Recent Theoretical Progress in Quarkonium Production in Heavy-Ion Collisions

Jiaxing Zhao (HFHF & Goethe Univ.)

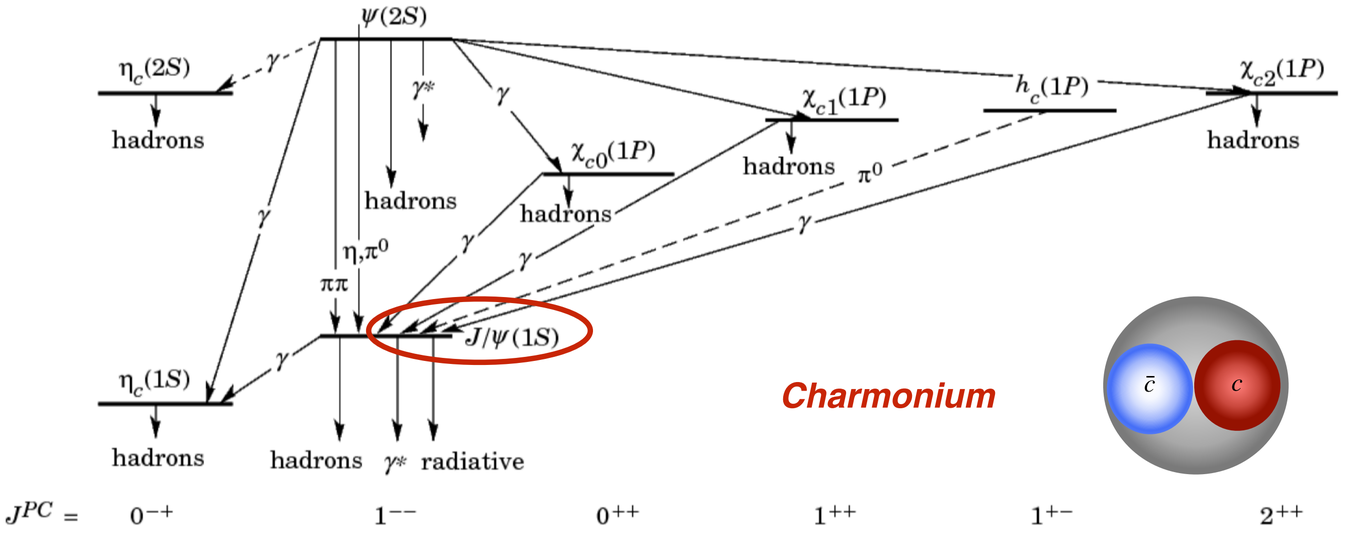
jzhao@itp.uni-frankfurt.de

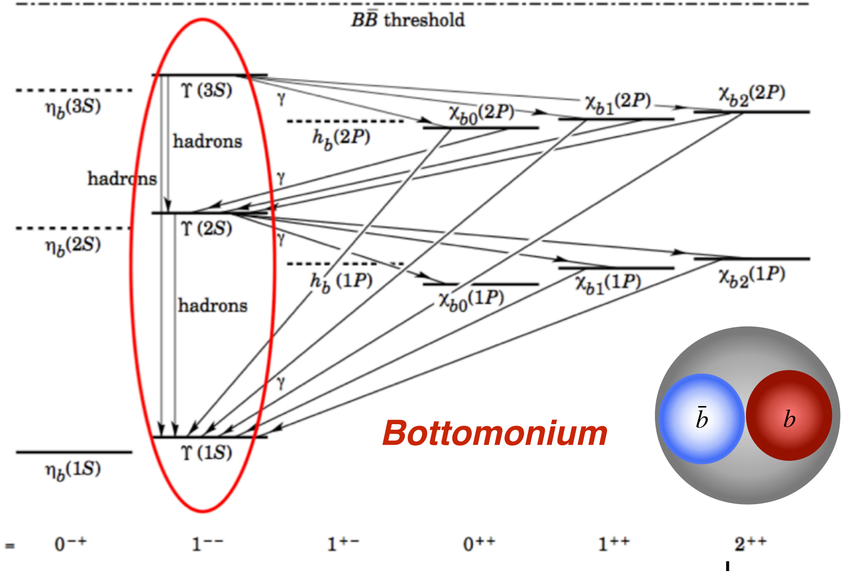
10/10/2026

HFHF Helmholtz Forschungsakademie Hessen für FAIR

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

CRC-TR 211 Strong-interaction matter under extreme conditions

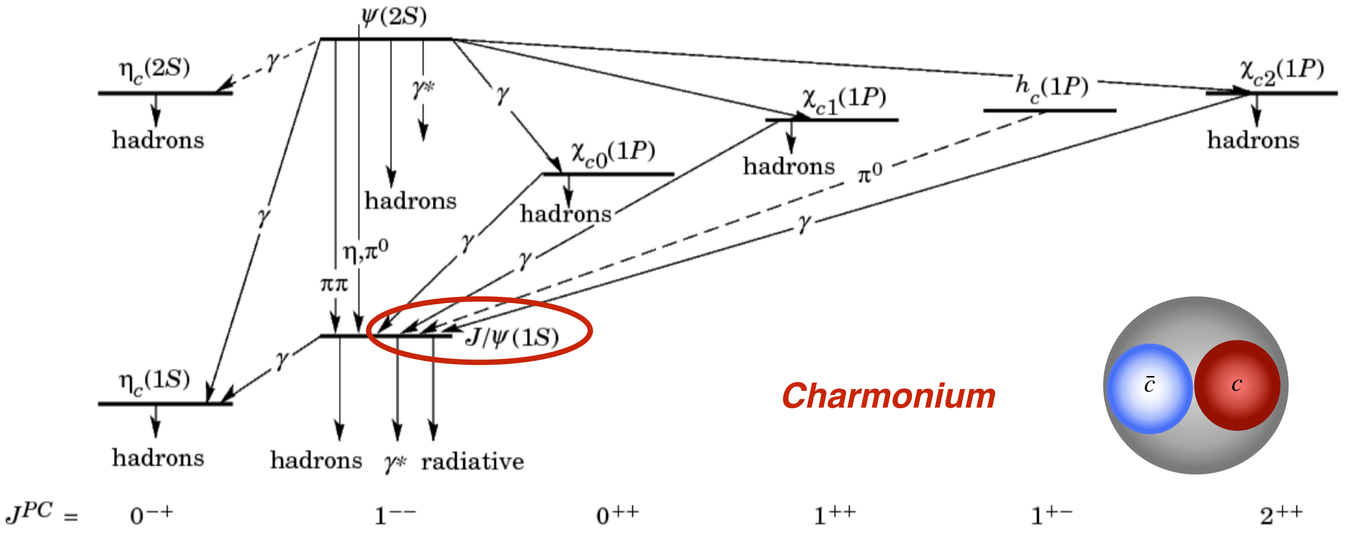




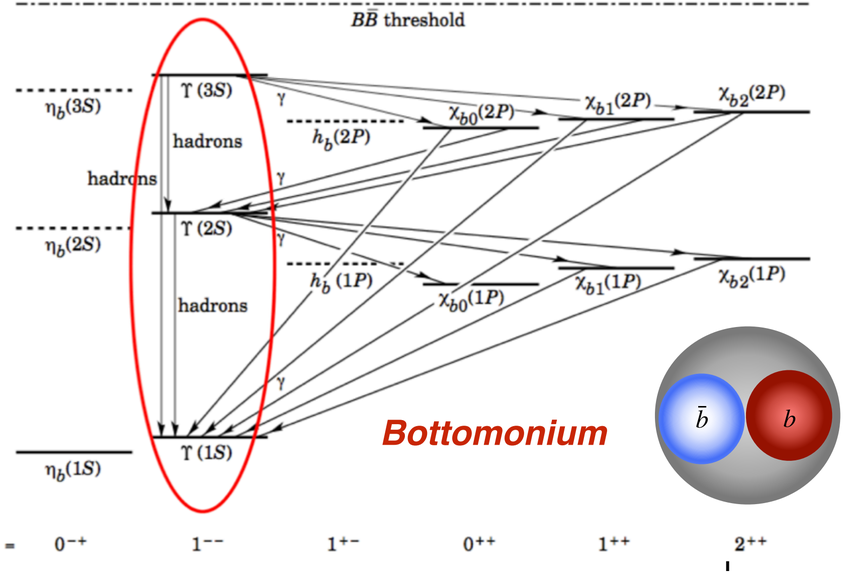
Charmonium

Bottomonium

How does quarkonium form and evolve in a thermal QCD medium, and what can we learn from it?



Charmonium



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- Quarkonia suppression has been considered as a smoking gun of the QGP (Matsui, Satz at 1986, …).

BROOKHAVEN NATIONAL LABORATORY

June 1986 BNL-38344

J/ψ SUPPRESSION BY QUARK-GLUON PLASMA FORMATION

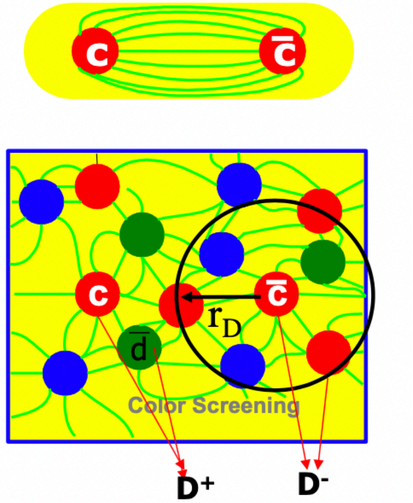
T. Matsui

Center for Theoretical Physics
Laboratory for Nuclear Science
Massachusetts Institute of Technology
Cambridge, MA 02139, USA

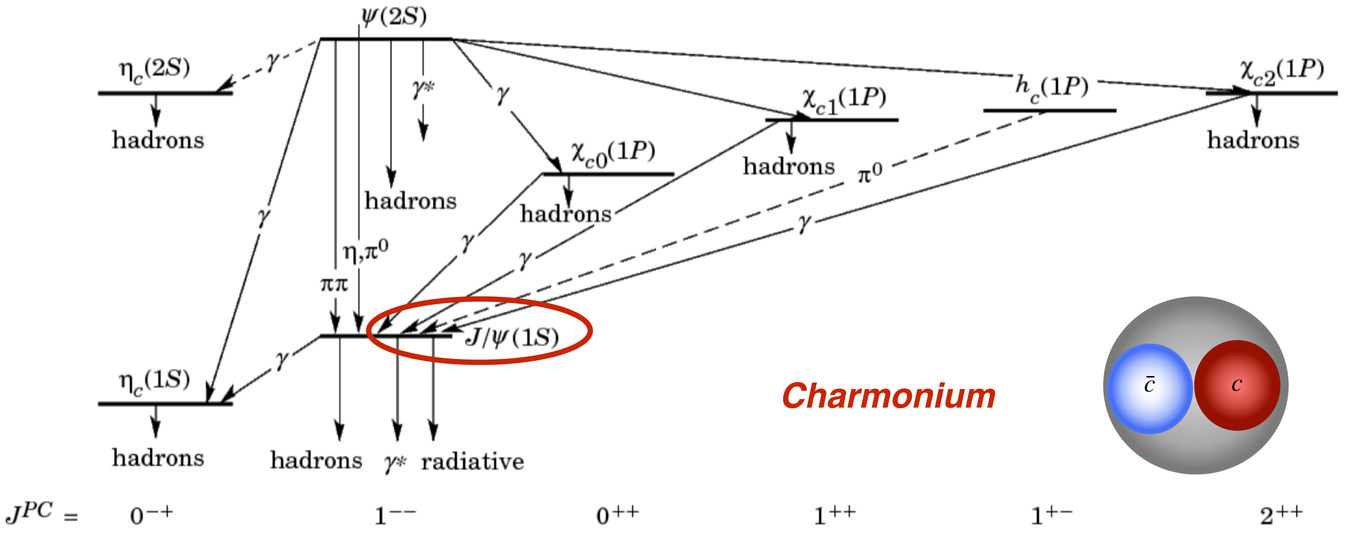
and

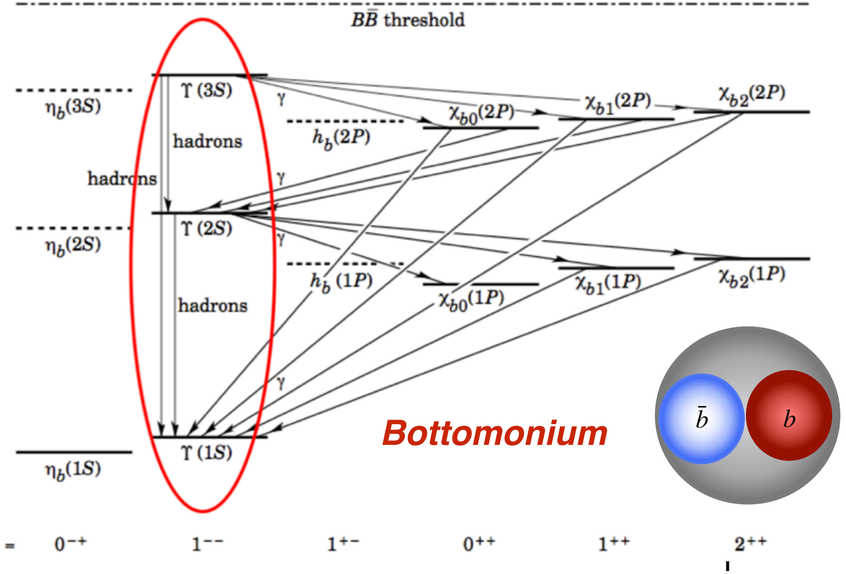
H. Satz

Fakultät für Physik
Universität Bielefeld, D-48 Bielefeld, F.R. Germany
and
Physics Department
Brookhaven National Laboratory, Upton, NY 11973, USA



QGP is well established in large collision systems (Au+Au, Pb+Pb), while quarkonium remains a sensitive probe of possible QGP formation in small systems (O+O, pp) !

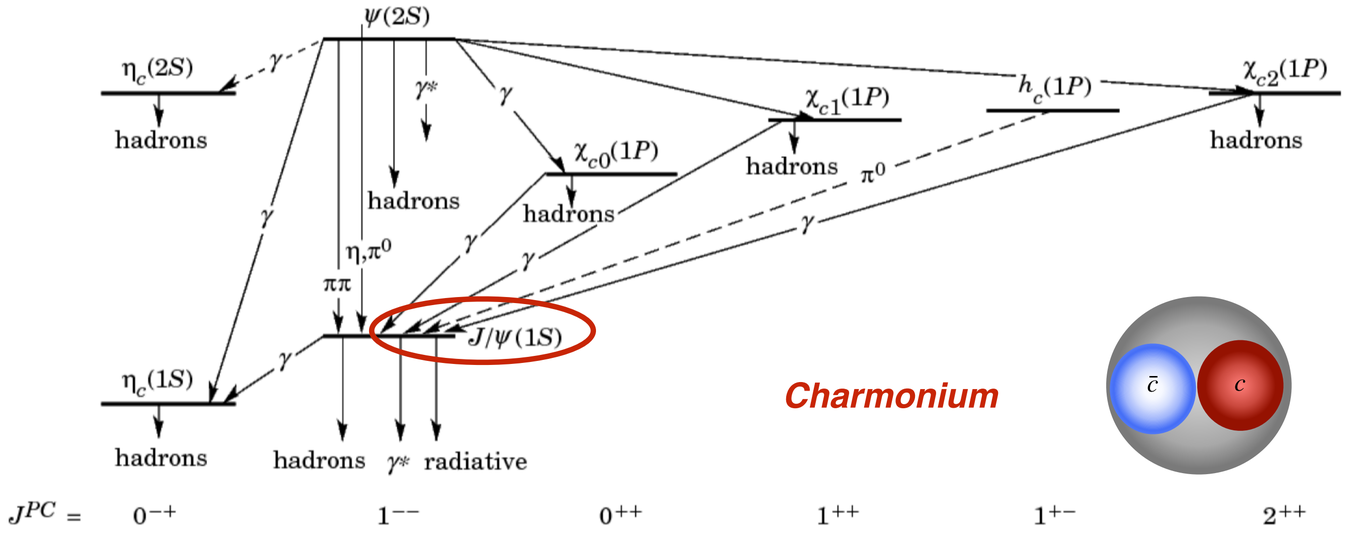


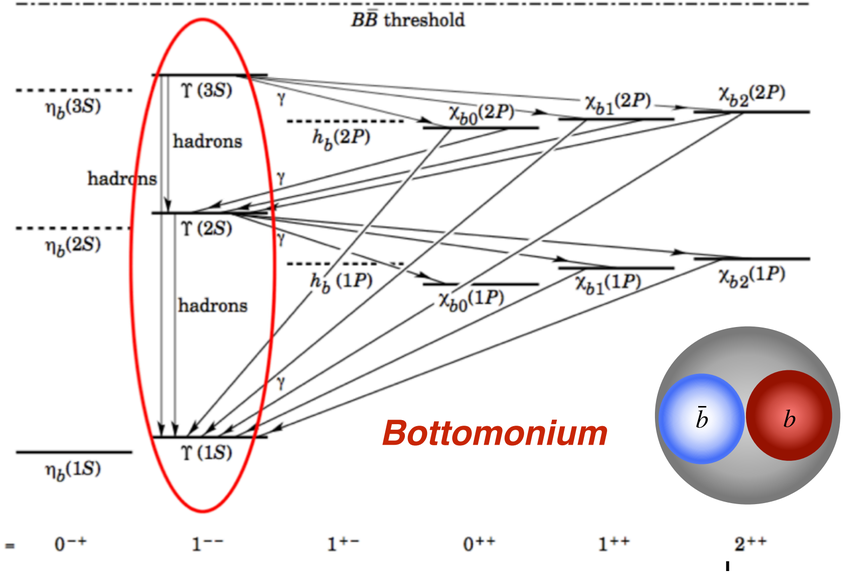


How does quarkonium form and evolve in a thermal QCD medium, and what can we learn from it?

- Quarkonia suppression has been considered as a smoking gun of the QGP (Matsui, Satz at 1986, …).
- Precision measurements of quarkonium production and distributions provide rich information:

QGP properties: temperature, transport coefficients, shear viscosity, …
QCD under extreme conditions: thermal QCD, strong electromagnetic fields, vorticity field,…
Nuclear structure: nuclear PDFs, shadowing, gluon saturation, …





How does quarkonium form and evolve in a thermal QCD medium, and what can we learn from it?

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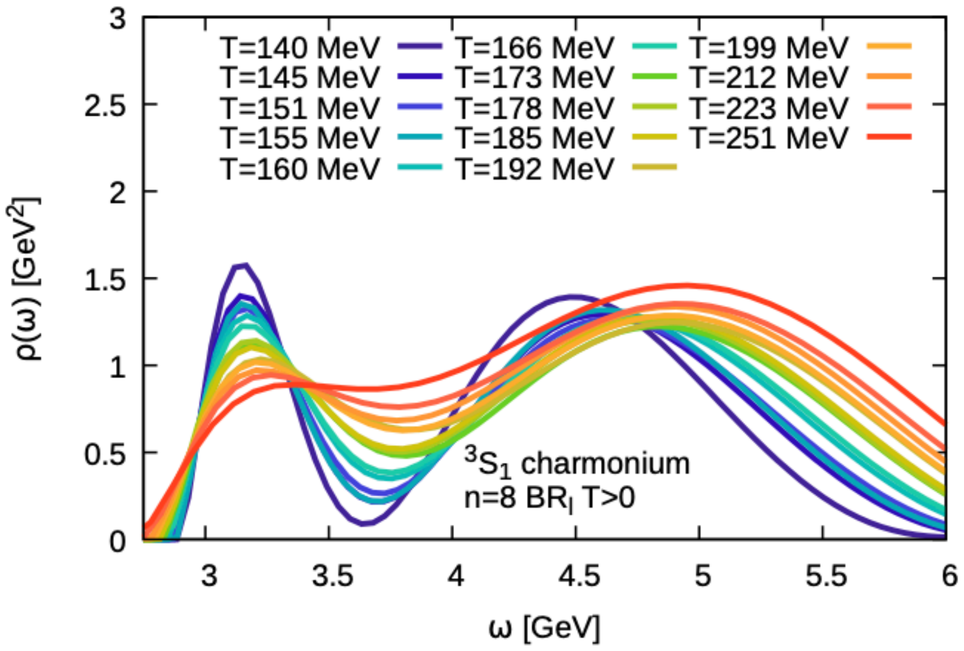
Outline:

- ➡ **Quarkonium in-medium properties: a changing picture**
- ➡ **From classical transport to quantum real-time evolution**

Quarkonium static properties in a hot QCD medium.

Quarkonium in the hot medium

All in-medium properties of quarkonium are encoded in their spectral function



S. Kim, P. Petreczky, A. Rothkopf, JHEP 11 (2018) 088

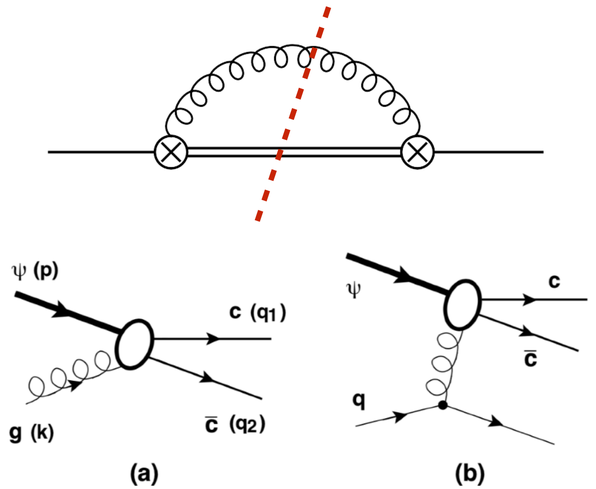
peak position shifts and becomes broader as temperature increases.

In the perturbative point of view:

Mass shift –> static color screening

Singlet-octet thermal break up –> gluon-dissociation

Landau damping–> inelastic scattering (quasifree limit)



N. Brambilla, M. Escobedo, J. Ghiglieri, M. Laine, O. Philipsen, P. Romatschke, M. Tassler, P. Petreczky, et al, JHEP 03, 054 (2007). PRD 78, 014017 (2008). JHEP 09, 038 (2010). JHEP 1112 (2011) 116…

Heavy Quark Potential at finite temperature

In-medium properties can be absorbed in a temperature-dependent heavy quark potential.

If the heavy quarks interact with the medium for a very long time, the potential is equivalent to the free energy.

How the heavy potential is modified at scales comparable to the internal time scale of quarkonium?

- **In the weak-coupling regime (High temperature–> HTL,…)** *M. Laine, O. Philipsen, P. Romatschke, M. Tassler, JHEP 03 (2007) 054*

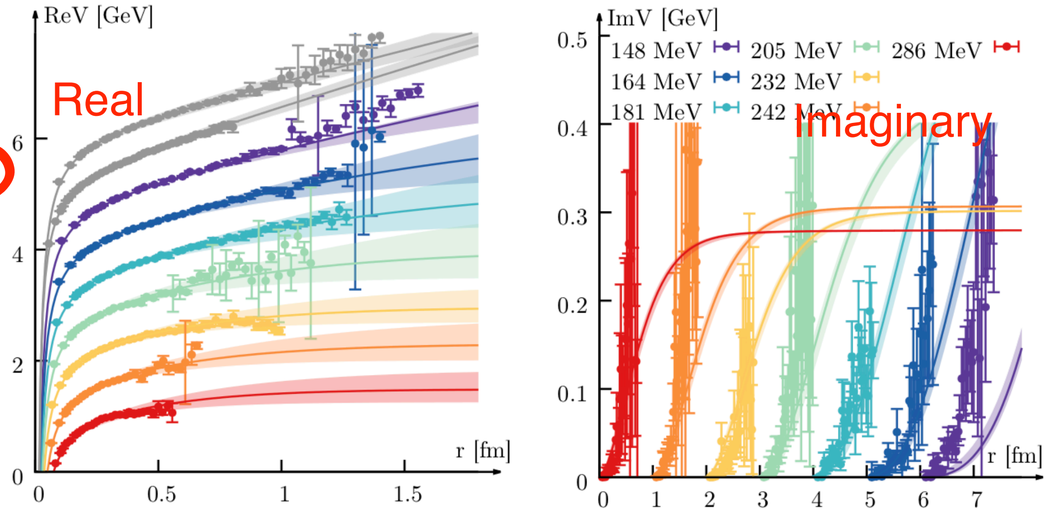
$$V(r,T) = -\frac{g^2 C_F}{4\pi}\left[m_{\rm D} + \frac{\exp(-m_{\rm D} r)}{r}\right] - \frac{i g^2 T C_F}{4\pi}\,\phi(m_{\rm D} r)$$

Real Imaginary

$$\phi(x) = 2\int_0^\infty \frac{{\rm d}z\, z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right]$$

- **In the strong-coupling regime (Lattice QCD,…)**

2015



D. Lafferty, A. Rothkopf, Phys.Rev.D 101 (2020) 5, 056010 .

Obvious screening for the real part potential, the imaginary part larger than HTL results.

Heavy Quark Potential at finite temperature

Masses and widths of bottomonium states are obtained via Lattice NRQCD from 150-350 MeV.

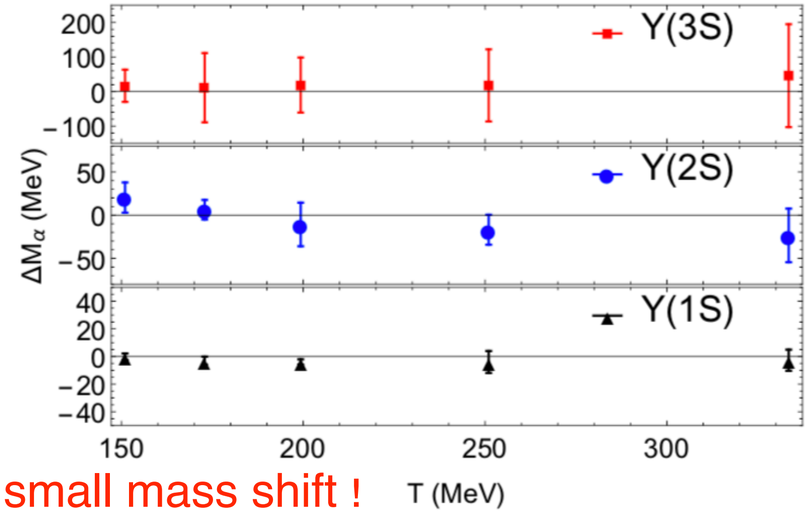
Excited bottomonia in quark-gluon plasma from lattice QCD

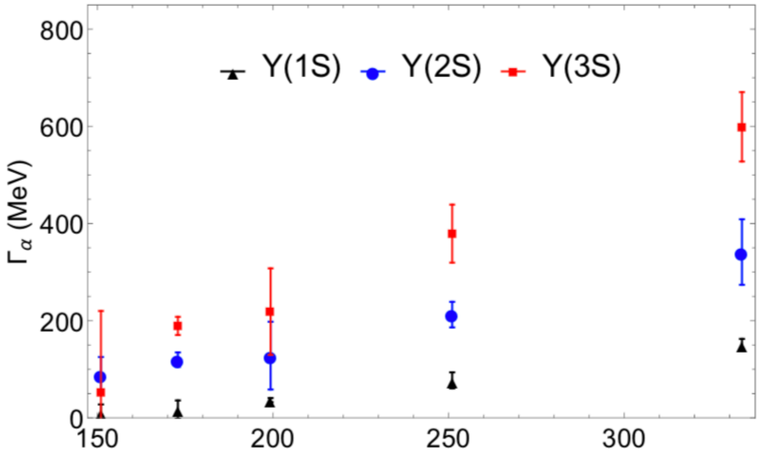
Rasmus Larsen[a], Stefan Meinel[b,c], Swagato Mukherjee[a], Peter Petreczky[a]

Gaussian spectral assumption.

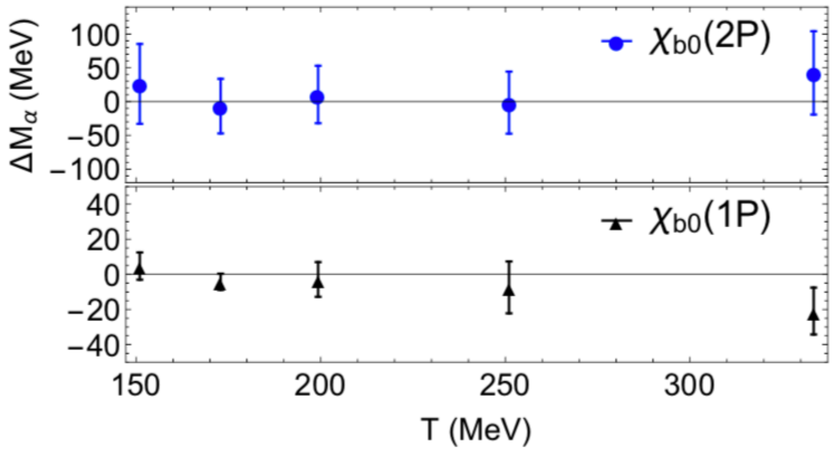
R. Larsen, S. Meinel, S. Mukherjee, and P. Petreczky, Phys. Lett. B 800, 135119 (2020).

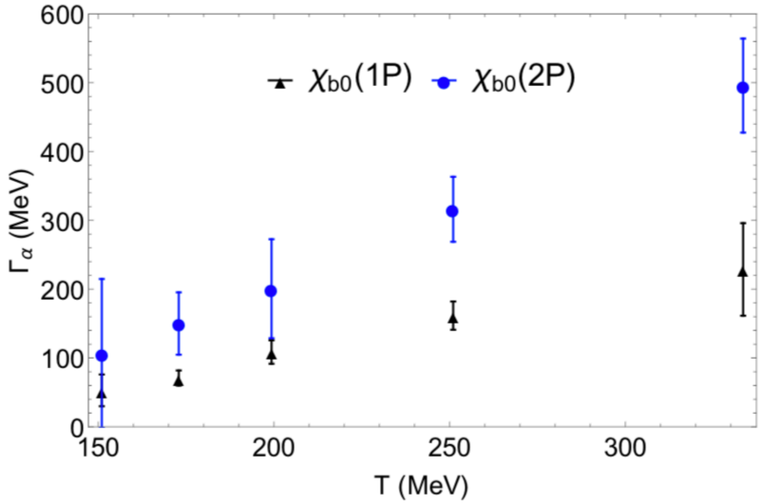
2020





Very small mass shift !





Heavy Quark Potential at finite temperature

Extraction of the HQ Potential from bottomonium mass and width (Lattice NRQCD)

Mass and Width



$M = 2m_b + Re[E]$

$\Gamma = -\,Im[E]$



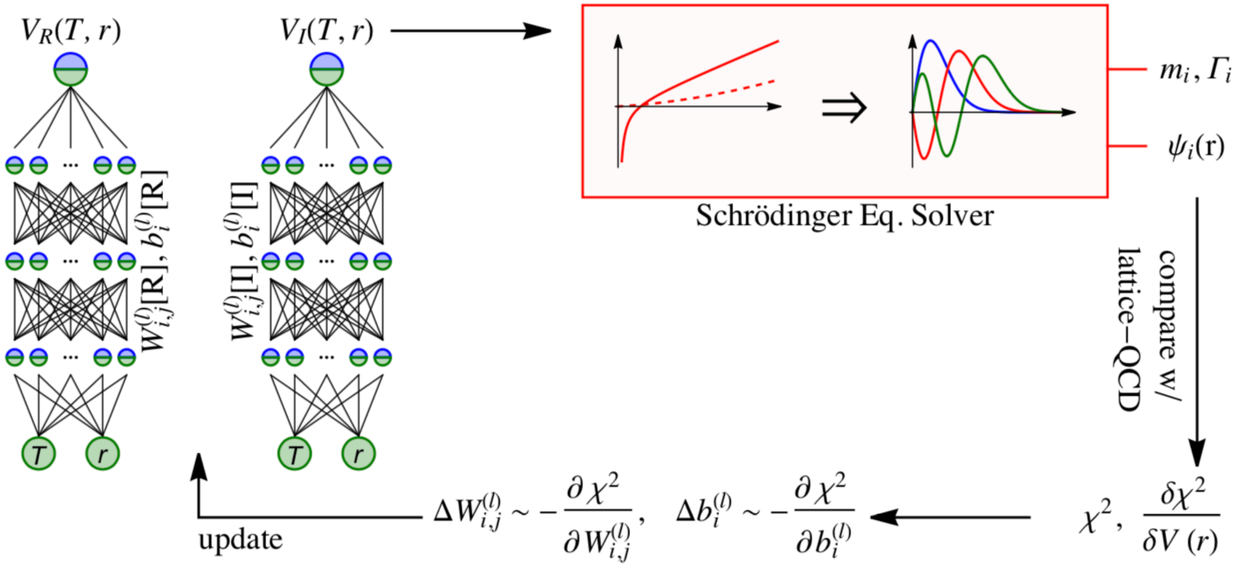
Inverse problem ?

Deep Neural Networks

Heavy quark potential

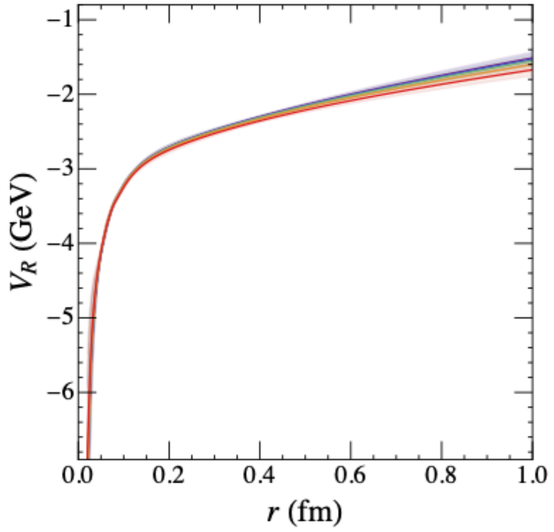
$$V(r,T) = V_R(r,T) + iV_I(r,T)$$

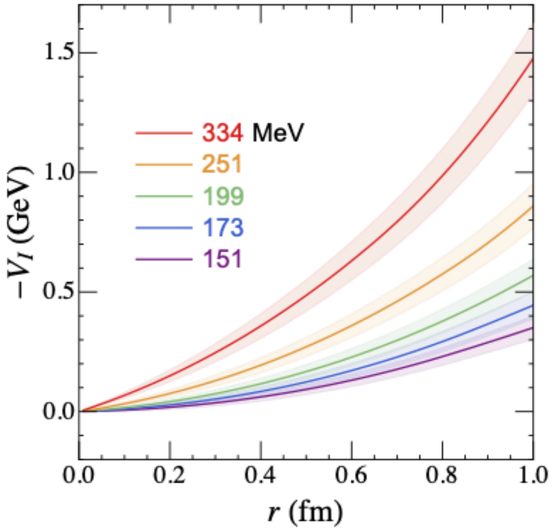
$$-\frac{\nabla^2}{m_b}\psi_n + [V_R(T,r) + i\,V_I(T,r)]\,\psi_n = E_n\psi_n$$



*S. Shi, K. Zhou, **JZ**, S. Mukherjee, and P. Zhuang. PRD 105 (2022) 1, 1.*

2021





Large imaginary part and very small screening effect !

Heavy Quark Potential at finite temperature

HQ potential with HTL resummed perturbation method within the Gribov-Zwanziger approach

Gauge fixing in the quantization is not complete, there exist still gauge copies.

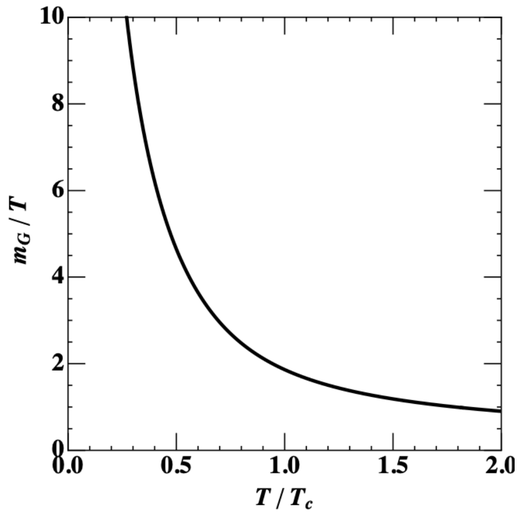
GZ approach for confinement of gluons, EOS of strongly coupled QCD medium.

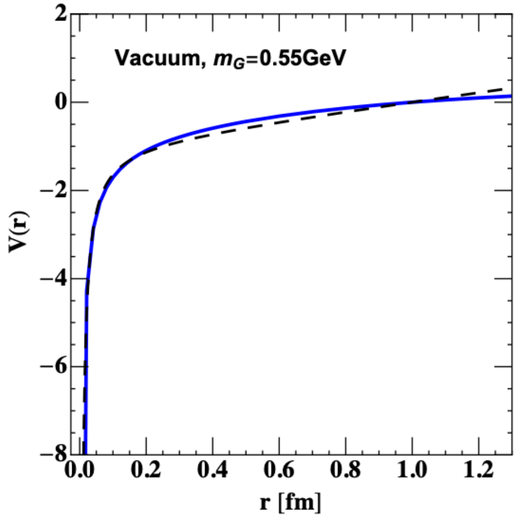
(infrared improved gluon propagator–chromomagnetic scale– nonperturbative)

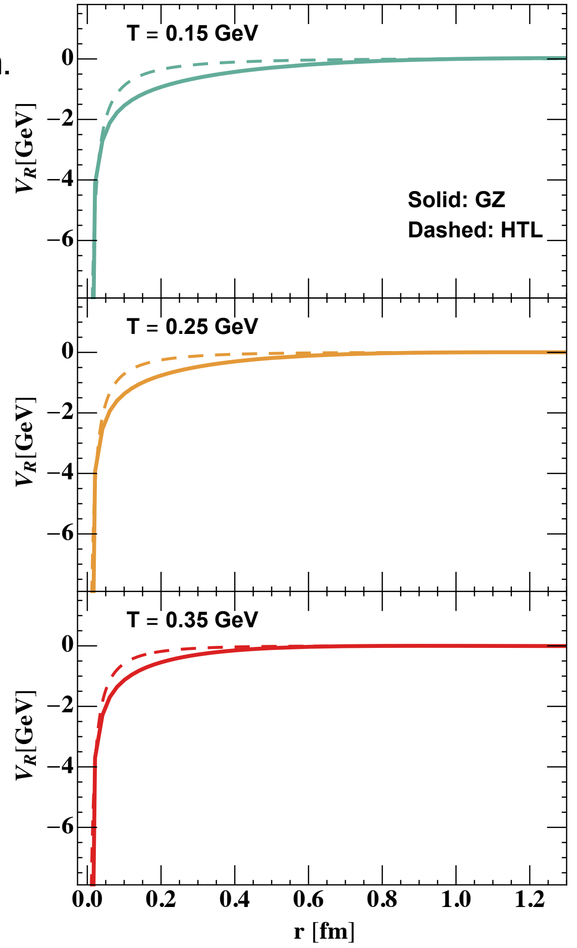
Gribov gluon propagator:

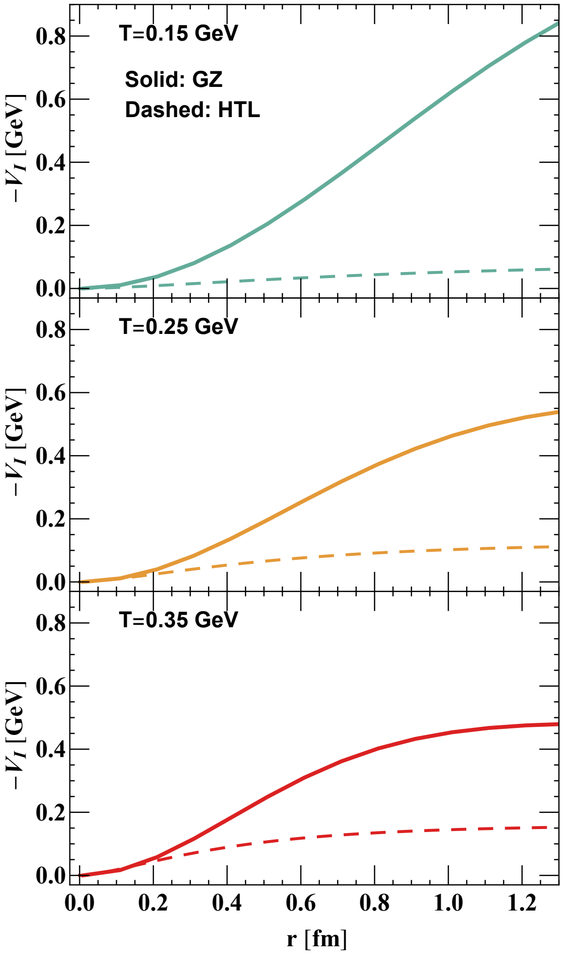
$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{p^2}{p^4 + m_G^4} \longrightarrow \text{Gribov mass}$$

2022









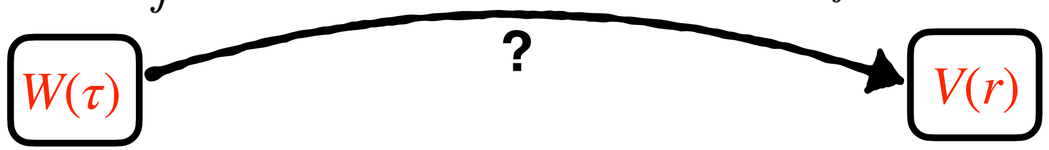
Real part shows less color screening at small r and color confinement at large r; larger imaginary part than HTL!

*W. Wu, G. Huang, **JZ**, P. ZHuang. PRD 107 (2023) 11, 114033*

M.Debnath, R.Ghosh and N.Haque, Eur.Phys.J.C 84 (2024) 3, 313

Heavy Quark Potential at finite temperature

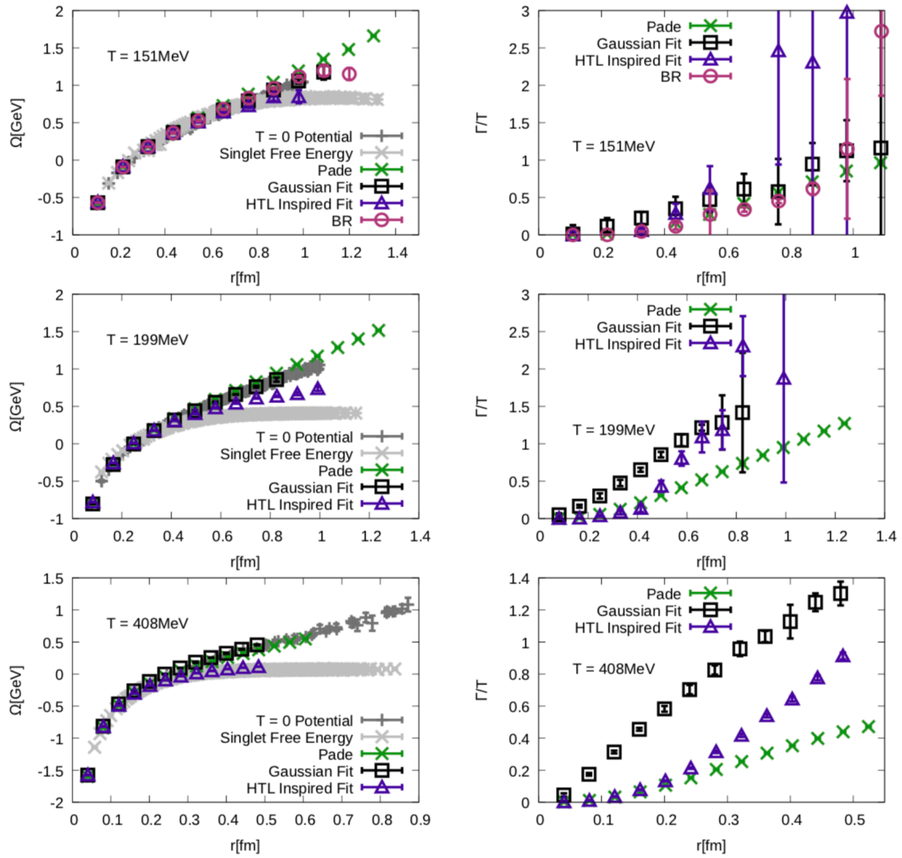
$$W(\tau) = \int d\omega e^{-\omega\tau}\rho(\omega) \qquad \rho(\omega) \qquad V(r) = \lim_{t\to\infty}\int d\omega\, \omega e^{-i\omega t}\rho(\omega,r)/\int d\omega\, e^{-i\omega t}\rho(\omega,r)$$



Reconstructing spectral functions through Euclidean correlation functions is an ill-posed inverse problem.
A big difference caused by the extraction strategy !

Extract the spectral functions from correlators with four different methods:

2022

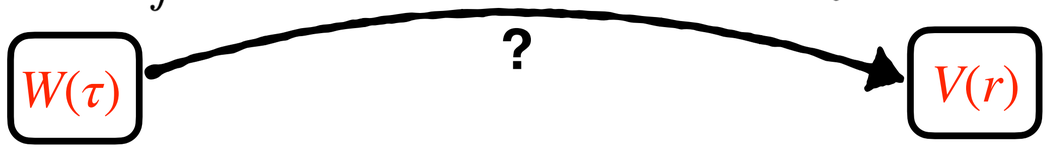


D. Bala et al, Phys.Rev.D 105, 054513 (2022).

1. Gaussian fit;
2. HTL inspired fit;
3. Pade fit;
4. Bayesian reconstruction (BR) method.

Heavy Quark Potential at finite temperature

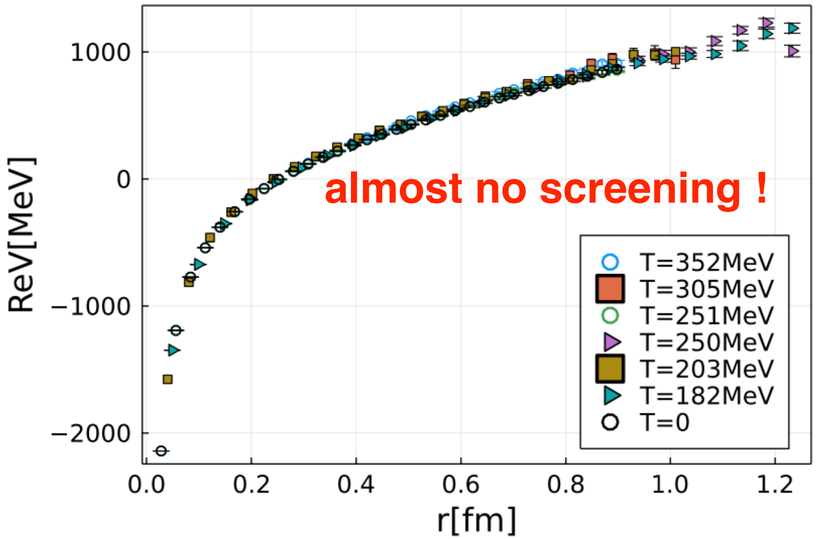
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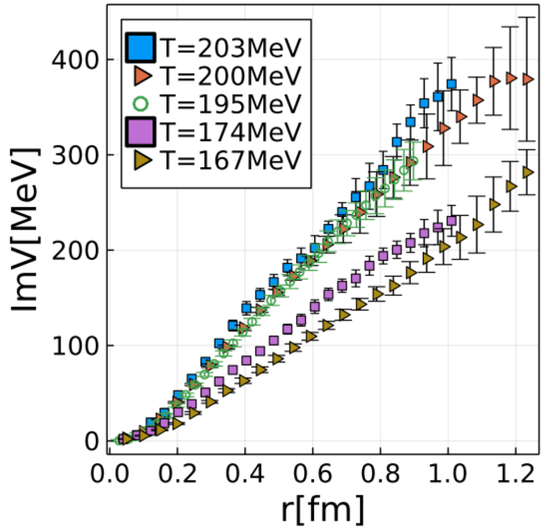


Reconstructing spectral functions through Euclidean correlation functions is an ill-posed inverse problem.
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A physically appealing parametrization of spectrum -> (cut) Lorentzian form:

2024



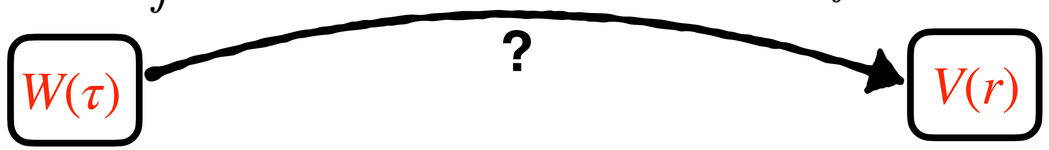


A.Bazavov, D. Hoying, O. Kaczmarek, R.N. Larsen, S. Mukherjee, P. Petreczky, A. Rothkopf, J.H. Weber , Phys.Rev.D 109, 074504 (2024)

Lattice QCD with dynamical fermions indicates no screening in static quark-antiquark potential !

Heavy Quark Potential at finite temperature

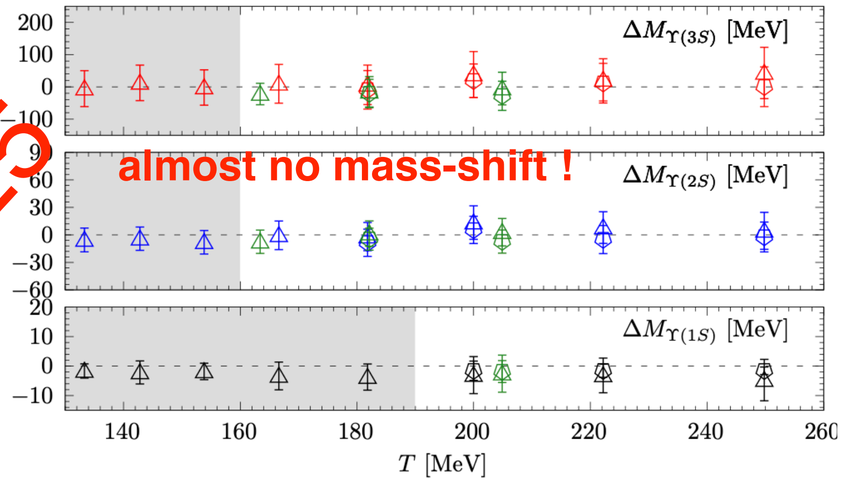
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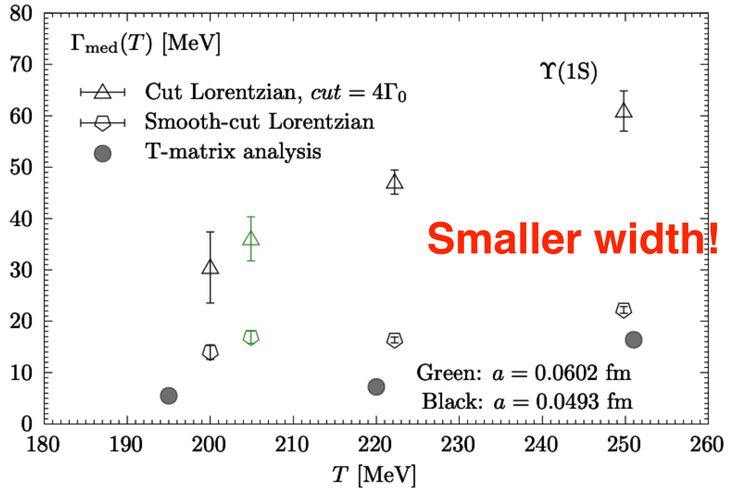


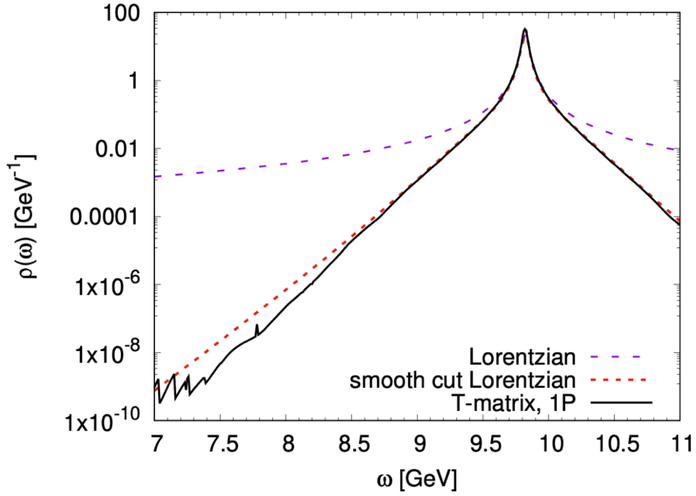
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2025





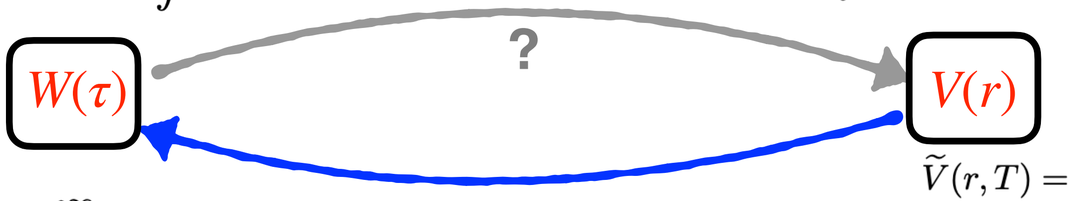


H. Ding ,W. Huang ,R. Larsen, S. Meinel ,S. Mukherjee , P. Petreczky, and Z. Tang, JHEP 05 (2025) 149.

Heavy Quark Potential at finite temperature

Extraction of the potential by fitting the Wilson line correlators and EOS in T-matrix approach

$$W(\tau) = \int d\omega e^{-\omega\tau}\rho(\omega) \qquad \rho(\omega) \qquad V(r) = \lim_{t\to\infty}\int d\omega\, \omega e^{-i\omega t}\rho(\omega,r)/\int d\omega\, e^{-i\omega t}\rho(\omega,r)$$



$$W(r,\tau,T) = \int_{-\infty}^{\infty} dE e^{-E\tau}\rho_{Q\bar{Q}}(E,r,T) \qquad \rho(\omega)$$

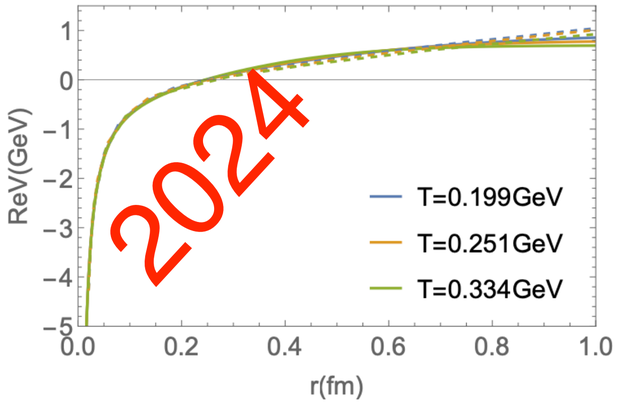
$$M_{\text{eff}}(r,\tau,T) = -\partial_\tau \ln W(r,\tau,T)$$

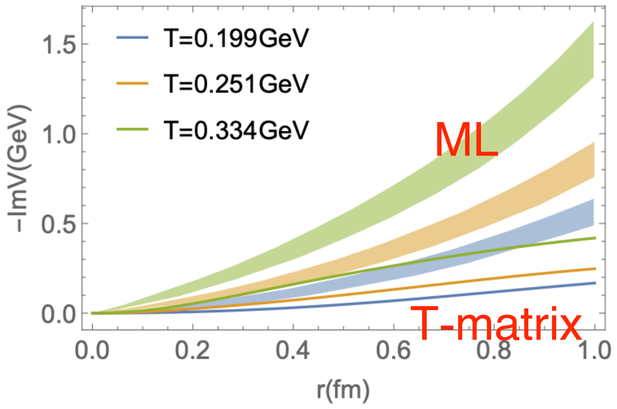
$$\widetilde{V}(r,T) = -\frac{4}{3}\alpha_s[\frac{e^{-m_d r}}{r} + m_d] - \frac{\sigma}{m_s}[e^{-m_s r-(c_b m_s r)^2} - 1]$$

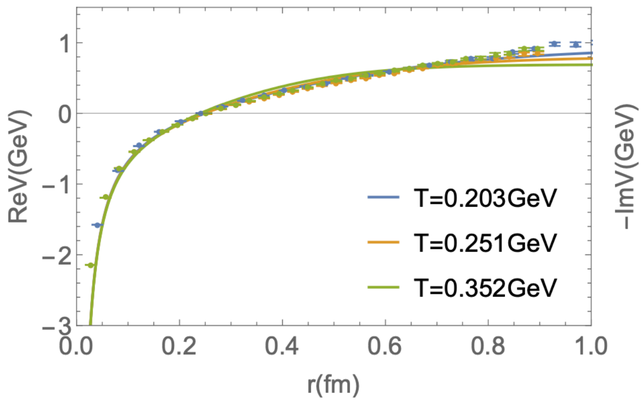
$$T_{ij}^{L,a}(z,p,p') = V_{ij}^{L,a}(p,p') + \frac{2}{\pi}\int_{-\infty}^{\infty} k^2 dk V_{ij}^{L,a}(p,k)G_{ij}^{0}(z,k)T_{ij}^{L,a}(z,k,p')\ .$$

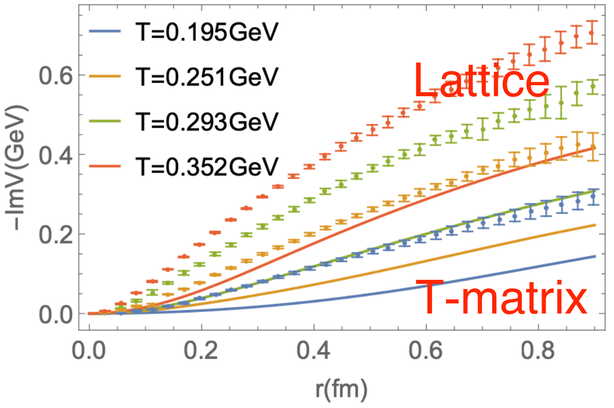
$$\rho_{Q\bar{Q}}(E,r,T) = \frac{-1}{\pi}\text{Im}\left[\frac{1}{E-\widetilde{V}(r,T)-\phi(r,T)\Sigma_{Q\bar{Q}}(E,T)}\right]$$

Z. Tang, S. Mukherjee, P. Petreczky, R. Rapp. Eur.Phys.J.A 60 (2024) 4, 92.









Fit the lattice results by adjusting the input in-medium potential.

Very small screening effect for the real part of the potential.

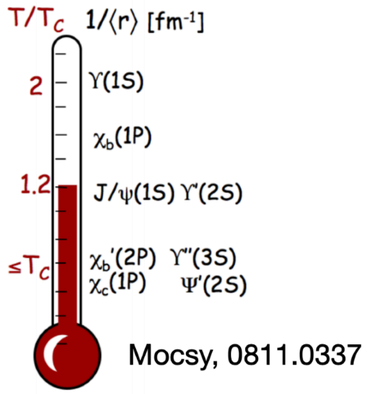
The imaginary part is smaller than the Lattice results (Lorentzian ansatz) and ML (Gaussian ansatz)!

Heavy Quark Potential at finite temperature

If color-screening is weak, what actually suppresses quarkonium in HICs?

Dynamical dissociation may play the dominant role in quarkonium suppression in the QGP.

The notion of a sharp dissociation temperature becomes less useful.



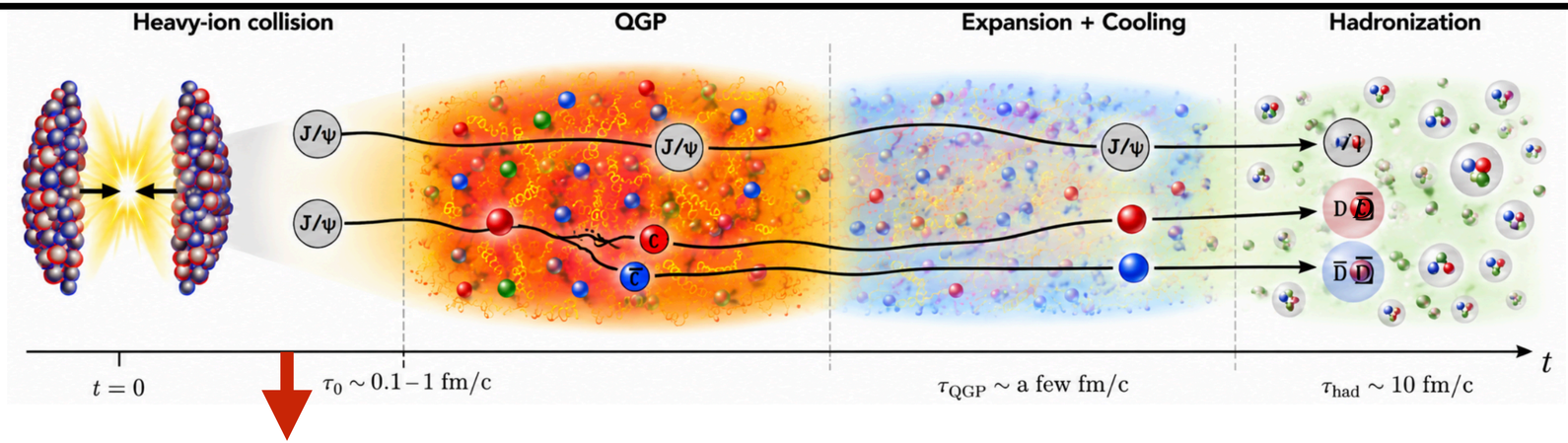
Can we distinguish it in the experiment? Which observable?

Pengfei Zhuang's talk.

Quarkonium real-time evolution in hot QCD medium

Key features of quarkonium in heavy-ion collisions

Key features of quarkonium in heavy-ion collisions



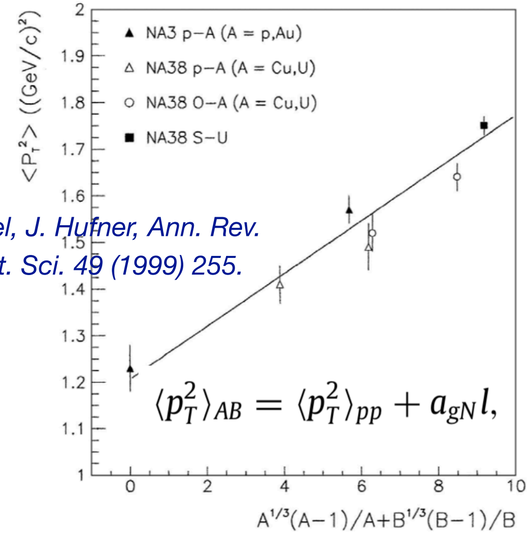
Heavy-ion collisions can be viewed as a superposition of pp collisions.
Cold nuclear effect: Shadowing effect, Cronin effect and Nuclear absorption

- Nuclear absorption : quarkonium has inelastic interaction with the surrounding nucleons and suffers from a suppression
- Cronin effect : Before two gluons fuse into a quarkonium, they acquire additional transverse momentum via multiscattering with the surrounding nucleons, and this extra momentum would be inherited by the produced quarkonium
- Shadowing effect: The distribution function for parton in a nucleus differs from a simple superposition of the distribution in a free nucleon

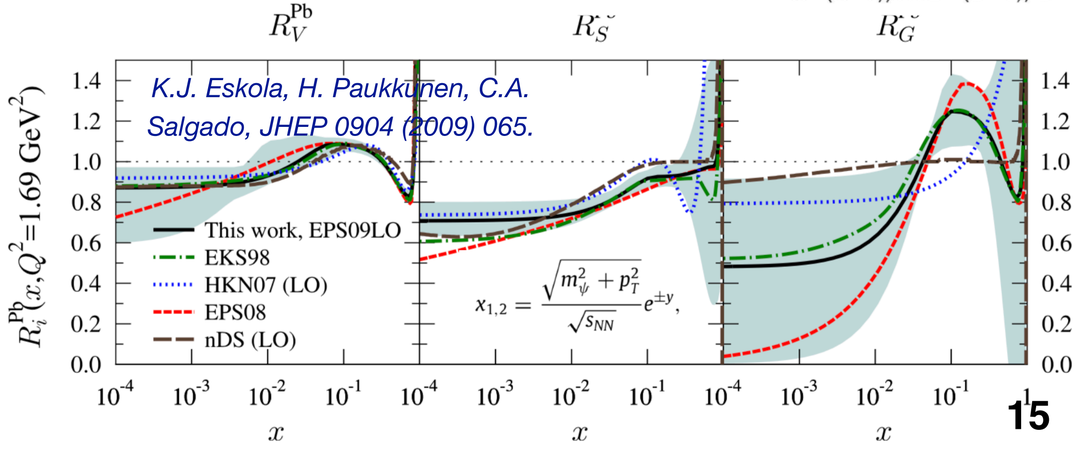
$$R_i^A(x, Q^2) = \frac{f_i^A(x, Q^2)}{A f_i(x, Q^2)}, \qquad i = q, \bar{q}, g$$

$$\mathcal{R} = 1 + A(R_i - 1)T_A(\mathbf{x}_T)/T_{AB}(0),$$

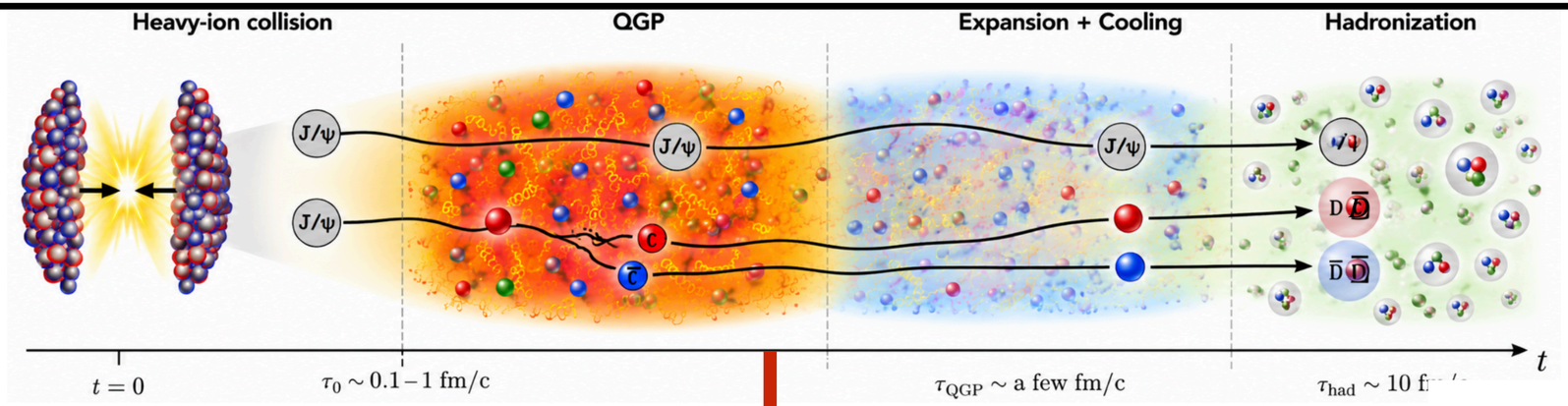
EPS09, EKS08, KHN07, NNPDF,…. Many model. Extracted from eA, pA,,,data



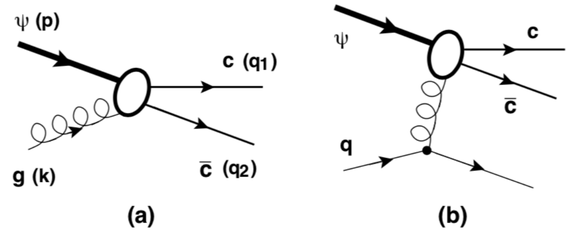
C. Gerschel, J. Hufner, Ann. Rev. Nucl. Part. Sci. 49 (1999) 255.



Key features of quarkonium in heavy-ion collisions

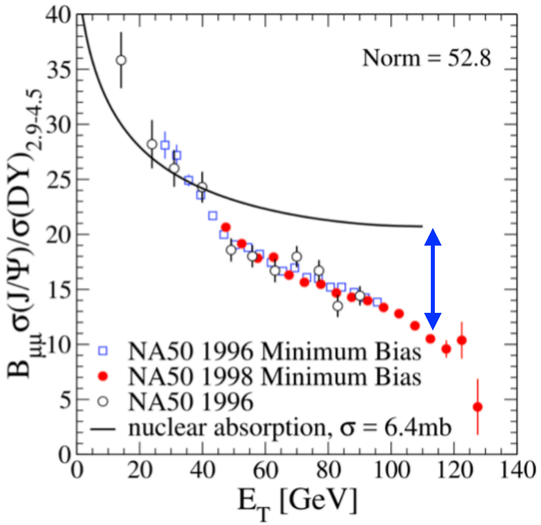


Hot nuclear effect: Anomalous suppression and regeneration



➡ Anomalous suppression: Quarkonium can be dissociated by color screening (?) and inelastic scattering with thermal partons in the QGP.

➡ Regeneration: Heavy quarks and antiquarks can recombine to form new color-singlet quarkonium states in QGP.




ELSEVIER

5 October 2000

"First regeneration paper"

PHYSICS LETTERS B

Physics Letters B 490 (2000) 196–202

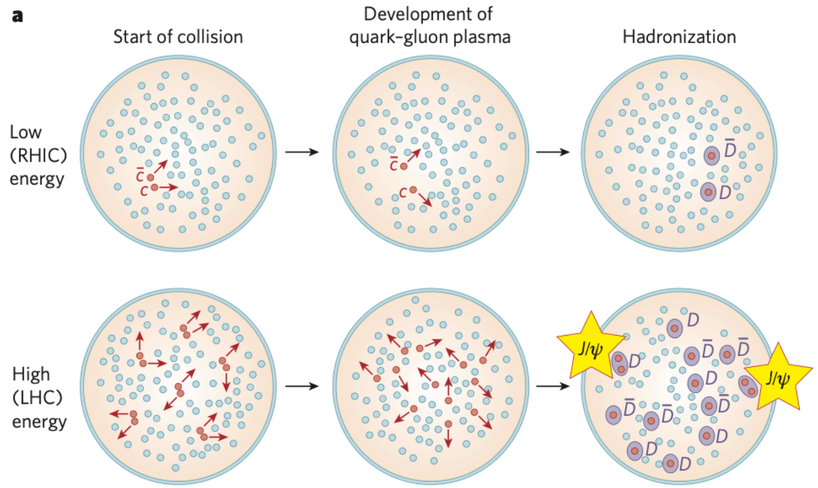
www.elsevier.nl/locate/npe

(Non) thermal aspects of charmonium production and a new look at J/ψ suppression ☆

P. Braun-Munzinger [a], J. Stachel [b]

[a] *Gesellschaft für Schwerionenforschung, D 64291 Darmstadt, Germany*
[b] *Physikalisches Institut der Universität Heidelberg, D 69120 Heidelberg, Germany*

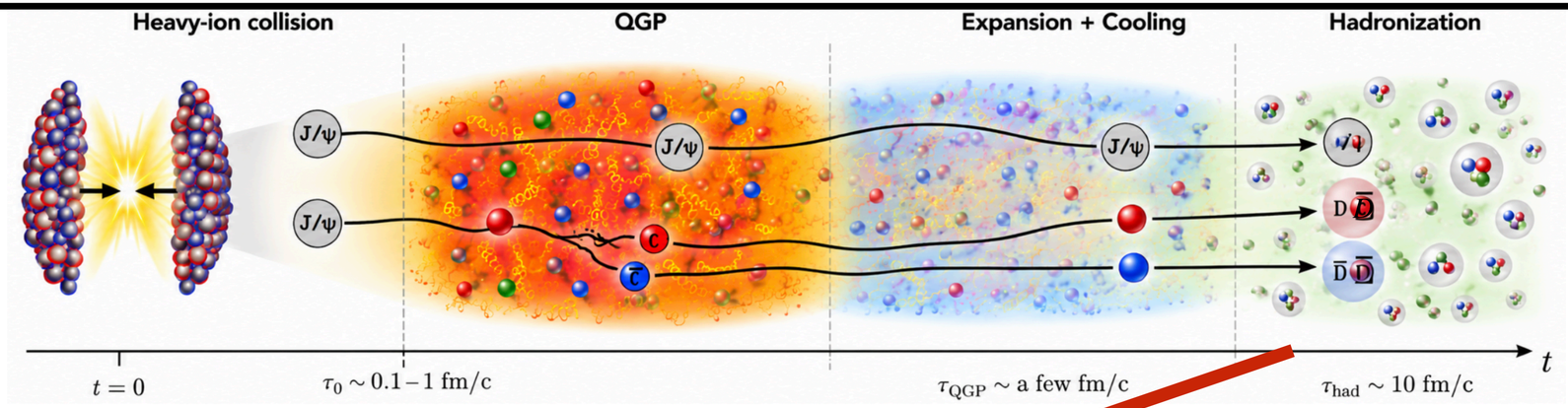
Received 26 July 2000; accepted 17 August 2000
Editor: J.-P. Blaizot

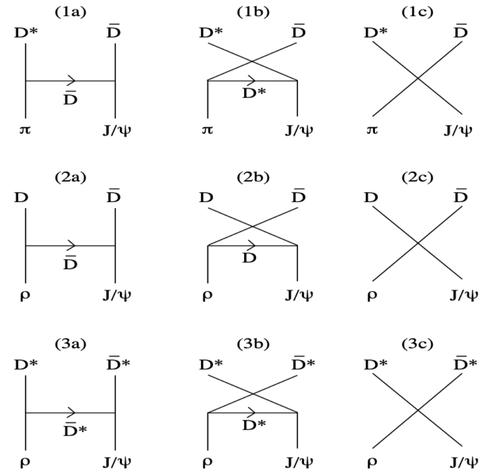


Also: Thews, Schroedter & Rafelski (2001), Phys.Rev.C 63 (2001) 054905.

P. Braun-Munzinger, J. Stachel, Nature 448 (2007) 302-309.

Key features of quarkonium in heavy-ion collisions



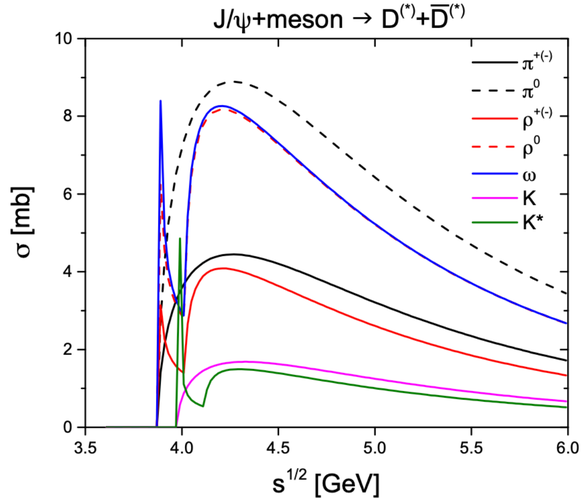


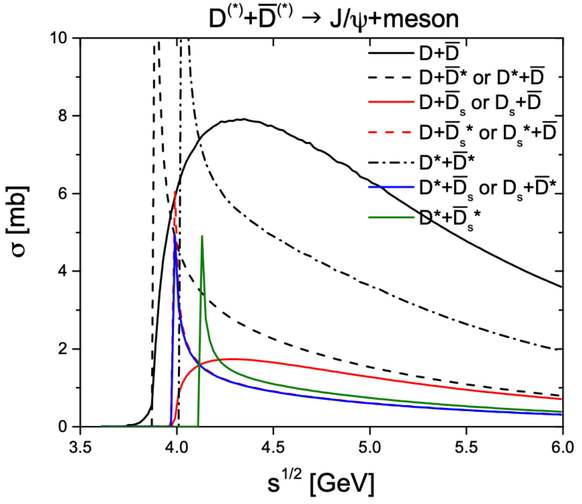
Hadronic scattering effect: suppression and regeneration

- Suppression: Quarkonium will be melted by scattering with light hadrons, π, ρ, ω, K,…
- Regeneration: The inverse processes.

Z. Lin and C.M.Ko. Phys.Rev.C 62 (2000) 034903.
K. L. Haglin and C. Gale, Phys. Rev. C 63, 065201 (2001). K. L. Haglin, Phys. Rev. C 61, 031902 (2000).
T. Song and S. H. Lee, Phys. Rev. D 72, 034002 (2005).

Cross sections remain poorly constrained, with large theoretical uncertainties. Experimental constraints are urgently needed.





*T. Song, **JZ**, J. Aichelin, E.Bratkovskaya, arXiv: 2605.29479.*

Quarkonium real-time evolution in hot QCD medium

Key features of quarkonium in heavy-ion collisions

Real-time evolution in heavy-ion collisions

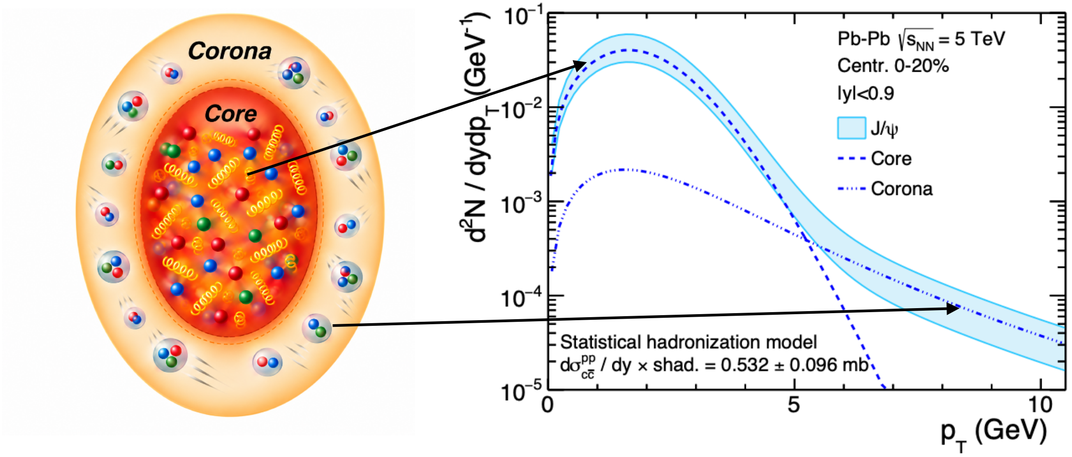
Statistic hadronisation model

Charmonium: fully dissociated in the QGP and exclusively generated at the QCD phase boundary.

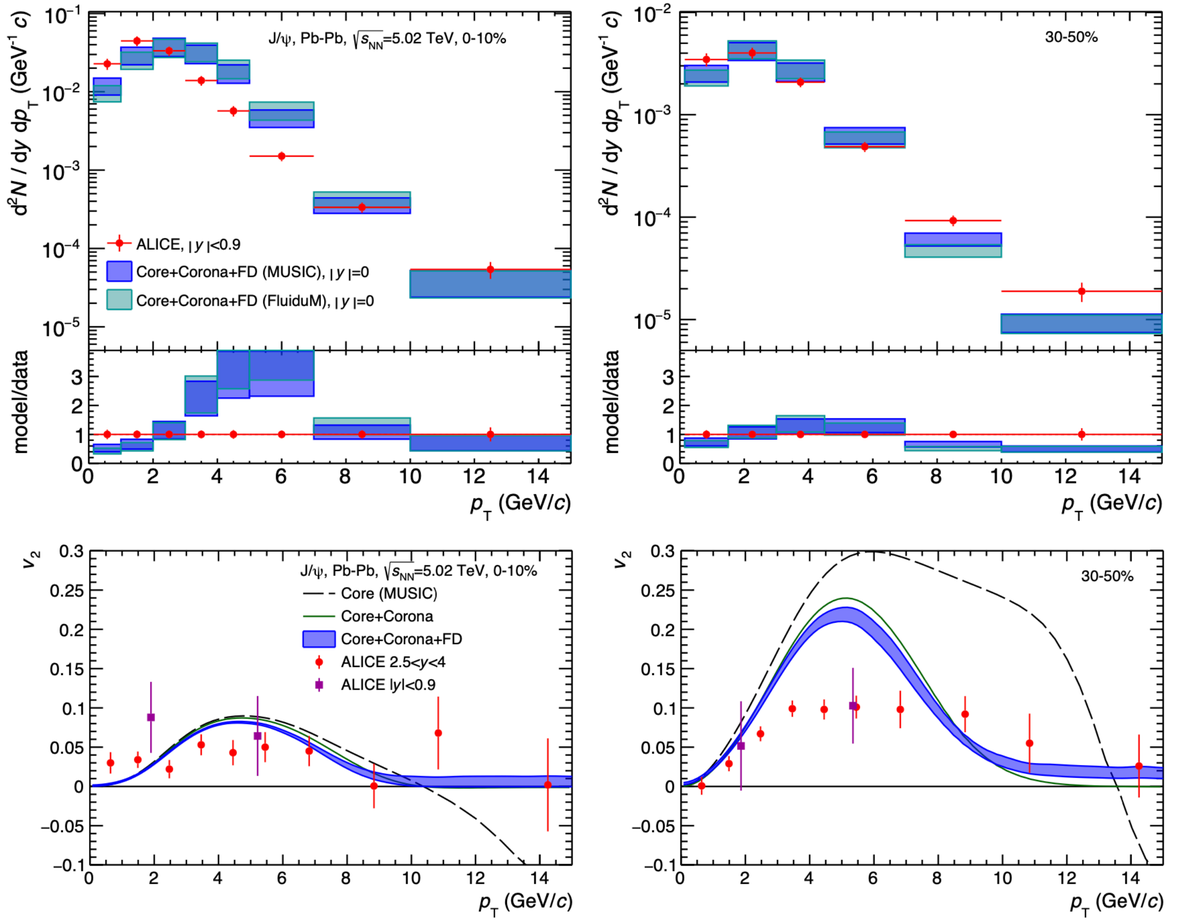
$$N_{c\bar{c}}^{dir} = \frac{1}{2} g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th}.$$

g_c : fugacity parameter

Recent: transverse momentum spectra are studied by a core-corona approach; core: Hydro+blast-wave/Cooper-Frye; corona: fit the spectra in pp



A. Andronic, P. Braun-Munzinger, H. Brunssen, J. Crkovska, J.Stachel, V. Vislavicius, and M. Volkl, JHEP 10 (2024) 229; Phys.Lett.B 797 (2019) 134836; JHEP 07 (2021) 035



p_T distribution, anisotropic flow coefficients v_2 and v_3 for charmonia are investigated.

Classical transport model

Charmonium are not fully dissociated. Dissociation and regeneration happen gradually in QGP.

➡ Transport description (Boltzmann equation -> Tsinghua model)

*P. Zhuang, X. Zhu, L.Yan, Y. Liu, K, Zhou, B. Chen, **JZ***



$$p^\mu \partial_\mu f_\psi = -\alpha E f_\psi + \beta E$$

$$\alpha = \frac{1}{2E_T}\int \frac{d^3\mathbf{p}_g}{(2\pi)^3 2E_g} W^{c\bar{c}}_{g\psi}(s) f_g(p_g, x)$$ Gluon-dissociation

$$\beta = \frac{1}{2E_T}\int \frac{d^3\mathbf{p}_g}{(2\pi)^3 2E_g}\frac{d^3\mathbf{p}_c}{(2\pi)^3 2E_c}\frac{d^3\mathbf{p}_{\bar{c}}}{(2\pi)^3 2E_{\bar{c}}} W^{g\psi}_{c\bar{c}}(s) f_c(p_c, x) f_{\bar{c}}(p_{\bar{c}}, x)(2\pi)^4\delta^{(4)}(p + p_g - p_c - p_{\bar{c}})$$ Regeneration

Dissociation and regeneration are related to each other via the detailed balance.

Analytic solution:

$$f(\mathbf{p}_T, y, \mathbf{x}_T, \eta, \tau) = f(\mathbf{p}_T, y, \mathbf{X}(\tau_0), H(\tau_0), \tau_0) e^{-\int_{\tau_0}^{\tau} d\tau' \alpha(\mathbf{p}_T, y, \mathbf{X}(\tau'), H(\tau'), \tau')/\Delta(\tau')} + \int_{\tau_0}^{\tau} d\tau' \beta(\mathbf{p}_T, y, \mathbf{X}(\tau'), H(\tau'), \tau')/\Delta(\tau') e^{-\int_{\tau'}^{\tau} d\tau'' \alpha(\mathbf{p}_T, y, \mathbf{X}(\tau''), H(\tau''), \tau'')/\Delta(\tau'')}$$

Cooper-Frye on hadronization hypersurface:

$$E\frac{dN}{d^3p} = \frac{g}{(2\pi)^3}\int d\Sigma_\mu(x) p^\mu f(\mathbf{p}_T, y, \mathbf{x}_T, \eta, \tau).$$

Recent developments: Implementation of 3+1D viscous hydrodynamics (MUSIC);
Extension to lower collision energies relevant for RHIC BES-II;
Extension to B_c mesons production with Langevin+Boltzmann eq.…

JZ, P. Zhuang, Phys.Rev.C 105 (2022) 6, 064907 ; Eur.Phys.J.A 61 (2025) 3, 41; JZ, B. Chen, P. Zhuang, Phys.Rev.C 105 (2022) 3, 034902.

Classical transport model

Charmonium are not fully dissociated. Dissociation and regeneration happen gradually in QGP.

➡ Transport description (Rate equation -> TAMU model)

R. Rapp, L. Grandchamp, X. Zhao, X. Du, M. He, B. Wu

$$\frac{dN_\psi(\tau)}{d\tau} = -\Gamma_\psi[N_\psi(\tau) - N_\psi^{\rm eq}(\tau)]$$

Dissociation rate

equilibrium limit of each state (Satisfied obviously.)

Include both gluon-dissociation and NLO (quasifree) process

$$N_\psi^{\rm eq} = g_c^2 N^{\rm eq}$$

Analytic solution:

$$N_\Psi(\tau) = N_\Psi(\tau_0)\exp\left(-\int_{\tau_0}^{\tau}\Gamma_\Psi(\tau')\,d\tau'\right) + \int_{\tau_{\rm reg}}^{\tau} d\tau'\,\Gamma_\Psi(\tau')N_\Psi^{\rm eq}(\tau')\exp\left(-\int_{\tau'}^{\tau}\Gamma_\Psi(\tau'')\,d\tau''\right).$$

Initial-suppression

Regeneration

Recent developments: Implementation of 3+1D viscous hydrodynamics (aHydro);
Reaction rates from thermodynamic T-matrix amplitudes, constrained by lattice-QCD WLCs;
Langevin for heavy quark + Boltzmann eq. for quarkonium to describe charmonium/bottomonium;
Extension to B_c , X(3872),… production,…

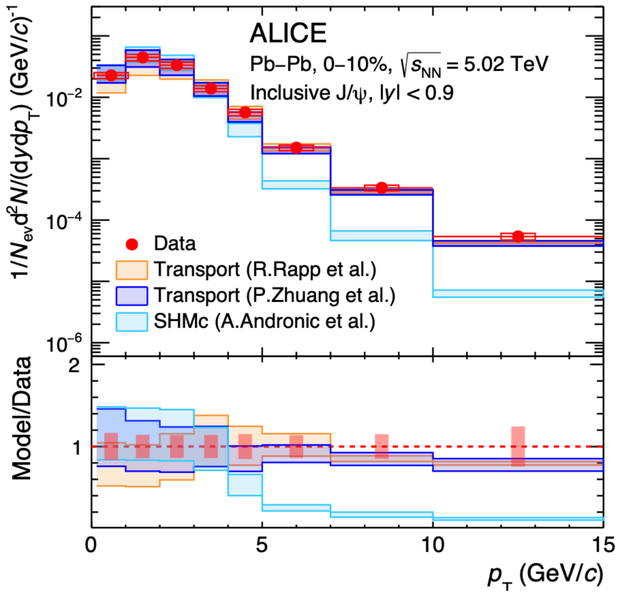
B. Wu, J. Boyd, R. Rapp, arXiv: 2608.21344.

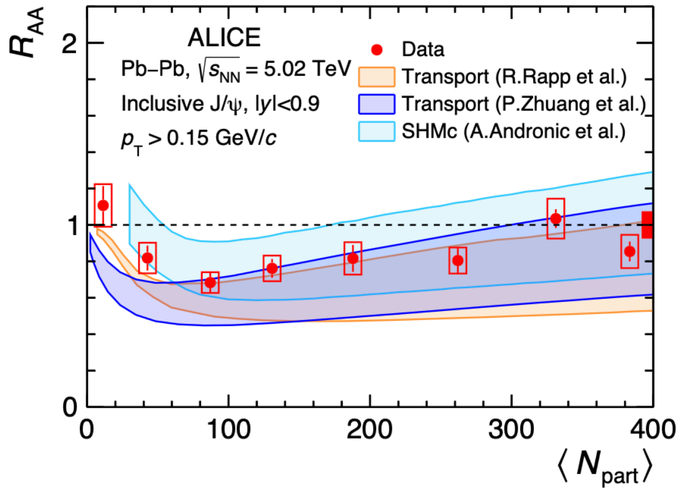
K. Fu, B. Wu and R. Rapp, Eur. Phys. J. A 62 (2026) 132. M. He, R. Rapp, Phys.Rev.Lett. 128 (2022) 16, 162301.

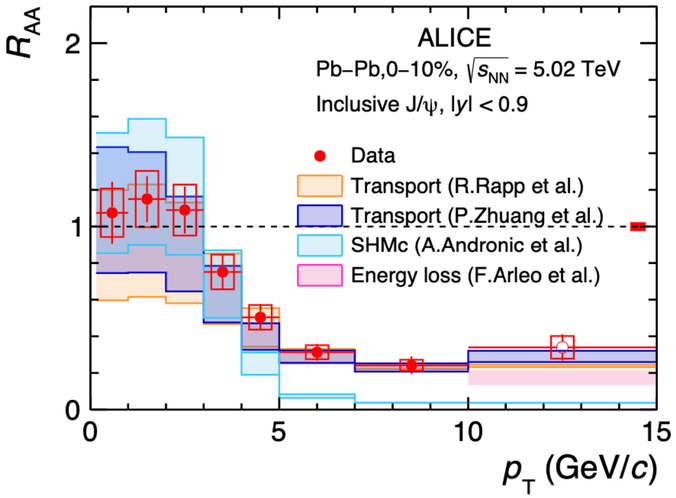
B. Wu, Z. Tang, M. He, R. Rapp, Phys. Rev. C 109(1), 014906 (2024).

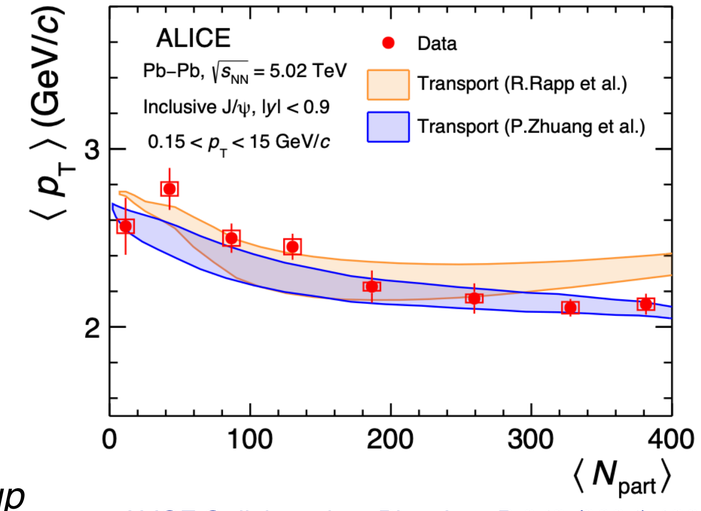
Classical transport model

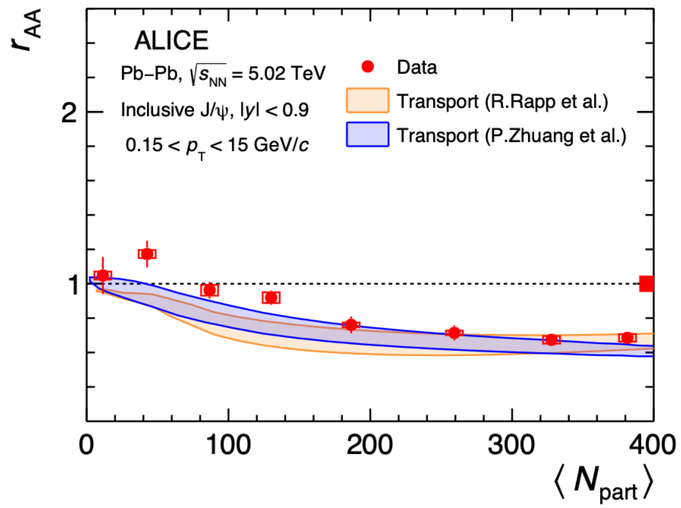
Models (such as SHMc, TAMU, Tsinghua) can explain the experimental observables, like charmonium, spectra, R_{AA}, v_2, r_{AA}, $\langle p_T \rangle$ …**compared to the recent ALICE results.**











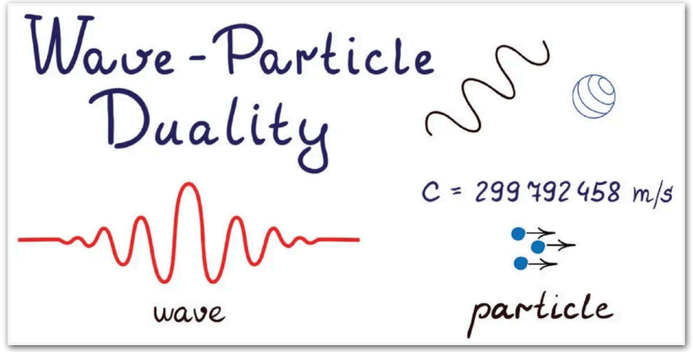


Thanks to ALICE and ALICE-China group

ALICE Collaboration, Phys.Lett.B 849 (2024) 138451

Classical vs. Quantum

Good, but is it enough?



➡ Is quarkonium a wave or a particle in heavy ion collisions ?

- Quantum coherence and decoherence

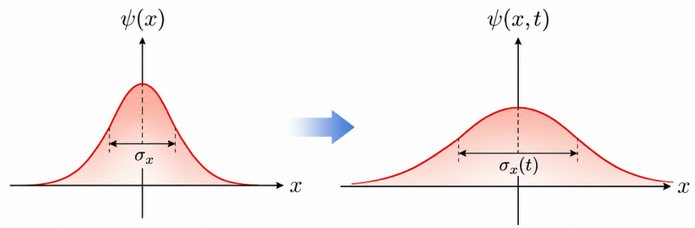
  Superposition state of various eigen states,…

  Usually absorbed into a phenomenological parameter “formation time” in the transport approach.

$$\Psi = \sum_i c_i \psi_{nl}$$

- Define and evolve a particle with a large width in the hot medium

$$\Gamma \gtrsim E$$



➡ Why the classical transport equation can be used to describe quarkonium evolution and production?

- QCD/pNRQCD –> Open quantum system (OQS) –> Boltzmann equation

  The goal is not to abandon transport, but to derive the transport description more systematically from quantum dynamics.

Open quantum system

"pure" state (**wavefunction**) -> "mixed" state (**density operator**)

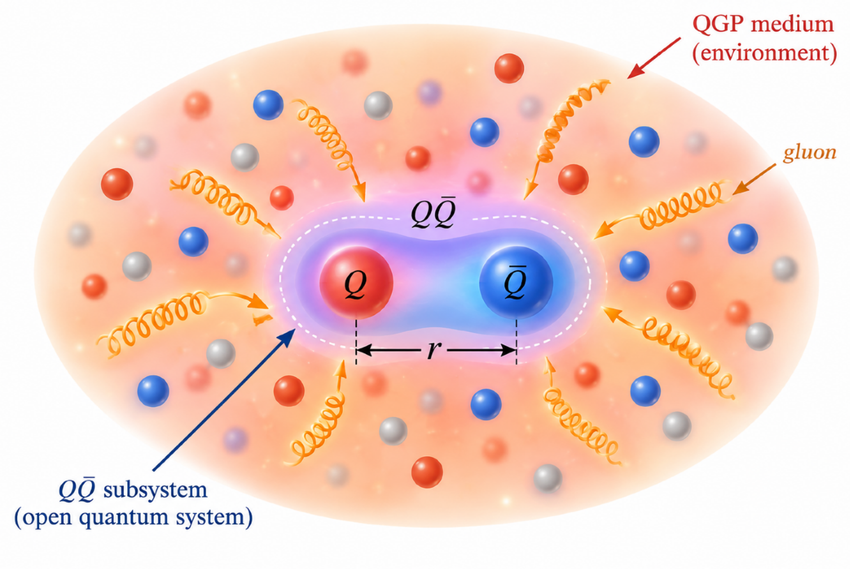
Open quantum system (OQS)

$$\hat{\rho}_{tot} = \sum_i p_i |\psi_i\rangle\langle\psi_i| \quad \text{von Neumann equation:} \quad \frac{d\hat{\rho}_{tot}}{dt} = -i[\hat{H}_{tot}, \hat{\rho}_{tot}]$$

$$\hat{H}_{tot} = \hat{H}_s \otimes I_e + I_s \otimes \hat{H}_e + \hat{H}_{int},$$

Subsystem **Environment** **Interaction**

Trace over the environment degrees of freedom :



Quantum master equation

$$i\hbar\dot{\hat{\rho}}_s(t) = \mathrm{Tr}_e[\hat{H}_{tot}, \hat{\rho}_{tot}] = [\hat{H}_s, \hat{\rho}_s] + \mathrm{Tr}_e[I_s \otimes \hat{H}_e + \hat{H}_{int}, \hat{\rho}_{tot}]$$

➡ Separation of time-scales:

Environment relaxation time scale $\tau_e \sim \frac{1}{\pi T}$.

Intrinsic time scale of subsystem $\tau_s \sim \frac{1}{E_{bind}}$.

Subsystem relaxation time scale $\tau_r \sim \frac{1}{\eta} \approx \frac{M}{T^2}$.

Open quantum system

$\tau_e \sim 1/(\pi T)$, $\tau_s \sim 1/E_{bind}$, $\tau_r \sim M/T^2$

Non-Markovian

Markovian approximation: $\tau_e \ll \tau_r$, memory loss; good for HIC

Quantum master equation –> Lindblad equation



Quantum Brownian Motion

A broader peak (e.g. excited states)

Short quantum decoherence time

Classical limit:

Fokker-Planck equation

Munich-Kent

Nantes-Paris-Saclay

Osaka

…

Quantum optical Limit

Well defined quarkonium state

Long quantum decoherence time

Classical limit:

Boltzmann equation/ Rate equation

TAMU

Tsinghua

Comover

Duke-MIT

…

Recent review papers: *R. Sharma, Eur.Phys.J.ST 230 (2021) 3, 697-718* *Y. Akamatsu, Prog.Part.Nucl.Phys.* 123 (2022) 103932. *X. Yao, Int.J.Mod.Phys.A 36 (2021) 20, 2130010.*

Progress sketch

Duke/MIT Approach

- pNRQCD+OQS works in **quantum optical limit** $M \gg Mv \gg Mv^2 \gtrsim T \gtrsim m_D$
- A semi-classical (gradient) expansion and w/o quantum effect anymore
- Used for bottomonium.

X. Yao, T. Mehen,W. Ke, Y. Xu, S.Bass, B. Muller. Phys.Rev.D 99 (2019) 9, 096028; JHEP 01 (2021) 046.

Gives a connection between the OQS and Boltzmann equation in the quantum optical limit!

Munich-KSU Approach

- pNRQCD+OQS works in **Quantum Brownian motion** Regime $M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E_{bind}$
- Expansion of E_{bind}/T from LO to NLO; the quantum jumps are now implemented.
- Used for bottomonium.

N.Brambilla, M.Escobedo, M.Strickland, A.Vairo, J.Weber, Phys.Rev.D 104 (2021) 9, 094049; JHEP 05 (2021) 136;JHEP 08 (2022) 303; Phys.Rev.D 108 (2023) 1, L011502.

The new results with quantum jumps and w/o color screening agree well with the R_{AA} and double ratios!

Nantes-Paris-Saclay Approach

- NRQCD+OQS works in **Quantum Brownian motion** Regime $M \gg T \sim m_D \gtrsim E_{bind}$
- Expansion of τ_e/τ_s.
- Used for bottomonium and charmonium in 1D.

J. Blaizot, M. Escobedo, JHEP 06, 034 (2018).
S. Delorme, T. Gousset, R. Katz, P.B. Gossiaux, Eur. Phys. J. A 58(10), 198 (2022); JHEP 06 (2024) 060

Beyond the dipole approximation; Equilibrium is checked.

Osaka Approach

- NRQCD+OQS works in **Quantum Brownian motion** Regime $M \gg T \sim m_D \gtrsim E_{bind}$
- Weak coupling (strict) and go beyond the weak coupling (approximation)
- Used for bottomonium.

T. Miura,Y. Akamatsu, M. Asakawa, et al, PRD 87 (2013) 045016; PRD 91 (2015) 5, 056002.; PRD97 (2018), 014003.; Phys.Rev.D 106 (2022) 7, 074001.

Beyond the dipole approximation; The dipole approximation is good, but depends on the initial condition!

Quarkonium real-time evolution in heavy-ion collisions

$\tau_e \sim 1/(\pi T)$, $\tau_s \sim 1/E_{bind}$, $\tau_r \sim M/T^2$

Non-Markovian

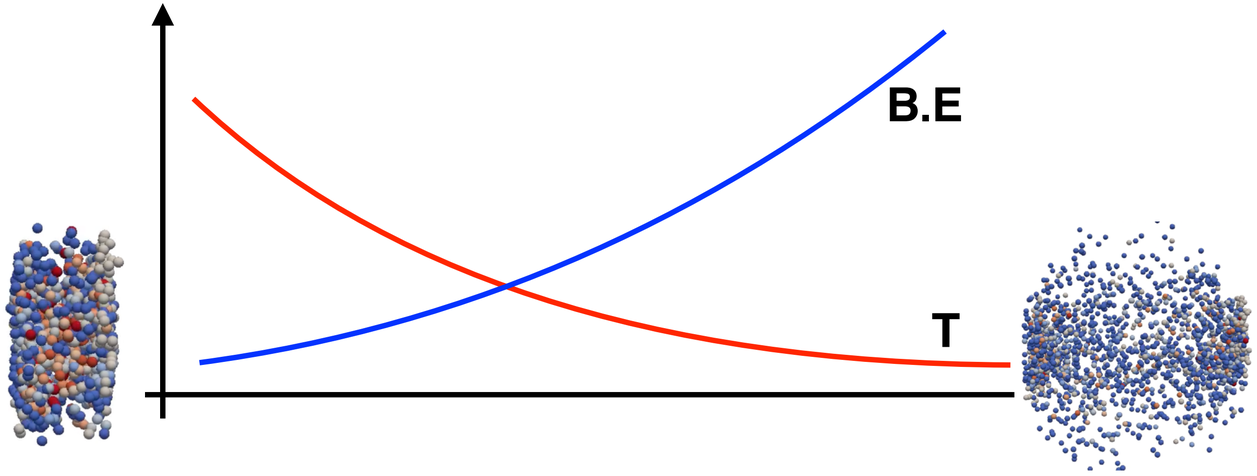
Markovian approximation: $\tau_e \ll \tau_r$, memory loss

Quantum master equation –> Lindblad equation



Quantum Brownian Motion

Quantum optical Limit



Further needs:

1. Connect the Quantum Brownian Motion at high temperture to the Quantum optical regime at low temperature.
2. One pair –> many pairs and regeneration from uncorrelated heavy quarks.

Universal Lindblad Equation proposed by Nathan and Rudne. *A. Hammou, et al, arXiv: 2608.15347.*

Frankfurt-Nantes approach

➡ Start from the OQS and works from the QBM to QOL

➡ N pairs of $Q\bar{Q}$. Used for bottomonium and charmonium.

*D. Villar, **JZ**, J. Aichelin, and P. Gossiaux, Phys.Rev.C 107 (2023) 5, 054913.*

*T. Song, J. Aichelin, **JZ**, P. Gossiaux and E. Bratkovskaya,PRC108 (2023) 5, 054908.*

*T. Song, **JZ**, J. Aichelin, and E. Bratkovskaya, arXiv:2605.29479.*

Open quantum system (OQS)

$$\hat{H}_{tot} = \hat{H}_s \otimes I_e + I_s \otimes \hat{H}_e + \hat{H}_{int},$$

Subsystem **Environment** **Interaction**

Probability that at time t the state Φ is produced:

$$P^{\Phi}(t) = \mathrm{Tr}[\rho^{\Phi}\hat{\rho}_{tot}(t)] \qquad \rho^{\Phi} = |\Phi><\Phi|$$

Rate of decay and formation of state Φ is:

$$\Gamma^{\Phi} = \frac{dP^{\Phi}}{dt} = \frac{d}{dt}\mathrm{Tr}[\rho^{\Phi}\rho_{tot}] \approx \frac{d}{dt}\prod\frac{d^3r_i d^3p_i}{(2\pi)^{3N}} W^{\Phi}(r,p) W_N(r_1,p_1,\ldots,r_N,p_N)$$

$$\Gamma_{\mathrm{coll}}(t) = \sum_k^2 \sum_{i\geq 3}^N \sum_n \delta(t - t_{ki}(n)) \int \prod_{j=1}^N d^3\mathbf{r}_j d^3\mathbf{p}_j W^{\Phi}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{p}_1,\mathbf{p}_2)\Big[W_N(\{\mathbf{r}_j\},\{\mathbf{p}_j\},t+\epsilon) - W_N(\{\mathbf{r}_j\},\{\mathbf{p}_j\},t-\epsilon)\Big]$$ **Collisional rate**

$$\Gamma_{\mathrm{local}}(t) = \int \prod_{j=1}^N d^3\mathbf{r}_j d^3\mathbf{p}_j \dot{W}^{\Phi}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{p}_1,\mathbf{p}_2,T(t)) W_N(\{\mathbf{r}_j\},\{\mathbf{p}_j\},t)$$ **Local rate**

Wigner density of quarkonium states is temperature or time dependent

Composite Particle Cross Sections from the Density Operator

E. A. Remler*,†

Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

Received April 22, 1981

Annals Phys. 136 (1981) 293

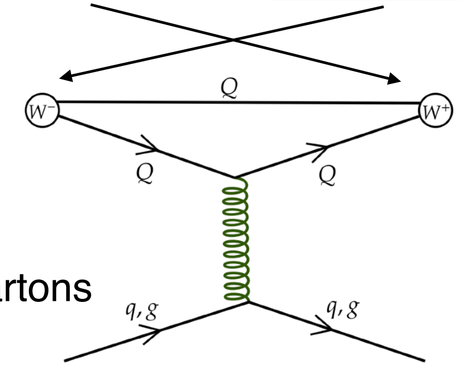
The general formula connecting composite particle cross sections to solutions of the time-dependent density is obtained. An overcounting problem is pointed out and solved. Formulae for composite particles, as approximated by both mean field and cascade theories, are derived.

Frankfurt-Nantes approach

N-body Wigner density is approximated as a classical phase space distribution

$$W_N \approx W_N^{\mathrm{C(classical)}} = \prod_{i=1}^{N} \delta(\mathbf{r}_i - \mathbf{r}_i^*(t))\delta(\mathbf{p}_i - \mathbf{p}_i^*(t))$$

$$\Gamma_{\mathrm{coll}} = \sum_{k}^{2} \sum_{i\geq 3}^{N} \sum_{n} \delta(t - t_{ki}(n)) \left[\omega_{k,3-k}(n, t+\epsilon) - \omega_{k,3-k}(n, t-\epsilon) \right]$$



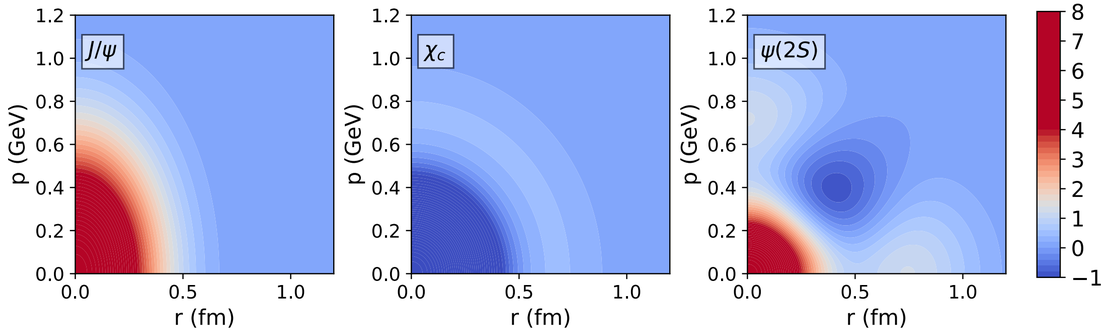
QGP thermal partons

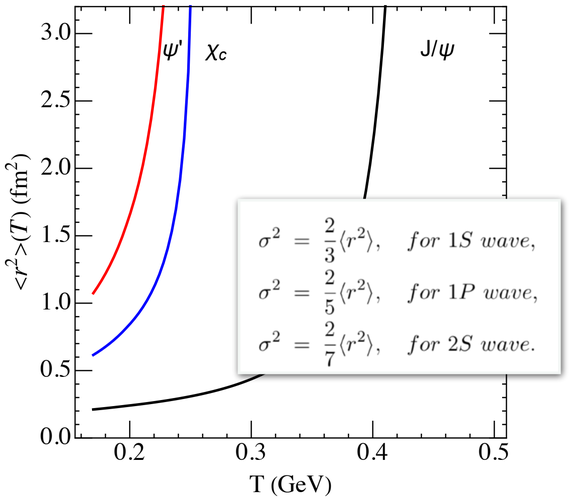
$$\omega_{1,2}(n,t) = \int \prod_{j=1}^{N} d^3\mathbf{r}_j d^3\mathbf{p}_j W^{\Phi}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2) W_N^C(\{\mathbf{r}_j\}, \{\mathbf{p}_j\}, t) = W^{\Phi}(\mathbf{r}_1(t), \mathbf{p}_1(t), \mathbf{r}_2(t), \mathbf{p}_2(t))$$

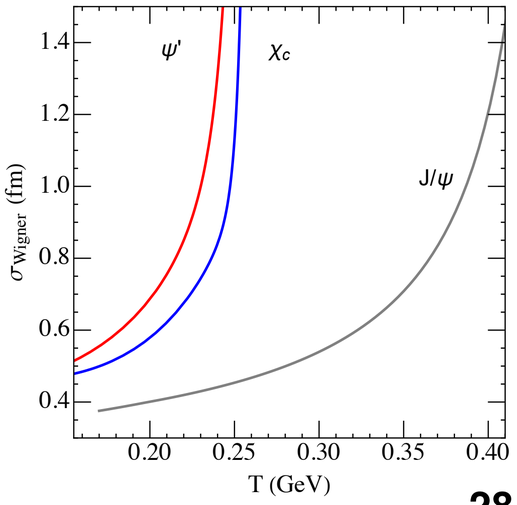
$$\Gamma_{\mathrm{local}} = \dot{\sigma}(T(t))\partial_\sigma W^{\Phi}(\mathbf{r}_1(t), \mathbf{p}_1(t), \mathbf{r}_2(t), \mathbf{p}_2(t), \sigma(T))$$

$$W^{1S}(r,p) = 8e^{-\frac{r^2}{\sigma^2} - \frac{p^2\sigma^2}{\hbar^2}}$$

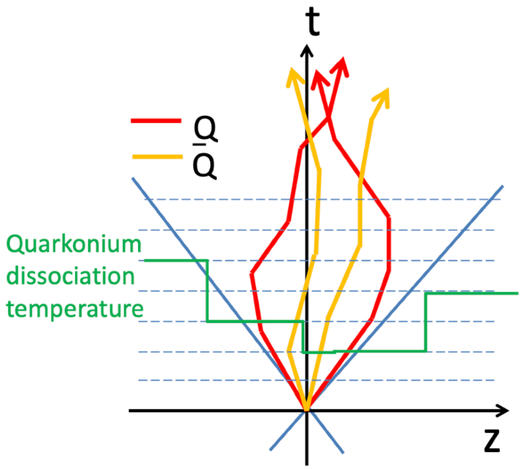
$$\Gamma_{\mathrm{local}}^{1S} = 16\dot{\sigma}(T(t)) \left(\frac{r^2}{\sigma^3} - \frac{\sigma p^2}{\hbar^2} \right) e^{-\frac{r^2}{\sigma^2} - \frac{p^2\sigma^2}{\hbar^2}}$$







Frankfurt-Nantes approach



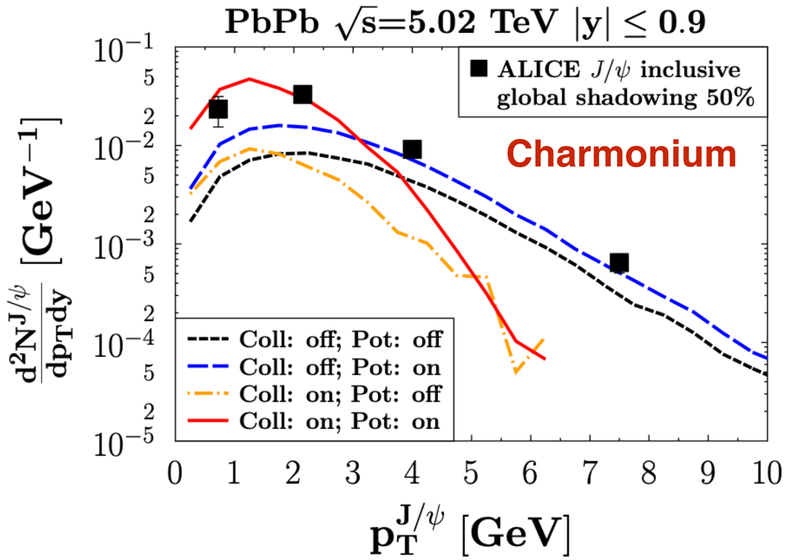
$$P^{\Phi}(t) = P^{\Phi}(t_{\rm ini}) + \int_{t_{\rm ini}}^{t} (\Gamma_{\rm local}(t') + \Gamma_{\rm coll}(t'))dt'$$

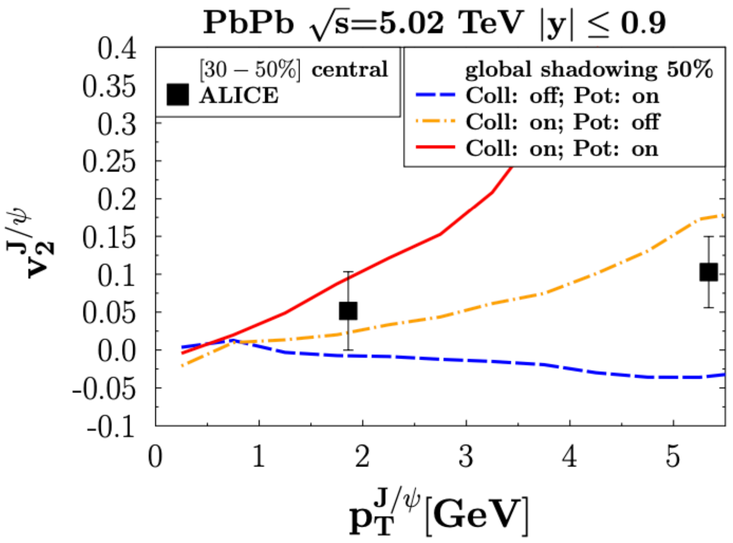
$$\frac{dN_{\Phi}}{dy} \approx P_{\Phi} \times \frac{d\tilde{N}_{Q}}{dy}\frac{d\tilde{N}_{\bar{Q}}}{dy}$$

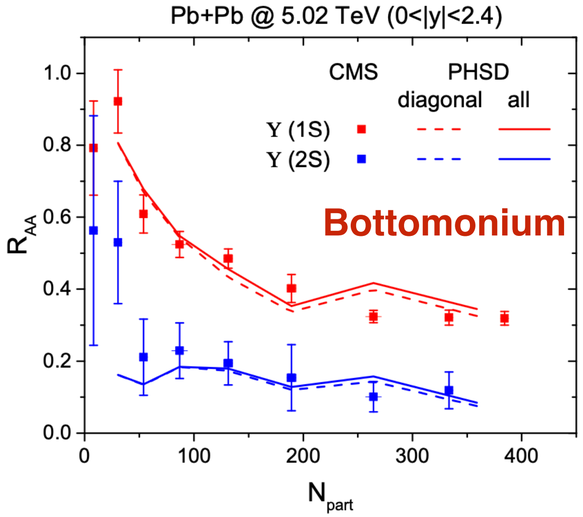
Initial probability for the production of a quarkonium once the local $Q\bar{Q}$ temperature falls below $T_{\rm diss}$

The interaction between heavy quarks in the evolution is also considered via a non-relativistic way.

At each time step, calculate the local and collisional rate.



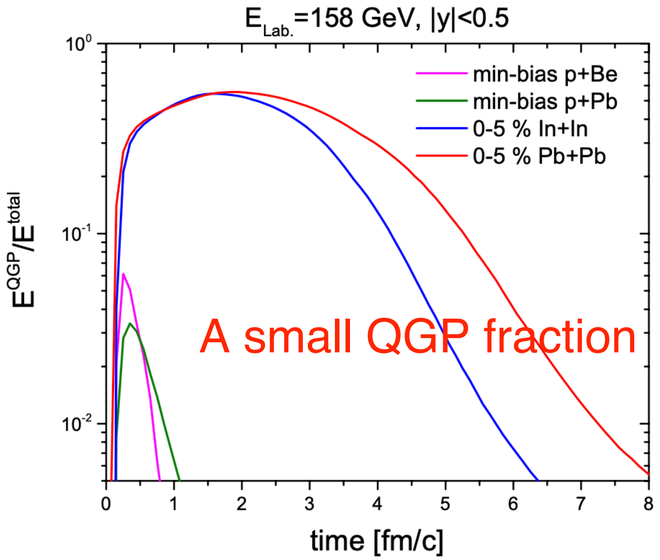


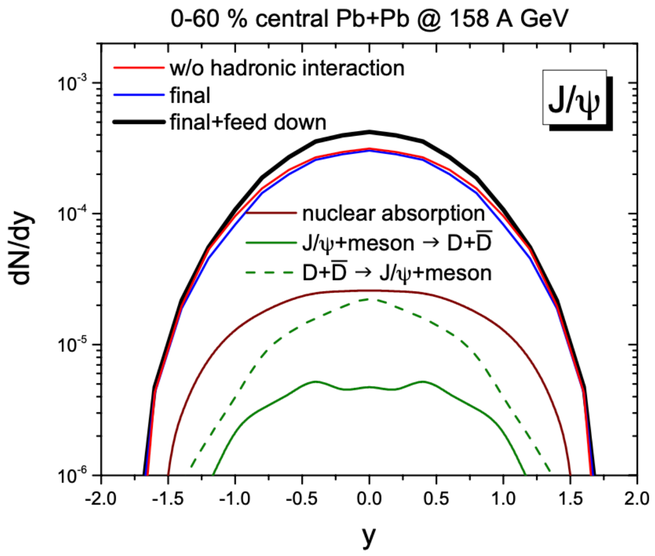


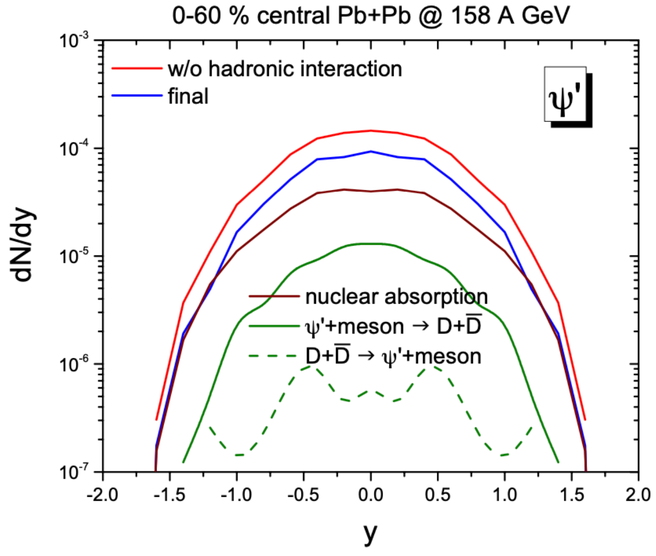
Frankfurt-Nantes approach

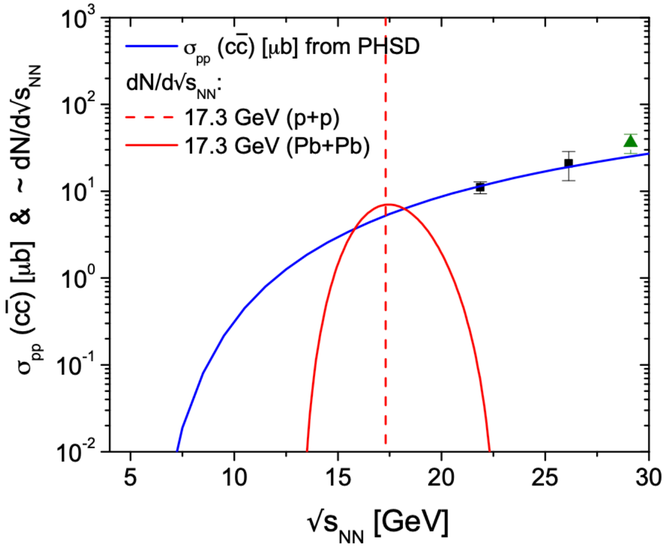
Dynamical charmonium formation in pp, pA, and AA collisions at SPS and FAIR energies in PHSD.

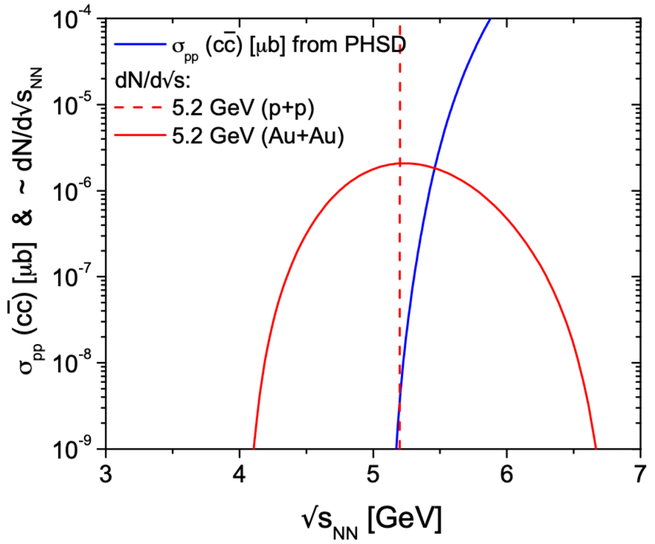
*T. Song, **JZ**, J. Aichelin, and E. Bratkovskaya, arXiv:2605.29479.*







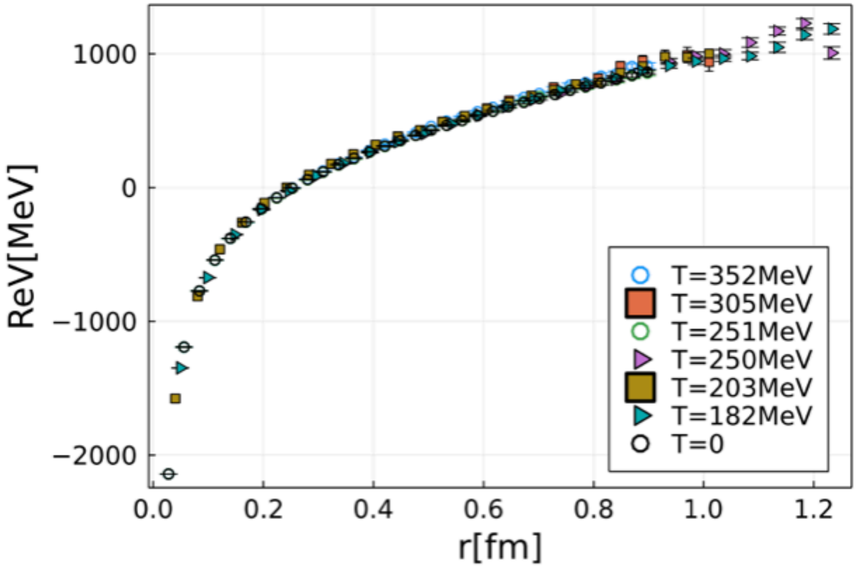


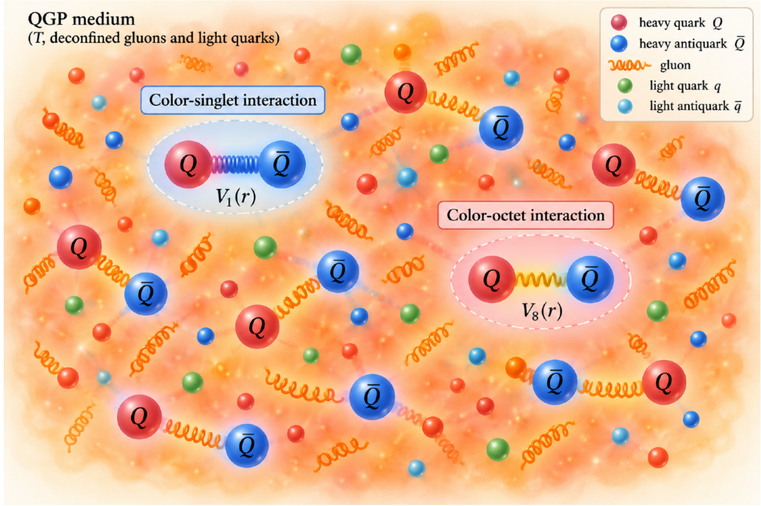


At SPS energies, nuclear absorption dominates suppression; $D\bar{D}$ regeneration is important for J/ψ, but negligible for ψ'.

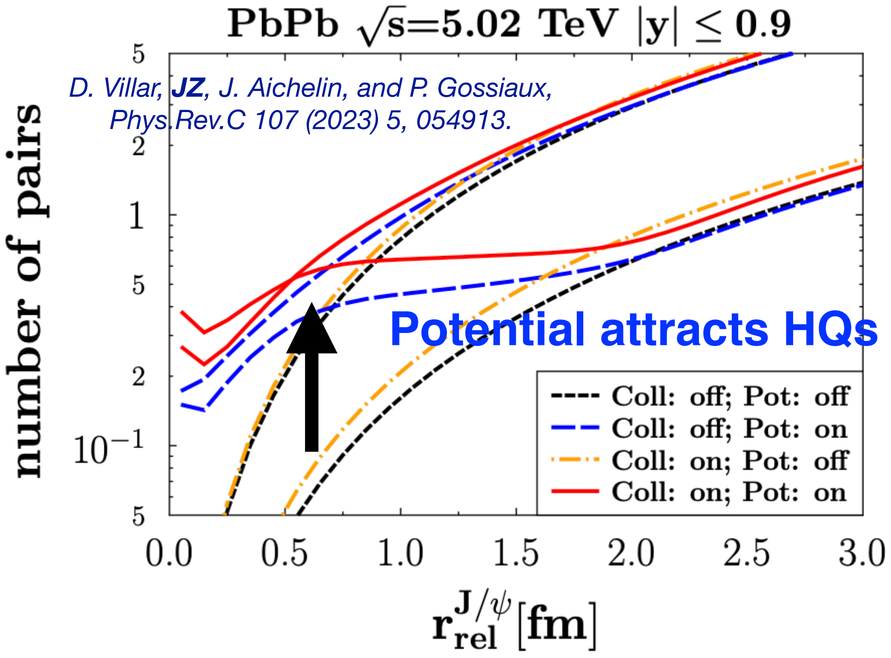
At FAIR energies, charm is produced extremely close to threshold → nuclear dynamics becomes crucial.

Do heavy quark interactions affect their trajectories?





At LHC energies, long QGP evolution + abundant charm production → strong impact on charm thermalization, J/ψ regeneration,…



*D. Villar, **JZ**, J. Aichelin, and P. Gossiaux, Phys.Rev.C 107 (2023) 5, 054913.*

Formulation of fully covariant Quantum-Molecular Dynamics for an N-body system with scalar and vector potentials

Jiaxing Zhao,[a,b] Joerg Aichelin,[c,d] Elena Bratkovskaya[e,a,b]

[a] *Helmholtz Research Academy Hessen for FAIR (HFHF),GSI Helmholtz Center for Heavy Ion Research. Campus Frankfurt, 60438 Frankfurt, Germany*

Will be implemented in EPOS4 soon!

Summary

- Quarkonia provide an **ideal probe of hot and dense QCD matter**. At the same time, their in-medium properties offer a unique opportunity to **test our theoretical understanding of thermal QCD**.

  “ Thermal real-time QCD & thermal imaginary-time QCD ”

- The in-medium properties of quarkonia can largely be encoded in a finite-temperature heavy-quark potential, which contains both real and imaginary parts. Recent studies suggest **weak or even negligible color screening in the real part, accompanied by a sizable imaginary part.**

- With the assumption of a classical particle of quarkonium, **the transport model as well as the statistical model (with corona effect) can describe quite well the experimental data**, which help us to understand the HQ in-medium potential, HQ energy loss, QGP properties,… Also extended to B_c and $X(3872)$.

- Aiming to include the quantum effects and to build a genuine first principles based real time evolution framework, **OQS is used and developed in different ways based on heavy quark effective theories. Much progress has been made.**

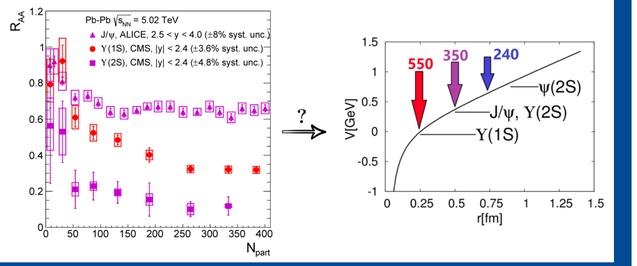
Latest model comparison paper

ExtreMe Matter Institute EMMI

EMMI Rapid Reaction Task Force

Suppression and (re)generation of quarkonium in heavy-ion collisions at the LHC

GSI, Darmstadt, Germany, December 16-20, 2019



This Rapid Reaction Task Force will scrutinize the theoretical and phenomenological description of quarkonium production in high-energy heavy-ion collisions, aimed at distinguishing between current approaches, narrow down uncertainties in the input parameters and set the path for future developments of reliable dynamical models of quarkonium transport. The overarching goal is to establish firm connections between heavy-quarkonium phenomenology in heavy-ion collisions and the microscopic properties of hot QCD matter.

OPEN SYMPOSIUM, December 16, 09:30, KBW Lecture Hall

Symposium Talks:
Quarkonium production at the LHC - experimental overview
Roberta Arnaldi (INFN Torino)

Theory of quarkonia in QCD matter
Jean-Paul Blaizot (CEA Saclay)

Phenomenological modelling of quarkonium in QGP, scope of RRTF
Michael Strickland (Kent State Univ.)

Organizers:
Anton Andronic (Münster Univ.)
Pol-Bernard Gossiaux (Subatech, IMT Atlantique)
Peter Petreczky (BNL, Upton, NY)
Ralf Rapp (Texas A&M Univ.)
Michael Strickland (Kent State Univ.)

Information:
www.gsi.de/emmi/rrtf

More about EMMI:
www.gsi.de/emmi

GSI HELMHOLTZ RESEARCH FOR GRAND CHALLENGES EMMI

Eur. Phys. J. A (2024) 60:88
https://doi.org/10.1140/epja/s10050-024-01306-6

THE EUROPEAN PHYSICAL JOURNAL A

Review

Comparative study of quarkonium transport in hot QCD matter

A. Andronic[1,a], P. B. Gossiaux[2,b], P. Petreczky[3,c], R. Rapp[4,d], M. Strickland[5,e], J. P. Blaizot[6], N. Brambilla[7], P. Braun-Munzinger[8,9], B. Chen[10], S. Delorme[11], X. Du[12], M. A. Escobedo[13,12], E. G. Ferreiro[12], A. Jaiswal[14], A. Rothkopf[15], T. Song[8], J. Stachel[9], P. Vander Griend[16], R. Vogt[17], B. Wu[4], J. Zhao[2], X. Yao[18]



Exploration of Quark Matter and the QCD Phase Boundary with Nuclear Collisions at the LHC

2020-08-26　点击：34



报告题目：Exploration of Quark Matter and the QCD Phase Boundary with Nuclear Collisions at the LHC

报 告 人：Peter Braun-Munzinger, GSI Helmholtz Zentrum fuer Schwerionenforschung (GSI)

报告时间：2015年9月24日16：00

报告地点：理科楼郑裕彤大讲堂

报告摘要：Quark Matter is a new state of matter which existed in the early universe until about 10 microseconds after the big bang. In this state the hadrons are all dissolved and the quarks and gluons are deconfined. We review how this state of matter is formed over a large space-time volume in a fireball in Pb-Pb collisions at the Large Hadron Collider LHC. The initially very hot and dense fireball expands collectively and its properties can be diagnosed with tomographic and hydrodynamic concepts. Particle production at the phase boundary between quark matter and hadronic matter (baryons and mesons) leads to a thermodynamic state with which can be compared with predictions from Quantum Chromodynamics. Analysis of hadron spectra and multiplicities (including quarkonia and hadrons containing heavy quarks) provides information on the quark matter phase boundary and on the degree of deconfinement reached. Future opportunities with the full energy and high luminosity collisions at the LHC will be discussed.

We share similar research interests in quarkonium, multi-charmed baryons (Ω_{ccc}, Ξ_{cc},), and related topics. I have learned a lot from him. Thank you!



Thanks for your attention !

Quarkonium real-time evolution in heavy-ion collisions

Models (such as TAMU, Tsinghua) can explain the experimental observables, like charmonium, spectra, R_{AA}, v_2, r_{AA}, $\langle p_T \rangle$ ….**new progress**

1. J/ψ v_2 with non-thermal charm distribution.
2. Directed flow v_1 and Triangular flow v_3 of J/ψ.
3. Quarkonium polarization.
4. Probe the initial nuclear deformation.
5. Quarkonium with EM fields.
6. B_c, X(3872)

…

*B. Chen, M. Hu, H. Zhang, and **JZ**, PLB802 (2020) 135271; **JZ**, B. Chen, and P. Zhuang, PRC 105 (2022) 3, 034902*
D.Yang and X.Yao, Phys.Rev.D 110 (2024) 7, 074037; Y. Zhao, X. Sheng, S. Li and D.Hou, JHEP 08 (2024) 070;
***JZ** and B. Chen, Eur.Phys.J.C 84 (2024) 8, 875.*
***JZ** and S. Shi, Eur.Phys.J.C 83 (2023) 6, 511.*

See review: S. Iwasaki, M. Oka and K. Suzuki, Eur.Phys.J.A 57 (2021) 7, 222;
***JZ**, K. Zhou, S. Chen, P. Zhuang, PPNP.* 114 (2020) 103801.

B. Wu, Z. Tang, M. He, R. Rapp, Phys. Rev. C 109(1), 014906 (2024).
*B. Wu, X. Du, M. Sibila, R. Rapp, Eur. Phys. J. A **57**(4), 122 (2021).*
***JZ** and P. Zhuang, Eur.Phys.J.A 61 (2025) 3, 41.*
A.Esposito, E. Ferreiro, A.Pilloni, A. Polosa and C. Salgado, Eur.Phys.J.C 81 (2021) 7, 669.
Y. Guo, X. Guo, J. Liao, E. Wang and H. Xing, Phys.Rev.C 110 (2024) 2, L021901.

Duke-MIT Approach

- **pNRQCD+OQS works in quantum optical limit** $M \gg Mv \gg Mv^2 \gtrsim T \gtrsim m_D$
- **A semi-classical (gradient) expansion and w/o quantum effect anymore**
- **Used for bottomonium.**

X. Yao, T. Mehen,W. Ke, Y. Xu, S.Bass, B. Muller. Phys.Rev.D 99 (2019) 9, 096028; JHEP 01 (2021) 046.

$$\rho_S(t) = \rho_S(0) + \sum_{a,b,c,d} \gamma_{ab,cd}(t)\left(L_{ab}\rho_S(0)L_{cd}^\dagger - \frac{1}{2}\{L_{cd}^\dagger L_{ab}, \rho_S(0)\}\right)$$

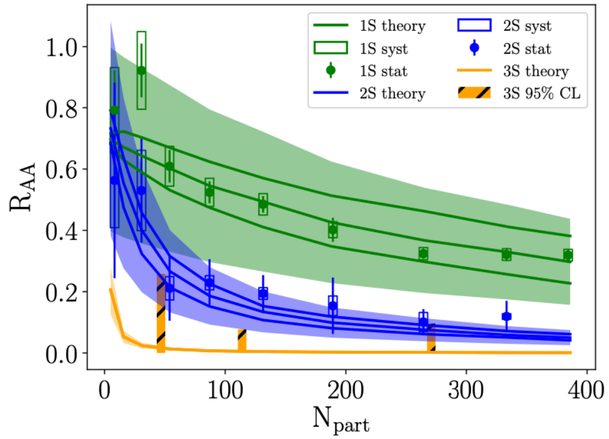
Lindblad equation

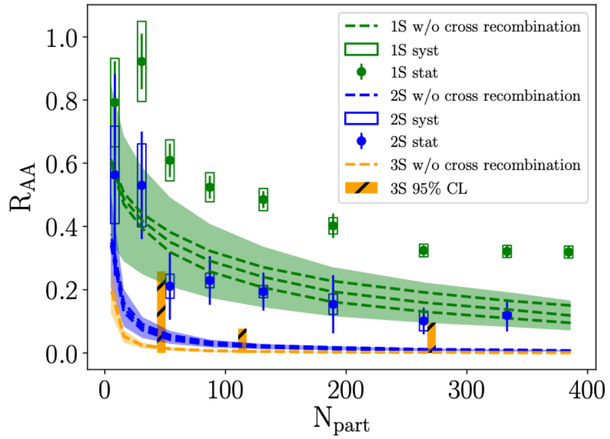
$$-i\sum_{a,b}\sigma_{ab}(t)[L_{ab}, \rho_S(0)] + \mathcal{O}(H_I^3)\,.$$

$$f_{nl}(\boldsymbol{x},\boldsymbol{k},t) \equiv \int \frac{\mathrm{d}^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \langle \boldsymbol{k}+\frac{\boldsymbol{k}'}{2}, nl, 1|\rho_S(t)|\boldsymbol{k}-\frac{\boldsymbol{k}'}{2}, nl, 1\rangle$$

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x},\boldsymbol{k},t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x},\boldsymbol{k},t) = \mathcal{C}_{nl}^{(+)}(\boldsymbol{x},\boldsymbol{k},t) - \mathcal{C}_{nl}^{(-)}(\boldsymbol{x},\boldsymbol{k},t)$$

Similar to the TAMU and Tsinghua model





Importance of recombination from correlated $b\bar{b}$!

Gives a connection between the OQS and Boltzmann equation in the quantum optical limit!

Munich-Kent Approach

- **pNRQCD+OQS works in Quantum Brownian motion Regime** $M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E_{bind}$
- **Expansion of** E_{bind}/T **from LO to NLO; the quantum jumps are now implemented.**
- **Used for bottomonium.**

N.Brambilla, M.Escobedo, M.Strickland, A.Vairo, J.Weber, Phys.Rev.D 104 (2021) 9, 094049; JHEP 05 (2021) 136;JHEP 08 (2022) 303; Phys.Rev.D 108 (2023) 1, L011502.

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] + \sum_n \left(C_n \rho(t) C_n^\dagger - \frac{1}{2}\left\{ C_n^\dagger C_n, \rho(t) \right\} \right)$$

$$\rho(t) = \begin{pmatrix} \rho_s(t) & 0 \\ 0 & \rho_o(t) \end{pmatrix},$$

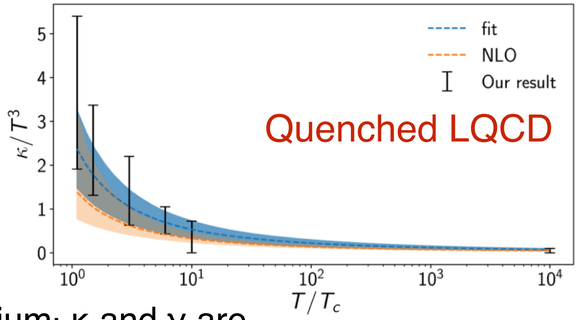
$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix},$$

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}} r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}} r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\kappa = \frac{g^2}{18}\int_0^\infty dt \left\langle \left\{ \tilde{E}^{a,i}(t,\mathbf{0}), \tilde{E}^{a,i}(0,\mathbf{0}) \right\} \right\rangle,$$

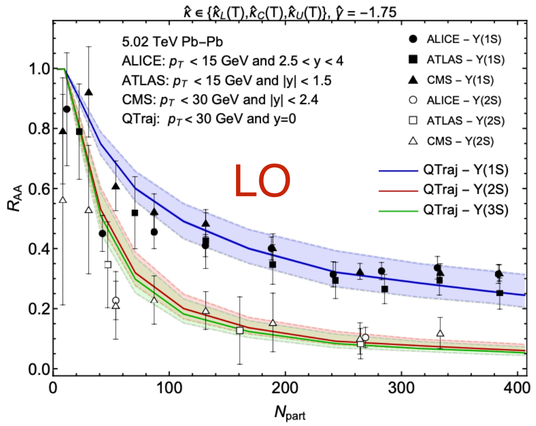
$$\gamma = -i\frac{g^2}{18}\int_0^\infty dt \left\langle \left[\tilde{E}^{a,i}(t,\mathbf{0}), \tilde{E}^{a,i}(0,\mathbf{0}) \right] \right\rangle$$



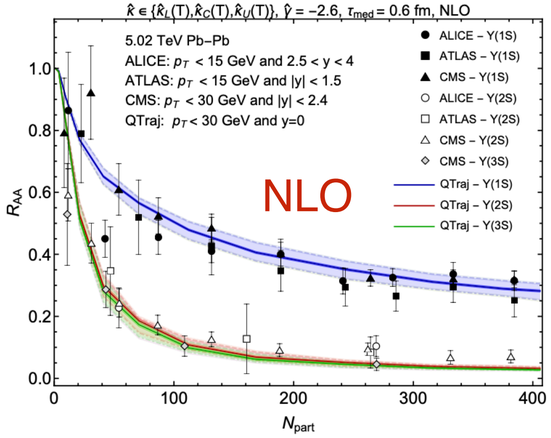
Strong coupling between heavy quark and medium; κ and γ are related to the thermal width Γ and mass shift δM of the bottomonium.

Quantum jump

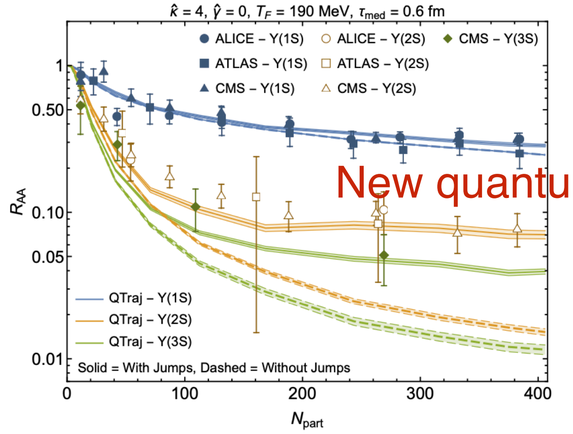
T_f = 250MeV



T_f = 190MeV



T_f = 190MeV, $\hat{\kappa} = 4, \hat{\gamma} = 0$ -> w/o screening



The new results with quantum jumps and w/o color screening agree well with the R_{AA} and double ratios!

Nantes-Paris-Saclay Approach

- **NRQCD+OQS works in Quantum Brownian motion Regime** $M \gg T \sim m_D \gtrsim E_{bind}$
- **Expansion of τ_e/τ_s.**
- **Used for bottomonium and charmonium in 1D.**

J. Blaizot, M. Escobedo, JHEP 06, 034 (2018).
S. Delorme, T. Gousset, R. Katz, P.B. Gossiaux, Acta Phys. Pol. B Proc. Suppl. 16, 1–112 (2023) ; Eur. Phys. J. A 58(10), 198 (2022); JHEP 06 (2024) 060.
A. Hammou, J. Blaizot, P.B. Gossiaux, T. Gousset, arXiv: 2608.15347.

$$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q, \qquad \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$$

Real part potential

$$\mathcal{L}_0\mathcal{D} = -i\,[H_Q, \mathcal{D}]$$

$$\mathcal{L}_1\mathcal{D} = -\frac{i}{2}\int_{xx'} V(x-x')\,[n_x^a n_{x'}^a, \mathcal{D}]$$

$$\mathcal{L}_2\mathcal{D} = \frac{1}{2}\int_{xx'} W(x-x')\,(\{n_x^a n_{x'}^a, \mathcal{D}\} - 2n_x^a \mathcal{D} n_{x'}^a)$$ fluctuations

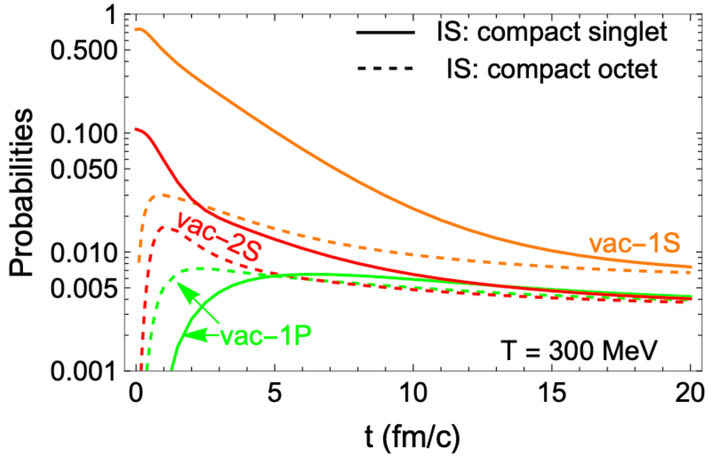
$$\mathcal{L}_3\mathcal{D} = -\frac{i}{4T}\int_{xx'} W(x-x')\left(\dot{n}_x^a \mathcal{D} n_{x'}^a - n_x^a \mathcal{D}\dot{n}_{x'}^a + \frac{1}{2}\{\mathcal{D}, [\dot{n}_x^a, n_{x'}^a]\}\right)$$ dissipation

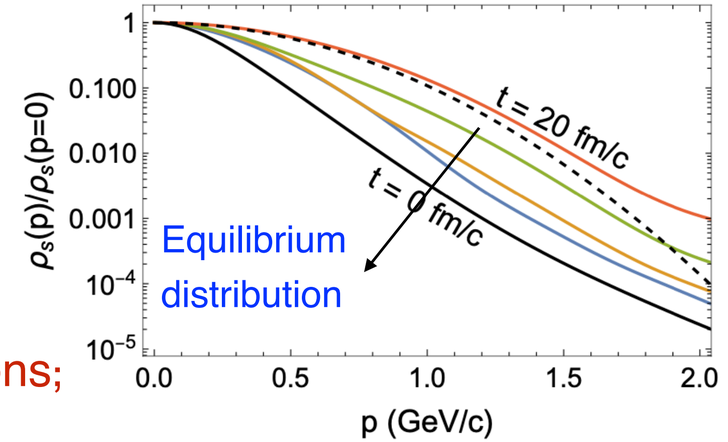
$$\mathcal{L}_4\mathcal{D} = \frac{1}{32T^2}\int_{xx'} W(x-x')\,(\{\dot{n}_x^a \dot{n}_{x'}^a, \mathcal{D}\} - 2\dot{n}_x^a \mathcal{D}\dot{n}_{x'}^a)\,.$$

preservation of positivity

Imaginary part potential

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s \\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{s},\mathbf{s}',t) \\ \mathcal{D}_o(\mathbf{s},\mathbf{s}',t)\end{pmatrix}, \qquad \mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so} \\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$





Beyond the dipole approximation;
The equations are solved with different initial states and medium configurations;
Equilibrium is checked.

Osaka Approach

- **NRQCD+OQS works in Quantum Brownian motion Regime** $M \gg T \sim m_D \gtrsim E_{bind}$
- **Weak coupling (strict) and go beyond the weak coupling (approximation)**
- **Used for bottomonium.**

T. Miura,Y. Akamatsu, M. Asakawa, et al, PRD 87 (2013) 045016; PRD 91 (2015) 5, 056002.; PRD97 (2018), 014003.; Phys.Rev.D 106 (2022) 7, 074001.

$$\frac{d}{dt}\rho_r(t) = -i[H_{\rm eff}^{(r)}, \rho_r] + \sum_{\vec{k}a}\left(2L_{\vec{k}a}^{(r)}\rho_r L_{\vec{k}a}^{(r)\dagger} - L_{\vec{k}a}^{(r)\dagger}L_{\vec{k}a}^{(r)}\rho_r - \rho_r L_{\vec{k}a}^{(r)\dagger}L_{\vec{k}a}^{(r)}\right),$$

$$H_{\rm eff}^{(r)} = \frac{\vec{p}^2}{M} + V(\vec{r})(t^a \otimes t^{a*}) - \frac{1}{4MT}\left\{\vec{p}, \vec{\nabla}D(\vec{r})\right\}(t^a \otimes t^{a*}),$$

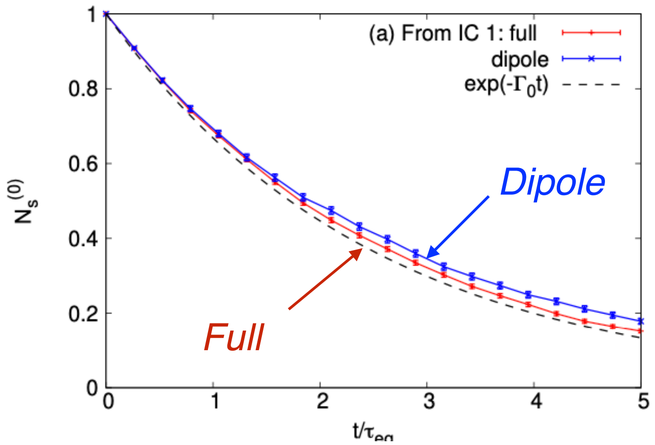
$$L_{\vec{k}a}^{(r)} = \sqrt{\frac{\tilde{D}(\vec{k})}{2L^3}}\left[1 - \frac{\vec{k}}{4MT}\cdot\left(\frac{1}{2}\vec{P}_{\rm CM} + \vec{p}\right)\right]e^{\frac{i\vec{k}\cdot\vec{r}}{2}}(t^a \otimes 1)$$

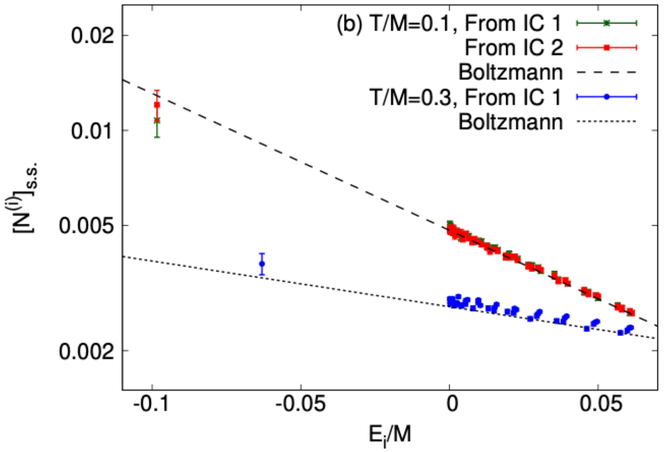
$$- \sqrt{\frac{\tilde{D}(\vec{k})}{2L^3}}\left[1 - \frac{\vec{k}}{4MT}\cdot\left(\frac{1}{2}\vec{P}_{\rm CM} - \vec{p}\right)\right]e^{-\frac{i\vec{k}\cdot\vec{r}}{2}}(1 \otimes t^{a*}).$$

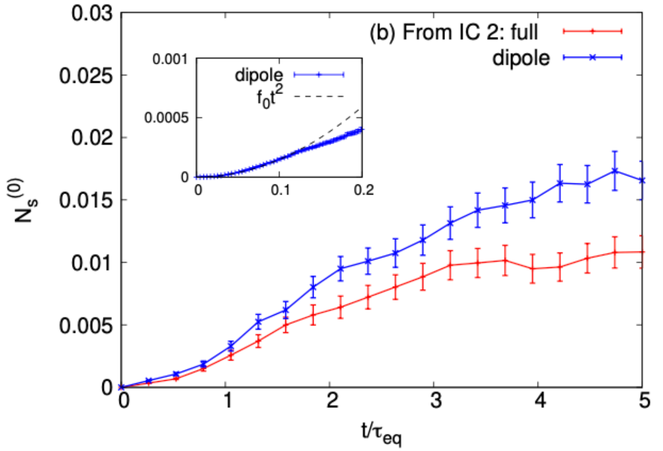
Beyond the weak coupling and assume the real and imaginary potential:

$$V(x) = -\frac{\alpha}{\sqrt{x^2 + x_{\rm c}^2}}{\rm e}^{-m_{\rm D}|x|},$$

$$D(x) = \gamma\exp(-x^2/\ell_{\rm corr}^2).$$







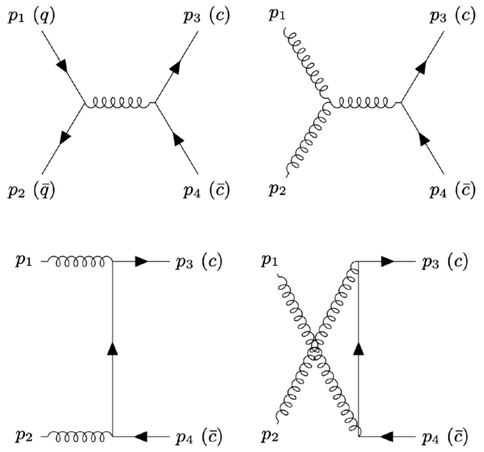
Beyond the dipole approximation;

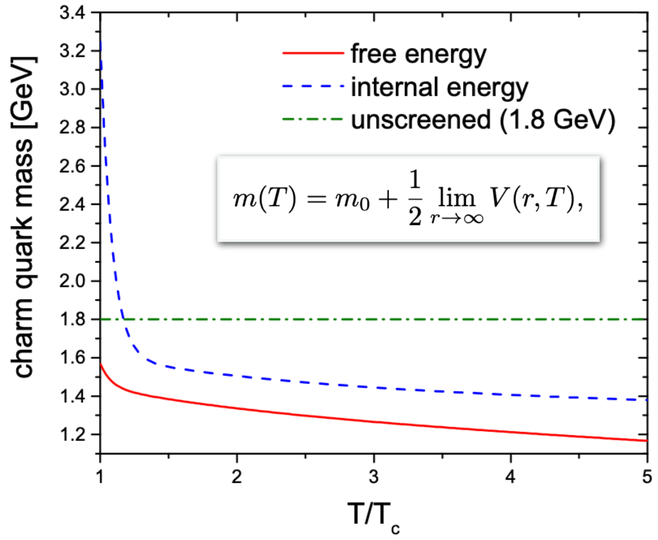
The dipole approximation is an efficient alternative method, but it depends on the initial condition!

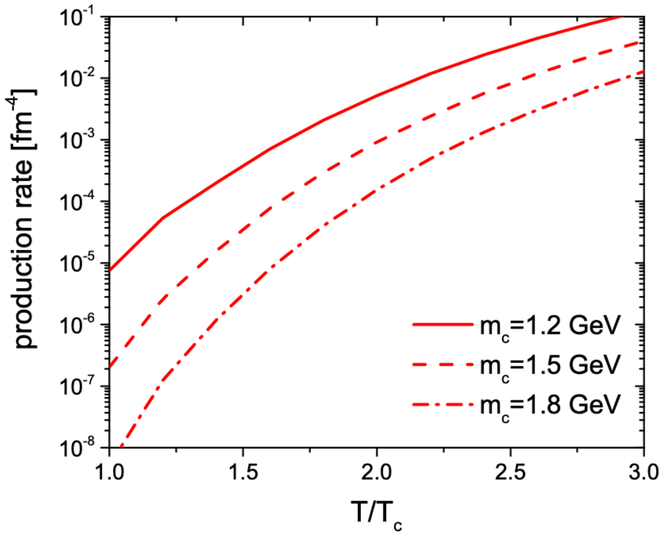
Equilibrium is satisfied.

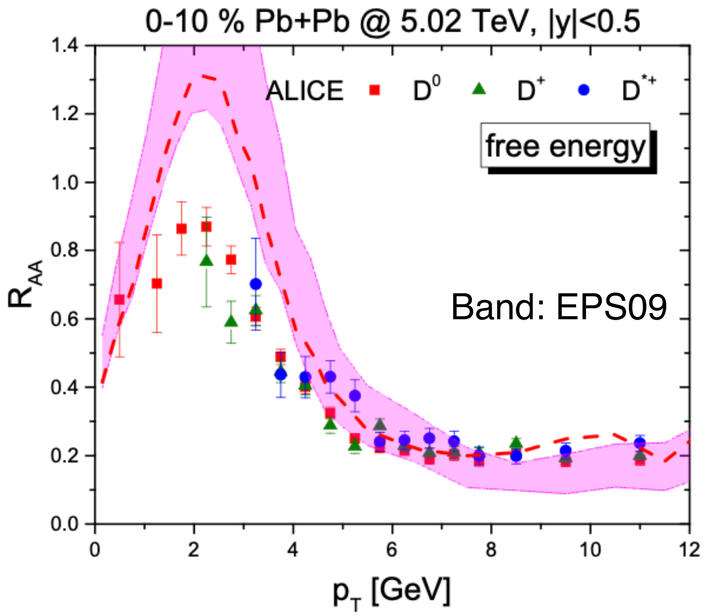
Heavy Quark Potential <-> Open heavy flavor production

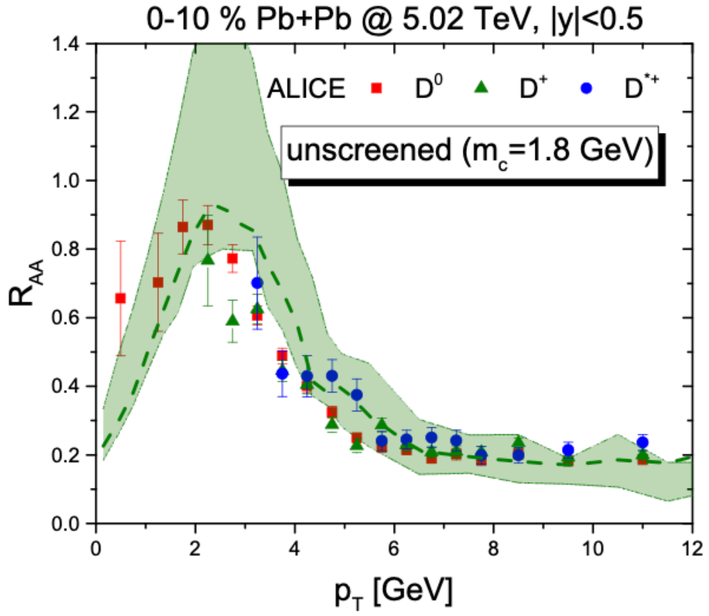
Heavy quark potential is related to heavy quark thermal mass, which affects the heavy quark thermal production in QGP.











*T. Song, **JZ**, I. Grishmanovskii,Eur.Phys.J.A 62 (2026) 1, 11.*

The open charm observables favor the unscreened in-medium potential!