



Collider Spin Tomography with Missing Neutrinos

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Outlines

- Quantum tomography and kinematic ambiguity
- Our goal and setup
- Identifiability analysis
- Self-consistent reconstruction
- Numerical results
- Summary

Why full spin tomography?

$$ee, pp \rightarrow f\bar{f}$$

- Tomography reconstructs the quantum state of the produced particle pair, not just one selected observable
 - **Spin information** is encoded in the production density matrix
 - Gives polarization $B_i^\pm$ and spin-correlation observables C_{ij}
 - Enables **entanglement and CHSH tests** at colliders
 - Probing **BSM parameters** in spin correlations

What does full tomography reconstruct?

- Differential spin density matrix $\rho(\hat{k})$ for two spin-half particles:

$$\rho(\hat{k}) = \frac{1}{4} \frac{d\sigma}{\sigma d\hat{k}} \left[\mathbb{I}_4 + B_i^+(\hat{k}) \sigma_i \otimes \mathbb{I}_2 + B_i^-(\hat{k}) \mathbb{I}_2 \otimes \sigma_i + C_{ij}(\hat{k}) \sigma_i \otimes \sigma_j \right]$$

- Differential production rate: $\mathcal{R}(\hat{k}) \equiv \frac{d\sigma}{\sigma d\hat{k}}$ **1**
- Single-particle polarizations: $B_i^\pm(\hat{k})$ **3+3=6**
- Spin-correlation matrix: $C_{ij}(\hat{k})$ in helicity basis **9**
- In total: $1+3+3+9=16$ differential components
- Once $\rho(\hat{k})$ is reconstructed, any polarization, correlation, entanglement, or Bell observable can be derived from it.

Normalization:
 $\int d\hat{k} \text{Tr}[\rho(\hat{k})] = 1$

Spin is measured through decay

- $e^+e^- \rightarrow \tau^+\tau^-$ production at $\hat{k}$ direction and its **spin density matrix**:

$$\rho_{\alpha\alpha';\beta\beta'}^{\text{prod}} = \frac{1}{|\overline{M}|^2} \sum_{\text{ini}} M(e^+e^- \rightarrow \tau_\alpha^+(\hat{k})\tau_\beta^-(-\hat{k})) M^*(e^+e^- \rightarrow \tau_{\alpha'}^+(\hat{k})\tau_{\beta'}^-(-\hat{k}))$$

- Colliders do not measure spin directly
- Spin information is transferred to decay-product momenta

- Two-body τ decay: $\tau^\pm \rightarrow \pi^\pm \nu^{(-)}$ and **spin measurement operator**:

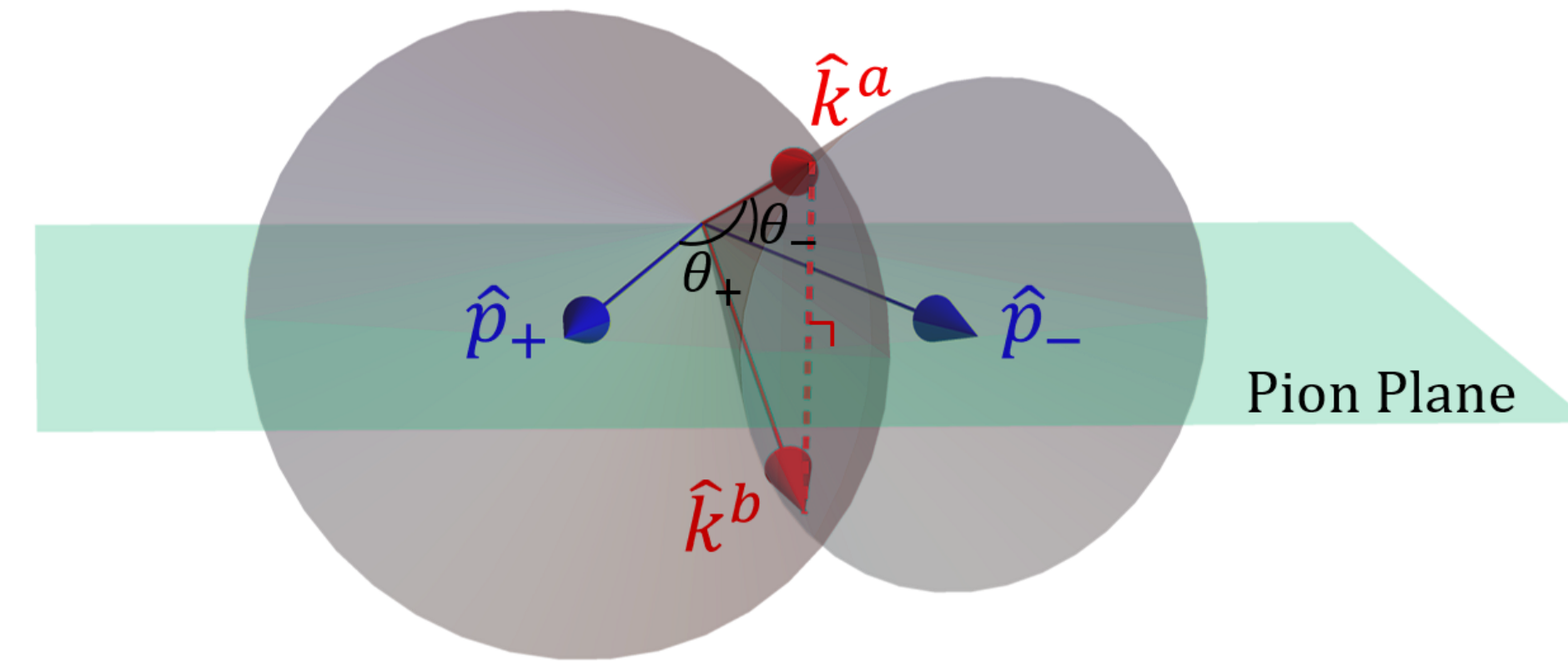
$$\hat{D}_{\alpha\alpha'}^\pm(\hat{q}_\pm) = \frac{f_\pi^2}{2m_\tau^3 |q|} M(\tau_{\alpha'}^\pm \rightarrow \pi^\pm(\hat{q}_\pm)\nu) M^*(\tau_\alpha^\pm \rightarrow \pi^\pm(\hat{q}_\pm)\nu) = \frac{1}{2}(\mathbf{I}_2 \mp \hat{q}_\pm \cdot \vec{\sigma})_{\alpha\alpha'}$$

- Differential $\pi^\pm$ distributions: $\frac{1}{\sigma} \frac{d\sigma}{d\hat{q}_+ d\hat{q}_-} = \text{Tr} \left[\rho_{\tau^+\tau^-}^{\text{prod}} \hat{E} \right] \equiv \text{Tr} \left[\rho_{\tau^+\tau^-}^{\text{prod}} \hat{D}^+(\hat{q}_+) \otimes \hat{D}^-(\hat{q}_-) \right]$

- Decay turns spin tomography into a momentum-distribution measurement

The real obstacle: invisible neutrinos

- Weak decay typically involves ν
 - E.g. τ , or clean leptonic decay of top
- We observe only the visible particle momenta, not p_ν
- The parent direction $\hat{k}$ is not uniquely reconstructed
 - **2-fold** $\tau\bar{\tau}$: $e^+e^- \rightarrow \tau^+(\hat{k})\tau^-(-\hat{k}) \rightarrow \pi^+(\hat{p}_+)\bar{\nu}_\tau\pi^-(\hat{p}_-)\nu_\tau$
 $\text{dof} = 8 (p_{\nu,\bar{\nu}}) - 4 (E/\vec{p}) - 2 (m_{\tau^+,\tau^-}) - 2 (m_{\nu,\bar{\nu}}) = 0$
 - **Multi-fold** leptonic $t\bar{t}$: $pp \rightarrow t\bar{t} \rightarrow b\ell_1^+\nu_{\ell_1} + \bar{b}\ell_2^-\bar{\nu}_{\ell_2}$
 $\text{dof} = 8 (p_{\nu,\bar{\nu}}) - 2 (\vec{p}_T) - 2 (m_{t,\bar{t}}) - 2 (m_{W^+,W^-}) - 2 (m_{\nu,\bar{\nu}}) = 0$



One visible event $\implies$ multiple possible kinematic configurations

Information incomplete at event-by-event level

Belle τ EDM: flat averaging over two solutions

BELLE
PLB 551 (2003) 16
JHEP 04 (2022) 110

- Belle searched for the τ EDM using classical optimal observables
- Each event admits two kinematically allowed τ directions $\hat{k}^{a,b}$
- The observables were evaluated for both solutions and averaged equally

- **Flat average:** $O_{\text{flat}}(\Phi) = \frac{1}{2} \left[O(\phi^a) + O(\phi^b) \right]$

See also
Li et al. 2605.01233
Zhou et al. 2605.26724

- $\Phi = \{\vec{p}_+, \vec{p}_-\}$: visible $\pi^\pm$ momentum ; dof = 6;
- $\phi = \{\hat{k}, \hat{q}_+, \hat{q}_-\}$ $\hat{k}$: τ direction in COM; $\hat{q}_\pm$: $\pi^\pm$ direction in tau rest frame
- Classical observable, not a reconstruction of the full spin density matrix

Flat averaging avoids choosing one branch,
but does not use the spin-dependent probabilities of the two solutions

BTW: for BELLE II, it may use extra vertex information to deal with tau pair twofold issue.

Top measurements: reduce or choose

ATLAS
Nature 633 (2024) 542
CMS
ROPP 87(2024)117801

- Dileptonic $t\bar{t}$: two neutrinos create reconstruction ambiguities
- ATLAS/CMS measured the entanglement marker
 - $D = -\frac{1}{3}\text{Tr}[C]$, using diagonals rather than the full ρ . No ρ_{SM} assumed.
- ATLAS: **Ellipse method** (one real solution in 85% of events) $\implies$ **Neutrino Weighting method** (assign weights to each possible solutions, 5% of events) $\implies$ **Simple pairing method** (ℓ +closest b-jet)
- CMS: **Best fit solution method** (lowest $m_{t\bar{t}}$, weighted by $m_{\ell b}$, 90% of events)
- These are powerful measurements, but **not unresolved-fold tomography**

Existing strategies either reduce the target observable
or select best kinematic branch

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Our Goal

- **Keep** all kinematic solutions $\{\phi^a, \phi^b, \dots\}$
- **Do not choose** the “best” branch
- **Do not assume** a production template: $\rho_{\text{SM}}^{\text{prod}}$
- **Reconstruct** ρ via data

visible event $\implies \{\phi^a, \phi^b\} \implies$ visible-data tomography

Event-by-event level

Global property level

Ambiguity is not the same as information loss!
What part of $\rho(\hat{k})$ is fixed by visible data alone?

Our benchmark channel

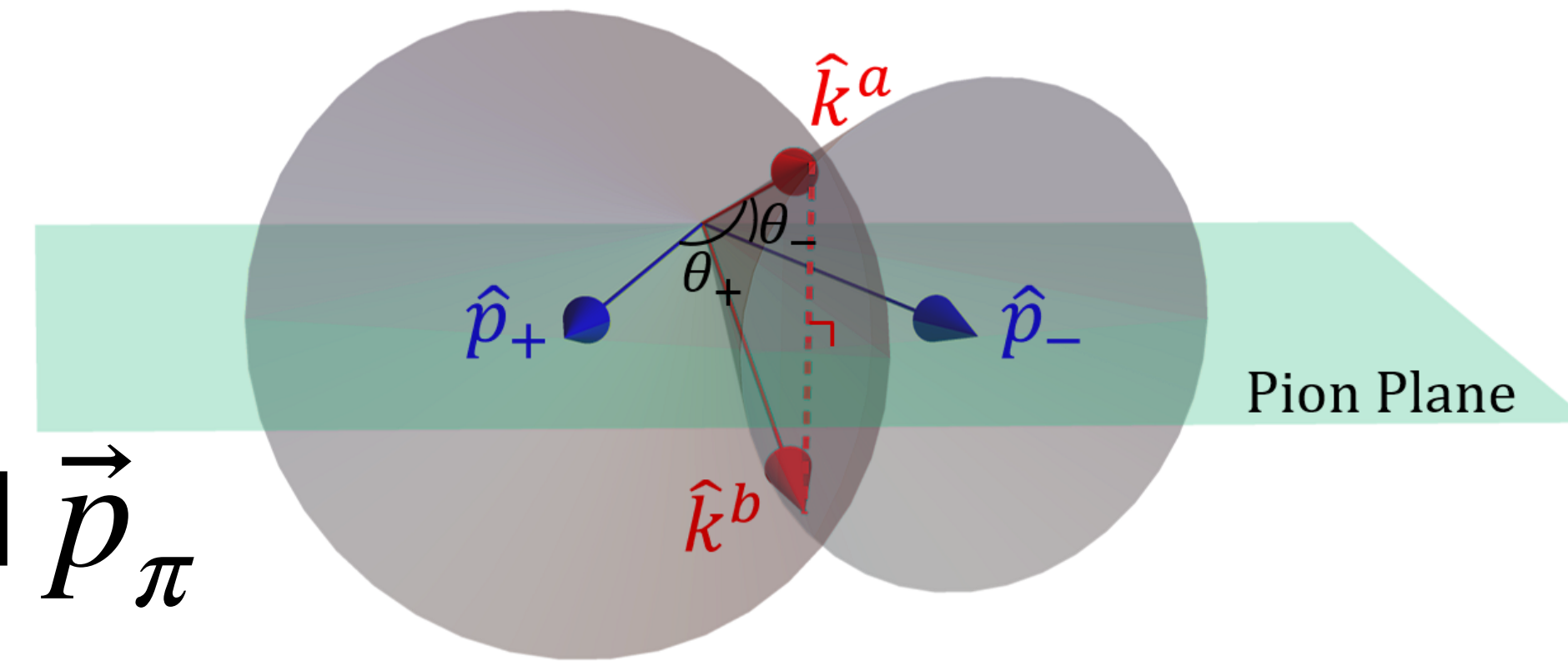
- $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\pi^- + \nu\bar{\nu}$
 - Clean production kinematics in an ee collider
 - $\tau^\pm \rightarrow \pi^\pm \nu$ provides maximal spin-analyzing power
 - Two neutrinos produce an event-by-event twofold ambiguity
 - Simple enough for an analytic identifiability analysis
 - Directly relevant to Belle I/II, BESIII, and future STCF studies

A minimal process containing both complete spin information
and unresolved kinematic ambiguity

Geometry of the twofold solution

- The τ energy is fixed by $e^+e^- \rightarrow \tau^+\tau^-$
- For each pion, $\hat{k}$ must lie on a cone around $\vec{p}_\pi$
- The two pion cones intersect at two points
- These two points give the two allowed τ directions

$$\hat{k}^{a,b} = u \hat{p}_+ + v \hat{p}_- \pm w \frac{\vec{p}_+ \times \vec{p}_-}{|\vec{p}_+ \times \vec{p}_-|}$$



The two solutions differ only by the sign of the component normal to the pion plane.
The twofold ambiguity is a mirror ambiguity in the parent τ direction.

From a visible event to hidden folds

- The detector measures only visible variables:

$$\Phi_{\text{visible}} = \{\vec{p}_+, \vec{p}_-\}; \phi = \{\hat{k}, \hat{q}_+, \hat{q}_-\}$$

- The full configuration has two possible branches:

$$\phi \implies \Phi \text{ (1-to-1); } \Phi \implies \{\phi^a(\Phi), \phi^b(\Phi)\}$$

- Each branch has its own τ direction and spin analyzers, the branch label is not observed

- **Ideal case** (no kinematic ambiguity):

- $f(\phi) \equiv \frac{d\sigma}{\sigma d\phi} = \text{Tr} \left[\rho_{\tau^+\tau^-} \hat{E} \right]$ internal full distribution;

- The decay as general quantum measurement $\hat{E}$

- Observed results from experiments: $\frac{d\sigma}{\sigma d\phi}$

- Extracting the spin information: $\langle O \rangle \implies \rho_{\tau^+\tau^-}$

From a visible event to hidden folds

- The detector measures only visible variables:

$$\Phi_{\text{visible}} = \{\vec{p}_+, \vec{p}_-\}; \phi = \{\hat{k}, \hat{q}_+, \hat{q}_-\}$$

- The full configuration has two possible branches:

$$\Phi \implies \{\phi^a(\Phi), \phi^b(\Phi)\}$$

- Variable relations:

- $f(\phi) \equiv \frac{d\sigma}{\sigma d\phi} = \text{Tr} \left[\rho_{\tau^+\tau^-} \hat{E} \right]$ internal distribution; $g(\Phi) \equiv \frac{d\sigma}{\sigma d\Phi}$ visible pion distribution

- Phase-space Jacobian $J^{-1}(\Phi) \equiv \left| \det \left(\frac{\partial \{\vec{p}_+, \vec{p}_-\}}{\partial \{\hat{k}, \hat{q}_+, \hat{q}_-\}} \right) \right| = \left| \det \left(\frac{\partial \{\Phi\}}{\partial \{\phi\}} \right) \right|$

- Distribution relation: $g(\Phi) = \sum_{t=a,b} J^t(\Phi) f(\phi^t) \equiv \int d\hat{k} \text{Tr} \left[\rho(\hat{k}) \bar{E} \right]$, with $\bar{E}(\Phi) = \sum_t J^t(\Phi) \hat{E}(\phi^t) \delta(\hat{k} - \hat{k}^t)$

The visible data provides coarse-grained measurements

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The theoretical question: what information is invisible?

- Define the visible-data map: $\mathcal{M} : \rho \mapsto g(\Phi)$
- Consider a deformation of the production density matrix: $\rho \longrightarrow \rho + \delta\rho$
- It is unidentifiable only when: $\delta g(\Phi) = \mathcal{M}[\delta\rho](\Phi) = 0$, for all visible Φ
 - Equivalently, $\delta\rho \in \ker \mathcal{M}$
- Multiple hidden solutions do not automatically imply information loss
- The visible map may remain informationally complete
- Only directions in $\ker \mathcal{M}$ are invisible to the data
- All directions outside the kernel remain reconstructible
 - **Two possible cases:**

$$\ker \mathcal{M} = 0 \implies \text{full identifiability}$$

$$\ker \mathcal{M} \neq 0 \implies \text{only the null directions are lost}$$

Information loss is determined by the null space—not by the number of kinematic solutions

Only one density-matrix direction is invisible

- Solving the null-space condition for $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\pi^- + \nu\bar{\nu}$

$$\rho(\hat{k}) = \frac{1}{4} \mathcal{R}(\hat{k}) \left[\mathbb{I}_4 + B_i^+(\hat{k}) \sigma_i \otimes \mathbb{I}_2 + B_i^-(\hat{k}) \mathbb{I}_2 \otimes \sigma_i + C_{ij}(\hat{k}) \sigma_i \otimes \sigma_j \right]$$

- Analytic result for $\delta g(\Phi) = 0$ when varying $\delta\rho$:

$$\delta \mathcal{R}(\hat{k}) = \delta \left(\frac{d\sigma}{\sigma d\hat{k}} \right) = 0, \quad \delta B_i^\pm(\hat{k}) = 0,$$

$$\delta C_{ij}(\hat{k}) = 0 \quad \text{except for, } \delta C_{nr}(\hat{k}) = -\delta C_{rn}(\hat{k}) = d_{\text{null}} / \mathcal{R}(\hat{k})$$

- where d_{null} is independent of $\hat{k}$.

- Therefore, the invisible combination is: $C_{nr}^- \equiv C_{nr} - C_{rn}$.

- The differential rate $\frac{d\sigma}{\sigma d\hat{k}}$, $B_i^\pm(\hat{k})$, 8 out of 9 C_{ij} remains identifiable

- Only one antisymmetric correlation direction C_{nr}^- is lost.

$$C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & C_{rn} \\ 0 & -C_{nr} & 0 \end{pmatrix}$$

The twofold ambiguity produces a 1D null space—not a general failure of tomography

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Without a production template, the fold probabilities are unknown

- For one visible event Φ , both folds are kinematically allowed: $\Phi \implies \{\phi^a, \phi^b\}$
- Their conditional fold weights are

$$\omega_t(\Phi; \rho) = \frac{J^t(\Phi) f(\phi^t; \rho)}{\sum_{s=a,b} J^s(\Phi) f(\phi^s; \rho)}, \quad t = a, b.$$

- The fold weights depend on the production density matrix ρ , but ρ is precisely what tomography aims to reconstruct
- Circular structure: $\rho \implies f(\phi^t; \rho) \implies \omega_t \implies \text{tomographic observables} \implies \rho$

Theory template

ρ_{template}

$\implies$ known weights,

Model-independent tomography

ρ unknown

$\implies$ weights must be learned from data

Production-model independence turns fold assignment into a self-consistency problem

The fixed-point family and self-consistent unfolding

- Define the vector $\mathbf{x} = \left\{ \frac{d\sigma}{\sigma d\hat{k}}|_{\alpha}, B_{i,\alpha}^+, B_{i,\alpha}^-, C_{ij,\alpha} \right\}$, 16 components binned for $\hat{k}$
- Data-driven iteration:
$$\mathbf{x}^{(n-1)} \Rightarrow \omega_t^{(n)} \Rightarrow \langle \widetilde{O} \rangle \Rightarrow \mathbf{x}^{(n)}$$
$$\mathbf{x}^{(n)} = T(\mathbf{x}^{(n-1)})$$
 - Start from a trial density matrix
 - Use it to compute the fold probabilities
 - Reconstruct all density-matrix parameters
 - Repeat until the update becomes self-consistent

The fixed-point family and self-consistent unfolding

- Fixed-point condition: $\mathbf{x}_* = T(\mathbf{x}_*)$
- The null space produces a continuous fixed-point family:

$$\mathbf{x}_*(d_{\text{null}}) = \mathbf{x}_{\text{truth}} + d_{\text{null}} \mathbf{n}_{\text{null}}$$

- where $\mathbf{n}_{\text{null}}$ is the direction associated with $C_{nr} - C_{rn}$
- $\mathbf{x}_{\text{truth}}$ is the 15 components with their true value
 - Every member of the family gives the same visible distribution $\delta g(\Phi) = 0$
 - All members coincide with the truth in the identifiable subspace
 - They differ only along the unidentifiable null direction
 - Positivity restricts which members correspond to physical ρ

Local convergence toward the fixed-point family

- Linearized iteration
 - Near a fixed point $\mathbf{x}_*$,

$$\delta \mathbf{x}^{(n)} \simeq D_T(\mathbf{x}_*) \delta \mathbf{x}^{(n-1)}, \text{ with } (D_T)_{ml} \equiv \left. \frac{\partial T_m}{\partial x_l} \right|_{\mathbf{x}_*}$$

- For an eigenmode $\mathbf{v}$,

$$D_T \mathbf{v} = \lambda \mathbf{v}, \quad \delta \mathbf{x}^{(n)} \propto \lambda^n$$

- Convergence criterion:

$$|\lambda| < 1 \quad \Rightarrow \quad \text{perturbation is damped,}$$

$$|\lambda| = 1 \quad \Rightarrow \quad \text{marginal direction .}$$

Local convergence toward the fixed-point family

- Null direction: analytic result

- Because every point along the null family is a fixed point,

$\mathbf{x}_*(d_{\text{null}}) = \mathbf{x}_{\text{truth}} + d_{\text{null}} \mathbf{n}_{\text{null}}$, differentiation $\mathbf{x}_* = T(\mathbf{x}_*)$ gives

$$D_T(\mathbf{x}_*) \mathbf{n}_{\text{null}} = \mathbf{n}_{\text{null}}$$

$$\lambda_{\text{null}} = 1$$

- Identifiable subspace: numerical result

- After removing the null mode, $\rho_{\text{id}}(D_T) < 1$

- For the least-damped identifiable mode, numeric results shows

$$\lambda_{\text{id}, \text{max}}^{\text{SM}} \simeq 0.9605, \quad \lambda_{\text{id}, \text{max}}^{\text{non-SM}} \simeq 0.9608$$

- Thus, all identifiable perturbations are locally damped, while the null direction has no restoring force.

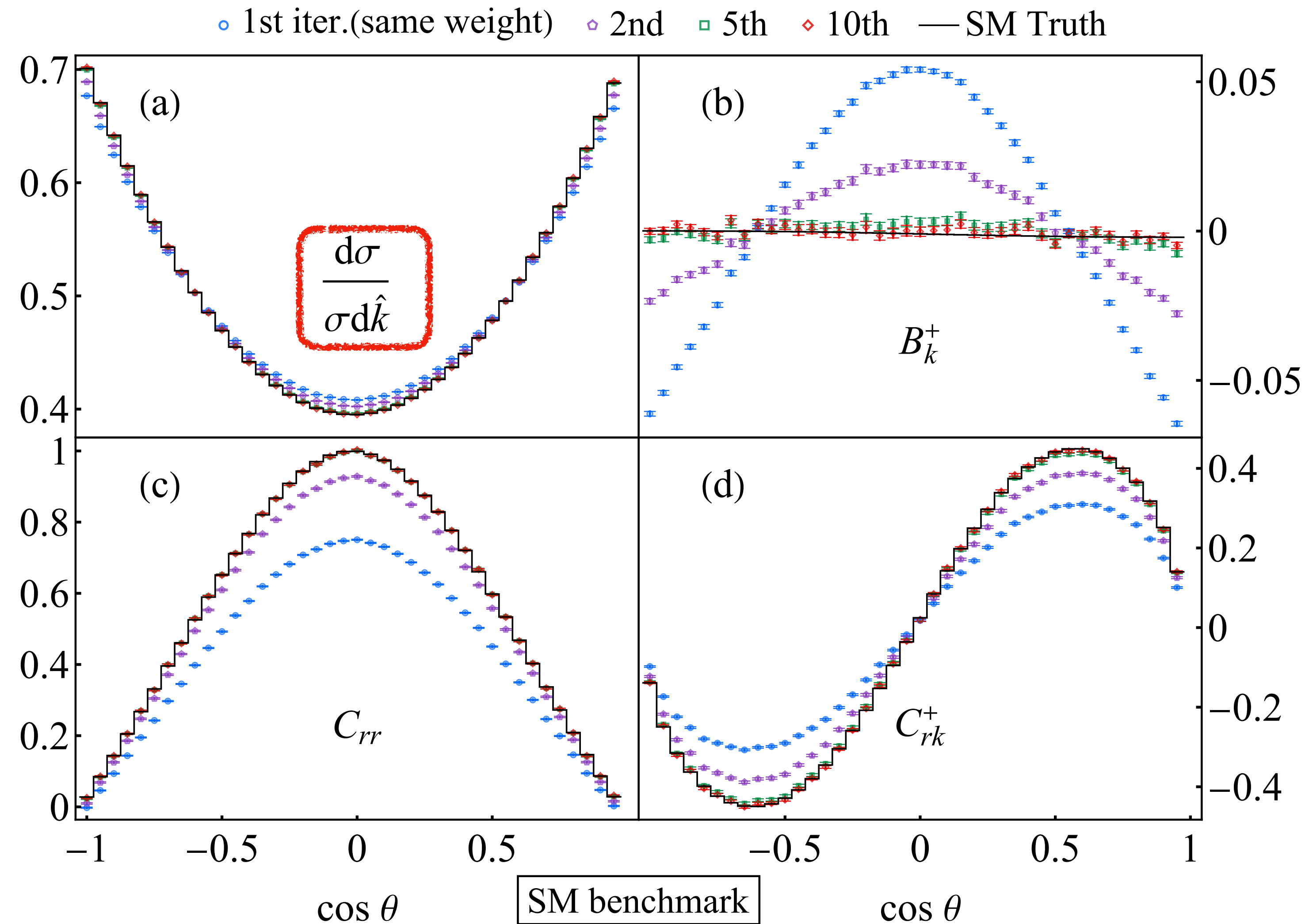
The identifiable components converge uniquely; the null component does not

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Closure test: Standard Model

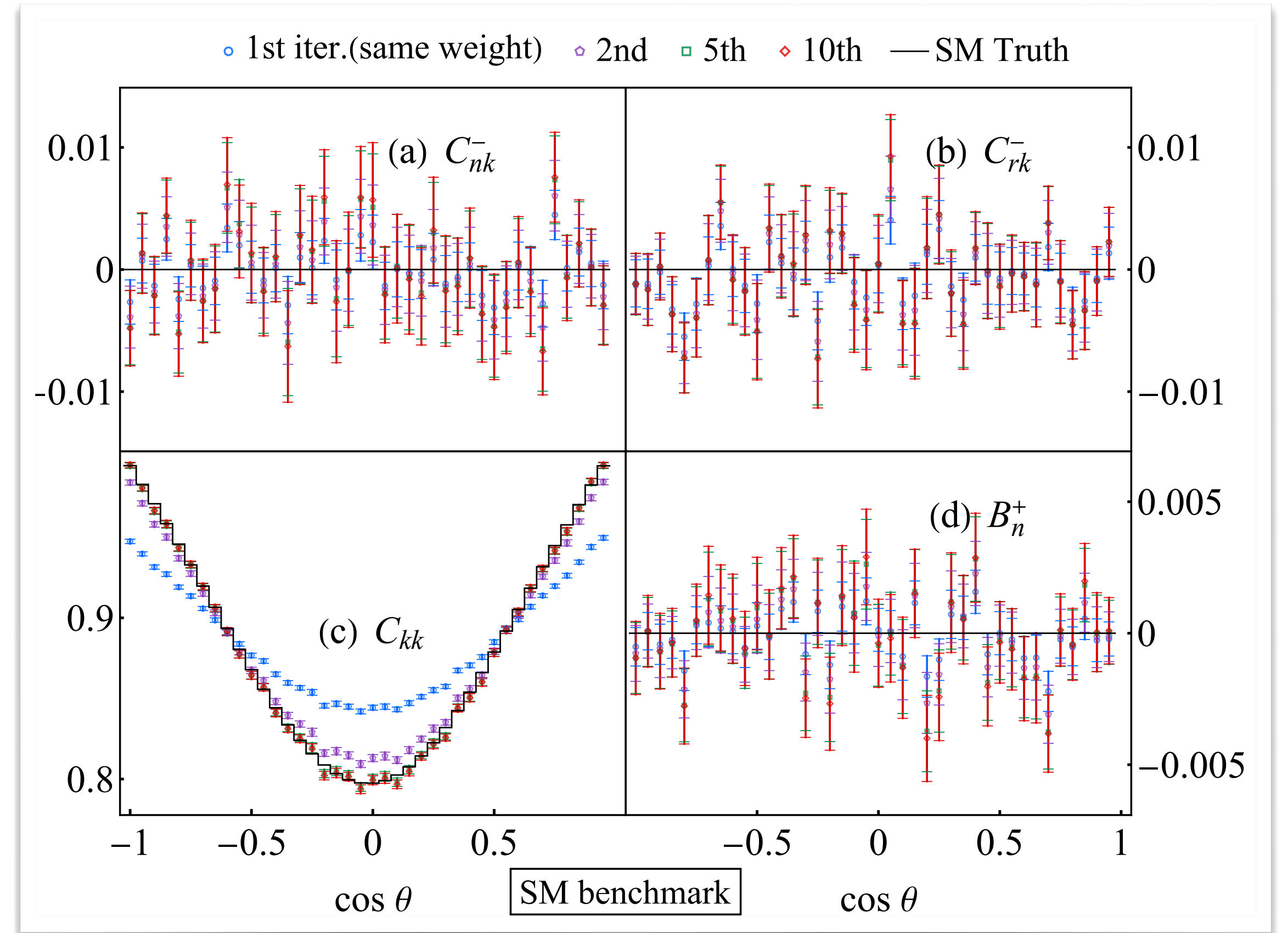
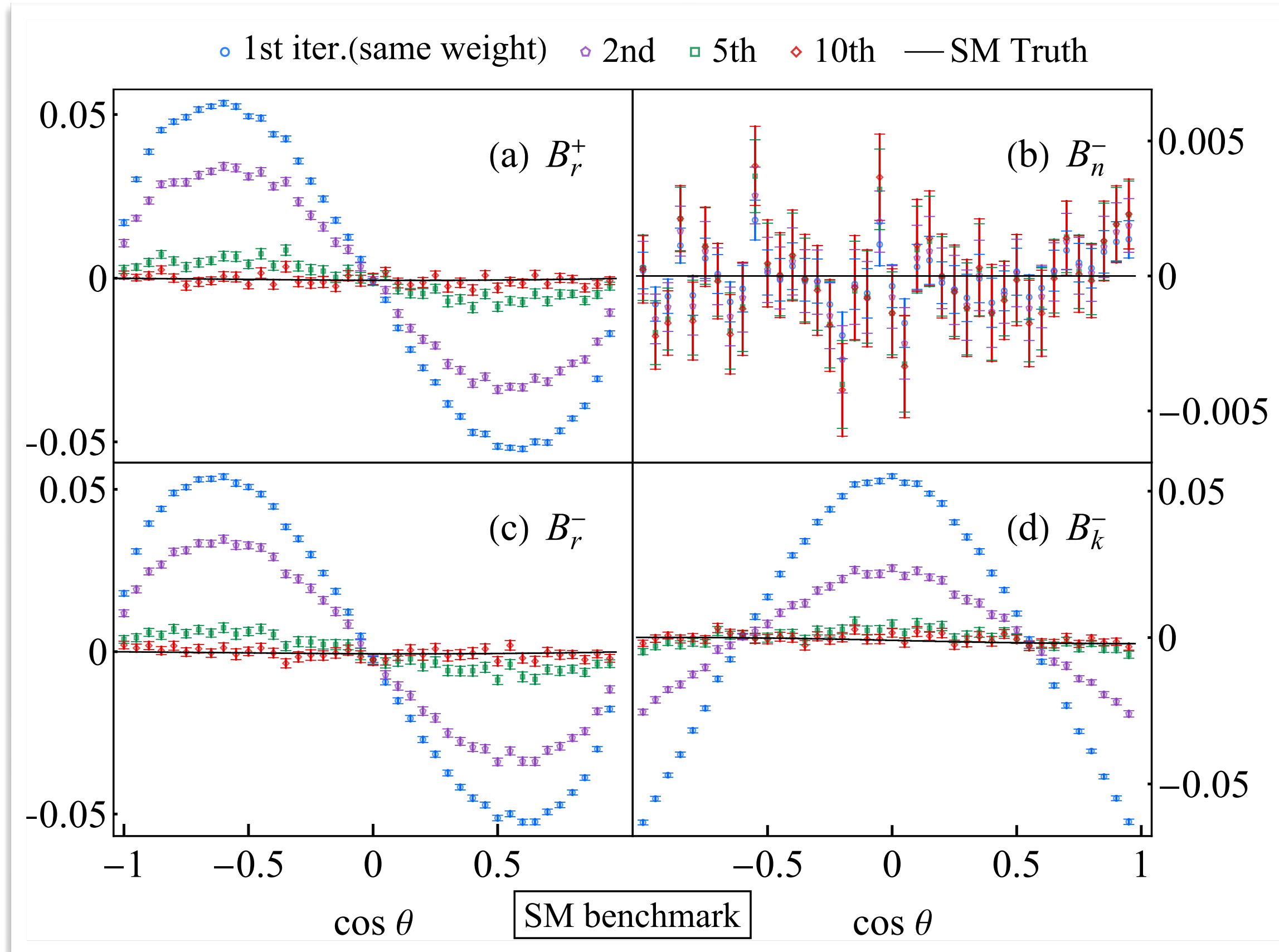
- $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\pi^- + \nu\bar{\nu}$
- Generate MC events from the SM truth density matrix
- Reconstruct using only visible pion momenta
- Start from a spin-independent trial density matrix
- The first update reproduces the flat 50% average
- Iterations move the identifiable components toward the truth
- Differential cross-section is successfully reconstructed



Flat averaging is not tomography; self-consistency removes the fold-induced bias

Closure test: Standard Model

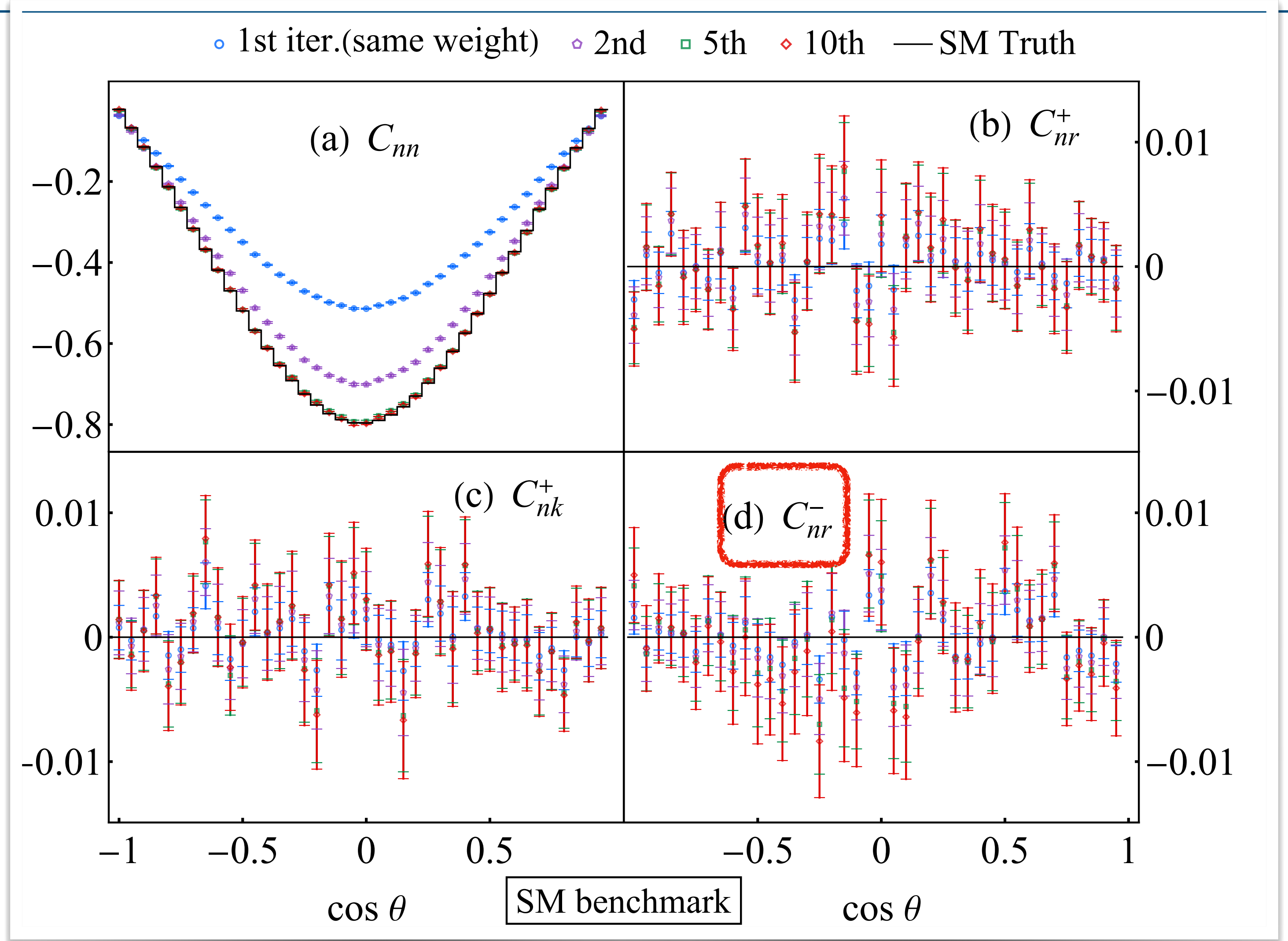
- Other spin-correlations reconstructed successfully



Flat averaging is not tomography; self-consistency removes the fold-induced bias

Closure test: Standard Model

- Others reconstructed successfully,
- but C_{nr}^- constructed?
See non-SM model



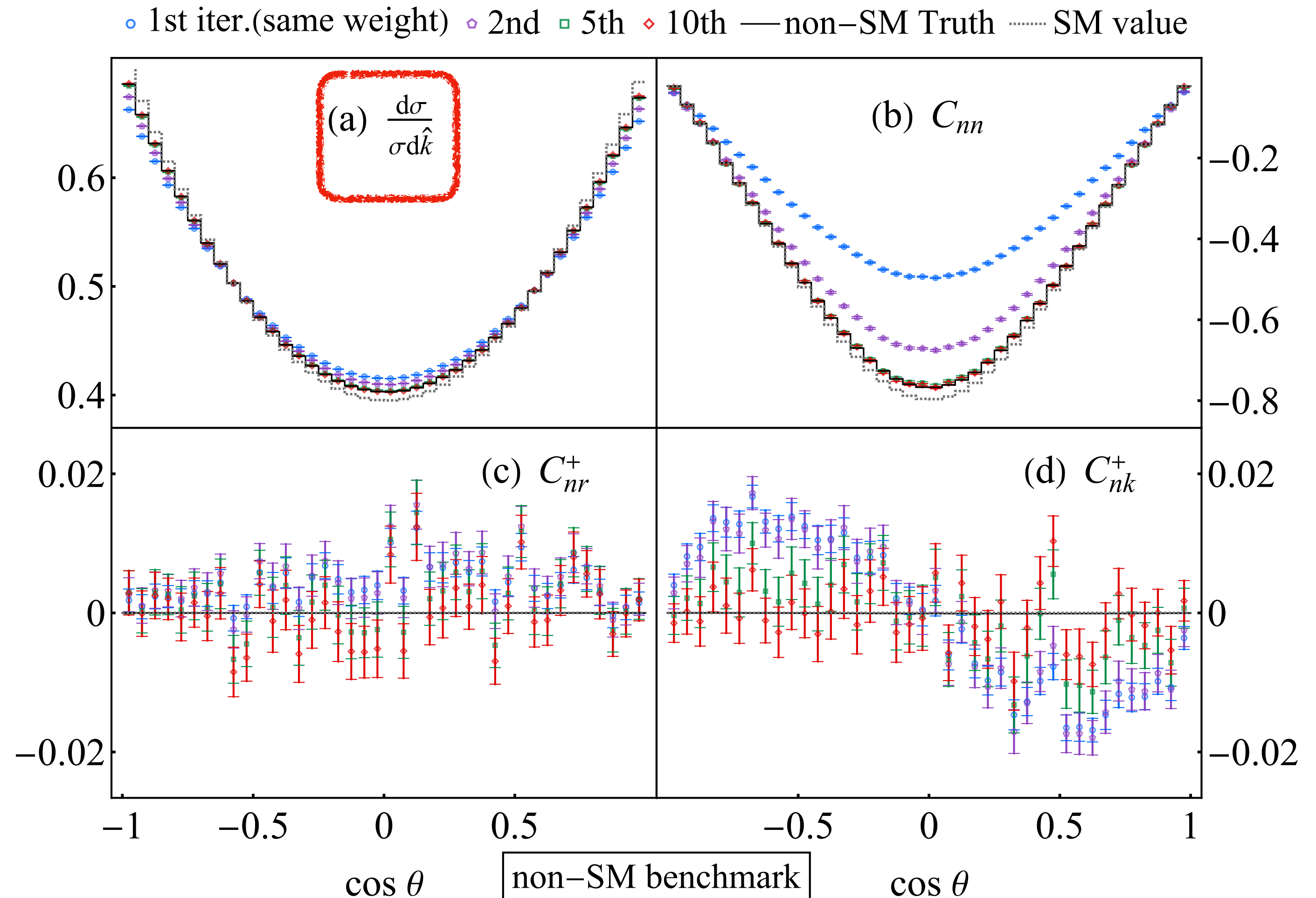
Flat averaging is not tomography; self-consistency removes the fold-induced bias

Stress test: anomalous τ dipoles

- Non-SM benchmark:

$$a_\tau = 0.01 - 0.02i, \quad \tilde{d}_\tau \equiv d_\tau \frac{2m_\tau}{e} = -0.05 + 0.03i.$$

- Generate events with a non-SM production density matrix
- Apply the same visible-only reconstruction and spin-independent initialization
- No SM or non-SM production template is supplied to the iteration
- Flat averaging remains biased
- All identifiable components evolve toward the non-SM truth

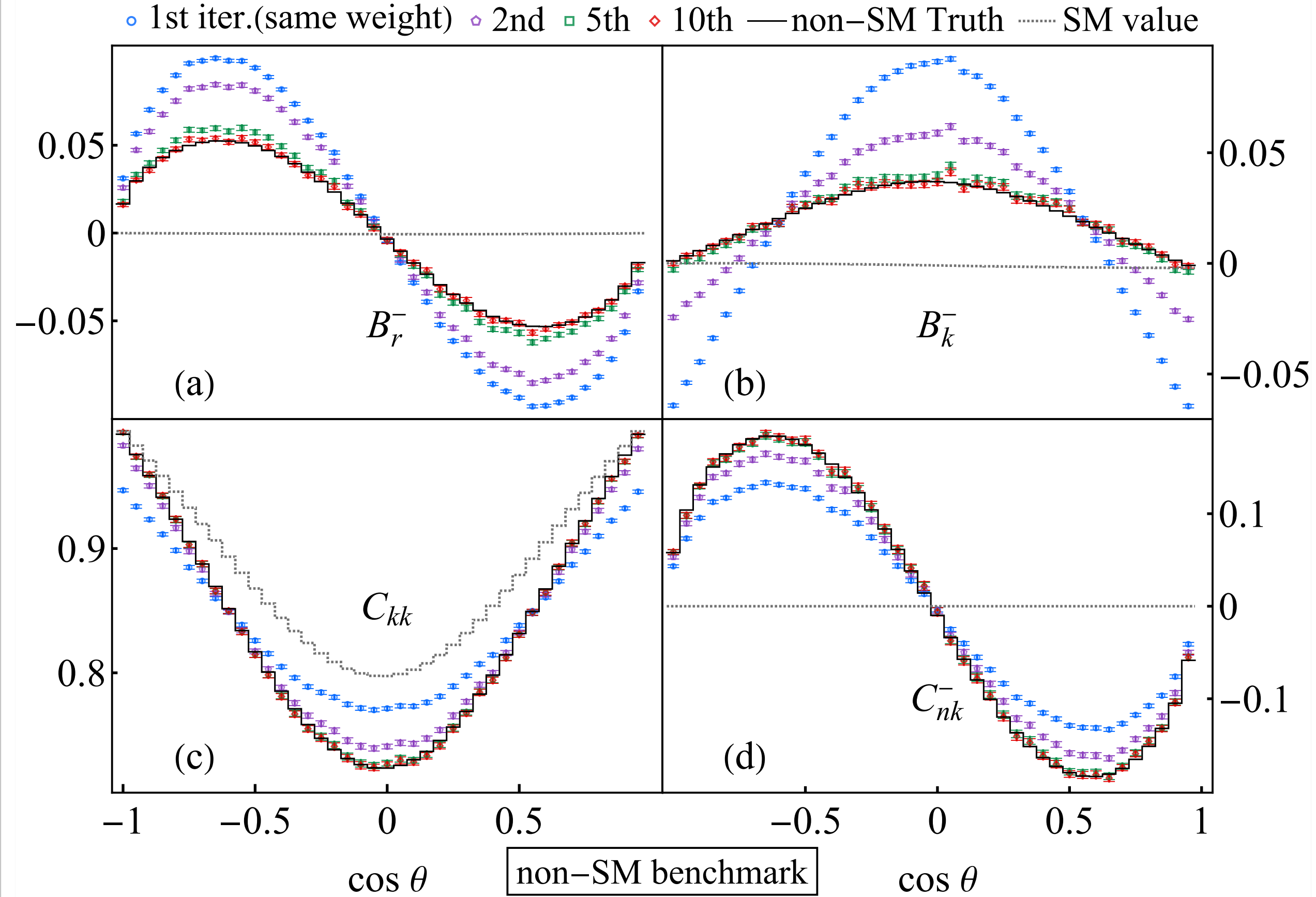
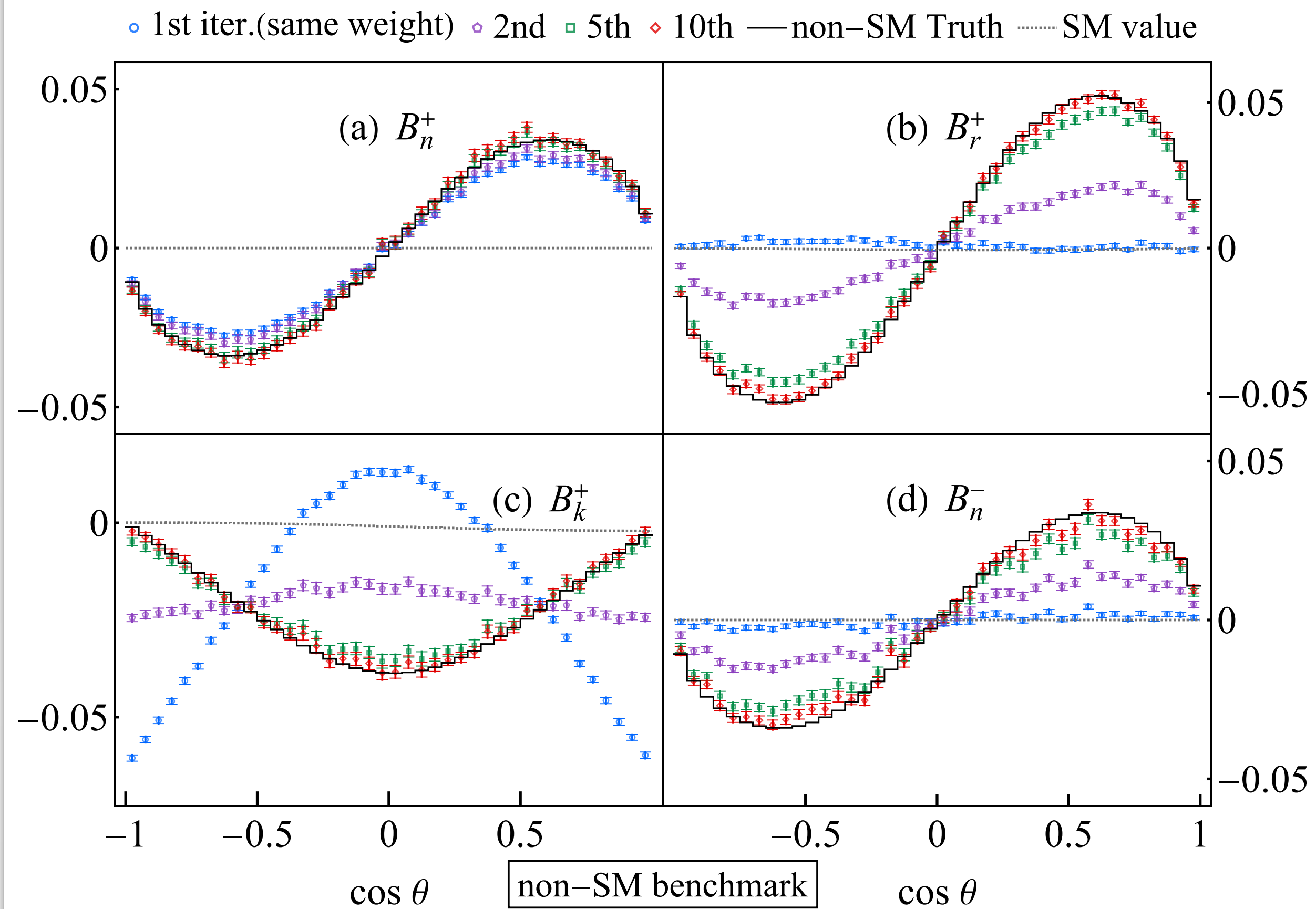


Model-independent algorithm + non-SM data $\Rightarrow$ non-SM density matrix

The iteration follows the density matrix encoded in the data—not an assumed SM template

Stress test: anomalous τ dipoles

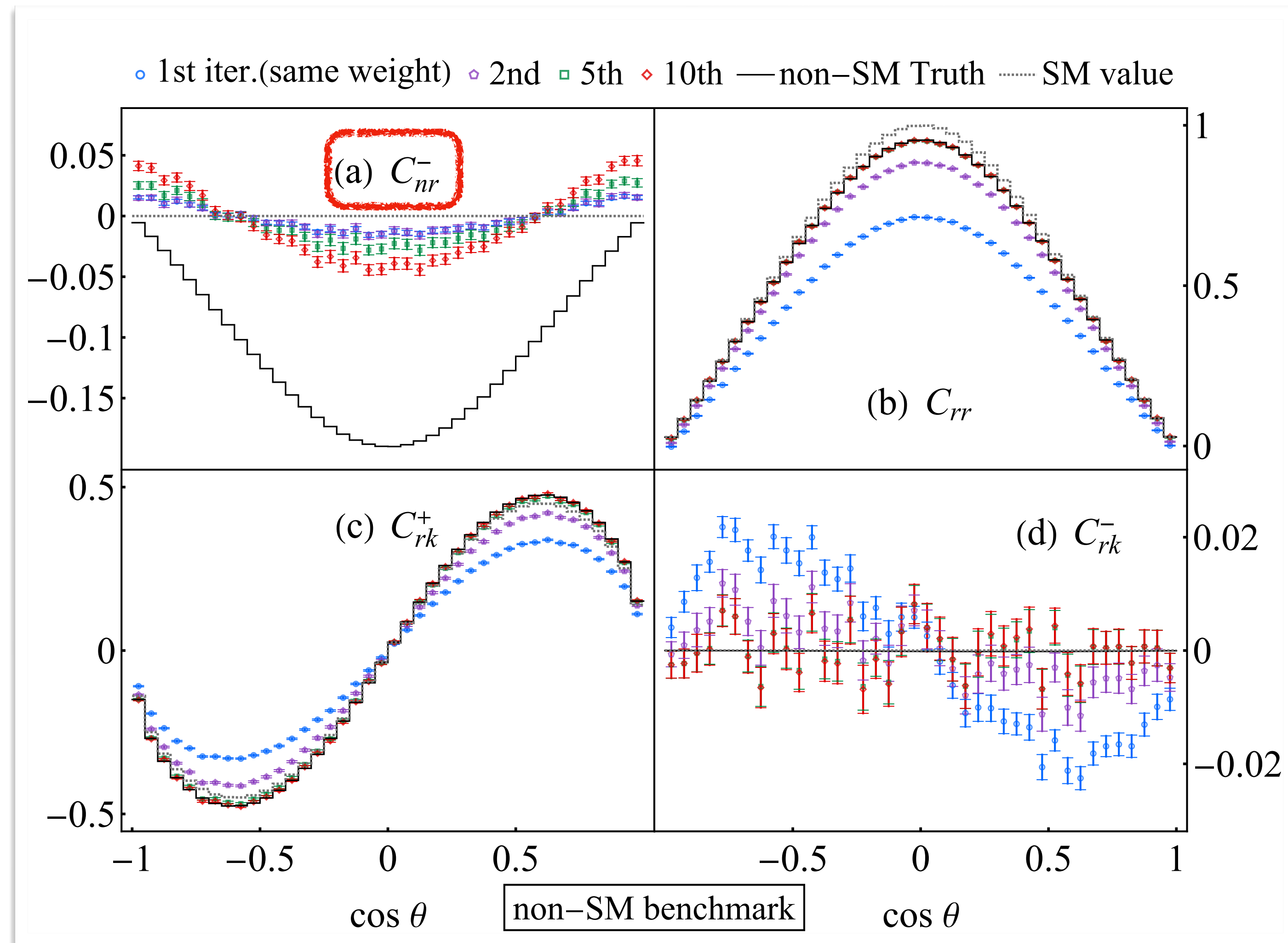
- Other spin-correlations reconstructed successfully



Model-independent algorithm + non-SM data $\Rightarrow$ non-SM density matrix
 The iteration follows the density matrix encoded in the data—not an assumed SM template

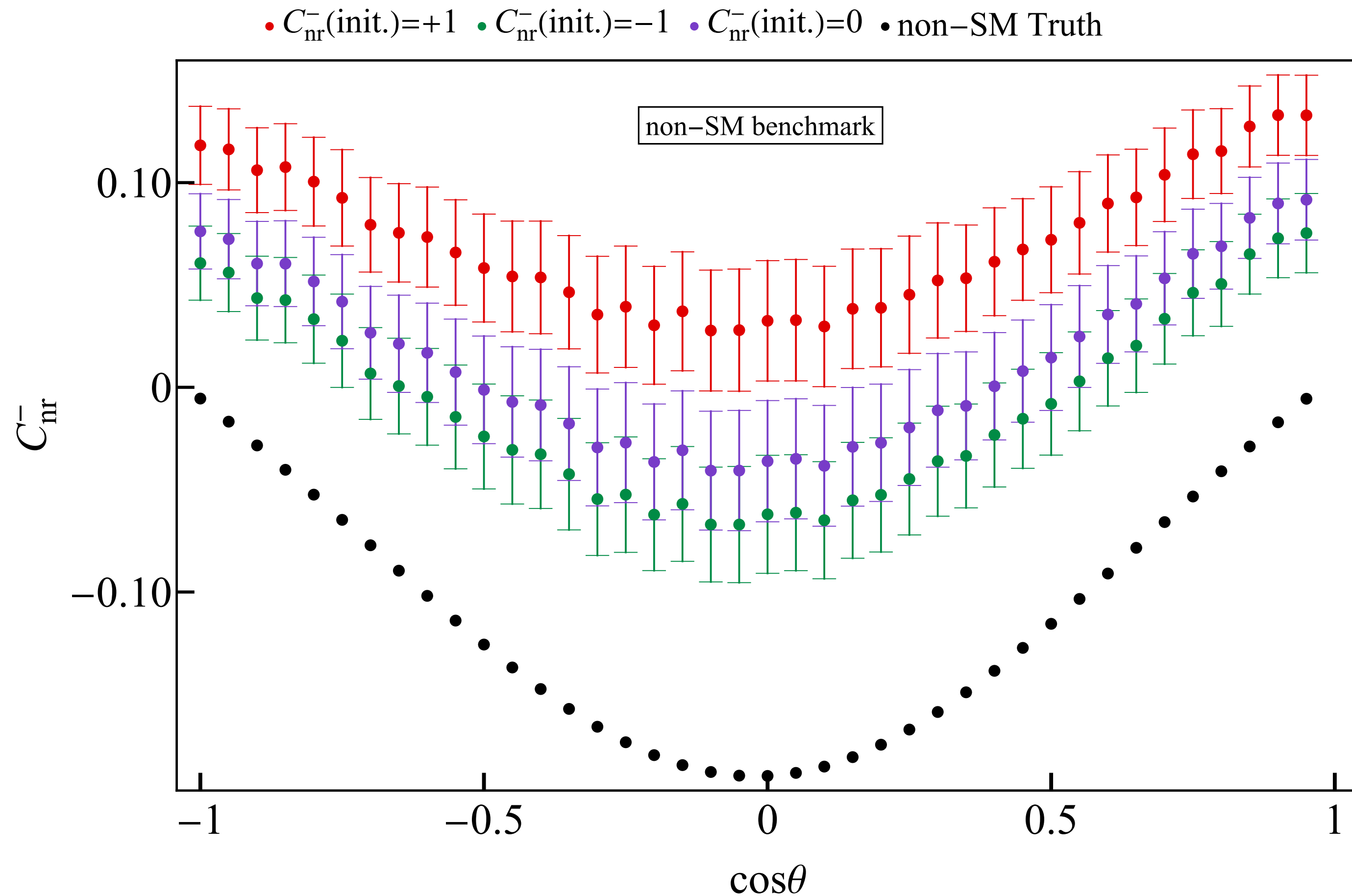
Stress test: anomalous τ dipoles

- Others reconstructed successfully
- But is C_{nr}^- constructed?
In non-SM model, it does not converge to the truth value.
- What if more iterations?



Model-independent algorithm + non-SM data $\Rightarrow$ non-SM density matrix
The iteration follows the density matrix encoded in the data—not an assumed SM template

The null component C_{nr}^- behaves differently



- Identifiable components converge toward the truth very quickly (< 10 iterations)
- The null component C_{nr}^- is not fixed by visible data
- Different initializations lead to different stabilizing point, but are consistent with the fixed-point family

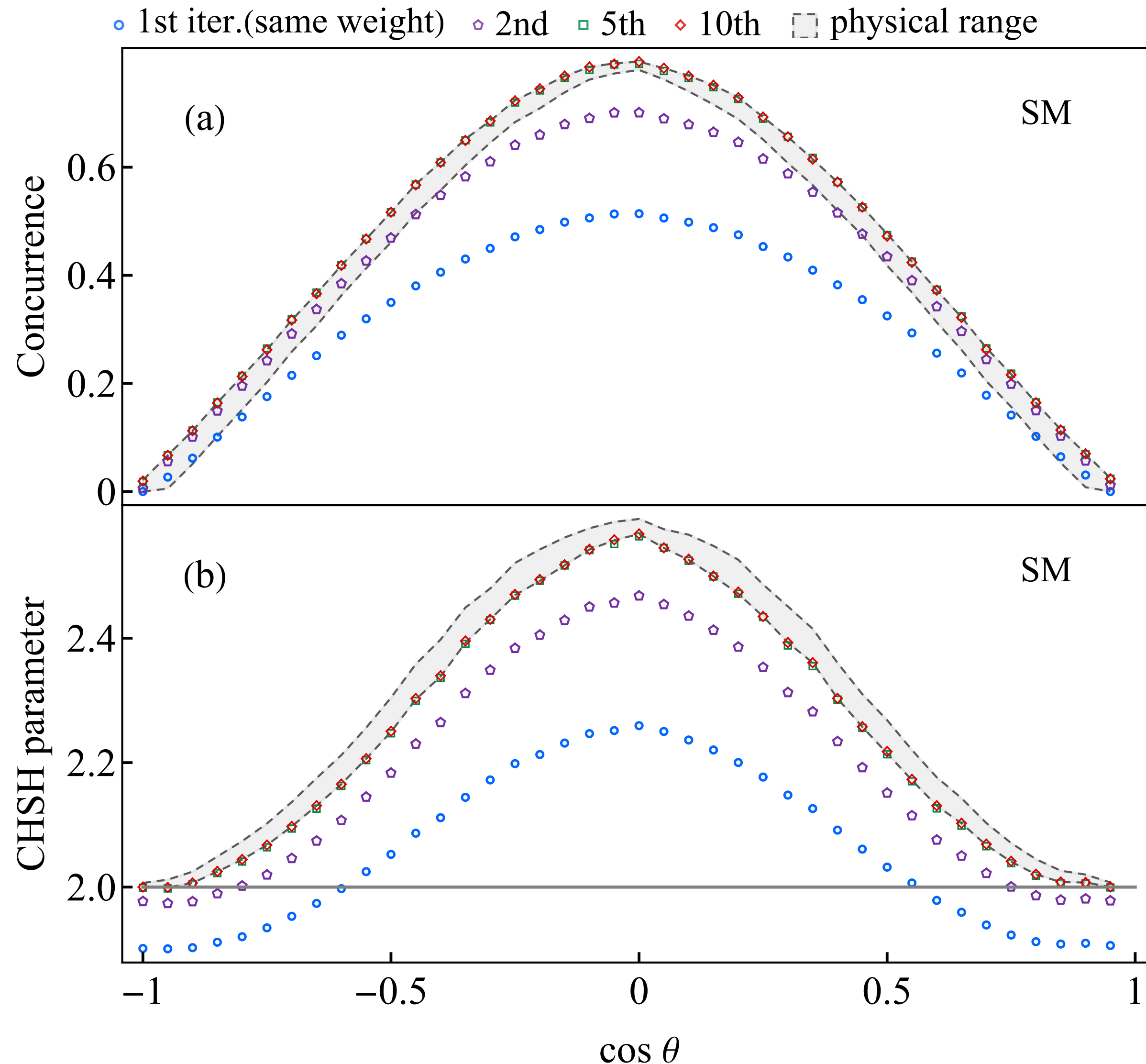
$$\mathbf{x}_*(d_{\text{null}}) = \mathbf{x}_{\text{truth}} + d_{\text{null}} \mathbf{n}_{\text{null}}$$

$$\delta C_{nr}(\hat{k}) = -\delta C_{rn}(\hat{k}) = d_{\text{null}}/\mathcal{R}(\hat{k}),$$

- 4000 iterations stabilizes

- $\text{Iter} - \text{truth} = \delta C_{nr}^-(\hat{k}) \simeq d_{\text{null}} \times \left(\frac{d\sigma}{\sigma d\hat{k}} \right)^{-1}$; The numeric difference is consistent with analytic results.

Positivity still constrains quantum observables



- Positivity restricts the fixed-point family
- Concurrence and CHSH need not have unique reconstructed values
- They can nevertheless be bounded using visible data and $\rho(\hat{k}) \geq 0$
- The positivity-constrained intervals contain the truth in the closure tests
- Flat averaging can underestimate the quantum observables

Unidentifiable does not mean unconstrained.

Implications for BELLE I/II, BESIII and STCF

- The procedure of model-independent reconstruction under kinetic ambiguity

visible event $\Phi \Rightarrow \{\phi^a, \phi^b\} \Rightarrow$ visible-map tomography $\Rightarrow$ $\begin{cases} \text{identifiable } \rho \\ \text{positivity ranges} \end{cases}$

- Practical next steps:
 - Include detector acceptance and efficiency
 - Include momentum resolution and backgrounds
 - Combine additional τ decay channels
 - Re-evaluate the null space for the full experimental measurement

**Kinematic ambiguity can be modeled,
not simply averaged away or branch-selected.**

Summary

- Missing neutrinos turn ideal spin measurements into coarse-grained visible measurements
- Information loss is determined by the null space of the visible map
- For $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\pi^-\bar{\nu}\nu$, only $C_{nr} - C_{rn}$ is unidentifiable
- All other differential spin information can be reconstructed without a production template
- Positivity gives controlled ranges for null-sensitive quantum observables

Kinematic ambiguity does not destroy tomography;
it defines which part of tomography is identifiable

Keep all folds $\implies$ find the null space $\implies$ reconstruct the identifiable density matrix

Thanks you!

