



PRECISION QCD WITH NNLOJET EVENT GENERATOR

**NNLO
JET**

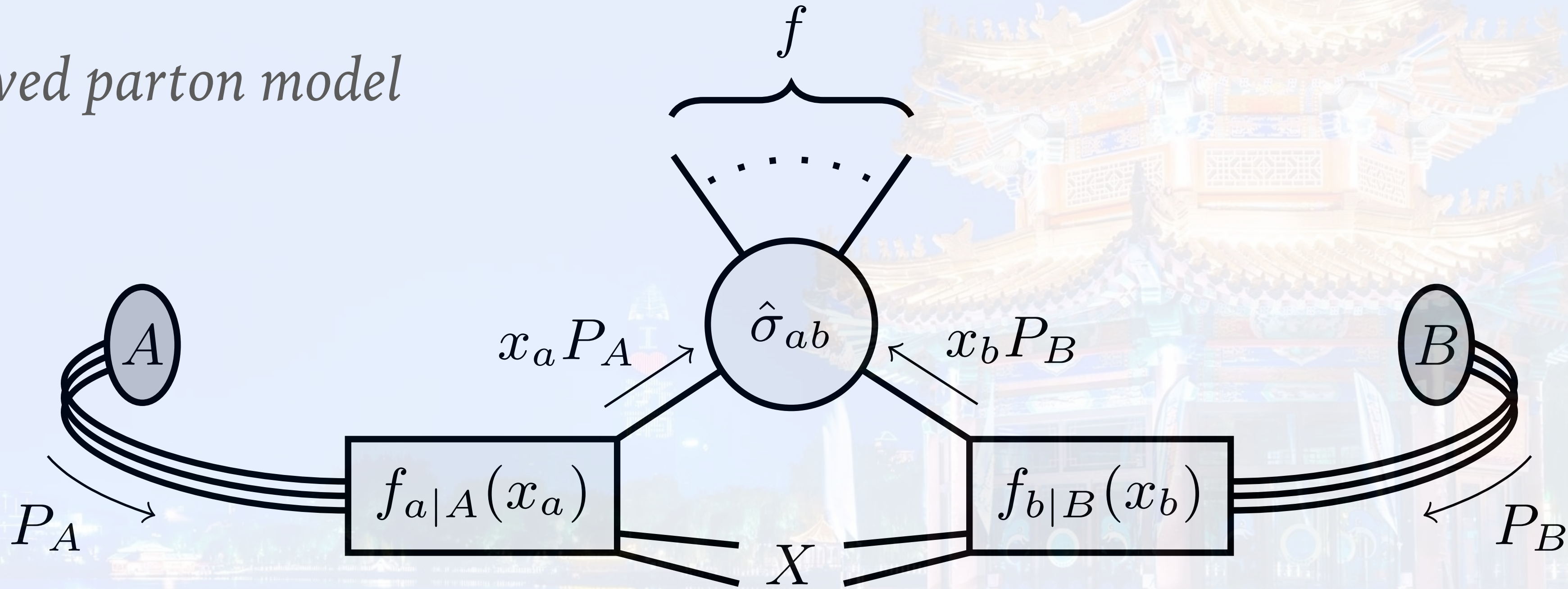
33th LHC Mini-Workshop

*Xuan Chen
Shandong University
Beijing, 19 September, 2026*



Precision Predictions at Hadron Collider

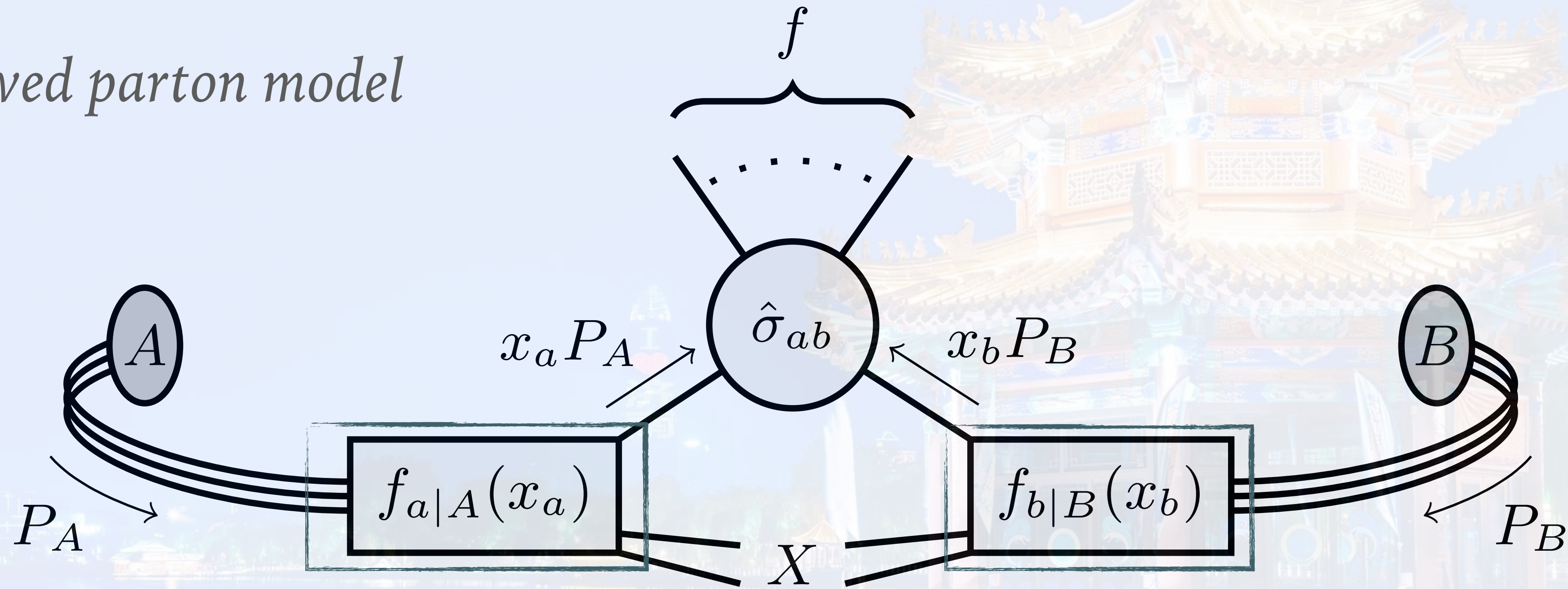
QCD improved parton model



$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b f_{a|A}(x_a) f_{b|B}(x_b) \hat{\sigma}_{ab}(x_a, x_b) (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

Precision Predictions at Hadron Collider

QCD improved parton model



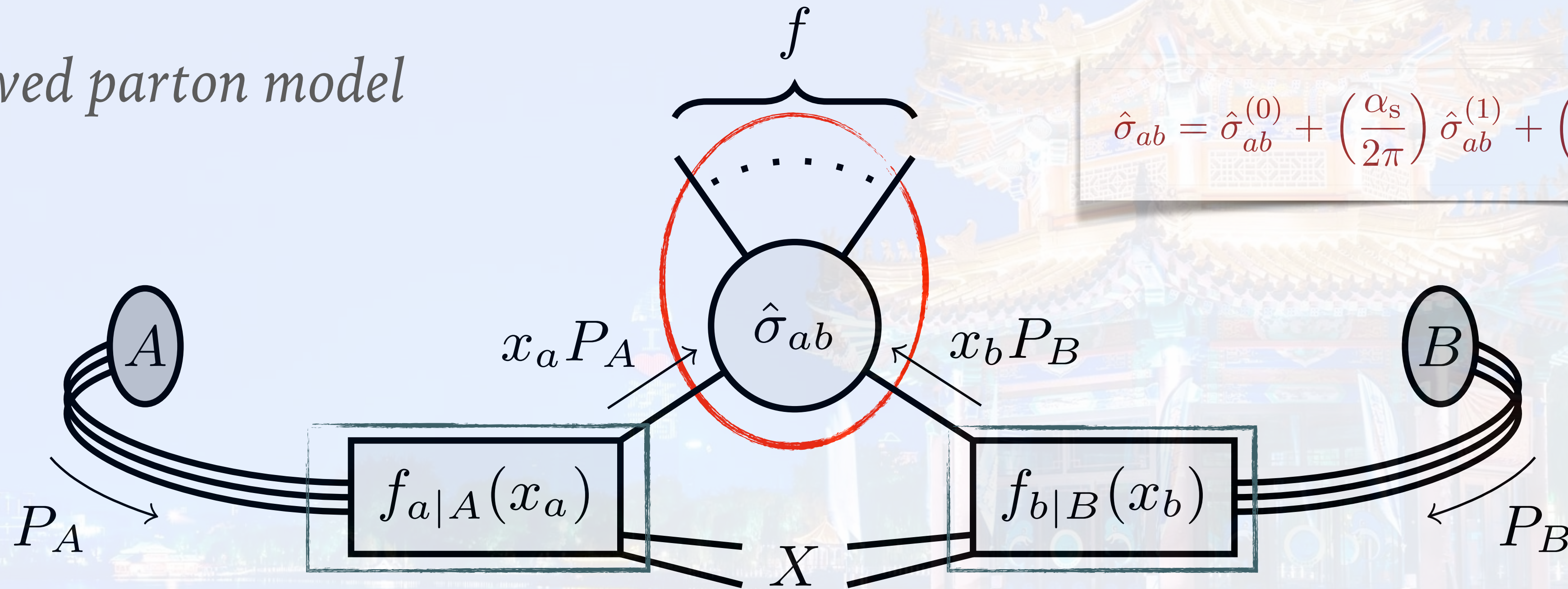
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Parton distribution functions
(Energy evolution from all exp.)

$\pm 3\text{-}5\%$ at LHC energy

Precision Predictions at Hadron Collider

QCD improved parton model



$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \dots$$

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Parton distribution functions
(Energy evolution from all exp.)

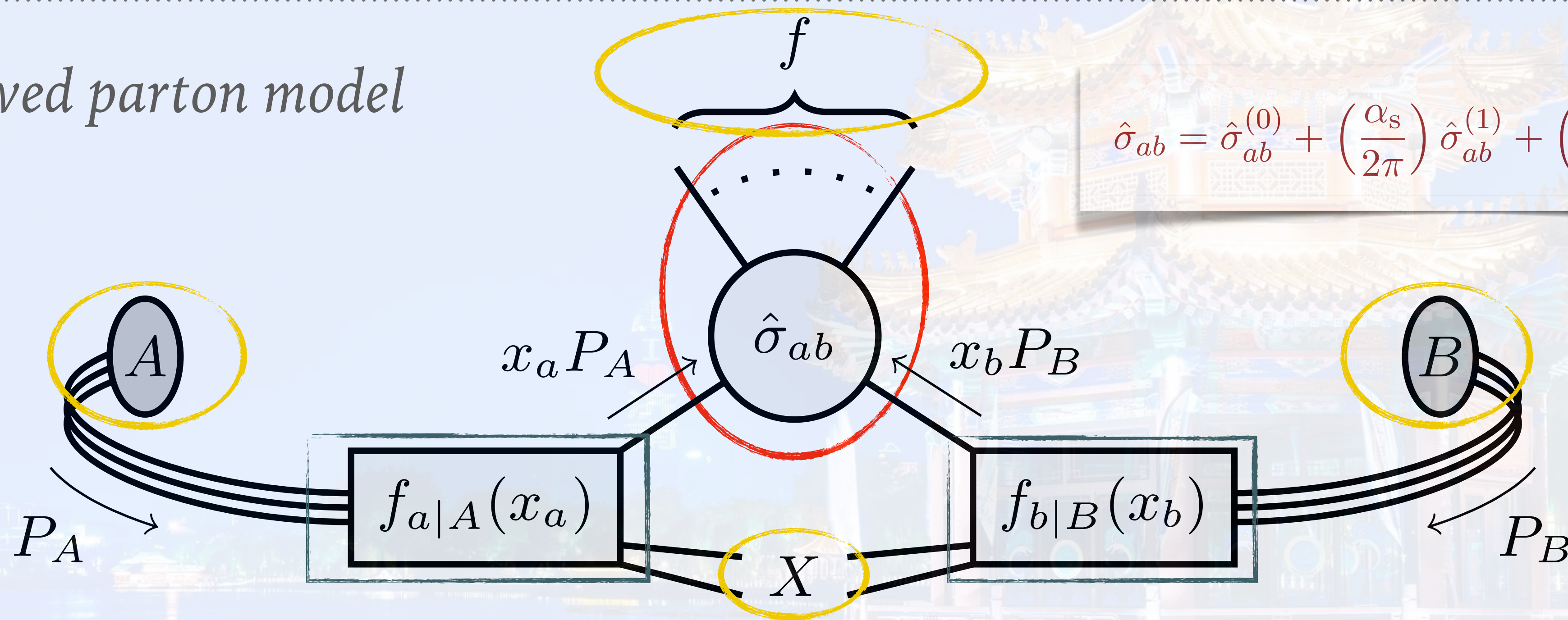
$\pm 3\text{-}5\%$ at LHC energy

Hard scattering
(Perturbative quantum field theory)

$\pm 1\sim 3\%$ level!

Precision Predictions at Hadron Collider

QCD improved parton model



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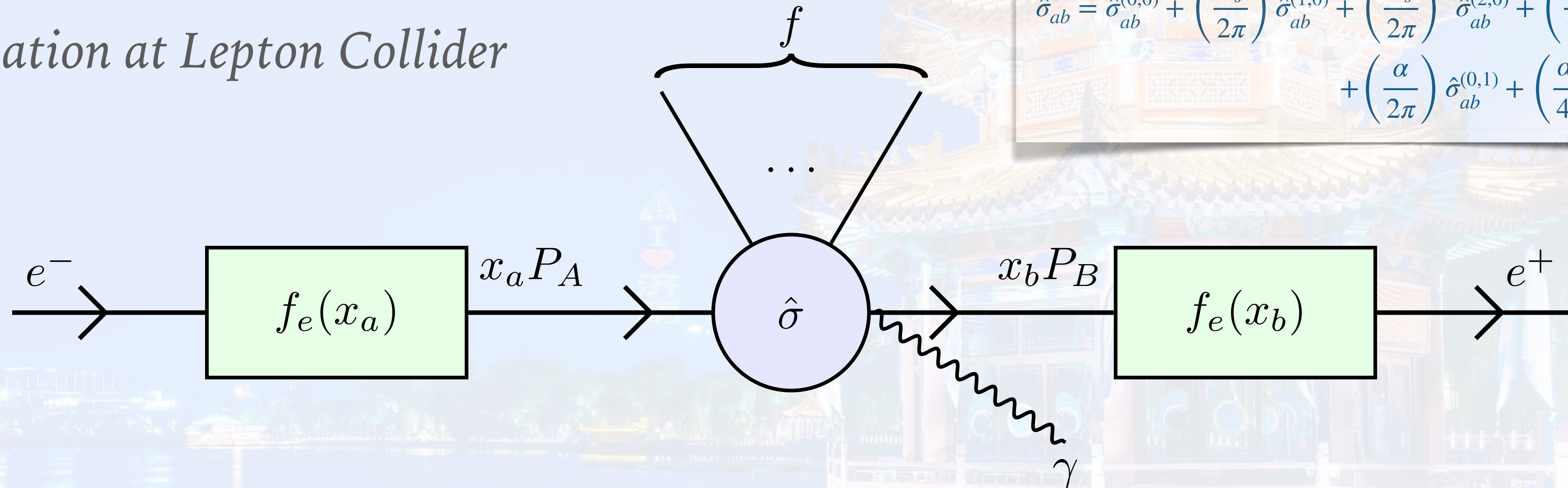
$\pm 1\sim 3\%$ level!

non-perturbative effects
(Fragmentation, hadronisation)

$\pm \Lambda_{\text{QCD}}/\sqrt{s}$

PRECISION PREDICTIONS AT LEPTON COLLIDER

Factorisation at Lepton Collider



$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0,0)} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}_{ab}^{(1,0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2,0)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \hat{\sigma}_{ab}^{(3,0)} + \dots$$

$$+ \left(\frac{\alpha}{2\pi}\right) \hat{\sigma}_{ab}^{(0,1)} + \left(\frac{\alpha\alpha_s}{4\pi^2}\right) \hat{\sigma}_{ab}^{(1,1)} + \dots$$

$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b f_{a|A}(x_a) f_{b|B}(x_b) \hat{\sigma}_{ab}(x_a, x_b) (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

Electron PDF at NLO+NLL
(Determined by first principle)
 $\pm 0.1 \sim 1 \%$ at $\mathcal{O}(10^2)$ GeV

Hard scattering (Perturbative QFT)
 $\pm 5 \sim 10 \%$ for jets
 $\pm 0.01 \sim 2 \%$ for leptons

non-perturbative effects
(Fragmentation, hadronisation)
 $\pm \Lambda/\sqrt{\hat{s}}, \delta(\mathcal{O} - c_{\mathcal{O}}/\sqrt{\hat{s}})$

State-of-the-Art QCD Calculations @ NNLO

Marcoli's slide @ Loop Summit 2

Sector improved residue subtraction,
STRIPPER

qT slicing, MATRIX

Antenna subtraction,
NNLOJET

pp→γγj

pp→Htt
soft approx.
massification (LC)

pp→Wtt
soft approx.
massification (LC)

pp→Wbb
massification (LC)

pp→Htt
soft approx.
massification (LC)

pp→γγγ
LC

qT slicing + PS, MiNNLOPS

pp→Zbb
massification (LC)

pp→jjj
LC, gluons

pp→γγj

pp→jjj
LC

pp→Wbb
LC, mb=0

pp→jjj
LC

pp→γγj
LC

pp→γγγ
LC

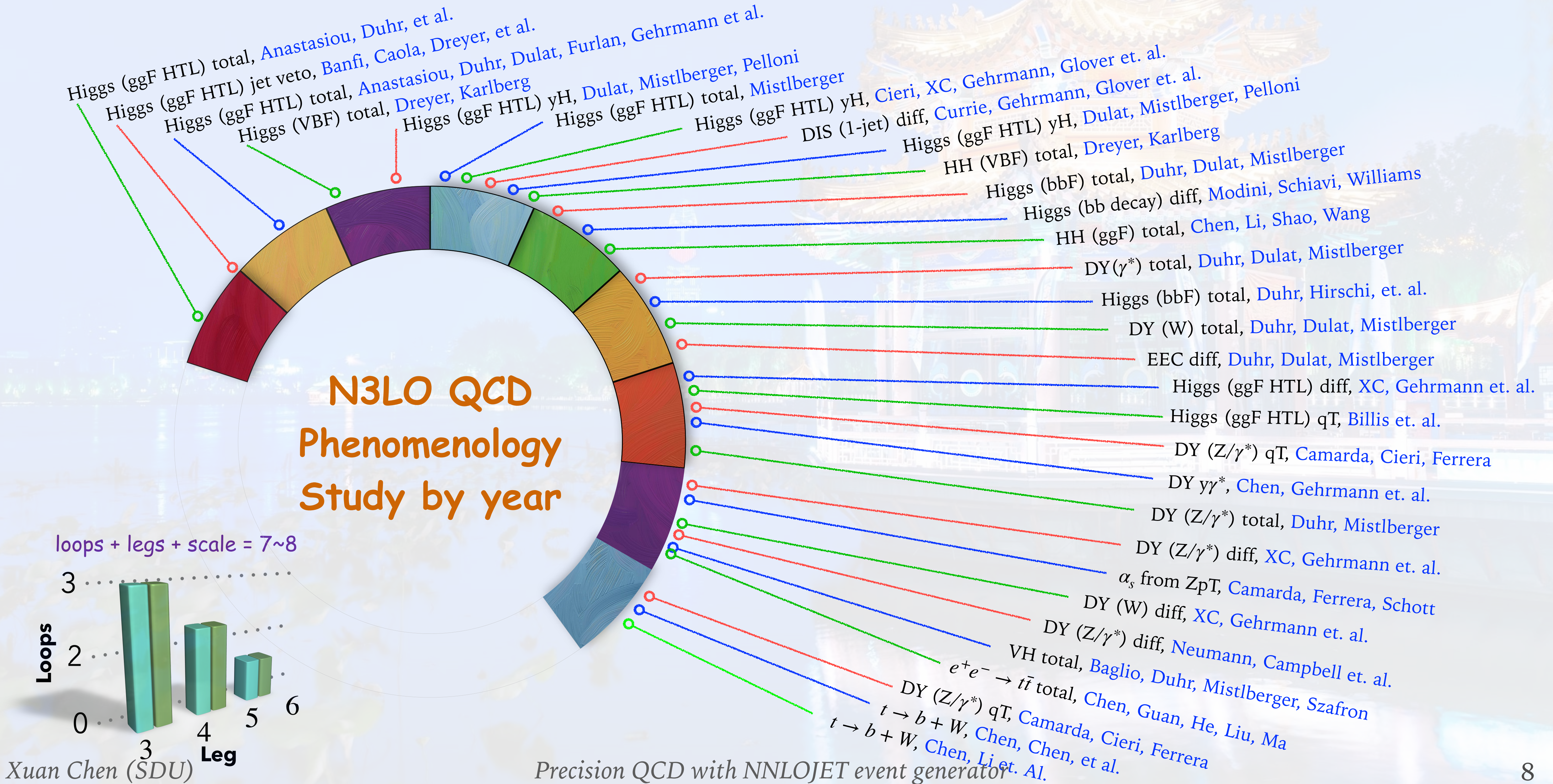
Precise predictions for $t\bar{t}H$ production at the LHC: inclusive cross section and differential distributions Simone Devoto^a, Massimiliano Grazzini^b, Stefan Kallweit^b, Javier Mazzitelli^c and Chiara Savoini^d ^a Dipartimento di Fisica e Astronomia, Università di Padova PHYSICAL REVIEW LETTERS 131, 231901 (2023)	Precise Predictions for the Associated Production of a W Boson with a Top-Antitop Quark Pair at the LHC Luca Buonocore^a, Simone Devoto^a, Massimiliano Grazzini^b, Stefan Kallweit^b, Javier Mazzitelli^c, Luca Rottoli^a and Chiara Savoini^d ^a Dipartimento di Fisica e Astronomia, Università di Padova PHYSICAL REVIEW D 107, 074032 (2023)	Associated production of a W boson and massive bottom quarks at next-to-next-to-leading order in QCD Luca Buonocore^a, Simone Devoto^a, Stefan Kallweit^b, Javier Mazzitelli^c, Luca Rottoli^a and Chiara Savoini^d ^a Dipartimento di Fisica e Astronomia, Università di Padova PHYSICAL REVIEW LETTERS 130, 111902 (2023)	Higgs Boson Production in Association with a Top-Antitop Quark Pair in Next-to-Next-to-Leading Order QCD Stefano Catani^a, Simone Devoto^a, Massimiliano Grazzini^b, Stefan Kallweit^b, Javier Mazzitelli^c and Chiara Savoini^d ^a INFN, Sezione di Firenze e Dipartimento di Fisica e Astronomia, Università degli Studi di Firenze PHYSICAL REVIEW LETTERS 127, 152001 (2021)	Triphoton production at hadron colliders in NNLO QCD Stefan Kallweit^a, Vasily Sotnikov^b, Marius Wiesemann^b ^a Dipartimento di Fisica G. Galilei, Università degli Studi di Milano-Bicocca and INFN, Piazza della Scienza 3, 20126 Milano, Italy ^b Max-Planck-Institut für Physik, Föhringer Ring 6, 80805 München, Germany ARTICLE INFO ABSTRACT Article history: Received 19 October 2020 Received in revised form 13 November 2020 Accepted 7 December 2020 Available online 9 December 2020 Editor: G.F. Giudice We present next-to-next-to-leading-order (NNLO) QCD corrections to the production of three isolated photons in hadronic collisions at the fully differential level. We employ q_T subtraction within MATRIX and an efficient implementation of analytic two-loop amplitudes in the leading-colour approximation to achieve the first on-the-fly calculation for this process at NNLO accuracy. Numerical results are presented for proton-proton collisions at energies ranging from 7 TeV to 100 TeV. We find full agreement with the 8 TeV results of Ref. [1] and confirm that NNLO corrections are indispensable to describe ATLAS 8 TeV data. In addition, we demonstrate the significance of NNLO corrections for future precision studies of triphoton production at higher collision energies. Precision studies have become of major importance in the rich physics programme at the Large Hadron Collider (LHC). Many LHC reactions, in particular $2 \rightarrow 1$ and $2 \rightarrow 2$ processes, are not only measured, but also predicted at a remarkable accuracy by now. Prime examples are colour singlet processes, such as vector-boson pair production, cf. for instance the recent $Z\gamma$ [2] and ZZ [3] mea- tor bosons gives direct access to anomalous quartic gauge couplings, e.g. the $Z \rightarrow \gamma\gamma\gamma$ decay has been constrained in Ref. [38]. Furthermore, the triphoton final state is important to constrain anomalous Higgs couplings in rare Higgs boson decays [39–41] or in the rare Higgs boson production process in association with a photon [42] with the Higgs boson decaying into a pair of pho- © 2020 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP³.
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pp→Hbb
massification

Precision QCD with NNLOJET event generator

Xuan Chen (SDU)

Perturbative QCD @ N3LO



State-of-the-Art QCD Calculations @ N3LO

- Several phenomenologically relevant results despite the extreme complexity.
- Available techniques are applicable to limited cases with high quality EXP data.
- New approaches must be developed for more complicated scattering.

(10 M → X00 k CPU hours)

- ◇ qT slicing
- ⊙ τ slicing
- * Projection-2-Born
- † Antenna subtraction

Precision QCD with NNLOJET event generator

PHYSICAL REVIEW LETTERS 128, 052001 (2022)

Dilepton Rapidity Distribution in Drell-Yan Production to Third Order in QCD

Xuan Chen^{1,2,3,*}, Thomas Gehrmann^{1,†}, Nigel Glover^{1,‡}, Alexander Huss^{1,§}, Tong-Zhi Yang^{1,||} and Hua Xing Zhu^{6,¶}

PHYSICAL REVIEW LETTERS 128, 252001 (2022)

Third-Order Fiducial Predictions for Drell-Yan Production at the LHC

Xuan Chen^{1,2}, Thomas Gehrmann³, Nigel Glover⁴, Alexander Huss⁵, Pier Francesco Monni⁵, Emanuele Re^{6,7}, Luca Rottoli³ and Paolo Torrielli⁸

Physics Letters B 840 (2023) 137876

Transverse mass distribution and charge asymmetry in W boson production to third order in QCD

Xuan Chen^{a,b,c,*}, Thomas Gehrmann^c, Nigel Glover^d, Alexander Huss^e, Tong-Zhi Yang^c, Hua Xing Zhu^f

Third order QCD predictions for fiducial W-boson production

John Campbell^a and Tobias Neumann^b

Fully differential Higgs boson pair production at N³LO with top quark mass effects

Xuan Chen^a, Yuesheng Dai^a, Hai Tao Li^a, Shi-Yuan Li^a, Hua-Sheng Shao^b and Jian Wang^{a,c}

Next-to-next-to-next-to-leading order QCD corrections to photon-pair production

Michał Czakon^{1,†}, Felix Eschment^{1,‡}, Terry Generet^{2,§} and Rene Poncelet^{3,§}

pp → $\gamma\gamma$

pp → $\gamma\gamma$

PHYSICAL REVIEW LETTERS 127, 072002 (2021)

pp → H

X. Chen^{1,2,3}, T. Gehrmann¹, E. W. N. Glover⁴, A. Huss⁵, B. Mistlberger⁶, and

Fully Differential Higgs Boson Production to Third Order in QCD

ep → J

Liang Dong^{1,*}, Shen Fang^{2,†}, Jun Gao^{1,‡}, Hai Tao Li^{3,§}, Ding Yu Shao^{2,4,5,¶}, Hua Xing Zhu^{6,5,**} and Yu Jiao Zhu^{7,††}

Two-Dimensional Transverse-Momentum Subtraction and Semi-Inclusive Deep-Inelastic Scattering at N³LO in QCD

N³LO predictions for the decay of the Higgs boson to bottom quarks

H → bb*

PHYSICAL REVIEW LETTERS 134, 251905 (2025)

H → JJ*

Jet Rates in Higgs Boson Decay at Third Order in QCD

Elliot Fox¹, Aude Gehrmann-De Ridder^{2,3}, Thomas Gehrmann³, Nigel Glover¹, Matteo Marcoli¹ and Christian T. Preuss⁴

Phys. Lett. B 869 (2025) 139804

Jet production at electron-positron colliders at next-to-next-to-next-to-leading order in QCD

Xuan Chen^a, Petr Jakubčík^{b,*}, Matteo Marcoli^c, Giovanni Stagnitto^d

e⁺e⁻ → JJ[†]

NNLOJET: Parton Level Event Generator



**A parton-level event generator
for jet cross sections at NNLO QCD accuracy**

About NNLOJET is a parton-level event generator for jet cross sections using the antenna subtraction method. It can be used to compute a large number of jet cross sections and related observables in e^+e^- , ep and pp collisions at next-to-next-to-leading order in QCD. NNLOJET contains routines for Monte Carlo phase-space integration, event handling and analysis.

Citation If you are using NNLOJET for a scientific paper, please cite:

A. Huss et al. (NNLOJET Collaboration)
NNLOJET: a parton-level event generator for jet cross sections at NNLO QCD accuracy
[arXiv:2503.22804](https://arxiv.org/abs/2503.22804) [INSPIRE]

Please also cite the relevant references for each process (as included in the .bib file which is automatically written when running NNLOJET through the automatic workflow)

License [GNU General Public License \(GPL\) v3.0](https://www.gnu.org/licenses/gpl-3.0.html)

Contact Please send comments, questions and suggestions to nnlojet-support@cern.ch



NNLOJET collaboration meeting 2026 CERN

- NNLO parton level event generator
 - Based on antenna subtraction
- Provides infrastructure
 - Process management
 - Phase space, histogram routines
 - Validation and testing
- Parallel computing (MPI) support
- Typical runtimes: 60 k ~ 250 k core-hours

<https://nnlojet.hepforge.org/index.html>

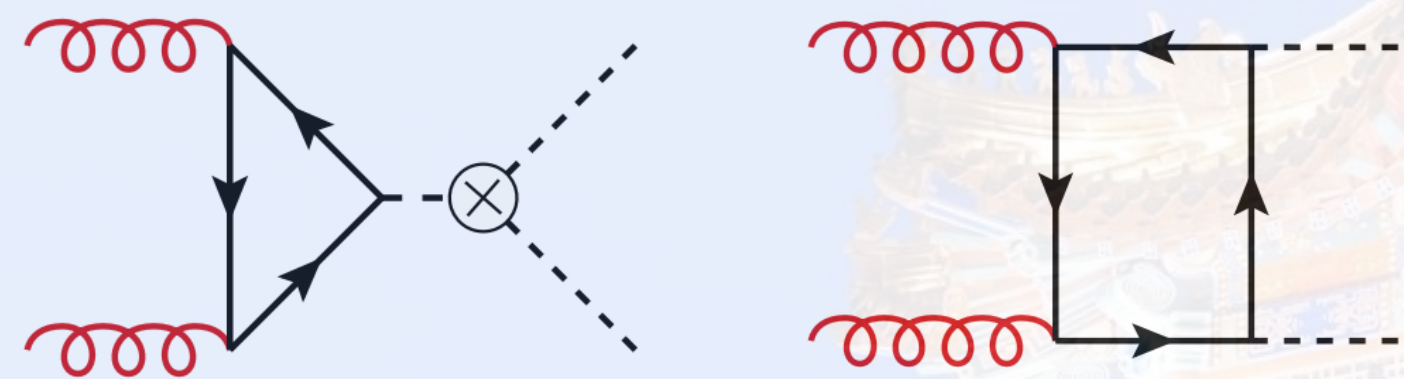
Application of **NNLOJET** at the LHC

$pp \rightarrow HH$ @ N3LO 2601.19990

$pp \rightarrow \gamma\gamma$ (+ jet) @ NN(N)LO 2501.14021

HH Production at N3LO QCD

► Feynman diagrams in the SM:



- Earlier Studies: **SM LO (1988)** $\Rightarrow \dots \Rightarrow$ **NLO (2016)**

[Nucl. Phys. B 309 \(1988\) 282-294](#)

[JHEP 10 \(2016\) 107](#)

[PRL 117 \(2016\) 1, 012001](#)

$\sqrt{s}$ (TeV)	NLO _{m_t} (fb)	NLO _{∞} (fb)
13	27.56 ^{+14%} _{-13%}	25.81 ^{+18%} _{-15%}
14	32.64 ^{+14%} _{-12%}	31.89 ^{+18%} _{-15%}
27	126.2 ^{+12%} _{-10%}	183.0 ^{+16%} _{-14%}
100	1119 ^{+13%} _{-13%}	3724 ^{+13%} _{-11%}

► Heavy Top Limit (HTL): Integrate out top quarks ($m_t \rightarrow \infty$)

► Effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} \left(C_h \frac{h}{v} - C_{hh} \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G^{a\mu\nu} + \mathcal{O}(h^k, k \geq 3)$$

- Couplings: $C_h = -\frac{\alpha_s}{3\pi}(1 + \delta)$, $C_{hh} = -\frac{\alpha_s}{3\pi}(1 + \delta + \eta)$

- Wilson Coefficients: $\delta = \sum_{i=0} \left(\frac{\alpha_s}{4\pi} \right)^i \delta^{(i)}$, $\eta = \sum_{i=0} \left(\frac{\alpha_s}{4\pi} \right)^i \eta^{(i)}$

HTL NLO (1998) $\Rightarrow$ NNLO (2013)

[Phys. Rev. D 58 \(1998\) 115012](#)

[Phys. Rev. Lett. 111 \(2013\) 201801](#)

[Nucl. Phys. B 888 \(2014\) 17-29](#)

$\Rightarrow$ NNLO FTa (2018) $\Rightarrow$ N3LO (2020)

[JHEP 05 \(2018\) 059](#)

[Phys. Lett. B 803 \(2020\) 135292](#)

[JHEP 03 \(2020\) 072](#)

► Feynman diagrams in the HTL:



- HH production: $2m_H < \sqrt{\hat{s}}$

- HTL Valid condition: $\sqrt{\hat{s}} \ll 2m_T$

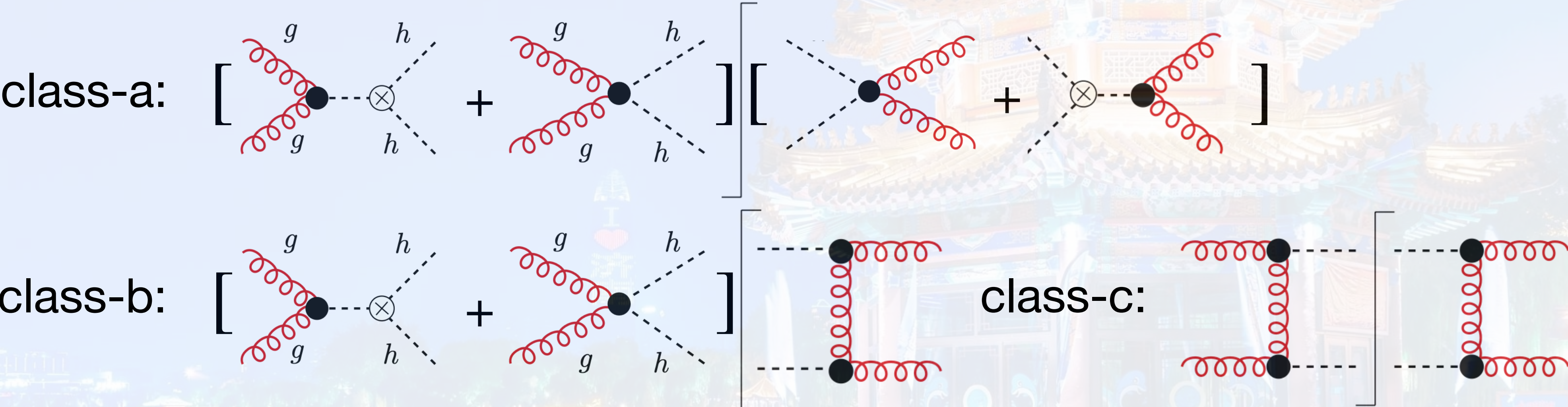
$\Rightarrow$ **Top-quark mass effects**

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
LO	13.80 ^{+31%} _{-22%}	17.06 ^{+31%} _{-22%}	98.22 ^{+26%} _{-19%}	2015 ^{+19%} _{-15%}
NLO	25.81 ^{+18%} _{-15%}	31.89 ^{+18%} _{-15%}	183.0 ^{+16%} _{-14%}	3724 ^{+13%} _{-11%}
NNLO	30.41 ^{+5.3%} _{-7.8%}	37.55 ^{+5.2%} _{-7.6%}	214.2 ^{+4.8%} _{-6.7%}	4322 ^{+4.2%} _{-5.3%}
N ³ LO	31.31 ^{+0.66%} _{-2.8%}	38.65 ^{+0.65%} _{-2.7%}	220.2 ^{+0.53%} _{-2.4%}	4439 ^{+0.51%} _{-1.8%}

Total cross section + m_{hh} distribution

HH Production at N3LO QCD

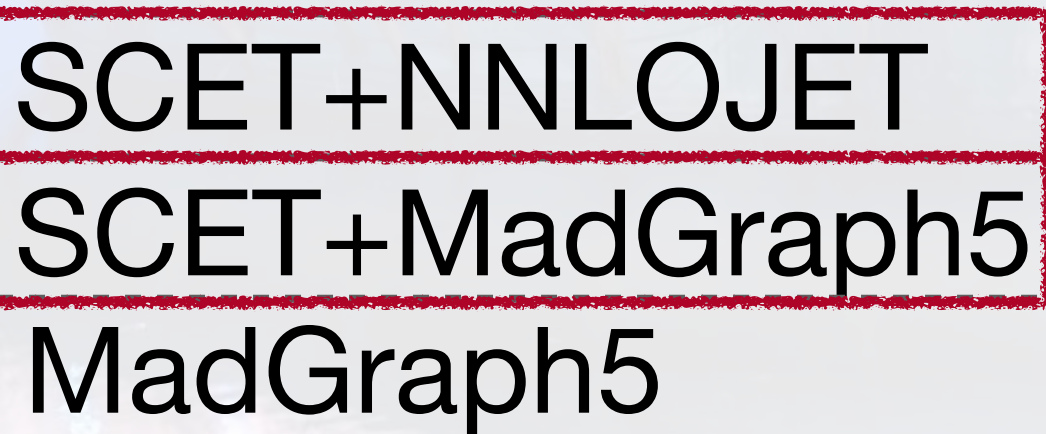
➤ According to the number of **effective** couplings: $d\sigma_{hh} = d\sigma_{hh}^a + d\sigma_{hh}^b + d\sigma_{hh}^c$



➤ Scheme of each class (fully differential):

	LO	NLO	NNLO	N ³ LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
class-a	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
class-b	0	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
class-c	0	0	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$

➤ Usage of packages:



NNLOJET (derived from single H) **qt-slicing:**

$$d\sigma_{hh} = d\sigma_{hh} \Big|_{p_T^{hh} < p_T^{\text{veto}}} + d\sigma_{hh} \Big|_{p_T^{hh} > p_T^{\text{veto}}}$$

HH Production at N3LO QCD

- Apply q_T -slicing at N3LO with **SCET factorisation** and expand to N3LO:

$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} \sum_p \sigma_{LO}^{HH} H_{gg} \left[\sum_k \int_{x_1}^1 \frac{dz_1}{z_1} I_{pk}(z_1, b_T^2, \mu) f_{k/h_1}(x_1/z_1, \mu) \right. \\ \left. \times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} I_{pj}(z_2, b_T^2, \mu) f_{j/h_2}(x_2/z_2, \mu) S(b_\perp, \mu) \right] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

- All factorised functions are known @ N3LO:

1) 3-loop hard function $H_{gg}^{(3)}$:

Baikov, Chetyrkin et. al., Phys.Rev.Lett. 102 (2009) 212002, Gehrmann, Glover et. al., JHEP 06 (2010) 094

2) TMD soft function $S(b_\perp, \mu)$ at α_s^3 : *Li, Zhu, Phys.Rev.Lett. 118 (2017) 022004*

3) Matching kernel of TMD beam function $I_{pk}(z, b_\perp, \mu)$ at α_s^3 :

Luo, Yang, Zhu, Zhu, Phys.Rev.Lett 124 (2020) 092001, Ebert, Mistlberger, Vita, JHEP 09 (2020) 146

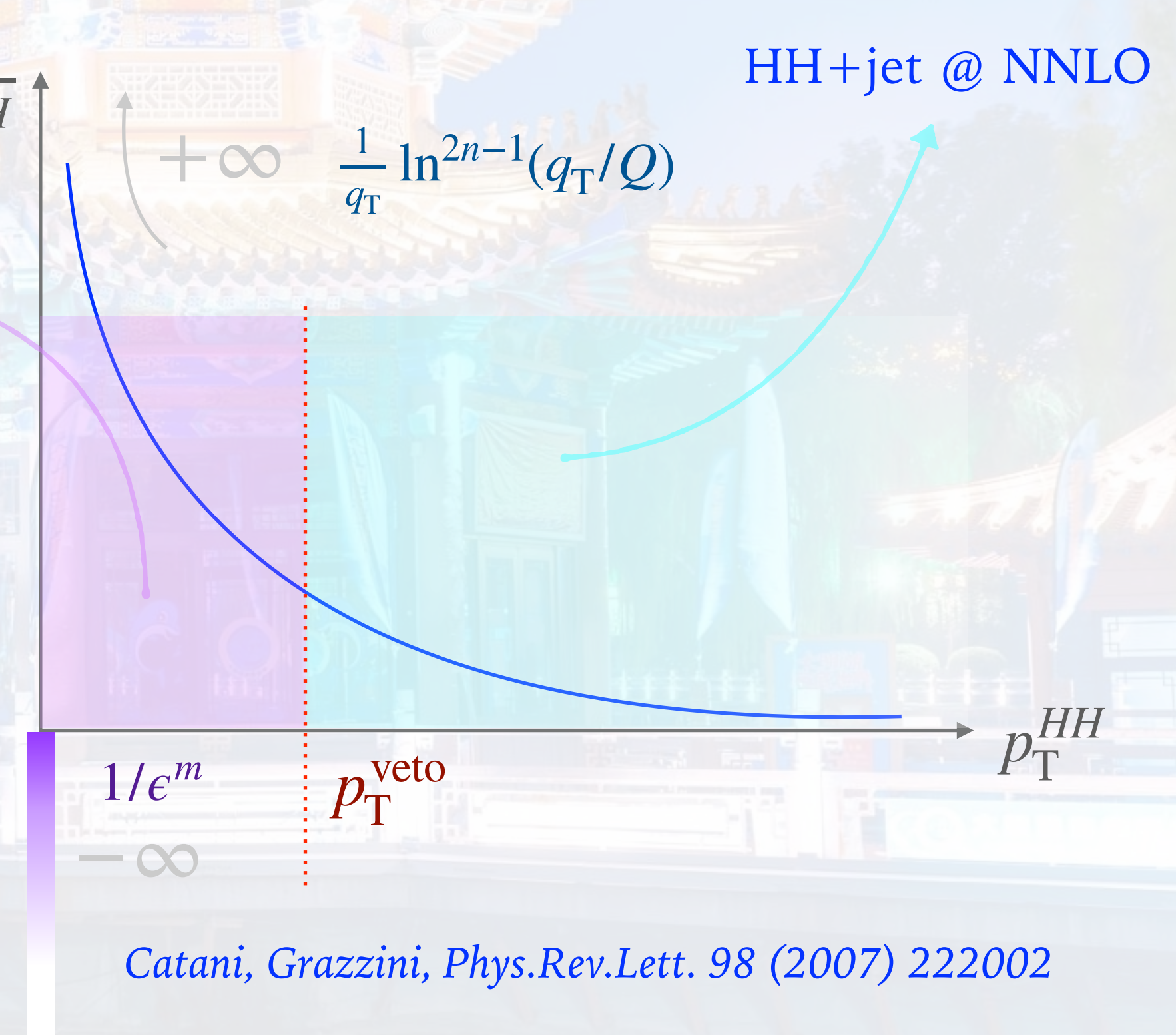
- Apply p_T^{veto} to factorise N3LO contribution into two parts:

$$d\sigma_{N^3LO}^{HH,a} = [\mathcal{H}^{HH} \otimes d\sigma^{HH}]_{N^3LO} \Big|_{\delta(p_T^{HH})} + [d\sigma_{NNLO}^{HH+jet} - d\sigma_{N^3LO}^{HH CT}]_{p_T^{HH} > p_T^{\text{veto}}} + \mathcal{O}((p_T^{\text{veto}}/Q)^2)$$

Competing interests: p_T^{veto} as small as possible $\leftrightarrow$ p_T^{veto} as large as possible

$\hookrightarrow$ suppress power corrections

$\hookrightarrow$ numerical stability & efficiency



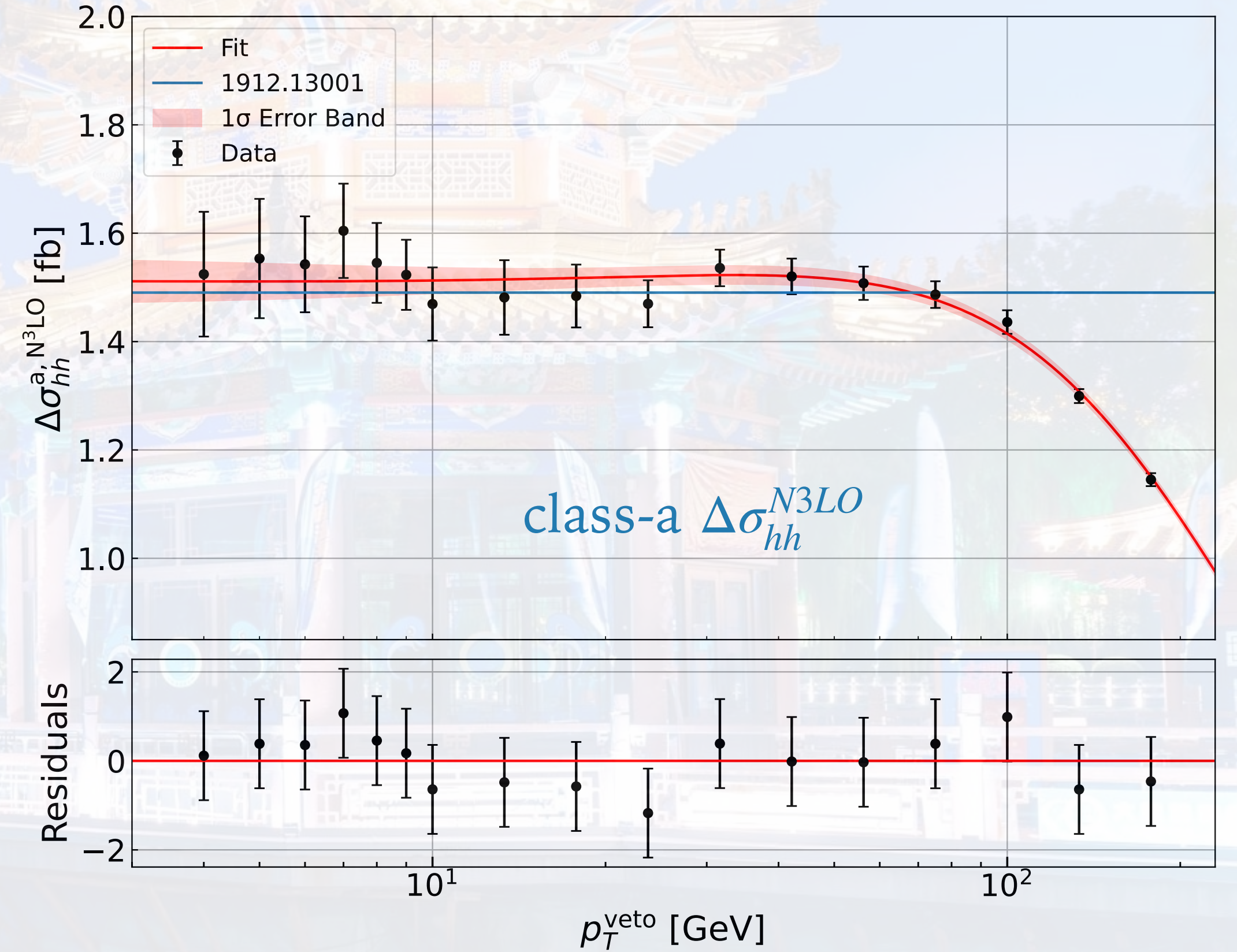
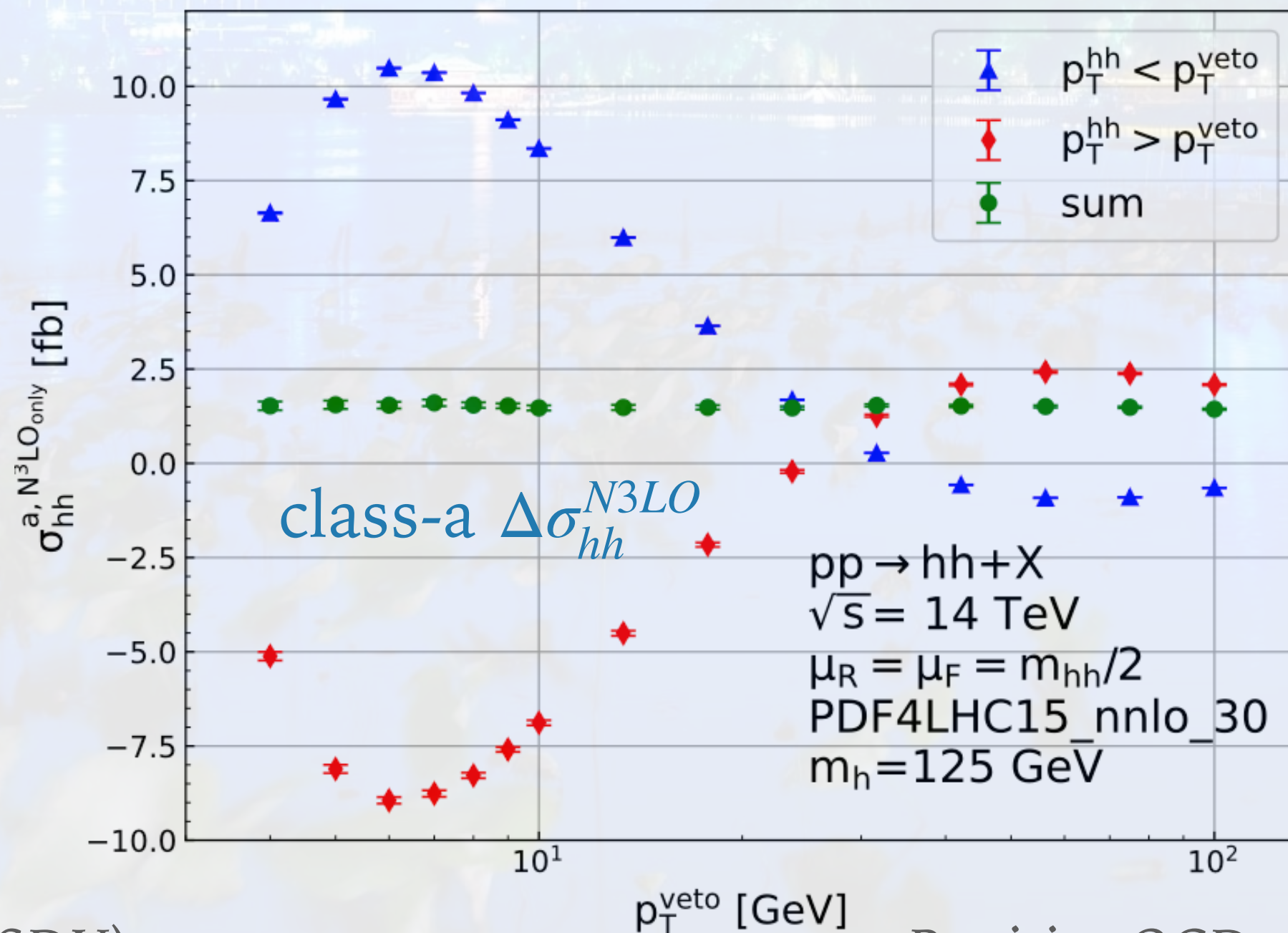
HH Production at N3LO QCD

- Class-a contribution achieve N3LO accuracy via q_T -slicing:

$$d\sigma_{hh} = d\sigma_{hh} \Big|_{p_T^{hh} < p_T^{\text{veto}}} + d\sigma_{hh} \Big|_{p_T^{hh} > p_T^{\text{veto}}}$$

- Large cancellation between above and below p_T^{veto} cut.
- Fit the p_T^{veto} power correction with χ^2 and functional form:

$$\Delta\sigma^{\text{N}^3\text{LO}}(p_T^{\text{veto}}) \sim \Delta\sigma_0^{\text{N}^3\text{LO}} + \frac{\alpha_s^3}{(2\pi)^3} \frac{p_T^{\text{veto}^2}}{m_{hh}^2} \sum_{n=0}^5 a_n^{(3)} \ln^n \frac{m_{hh}}{p_T^{\text{veto}}}$$



XC, Y.H. Dai, H.T. Li, S.Y. Li, H.S. Shao, J. Wang, JHEP 06 (2026)

- p_T^{veto} plateau observed $< 10 \text{ GeV}$.

- Excellent agreement with inclusive total XS:

N3LO only ($p_T^{\text{veto}} \rightarrow 0$): **1.513(47) fb**

N3LO only reference: **1.4904(1) fb** (based on iHixs 2)

F. Dulat, A. Lazopolulos, B. Mistlberger, Comput.Phys.Commun 233 (2018)

HH Production at N3LO QCD

► Differential XS calculation setup:

- Fiducial cuts:

$$p_T^{h1st} > 30 \text{ GeV} \quad p_T^{h2nd} > 20 \text{ GeV} \quad |y_h| < 2.4$$

- Centre of mass energy: $\sqrt{s} = 14 \text{ TeV}$

- PDF: NNPDF40_an3lo_as_01180

- SM Parameters:

$$\nu = 246.2 \text{ GeV} \quad m_h = 125 \text{ GeV} \quad m_t = 173 \text{ GeV}$$

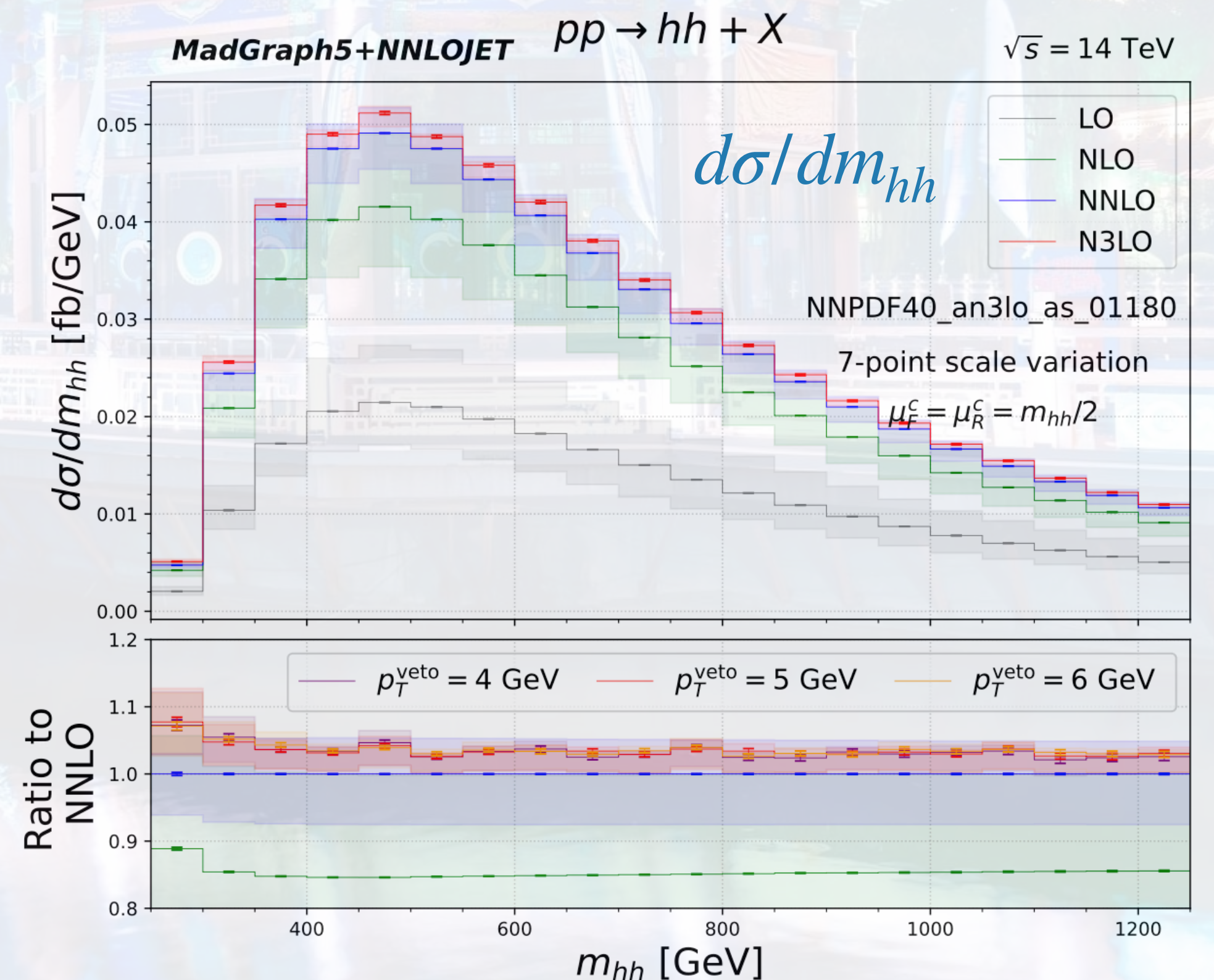
- Central scale: $\mu_F = \mu_R = m_{hh}/2$

- Scale variation: $\mu_R \rightarrow k_{\mu_R} \mu_R, \mu_F \rightarrow k_{\mu_F} \mu_F$

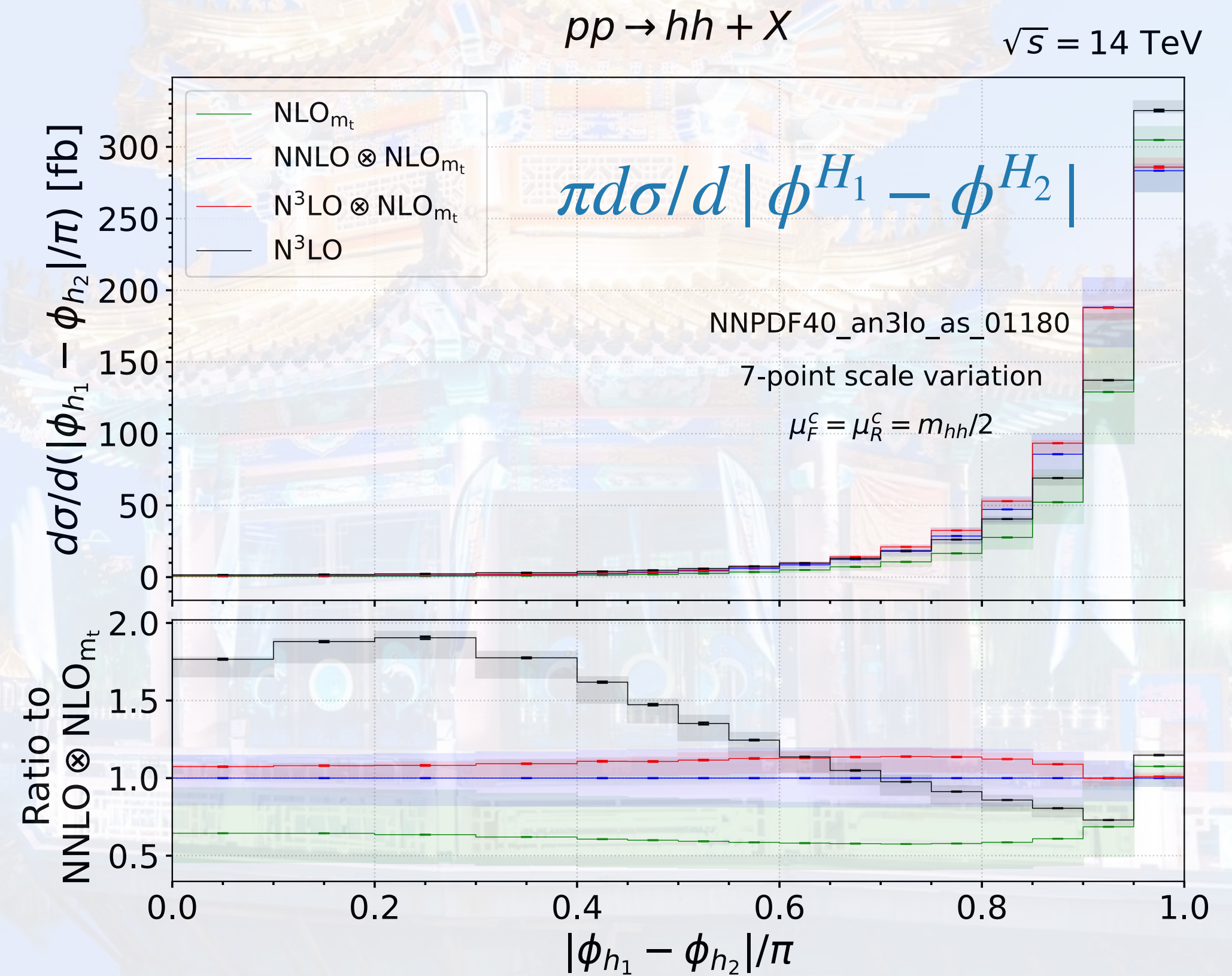
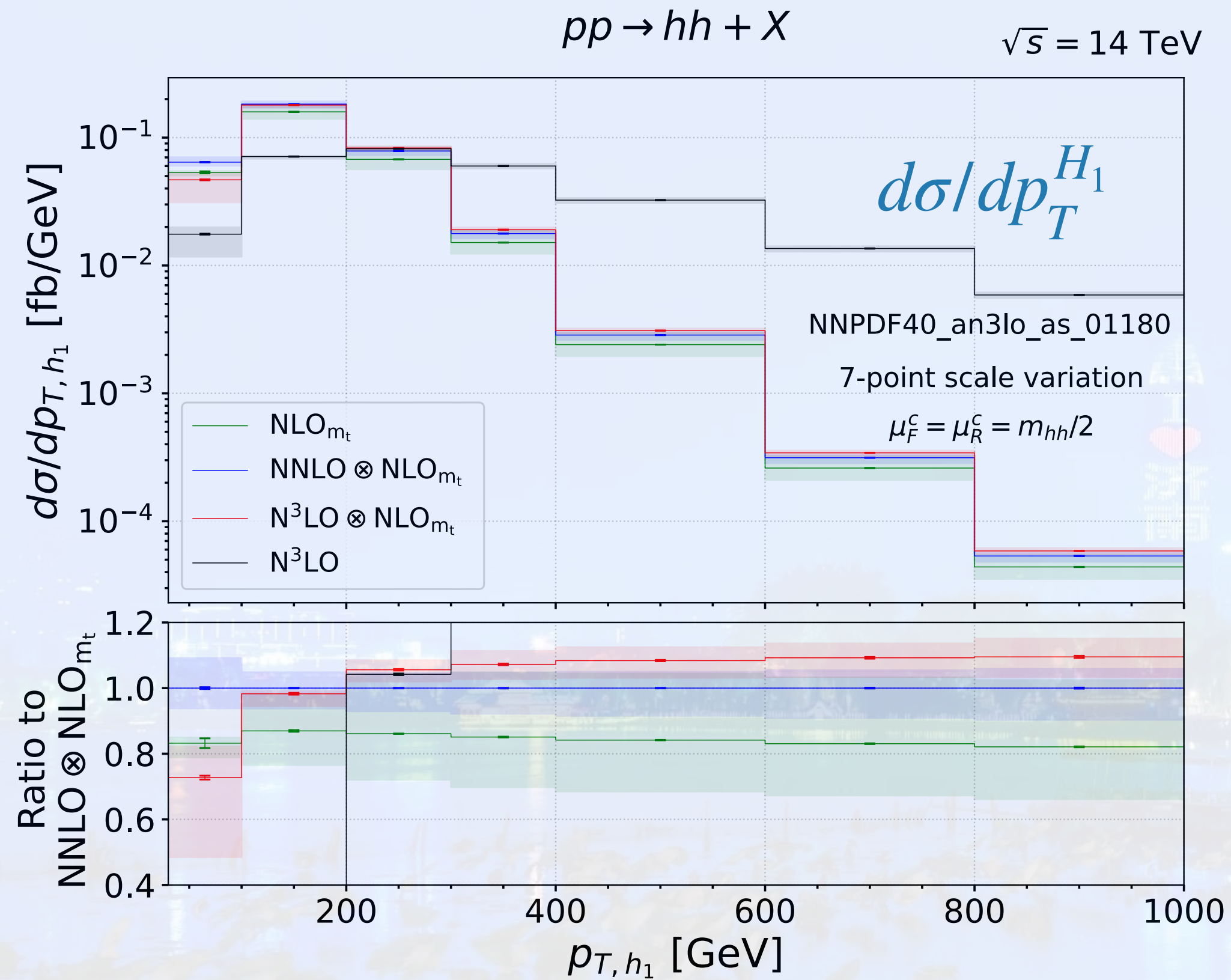
$$(k_{\mu_R}, k_{\mu_F}) \in \{(1,1), (1,1/2), (1,2), (1/2,1), (2,1), (1/2,1/2), (2,2)\}$$

XC, Y.H. Dai, H.T. Li, S.Y. Li, H.S. Shao, J. Wang, JHEP 06 (2026)

	σ_{hh}^{LO} [fb]	σ_{hh}^{NLO} [fb]	$\sigma_{hh}^{\text{NNLO}}$ [fb]	$\sigma_{hh}^{\text{N}^3\text{LO}}$ [fb]
Sum	$14.85^{+30\%}_{-22\%}$	$27.87^{+18\%}_{-15\%}$	$32.74^{+5.2\%}_{-7.3\%}$	$34.00(4)^{+1.4\%}_{-2.9\%} \pm 0.05\%$
Class-a	$14.85^{+30\%}_{-22\%}$	$28.25^{+18\%}_{-15\%}$	$33.62^{+5.7\%}_{-7.7\%}$	$35.20(4)^{+1.7\%}_{-3.4\%} \pm 0.02\%$
Class-b	0	$-0.3758(2)^{+42\%}_{-28\%}$	$-0.8854(2)^{+29\%}_{-12\%}$	$-1.216(3)^{+13\%}_{-14\%} \pm 1\%$
Class-c	0	0	$0.003065(3)^{+55\%}_{-34\%}$	$0.008865(3)^{+41\%}_{-28\%}$



HH Production at N3LO QCD



- Fully differential predictions from event generators with nontrivial N3LO corrections to the shape of distributions.
- Excellent fixed order convergence with scale uncertainties **reduced by more than 60%**.
- Reference $p_T^{veto} = 5$ GeV with stable predictions by varying p_T^{veto} at 4 and 6 GeV (negligible p_T^{veto} dependence)
- **Top mass effects** at NLO included by reweighting (available from ggxy package) [Davies et. al. Comput.Phys.Commun. 320 \(2026\) 109933](#)
- Included in **LHCHWG YR5 report** as inclusive and differential benchmarks [Ajath, Alioli et. al. 2609.04868](#)

Application of **NNLOJET** at the LHC

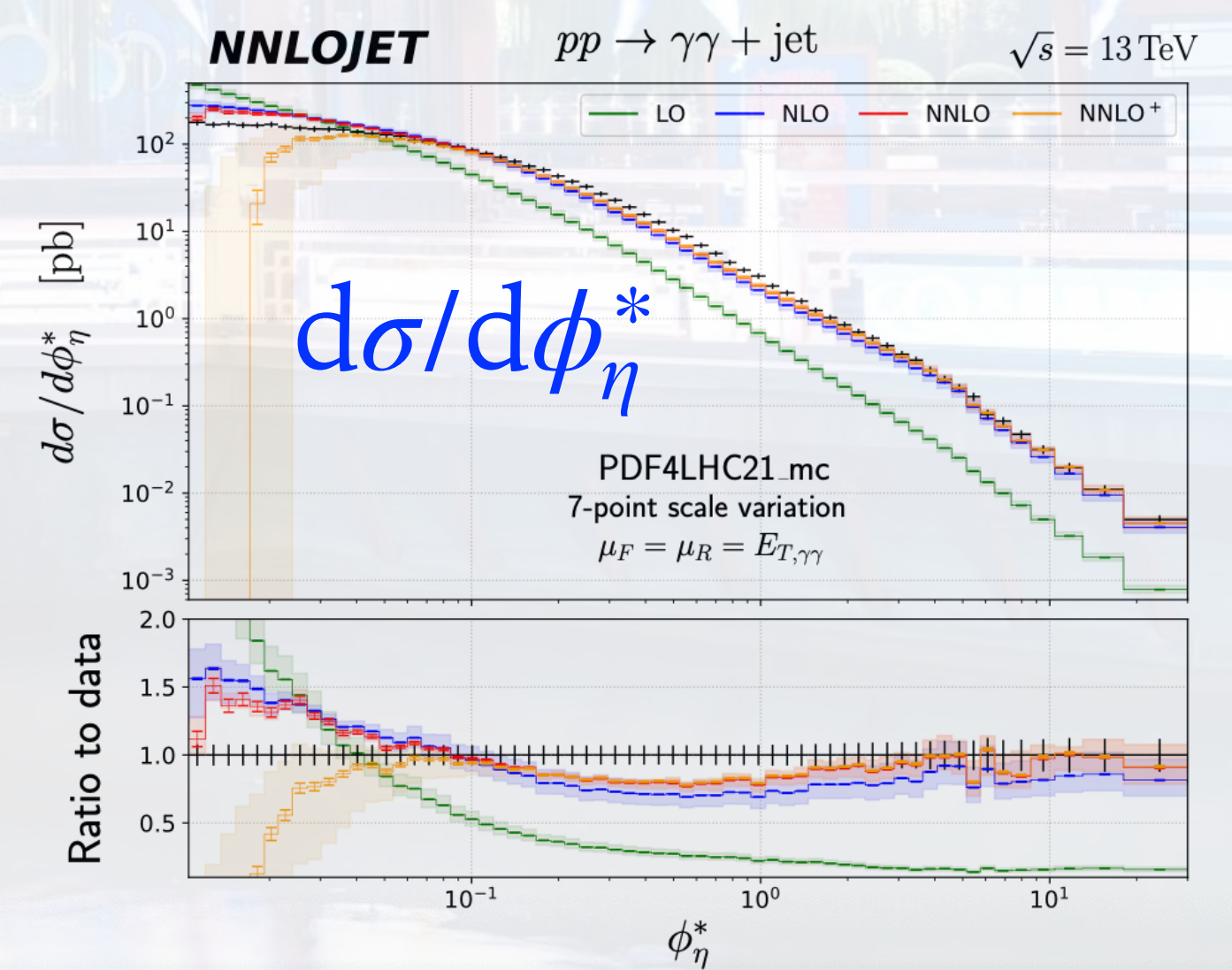
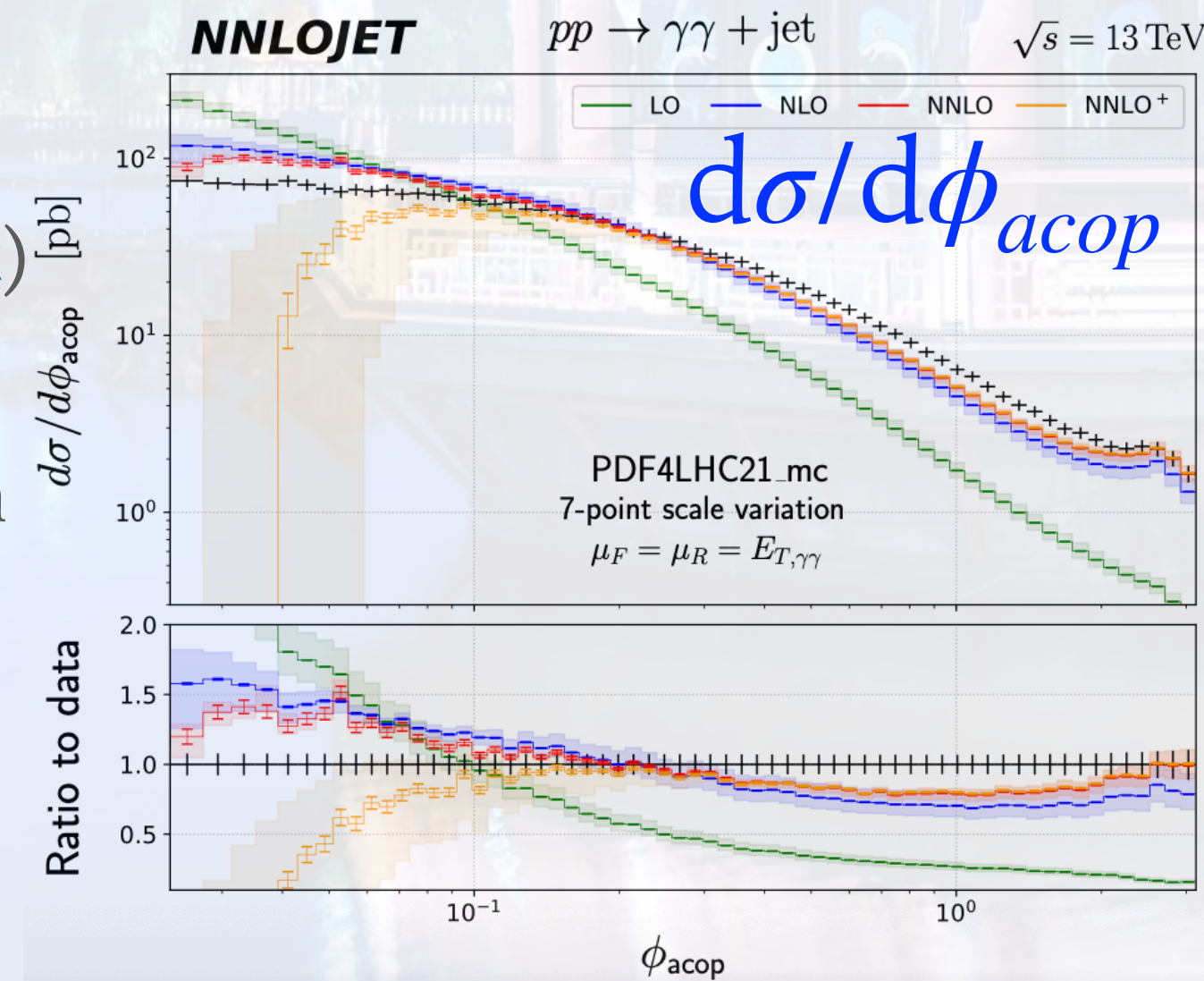
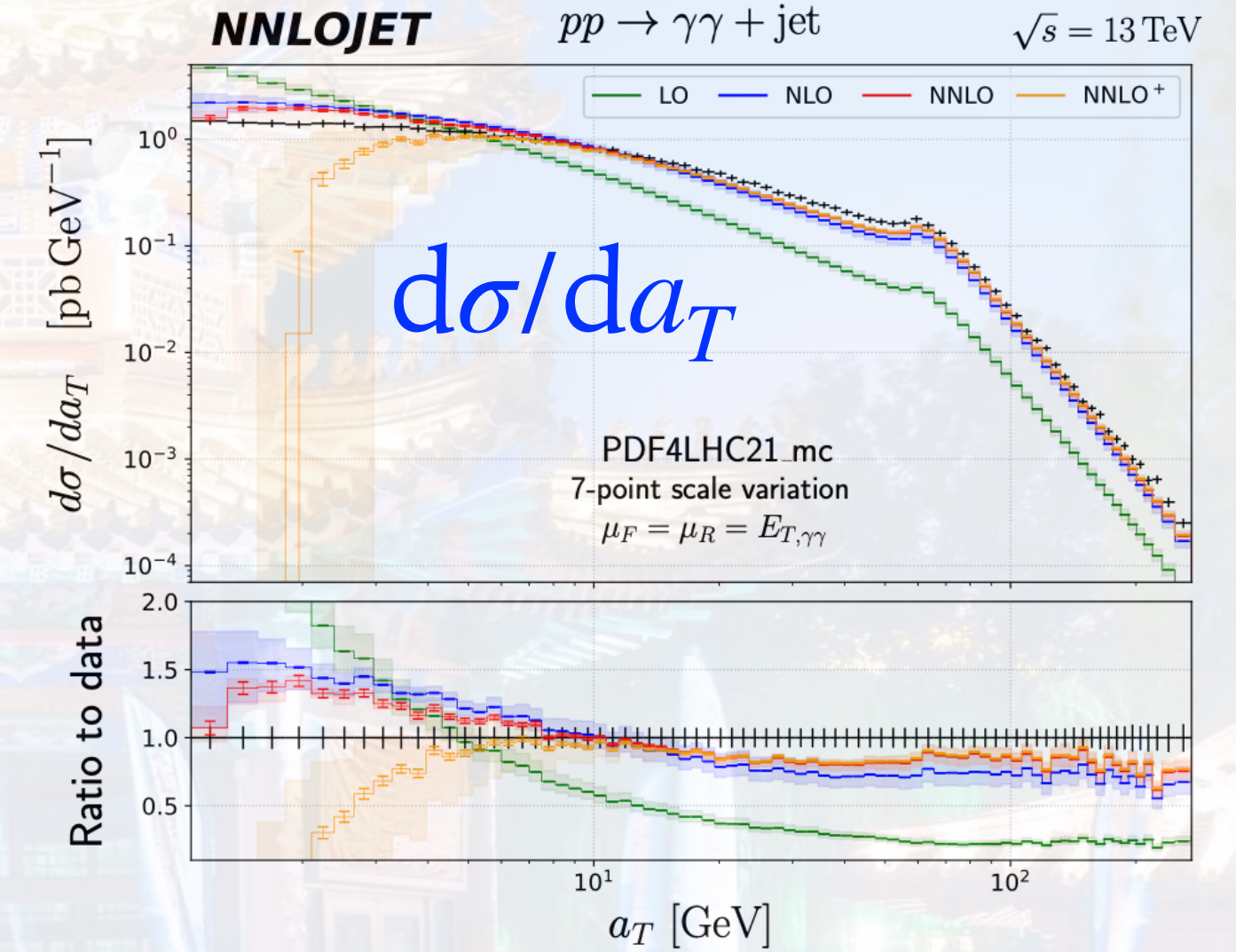
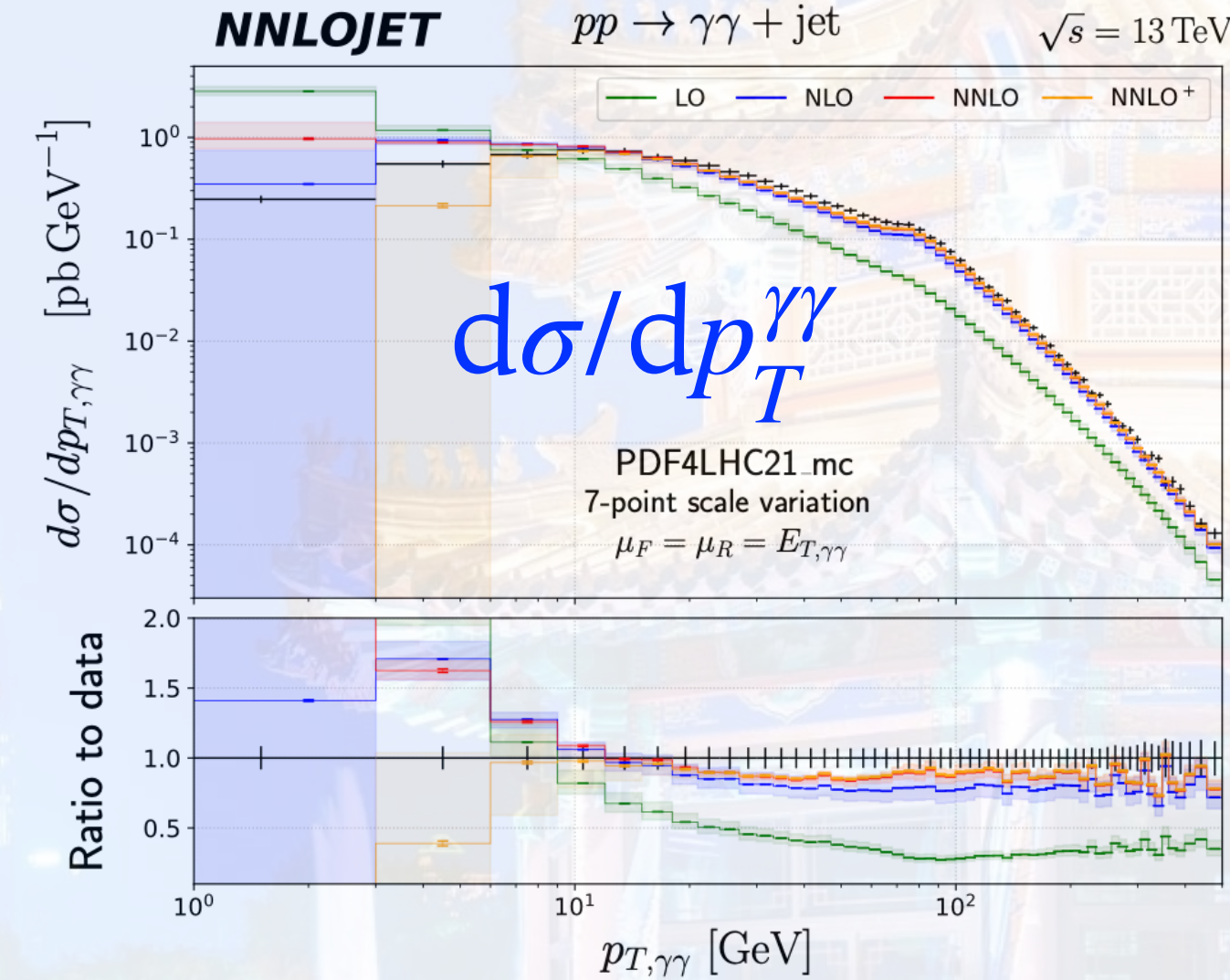
$pp \rightarrow HH$ @ N3LO 2601.19990

$pp \rightarrow \gamma\gamma (+ \text{jet})$ @ NN(N)LO 2501.14021

$\gamma\gamma$ + Recoil Production at NNLO QCD

$pp \rightarrow \gamma\gamma + \text{recoil} @ \text{NNLO QCD}$

- Di-photon production at the LHC with $p_T^{\gamma\gamma} > 1 \text{ GeV}$
 - The main SM background of $H \rightarrow \gamma\gamma$
 - Large pQCD and photon fragmentation corrections
 - **NNLO** = $\mathcal{O}(\alpha_s^3)$ terms:
 - NNLO in $q\bar{q}$ and qg channels:
Tree and 1-loop ME: OpenLoops
Two-loop, five-point **full colour contributions**
 - **LO** loop-induced process for gg channel
 - **NNLO⁺** = NNLO + NLO (loop-induced new result)
- Comparison with ATLAS 13 TeV data:
 - Better agreement but still with systematic deviation
 - Loop-induced corrections are crucial for the reduction of systematic uncertainties.
- Future plan (to be included in public release) :
 - Impact of various photon-isolation algorithm
 - Extend to $p_T^{\gamma\gamma} \in [0,1] \text{ GeV}$ to complete N3LO

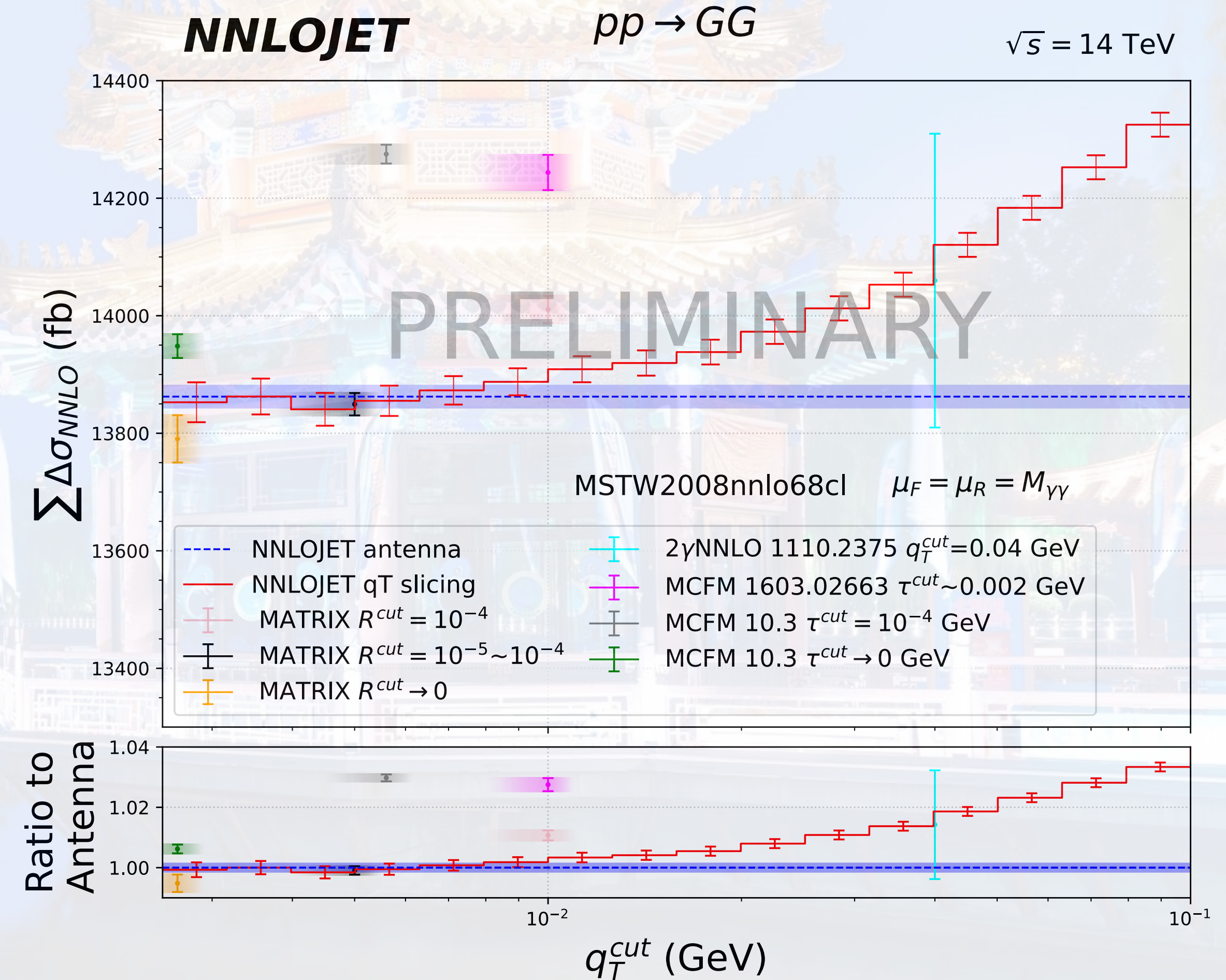


F. Buccioni, XC, W.J. Feng, T. Gehrmann et. al., Phys. Rev. Lett. 134 (2025) 17

$\gamma\gamma$ Production up to N3LO QCD

$pp \rightarrow \gamma\gamma$ @ NNLO QCD (α_s^2)

- Achieved by multiple groups with slicing and local subtraction
 - **1st** calculation in 2011 by 2 γ NNLO (erratum in 2016):
 - $\sqrt{s} = 14$ TeV, MSTW2008 PDF with match fixed order.
 - **Smooth γ isolation**: $E_T^{\text{iso}} = p_T^\gamma/2$, $\Delta R \leq 0.4$, $n = 1$
 - $M_{\gamma\gamma} \in [20, 250]$ GeV, $|y^\gamma| \in [0, 2.5]$, $\alpha = 1/137$
 - $p_T^{\gamma_1} \geq 40$ GeV, $p_T^{\gamma_2} \geq 25$ GeV, $q_T^{\text{cut}} \geq 4 \times 10^{-2}$ GeV
 - **2nd** calculation in 2016 by MCFM:
 - Find disagreement with 2 γ NNLO $\rightarrow$ erratum
 - $q_T^{\text{cut}} \geq 1 \times 10^{-2}$ or $\tau^{\text{cut}} \geq 4 \times 10^{-4}$ GeV extrapolated to 0
 - **3rd** calculation in 2017 by MATRIX:
 - $r^{\text{cut}} \geq 5 \times 10^{-4}$ ($r^{\text{cut}} = q_T^{\text{cut}}/M_{\gamma\gamma}$) and extrapolation
 - **4th** calculation in 2020 by NNLOJET:
 - First **local subtraction** implementation (no slicing dependence of r^{cut} , q_T^{cut} , τ^{cut})
 - **Including** 2-loop colour singlet and gluon induced contribution



Translate all slicing parameters to q_T^{cut} , all published results are questionable!

Newer version of public tools can make correct predictions

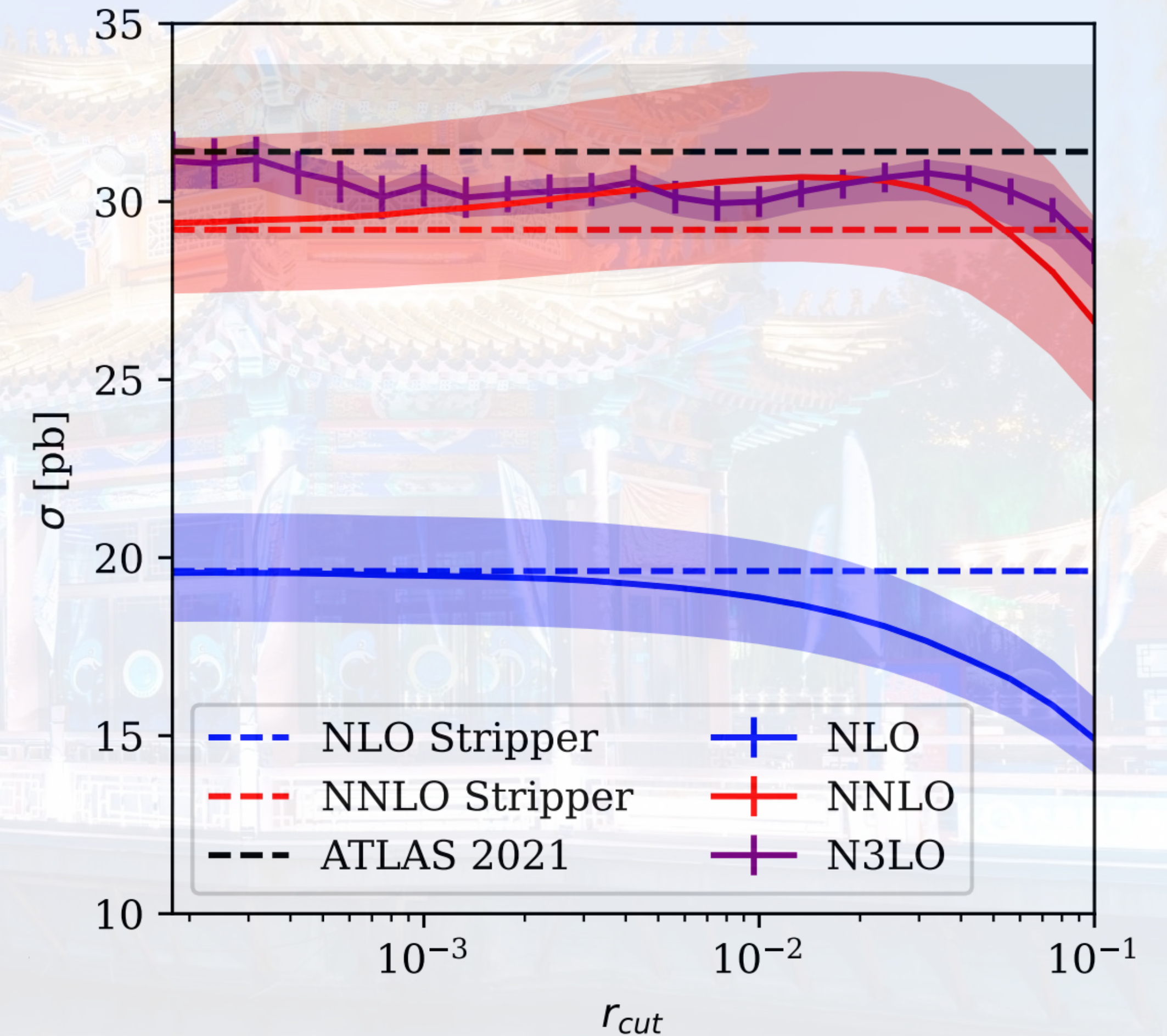
$\gamma\gamma$ Production up to N3LO QCD

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- Achieved by multiple groups with slicing and local subtraction
 - **1st** calculation in 2011 by 2 γ NNLO (erratum in 2016):
 - $\sqrt{s} = 14$ TeV, MSTW2008 PDF with match fixed order.
 - **Smooth γ isolation**: $E_T^{\text{iso}} = p_T^\gamma/2$, $\Delta R \leq 0.4$, $n = 1$
 - $M_{\gamma\gamma} \in [20, 250]$ GeV, $|y^\gamma| \in [0, 2.5]$, $\alpha = 1/137$
 - $p_T^{\gamma_1} \geq 40$ GeV, $p_T^{\gamma_2} \geq 25$ GeV, $q_T^{\text{cut}} \geq 4 \times 10^{-2}$ GeV
 - **2nd** calculation in 2016 by MCFM:
 - Find disagreement with 2 γ NNLO → erratum
 - $q_T^{\text{cut}} \geq 1 \times 10^{-2}$ or $\tau^{\text{cut}} \geq 4 \times 10^{-4}$ GeV extrapolated to 0
 - **3rd** calculation in 2017 by MATRIX: *MATRIX: M. Grazzini, S. Kallweit, M. Wiesemann, EPJC. 78 (2018) 537*
 - $r^{\text{cut}} \geq 5 \times 10^{-4}$ ($r^{\text{cut}} = q_T^{\text{cut}}/M_{\gamma\gamma}$) and extrapolation

$pp \rightarrow \gamma\gamma$ @ N3LO QCD (α_s^3)

- The first di-photon production with N3LO QCD correction
 - Also with **smooth γ isolation** and **q_T -slicing** at $r^{\text{cut}} \geq 2 \times 10^{-4}$
 - Excellent numerical stability control with **10M CPU hours**



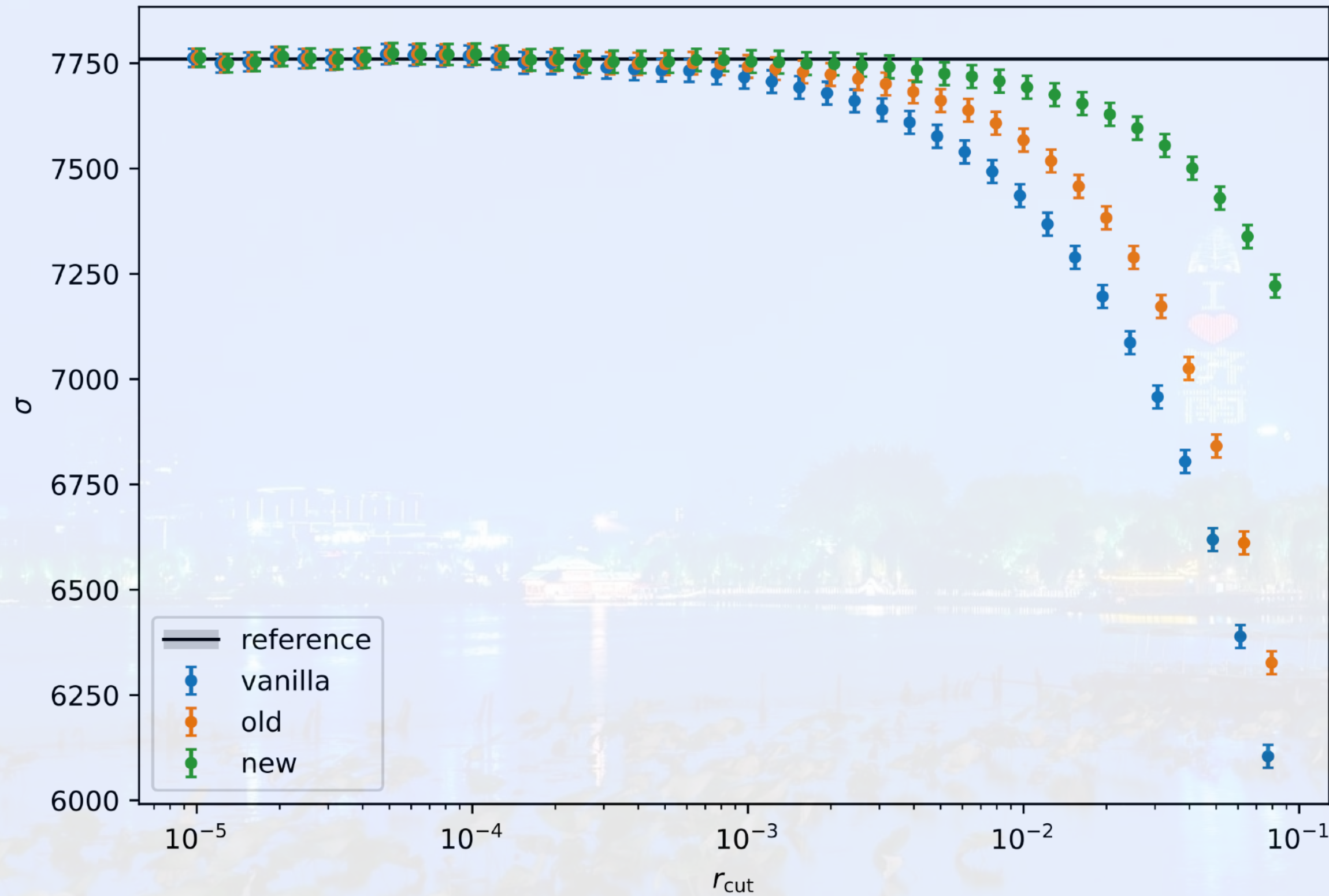
M. Czakon, F. Eschment, T. Generet, R. Poncelet, 2604.12613

$$\begin{aligned}
 d\sigma_{N^3LO}^{\gamma\gamma} = & [\mathcal{H}^{\gamma\gamma} \otimes d\sigma^{\gamma\gamma}]_{N^3LO} \Big|_{\delta(p_T^{\gamma\gamma})} + [d\sigma_{NNLO}^{\gamma\gamma+jet} - d\sigma_{N^3LO}^{\gamma\gamma CT}]_{p_T^{\gamma\gamma} > p_T^{\text{veto}}} \\
 & + \mathcal{O}((p_T^{\text{veto}}/Q)^1) + \mathcal{O}((p_T^{\text{veto}}/Q)^2) \\
 & \text{Smooth isolation} \quad q_T \text{ factorisation} \\
 & \text{power correction} \quad \text{power correction}
 \end{aligned}$$

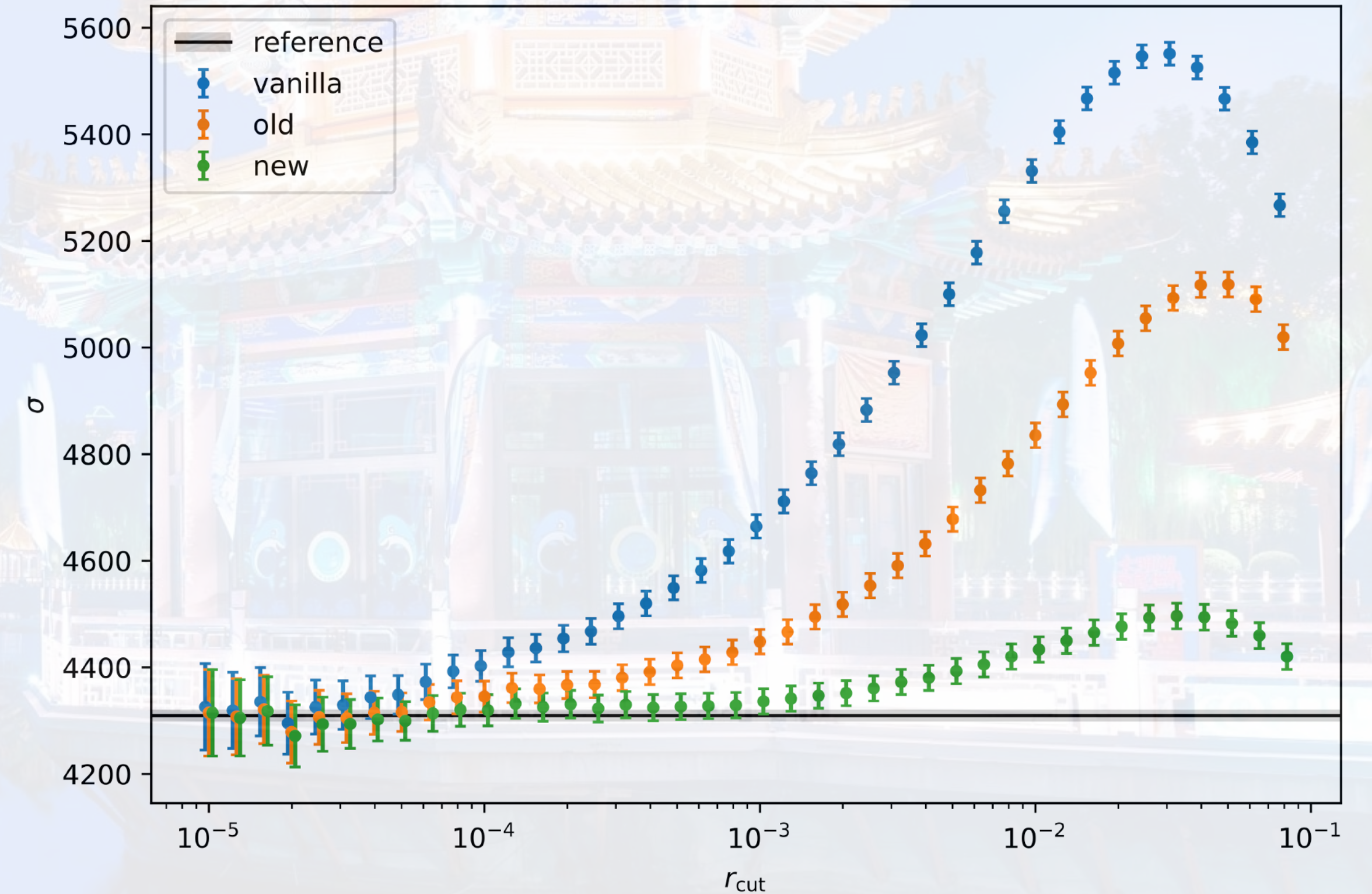
$\gamma\gamma$ Production up to N3LO QCD

$pp \rightarrow \gamma\gamma$ @ NNLO QCD (α_s^2)

NLO_only: tot



NNLO_only: tot



Vanilla: Apply simple recoil prescription

1507.06937

2203.01565

Old: Address single quark emission

2408.05265

New: Address all linear power correction

$$d\sigma_{N^3LO}^{\gamma\gamma} = [\mathcal{H}^{\gamma\gamma} \otimes d\sigma^{\gamma\gamma}]_{N^3LO} \Big|_{\delta(p_T^{\gamma\gamma})} + [d\sigma_{NNLO}^{\gamma\gamma+jet} - d\sigma_{N^3LO}^{\gamma\gamma CT}]_{p_T^{\gamma\gamma} > p_T^{veto}} + \mathcal{O}((p_T^{veto}/Q)^1) + \mathcal{O}((p_T^{veto}/Q)^2)$$

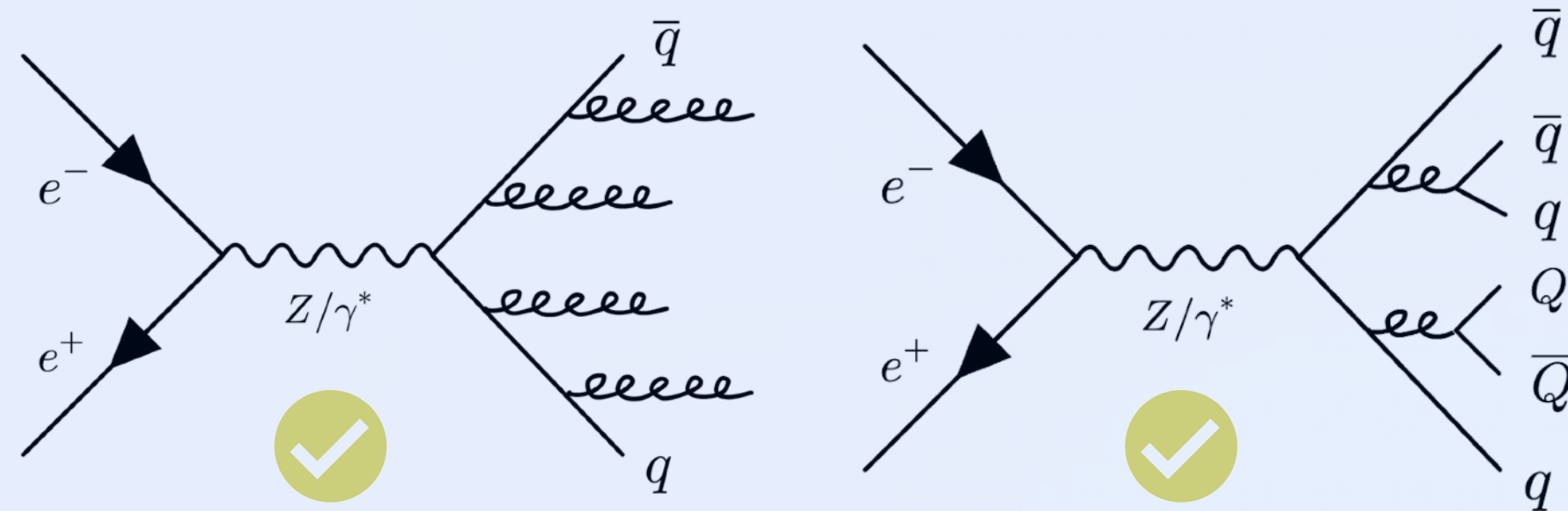
Smooth isolation power correction q_T factorisation power correction

Application of **NNLOJET** in e^+e^- Colliders

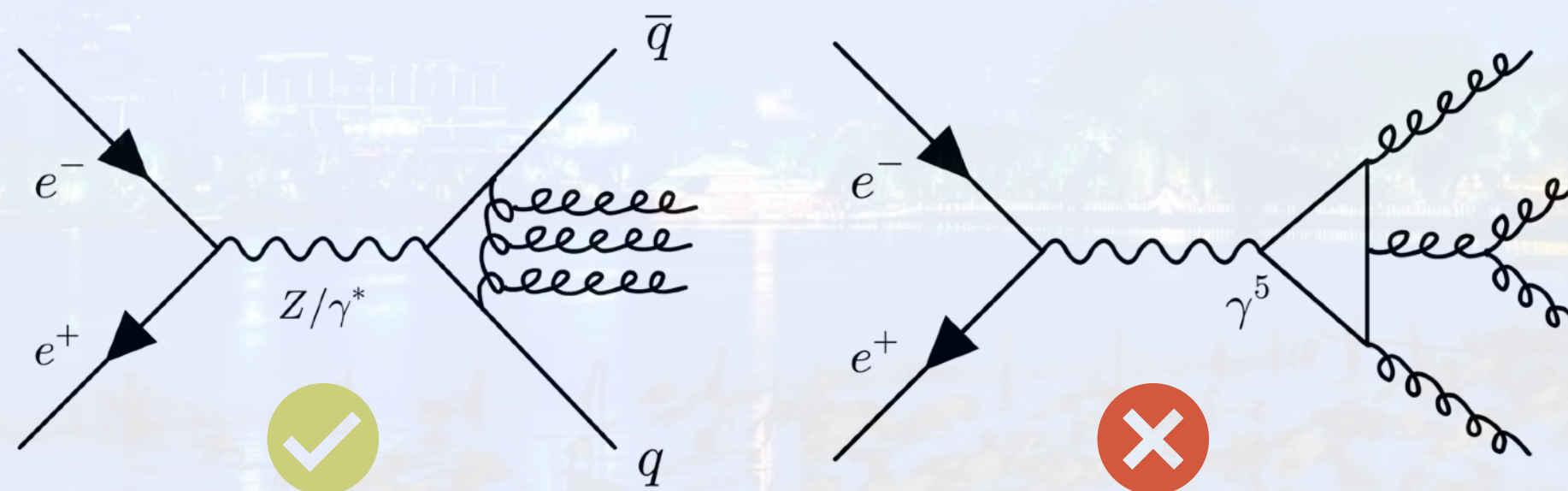
$e^+e^- \rightarrow 4 \text{ jets @ NNLO}$ 2602.18185

$e^+e^- \rightarrow \text{JJJJ} @ \text{NNLO}$

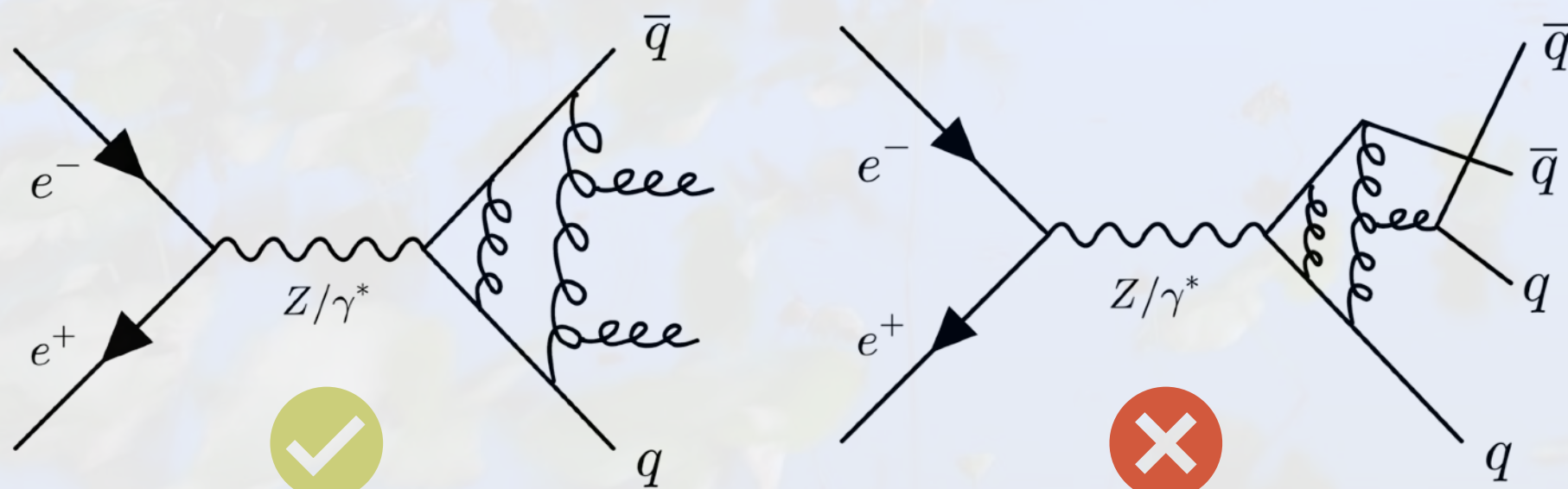
- Double-real matrix element examples:



- Real-virtual matrix element examples:



- Double-virtual matrix element examples

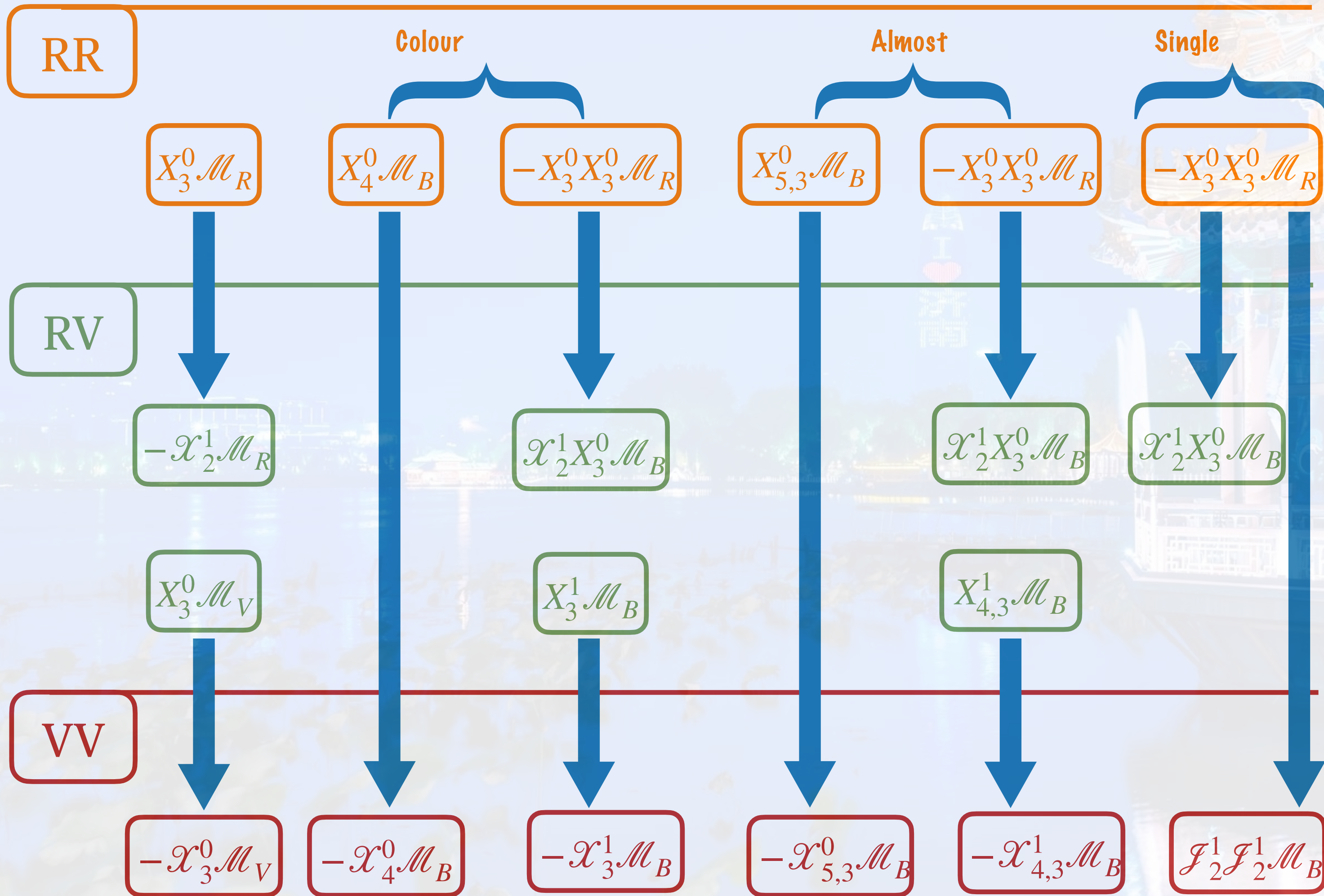


- Massless quark with $n_f = 5$ flavour scheme
- Tree and one-loop matrix elements are obtained from **OpenLoops2**: *Buccioni, Lang, Lindert, Maierhoefer et. al., EPJC. 79 (2019) 10, 866*
 - Efficient and stable numerical evaluations in **IR regions**
 - Axial-vector coupling of the Z to closed quark loops **ignored**
- Two-loop matrix elements finite remainders obtained from **PentagonFunctions++** and **FivePointAmplitudes++**: *De Laurentis, Ita, Page, Sotnikov, JHEP. 06 (2025) 093* *Abreu, et. al., Phys. Rev. Lett. 132 (2024) 141601*
 - Solve the canonical differential equations with one-mass pentagon functions and cross kinematics to **decay channel**
 - Use **leading colour approximation** in each partonic channel
 - Vector coupling with singlet contribution (d_{abc}) **ignored**
 - Finite remainders defined in **Catani's scheme**
- $e^+e^- \rightarrow 5 \text{ Jets} @ \text{NLO}$ **validation** with SHERPA:

y_{cut}	NNLOJET	Sherpa	Deviation
$10^{-2.5}$	$-1.13(2) \cdot 10^2 (\text{RS})$ $6.96(1) \cdot 10^2 (\text{VI})$ $5.84(2) \cdot 10^2$	$1.843(8) \cdot 10^3 (\text{RS})$ $-1.262(3) \cdot 10^3 (\text{VI})$ $5.84(4) \cdot 10^2$	0.11σ
$10^{-3.5}$	$-5.87(6) \cdot 10^3 (\text{RS})$ $1.419(3) \cdot 10^4 (\text{VI})$ $8.32(7) \cdot 10^3$	$3.42(3) \cdot 10^4 (\text{RS})$ $-2.60(1) \cdot 10^4 (\text{VI})$ $8.35(9) \cdot 10^3$	0.26σ

$e^+e^- \rightarrow JJJJ @ NNLO$

Generalized antenna subtraction to handle IR divergences



Fox, Glover, Marcoli, JHEP 12 (2024) 225

```
XX:=
+D30FF(a,i,j)*B3g0Z([a,i],[i,j],k,l,b,1,2)*JET45([a,i],[i,j],k,l,b)*a1
+F30FF(i,j,k)*B3g0Z(a,[i,j],[j,k],l,b,1,2)*JET45(a,[i,j],[j,k],l,b)*a2
+F30FF(j,k,l)*B3g0Z(a,i,[j,k],[k,l],b,1,2)*JET45(a,i,[j,k],[k,l],b)*a3
+D30FF(b,l,k)*B3g0Z(a,i,j,[k,l],[l,b],1,2)*JET45(a,i,j,[k,l],[l,b])*a4

+D40FF(a,i,j,k)*B2g0Z([a,i,j],[i,j,k],l,b,1,2)*JET44([a,i,j],[i,j,k],l,b)*a5
-D30FF(
a,i,j)*D30FF([a,i],[i,j],k)*B2g0Z([a,i],[i,j],[i,j],k,l,b,1,2)*JET44([a,i],[i,j],[i,j],k,l,b)*a6
-F30FF(k,j,i)*D30FF(
a,[i,j],[j,k])*B2g0Z([a,[i,j]],[[i,j],[j,k]],l,b,1,2)*JET44([a,[i,j]],[[i,j],[j,k]],l,b)*a7
+F40FF(i,j,k,l)*B2g0Z(a,[i,j,k],[j,k,l],b,1,2)*JET44(a,[i,j,k],[j,k,l],b)*a8
-F30FF(i,j,k)*F30FF([i,j],[j,k],l)*B2g0Z(a,[i,j],[j,k],[j,k,l],b,1,2)*JET44(
a,[i,j],[j,k],[j,k,l],[j,k,l],b)*a9
-F30FF(l,k,j)*F30FF([l,k],[k,j],i)*B2g0Z(a,[i,j,k],[[j,k],[k,l]],b,1,2)*JET44(
a,[i,j,k],[[j,k],[k,l]],b)*a10
+D40FF(b,l,k,j)*B2g0Z(a,i,[j,k,l],[k,l,b],1,2)*JET44(a,i,[j,k,l],[k,l,b])*a11
-D30FF(b,l,k)*D30FF([b,l],[l,k],j)*B2g0Z(a,i,[j,k,l],[[k,l],[l,b]],1,2)*JET44(
a,i,[j,k,l],[[k,l],[l,b]],b)*a12
-F30FF(j,k,l)*D30FF(b,[l,k],[k,j])*B2g0Z(a,i,[j,k],[k,l],[k,l],b,1,2)*JET44(
a,i,[j,k],[k,l],[k,l],b)*a13

-D30FF(a,i,j)*F30FF([i,j],k,l)*B2g0Z([a,i],[i,j],k,[k,l],b,1,2)*JET44([a,i],[i,j],k,[k,l],b)*dip(1)*a14
-F30FF(l,k,j)*D30FF(a,i,[j,k])*B2g0Z([a,i],[i,j,k],[k,l],b,1,2)*JET44([a,i],[i,j,k],[k,l],b)*dip(-1)*a15

-D30FF(b,l,k)*F30FF([l,k],j,i)*B2g0Z(a,[i,j],[j,k,l],[l,b],1,2)*JET44(a,[i,j],[j,k,l],[l,b])*dip(1)*a16
-F30FF(i,j,k)*D30FF(b,l,[j,k])*B2g0Z(a,[i,j],[l,j,k],[l,b],1,2)*JET44(a,[i,j],[l,j,k],[l,b])*dip(-1)*a17

+D30FF(a,i,j)*D30FF(b,l,k)*B2g0Z([a,i],[i,j],[k,l],[l,b],1,2)*JET44([a,i],[i,j],[k,l],[l,b])*a18
-D30FF(a,i,j)*D30FF(b,l,k)*B2g0Z([a,i],[i,j],[k,l],[l,b],1,2)*JET44([a,i],[i,j],[k,l],[l,b])*a19
-D30FF(b,l,k)*D30FF(a,i,j)*B2g0Z([a,i],[i,j],[k,l],[l,b],1,2)*JET44([a,i],[i,j],[k,l],[l,b])*a20

+D530FF(a,i,j,k,l)*B2g0Z([a,i,j],[a,i,j,k,l],[j,k,l],b,1,2)*JET44([a,i,j],[a,i,j,k,l],[j,k,l],b)*a21
+D530FF(b,l,k,j,i)*B2g0Z(a,[k,j,i],[i,j,k,l,b],[b,l,k],1,2)*JET44(a,[k,j,i],[i,j,k,l,b],[b,l,k])*a22
```

22 subtraction terms for leading colour $\mathcal{M}_{RR}^0(q, g, g, g, g, \bar{q})$

```
XX:=
-(+J21QGFF(3,i)+J21GGFF(i,j)+J21GGFF(j,k)+J21QGFF(k,4))*B3g0Z(3,i,j,k,4,1,2)*JET45(3,i,j,k,4)*a1

+J21QGFF(3,i)*D30FF(3,i,j)*B2g0Z([3,i],[i,j],k,4,1,2)*JET44([3,i],[i,j],k,4)*a2
+J21GGFF(i,j)*D30FF(3,i,j)*B2g0Z([3,i],[i,j],k,4,1,2)*JET44([3,i],[i,j],k,4)*a3
+J21GGFF(j,k)*D30FF(3,i,j)*B2g0Z([3,i],[i,j],k,4,1,2)*JET44([3,i],[i,j],k,4)*dip(-1)*a4
+J21QGFF(k,4)*D30FF(3,i,j)*B2g0Z([3,i],[i,j],k,4,1,2)*JET44([3,i],[i,j],k,4)*a5

+J21QGFF(3,i)*F30FF(i,j,k)*B2g0Z(3,[i,j],[j,k],4,1,2)*JET44(3,[i,j],[j,k],4)*dip(1)*a6
+J21GGFF(i,j)*F30FF(i,j,k)*B2g0Z(3,[i,j],[j,k],4,1,2)*JET44(3,[i,j],[j,k],4)*a7
+J21GGFF(j,k)*F30FF(i,j,k)*B2g0Z(3,[i,j],[j,k],4,1,2)*JET44(3,[i,j],[j,k],4)*a8
+J21QGFF(k,4)*F30FF(i,j,k)*B2g0Z(3,[i,j],[j,k],4,1,2)*JET44(3,[i,j],[j,k],4)*dip(-1)*a9

+J21QGFF(3,i)*D30FF(4,k,j)*B2g0Z(3,i,[j,k],[k,4],1,2)*JET44(3,i,[j,k],[k,4])*a10
+J21GGFF(i,j)*D30FF(4,k,j)*B2g0Z(3,i,[j,k],[k,4],1,2)*JET44(3,i,[j,k],[k,4])*dip(-1)*a11
+J21GGFF(j,k)*D30FF(4,k,j)*B2g0Z(3,i,[j,k],[k,4],1,2)*JET44(3,i,[j,k],[k,4])*a12
+J21QGFF(k,4)*D30FF(4,k,j)*B2g0Z(3,i,[j,k],[k,4],1,2)*JET44(3,i,[j,k],[k,4])*a13

+D30FF(3,i,j)*B2g1Z([3,i],[i,j],k,4,1,2)*JET44([3,i],[i,j],k,4)*a14
+F30FF(i,j,k)*B2g1Z(3,[i,j],[j,k],4,1,2)*JET44(3,[i,j],[j,k],4)*a15
+D30FF(4,k,j)*B2g1Z(3,i,[j,k],[k,4],1,2)*JET44(3,i,[j,k],[k,4])*a16

+D31FF(3,i,j)*B2g0Z([3,i],[i,j],k,4,1,2)*JET44([3,i],[i,j],k,4)*a17
+F31FF(i,j,k)*B2g0Z(3,[i,j],[j,k],4,1,2)*JET44(3,[i,j],[j,k],4)*a18
+D31FF(4,k,j)*B2g0Z(3,i,[j,k],[k,4],1,2)*JET44(3,i,[j,k],[k,4])*a19

+D431_gFF(3,i,j,k)*B2g0Z([3,i],[i,j],k,4,1,2)*JET44([3,i],[i,j],k,4)*a20
+F431_gFF(k,j,i,3)*B2g0Z(3,[i,j],[j,k],4,1,2)*JET44(3,[i,j],[j,k],4)*a21
+F431_gFF(i,j,k,4)*B2g0Z(3,[i,j],[j,k],4,1,2)*JET44(3,[i,j],[j,k],4)*a22
+D431_gFF(4,k,j,i)*B2g0Z(3,i,[k,j],[k,4],1,2)*JET44(3,i,[k,j],[k,4])*a23
```

23 subtraction terms for leading colour $\mathcal{M}_{RV}^1(q, g, g, g, g, \bar{q})$

$e^+e^- \rightarrow \text{JJJJ} @ \text{NNLO}$

➤ Application to jet rates @ α_s^4 :

$$R_n^{(4)}(y_{cut}) = \frac{\Gamma_{Z/\gamma^* \rightarrow n \text{ jets}}^{(4)}(y_{cut})}{\Gamma_{Z/\gamma^* \rightarrow hadrons}^{(4)}}$$

$$R_4^{(4)}(y_{cut})\Gamma_{Z/\gamma^* \rightarrow hadrons}^{(4)} = \int_{y_{cut}}^1 \frac{d\sigma}{dy_{34}} dy_{34} - \int_{y_{cut}}^1 \frac{d\sigma}{dy_{45}} dy_{45}$$

$R_n(\alpha_s)$	α_s^0	α_s^1	α_s^2	α_s^3	α_s^4	
n=2		<u>1977</u>	<u>2004, 2004, 2006</u>	<u>2008, 2025</u>		FO
n=3		<u>1976</u>	<u>1980, 1981, 1981, 1982</u>	<u>2008, 2008, 2014, 2025</u>		N4LO
n=4			<u>1979, 1980</u>	<u>1996, 1997, 1998, 1999</u>	2026 This work	N3LO
n=5				<u>1989, 1989, 1989</u>	<u>2010, 2011</u>	NNLO
n=6					<u>1999</u>	NLO
						LO

Table 4.2: Wish-list for calculations of missing higher-order perturbative QCD $\mathcal{O}(\alpha_s^n)$ and/or EW $\mathcal{O}(\alpha^n)$ to match the expected experimental uncertainty at future e^+e^- and ep colliders.

Observable	Missing higher-order & power-suppressed corrections
Hadronic Z width	$\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha_s^6)$, $\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha_s \alpha^3)$, $\mathcal{O}(\alpha_s^2 \alpha^2)$
Hadronic W width	$\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha_s^2 \alpha)$
Hadronic τ width	$\mathcal{O}(\alpha_s^5)$
Hadronic event shapes (Z, W, H decays)	N ³ LO differential, N ^{3,4} LL resummation, power corrections
Inclusive jet rates	3-jet cross-sections at N ³ LO, 4-jets at N ² LO, 5-jets at NLO
Lattice QCD results (α_s extr.; quark masses m_c, m_b)	$\mathcal{O}(\alpha_s^6)$ β -function; $\mathcal{O}(\alpha_s^5)$ heavy quark decoupling; $\mathcal{O}(\alpha_s^4)$ static potential
$\sigma(e^+e^- \rightarrow W^+W^-)$ vs. $\sqrt{s}$	EW N ² LO: $\mathcal{O}(\alpha^2)$, Mixed EW-QCD: $\mathcal{O}(\alpha_s \alpha^2)$, $\mathcal{O}(\alpha_s^2 \alpha)$
$\sigma(e^+e^- \rightarrow t\bar{t})$ vs. $\sqrt{s}$	NRQCD: $\mathcal{O}(\alpha_s^5)$, Non-resonant: $\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha_s^3)$ differential; QED: $\mathcal{O}(\alpha^3)$ at NNLL
$H \rightarrow b\bar{b}$ width	N ⁴ LO ($m_b \neq 0$); N ⁴ LO differential ($m_b = 0$)
$H \rightarrow gg$ width	N ⁵ LO (heavy-top limit), N ⁴ LO ($m_t \neq 0$); N ⁴ LO differential, N ³ LO differential ($m_t \neq 0$)
MC simulations for $e^+e^- \rightarrow X$ processes	N ^{2,3} LO matched to N ^{2,3} LL PS. Per mille control of non-pQCD effects (hadronization, CR, ...)
$ep \rightarrow \text{hadrons}$ (PDF and α_s determ.)	N ^{3,4} LO evolution equations and inclusive cross-sections
$ep \rightarrow \text{jets}$ (α_s determ.)	N ³ LO cross-sections

Physics Briefing Book: Input for the 2026 update of the European Strategy for Particle Physics

➤ JJJJ@NNLO is also genuine NNLO contribution to event shapes:

$$D = 27\lambda_1\lambda_2\lambda_3, \quad C = 3\lambda_1\lambda_2\lambda_3, \quad \text{from } \Theta^{ij} = \frac{\sum_k \frac{p_k^i p_k^j}{|\vec{p}_k|}}{\sum_k |\vec{p}_k|}$$

$$\frac{M_L^2}{s} = \frac{1}{s} \min \left\{ \left(\sum_{\vec{k}_k \in H_1} p_k \right)^2, \left(\sum_{\vec{k}_k \in H_2} p_k \right)^2 \right\}$$

$$A = \frac{3}{2}\lambda_3, \quad \text{from } \Phi^{ij} = \frac{\sum_k p_k^i p_k^j}{\sum_k |\vec{p}_k|^2}$$

$e^+e^- \rightarrow JJJJ @ \text{NNLO}$

► Application to 4 jet rate up to NNLO:

► Computational setup:

► $m_Z = 91.2 \text{ GeV}$, $\alpha_s(m_Z) = 0.118$

► $\mu_R^c = \sqrt{s}$ with $\mu_R \in [\sqrt{s}/2, 2\sqrt{s}]$

► $\Gamma_{Z/\gamma^* \rightarrow \text{hadrons}}^{\alpha_s}$ with matching α_s order.

► **Perturbative region** $10^{-3} \leq y_{\text{cut}} \leq 10^{-1}$ for $\sqrt{s} = m_Z$:

► Remarkable convergence with NLO.

► Excellent agreement with ALEPH data.

► Scale uncertainty reduced to $\pm(3\% - 5\%)$ smaller than LEP systematic errors.

► Symmetrization of scale uncertainty.

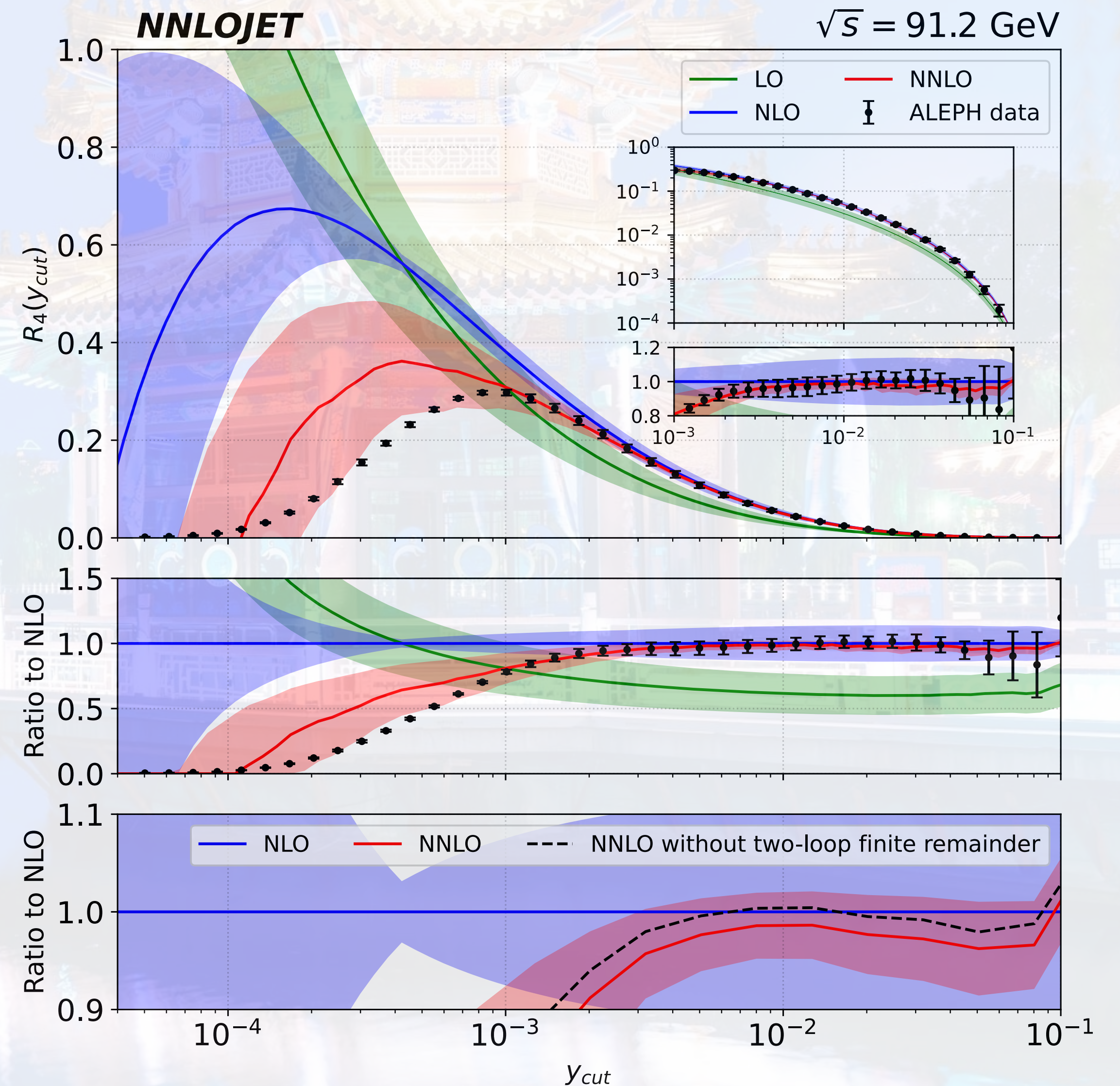
► Two-loop finite reminder has size above $-2\% \sim -5\%$

► LCA is expected to have $\sim 0.5\%$ modification

► **Resummation and none-perturbative region:**

► NNLO peak moves to larger y_{cut} value than NLO

► Jet rate turns negative around $y_{\text{cut}} = 10^{-4}$



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$e^+e^- \rightarrow JJJJ @ \text{NNLO}$

➤ Application to 4 jet rate up to NNLO:

➤ Computational setup:

- $m_Z = 91.2 \text{ GeV}$, $\alpha_s(m_Z) = 0.118$
- $\mu_R^c = \sqrt{s}$ with $\mu_R \in [\sqrt{s}/2, 2\sqrt{s}]$
- $\Gamma_{Z/\gamma^* \rightarrow \text{hadrons}}^{\alpha_s}$ with matching α_s order.

➤ Perturbative region:

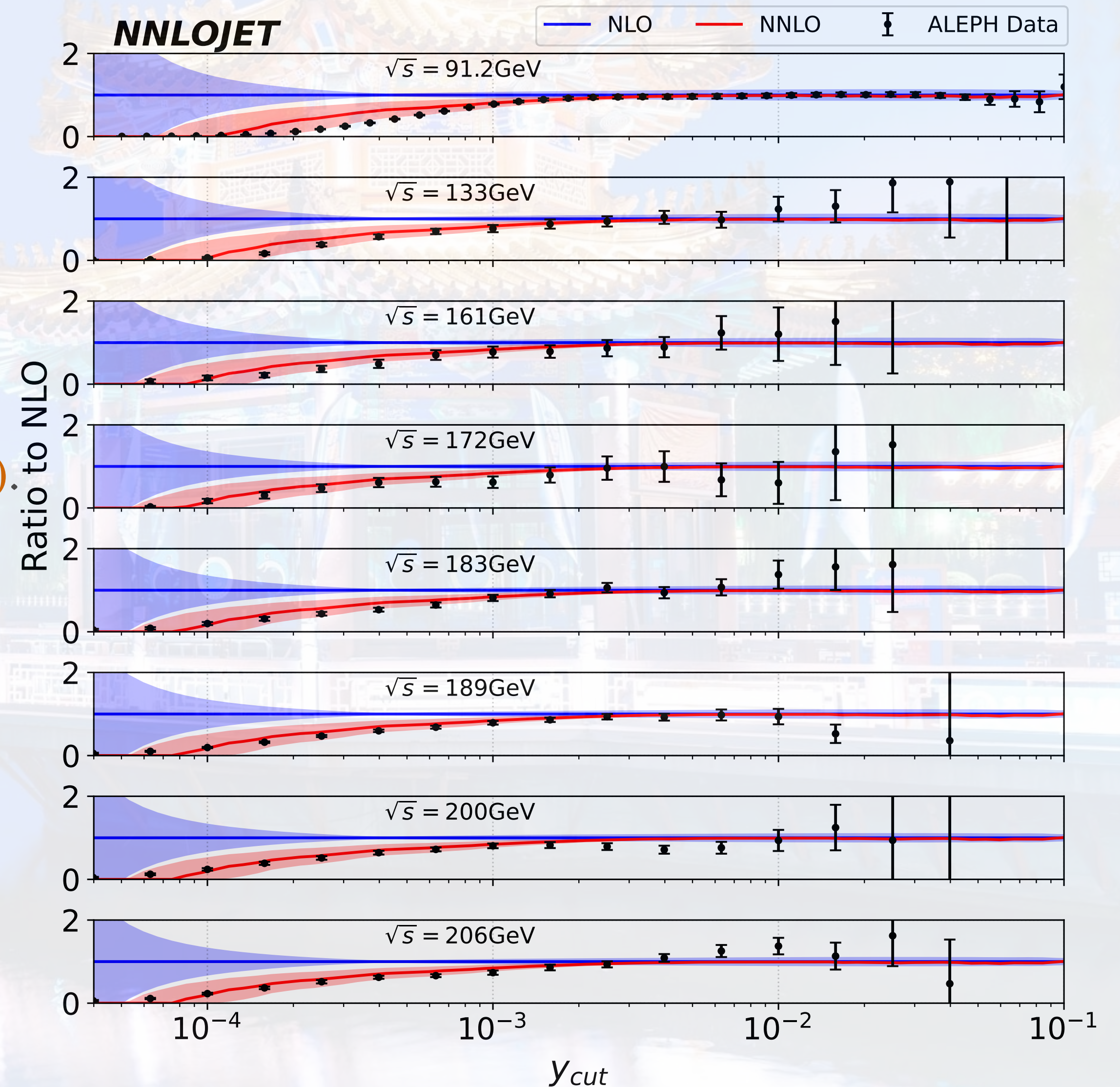
- $R_4(y_{\text{cut}})$ dependence of $\sqrt{s}$ is **only via running of $\alpha_s(\mu_R)$** .
- **FO prediction extends** to smaller y_{cut} as $\sqrt{s}$ increase.
- Excellent agreement with ALEPH data for $y_{\text{cut}} \sim 10^{-4}$
- Two-loop finite remainder is largely insensitive to $\sqrt{s}$.

➤ Further validation with data:

- Reanalyzing existing measurements
- Future electron-positron colliders

➤ Resummation and none-perturbative region:

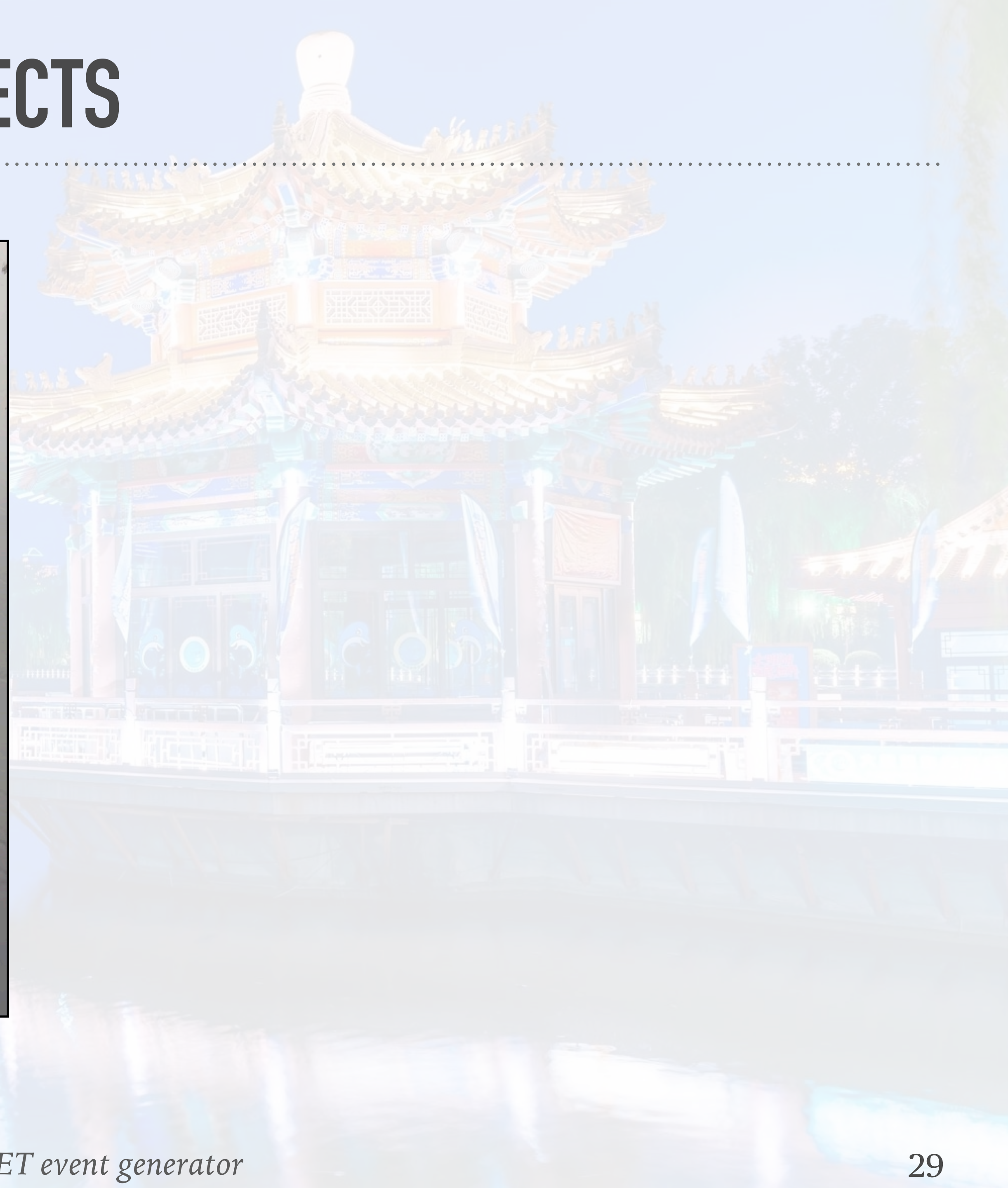
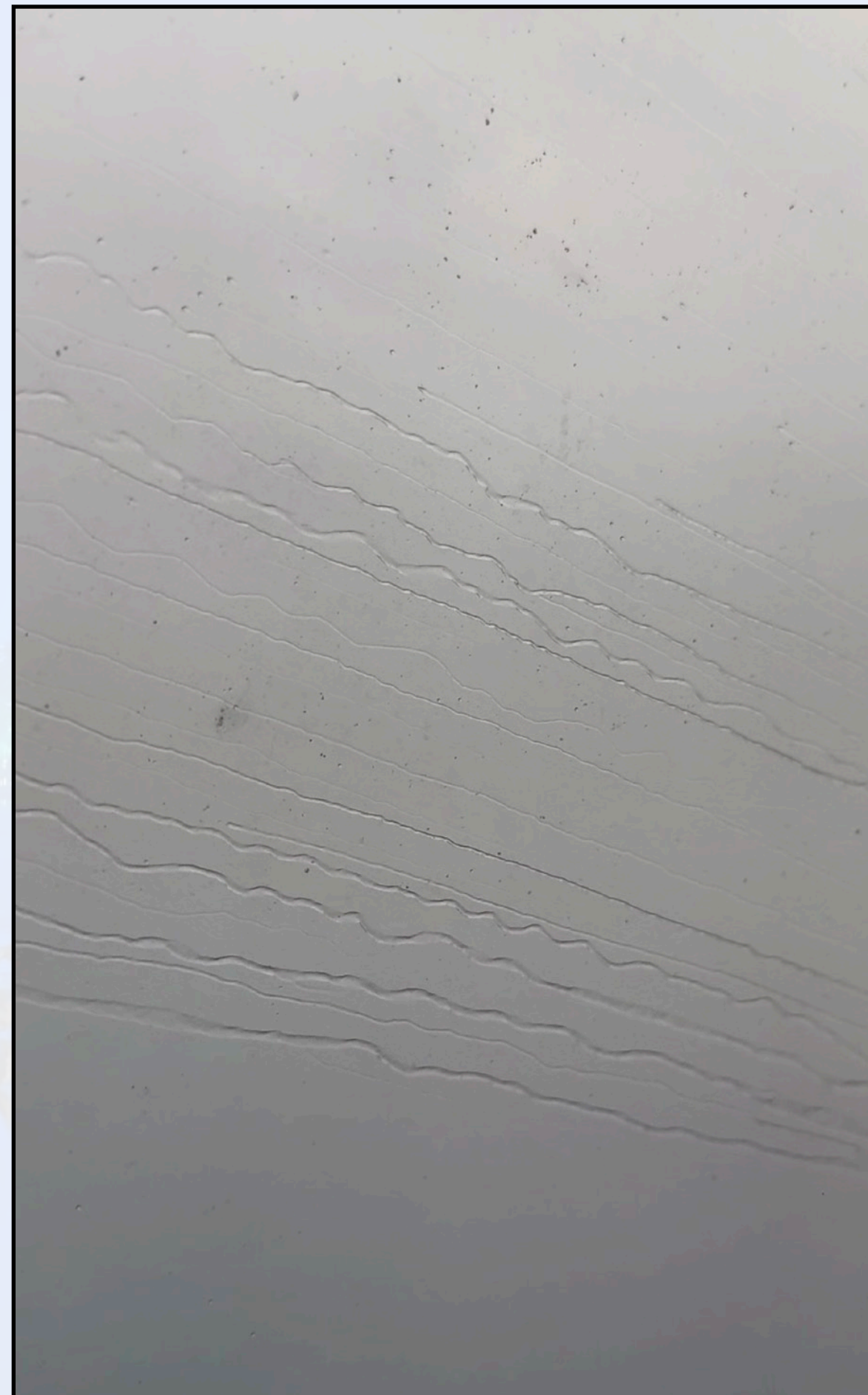
- NLO and NNLO results deviate for $y_{\text{cut}} \leq 10^{-3}$
- Need resummation to reduce dominant scale uncertainty



XC, Chicherin, Fox, Glover, Marcoli, Sotnikov, Sun,
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CONCLUSIONS AND FUTURE PROSPECTS

- Precision phenomenology requires improvements in multiple frontier (PDFs, α_s , quark mass etc.)
- **Event generators** serve as Swiss knives to push the prediction power to new level of accuracy.
- First few applications of **fully-differential N3LO QCD calculation** are available with nontrivial corrections.
- NNLO QCD predictions maybe solved, but still not easily accessible. The **public release of NNLOJET** aims to improve the accessibility.



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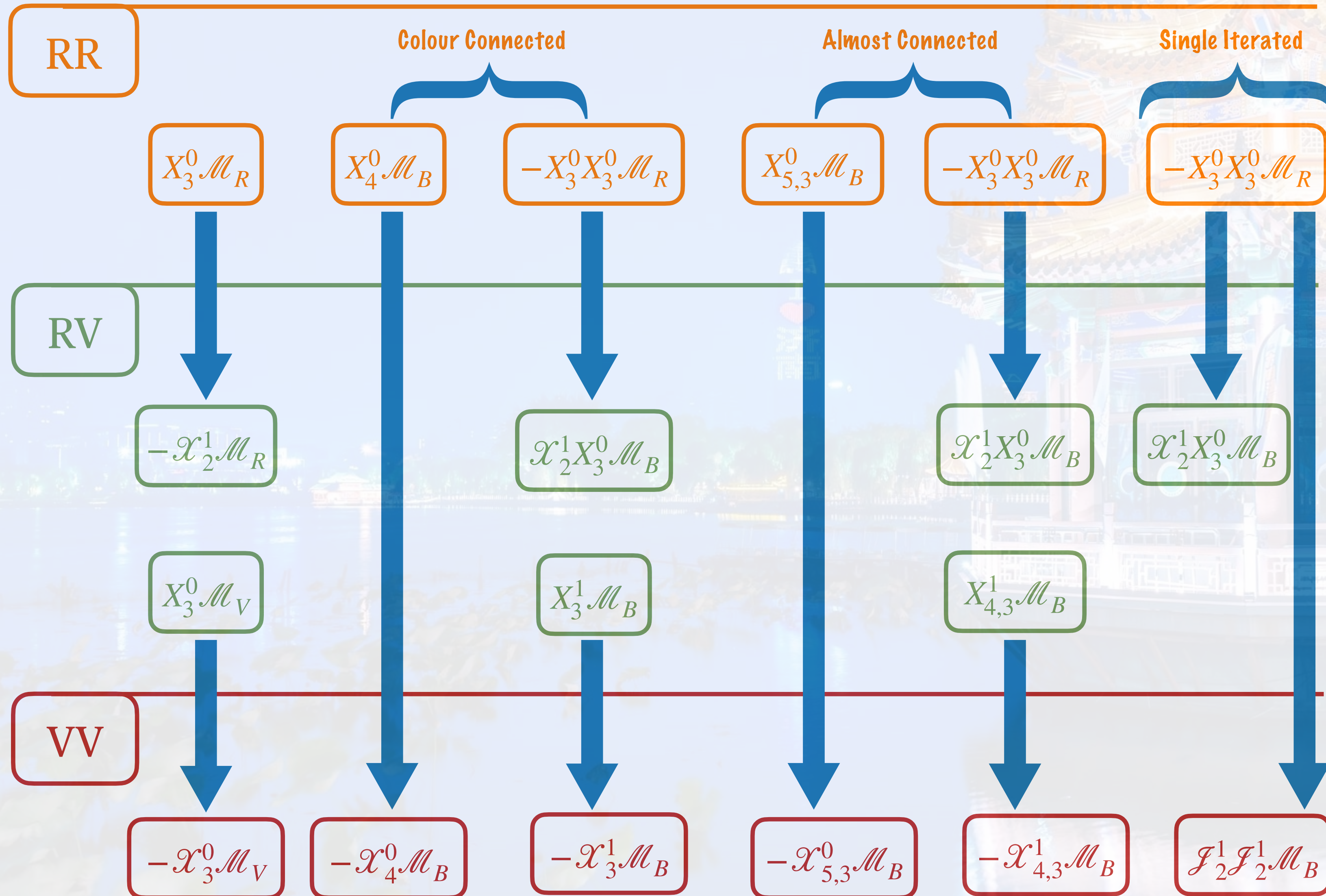
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Thank You for Your Attention

BACK UP SLIDES

Generalized Antenna @ NNLO



Fox, Glover, Marcoli, JHEP 12 (2024) 225

ANTENNA SUBTRACTION @ NNLO

	Feature	Reference
Traditional (NNLOJET v.1.0)	- Built from scattering matrix element (di-pole) All collider types available: LHC, LEP, DIS etc. - Most 2 to 2 scattering and crossing available Branch to Trunk and Trunk to Branch algorithms * Partially automated (LHC 3 jets production) * (Massive/heavy quark) * (Identified hadron, photon) $X_3^0(i, j, k)$$X_4^0(i, j, k, l)$$X_3^1(i, j, k)$	- Original: hep-ph/0505111 hep-ph/0612257 0710.0346 0912.0374 1006.1849 1112.3613 1211.2710 1301.4693 Massive: 0904.3297 1102.2443 1105.0530 1112.4736 1207.6546 1309.6887 1409.3124 Identified: 2201.06982 2208.02650 2406.09925 Automation: 2203.13531 2310.19757
Idealised	- Built from desired IR limits - Based on two hard partons (di-pole) - Currently only final state partons for NNLO	2302.12787 2307.14999 2308.10829
Generalised	- Built from desired IR limits - Based on three hard partons (tri-pole) - Currently only final state partons for NNLO - Possible extension to N3LO with sector mapping Efficiency boost at x5 ~ x10 $X_{5,3}^0(i, j, k, l, m)$$X_{4,3}^1(i, j, k, l)$	2410.12904 2507.12537

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^{3(4)}LL}$

FO	α_s^n	$H(m_V, \mu)$	$I_{ij}^{(n)}(x, b)$	$\ln W(x_a, x_b, m_V, \vec{b}, \mu = b_0/b) \sim \int_{\mu_h}^{\mu} d\bar{\mu} / \bar{\mu} (A(\alpha_s(\bar{\mu})) \ln \frac{m_V^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu})))$						
$\frac{d\hat{\sigma}_{NLO}^V}{dq_T}$	NLO	✓	✓	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1				
$\frac{d\hat{\sigma}_{NNLO}^V}{dq_T}$	N2LO	✓	✓	$\ln^3(b^2 m_V^2)$	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1			
$\frac{d\hat{\sigma}_{N^3LO}^V}{dq_T}$	N3LO	✓	✓	$\ln^4(b^2 m_V^2)$	$\ln^3(b^2 m_V^2)$	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1		
$\frac{d\hat{\sigma}_{N^4LO}^V}{dq_T}$	N4LO	✓	✗	$\ln^5(b^2 m_V^2)$	$\ln^4(b^2 m_V^2)$	$\ln^3(b^2 m_V^2)$	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1	
...	...			...	...	...	...	...	...	...
$\frac{d\hat{\sigma}_{N^kLO}^V}{dq_T}$	NKLO			$\ln^{k+1}(b^2 m_V^2)$	$\ln^k(b^2 m_V^2)$	$\ln^{k-1}(b^2 m_V^2)$	$\ln^{k-2}(b^2 m_V^2)$	$\ln^{k-3}(b^2 m_V^2)$	...	...
...	...			...	...	...	...	...	...	...
Resum				LL	NLL	NNLL	N3LL	N4LL	...	$N^{k+1}LL$
A				A1 ✓	A2 ✓	A3 ✓	A4 ✓	A5 ✗	...	A_{k+2}
B					B1 ✓	B2 ✓	B3 ✓	B4 ✓	...	B_{k+1}