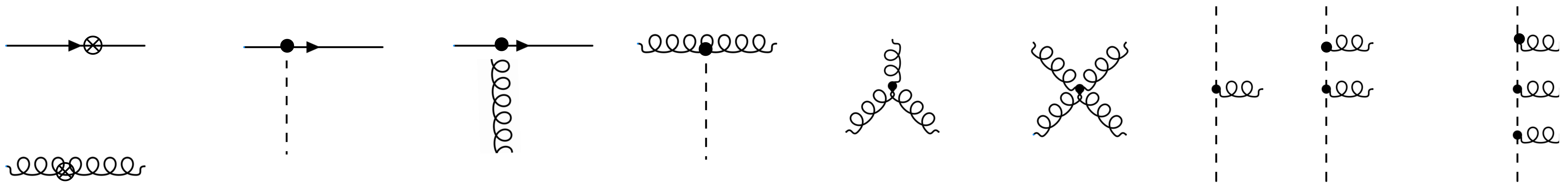
modelling hybrids

Hamiltonian QCD

Work in Coulomb gauge

$$H_{QCD} = \int d^3x \left[\psi^\dagger\left(-i\alpha\cdot\nabla + \beta m\right)\psi + \frac{1}{2}\left(\mathcal{J}^{-1/2}\Pi\mathcal{J}\cdot\Pi\mathcal{J}^{-1/2} + B\cdot B\right) - g\psi^\dagger\alpha\cdot A\psi\right] + H_C$$

$$H_C = \frac{1}{2}\int d^3x\, d^3y\, \mathcal{J}^{-1/2}\rho^A(\mathbf{x})\,\mathcal{J}^{1/2}\,\langle \mathbf{x}, A\,|\frac{g}{\nabla\cdot\mathbf{D}}(-\nabla^2)\frac{g}{\nabla\cdot\mathbf{D}}\,|\,\mathbf{y}, B\rangle\,\mathcal{J}^{1/2}\rho^B(\mathbf{y})\mathcal{J}^{-1/2}$$



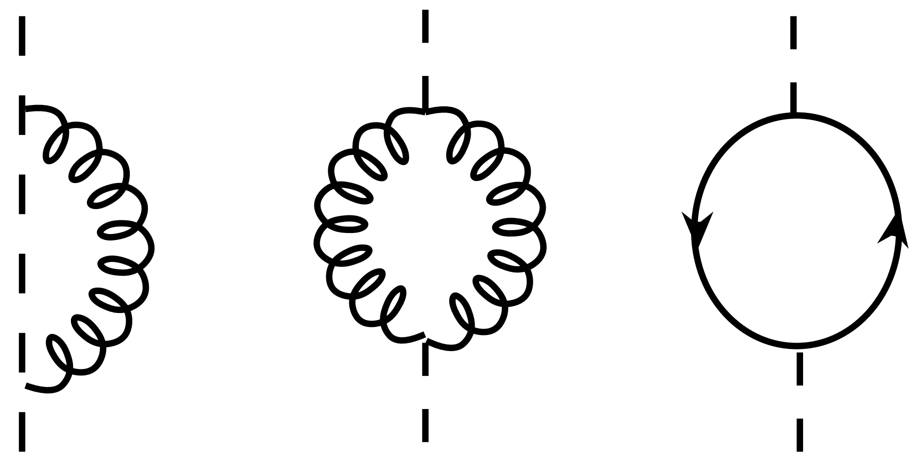
Swanson and Szczepaniak, Phys. Rev. D 59, 014035 (1998).

Hamiltonian QCD

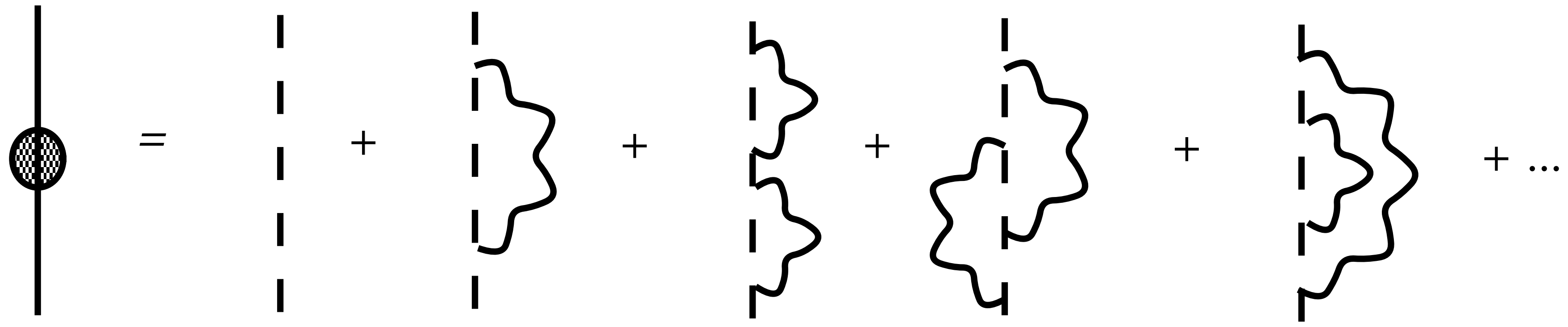
vacuum Ansatz

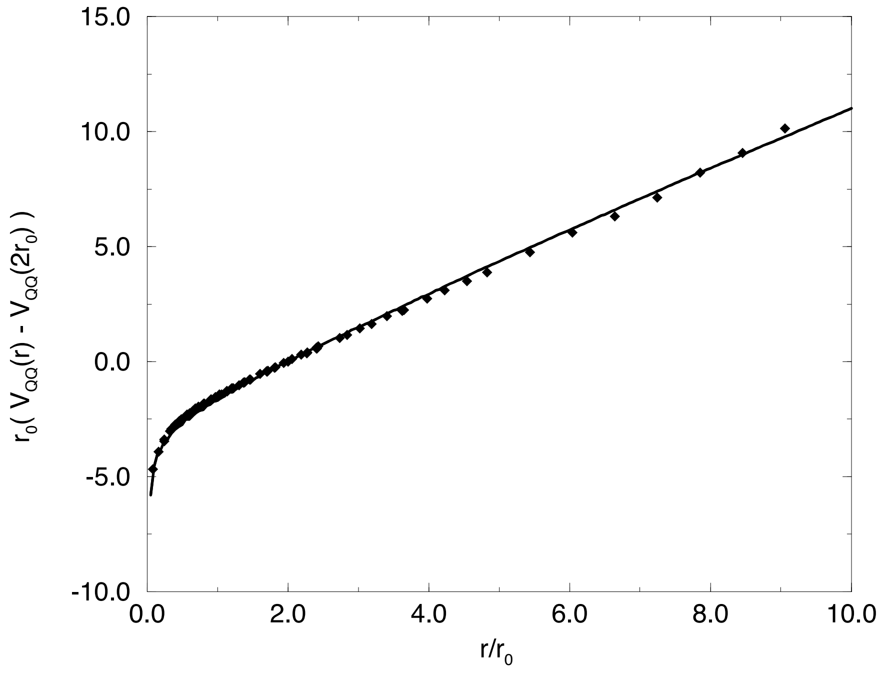
$$\Psi_0[\mathbf{A}] = \langle \mathbf{A}|\omega\rangle = \exp\left[-\frac{1}{2}\int \frac{d\mathbf{k}}{(2\pi)^3}\mathbf{A}^a(\mathbf{k})\,\omega(k)\mathbf{A}^a(-\mathbf{k})\right]$$

running coupling

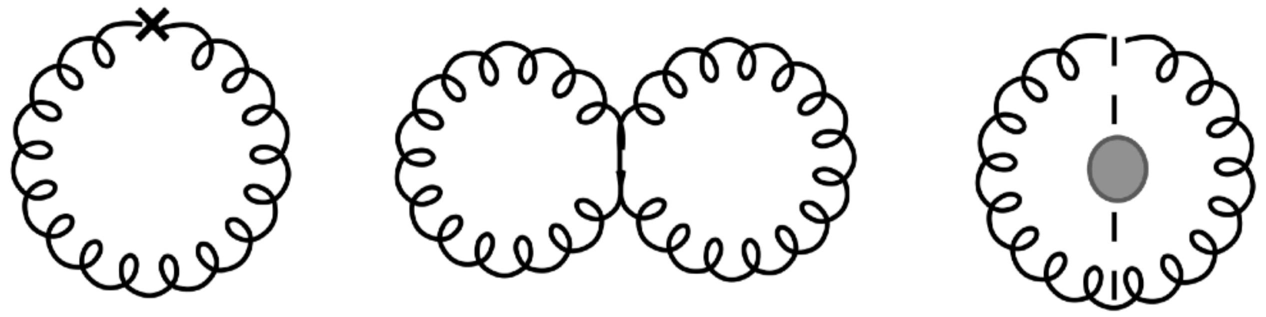


nonperturbative potential

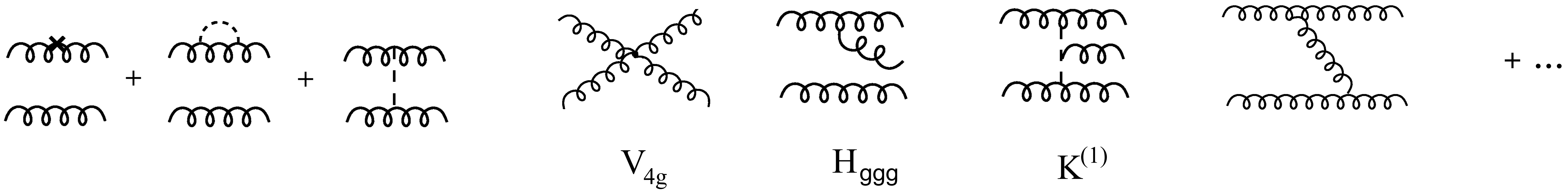


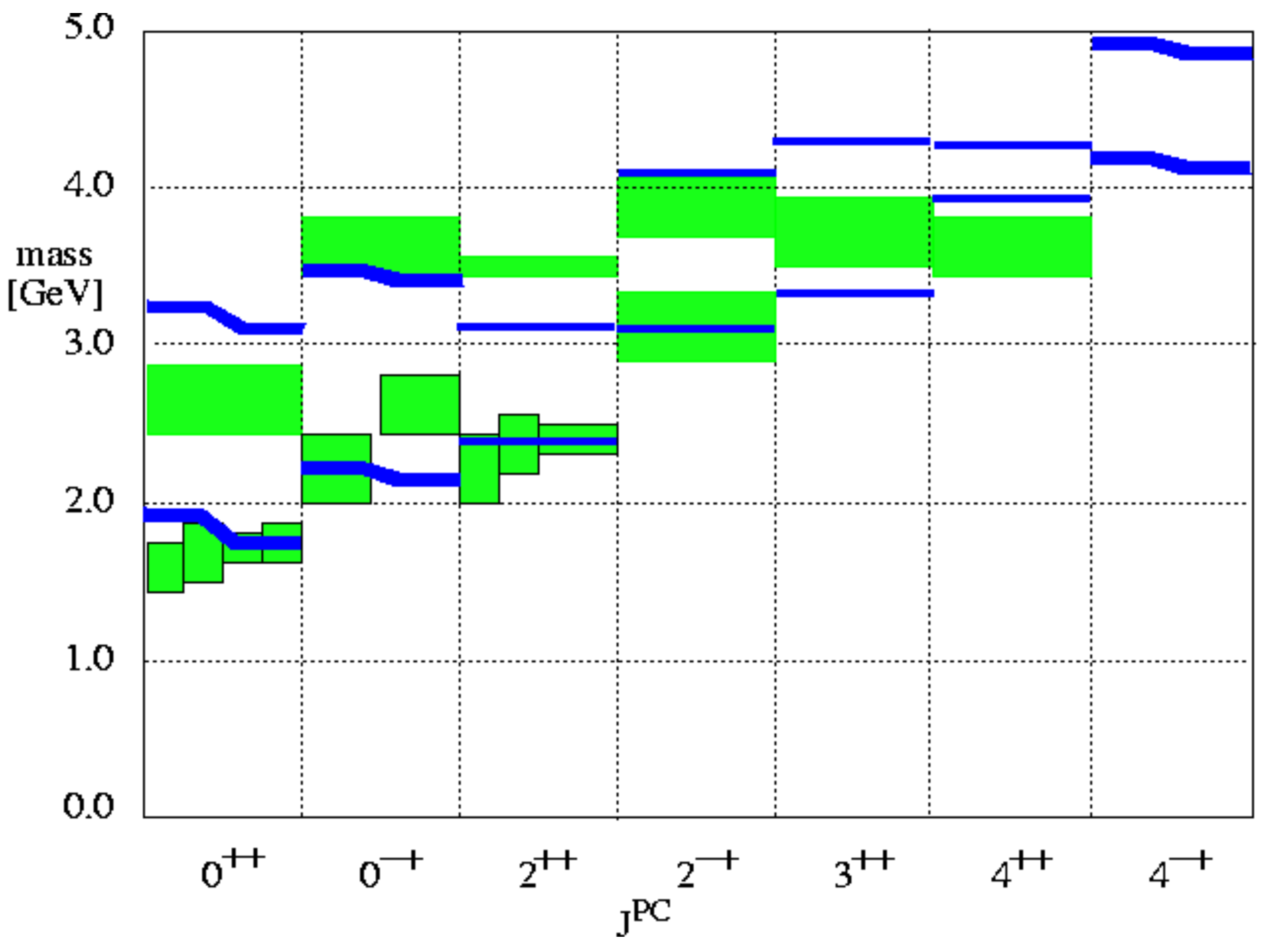


gluonic gap equation

$\frac{\delta}{\delta\omega}$  $= 0$

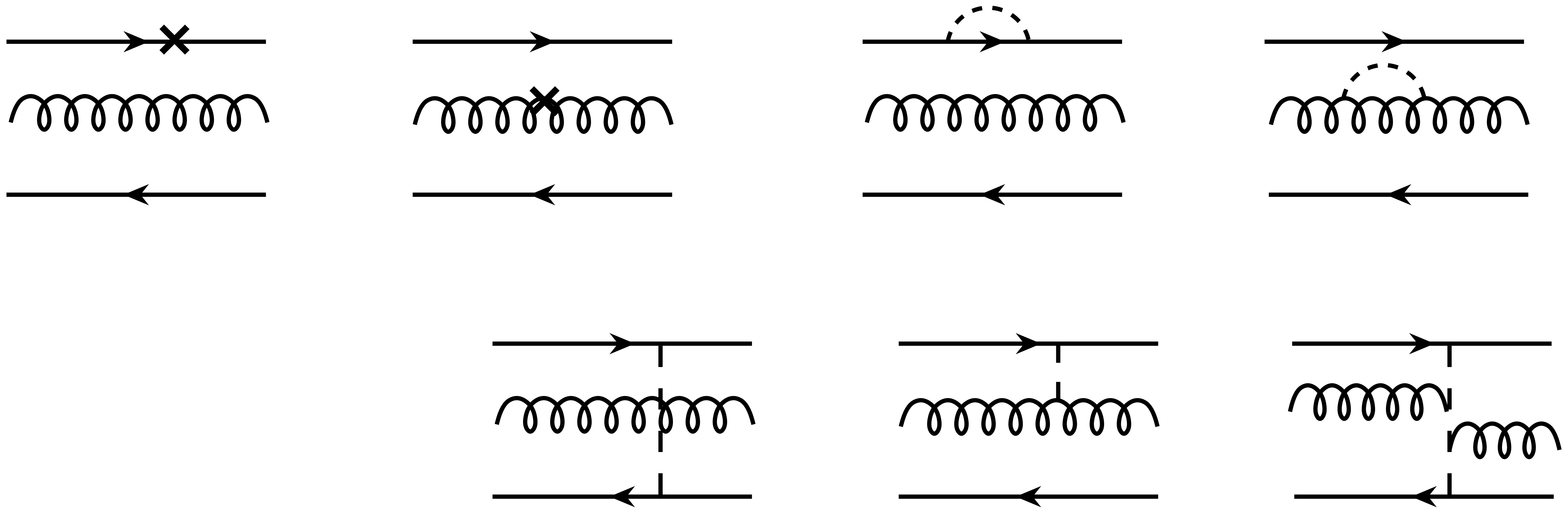
exotic states - glueballs





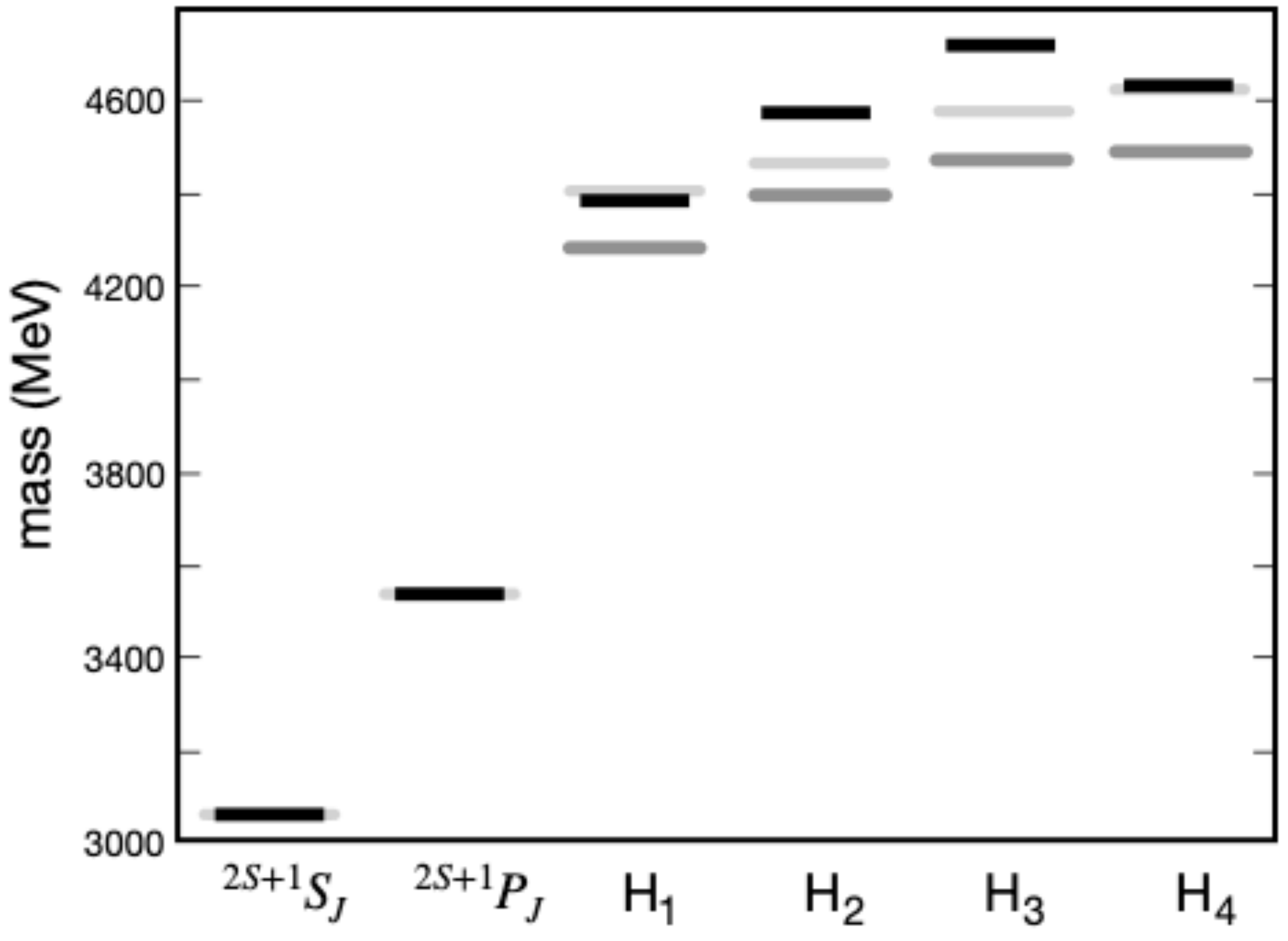
Swanson & Szczepaniak, arXiv:hep-ph/0308268

exotic states - hybrids



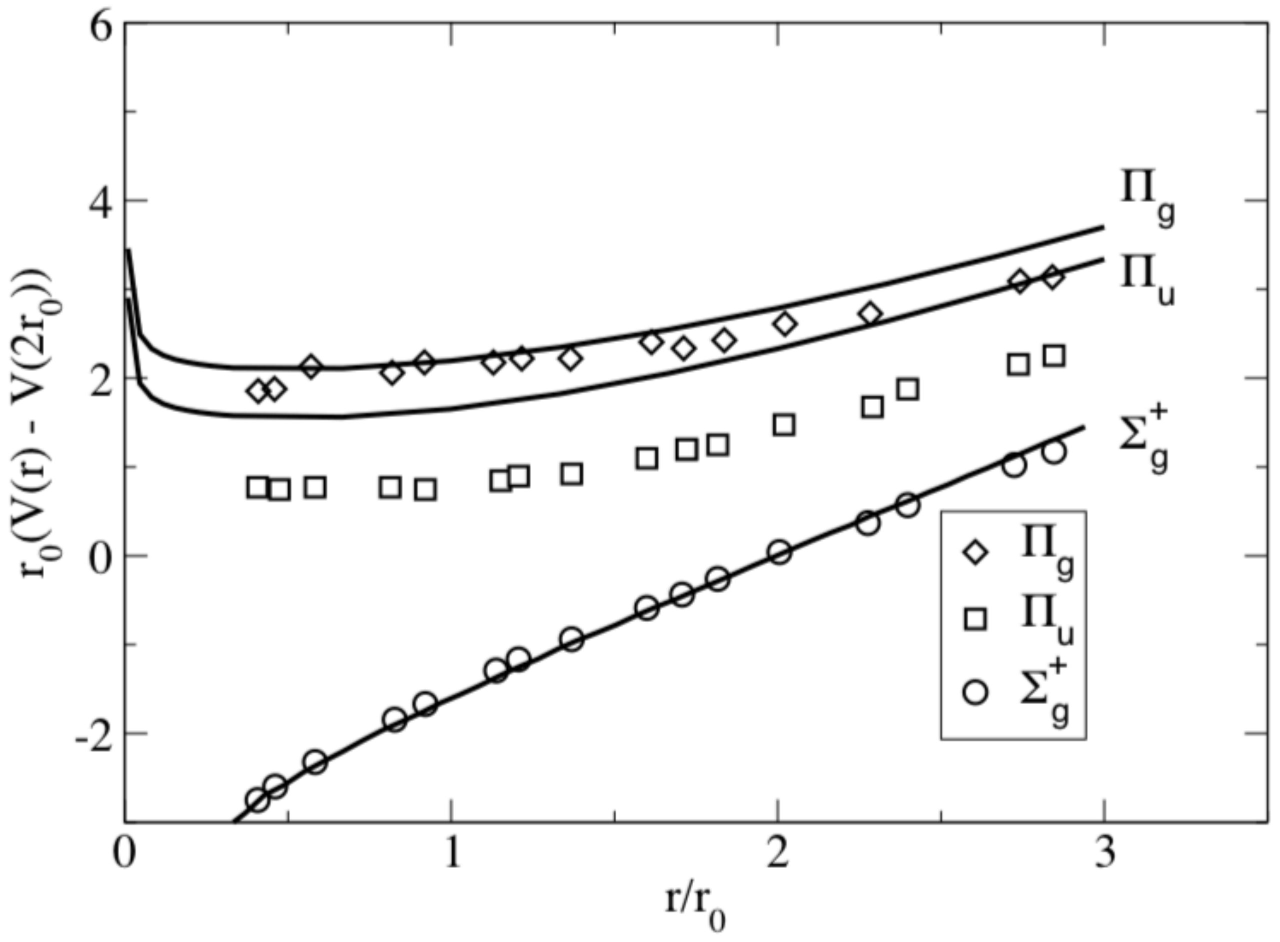
+ ...

$c\bar{c}g$ spectrum



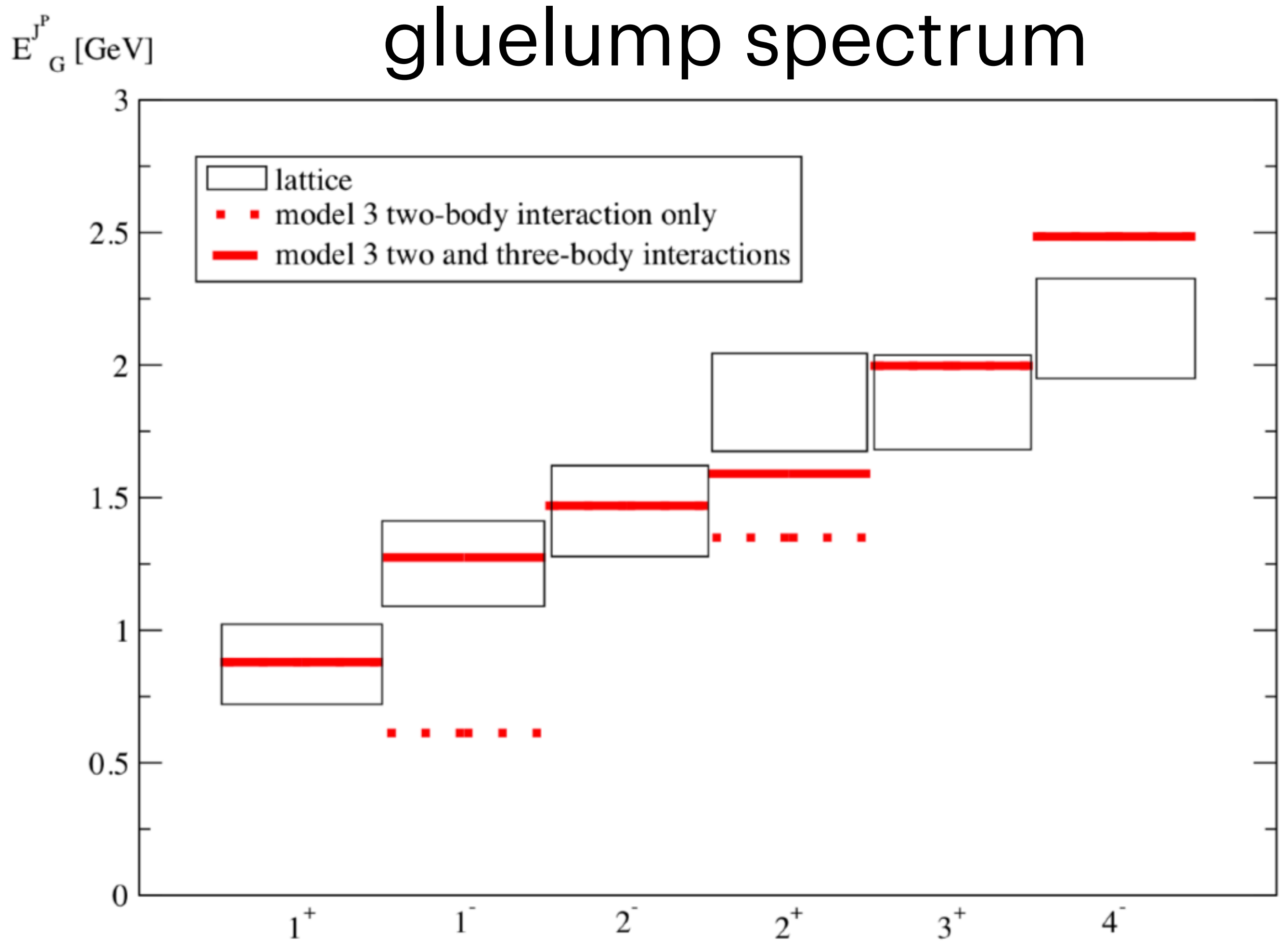
C. Farina et al, arXiv:2005.10850

adiabatic potentials



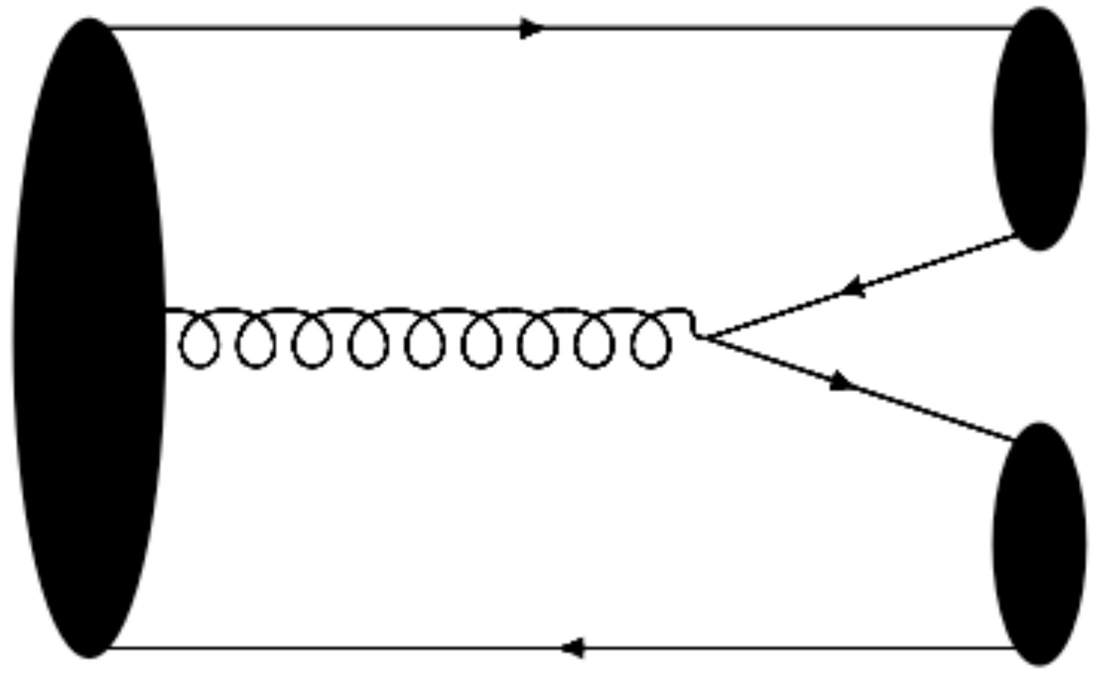
A. Szczepaniak and P. Krupinski, Phys. Rev. D 73, 116002 (2006).

gluelump spectrum



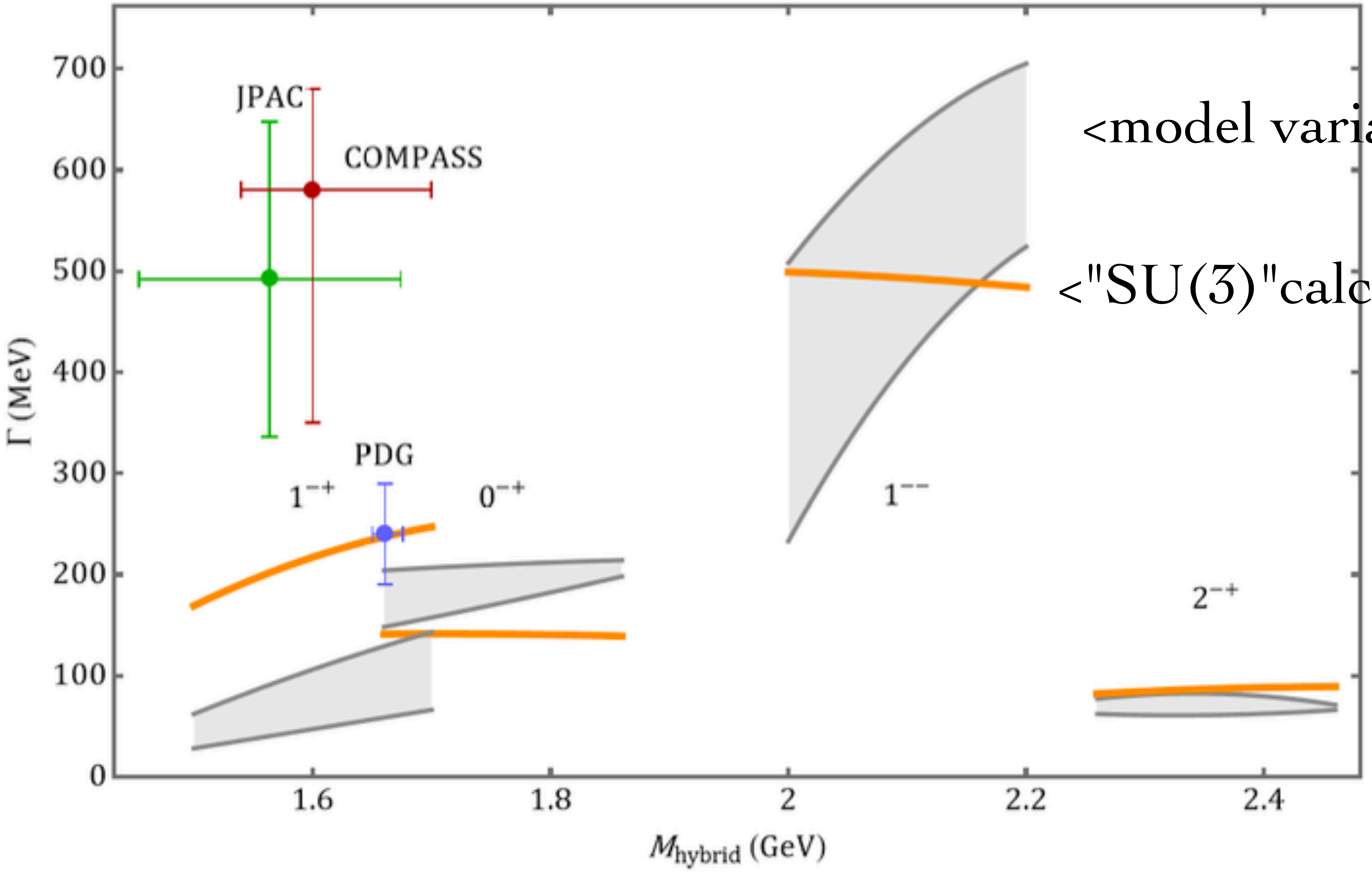
P. Guo, et al, arXiv:0707.3156

hybrid decays



spin selection rule

S+P selection rule



<model variation

<"SU(3)"calculation

C. Farina & E. Swanson, arXiv:2512.00459

C. Farina & E. Swanson, arXiv:2312.05370

C. Farina et al, arXiv:2005.10850

P. Page, E.S. Swanson, A. Szczepaniak, arXiv:hep-ph/9808346

E.S. Swanson & A. Szczepaniak, arXiv:hep-ph/9704434

hybrid flavour mixing

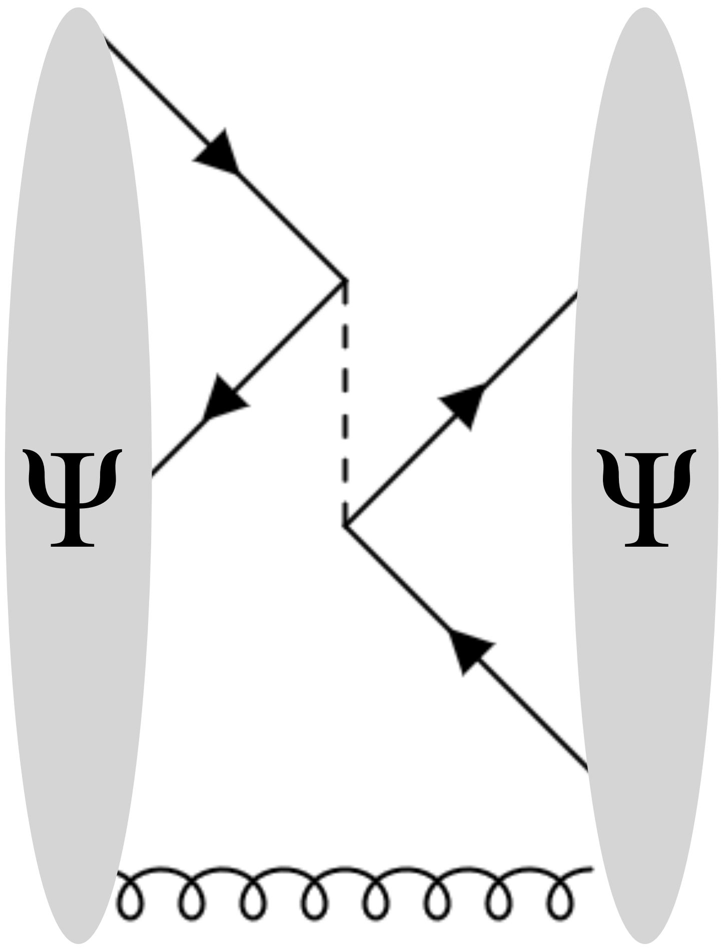
$$|u\bar{u}\rangle \quad |d\bar{d}\rangle \quad |s\bar{s}\rangle \quad |gg\rangle$$

$$H_{uds} = \begin{pmatrix} m + A_{nn} & A_{nn} & A_{ns} & \mathscr{A}_n^{(0)} \\ A_{nn} & m + A_{nn} & A_{ns} & \mathscr{A}_n^{(0)} \\ A_{ns} & A_{ss} & m + \Delta m + A_{ss} & \mathscr{A}_s^{(0)} \\ \mathscr{A}_n^{(0)} & \mathscr{A}_n^{(0)} & \mathscr{A}_s^{(0)} & M_{gb} \end{pmatrix}.$$

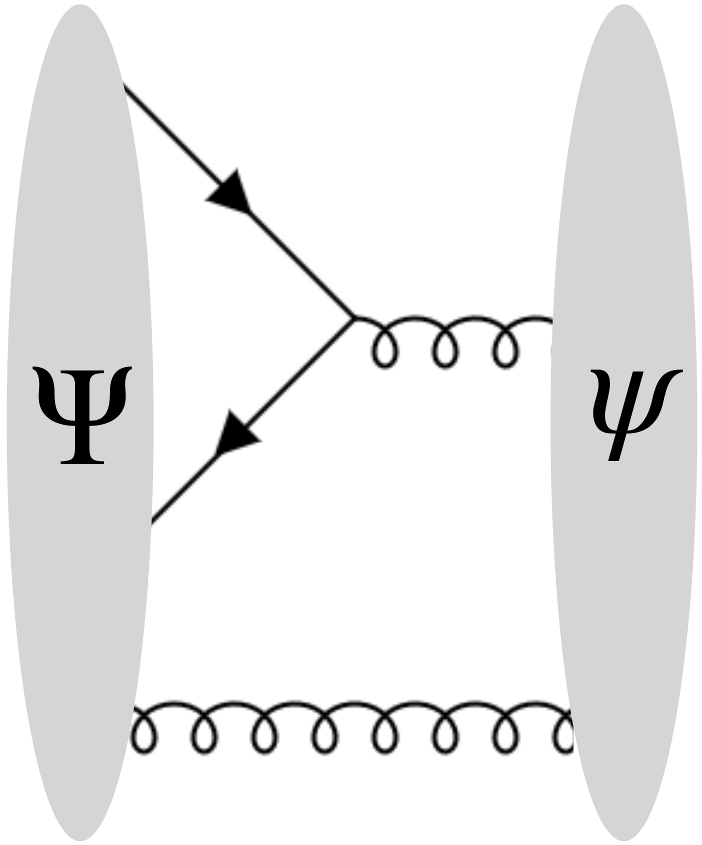
☞

$$H_{iso} = \begin{pmatrix} m & 0 & 0 & 0 \\ 0 & m + 2A_{nn} & \sqrt{2}A_{ns} & \sqrt{2}\mathscr{A}_n^{(0)} \\ 0 & \sqrt{2}A_{ns} & m + \Delta m + A_{ss} & \mathscr{A}_s^{(0)} \\ 0 & \sqrt{2}\mathscr{A}_n^{(0)} & \mathscr{A}_s^{(0)} & M_{gb} \end{pmatrix}.$$

E.S. Swanson, arXiv: 2302.10372

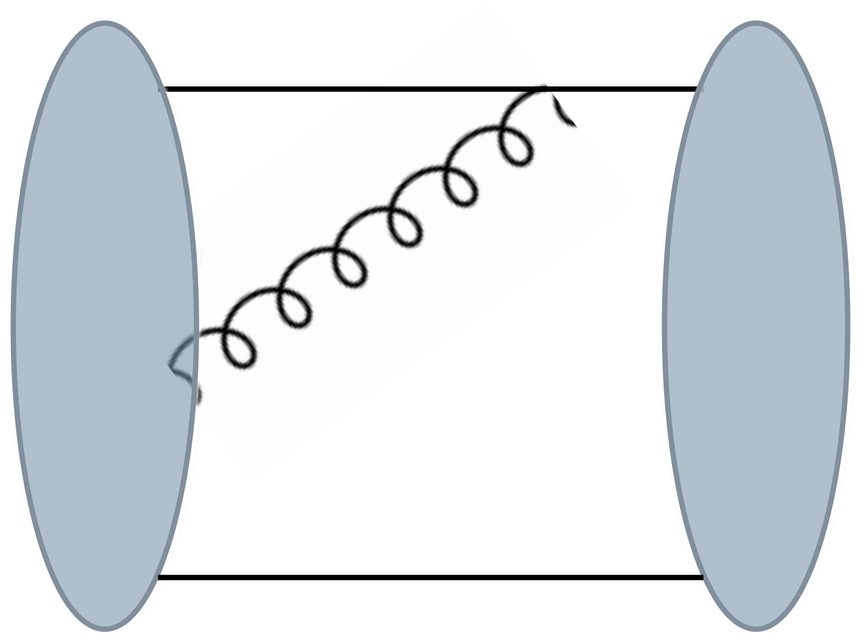


$\ell = 0;\ S = 1;\ H_1$ only



[neglect mixing through the continuum ~ the "OZI puzzle"]

hybrid-vector mixing



$$\mathcal{H}_{1S} = -ig \begin{cases} 84\ \mathrm{MeV}^2/m_q, & \rho \\ 190\ \mathrm{MeV}^2/m_c, & J/\psi \\ 225\ \mathrm{MeV}^2/m_b, & \Upsilon \end{cases} \approx -i \begin{cases} 210\ \mathrm{MeV}, & \rho \\ 60\ \ \mathrm{MeV}, & J/\psi \\ 20\ \ \mathrm{MeV}, & \Upsilon \end{cases}.$$

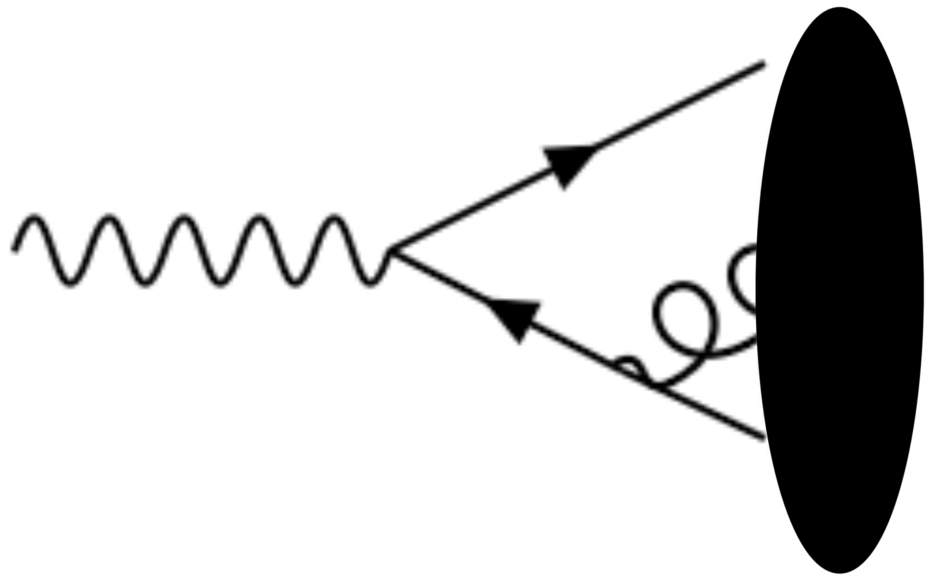
Hybrid configuration content of heavy S-wave mesons,
[MILC] T. Burch & D. Toussaint, Phys. Rev. 68, 094504 (2003).

$$\mathcal{H}_{NRQCD} \approx 170\ \mathrm{MeV}\ (J/\psi)$$

$$\mathcal{H}_{NRQCD} \approx 70\ \mathrm{MeV}\ \ (\Upsilon).$$

E.S. Swanson, arXiv: 2302.10372

vector hybrid decay constant



$$f_H = \frac{1}{\sqrt{M_H}} \sum_{n \neq H} \sqrt{M_n} f_V^{(n)} C_n$$

$$f_V^{(n)} = \sqrt{\frac{3}{M_n}} \int \frac{d^3k}{(2\pi)^3} \psi^{(n)}(\vec{k}) \sqrt{1 + \frac{m_q}{E_k}} \sqrt{1 + \frac{m_{\bar{q}}}{E_{\bar{k}}}} \left(1 + \frac{k^2}{3(E_k + m_q)(E_{\bar{k}} + m_{\bar{q}})} \right).$$

$$C_n = \langle nS | H_1(1^{--}) \rangle$$

$$\delta H = \begin{pmatrix} m_1 & & & & & & \mathcal{H}_{1S} \\ & m_2 & & & & & \mathcal{H}_{2S} \\ & & m_3 & & & & \mathcal{H}_{3S} \\ & & & m_4 & & & \mathcal{H}_{4S} \\ & & & & m_5 & & \mathcal{H}_{5S} \\ & & & & & m_6 & \mathcal{H}_{6S} \\ \mathcal{H}_{1S} & \mathcal{H}_{2S} & \mathcal{H}_{3S} & \mathcal{H}_{4S} & \mathcal{H}_{5S} & \mathcal{H}_{6S} & M_H \end{pmatrix}.$$

$$f_{H_1(1^{--})} \approx 20 \text{ MeV}.$$

E.S. Swanson, arXiv: 2302.10372

hybrid phenomenology

lattice light hybrid spectrum

11

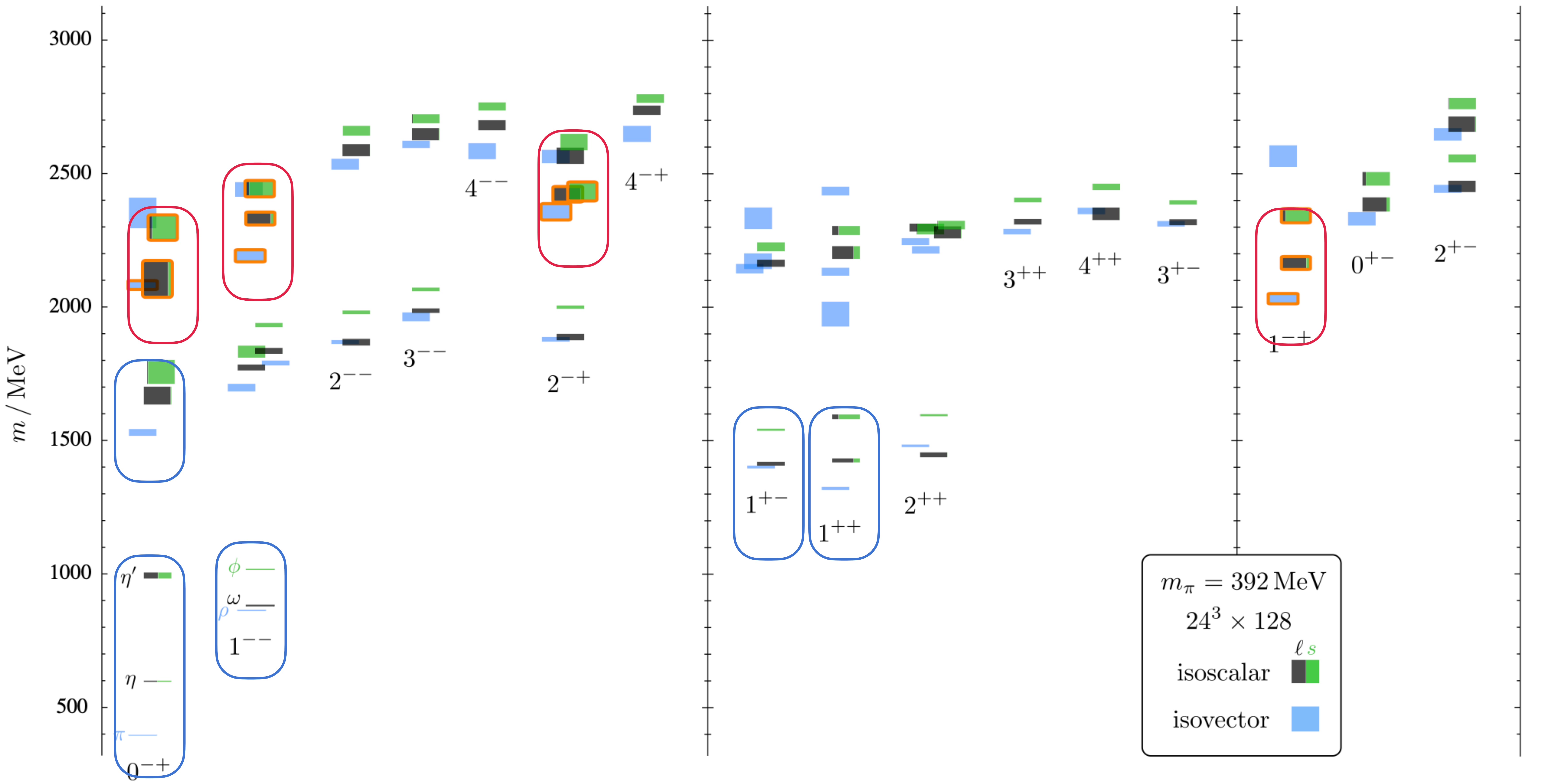
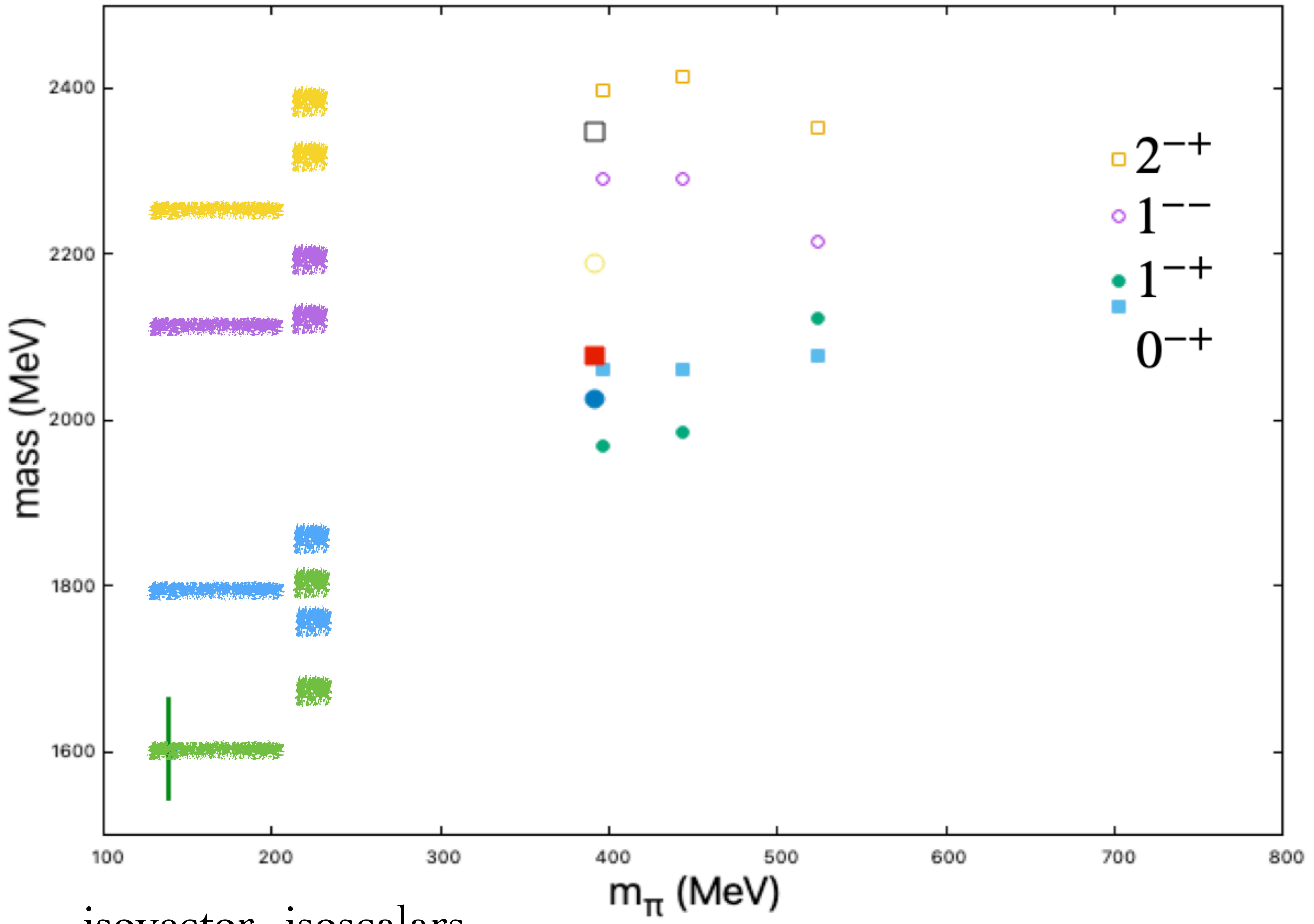


FIG. 11: Isoscalar (green/black) and isovector (blue) meson spectrum on the $m_\pi = 391$ MeV, $24^3 \times 128$ lattice. The vertical height of each box indicates the statistical uncertainty on the mass determination. States outlined in orange are the lowest-lying states having dominant overlap with operators featuring a chromomagnetic construction – their interpretation as the lightest hybrid meson supermultiplet will be discussed later.

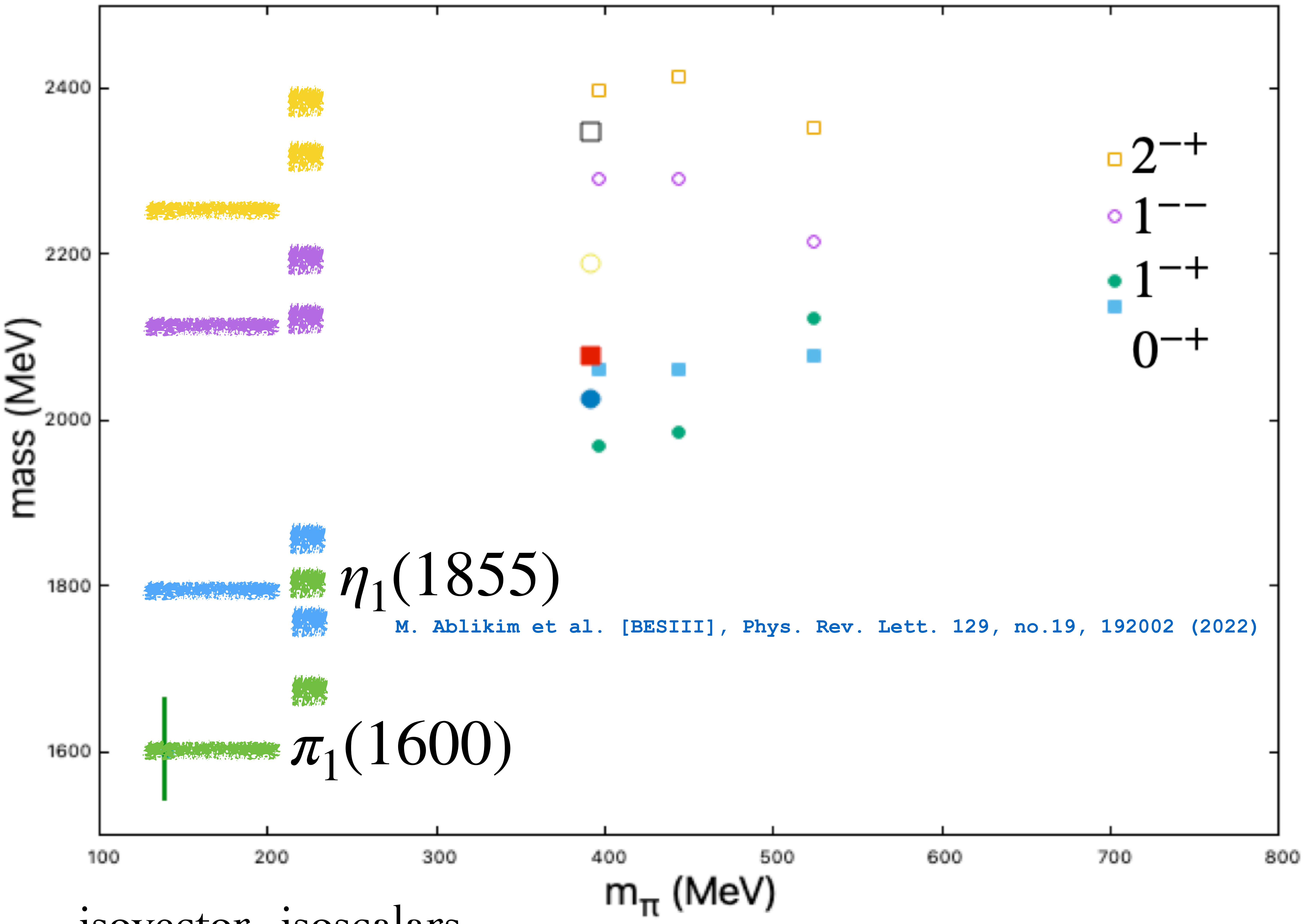
Toward the excited isoscalar meson spectrum from lattice QCD, [HadSpec] J.J. Dudek et al. Phys.Rev.D 88 (2013) 9, 094505.

lattice light hybrid spectrum



isovector   isoscalars

lattice light hybrid spectrum



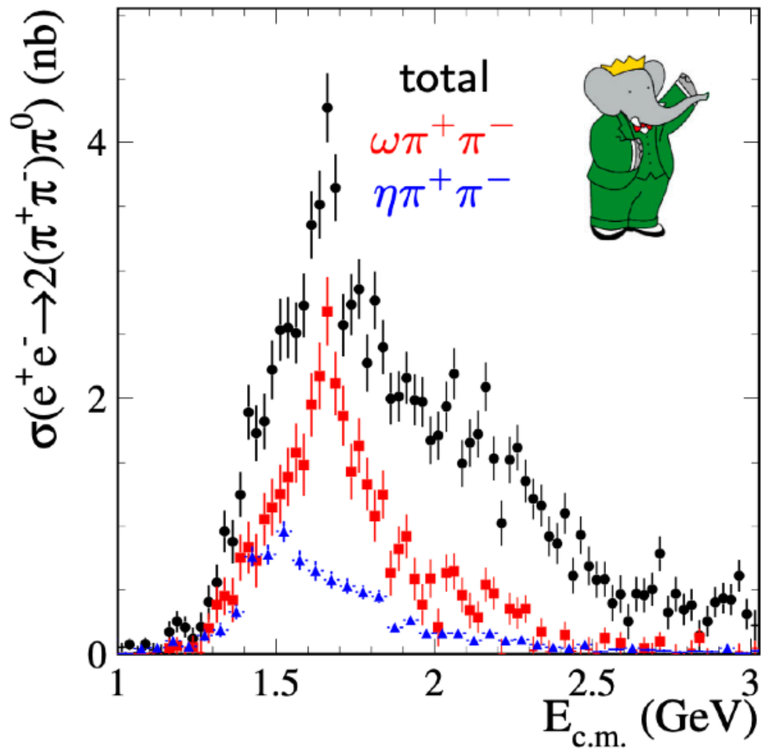
isovector isoscalars

$H_1(1^{--})$ 2100/2100/2200

State	Ref.	Mass	Width	Model (iv)	Mass	Model (v)	Mass
$\rho(770)$	PDG	775.2 ± 0.2	147.4 ± 0.8	1^3S_1	720	1^3S_1	730
$\rho(1450)$	PDG	1465 ± 25	400 ± 60	2^3S_1	1440	2^3S_1	1390
$\rho(1570)$	PDG	1570 ± 70	144 ± 90	1^3D_1	1510	1^3D_1	1480
$\rho(1700)$	PDG	1720 ± 20	250 ± 100	$H_1(1^{--})$	1760	3^3S_1	1780
						2^3D_1	1845
				3^3S_1	1850		
$\rho(1900)$	[33]	1900 ± 30	50 ± 30	2^3D_1	1910	4^3S_1	2090
$\rho(2150)$	[34]	2034 ± 16	234 ± 39	4^3S_1	2170	$H_1(1^{--})$	2100
				3^3D_1	2220	3^3D_1	2140

both scenarios find support in LGT and QMs

$e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^0$

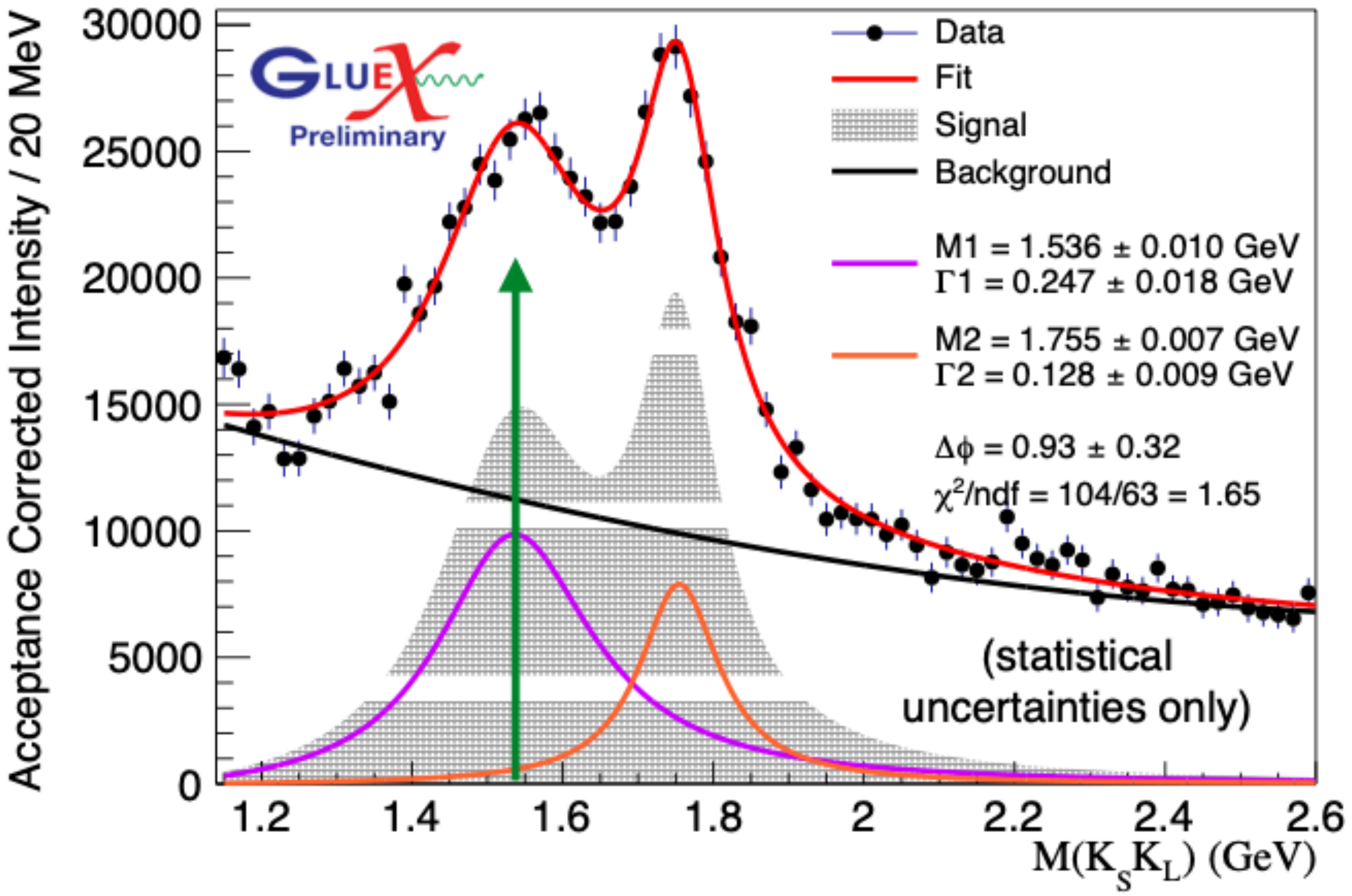


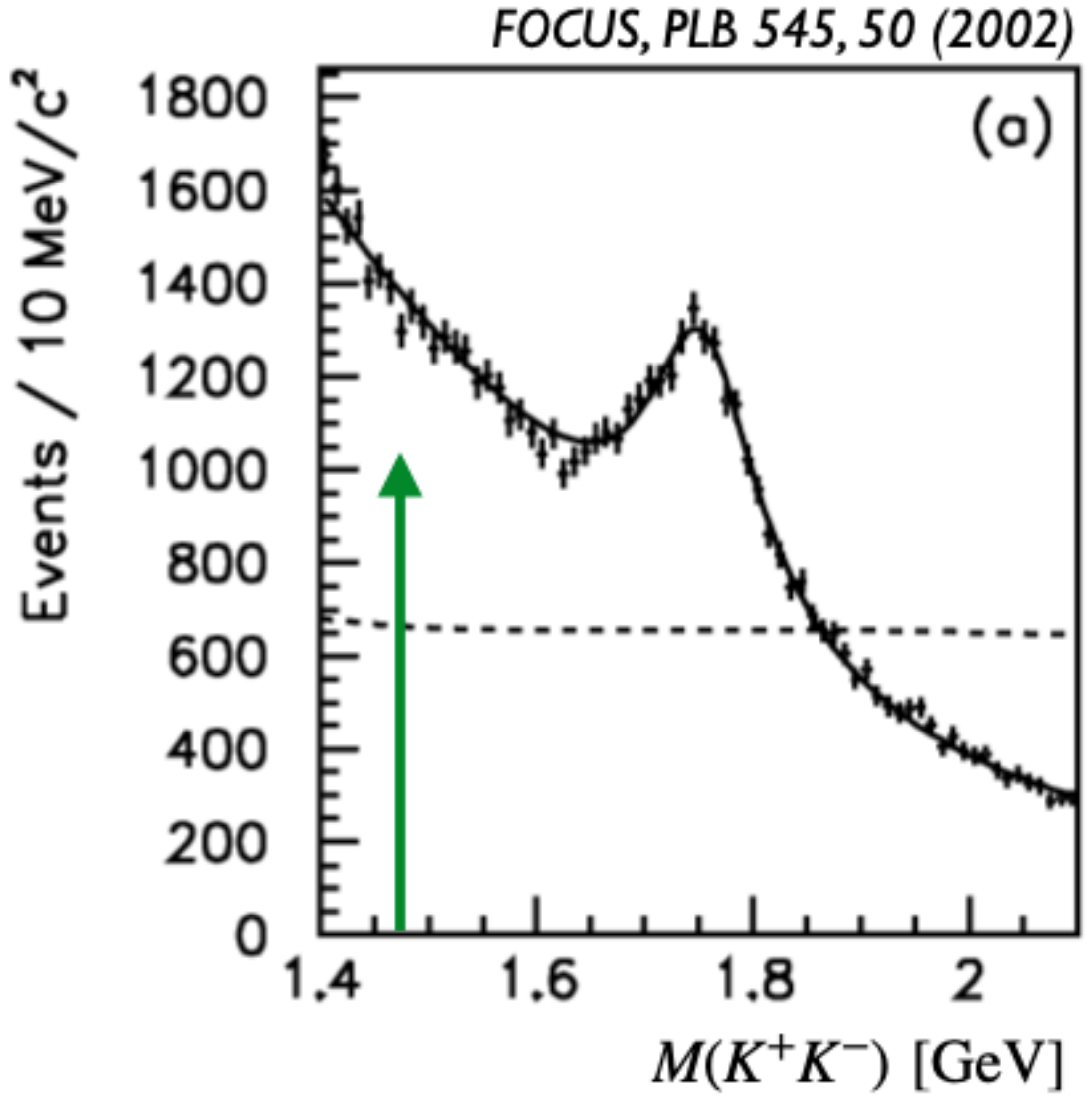
Phys.Rev. D76 (2007) 092005 [3]

Model	$m(m_s)$ (MeV)	a_S	σ (GeV^2)	$\mathcal{C}$ (MeV)	α_H	b_0 (GeV^{-1})	r_0(GeV^{-1})	ϵ	Rel error	Avg deviation (MeV)
i. [hyp]	335	0.59	0.16	−697	...	...	...	0	9%	94
ii. [SI]	300	1.52	0.071	110	...	...	...	0	7%	66
iii. [SD]	400	1.8	0.06	230	1.3	0.60	4.7	0	5%	54
iv. [SD]	330	2.1	0.055	385	1.3	0.53	5.2	0	5%	54
v. [30]	375 (525)	1.53	0.059	168	1.2	0.56	7.5	0.25	7%	79
Rel/SI	200	0.59	0.14	−246	...	...	...	0	6%	59
Rel/SD	400	0.72	0.14	−359	1.1	0.20	4.4	0	5%	55

$H_1(1^{--})$ 2100/2100/2200

$\gamma p \to K_S K_L p$





Second peak is consistent with FOCUS X(1750). Where is the first peak!?

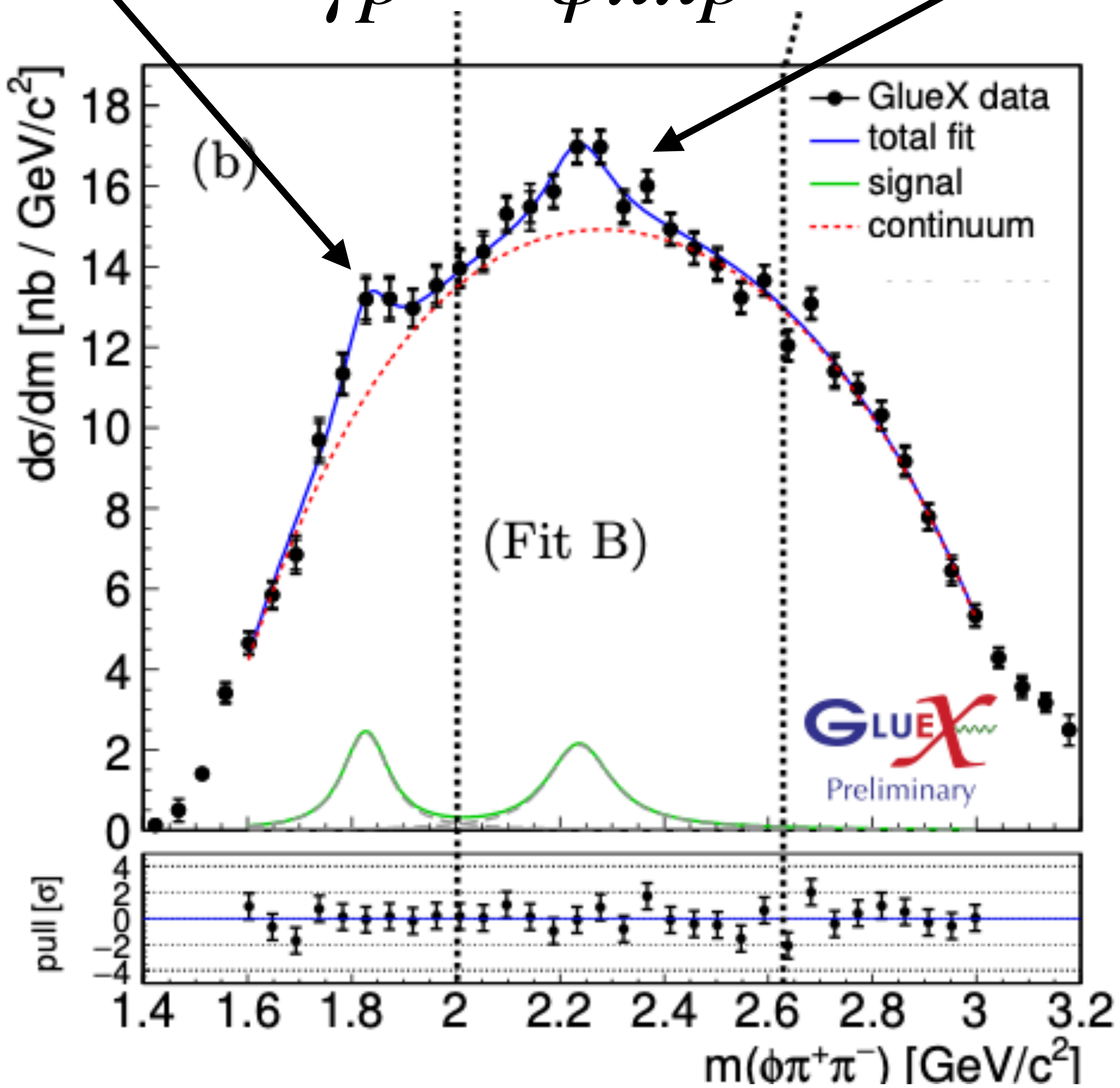
$H_1(1^{--})$ 2100/2100/2200

Seek BaBar/Belle $\varphi(2170)$

new!

consistent with BESIII $\varphi(2170) \to \varphi(2240)$

$\gamma p \to \varphi \pi \pi p$



arXiv:2512.04136

$H_1(0^{-+})$ 1800/1780/1860

source	π masses				η masses					
RPP[29]	139	1300	1800	–	548	958	1294	1409	1475	2221
LGT[11]	391	1520	2100^a	2350	600	1000	1680	1750	2100^a	2300^a
GI[38]	150	1300	1880		520	960	1440	1630		
fit A	560	1365	1720	1990	560	730	1365	1580	1990	2200
fit A'	600	1390	1750	2030	600	820	1390	1580	1750	1940

[a]gluonic content

not a good agreement!

isoscalars

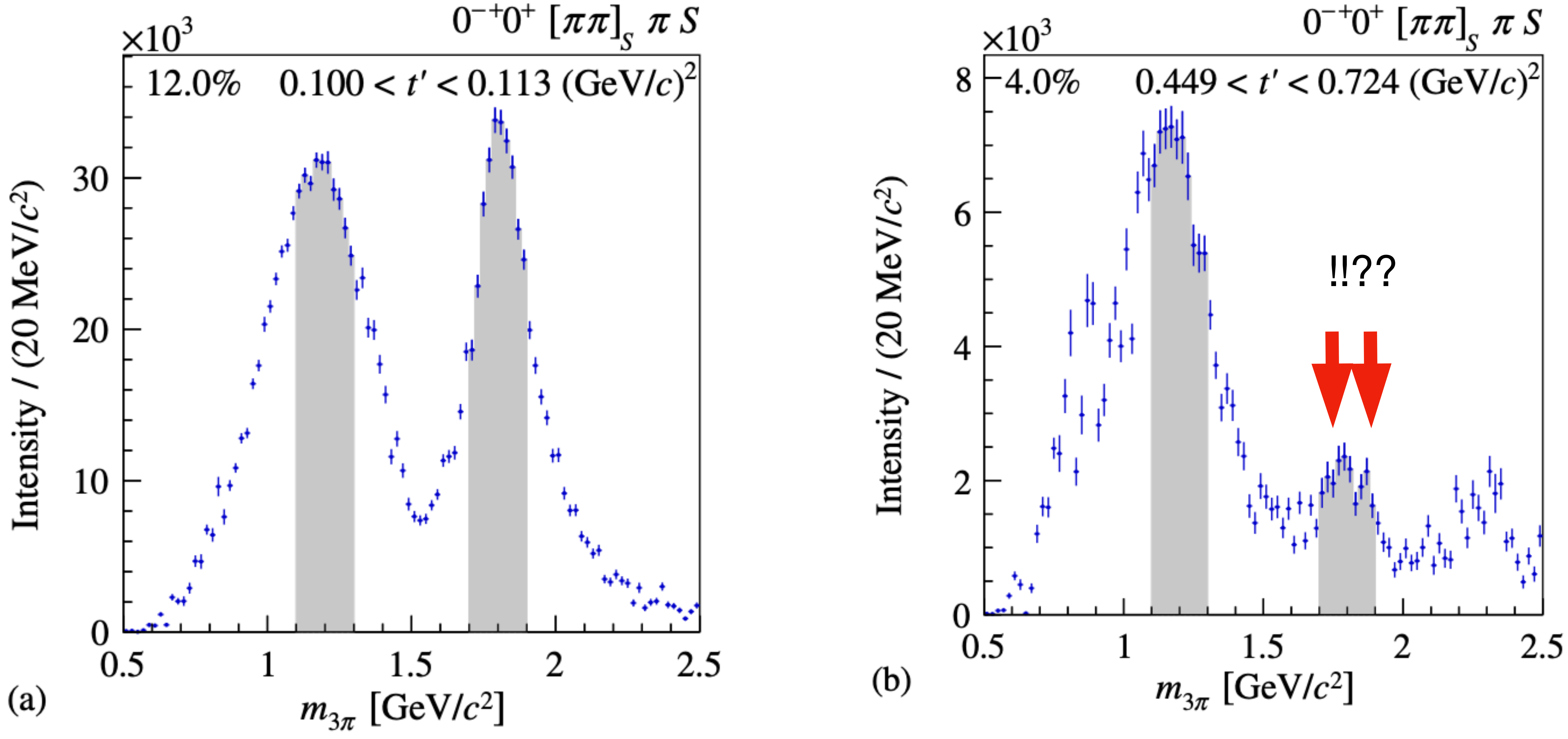
0^{-+}	$f_0(500)\eta$	$33.3\cos^2\theta$	$30.2\sin^2\theta$
$(1750-1850)$	$K^*\bar{K}$	$(\cos\theta\sqrt{0.8}-\sin\theta\sqrt{2})^2$	$(\sin\theta\sqrt{1.1}+\cos\theta\sqrt{2.7})^2$
	$a_2(1320)\pi$	$1.0\cos^2\theta$	$2.5\sin^2\theta$
	$f_0(980)\eta$	$42.8\cos^2\theta$	$44.9\sin^2\theta$
	$a_0(1450)\pi$	$34.5\cos^2\theta$	$54.6\sin^2\theta$
	$\phi\eta$	x	x
	$f_2(1270)\eta$	ø	x
	$\phi\eta'$	ø	ø
	$K^*(1410)\bar{K}$	ø	ø
	$f_0(1370)\eta$	ø	ø
	$K_0^*(1430)\bar{K}$	ø	ø
	$K_2^*(1430)\bar{K}$	ø	ø
	$f_2'(1525)\eta$	ø	ø
Γ_{tot}		$112\cos^2(\theta)-2.5\cos(\theta)\sin(\theta)+2\sin^2(\theta)$	$2.7\cos^2(\theta)+3.4\cos(\theta)\sin(\theta)+133\sin^2(\theta)$

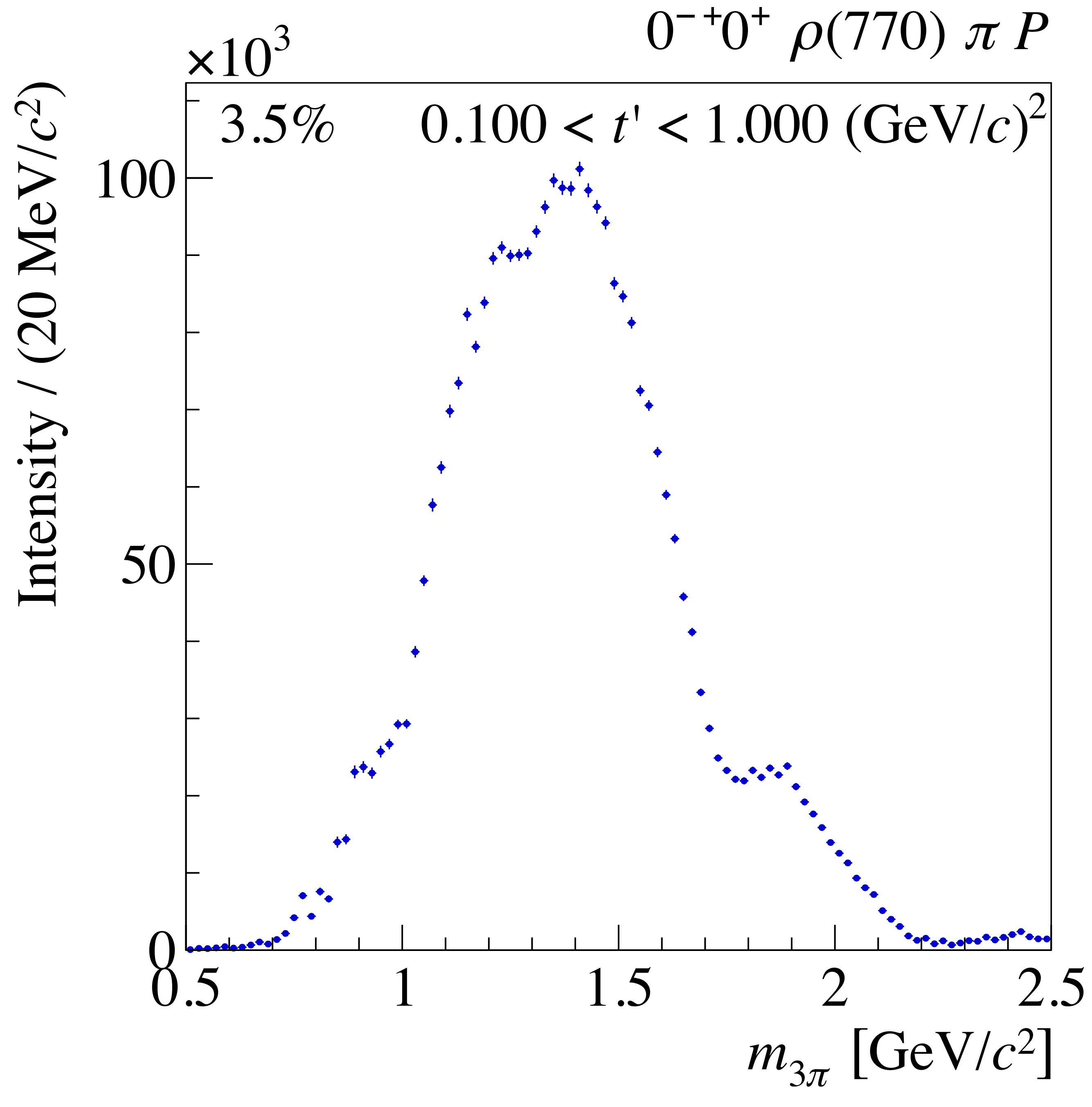
$\Gamma_{s\bar{s}g}\sim 10s$ MeV

state	$f_0(500)\pi$	$\rho\pi$	$f_0(980)\pi$	$K^*\bar{K}$	$b_1(1235)\pi$	$f_2(1270)\pi$	$f_0(1370)\pi$	$\omega\rho$	Γ_{tot}	Γ_{PDG}
$\pi(2S)$	1	230	2	ø	–	ø	ø	ø	233	200-600
$\pi(3S;1850)$	12	117	x	64	–	30	1	109	333+	215
$H_1(\pi)$	51	9	73	0	57	x	22	x	212	–

ratios of partial widths are a powerful discriminant

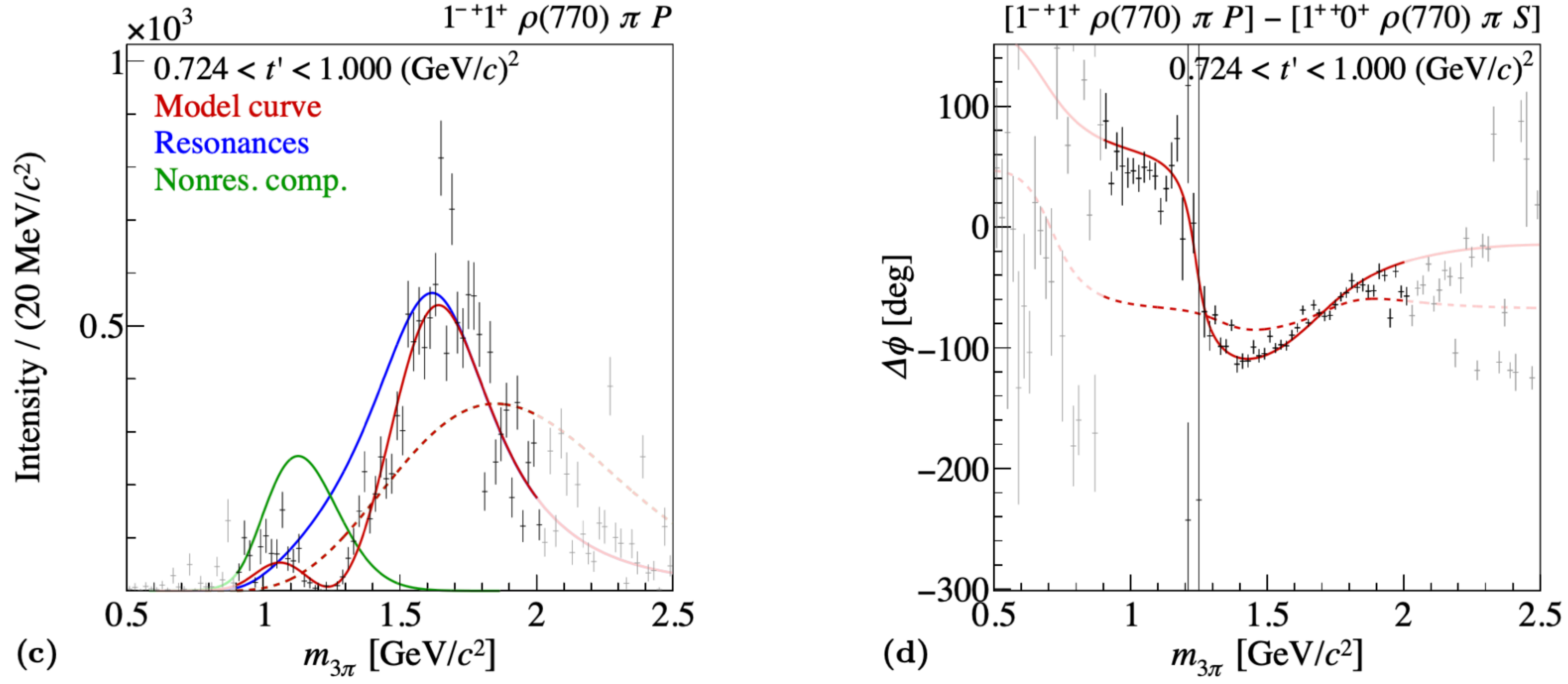
$H_1(0^{-+})$ 1800/1780/1860





$H_1(1^{-+})$ 1600/1680/1820

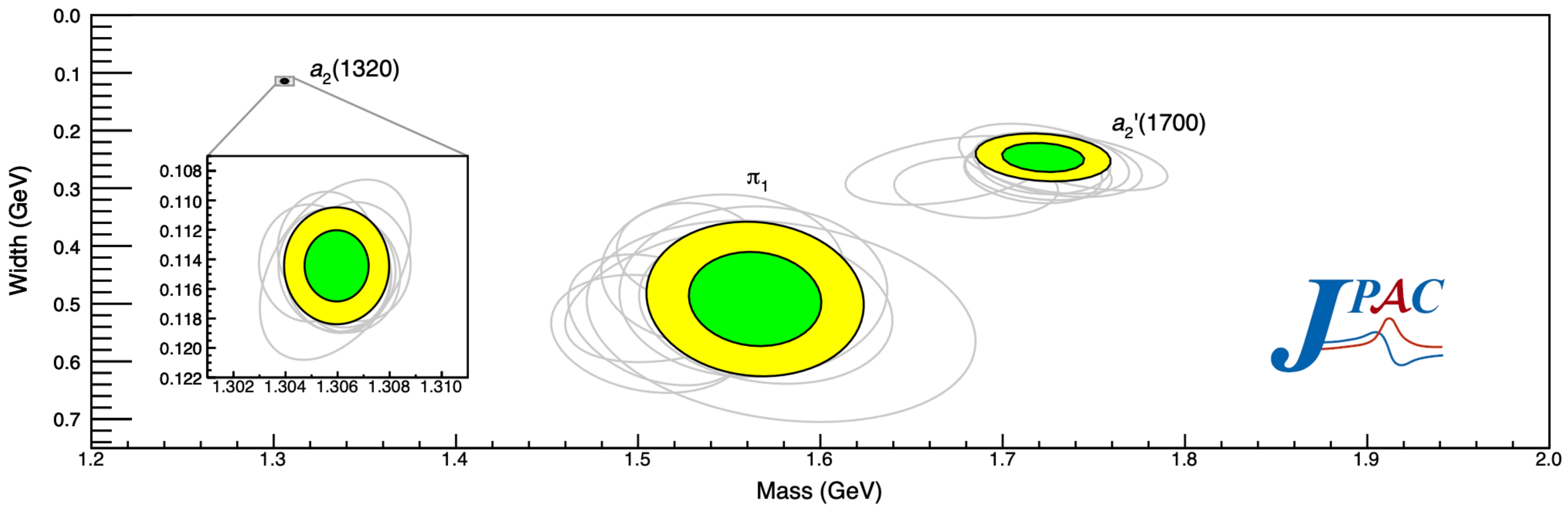
COMPASS reanalysis



COMPASS 1802.05913; 2108.01744

JPAC reanalysis

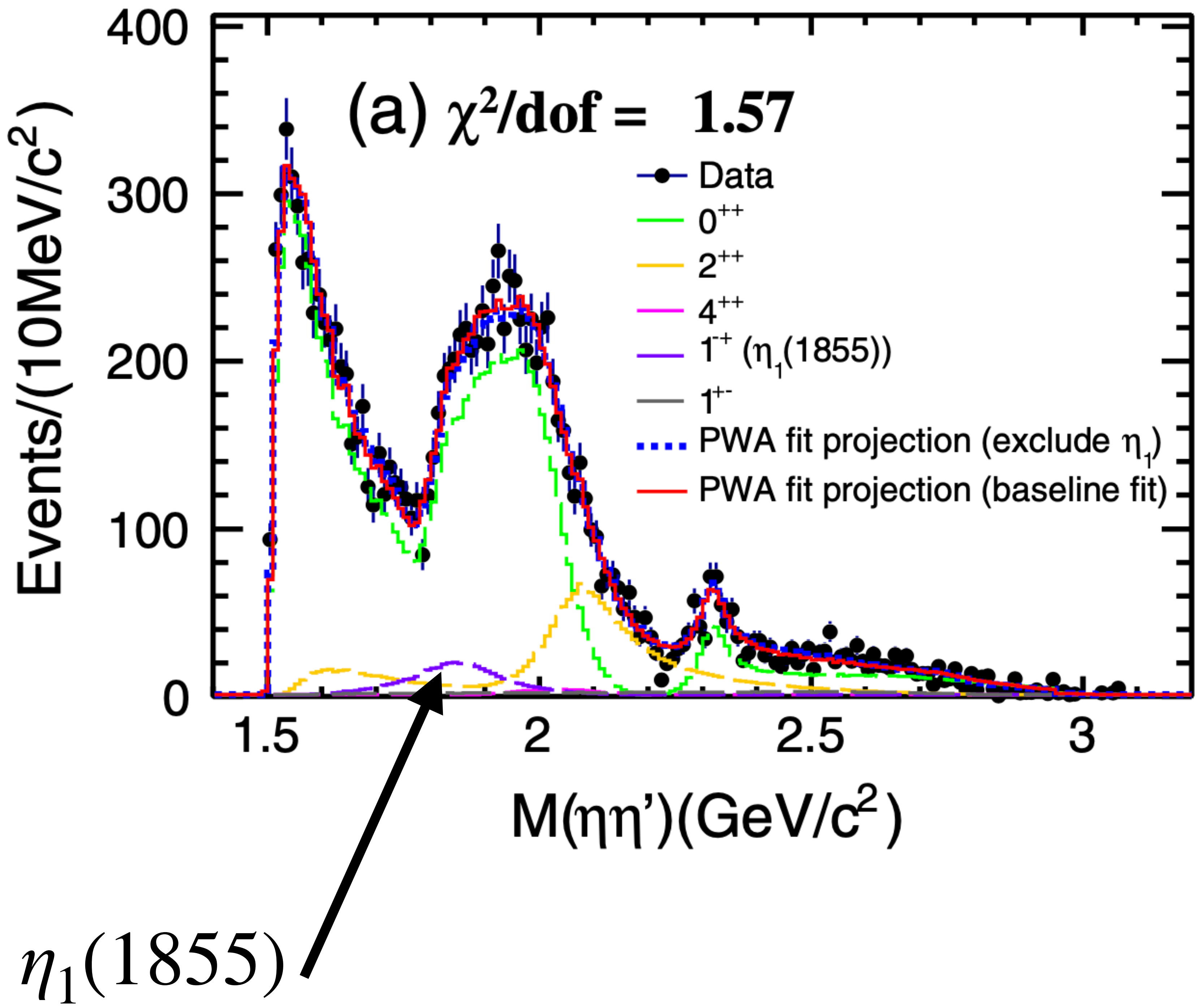
A. Rodas et al. [JPAC],
Determination of the pole position of the lightest hybrid meson candidate
Phys. Rev. Lett. **122**, 042002 (2019).



$H_1(1^{-+})$ 1600/1680/1820

BESIII discover a possible partner state!

BESIII



$\eta_1(1855)$

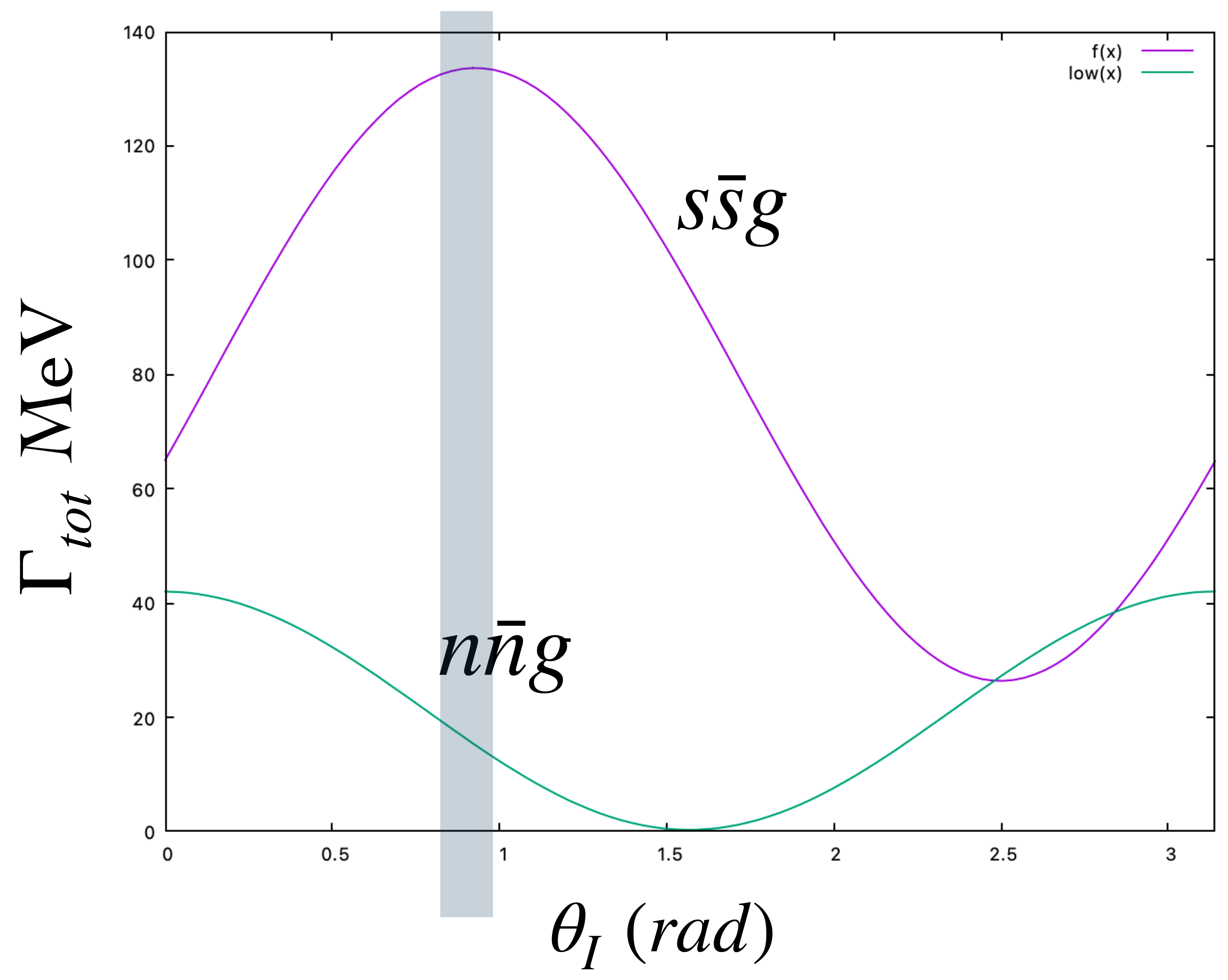
$$M = 1855 \pm 9^{+6}_{-1}\ \text{MeV}$$

$$\Gamma = 188 \pm 18^{+3}_{-8}\ \text{MeV}$$

BESIII, PRL **129**, 192002 (2022).

$H_1(1^{-+})$ 1600/1680/1820

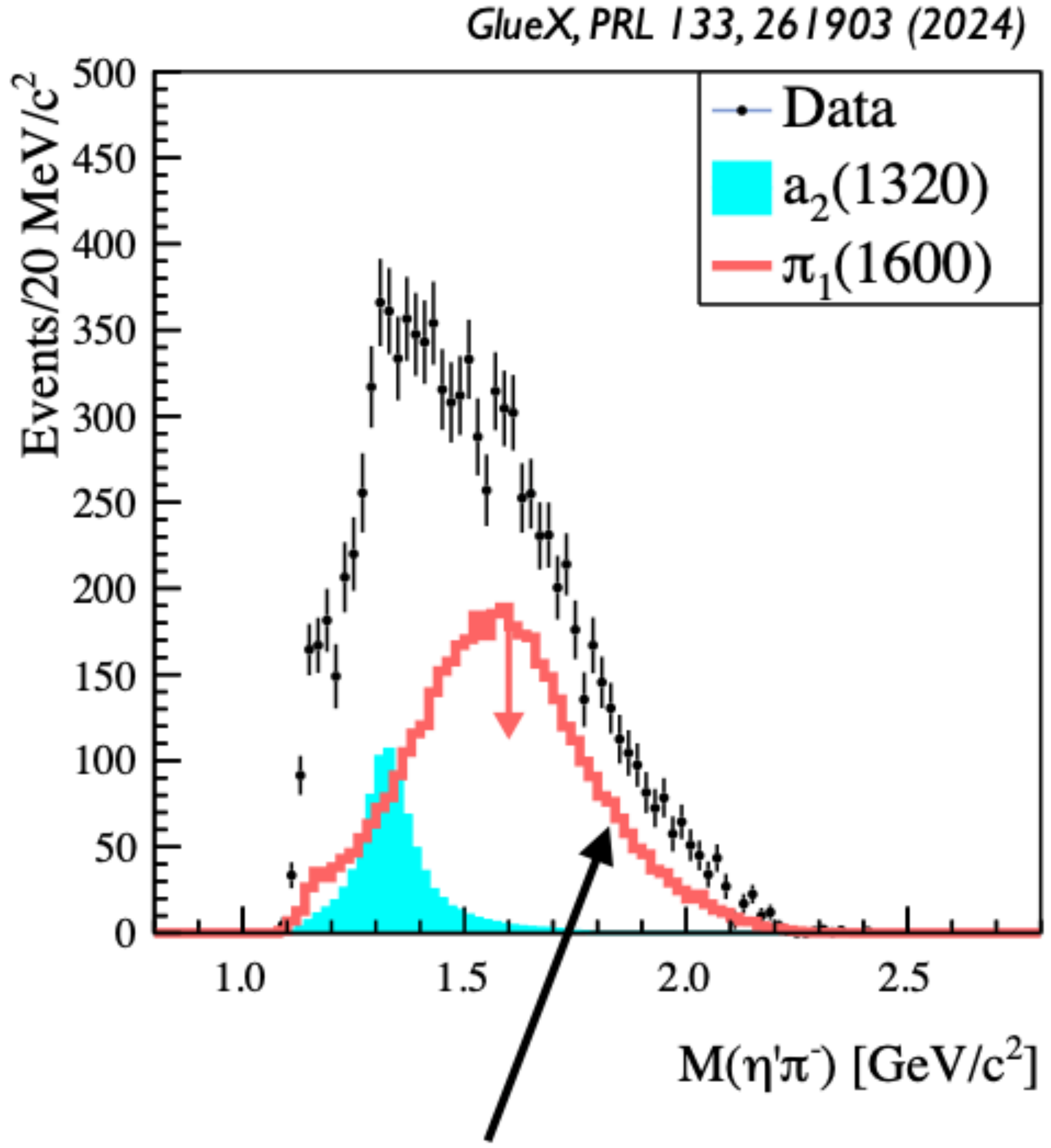
Other partners



With these observations one expects a light isoscalar exotic with mass near 1680 MeV with a total width of approximately 30 MeV and dominant decays to $a_1\pi$ and $K_1\bar{K}$.

$H_1(1^{-+})$ 1600/1680/1820

GlueX search reports an upper limit
(uses HadSpec decay calculation)



Upper limit using LQCD
[HadSpec, PRD 103, 054502 (2021)]
and GlueX data to constrain $\pi_1 \to b_1\pi$

$H_1(2^{-+})$ 2250/2300/2400

$\Gamma_{tot}(H_1(2^{-+}); I = 1) \approx 80$ MeV

$\theta_I \approx 10(5)$ deg

$\Gamma_{tot}(H_1(2^{-+}); I = 0) \approx 75$ MeV

$\Gamma_{tot}(H_1(2^{-+}); I = 0) \approx 60$ MeV

source	π_2 masses				η_2 masses	
RPP[29]	1670	1874	1963[a]	2090[a]	1617	1842
LGT[11]	1900	2350[b]	2550	–	1900	2000
GI[38]	1680	2130	–	–	1680	1890
fit A	1580	1880	2130	2350	1580	1785
fit A'	1590	1865	2100	2300	1590	1820

[a]not established
[b]gluonic content

2^{-+}	$f_0(500)\pi$	0.5			
(2360)	$\rho\pi$	3.7			
	$K^*\bar{K}$	0.8			
	$f_2(1270)\pi$	19	(18,43)	(20,15)	(32,**34**,36)
	$f_1(1285)\pi$	0.6			
	$f_0(980)\pi$	0.4			
	$f_0(1370)\pi$	x			
	$\rho(1450)\pi$	7.2			
	$K_{1L}\bar{K}$	x			
	$K_{1H}\bar{K}$	0.1			
	$a_1(1260)\eta$	0.2			
	$a_2(1320)\eta$	12	(10,23)	(13,11)	(15,**16**,16)
	$K^*(1410)\bar{K}$	0.3			
	$K_0^*(1430)\bar{K}$	x			
	$K_2^*(1430)\bar{K}$	39	(33,66)	(37,39)	(32,**34**,36)
	$a_0(1450)\eta$	x			
Γ_{tot}		**83.8**	(61,132)	(84,82)	(82,**87**,89)

$H_1(2^{-+})$ 2250/2300/2400

state	$\pi\rho$	$\omega\rho$	$f_2\pi$	$K^*\bar{K}$	$\rho(1450)\pi$	$K_2^*\bar{K}$	Γ_{tot}	Γ_{PDG}
$\pi_2(1S)$	117	25	85	20	3	–	250	258(8)
$\pi_2(2S)$	86	24	31	3	19	9	172+	237(30)
$\pi_2(3S;2250)$	12	10	16	6	35	8	87+	–
$H_1(\pi_2)$	4	0	20	0	7	40	71	–

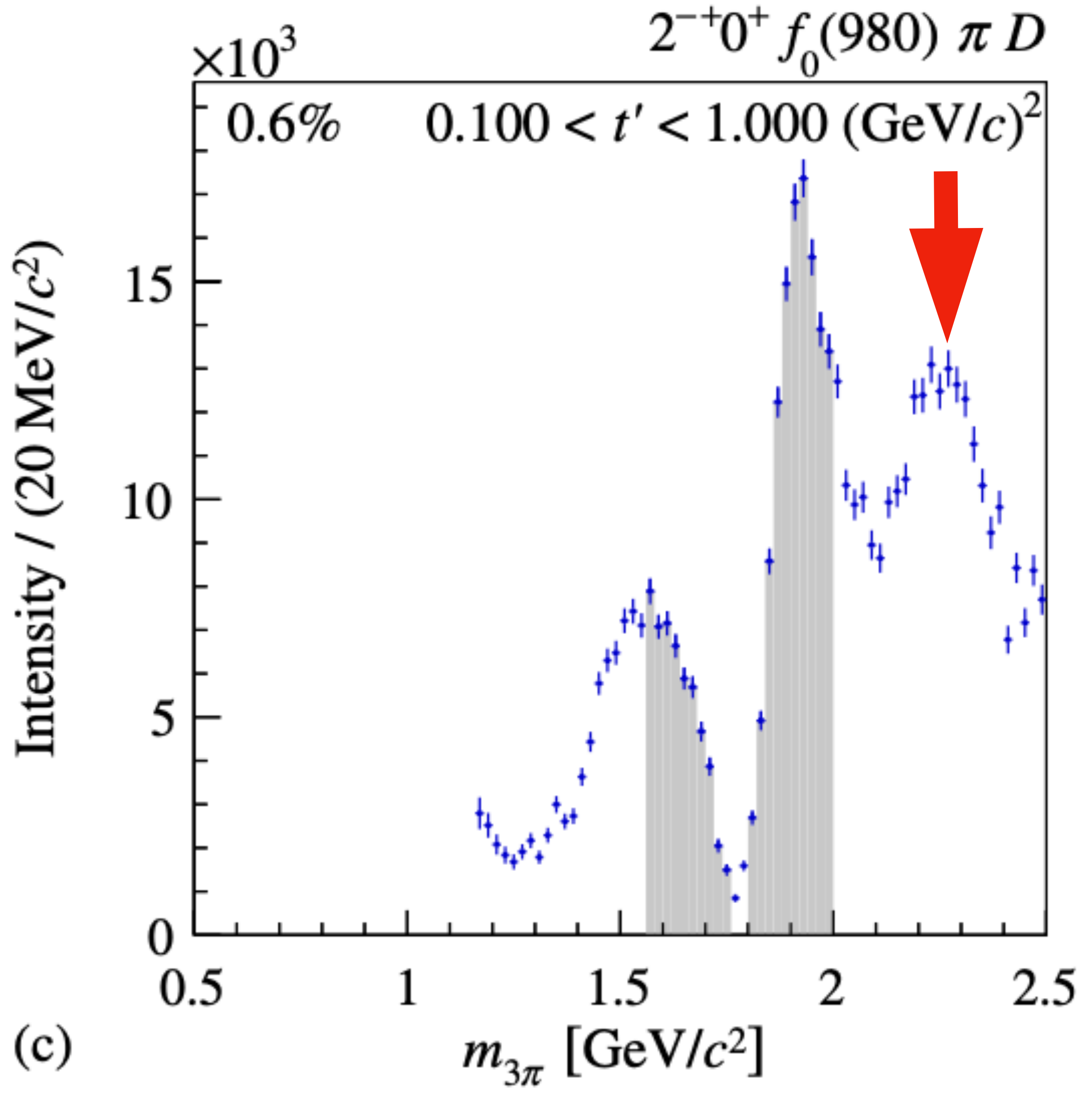
perhaps $f_2\pi/\rho\pi$ is the best diagnostic

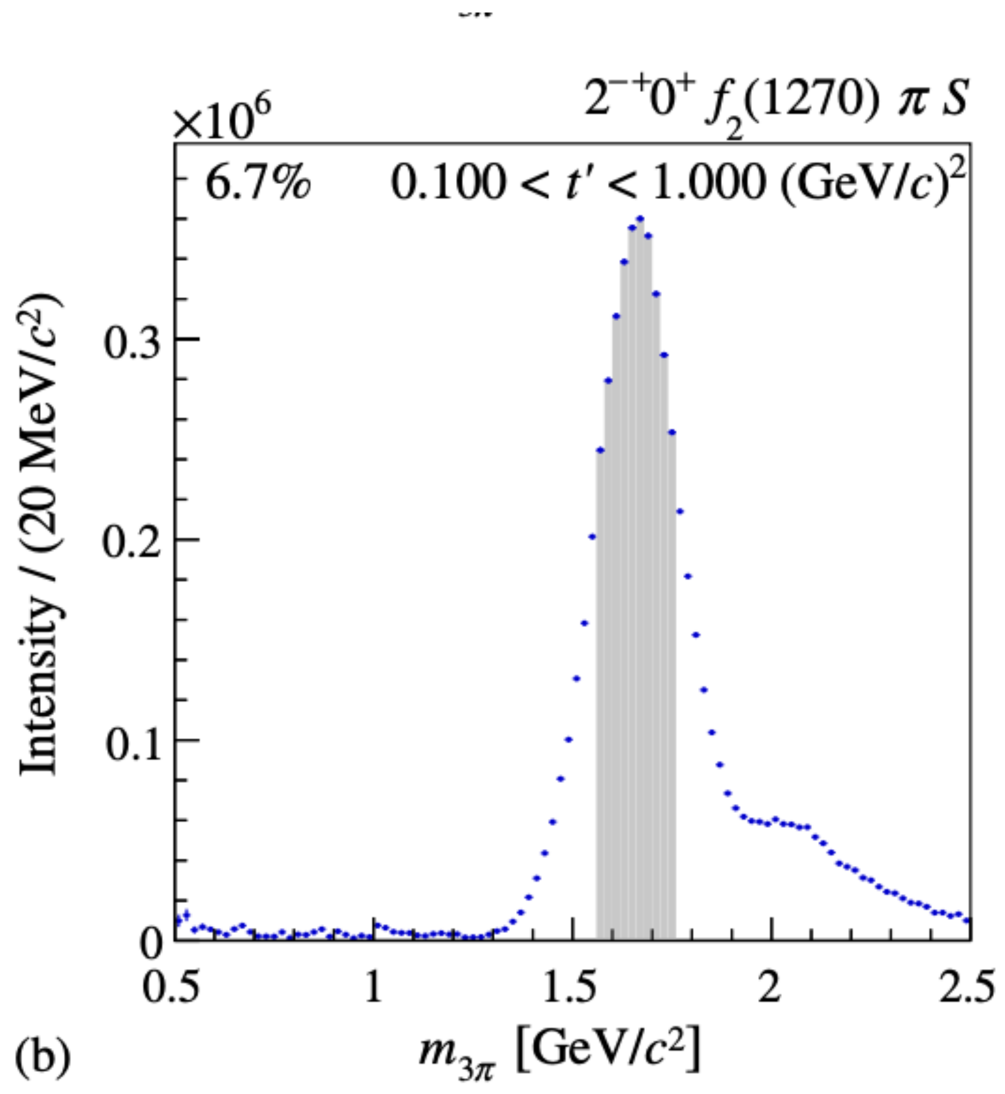
S-D-G amplitude ratios for $\pi_2 \to f_2\pi$ are approximately 1:0:0 for a hybrid but are 1 : 0.14 : 0.003 for the $q\bar{q}(1S)$ state, 1 : 0.62 : 0.06 for the $q\bar{q}(2S)$ state, and -0.24 : 1 : -0.38 for a $q\bar{q}(3S)$ state near 2250 MeV.

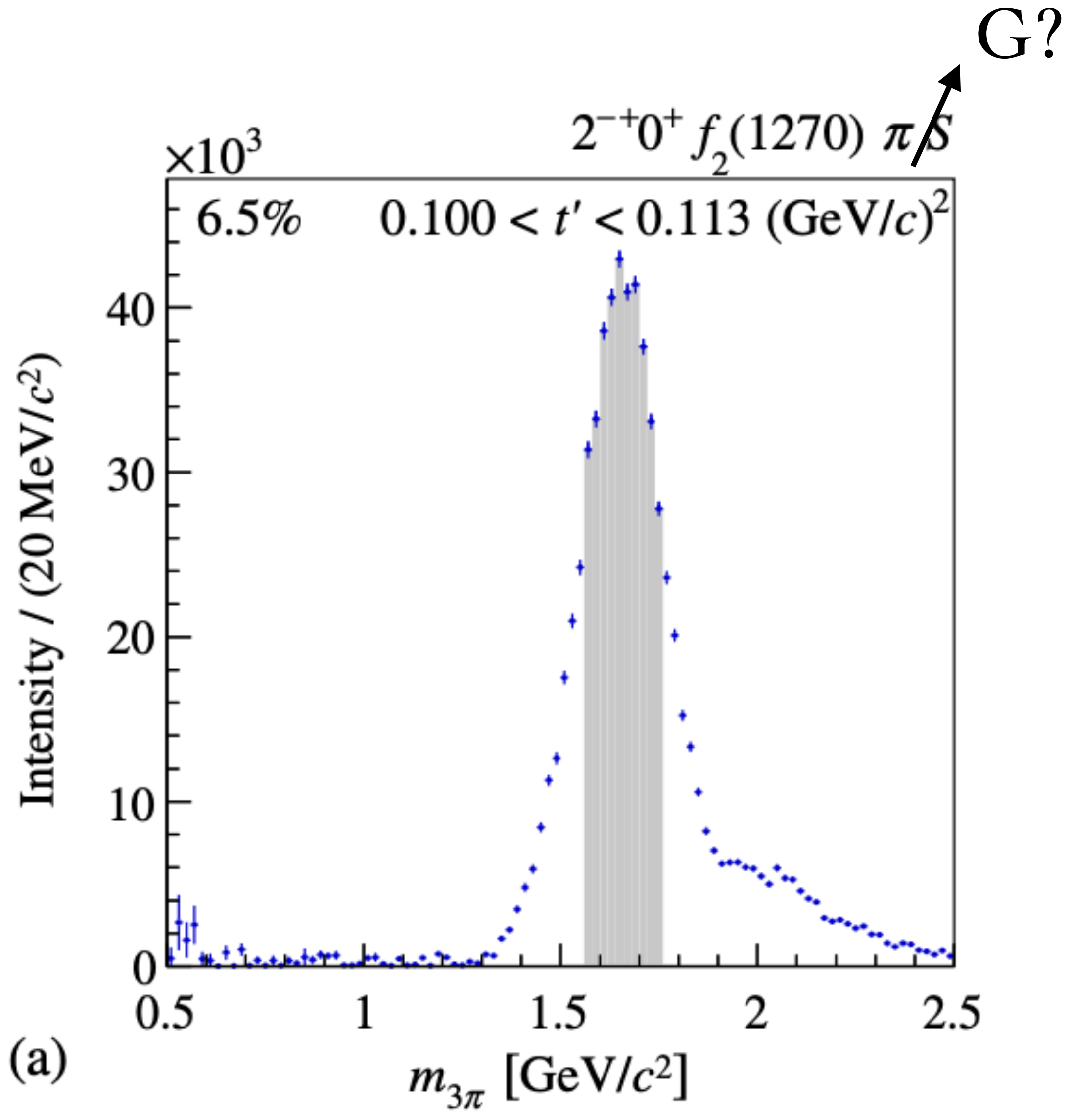
COMPASS: $\pi_2(1670) \to (f_2\pi)_S \quad \to 2S$

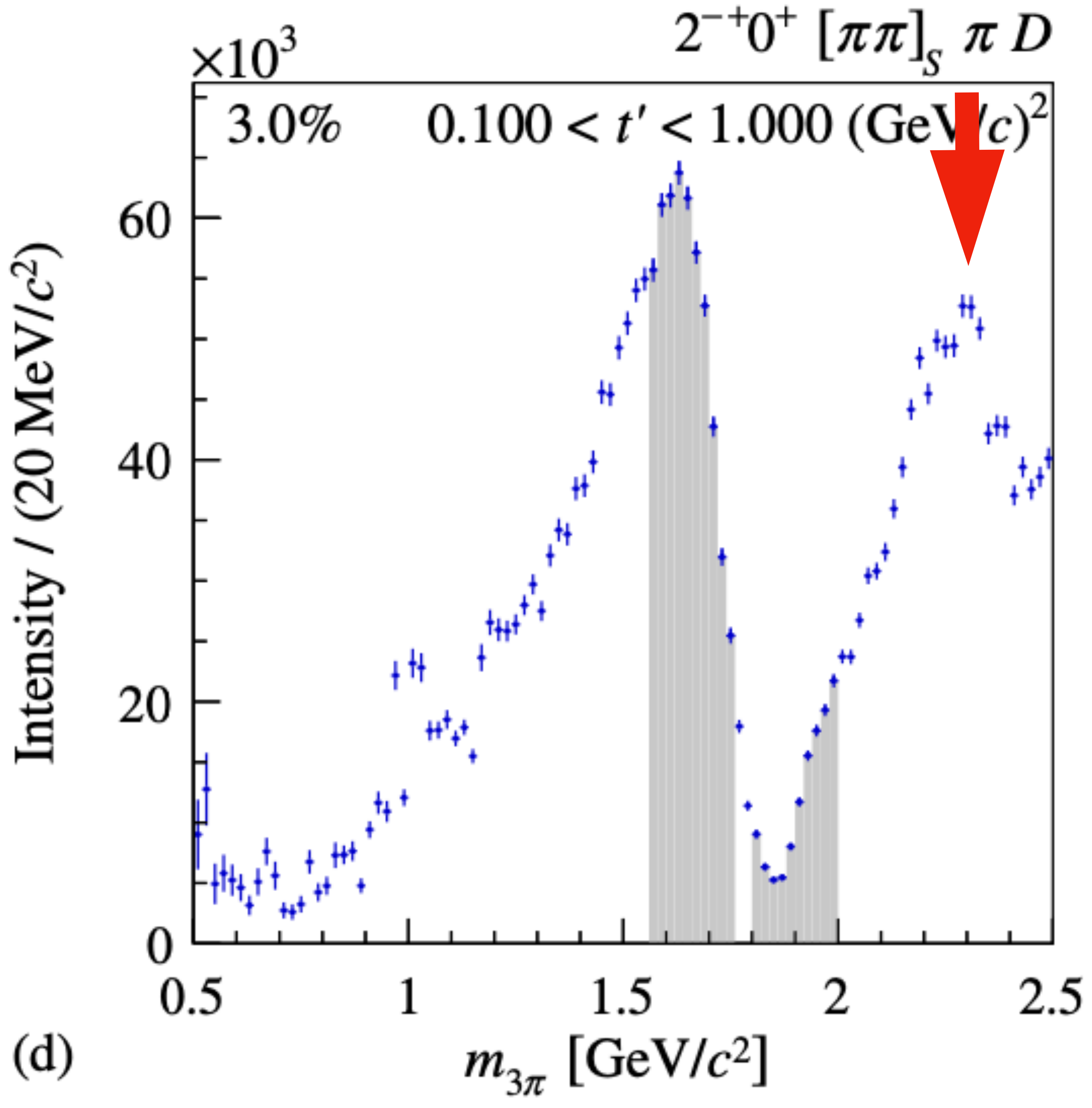
$\pi_s(1880) \to (f_2\pi)_D \quad \to 3S$

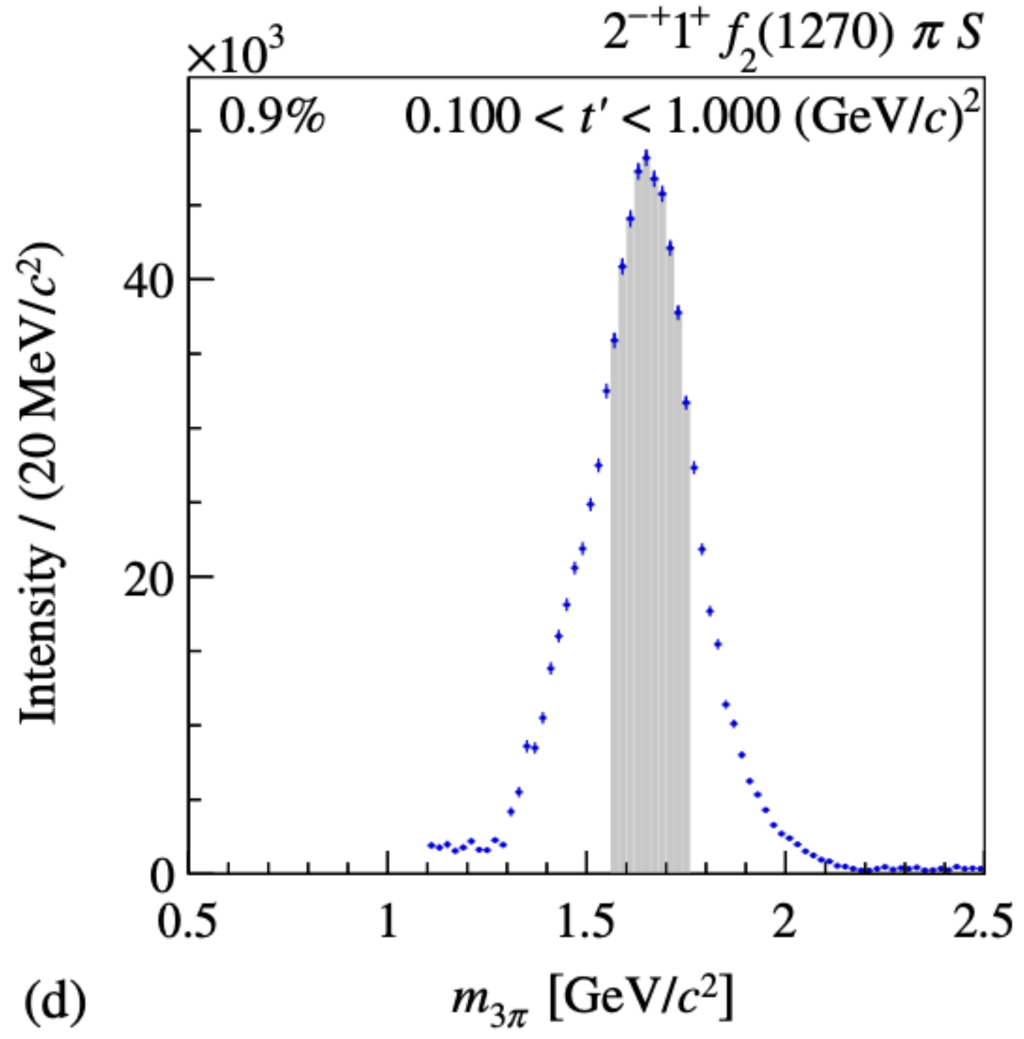
$H_1(2^{-+})$ 2250/2300/2400

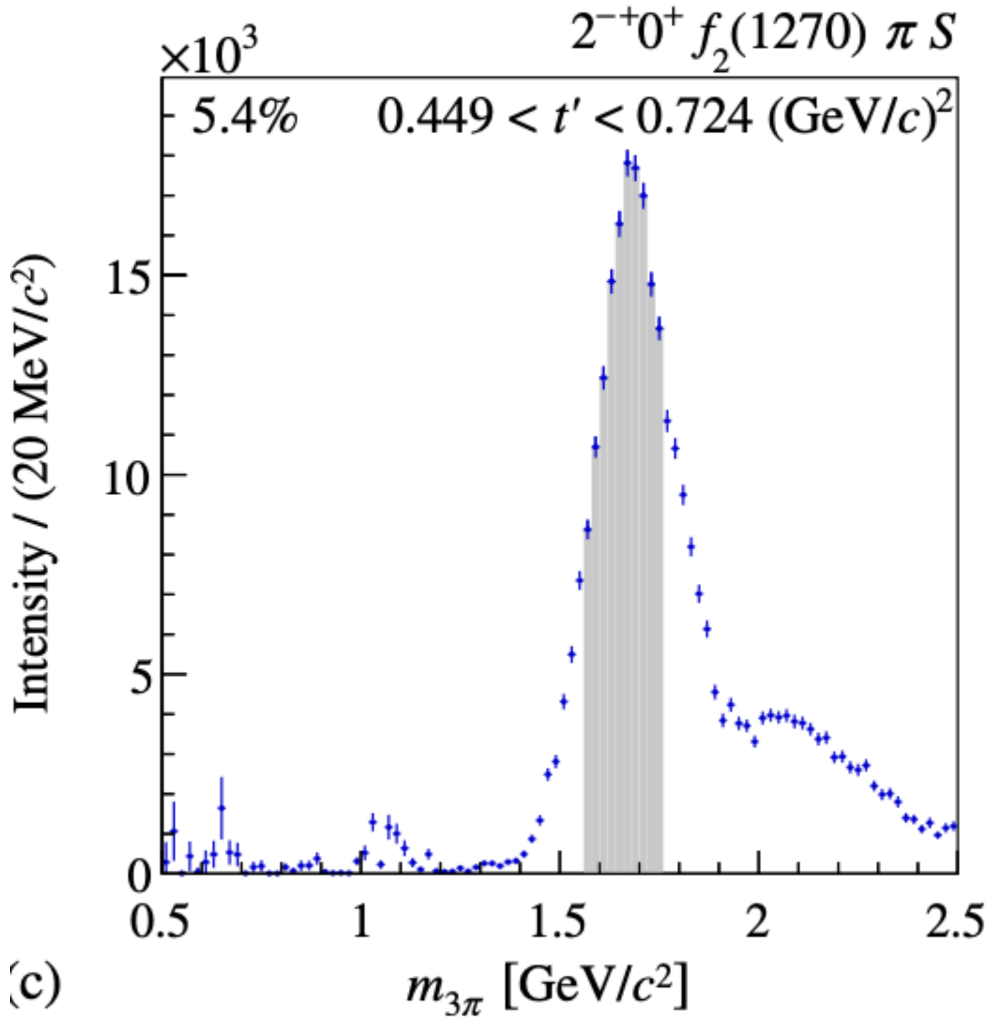












summary

- vector hybrid near 2100 w/ decay constant ~ 20 MeV. Likely too broad to be easily seen

- partner states at 2100-2250 and 2220-2350.

- with a $\pi_1(1600)$ expect partner states at 1750-1780 and ~1900 (near the $\eta_1(1855)$).

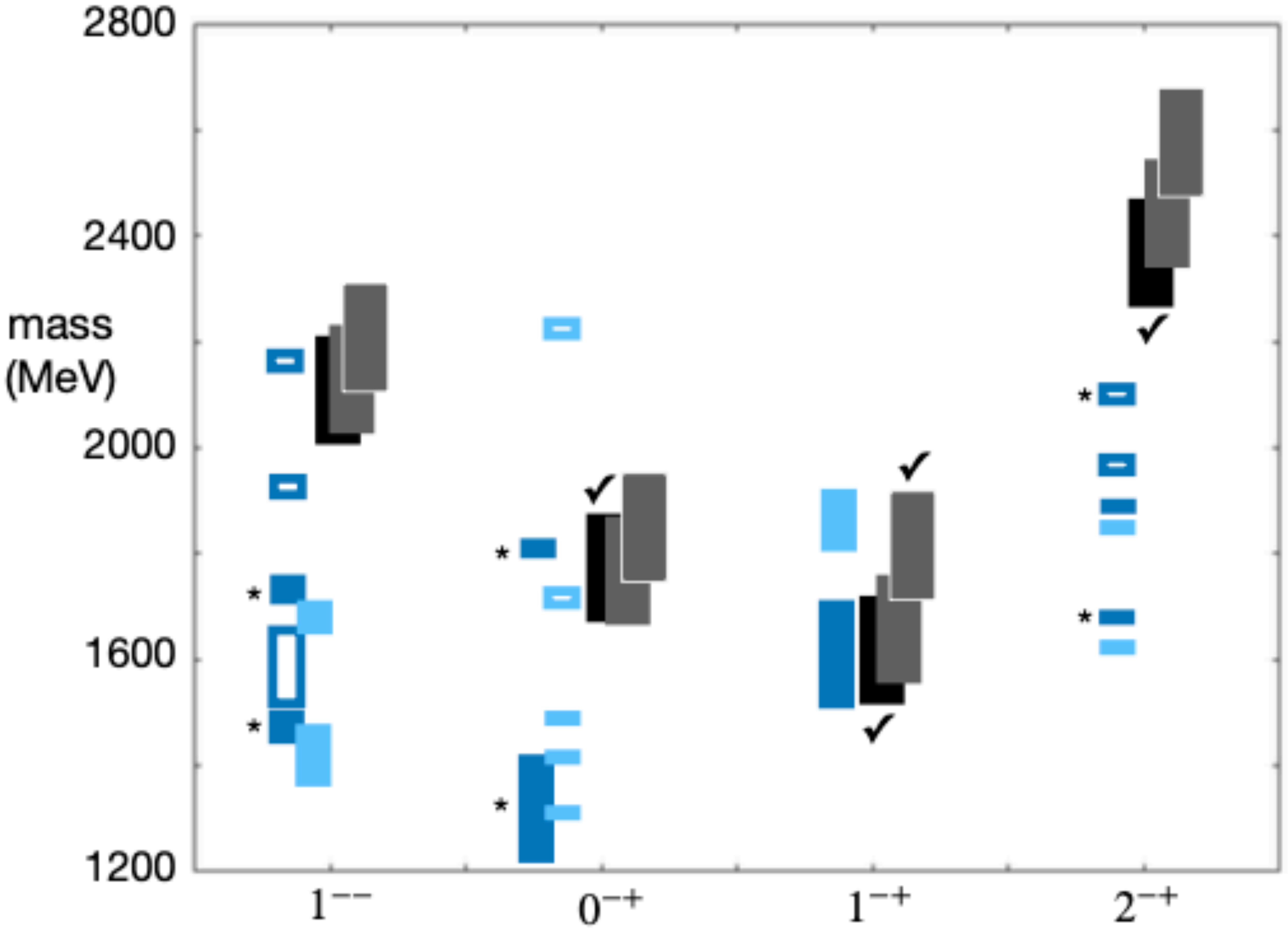
- the $\pi(1800)$ is a good hybrid candidate. The $\pi(3S)$ should be nearby

- it is possible that the hybrid $\pi_2(2360)$ has been observed!

- think about strange hybrids!

- perhaps the low lying hybrid spectrum is emerging (& consistent with expectations)

A Speculative Summary



*adjusted

LGT I=1

LGT I=0

hybrids

PDG I=1

PDG I=0

hollow box = unverified

* agrees well w/ QMs

discovering exotic particles

the traditional approach to "discovery"

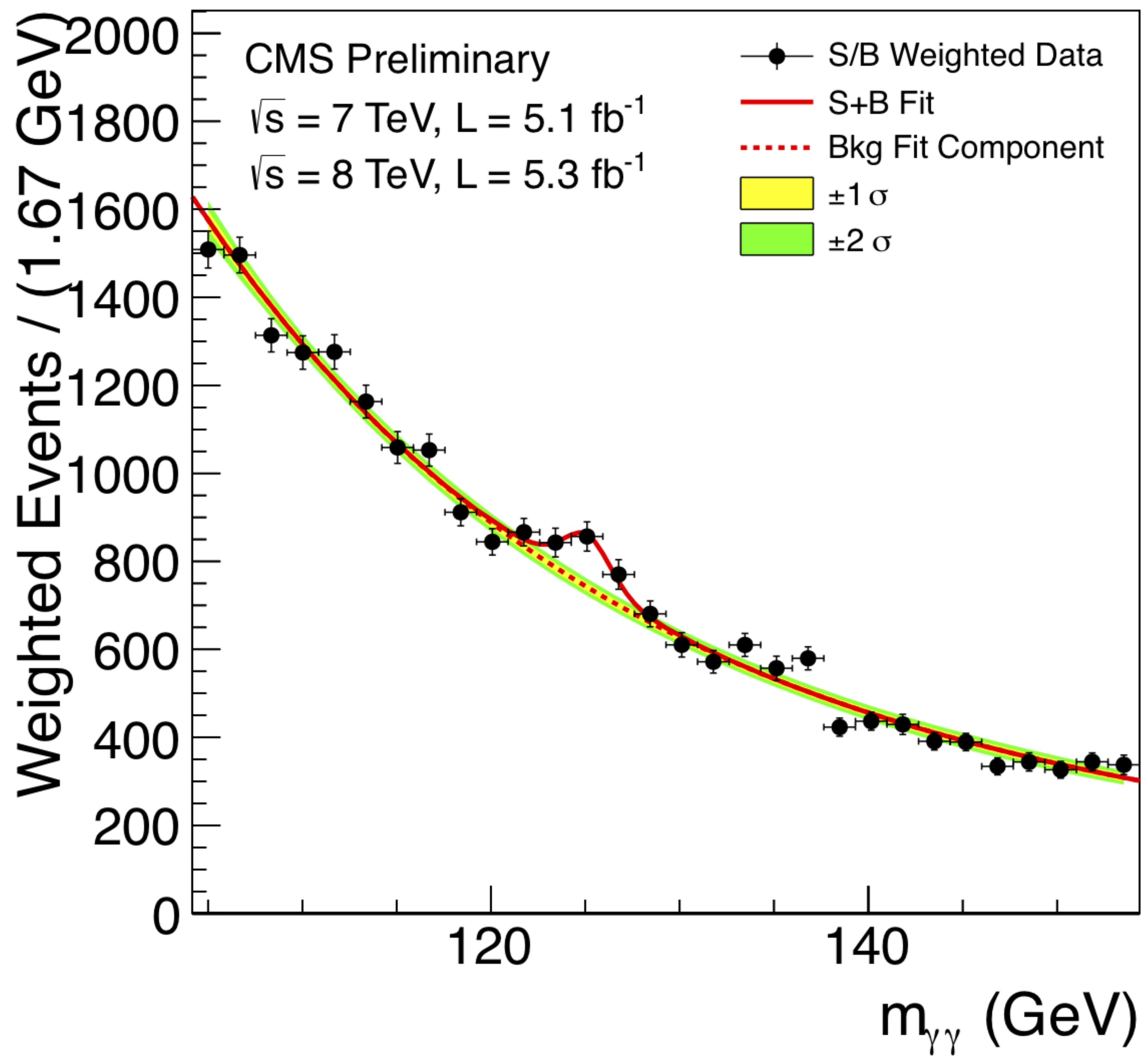
i. fit two model amplitudes to data, $M_0(\dots)$ and $M_1(\dots; m_R, \Gamma_R)$

ii. determine the P-value ~ the probability of obtaining the observed effect (or greater) given that the null hypothesis is true.

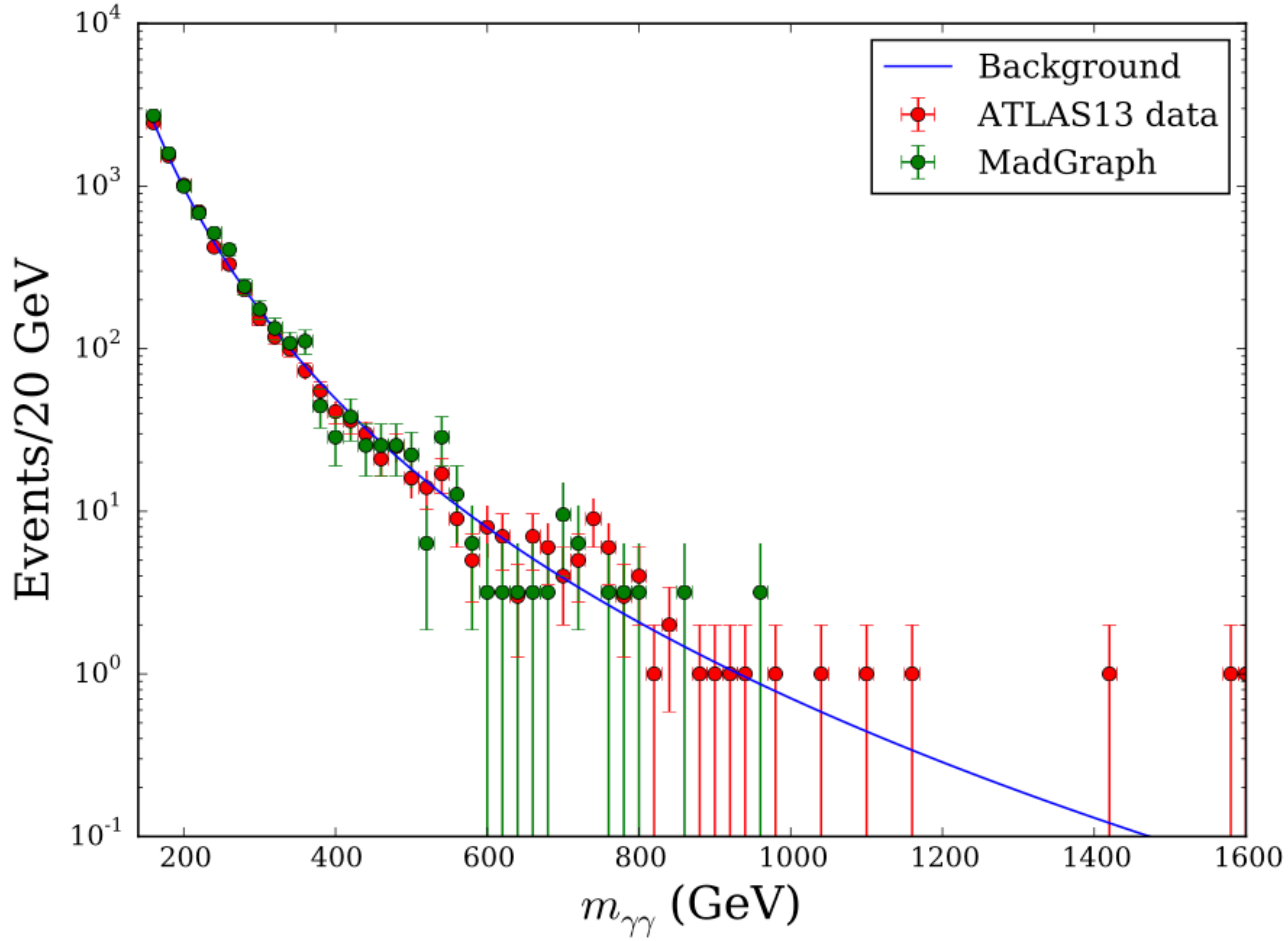
$$2 \log \frac{L_1}{L_0} \to \chi^2_{d_1 - d_0}, \quad L_0 \subset L_1.$$ (Wilks's theorem)

iii. declare a discovery if $P < 3 \cdot 10^{-7} \to 5\sigma$ $[P < 0.0027 \to 3\sigma]$. The new physics is described by the fit parameters, m_R, Γ_R.

Higgs

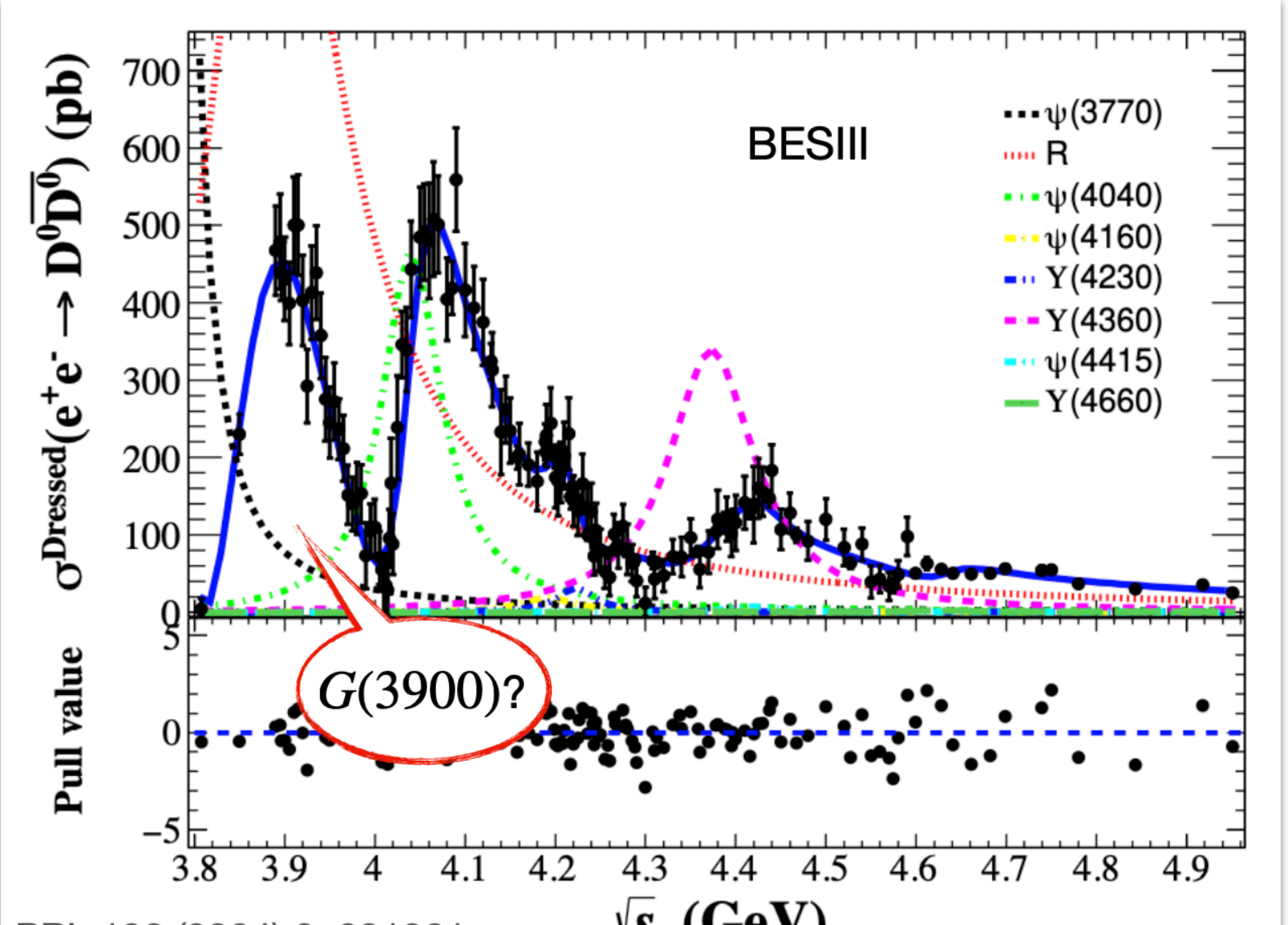


digamma



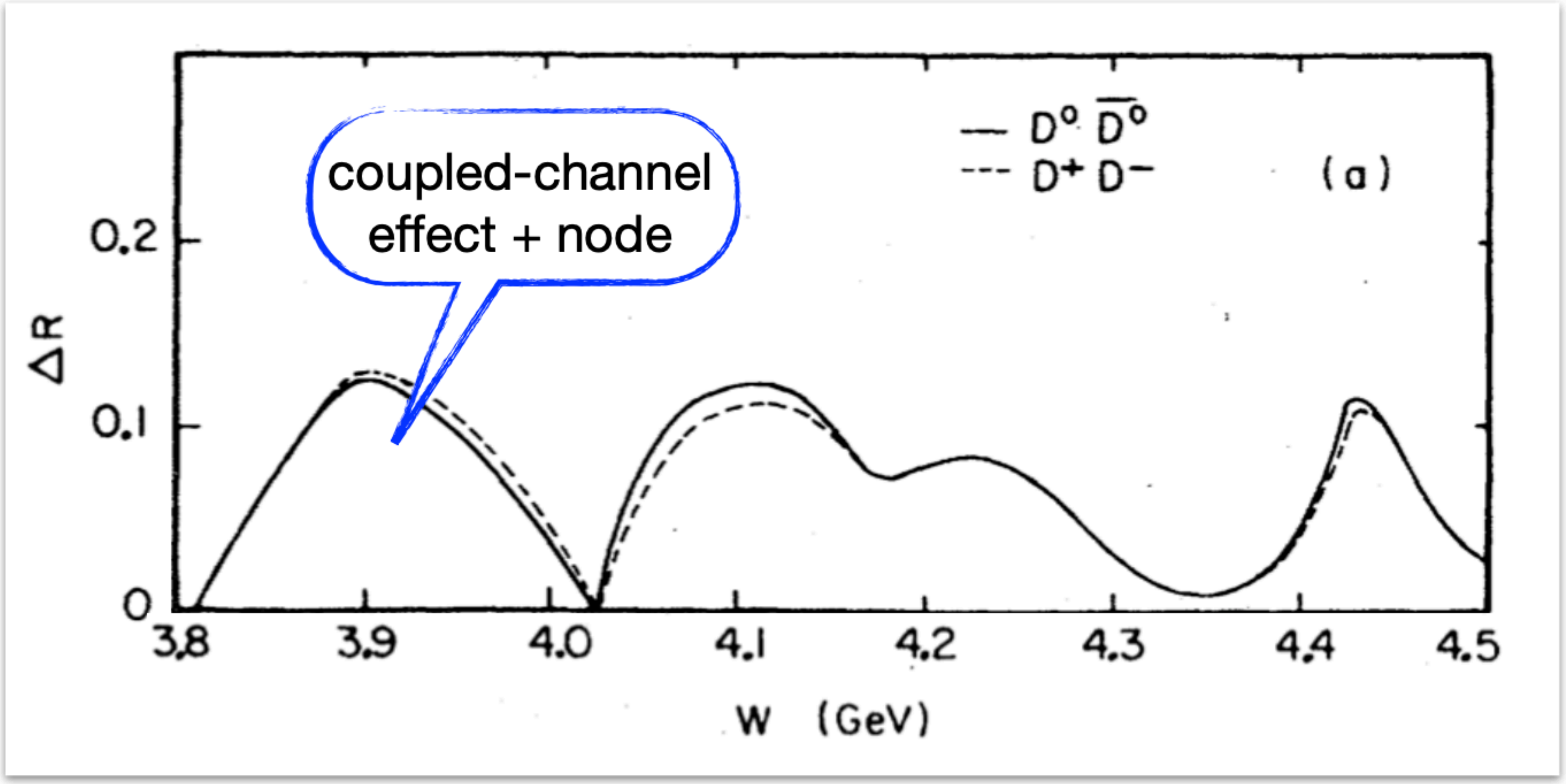
G(3900)

$e^+e^- \to D\bar{D}$



PRL 133 (2024) 8, 081901

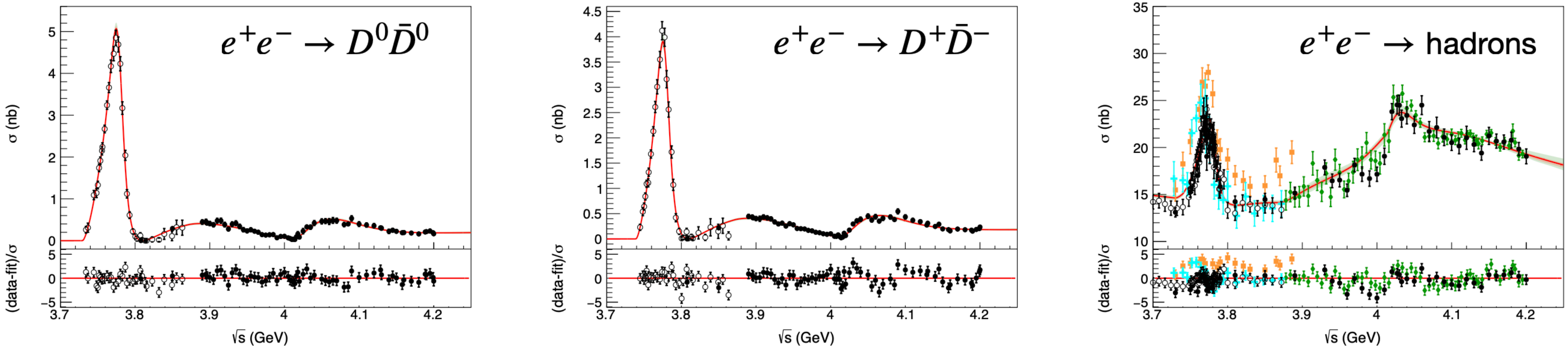
Eichten et al., Phys. Rev. D 21 (1980) 203

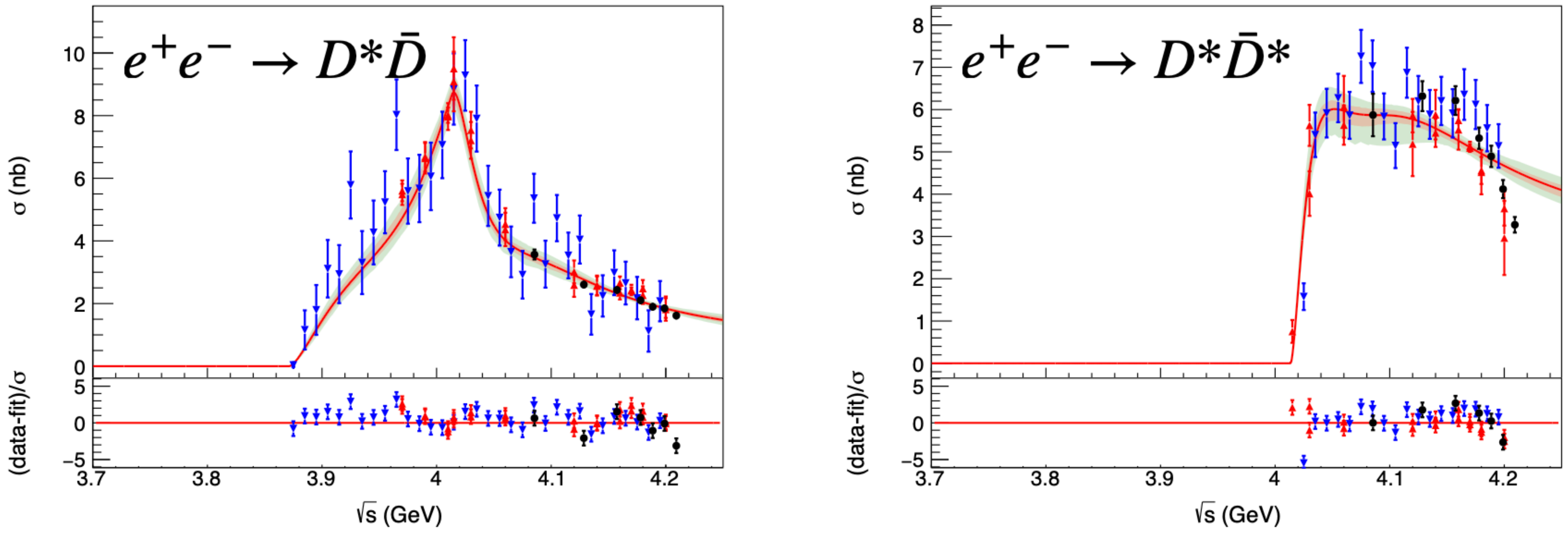


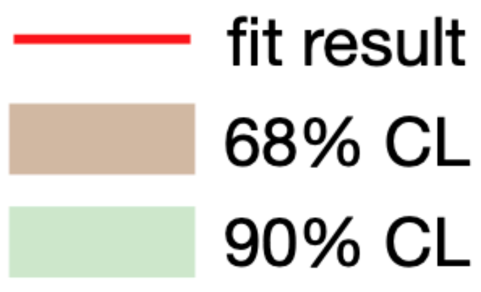
In our calculation there is some weak structure in the 3.9–4.0 GeV region. It does not arise from a $c\bar{c}$ resonance, but from the opening of the $D\bar{D}^* + D^*\bar{D}$ channel and a decrease in the $D\bar{D}$ channel due to a nearby zero in the 3S decay amplitude.

N. Hüsken, R.F. Lebed, R.E. Mitchell, E.S. Swanson, Y-Q Wang, 2404.03896

Refit with additional information and coupled channels

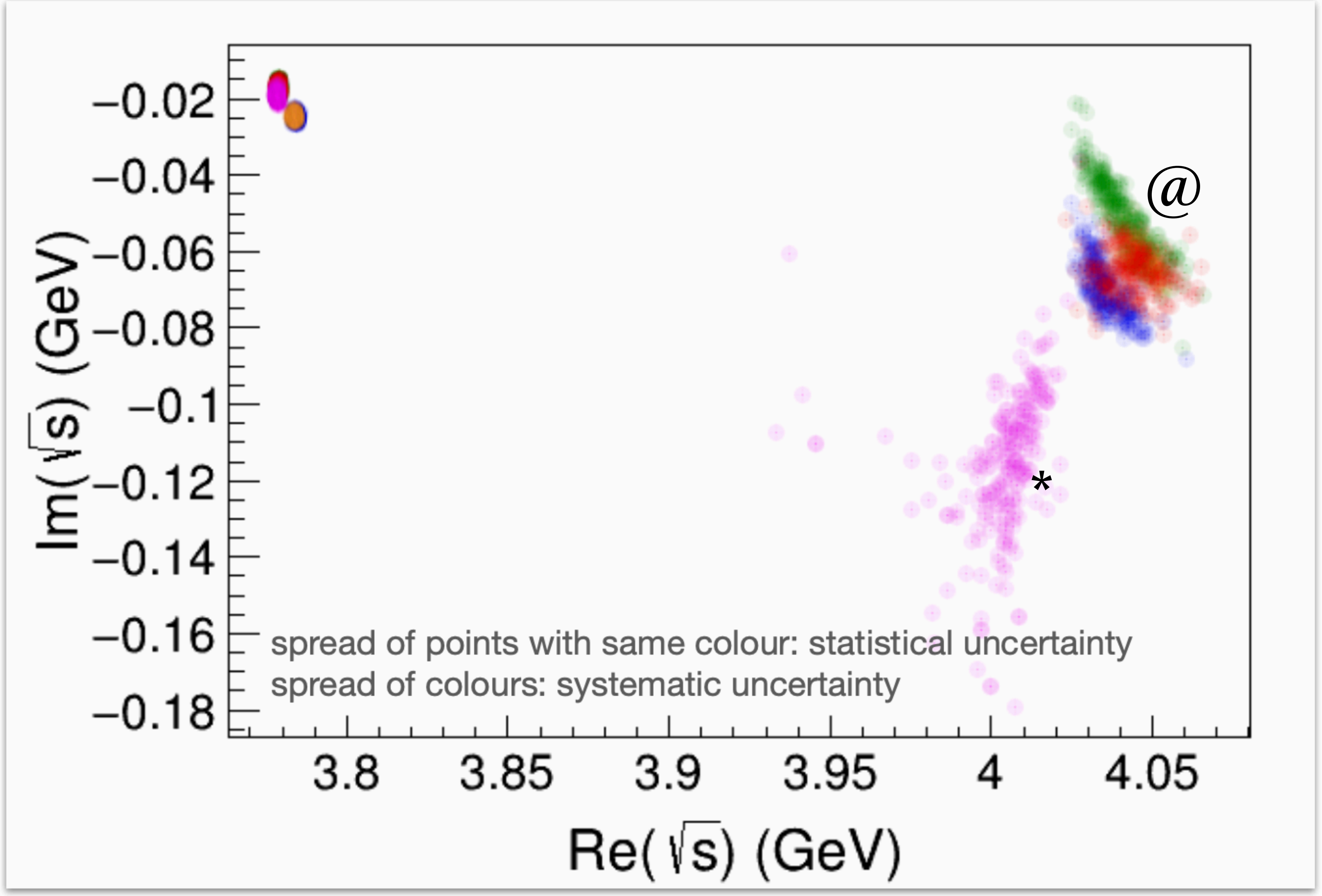






Data from:

BESIII (unoff.) Andy Julin, University of Minnesota
BESIII: PRL 133 (2024) 8, 081901
BESIII: JHEP 05 (2022) 155
Belle: Phys.Rev.D 97 (2018) 1, 012002
CLEO: Phys.Rev.D 80 (2009) 072001
BES: PRL 88, 101802 (2002)
BESII: PRL 97, 262001 (2006)
SPEAR: PRL 39, 526 (1977);
A. Osterheld et al. 86; Schindler 79



the devil's in the details:

- given our choices of K_{ij}, $n(s)$, Σ, we find described without a $G(3900)$ pole
- details we varied:
  - isospin-constraints between D^+D^- and $D^0\bar{D}^0$
  - threshold opening of channel that absorbs *missing* intensity
  - node in $\hat{n}(s) = n(s) \cdot (1 - k^2/k_0^2)$
- details we did not vary:
  - type of barrier factor $b_L(s)$
  - choice of Σ
- very valuable that other groups use different assumptions entirely different approaches

PRD 109 (2024) 11, 114015
PRL 133 (2024) 24, 241903
PRD 112 (2025) 5, 054027
PRD 112 (2025), 016015
arXiv:2509.17679

some find a $G(3900)$, some do not…

+ more in other energy regions

(@) 4 models w/ 24-30 parameters

(*) model with node

problems with the traditional approach

problems with the traditional approach

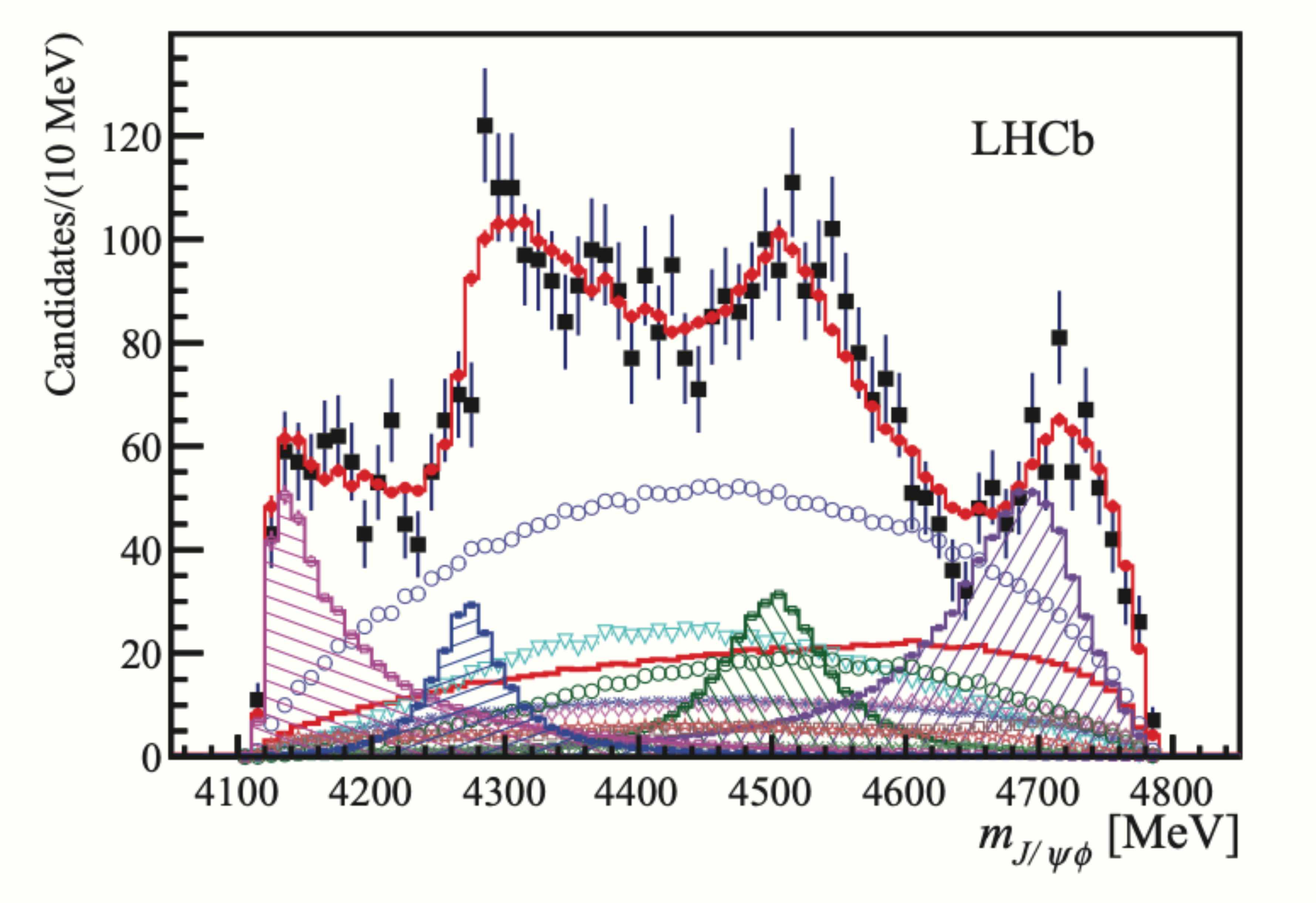
i. fluctuations in the data set may be important

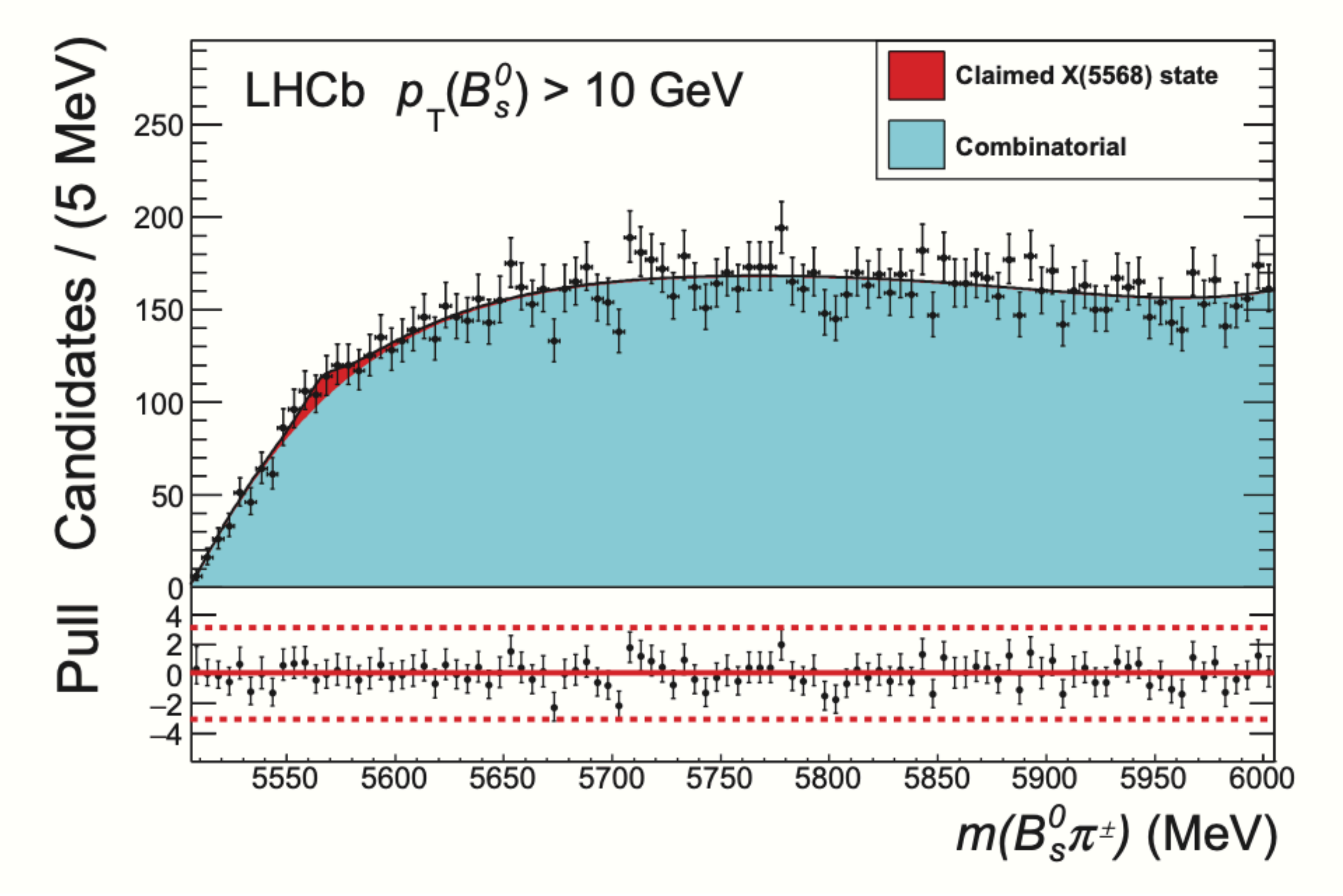
ii. models M0 and M1 are wrong!

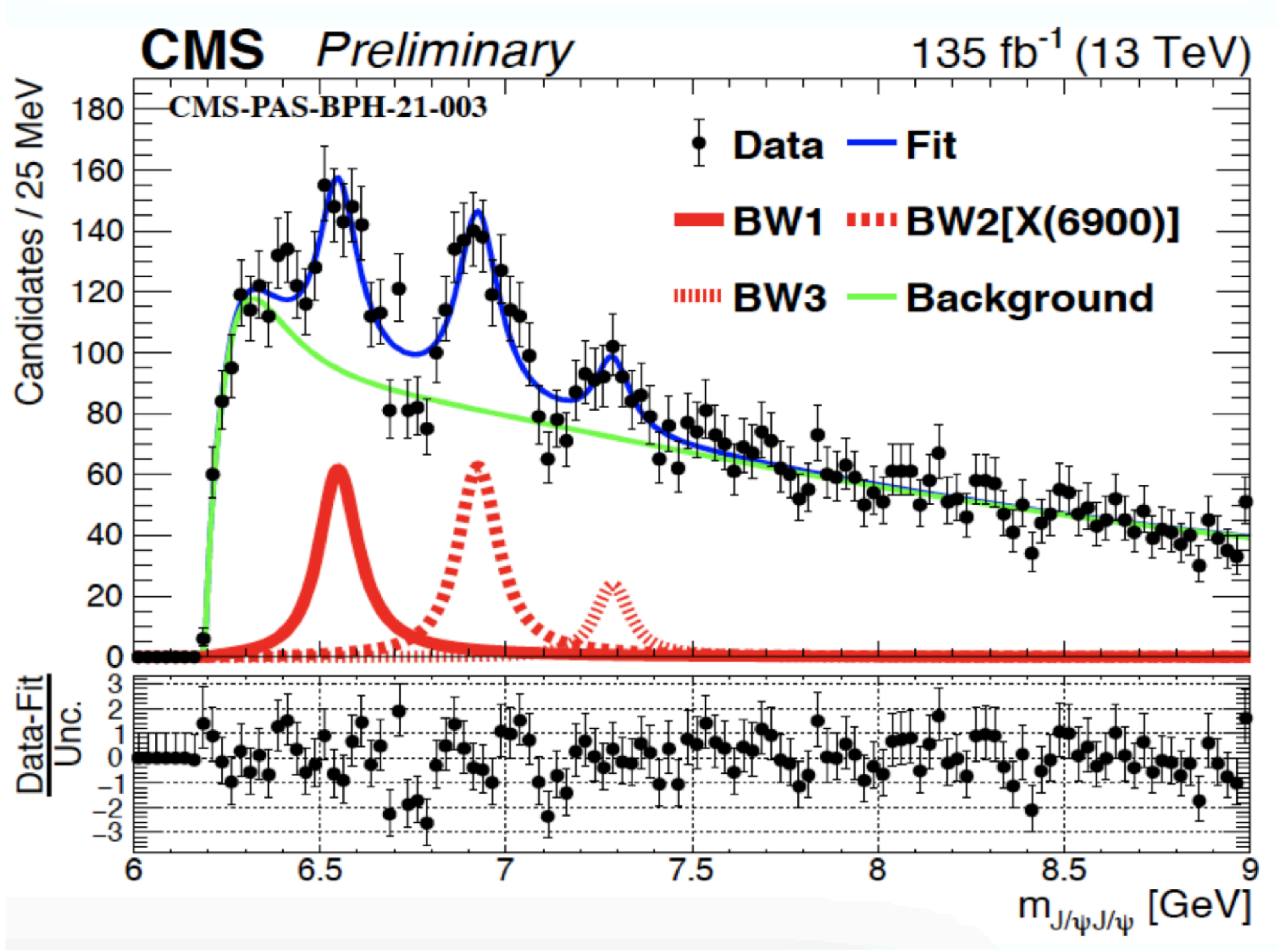
iii. systematic errors are often underestimated

iv. problematic overfitting

v. we wish to extract model structure, not assume it







problems with the traditional approach

$$\mathrm{BW}(m|M_0,\Gamma_0) = \frac{1}{M_0^2 - m^2 - iM_0\Gamma(m)}\,,$$

$$\Gamma(m) = \Gamma_0 \left(\frac{q}{q_0}\right)^{2\,L_{\Lambda^*}+1} \frac{M_0}{m} B'_{L_{\Lambda^*}}(q,q_0,d)^2\,.$$

i. fluctuations in the data set may be important

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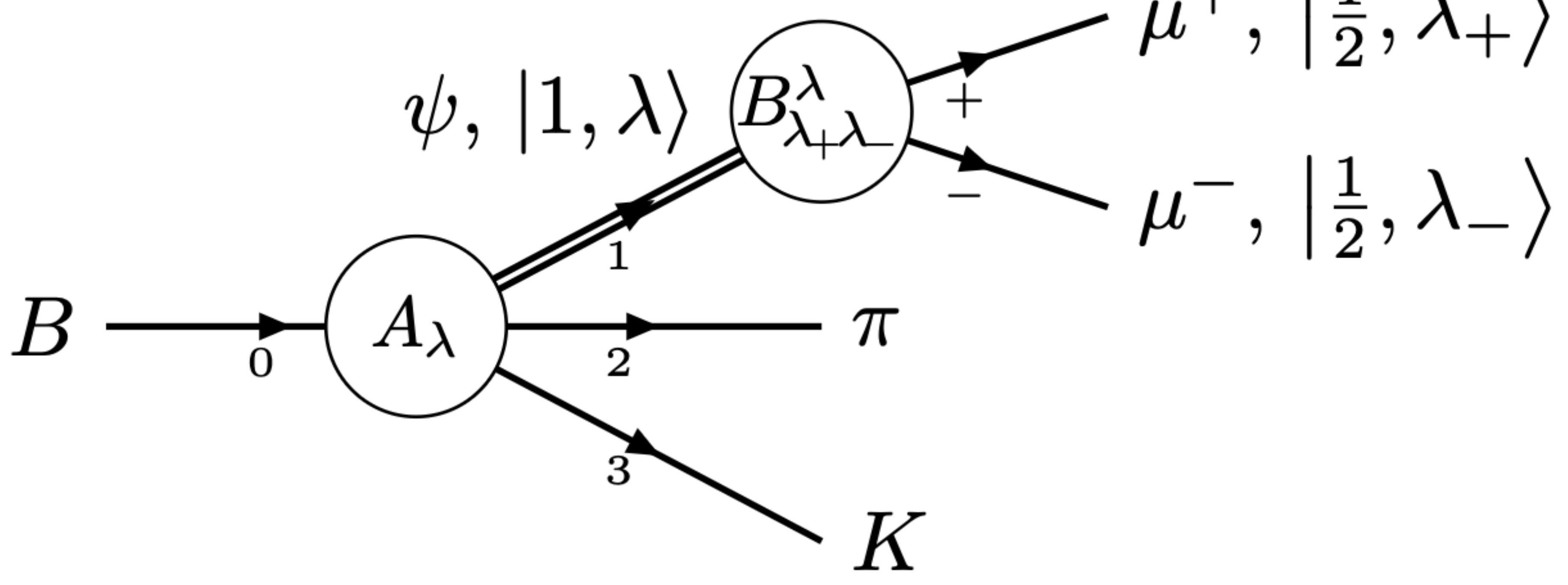
$$R_{\Lambda_n^*}(m_{Kp}) = B'_{L^{\Lambda_n^*}_{\Lambda_b^0}}(p,p_0,d)\left(\frac{p}{M_{\Lambda_b^0}}\right)^{L^{\Lambda_n^*}_{\Lambda_b^0}} \mathrm{BW}(m_{Kp}|M_0^{\Lambda_n^*},\Gamma_0^{\Lambda_n^*})\, B'_{L_{\Lambda_n^*}}(q,q_0,d)\left(\frac{q}{M_0^{\Lambda_n^*}}\right)^{L_{\Lambda_n^*}}.$$

$$\mathcal{M}^{\Lambda^*}_{\lambda_{\Lambda_b^0},\,\lambda_p,\,\Delta\lambda_\mu} = \sum_n R_{\Lambda_n^*}(m_{Kp})\,\mathcal{H}^{\Lambda_n^*\to Kp}_{\lambda_p}\sum_{\lambda_\psi} e^{i\,\lambda_\psi\phi_\mu}\; d^{\,1}_{\lambda_\psi,\Delta\lambda_\mu}(\theta_\psi)$$
$$\times\sum_{\lambda_{\Lambda^*}}\mathcal{H}^{\Lambda_b^0\to\Lambda_n^*\psi}_{\lambda_{\Lambda^*},\,\lambda_\psi}\; e^{i\,\lambda_{\Lambda^*}\phi_K}\; d^{\,\frac{1}{2}}_{\lambda_{\Lambda_b^0},\,\lambda_{\Lambda^*}-\lambda_\psi}(\theta_{\Lambda_b^0})\; d^{\,J_{\Lambda_n^*}}_{\lambda_{\Lambda^*},\,\lambda_p}(\theta_{\Lambda^*}).$$



unitary?

correct analytic structure?

crossing symmetric?

non-perturbative?

problems with the traditional approach

i. fluctuations in the data set may be important

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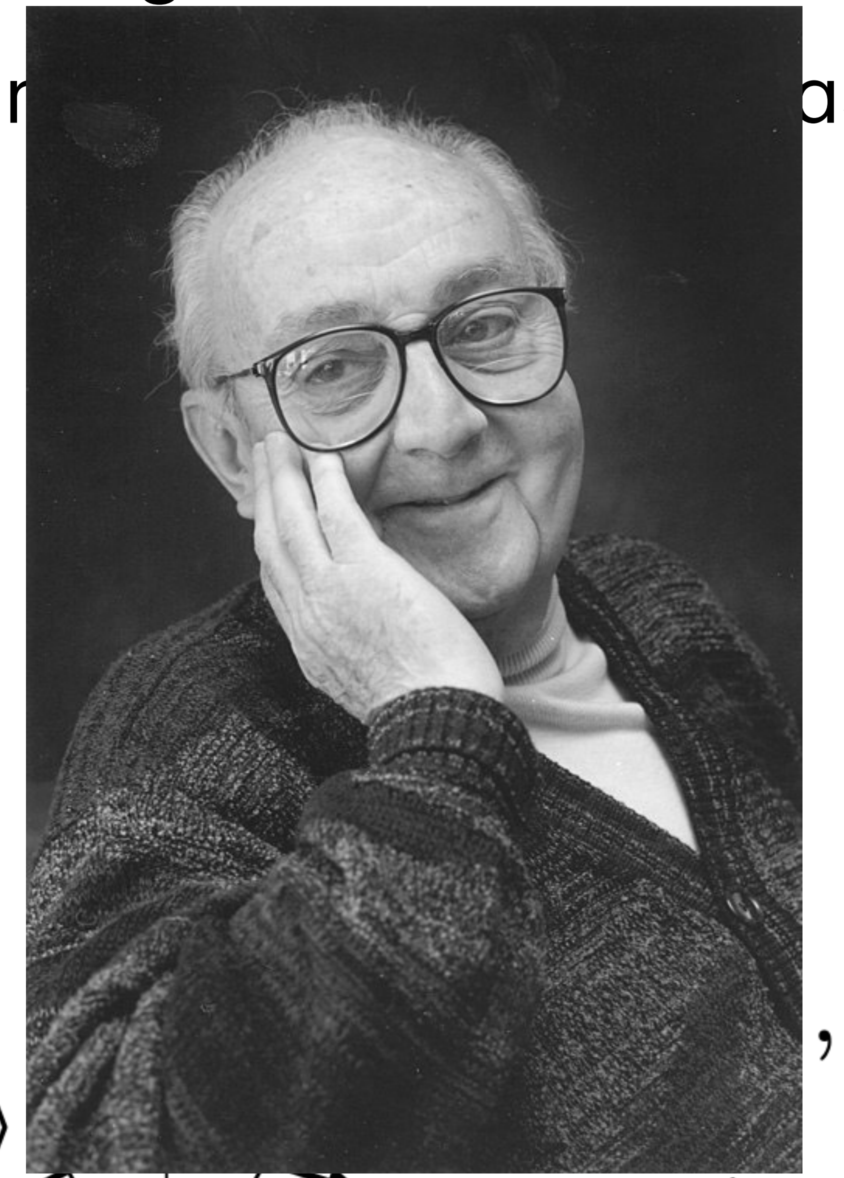
v. we wish to extract r … assume



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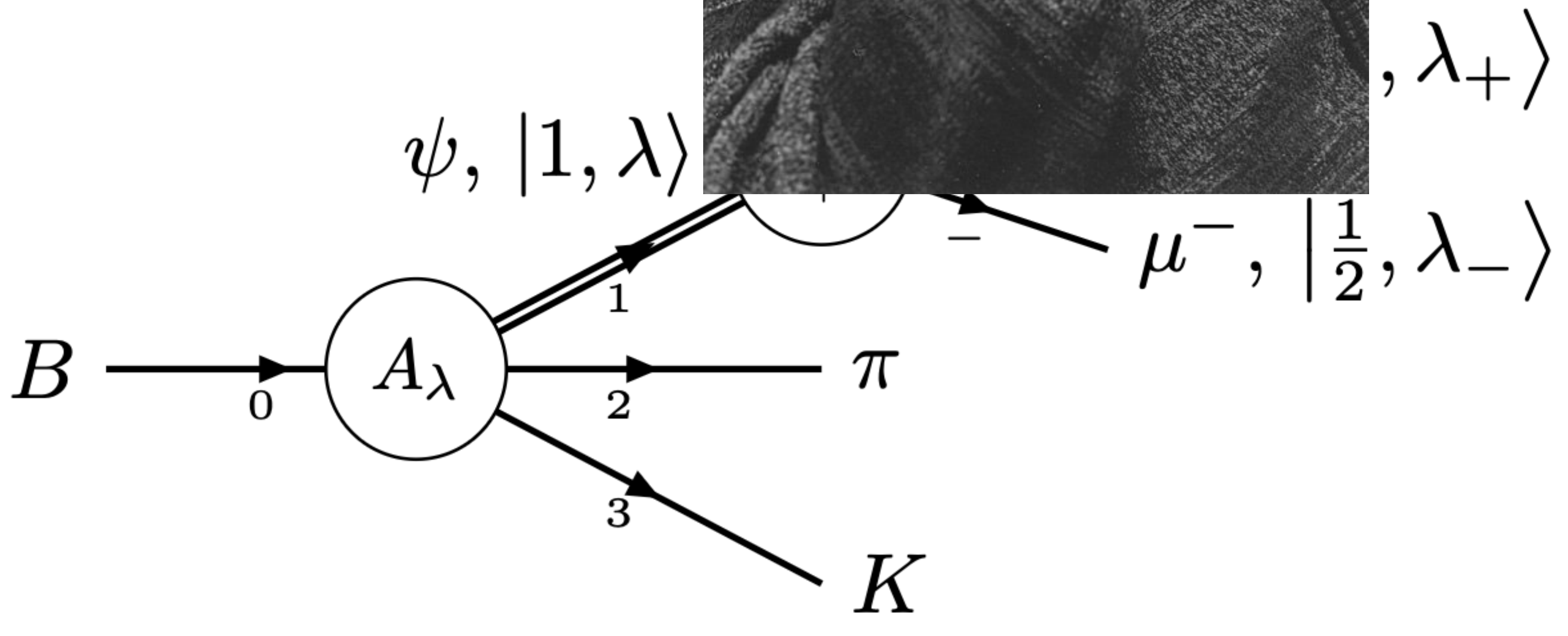
$$\times \sum_{\lambda_{\Lambda^*}} \mathcal{H}^{\Lambda_b^0\to\Lambda_n^*\psi}_{\lambda_{\Lambda^*},\lambda_\psi}\, e^{i\lambda_{\Lambda^*}\phi_K}\, d^{\frac{1}{2}}_{\lambda_{\Lambda_b^0},\lambda_{\Lambda^*}-\lambda_\psi}(\theta_{\Lambda_b^0})\, d^{J_{\Lambda_n^*}}_{\lambda_{\Lambda^*},\lambda_p}(\theta_{\Lambda^*}).$$



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i. fluctuations in the data set may be important

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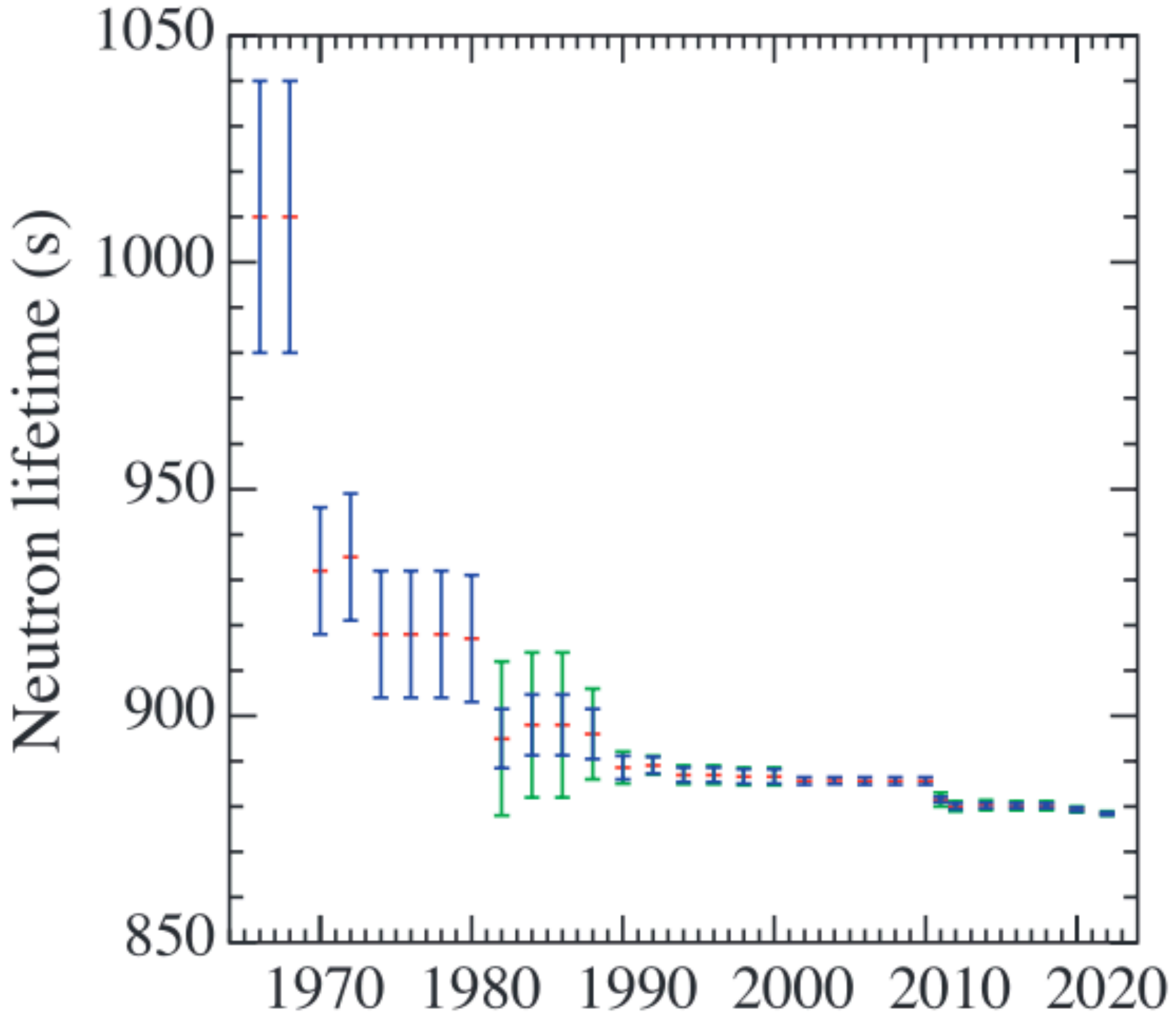
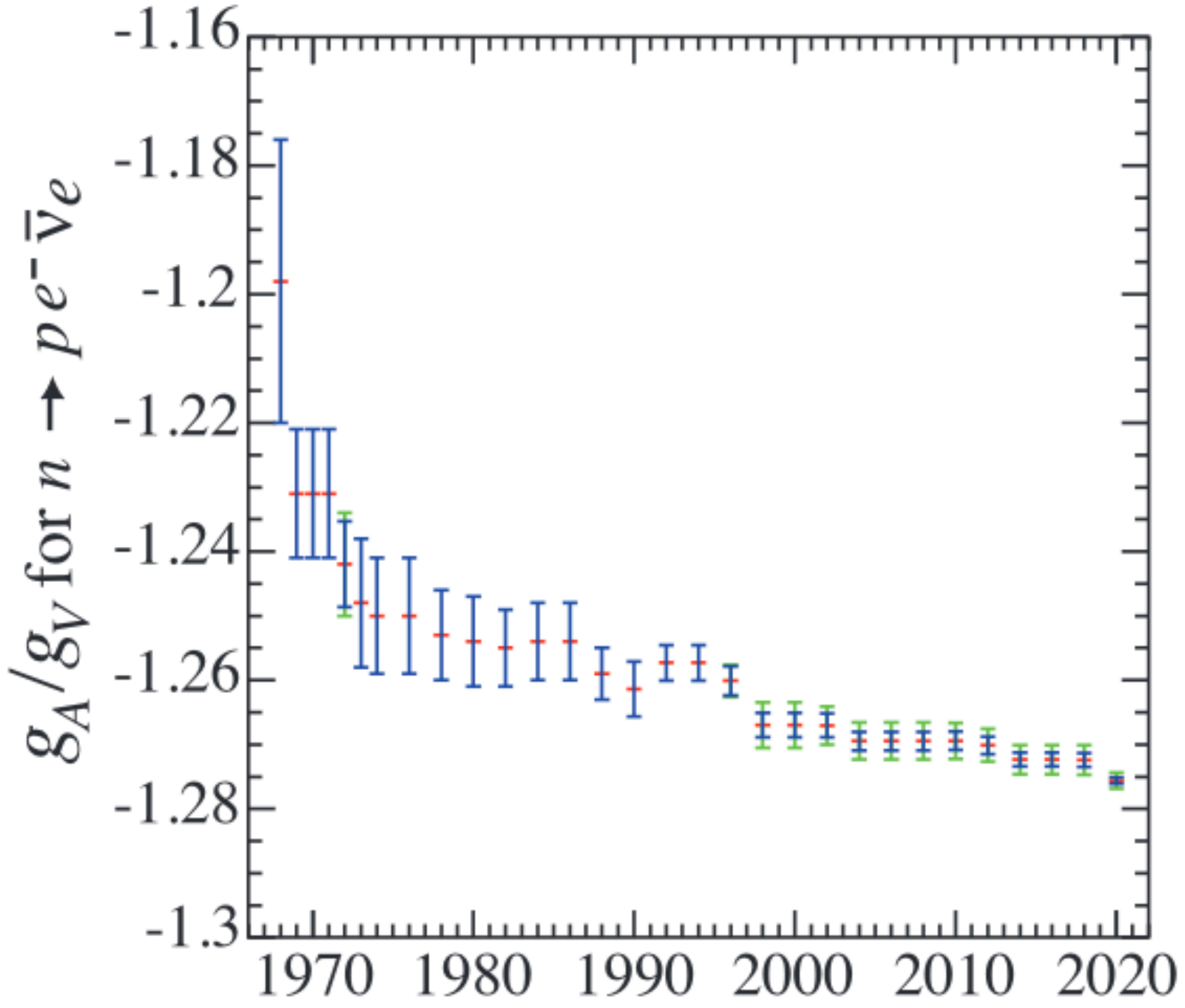
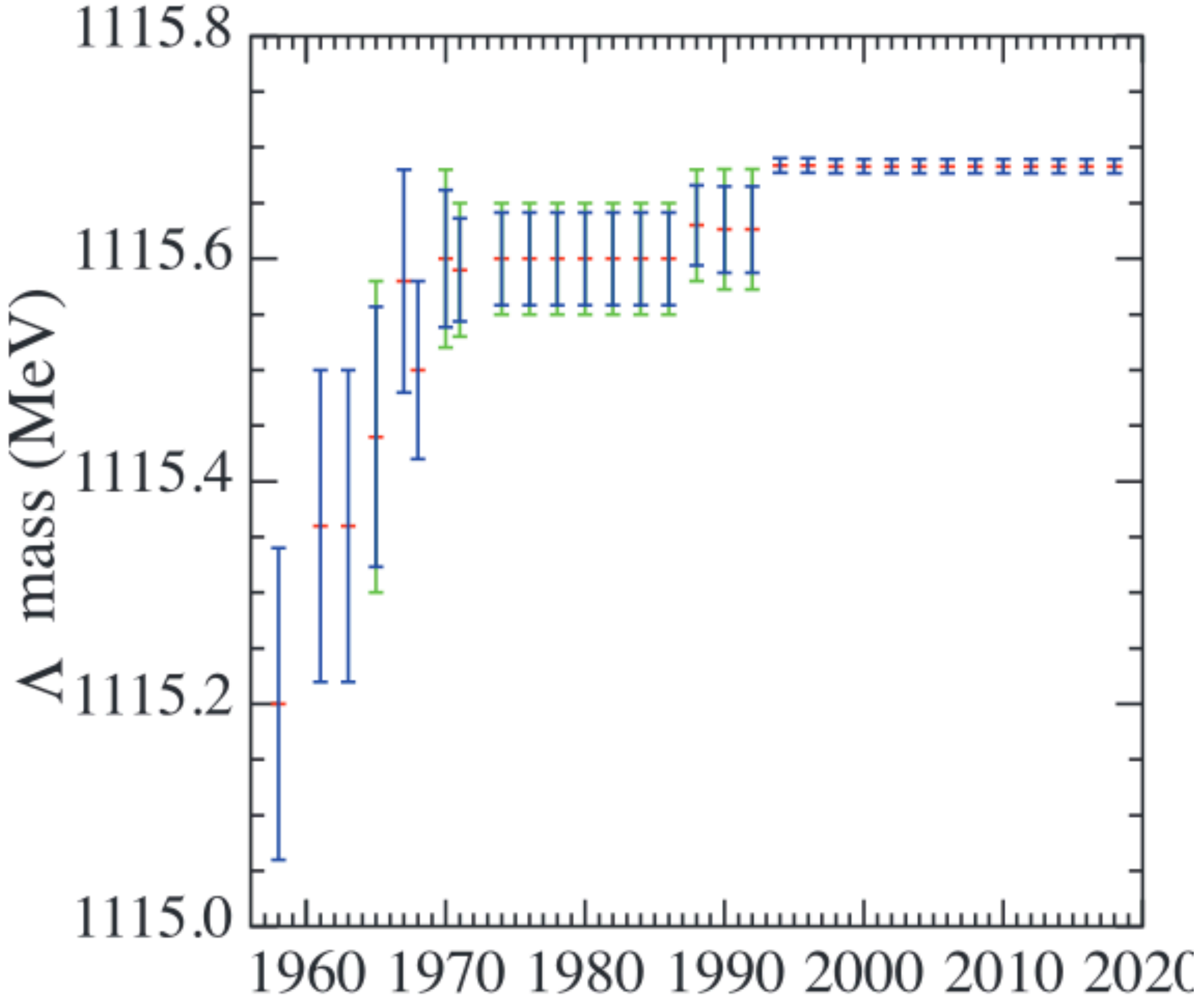
v. we wish to extract model structure, not assume it

i.e. over-confidence in one's model.

i.e. one acts as if the model generated the data.







problems with the traditional approach

i. fluctuations in the data set may be important

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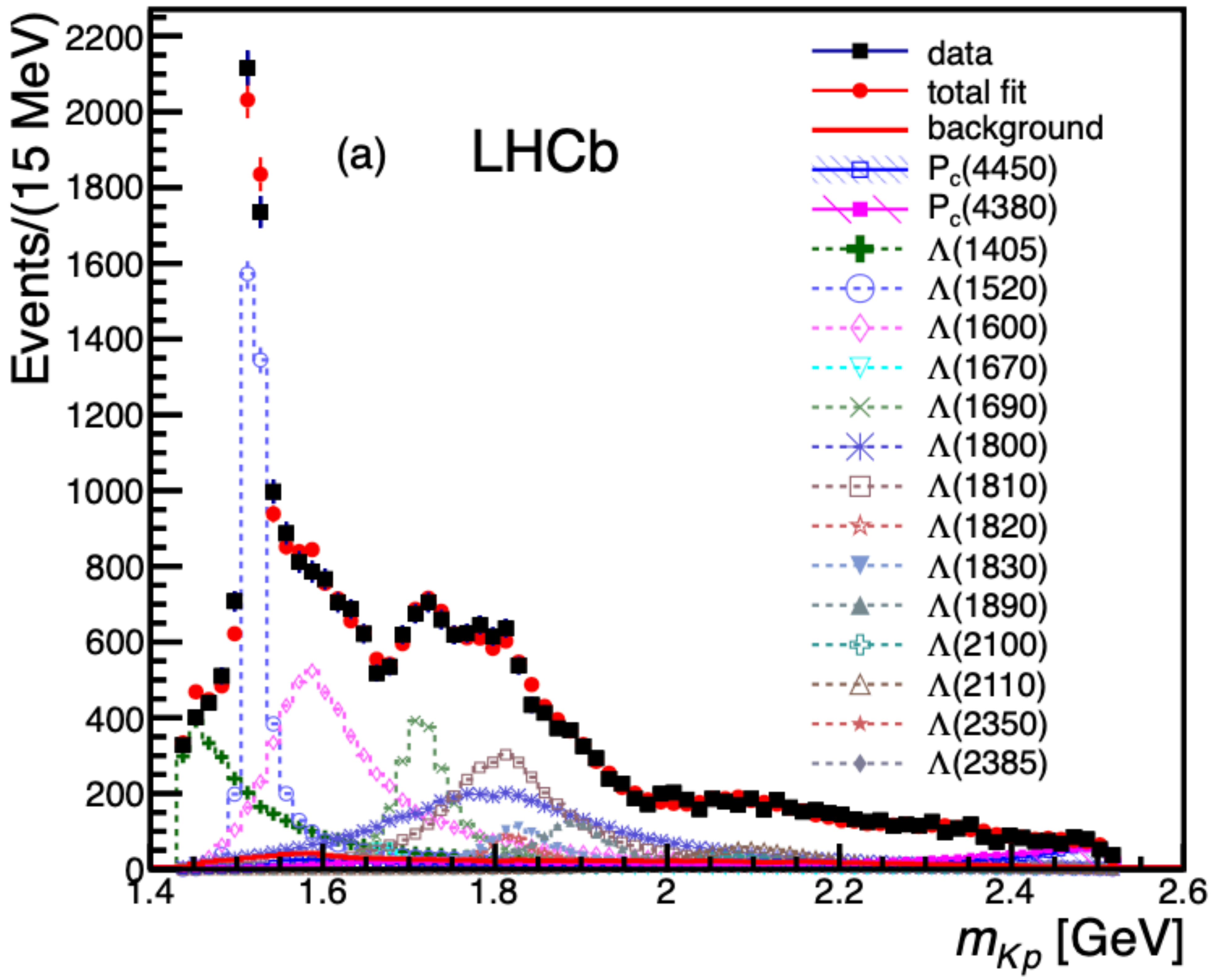
v. we wish to extract model structure, not assume it

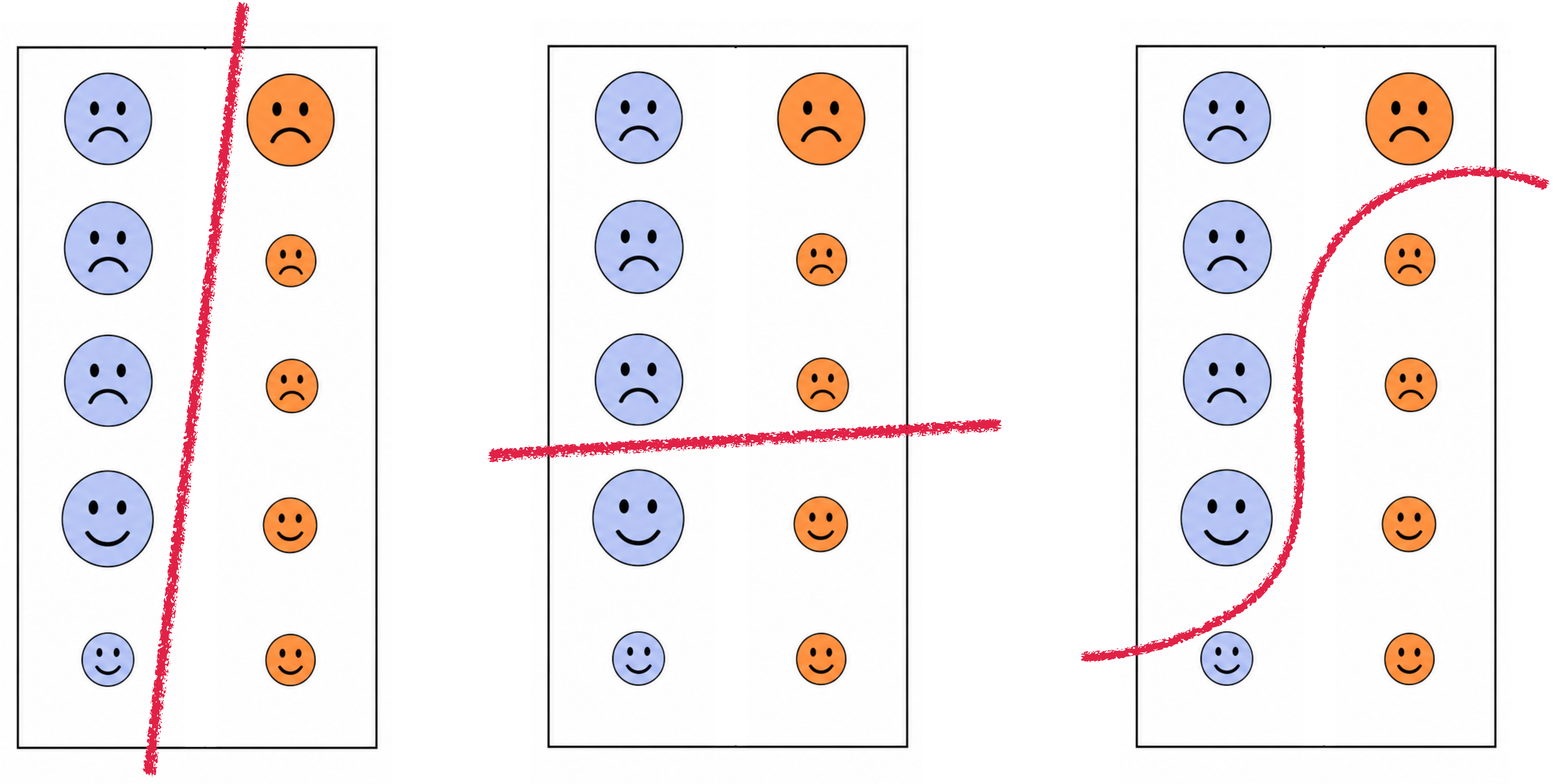
"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

→

→

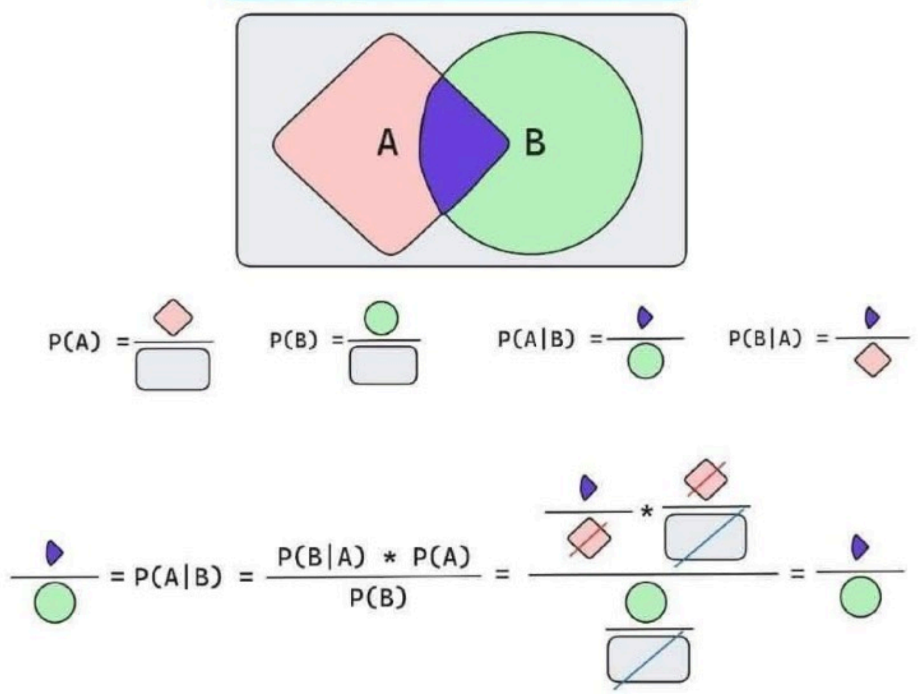


Your fit heuristic is domain-dependent!



h/t Randall Munroe

problems with the traditional approach



i. fluctuations in the data set may be important

ii. models M0 and M1 are wrong!

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a. LASSO

b. AIC

$$E_g(2k - 2\log f(\mathscr{D}\,|\,\hat{\theta})) \to -2E_g(\log(f(\mathscr{D}\,|\,\theta))$$

c. BIC

$$p(M\,|\,\mathscr{D}) \approx \exp\left[-\frac{1}{2}(k\ln n - 2\log L(\hat{\theta})) + O(1)\right] p(M)$$

d. Bayesian Model Averaging :

$$p(M\,|\,\mathscr{D}) = \int d\theta\, p(\mathscr{D}\,|\,\theta; M)\, p(\theta\,|\,M)\, p(M)$$

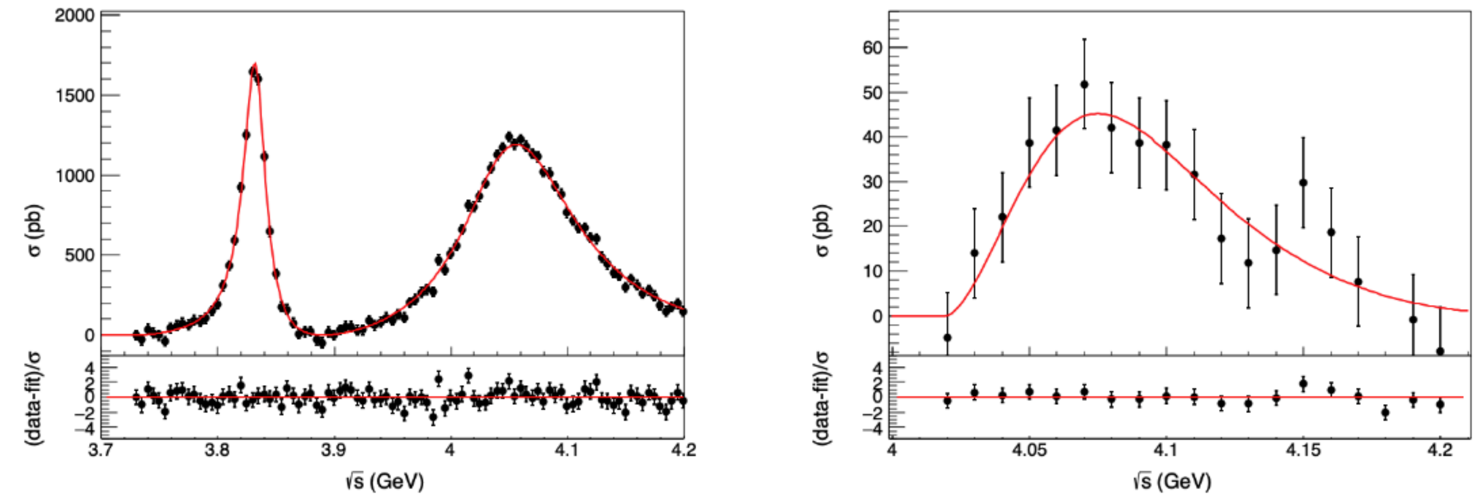
Priors are problematic! What is a uniform prior? Lack of reparametrization invariance. What is the totality of causative agents?

additional concerns

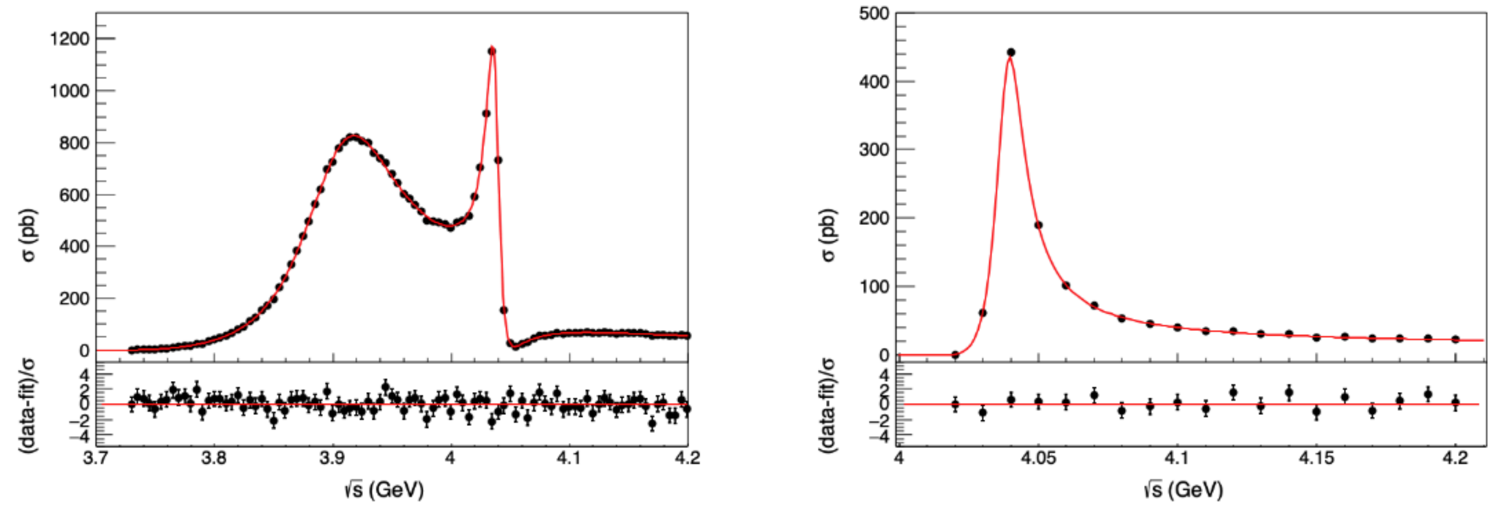
vi. model parameters are (approximately?) meaningless

vii. we are not interested in the model, but rather the analytic structur

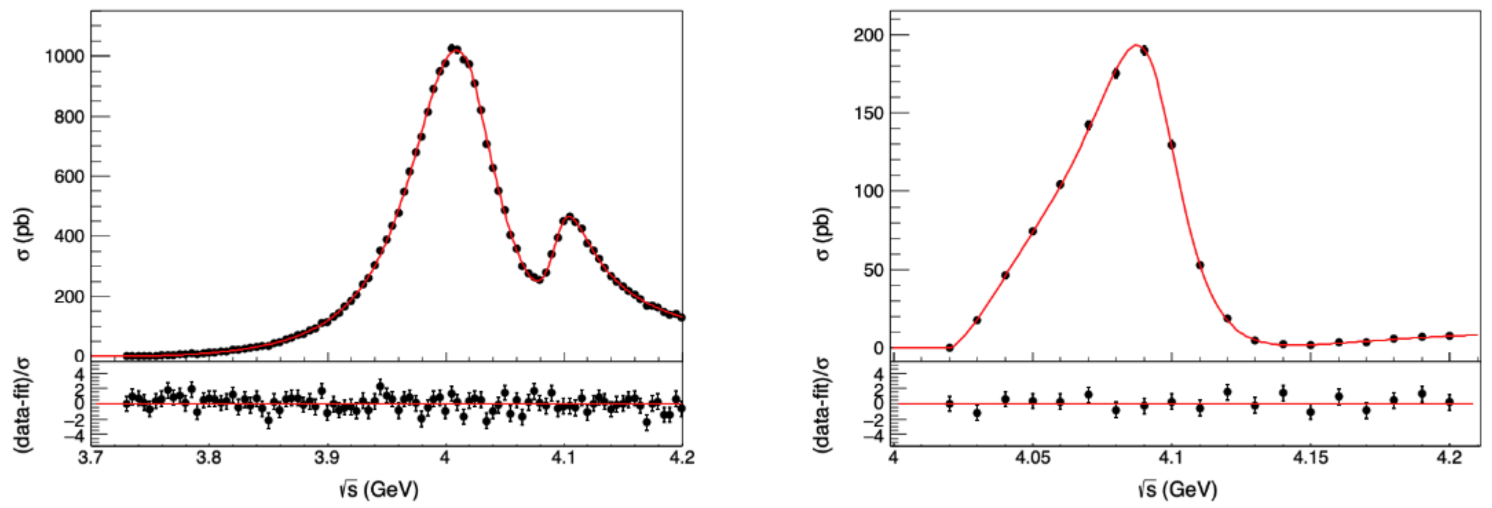
A



B



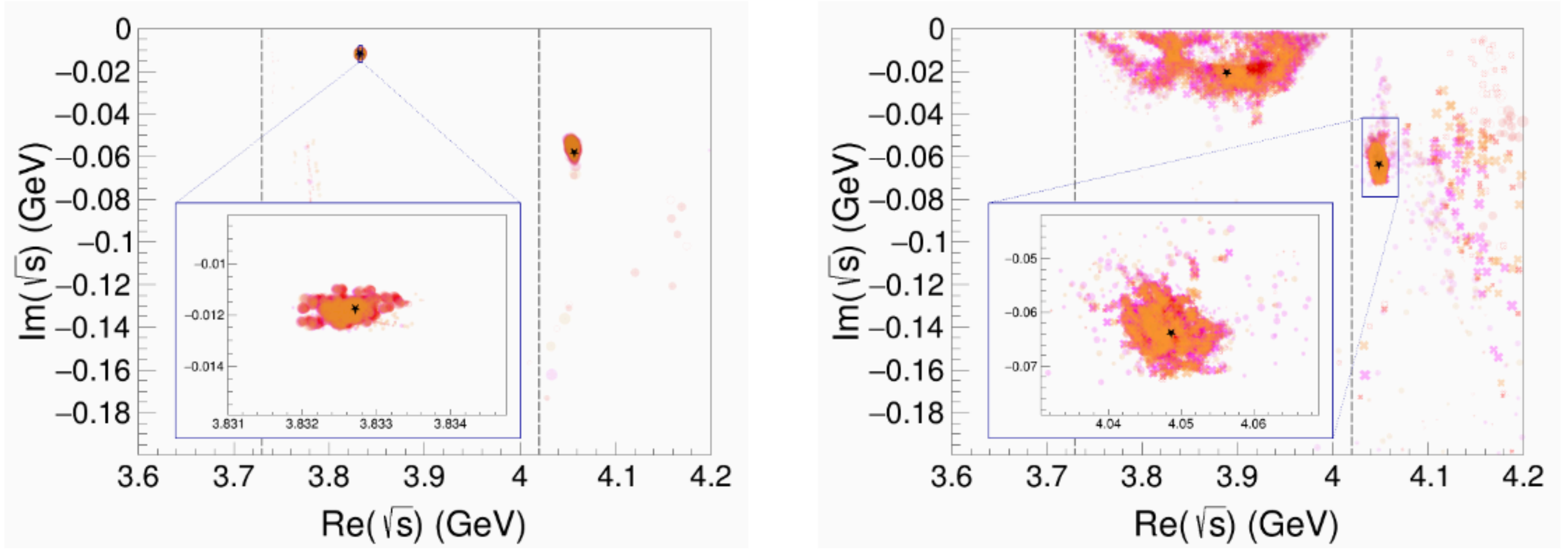
C



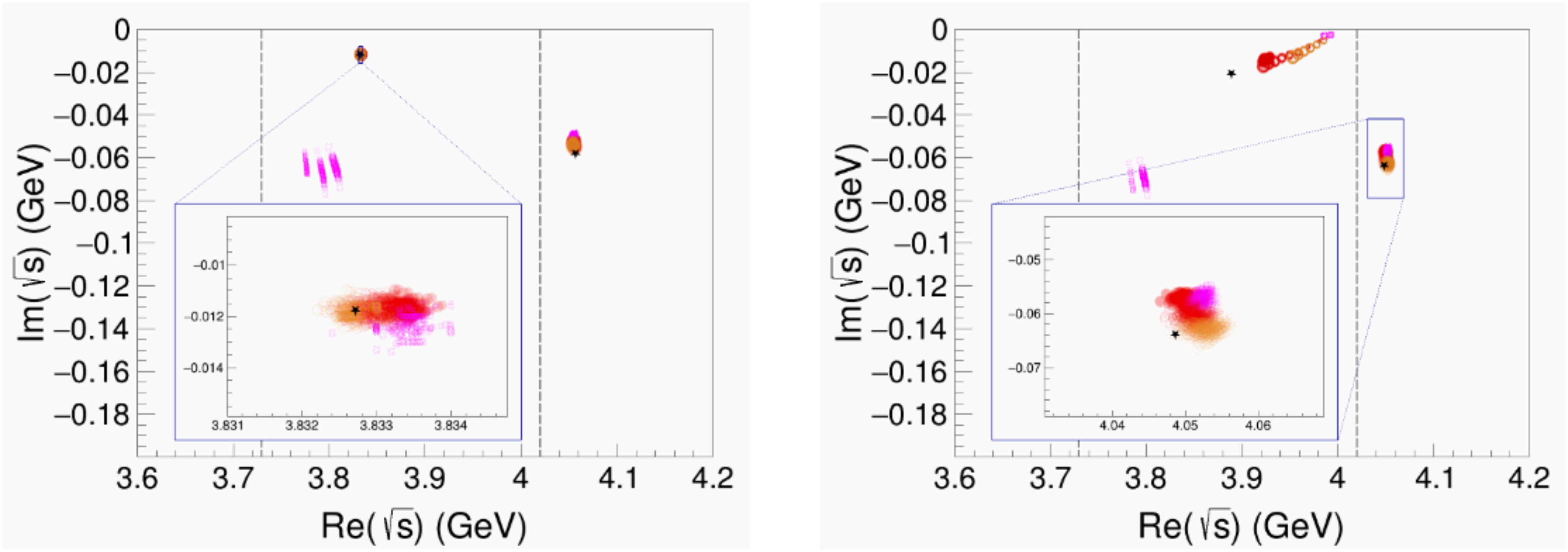
sheet II

sheet III

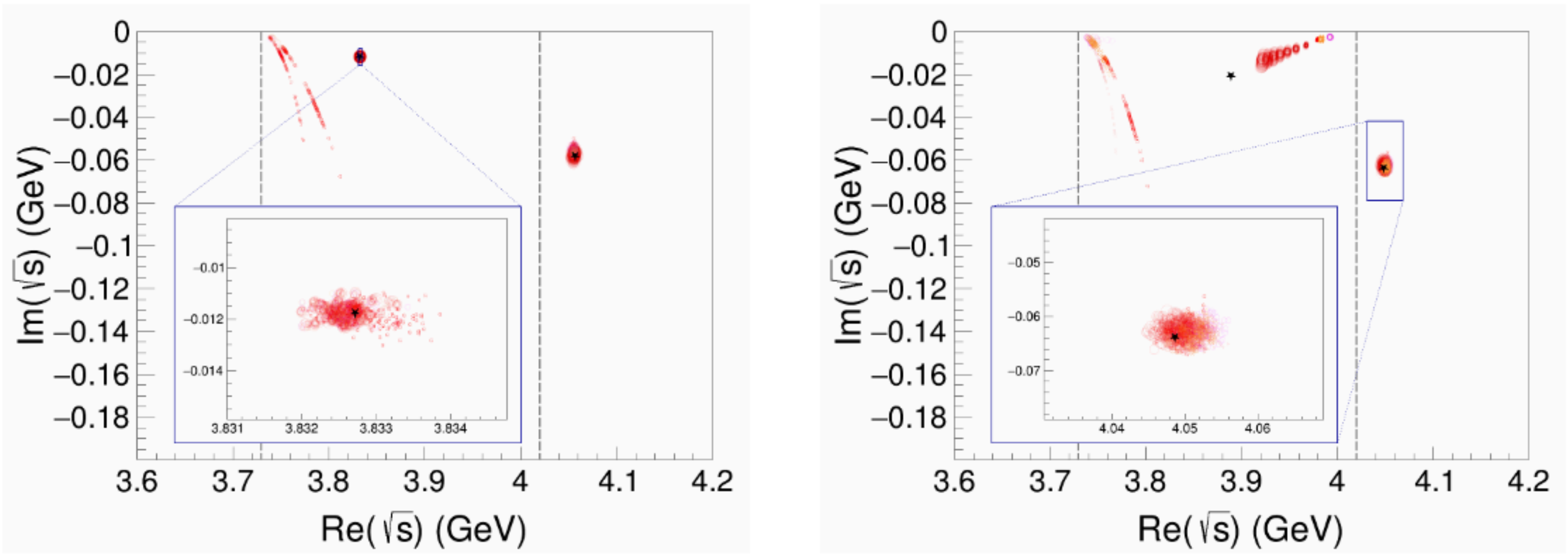
2 bare poles



1 bare pole



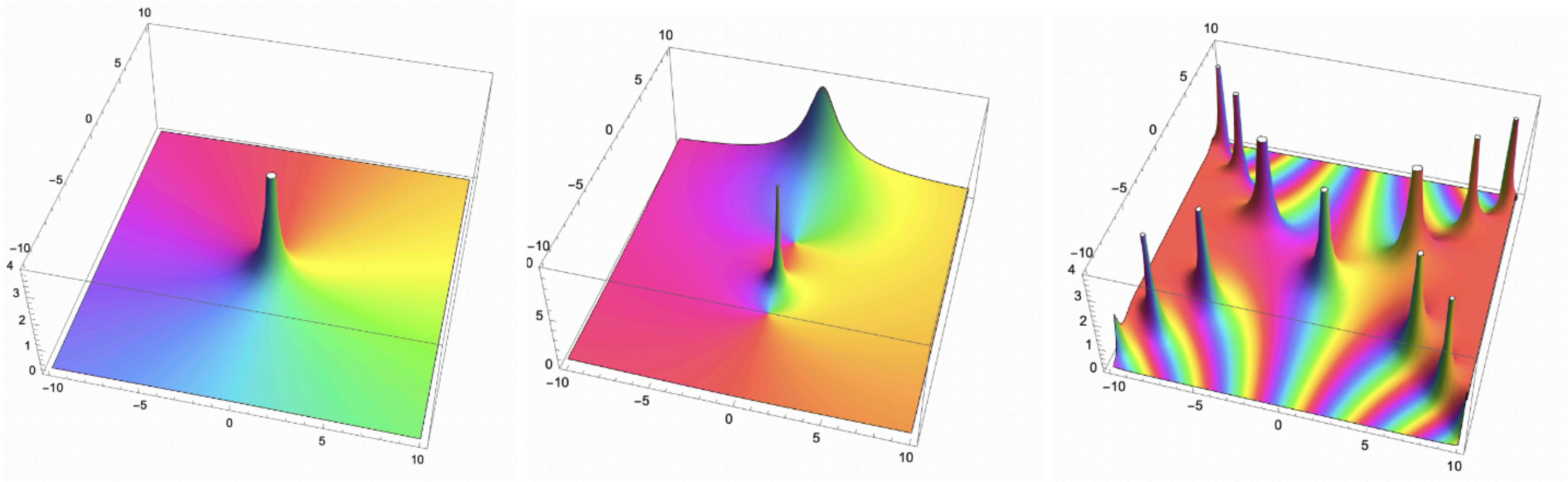
0 bare poles



additional concerns

vi. model parameters are (approximately?) meaningless

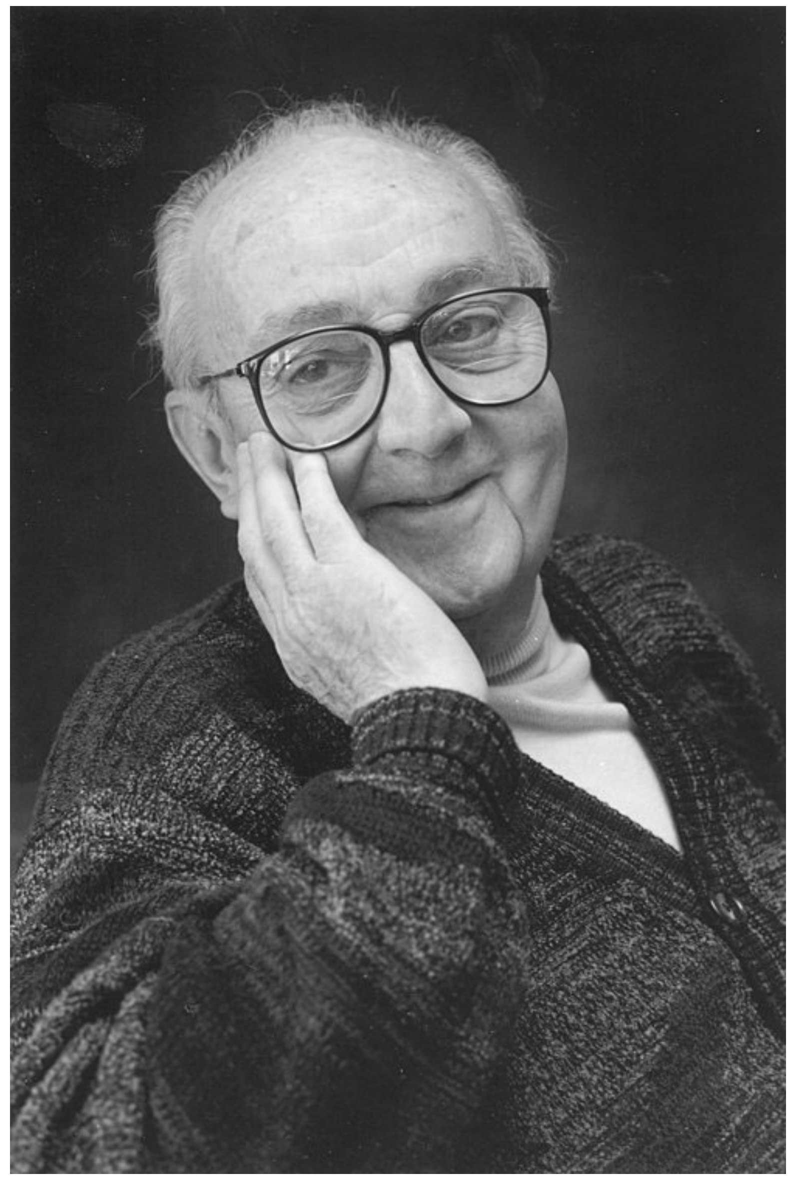
vii. we are not interested in the model, but rather the analytic structure of the model (or, at least, should be!)

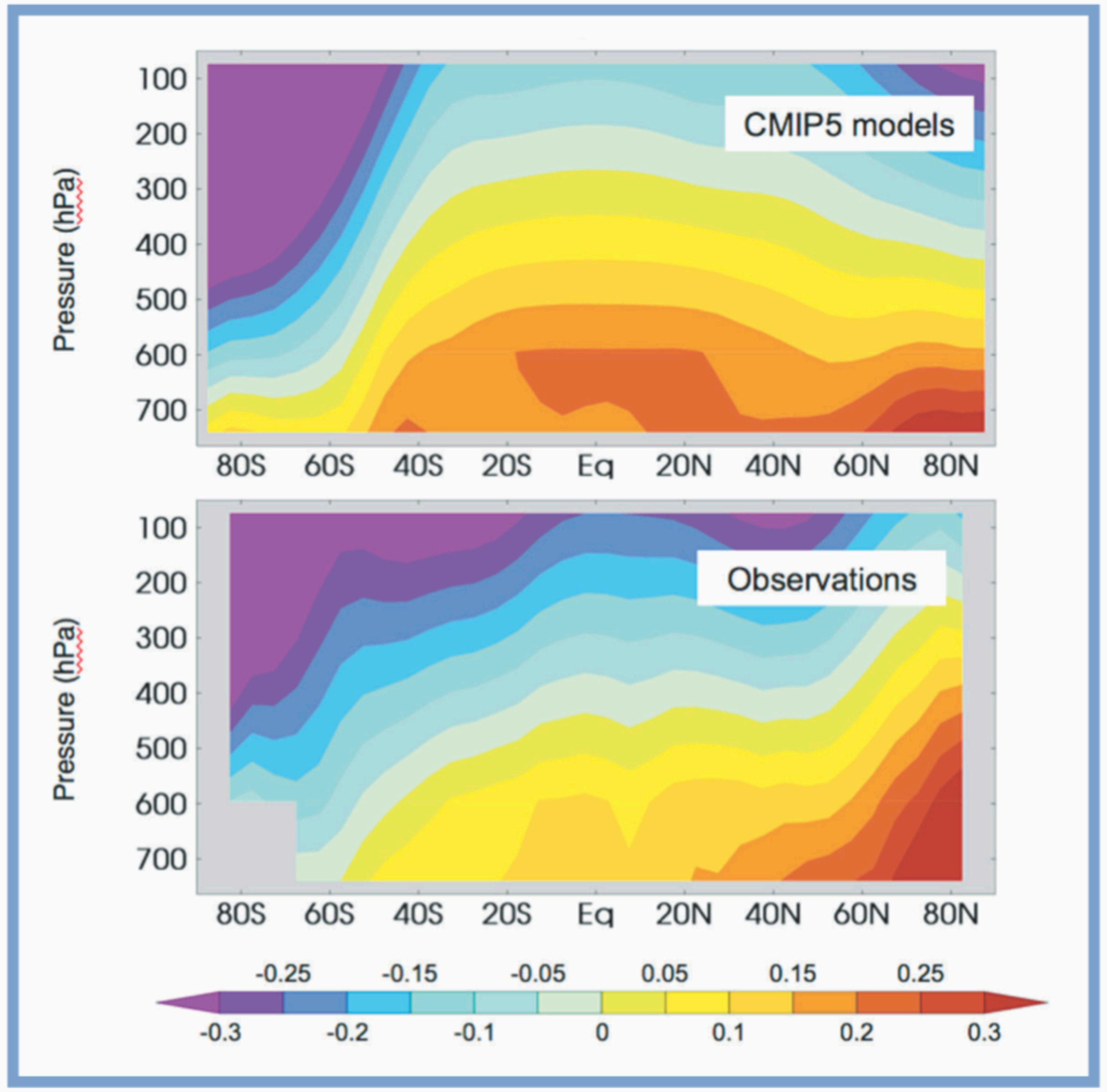


bigger problems

viii. what does it mean to model?

ix. what does it mean to minimize?





"Fingerprinting" with changes in the vertical structure of atmospheric temperature: The average of eight CMIP-5 models with anthropogenic forcing (upper panel) and satellite observations from Remote Sensing Systems (lower panel) both show coherent warming of the troposphere and cooling of the stratosphere. (Source: Santer et al., Proceeding of the National Academy of Sciences, submitted)

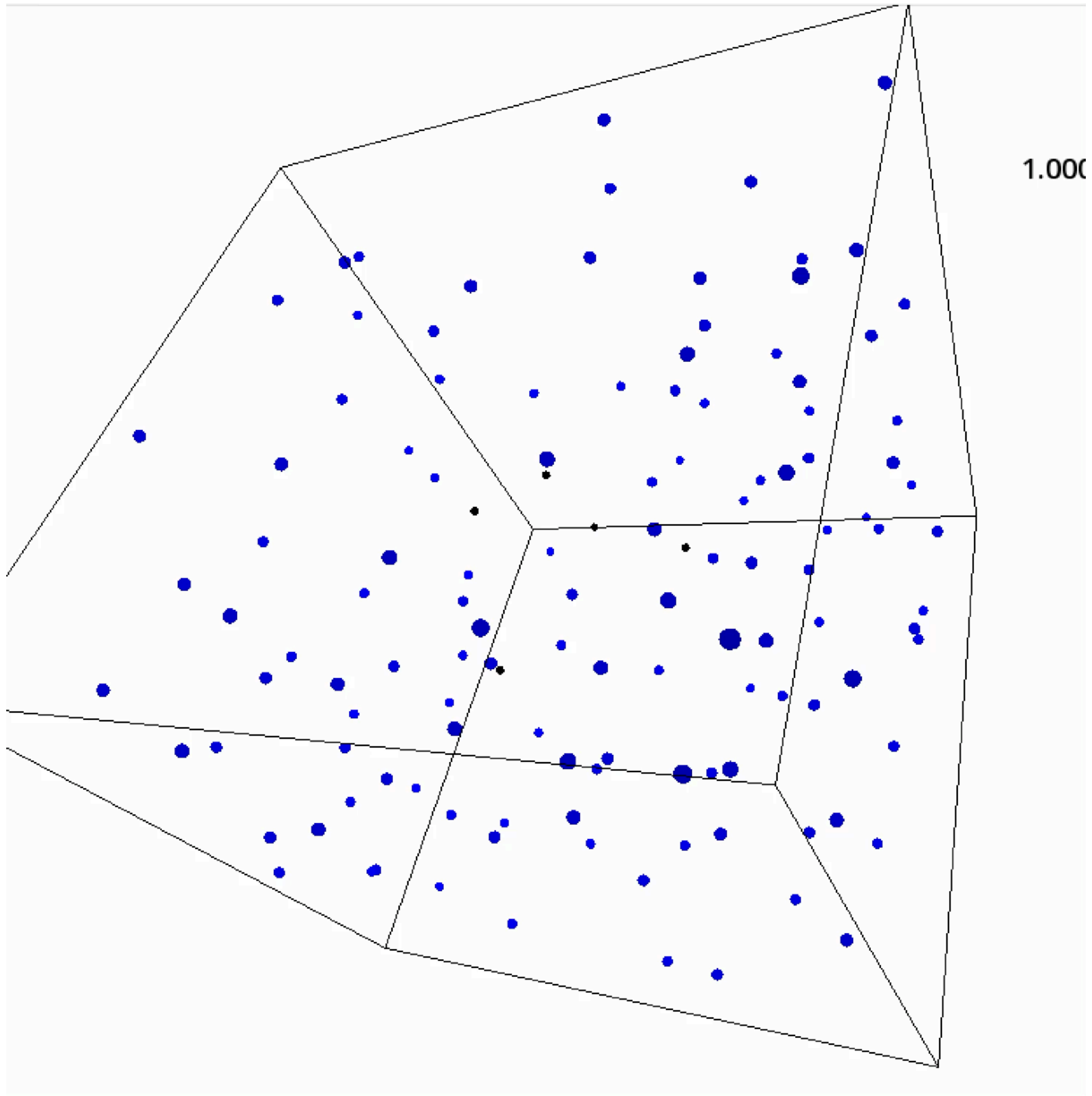
"~~Statisticians~~ Physicists, like artists, have the bad habit of falling in love with their models."

bigger problems

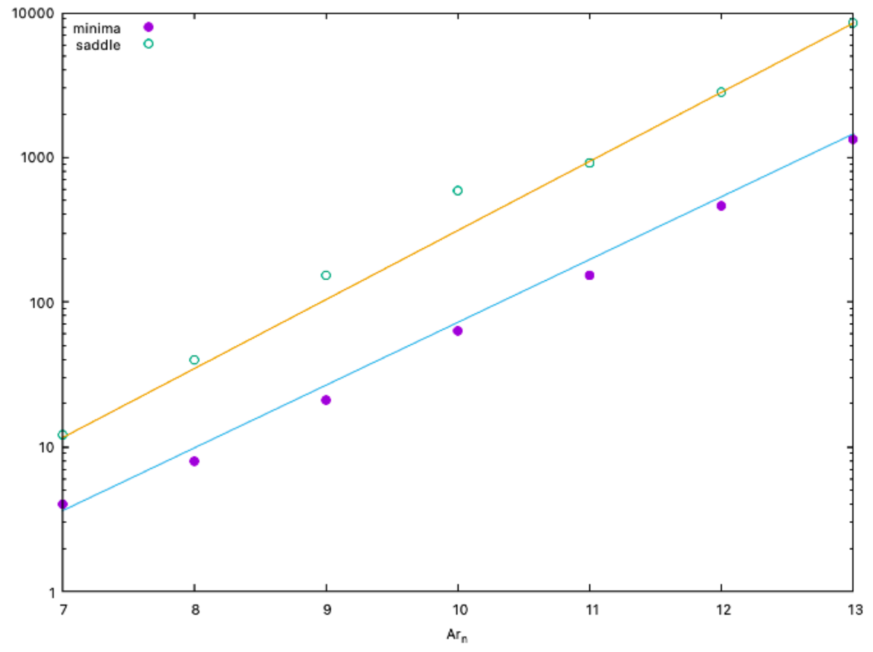
viii. what does it mean to model?

ix. what does it mean to minimize?

A 38 element Lennard-Jones system has ~ 10^{14} local minima (!!)



C.J. Tsai and K.D. Jordan, J. Phys Chem, **97** 227 (1993).



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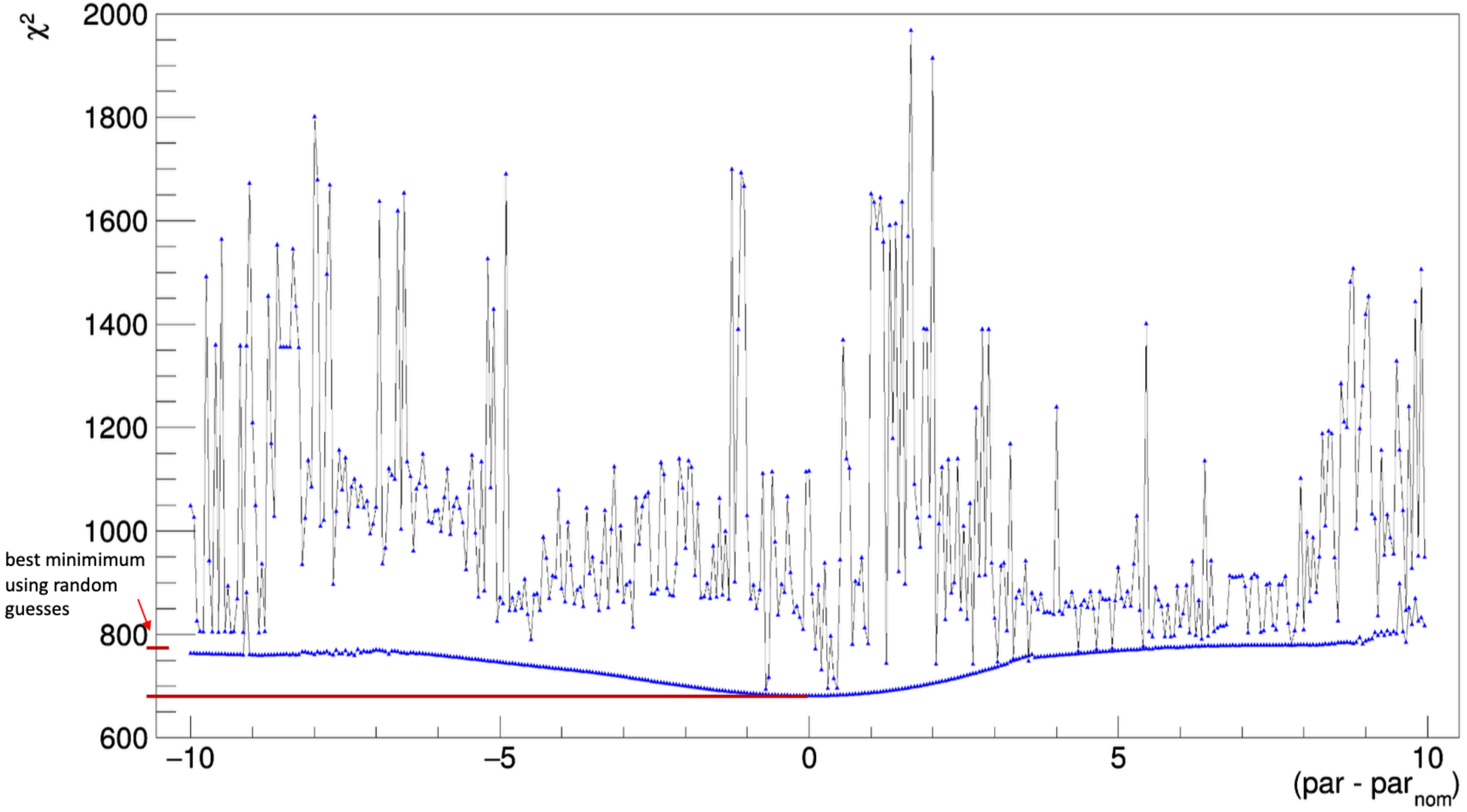


a way forward
(i) seat of the pants method

a way forward:

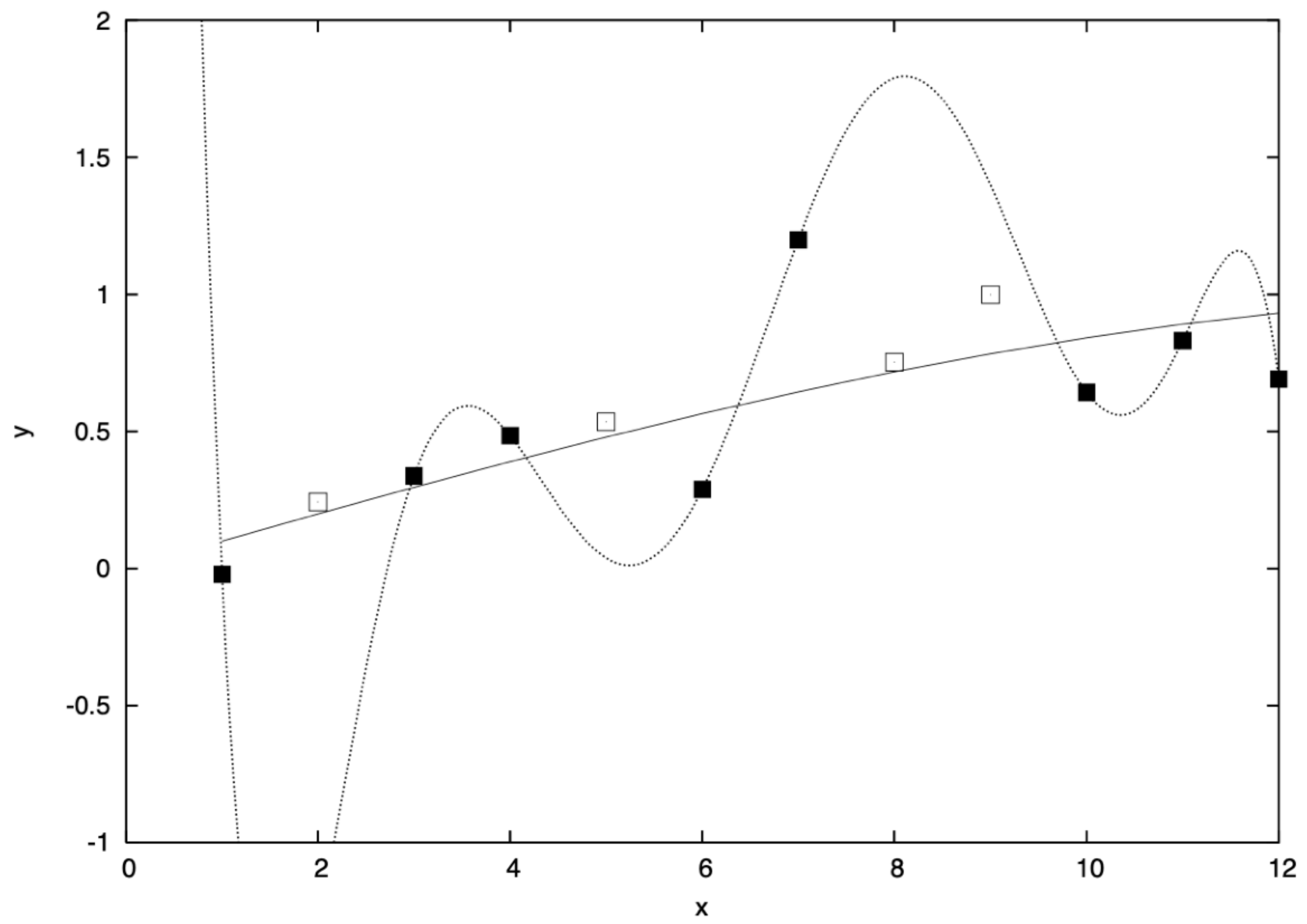
- One way in which one can claim the discovery of a state is if that knowledge permits better, or *predictive*, statements to be made about future experiments.

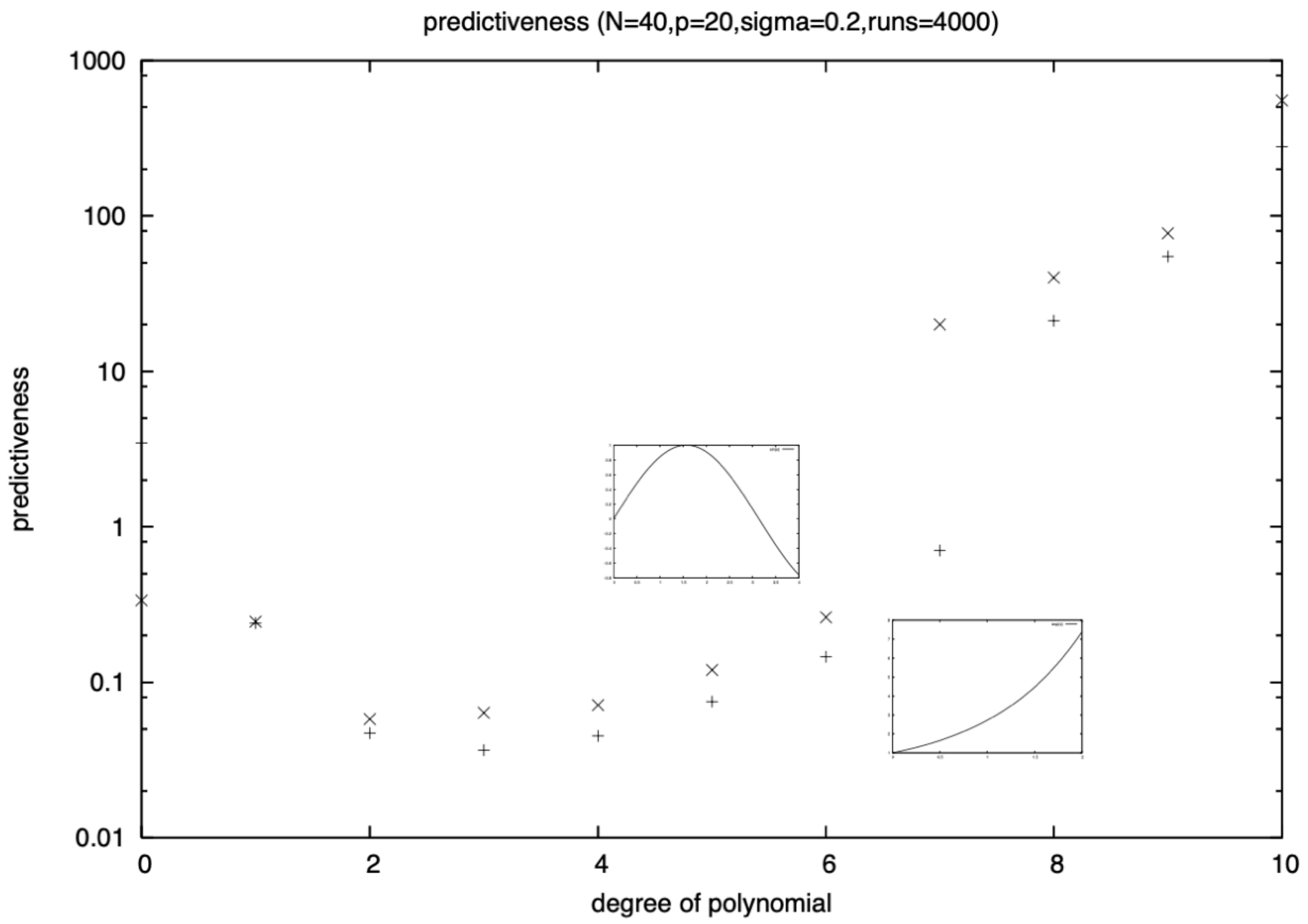
"How well does my model fit the data?" ➔ "How well can I predict the outcomes of future experiments?"

- We can use cross-validation to avoid overfitting.

- Combine these ideas by splitting the data set into training (ante) and validation (post) sets.

A simple example





Predictiveness

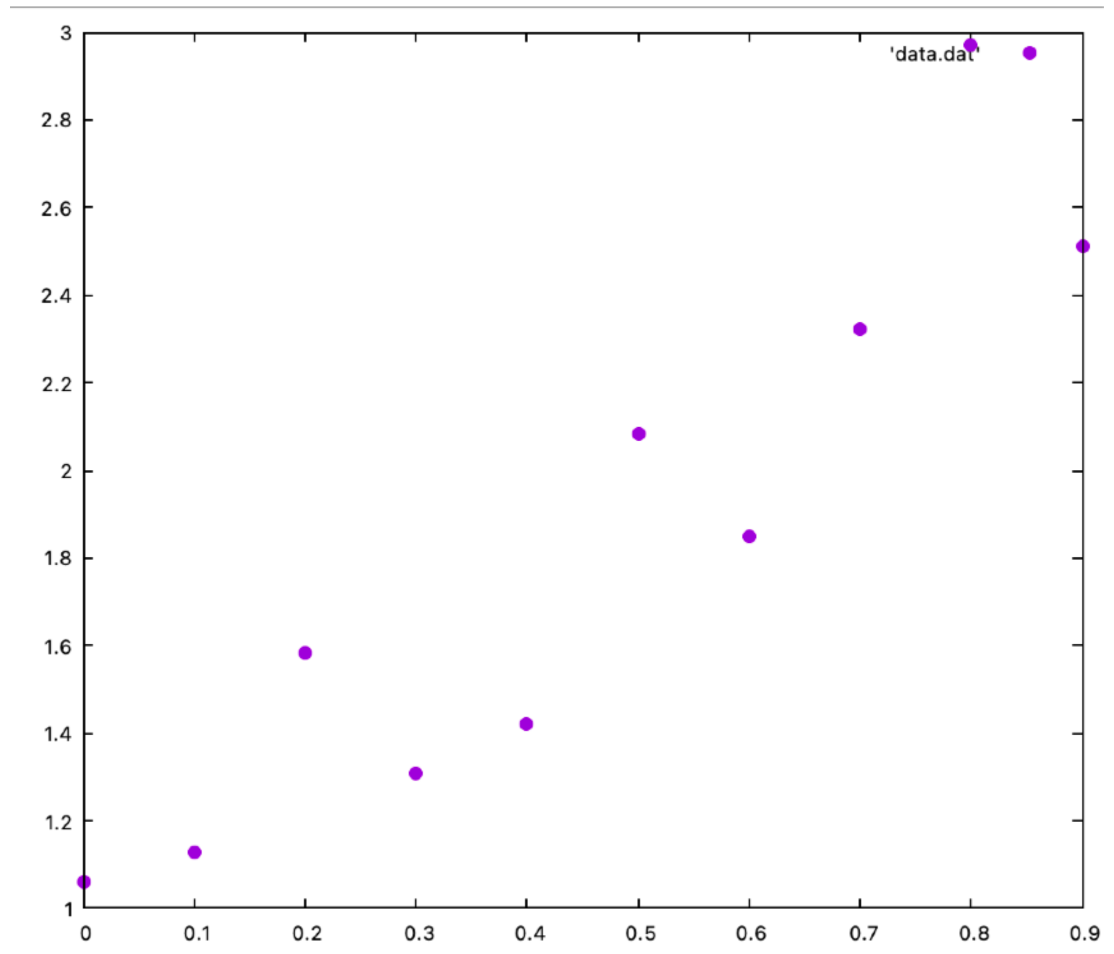
- abandon the idea that we "know" the model -- work in "super-model space" (which, of course, is a model; but now we seek a degree of agnosticism).
- stochastically explore model space. [One could model average by averaging over the trajectory. This is not our primary goal here.]
- explore the continuous portion of model space with Markov chain Monte Carlo. Metropolis-Hastings update according to

$$p(\theta \to \theta') = \min\left(1, \frac{p(\theta'|\mathscr{D})}{p(\theta|\mathscr{D})}\frac{f_{\theta,\theta'}}{f_{\theta',\theta}}\right) \qquad p(\theta|\mathscr{D}) = \frac{p(\mathscr{D}|\theta)\,p(\theta)}{p(\mathscr{D})}$$

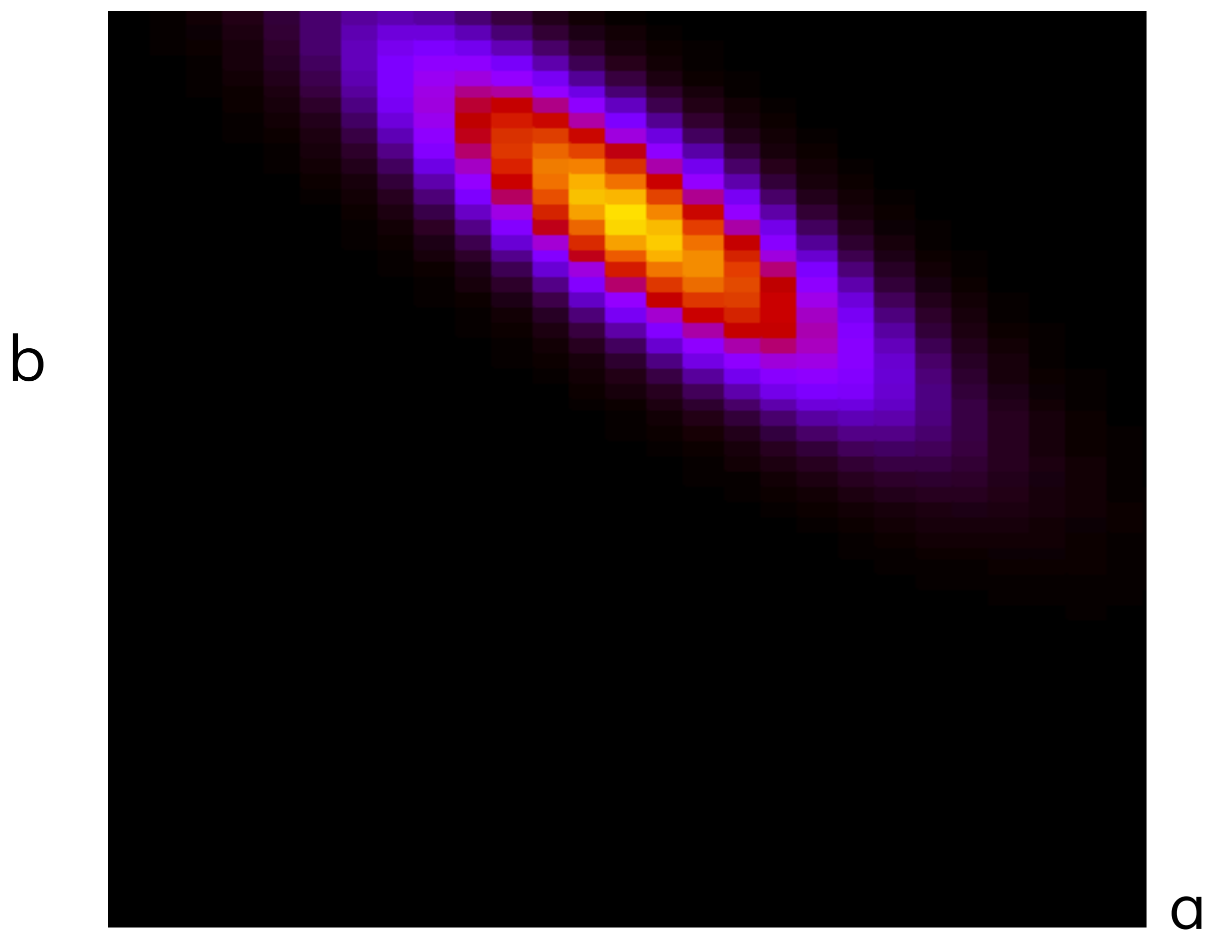
1000 MCMC steps starting at (0.5,0.5); uniform step of size $\in [-0.1, 0.1]$

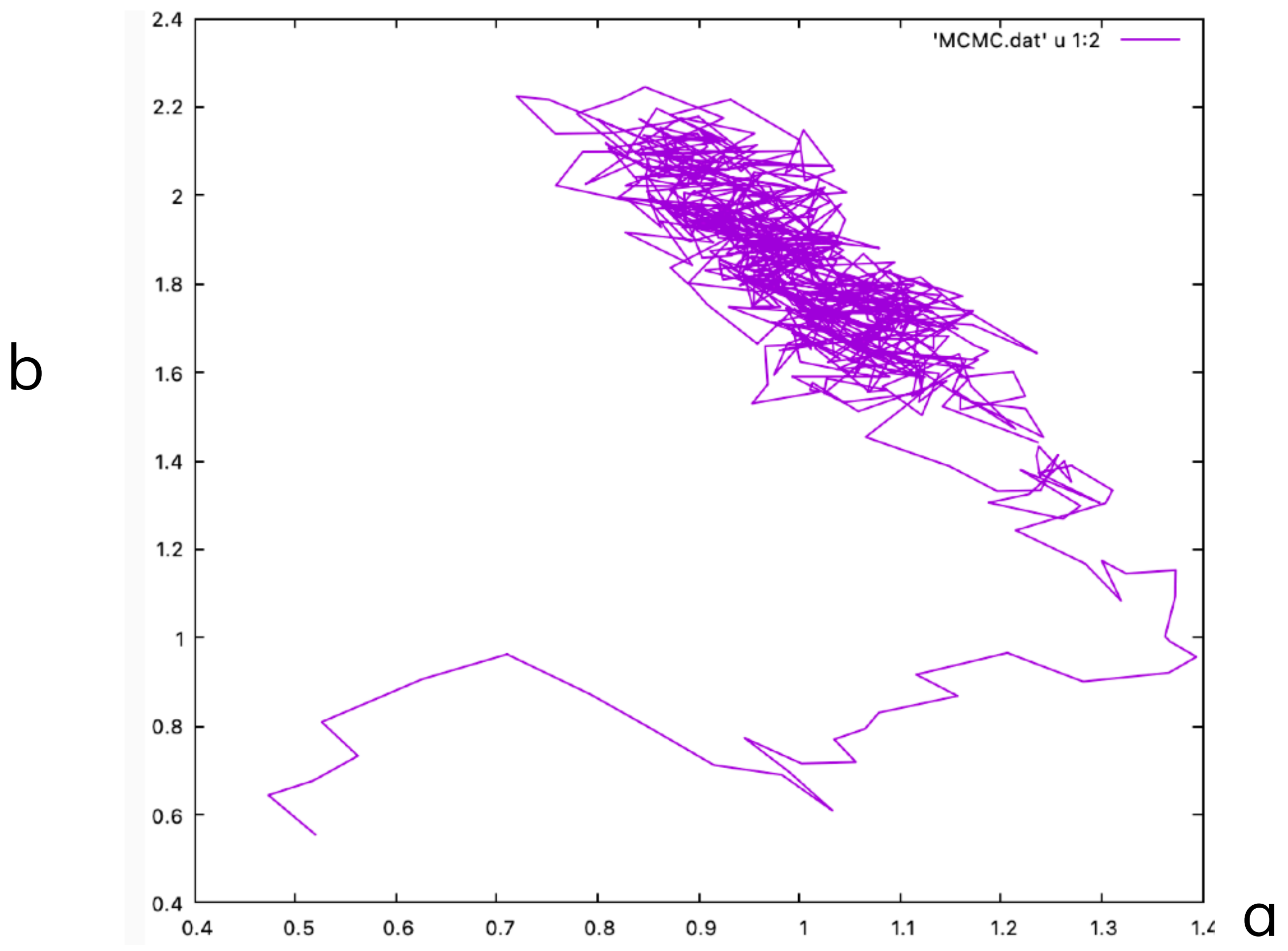
Every point in this space is a model, some are just better than others.

$\mathscr{D}(1 + 2x + \hat{\eta}(0.2))$

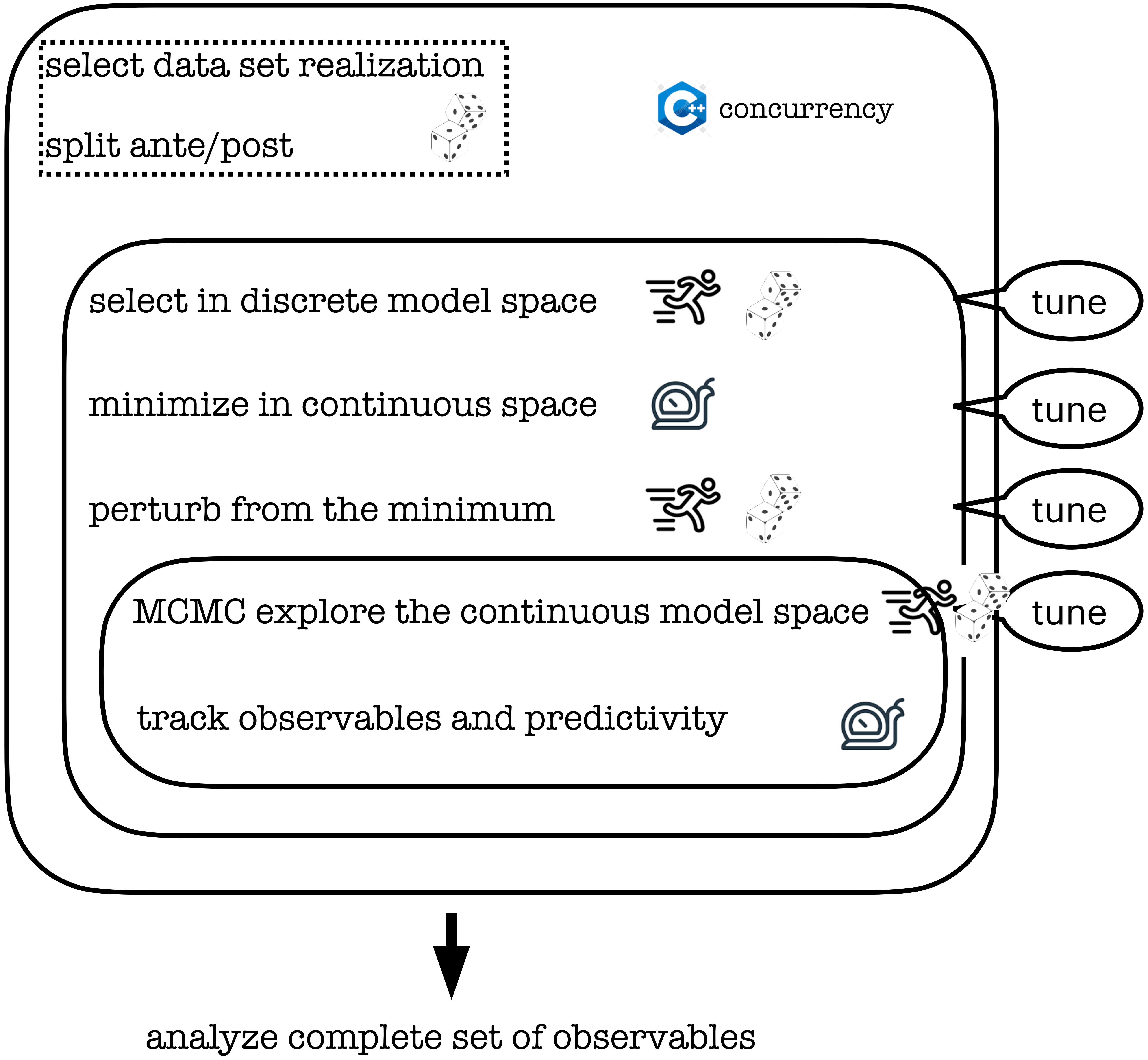


scan of the posterior





Predictivity - Algorithm



focus on predictiveness

de-emphasize fitting and fit quality

explore a large model space to enhance the reliability of the conclusions

use data realizations to enhance the reliability of the conclusions

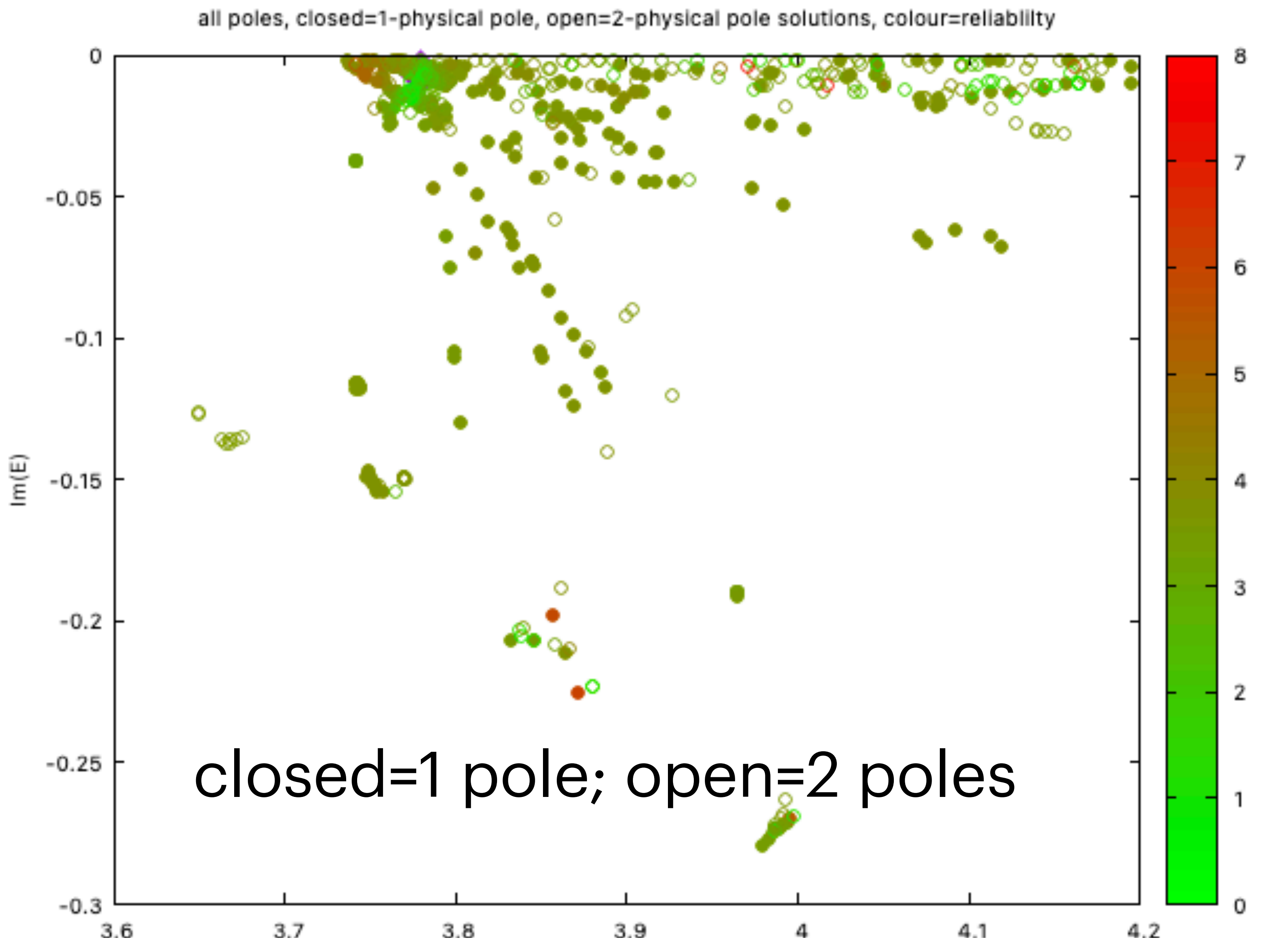
be as agnostic wrt priors and models as possible

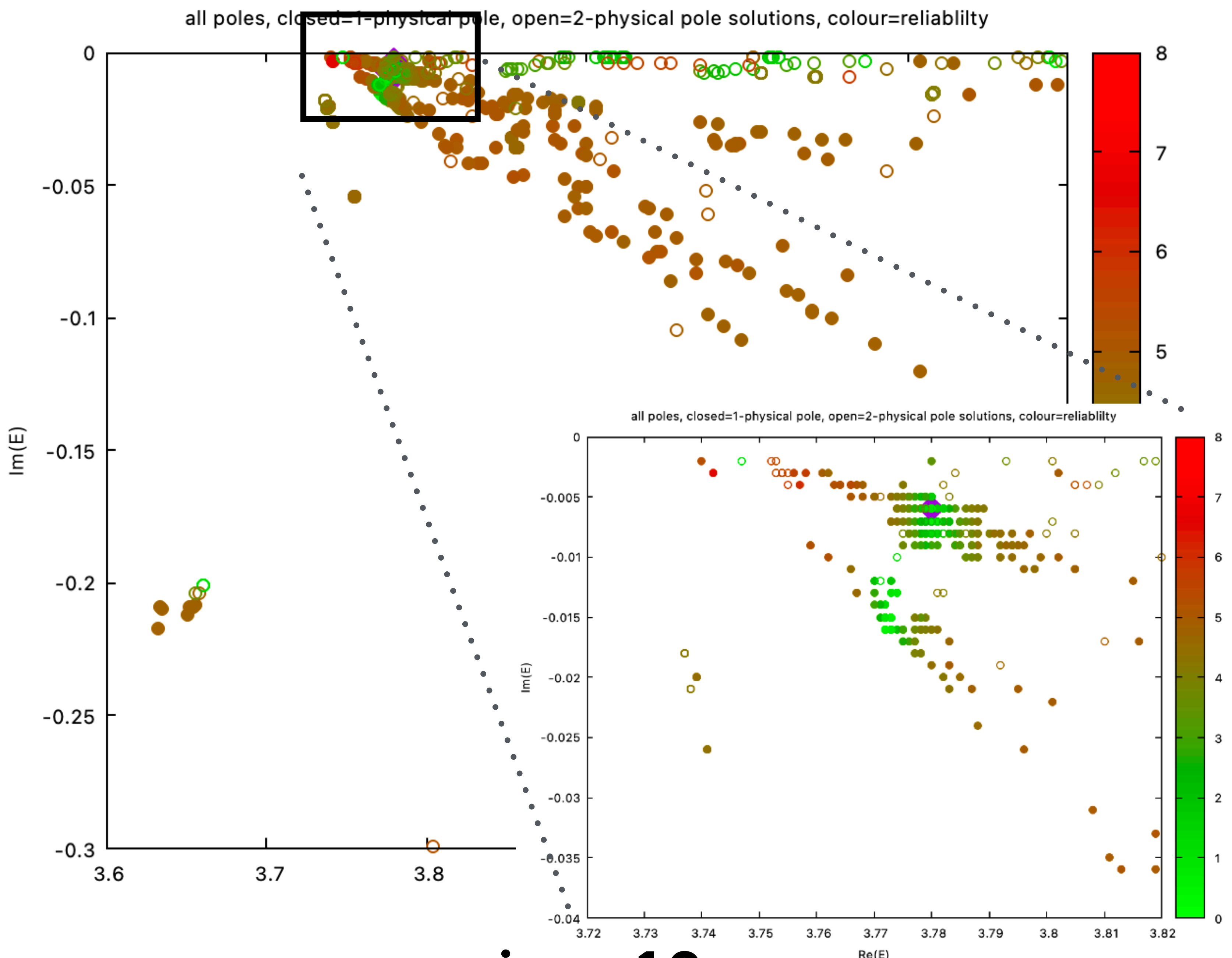
model parameters are not physical

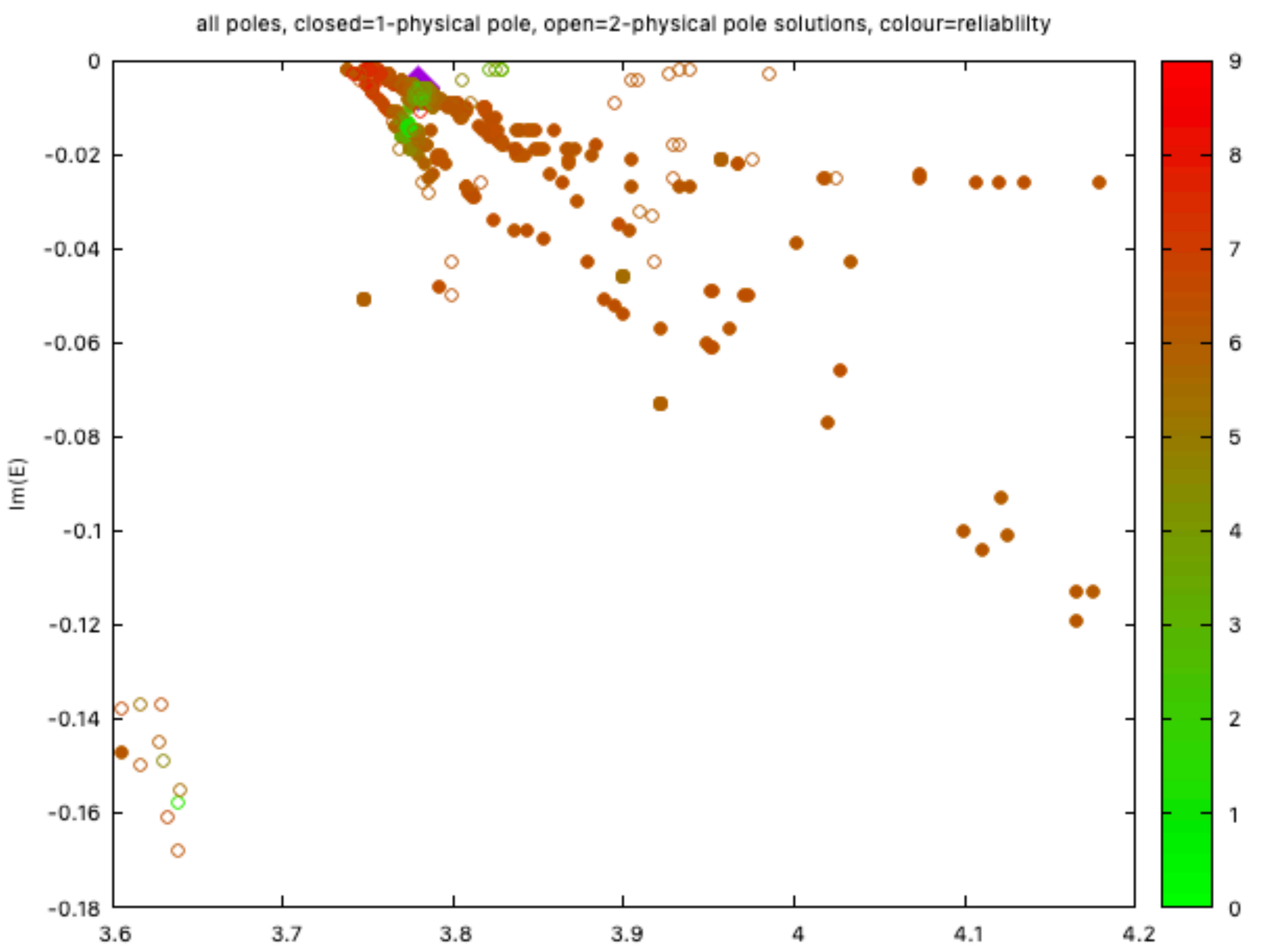
Pole Positions

Model space excludes the generating model.

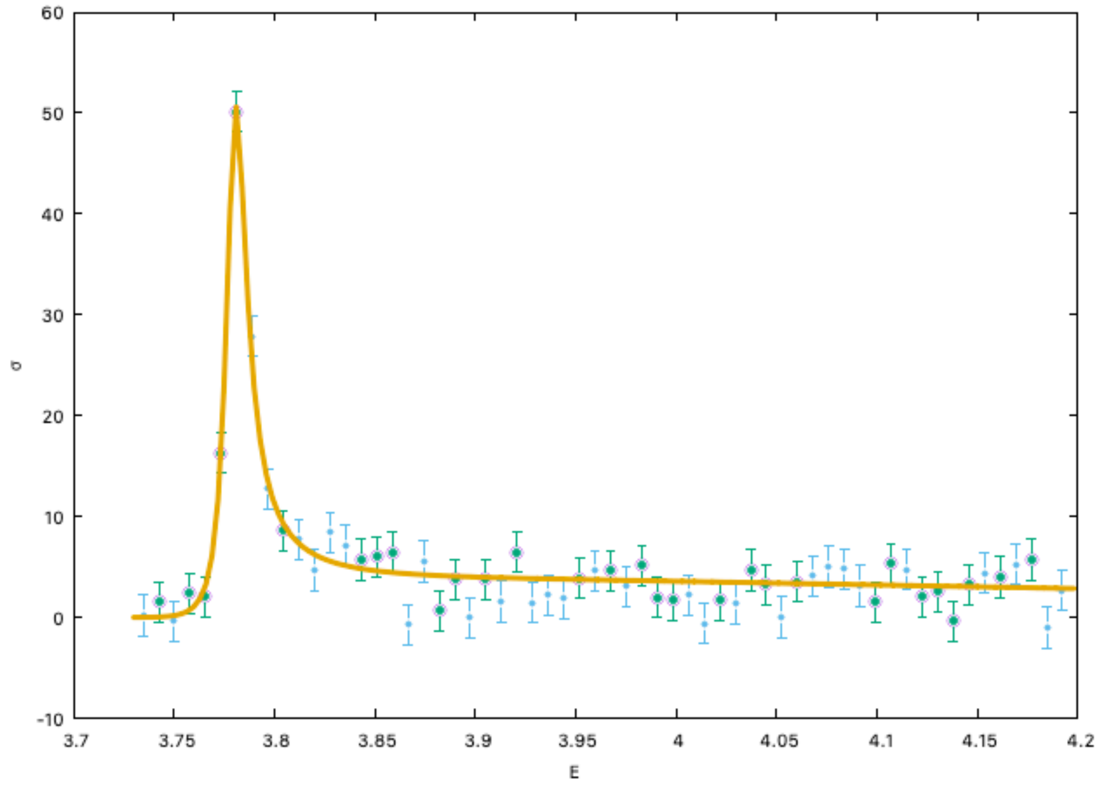
colour = "reliability":= post(ante)*post(post) = exp(-1/2 LP(a)-1.2LP(p)). This is log(1/2LP(a)+1/2LP(p))



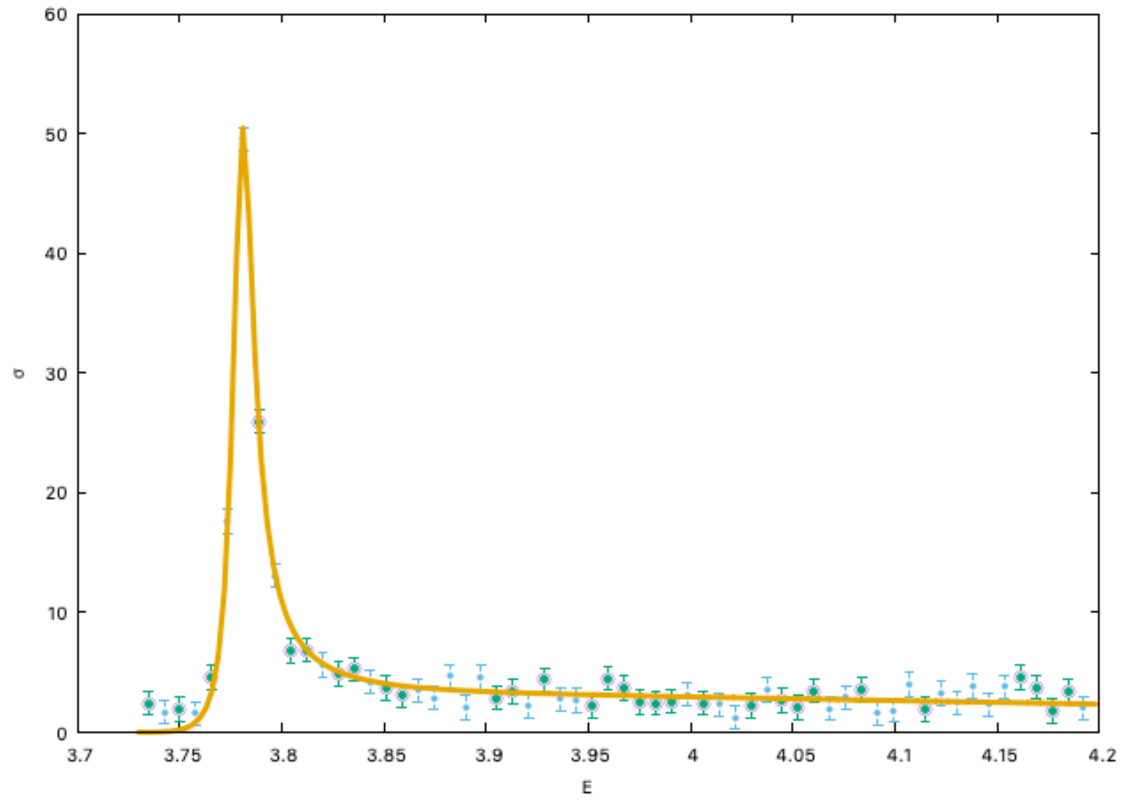




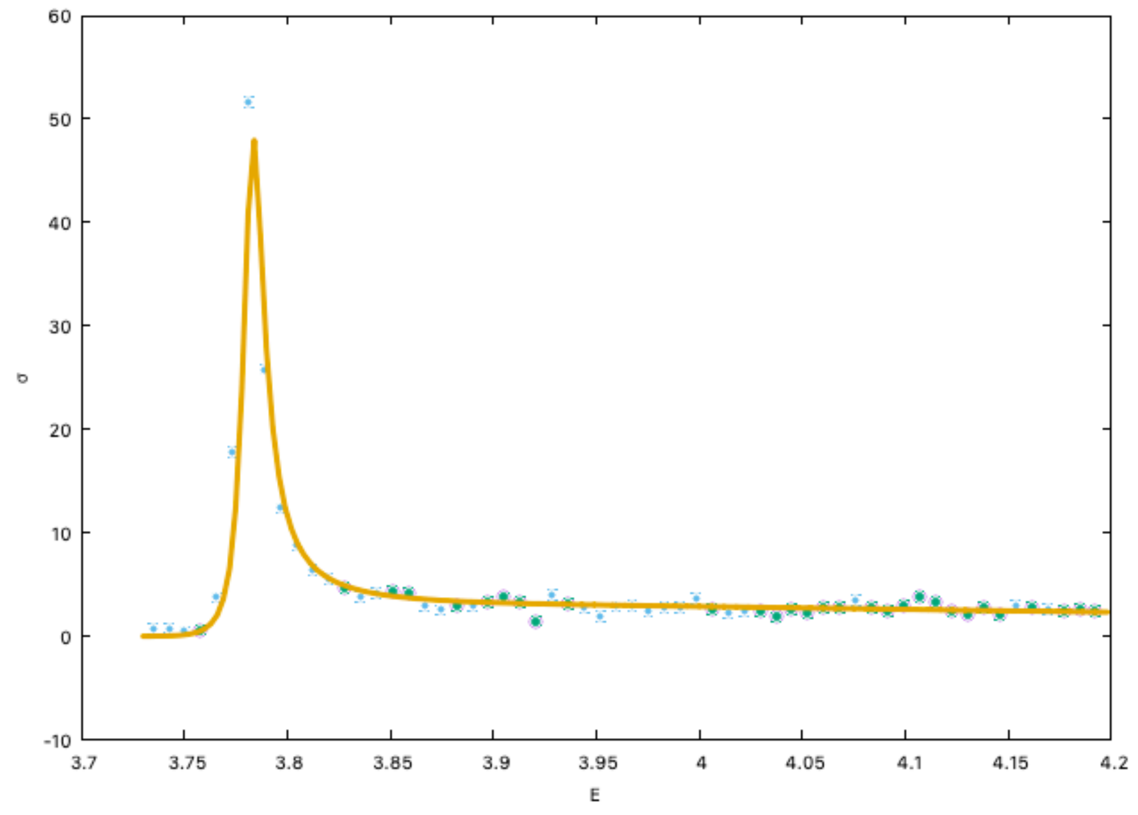
sigma=2.0



sigma=1.0



sigma=0.5



a way forward
(ii) Bayesian inference

Bayesian Inference

$$p(t\,|\,D) = \sum_M \int \overbrace{d\theta_M \, p(t\,|\,\theta_M, M, D)\, p(\theta_M\,|\,M, D)}^{\text{posterior predictive}} \cdot \overbrace{\int d\theta' \, \overbrace{p(D\,|\,\theta'_M, M)}^{\text{likelihood}}\, p(\theta'_M\,|\,M)\, p(M)/\overbrace{p(D)}^{\text{evidence}}}^{\text{model evidence}}$$

$$\to p(D\,|\,\hat{\theta}_M, M)\, p(\hat{\theta}_M\,|\,M)\, \underbrace{(2\pi)^{d/2}\sqrt{\det \Sigma}}_{\text{Occam factor}}\, p(M)/p(D)$$

Bayes theorem

marginalize, implements Occam's razor

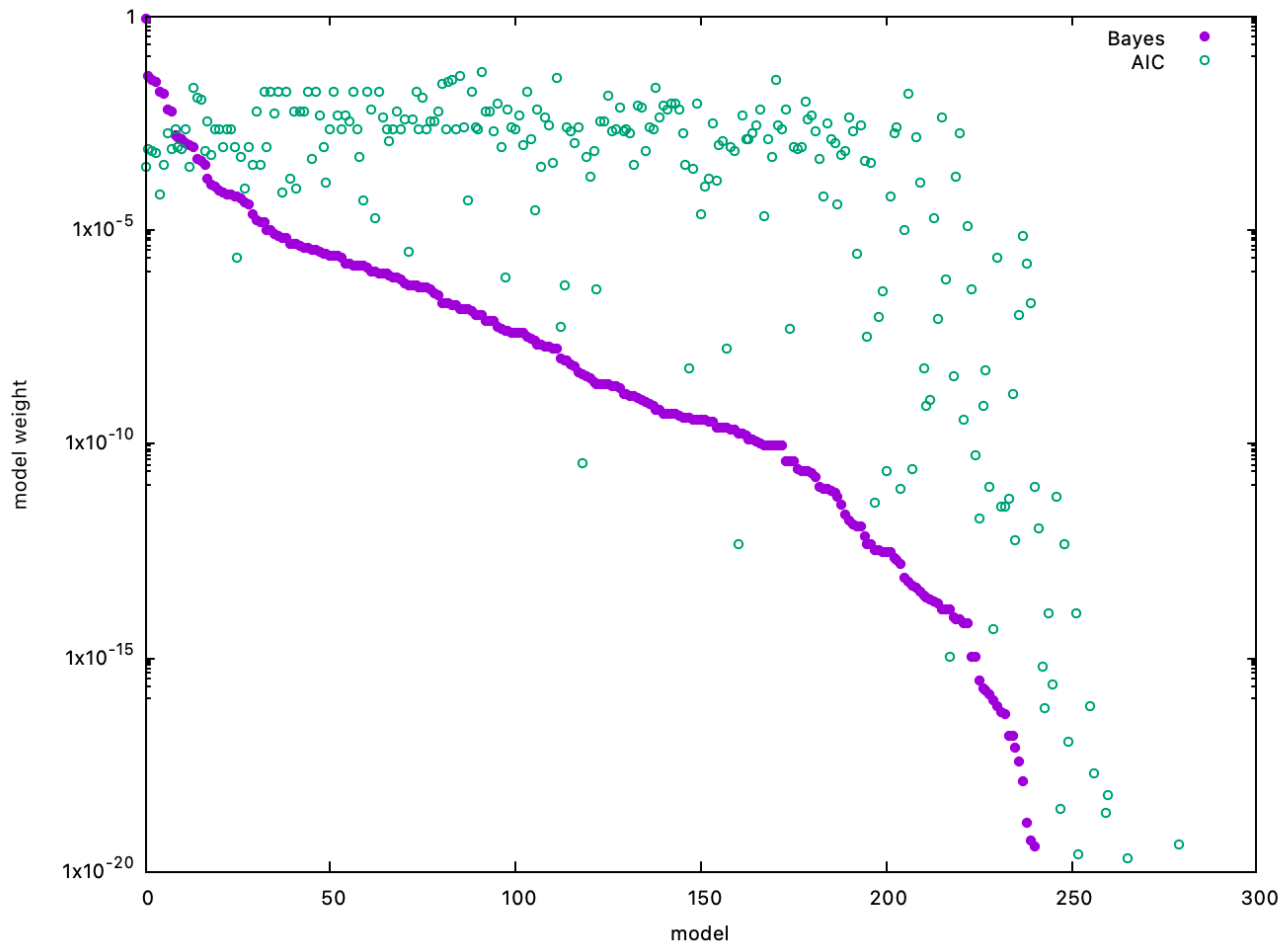
Laplace approximation (steepest descent)

No need to bootstrap data

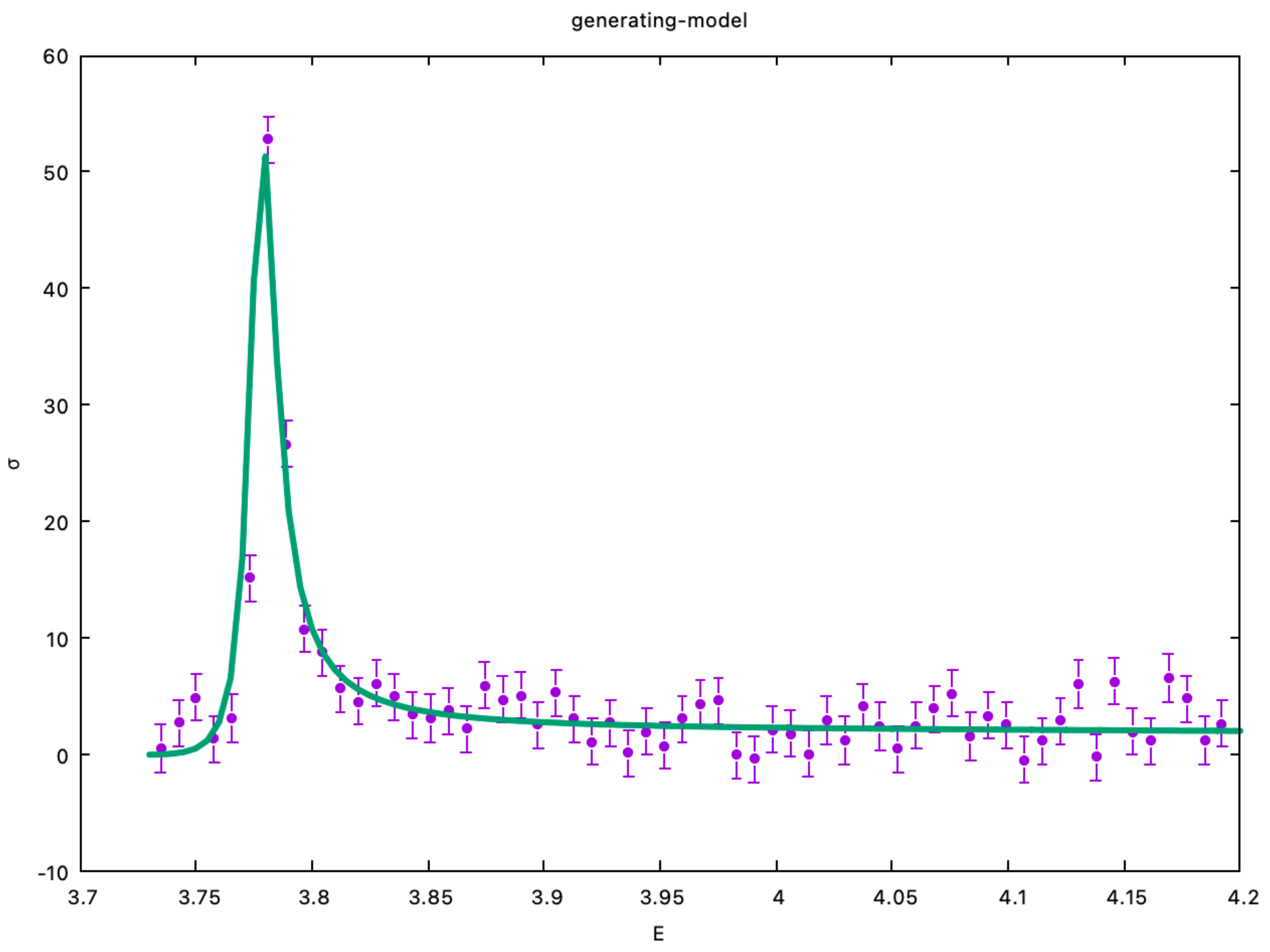
No need to explicitly penalize model complexity

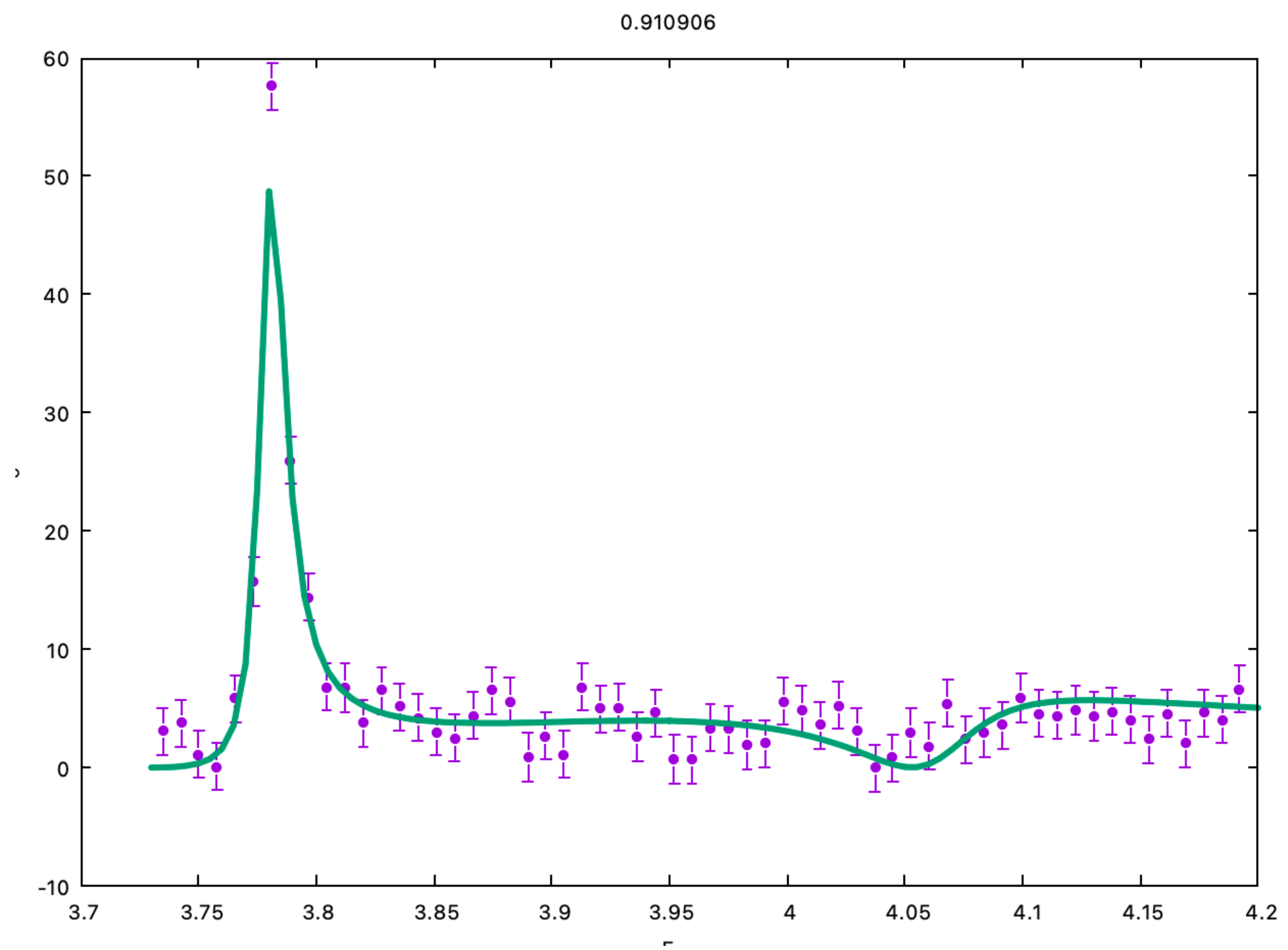
Single Channel K-matrix

compare Bayes and AIC weighting for 1000 models



Single Channel K-matrix





ModAvg/MAK5.cpp 1000 models

Single Channel K-matrix

```
generating-model poles:
pole = (3.778,-0.007) residue = (-15.4954,6.44069)
```

Bayes pole count probabilities

```
0 0%
1 99.2358%
2 0.59453%
3 0%
4 0%
```

Bayes pole count probabilities (no sheet I poles)

```
0 0%
1 96.1141%
2 0%
3 0%
4 0%
<pole>(1) = (3.778,-0.007) +/- (delR, delI, delRI) 0 0 4.5139e-36
<res>(1) = (-14.6995,6.29522) +/-  (delR, delI, delRI) 1.09651 0.277913 0.0706413
```

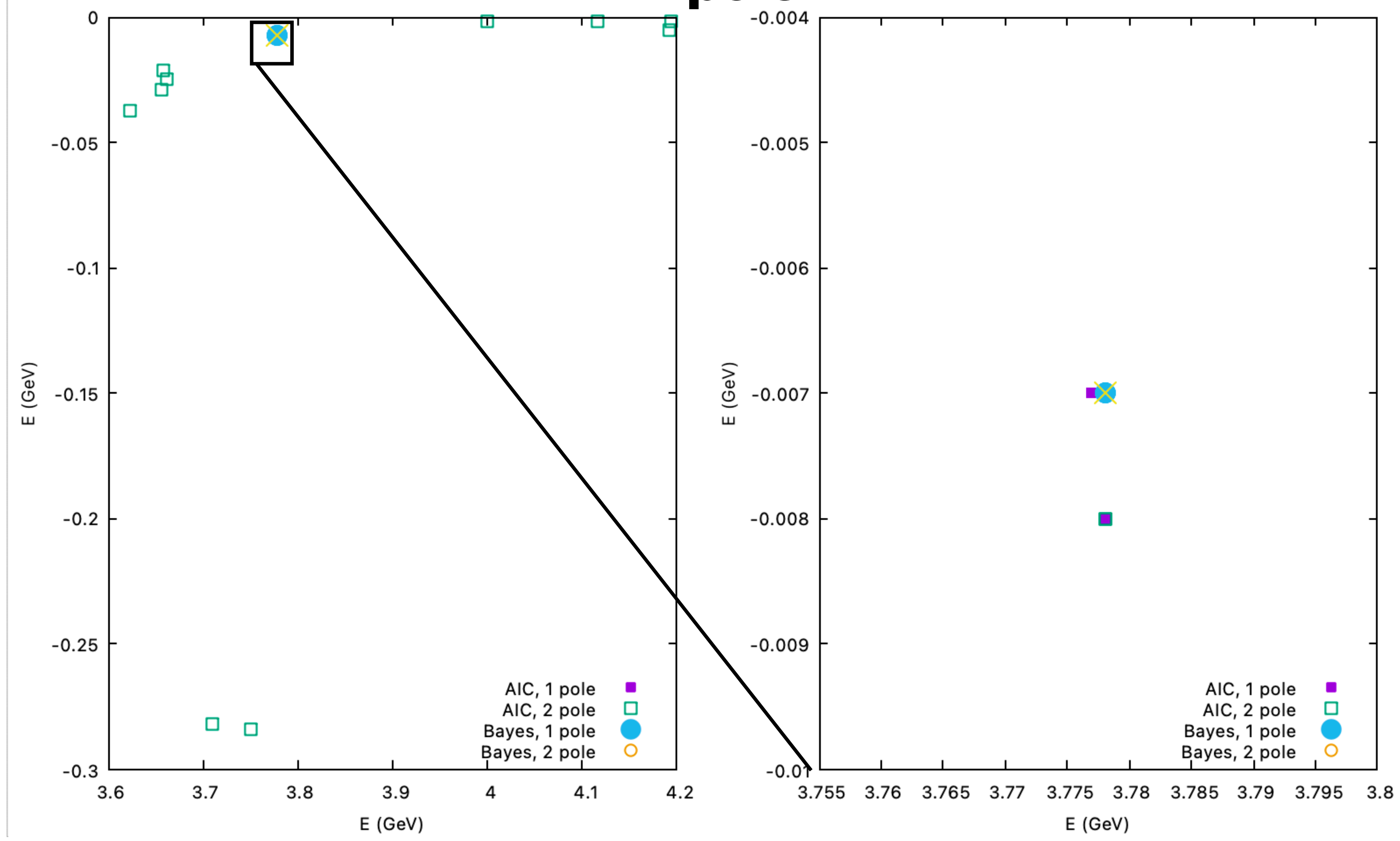
AIC pole count probabilities

```
0 0%
1 94.3365%
2 2.60253%
3 0.100984%
4 0%
```

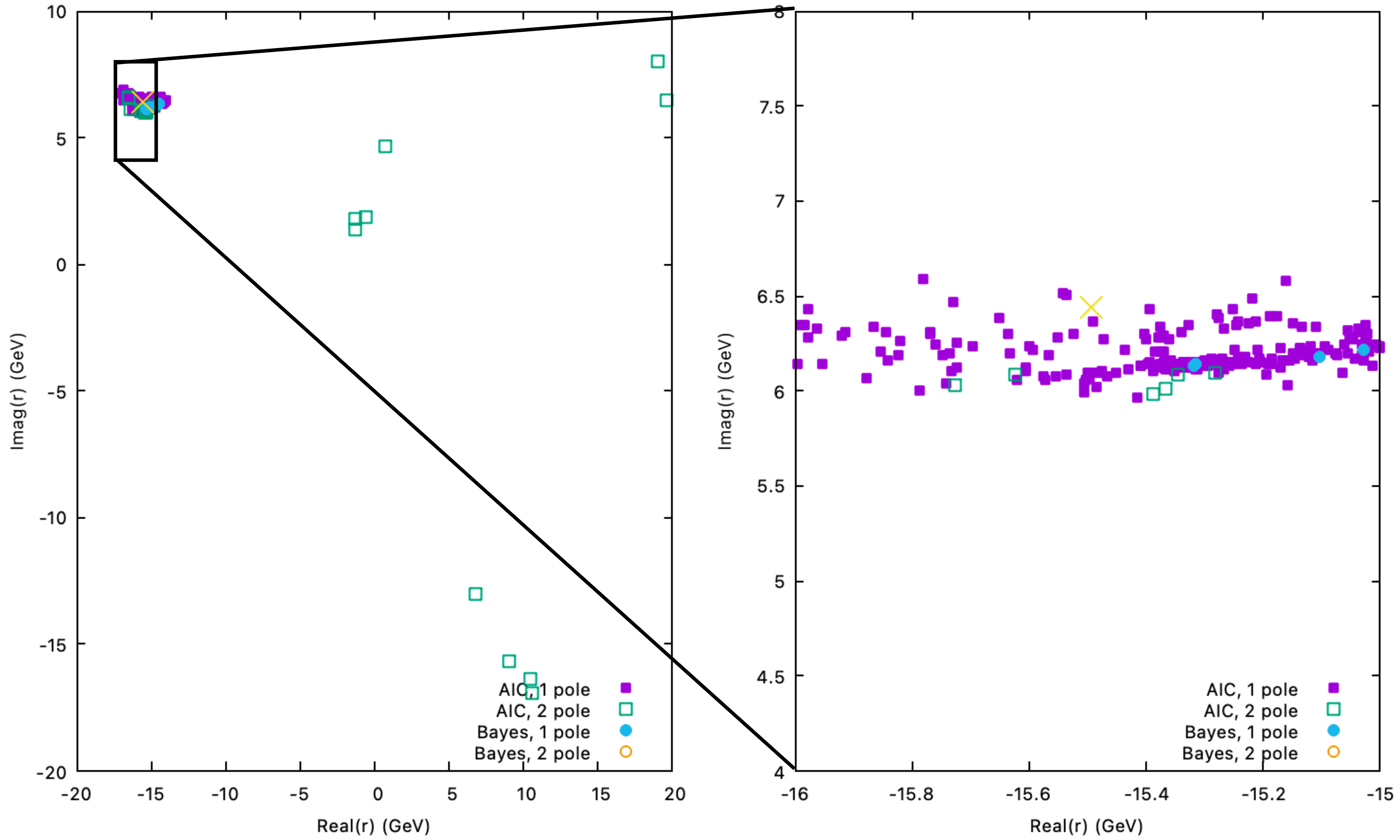
AIC pole count probabilities (no sheet I poles)

```
0 0%
1 77.6579%
2 0.935699%
3 0.0287494%
4 0%
<pole>(1) = (3.77798,-0.00700587) +/- (delR, delI, delRI) 1.36091e-05 -1.7134e-07 6.92959e-06
<res>(1) = (-15.3122,6.24651) +/-  (delR, delI, delRI) 70.7888 -1.24645 7.54011
```

pole

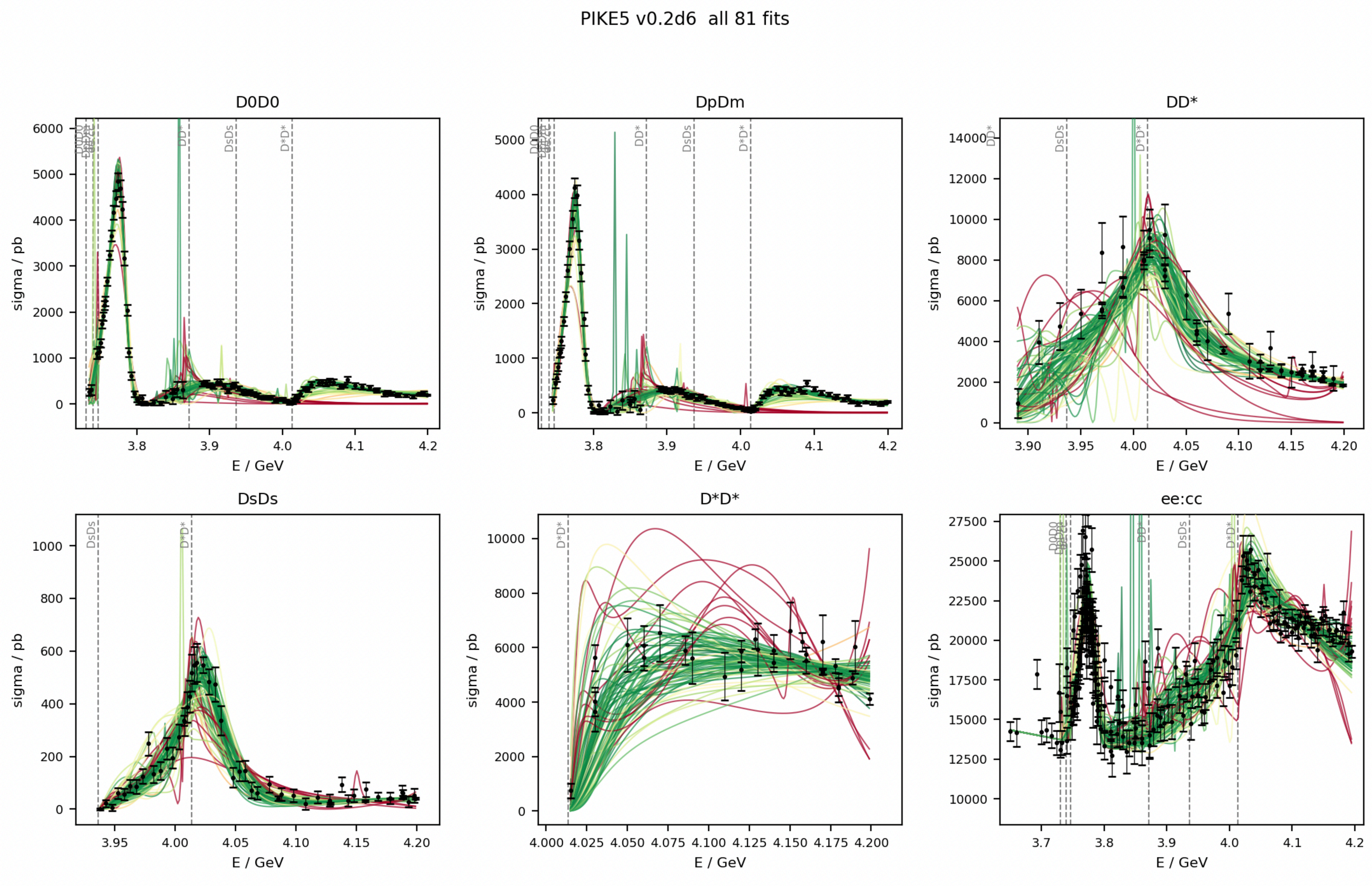


residue



G(3900)

81 models x 6 bootstraps



G(3900)

problems:

fitting is extraordinarily difficult

LASSO is little help

pole identification can be tricky

Laplace approximation is often not good; the Hessian condition number is very large (indicating a nonGaussian minimum)

attempts at Monte Carlo methods are bedeviled by very peaky multimodal space (a random step raises chi-2 by many orders of magnitude)

summary

- focus on predictiveness
- de-emphasize fitting and fit quality
- explore a large model space to enhance the reliability of the conclusions (we need to admit model uncertainty!)
- use data realizations to enhance the reliability of the conclusions
- be as agnostic wrt priors and models as possible
- model parameters are not physical
- problems wrt model optimization and finding global minima are reduced.

application to unbinned data?

make models completely agnostic with NNs? Cf, use a VAE to model the posterior...

✢ ÆRIC MEC HEHT GEWYRCAN