

Hubble delayed vacuum decay

and its Gravitational Wave Signals

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第二届TeV物理前沿研讨会暨第33届LHC mini-Workshop

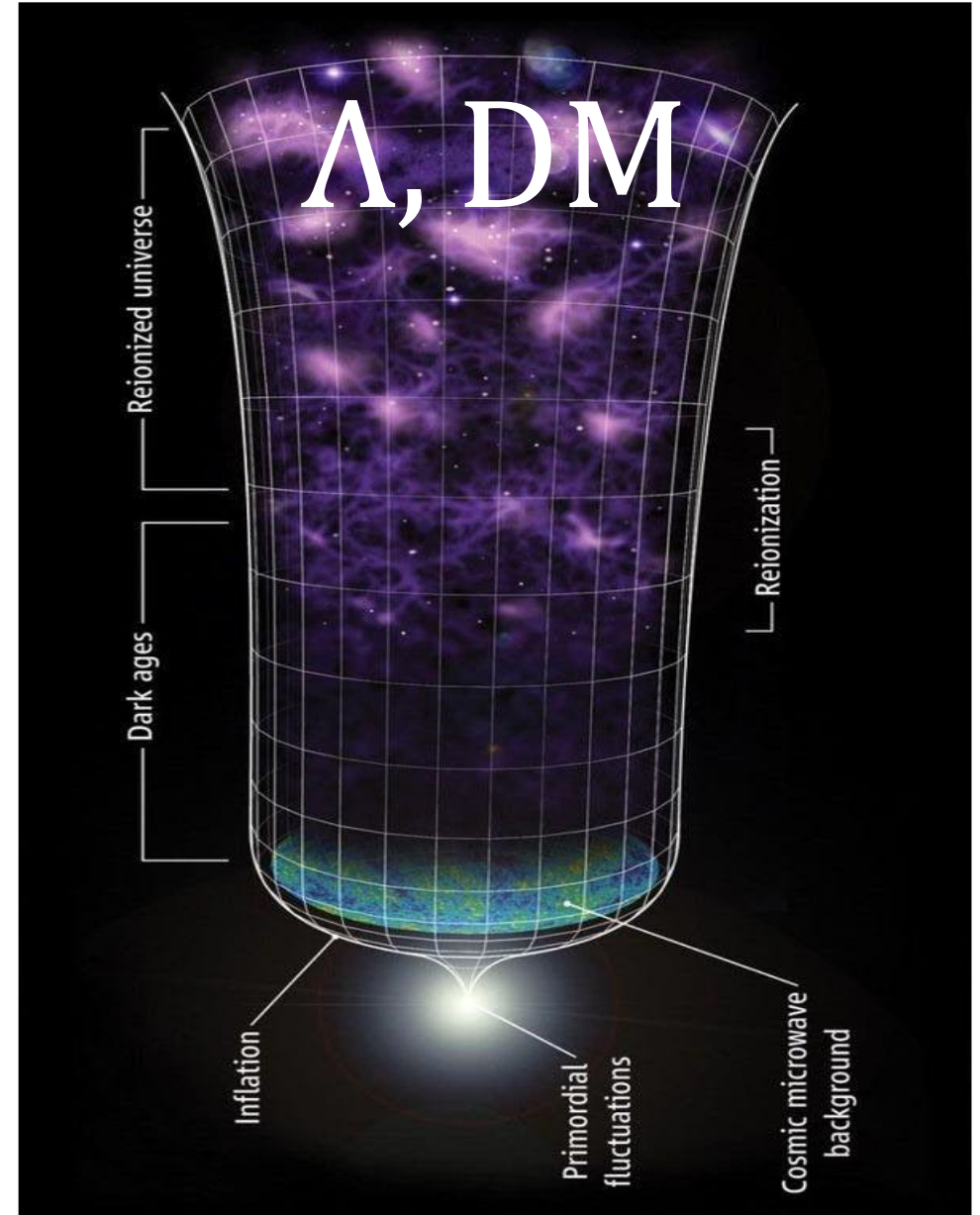
2026年9月18-20日 @ 北京

HA, Tingyu Li and Chen Yang, 2601.14366

HA, Tingyu Li 2606.06587

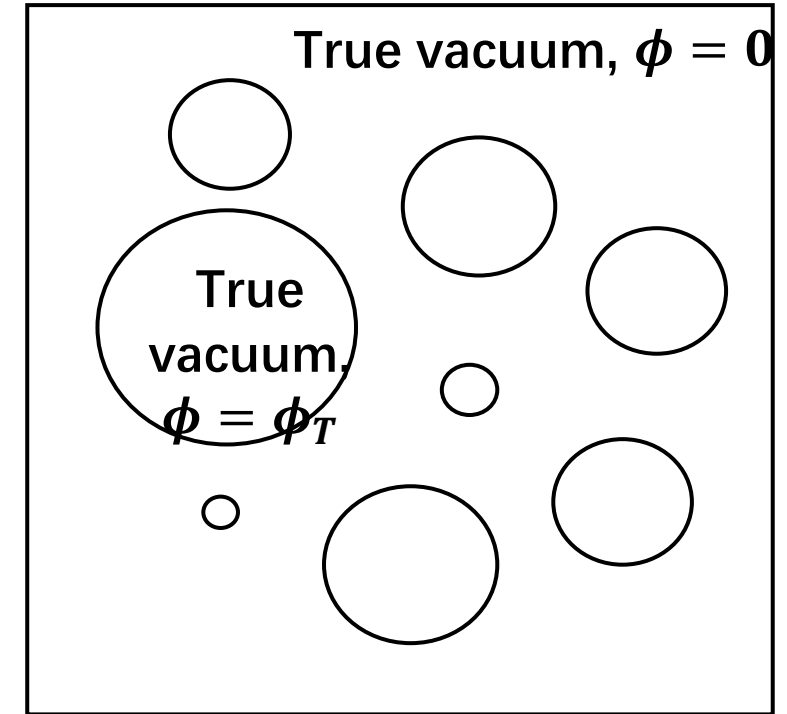
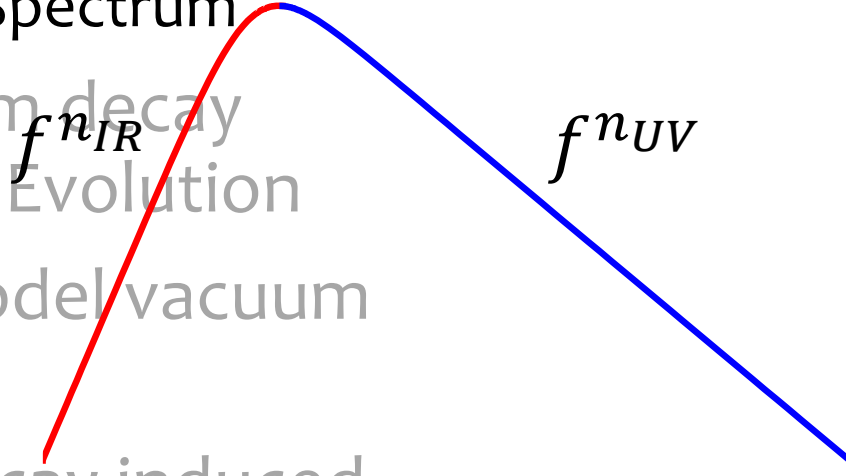
Outline

- Motivations
- GWs from a first-order phase transition
 - Strength Frequency Spectrum
- Standard Model vacuum decay induced by dark sector Evolution
- GWs from Standard Model vacuum decay
- Dark sector vacuum decay induced by Standard Model evolution
- Summary



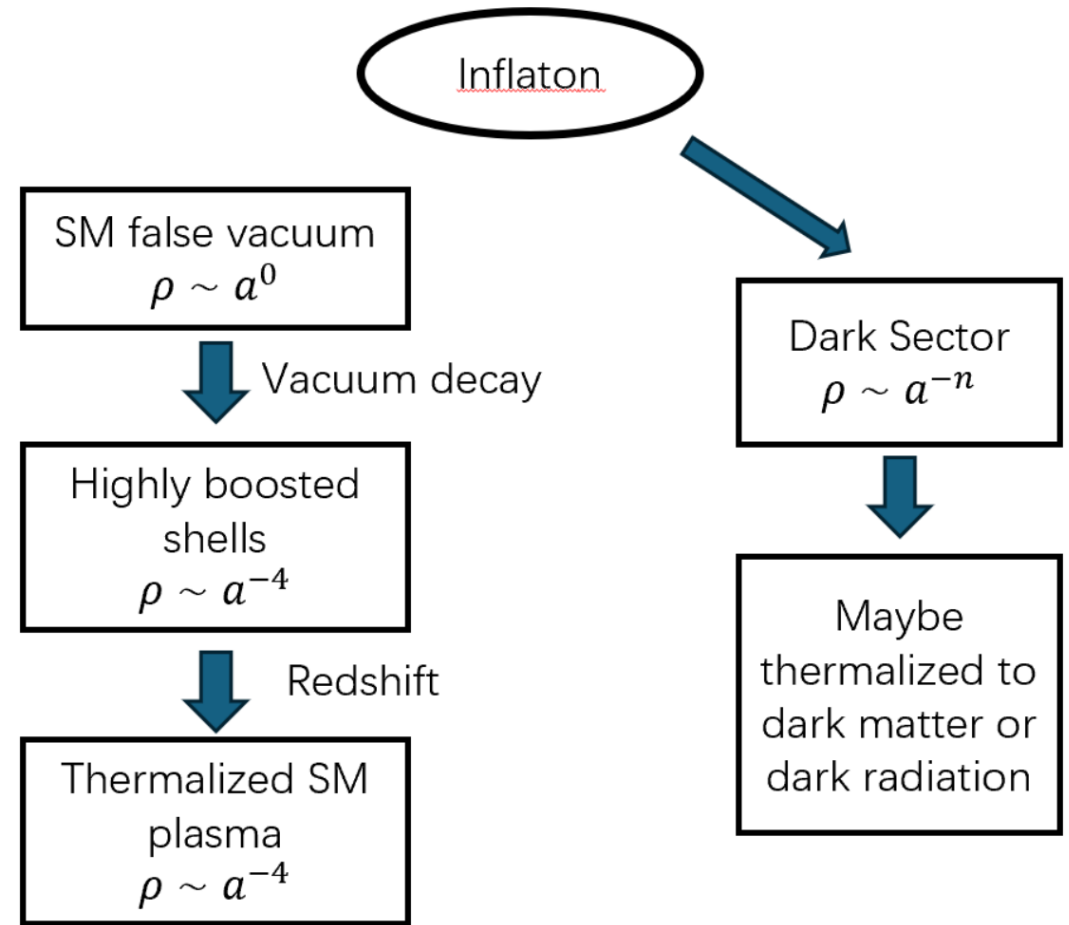
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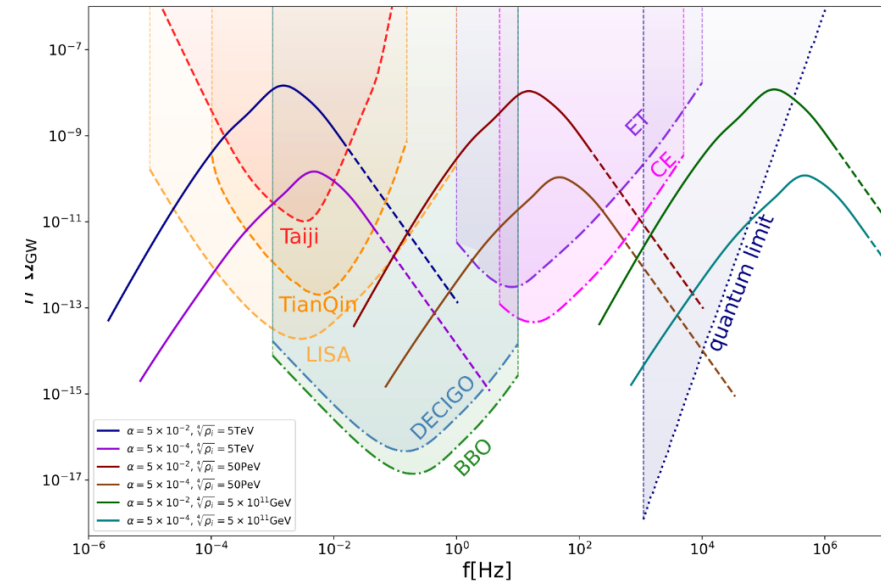
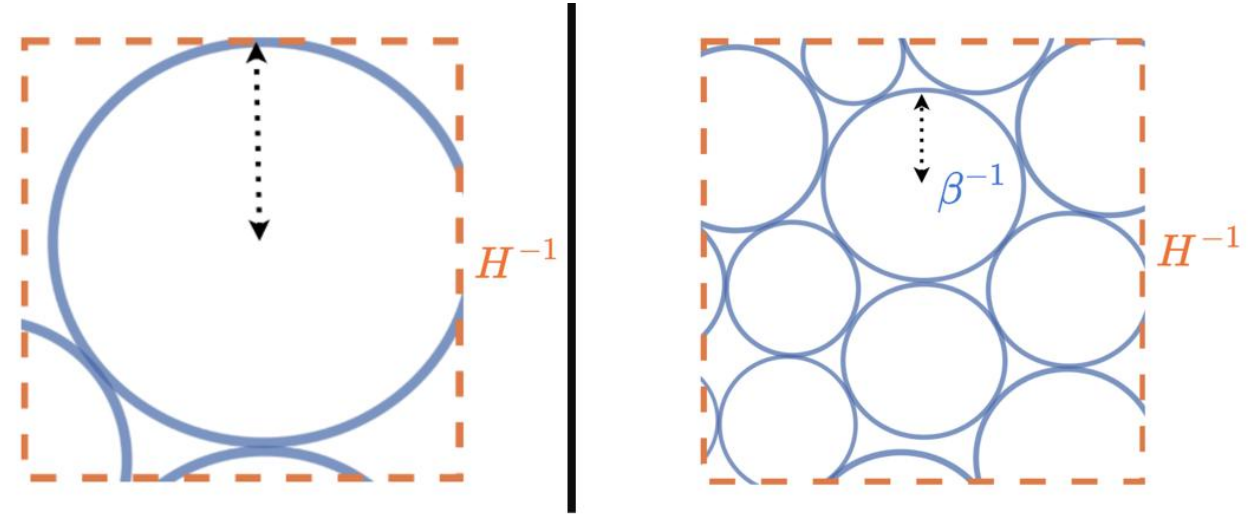
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The New Origin of the Big Bang

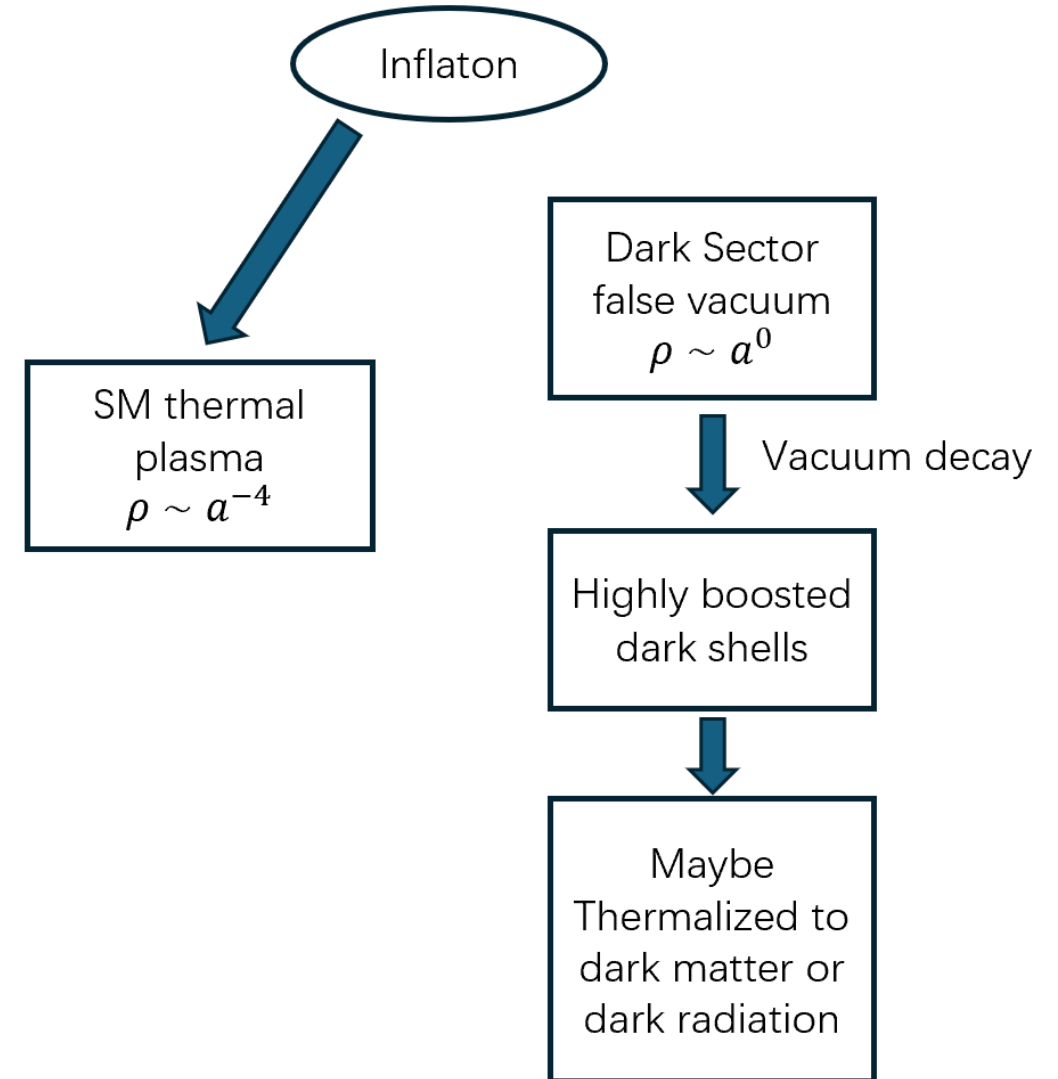
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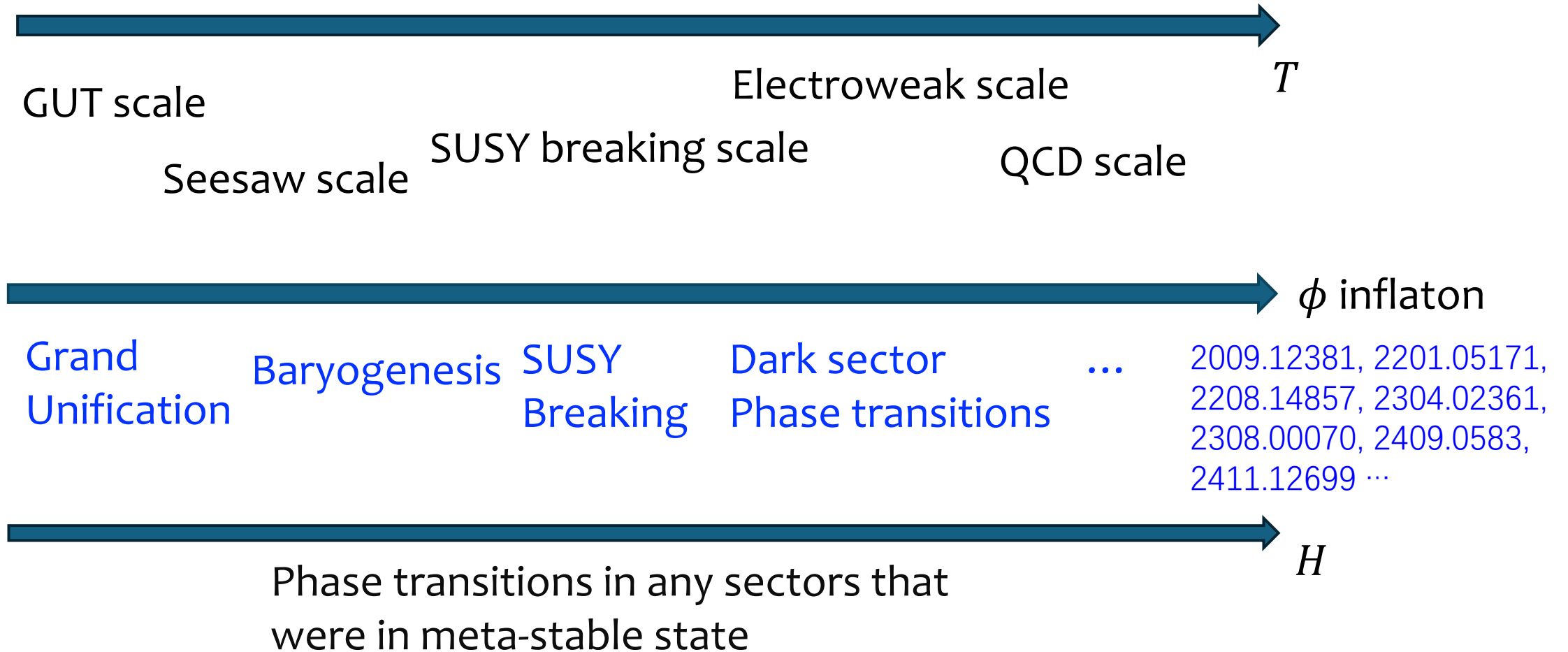
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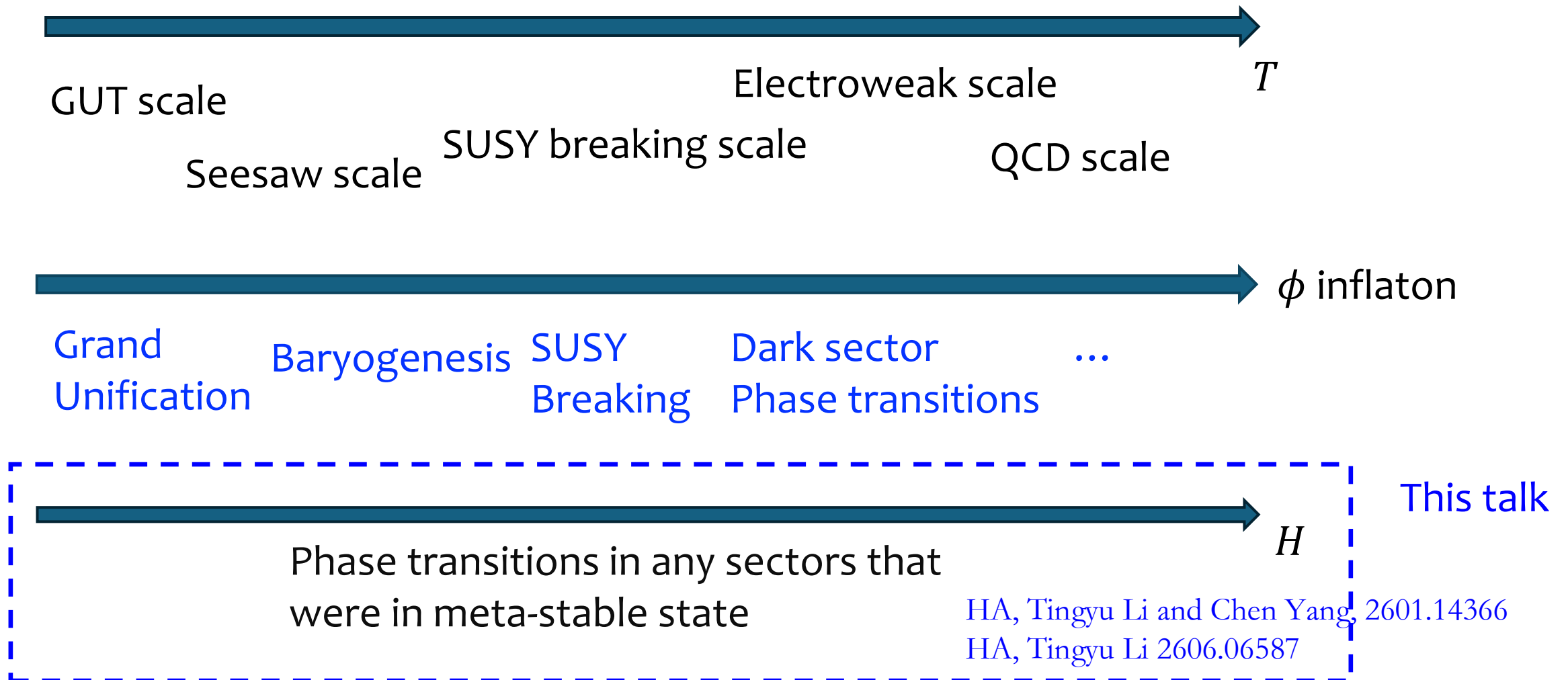
Motivations

- Phase transitions can be triggered by anything that evolves.



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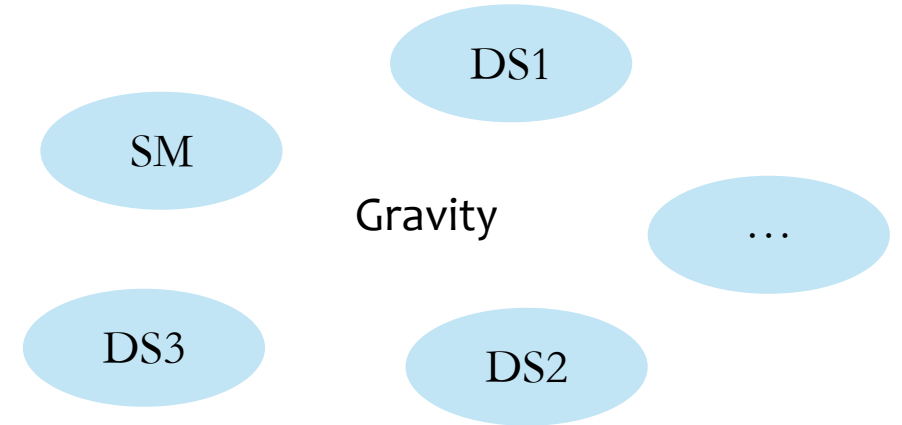


Motivations

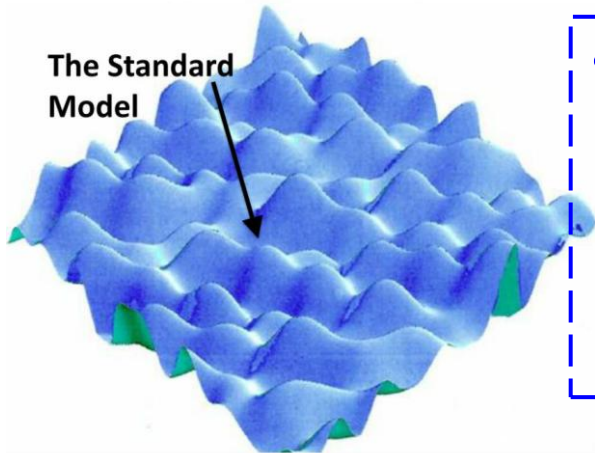
- The **Cosmological Constant** problem is still the deepest mystery in theoretical physics!
- The **anthropic principle** may be the only solution.
- Landscape is required for anthropic principle to work.



- The dark matter may only couple to the SM sector via gravity.



The String Theory Landscape



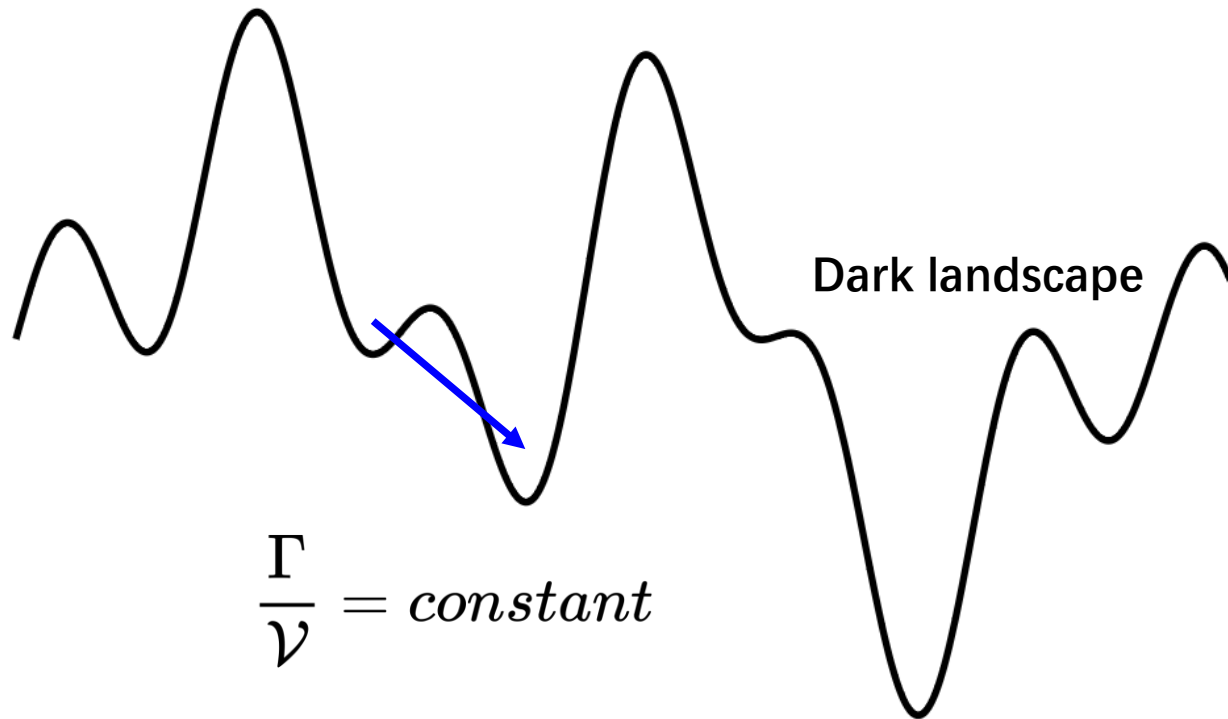
- After inflation, the inflaton field reheats some of the independent sectors.

- It is possible that each sector has an independent landscape of vacua.

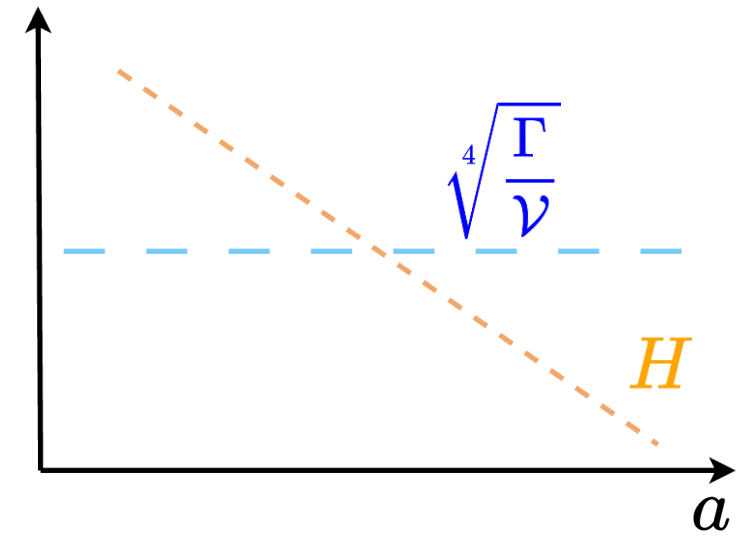
Motivations

- Meta stable vacuum decay

Phase transition condition: $\frac{\Gamma}{\mathcal{V}} \sim H^4$



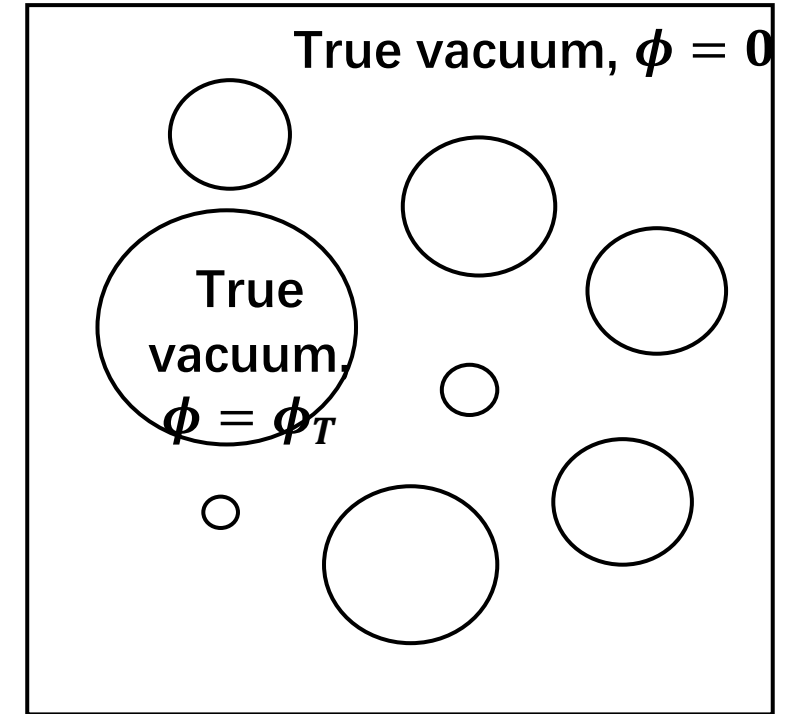
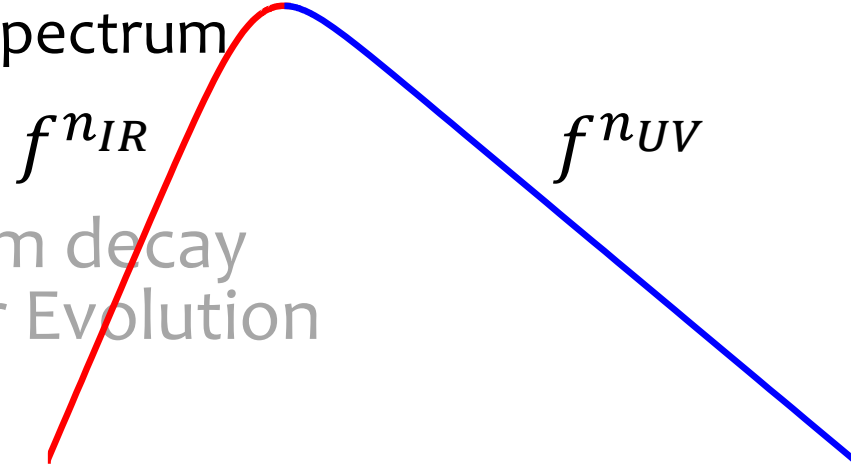
- The meta stable vacuum waits for the Hubble parameter to be small enough such that the phase transition will complete.



$$H \sim \frac{T^2}{M_{pl}} \sim a^{-2}$$

Outline

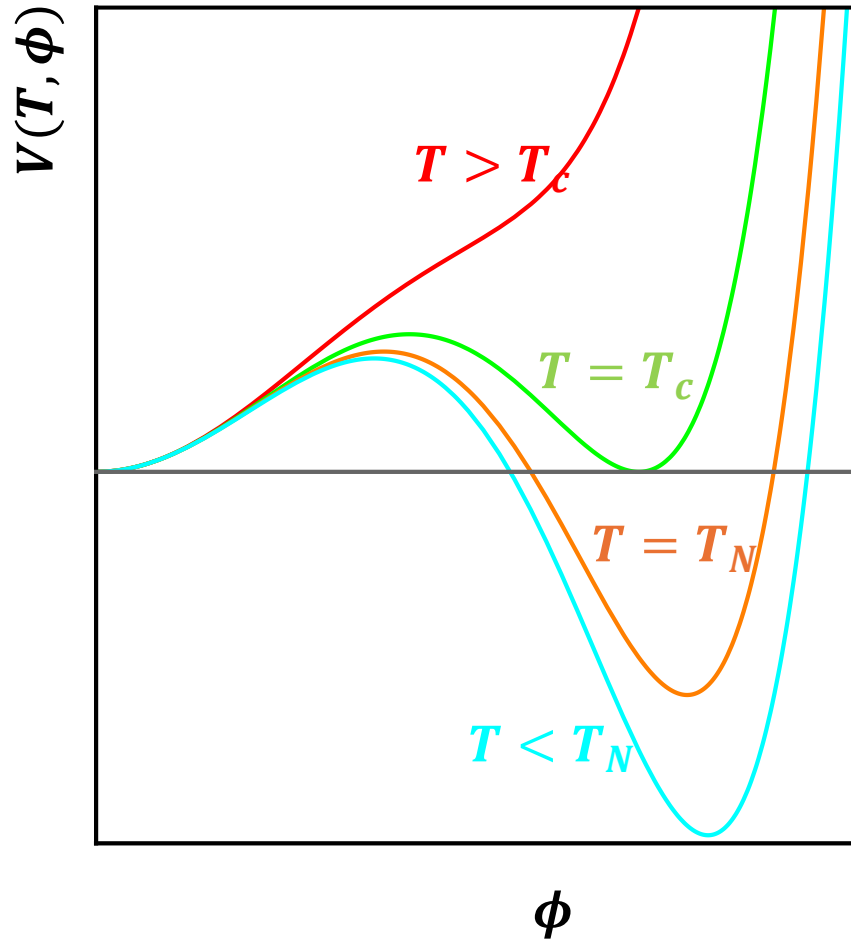
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GWs from a first-order phase transition

- Phase transition starts to complete when $\Gamma/V = H^4$ at $T = T_N$.

Coleman 1977, Callan and Coleman 1977, Linde 1981



$$\frac{\Gamma}{V} = m^4 e^{-S_3/T}$$
$$\frac{S_3}{T} \approx \frac{S_3(T_N)}{T_N} + \beta(t - t_N)$$
$$\beta = -\frac{d}{dt} \left(\frac{S_3}{T} \right)_{T=T_N}$$

β determines scales of the phase transition:

- (1) Phase transition completes $t \sim t_N + \beta^{-1}$.
- (2) Bubble radius $\sim \beta^{-1}$.
- (3) GW wavelength $\sim \beta^{-1}$.

$$\beta \sim H \left(\frac{S_3}{T} \right) \sim 150H$$

GW frequency

- $\beta = -\frac{d}{dt} \left(\frac{S_3}{T} \right) = -\frac{dT}{T dt} \frac{d}{d \log T} \left(\frac{S_3}{T} \right) = H \frac{d}{d \log T} \left(\frac{S_3}{T} \right) \sim H \left(\frac{S_3}{T} \right) \sim 150H$ for EWPT

- $\lambda_{today} \sim \lambda_{phys} \left(\frac{T_N}{T_{CMB}} \right) \sim \frac{M_{pl}}{T_N^2} \times \left(\frac{S_3(T_N)}{T_N} \right)^{-1} \times \frac{T_N}{T_{CMB}} \sim 10^7 \text{ km} \times \left(\frac{1 \text{ TeV}}{T_N} \right)$


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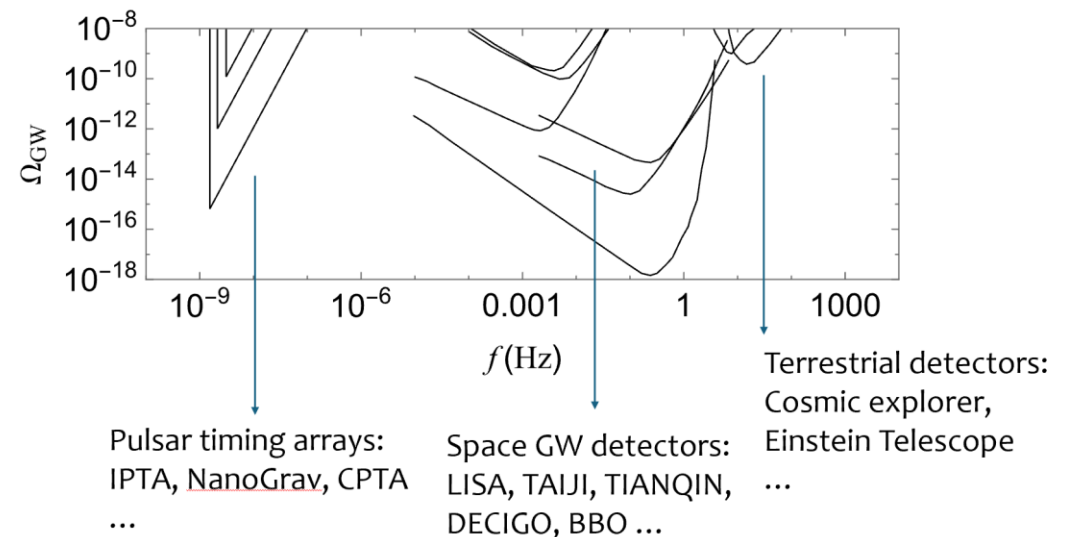
GW strength

- The metric around flat space-time
 $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- The gravitational waves are in h_{ij}^{TT} ,
the traceless and transverse part of
 h_{ij} .
- The gravitational potential are in h_{00} .
- In a relativistic system, we can use
 h_{00} to estimate h_{ij}^{TT} .

$$\rho_{\text{GW}} \sim \frac{GM^2}{R^4} \sim \frac{G\Delta\rho^2}{\beta^2} \quad H^2 \sim G\rho_R$$

$$\Omega_{\text{GW}} = \Omega_R \times \frac{\rho_{\text{GW}}}{\rho_R} \sim \Omega_R \times \frac{H^2}{\beta^2} \times \frac{\Delta\rho^2}{\rho_R^2}$$

$$\Omega_R \approx 10^{-5}$$



GW spectrum

- The metric in comoving coordinate with conformal time:

$$ds^2 = a^2(\eta)(-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j)$$

- The linearized Einstein Equation:

$$h''_{\mathbf{k},\lambda} + 2\mathcal{H}h'_{\mathbf{k},\lambda} + k^2 h_{\mathbf{k},\lambda} = 16\pi G a^2 e_{\lambda}^{ij} \Lambda_{ij}^{ab}(\hat{k}) T_{ab,\mathbf{k}},$$

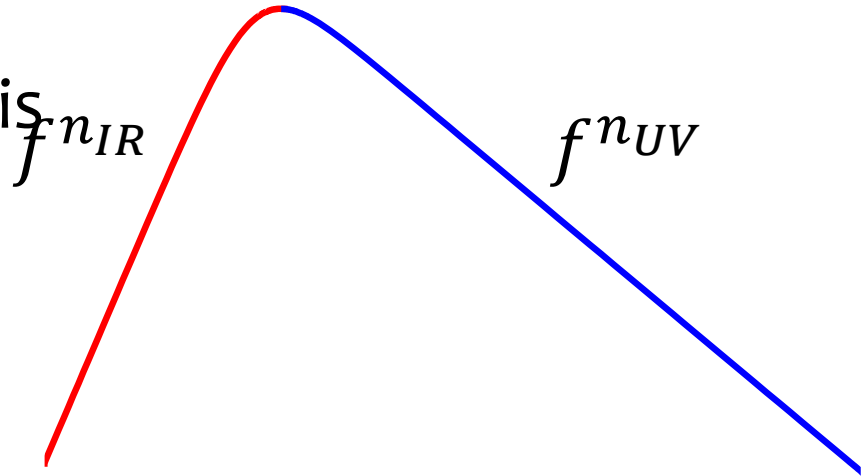
- The GW spectrum

$$\begin{aligned} \Omega_{\text{GW}}(k) = & \frac{k^3}{12\pi^2 a^4 H^2 M_{\text{Pl}}^4} \int_0^\eta d\eta_1 \int_0^\eta d\eta_2 a^3(\eta_1) a^3(\eta_2) \\ & \times \cos k(\eta_1 - \eta_2) \Lambda_{ab}^{cd}(\hat{k}) \langle T_{\mathbf{k}}^{ab}(\eta_1) T_{cd,\mathbf{p}}^*(\eta_2) \rangle'_{\mathbf{p}=\mathbf{k}}. \end{aligned}$$

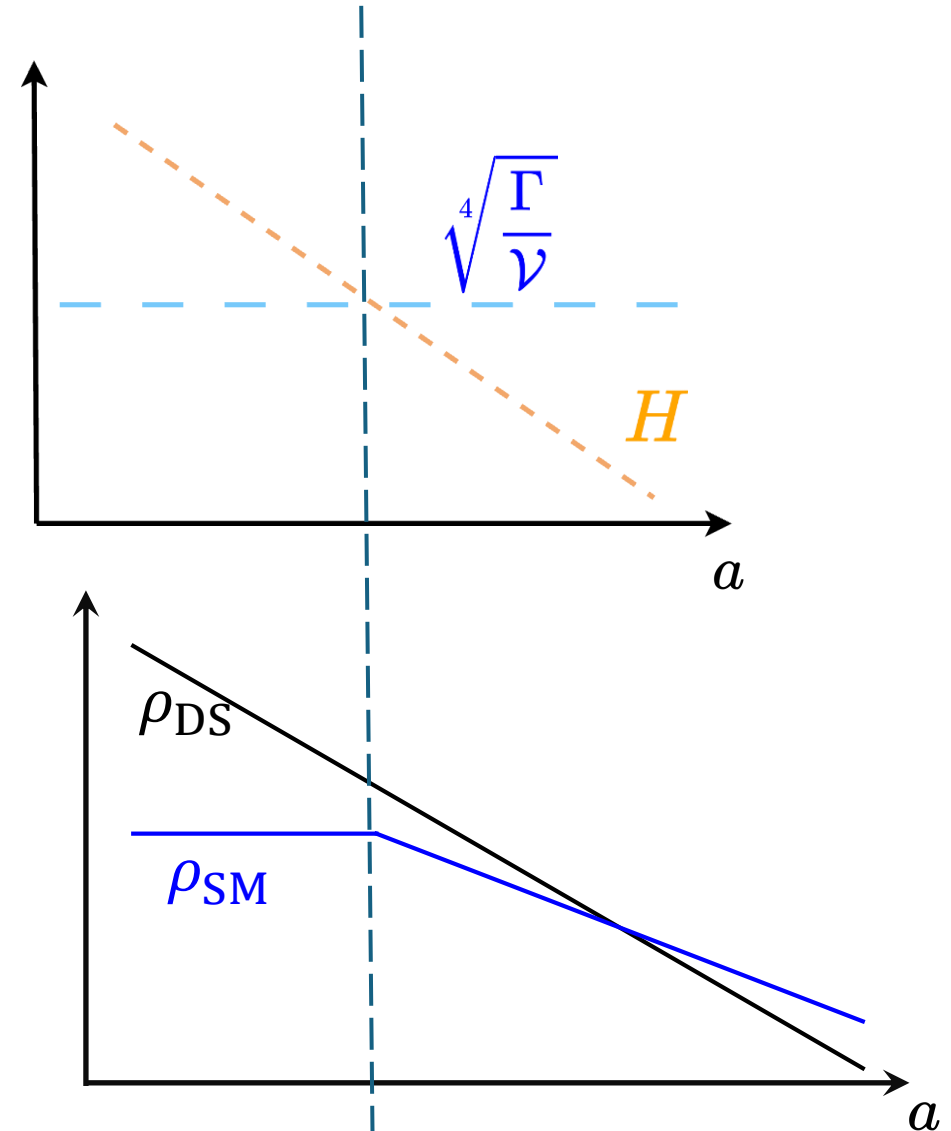
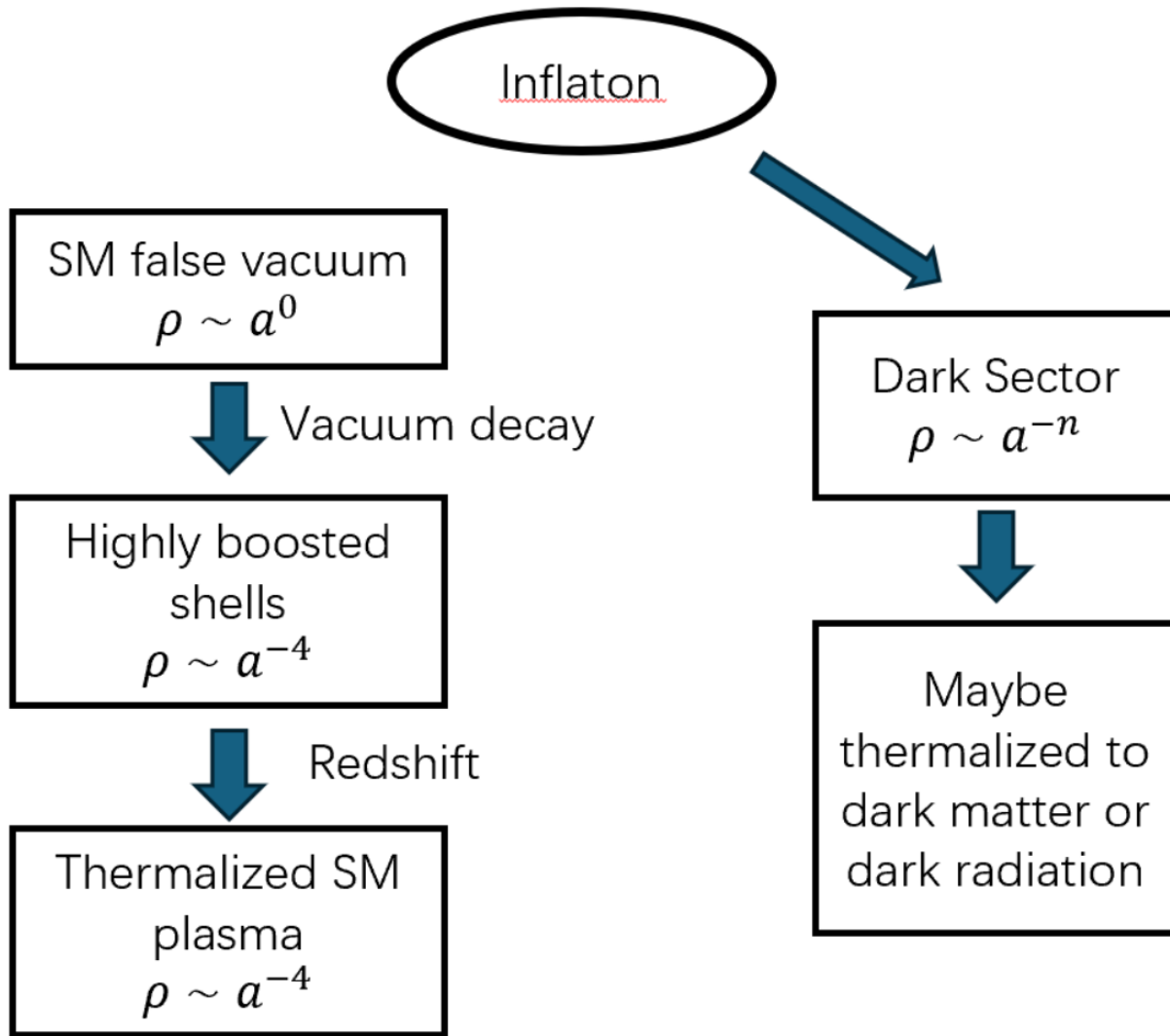
GW spectrum

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- $\beta^{-1} \ll H^{-1}$ {
 - $a(\eta)$ does not change much.
 - T_{ab} vanishes for $t > t_N + \beta^{-1}$.
 - For $k_{\text{phys}} < \beta$, the η_1, η_2 integral is independent of k .



SM vacuum decay induced by DS evolution



The New Origin of the Big Bang

DS equation of state

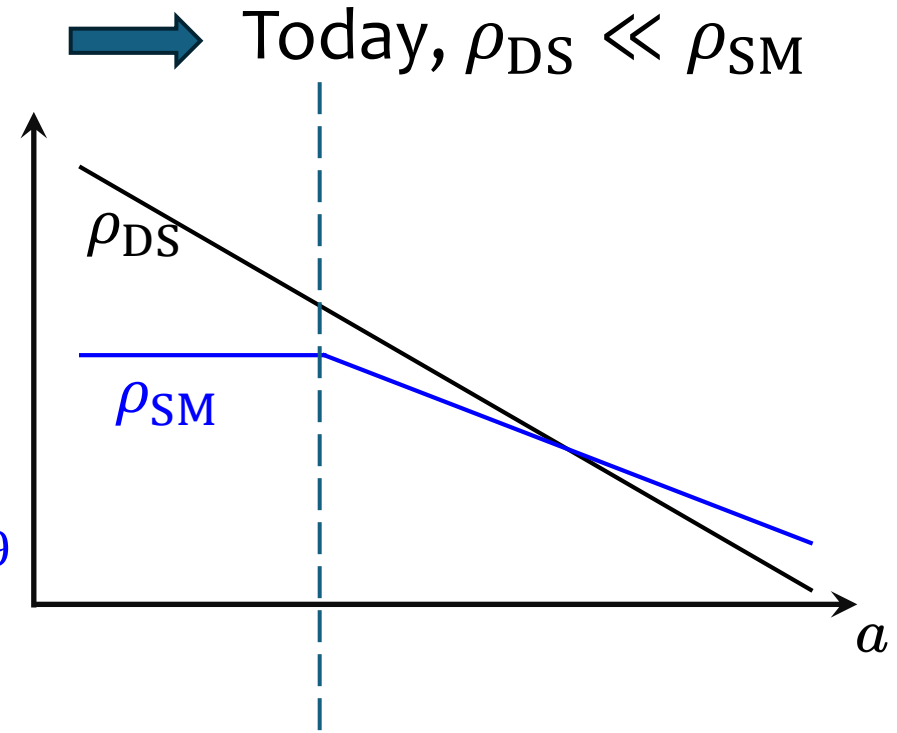
- During phase transition $\rho_{\text{DS}} \gg \rho_{\text{SM}}$ (PBH bound).
- Energy in DS today must satisfy the ΔN_{eff} bound $\Rightarrow$ Today, $\rho_{\text{DS}} \ll \rho_{\text{SM}}$

$$\rho_{\text{DS}} \propto a^{-n} \text{ with } n > 4.$$

- It can be achieved. However, to my knowledge **only kination (n=6)** model is “well-motivated”.

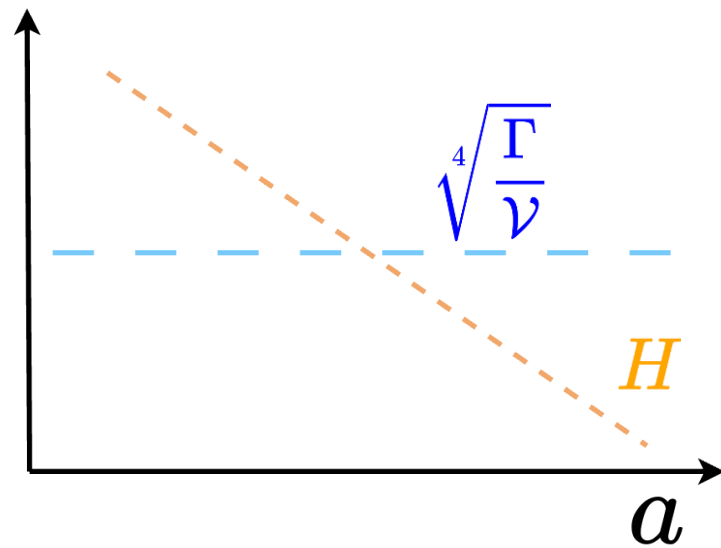
Spokoiny PLB 1993, Joyce PRD 1997, Peebles and Vilenkin PRD 1999

- The most natural way to realize kination domination is to make it happen right after inflation.



Evolution of the bubbles in Hubble delayed PT

- No concept of β
- $\dot{H}/H \sim H$
- Bubble radius $\sim H^{-1}$



Hubble delayed PT:

$$\frac{\Gamma}{\mathcal{V}} = \text{constant}$$

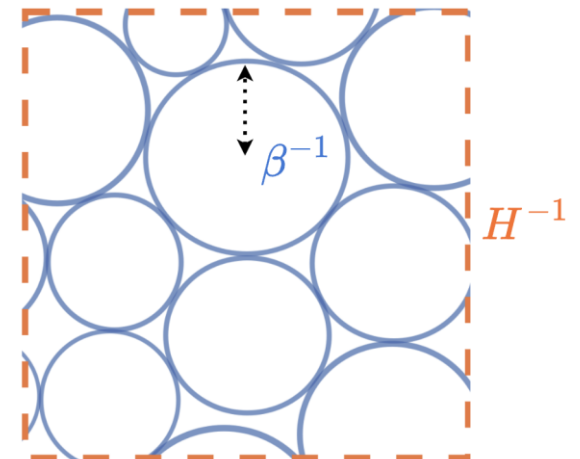
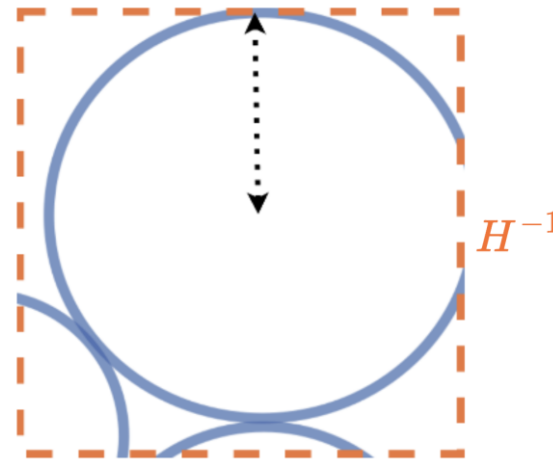
(No fluid, no plasma)

Thermal PT:

$$\frac{\Gamma}{\mathcal{V}} \sim \exp(\beta t)$$

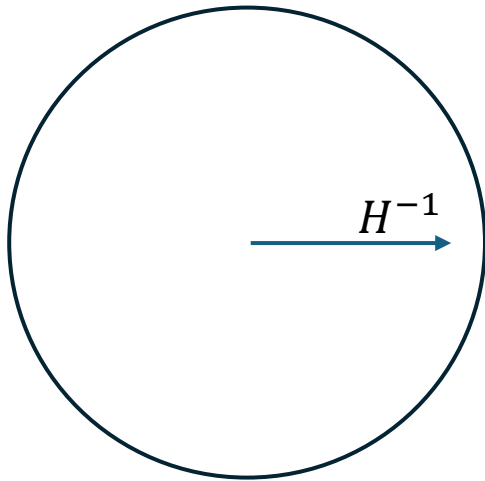
Phase transition condition:

$$\frac{\Gamma}{\mathcal{V}} \sim H^4$$



Evolution of the bubbles in Hubble delayed PT

- No plasma or fluid
- No frictions to bubble expansion
- The boost can be very large
- The walls are very thin.



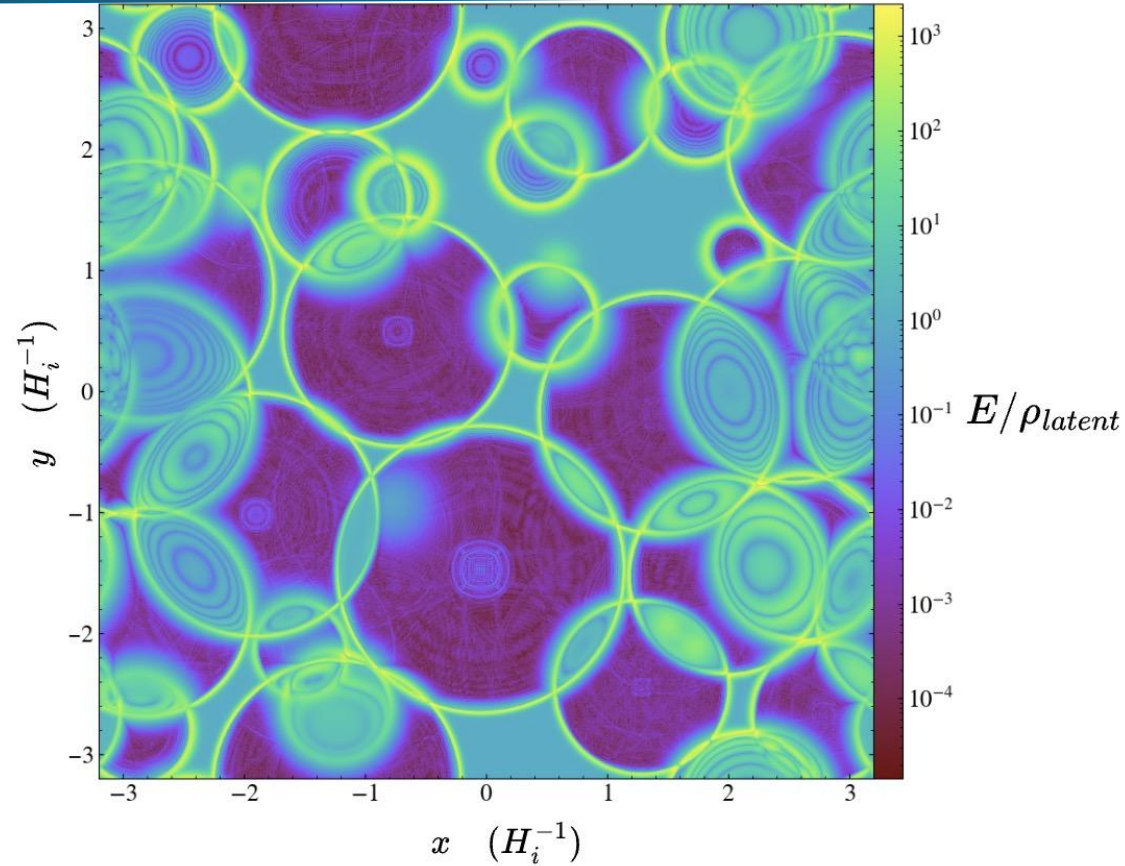
- Boost: $\gamma \sim \frac{\Delta\rho^{\frac{1}{4}}}{H} \sim \left(\frac{\Delta\rho}{\rho}\right)^{\frac{1}{4}} \frac{M_{pl}}{T} \sim 10^{15} \left(\frac{\Delta\rho}{\rho}\right)^{\frac{1}{4}} \quad (\text{EW})$
- The characteristic energy of each particle in the walls approaches M_{pl} .

Bubble evolution after collisions

- Long lived shell structure after collision
- EoM of the scalar field:

$$\underbrace{\frac{\partial^2 \phi}{\partial \eta^2}}_{\gamma^2} + 2\mathcal{H} \underbrace{\frac{\partial \phi}{\partial \eta}}_{\gamma^1} - \underbrace{\frac{1}{r} \frac{\partial^2}{\partial r^2}}_{\gamma^2} (r\phi) + a^2 \underbrace{\frac{dV}{d\phi}}_{\gamma^0} = 0$$

- The shells still form spherical structures.
- Potential terms are negligible.
- High energy shells go through each other in subsequent collisions.



Thermalization is very slow with scalar potential interactions.

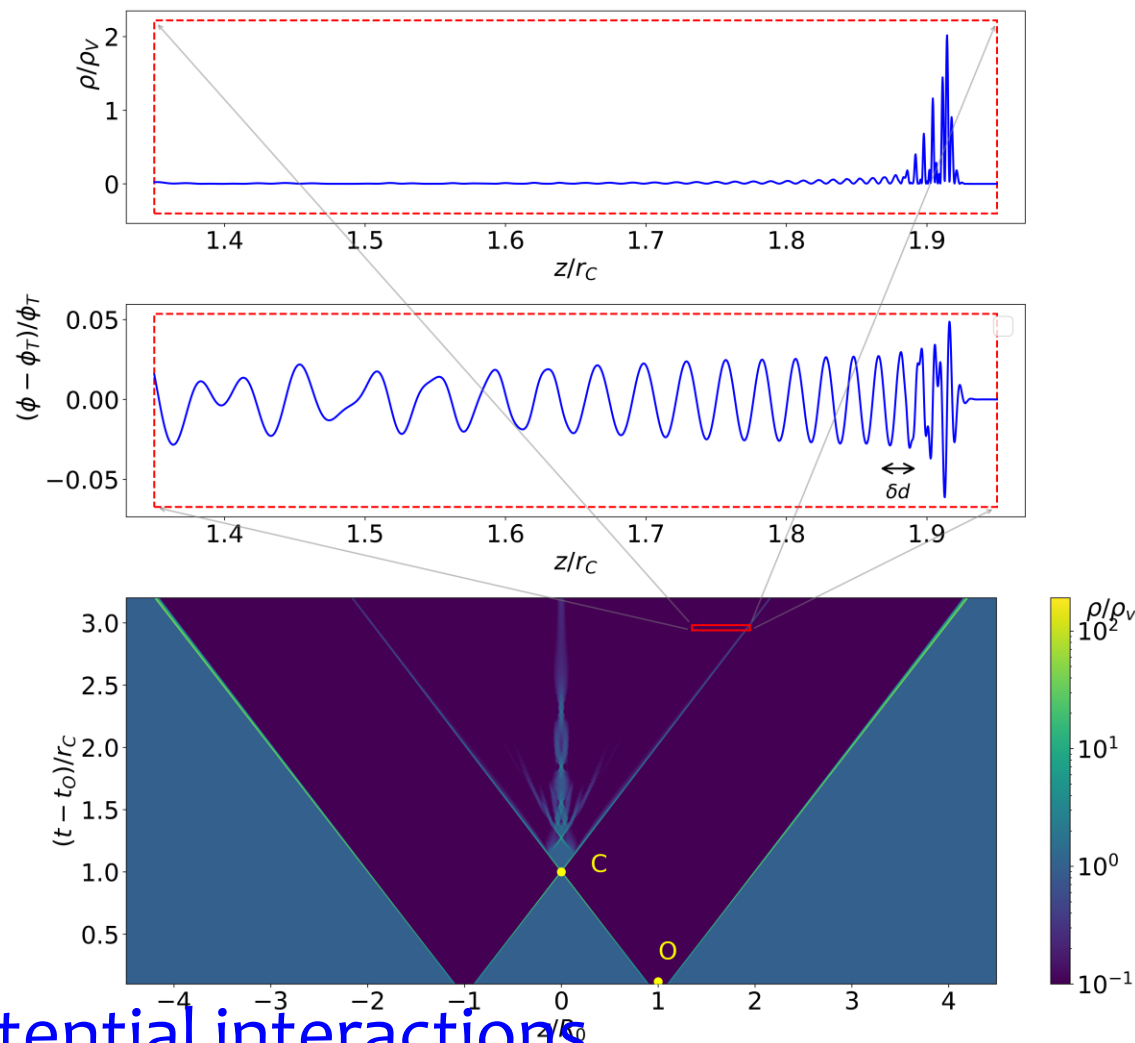
Properties of the GWs from DS vacuum decay

- Long lived shell structure after collision
- EoM of the scalar field:

$$\underbrace{\frac{\partial^2 \phi}{\partial \eta^2}}_{\gamma^2} + 2\mathcal{H} \underbrace{\frac{\partial \phi}{\partial \eta}}_{\gamma^1} - \underbrace{\frac{1}{r} \frac{\partial^2}{\partial r^2}}_{\gamma^2} (r\phi) + a^2 \underbrace{\frac{dV}{d\phi}}_{\gamma^0} = 0$$

- The shells still form spherical structures.
- Potential terms are negligible.
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Thermalization is very slow with scalar potential interactions.



Thermalization with gauge interactions

- Interactions that the cross section is not suppressed by large γ .

$$HH \rightarrow W^+ W^-$$



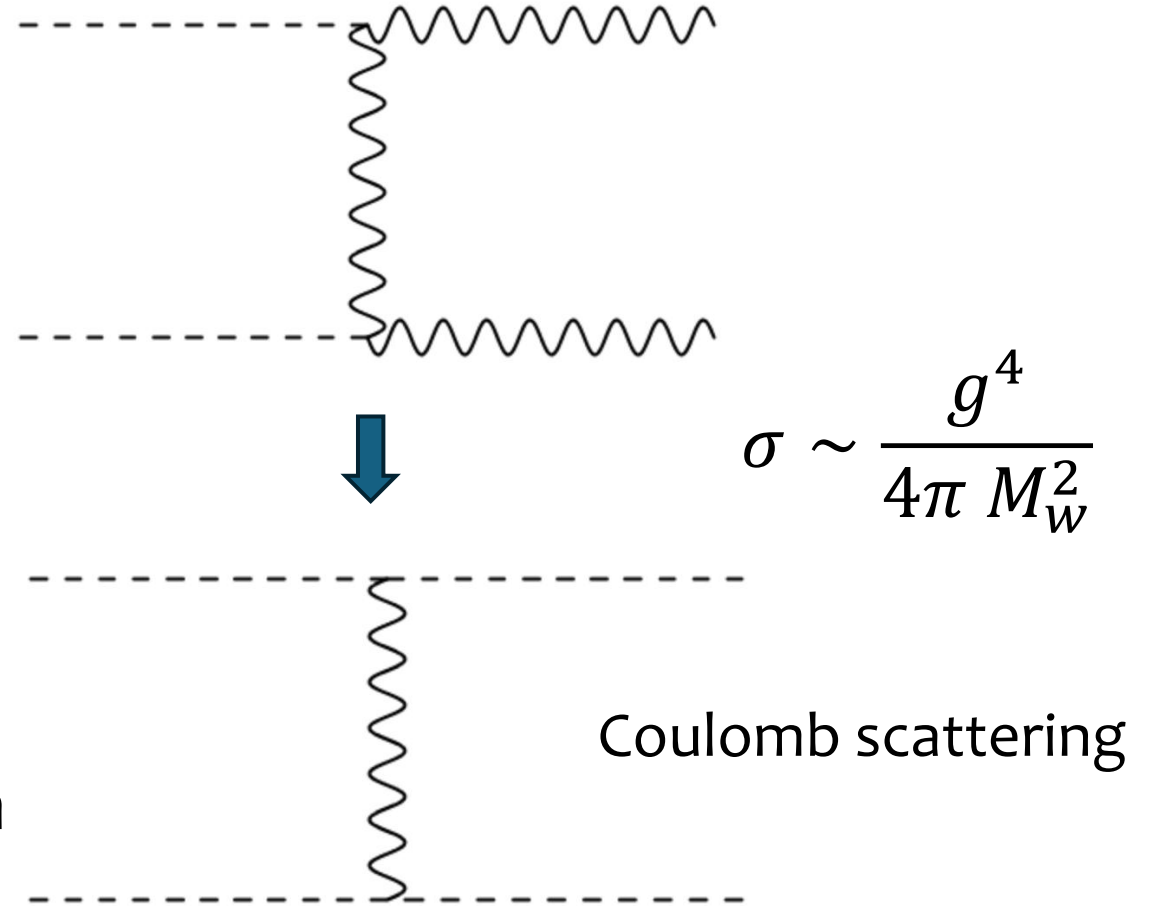
$$HH \rightarrow W_L^+ W_L^-$$



$$HH \rightarrow \phi\phi$$

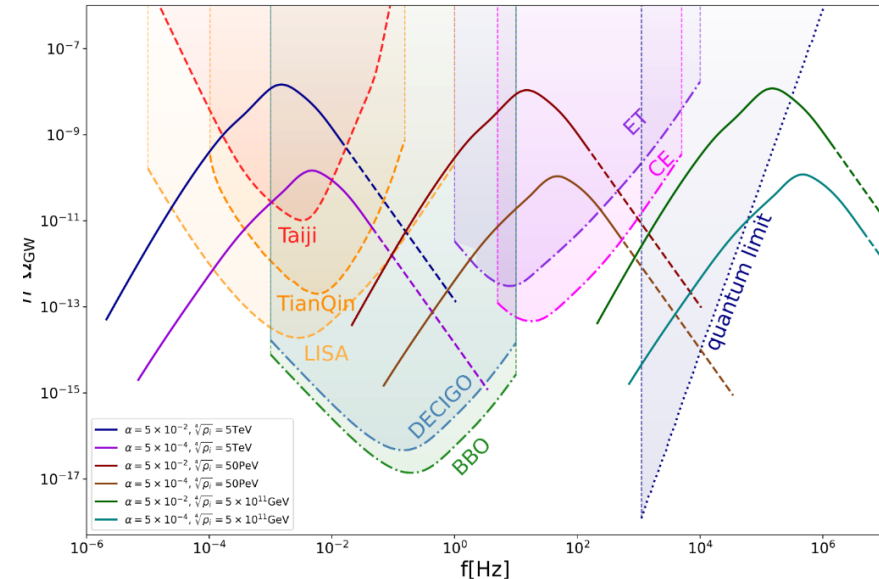
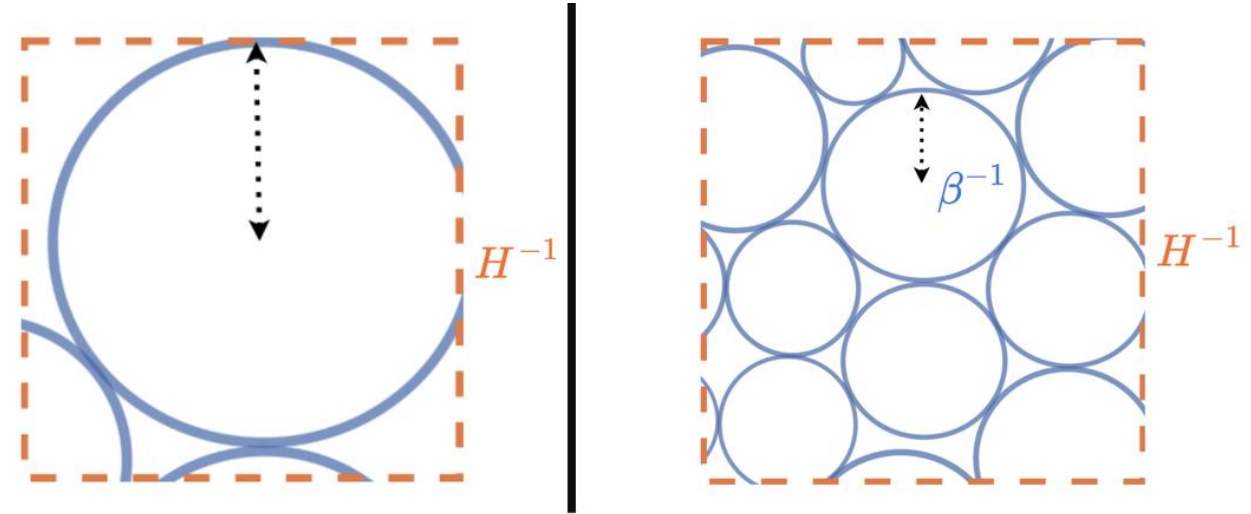
Goldstone
equivalence
theorem

- W bosons will decay into quarks.
- Once most of the energy are in the form of quarks, thermalization will complete.
- It needs about $4\pi/g^4$ collisions to achieve thermalization.



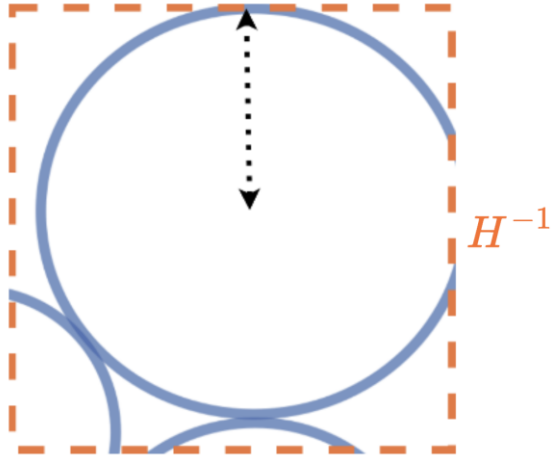
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Dark sector triggered SM sector vacuum decay

- GW strength



$$\rho_{\text{GW}} \sim \frac{GM^2}{R^4} \sim G\Delta\rho^2 R^2 \sim \frac{G\Delta\rho^2}{H^2} \quad \Delta\rho \sim \rho_R$$

$$\Omega_{\text{GW}} = \Omega_R \times \frac{\rho_{\text{GW}}}{\rho_R} \sim \Omega_R \times \frac{G\Delta\rho^2}{H^2 \rho_R} \sim \Omega_R \times \frac{\Delta\rho^2}{\rho_{\text{tot}} \rho_R} \sim \Omega_R \times \frac{\Delta\rho}{\rho_{\text{tot}}}$$

$$\Omega_{\text{GW}} = C\Omega_R \times \frac{\Delta\rho}{\rho_{\text{tot}}} \approx 3 \times 10^{-8} \times \frac{\alpha}{0.05}$$

$C \approx 0.01$ (numerical simulation)

$$\alpha = \frac{\Delta\rho}{\rho_{\text{tot}}} < 0.05 \quad \text{Primordial black hole bound}$$

GW spectrum

- $$\tilde{h}_{+,\times}(\eta, \mathbf{k}) = 16\pi G \int_{-\infty}^{\infty} d\eta' G(\eta, \eta', \mathbf{k}) a(\eta') \tilde{T}_{+,\times}(\eta', \mathbf{k})$$

$$G(\eta, \eta', \mathbf{k}) = y_1(\eta, \mathbf{k})y_2(\eta', \mathbf{k}) - y_2(\eta, \mathbf{k})y_1(\eta', \mathbf{k})$$

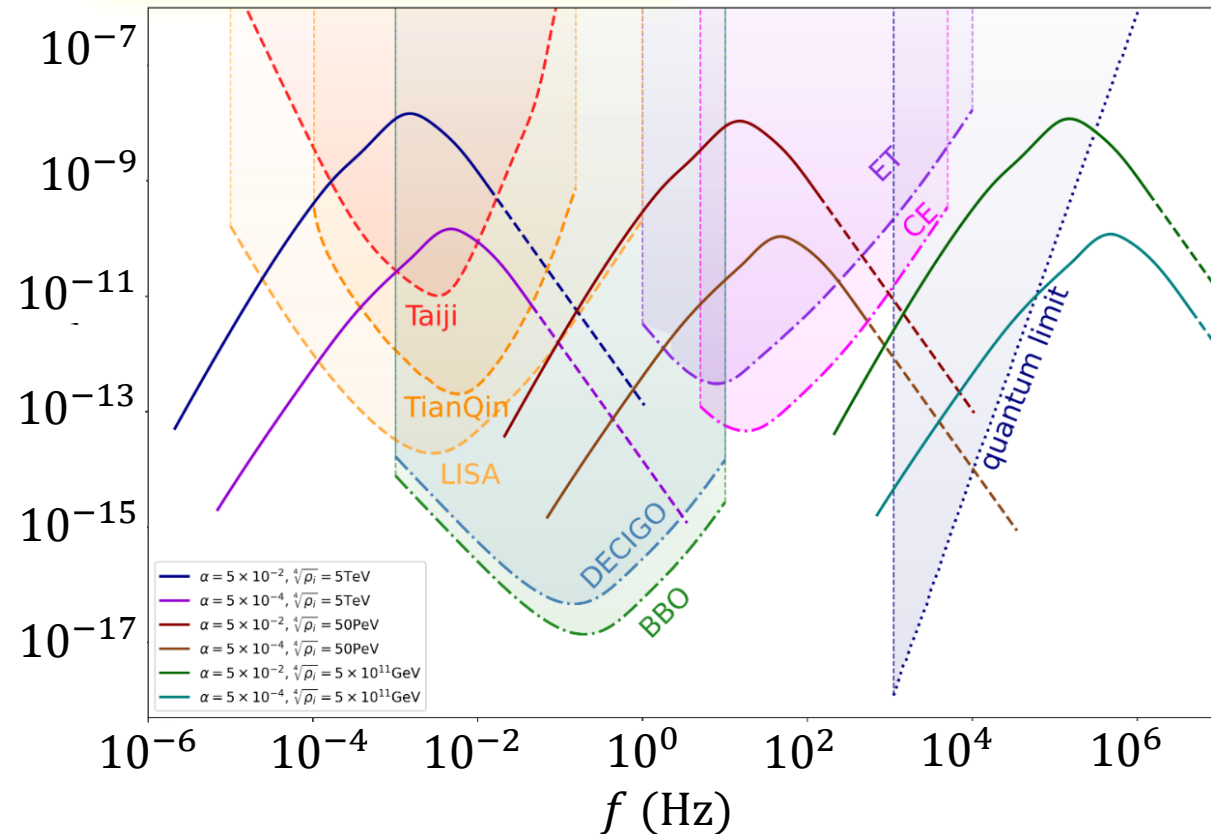
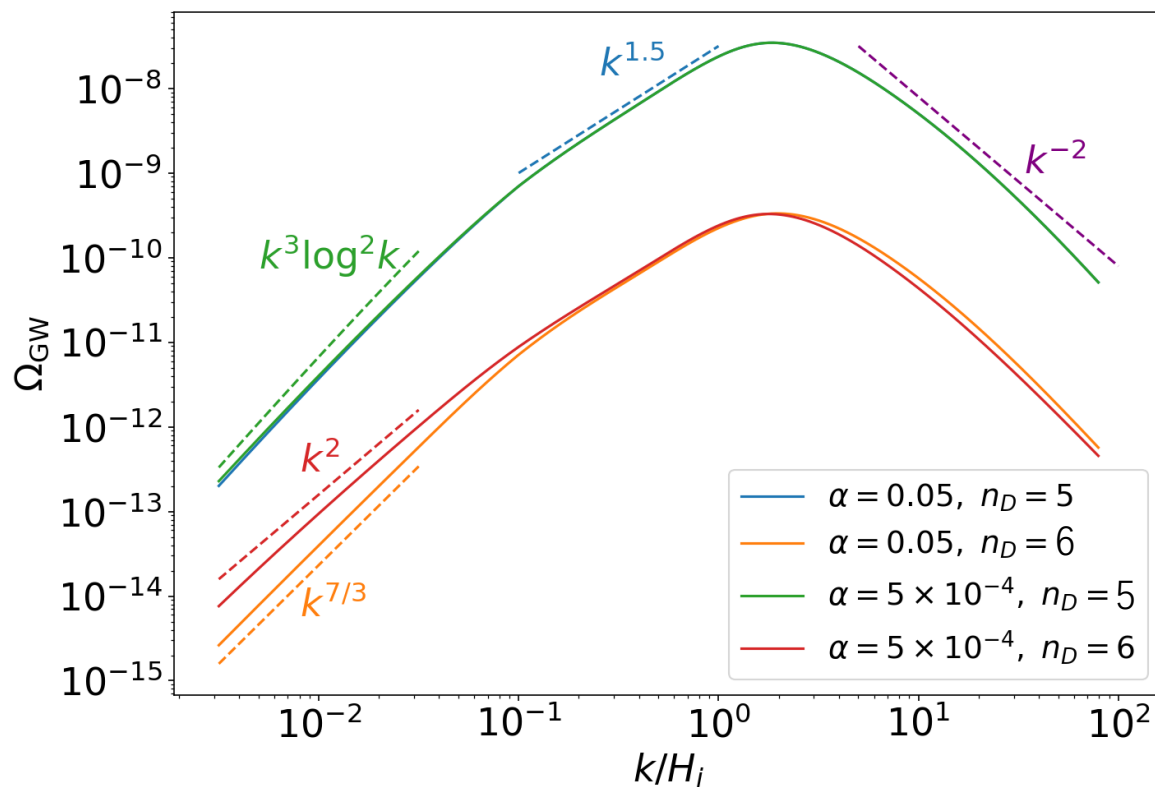
Expansion of the Universe is important

lasts at least
several e-folds

- From the GW spectrum we can get the detailed information of the expansion of the universe.

GW spectrum

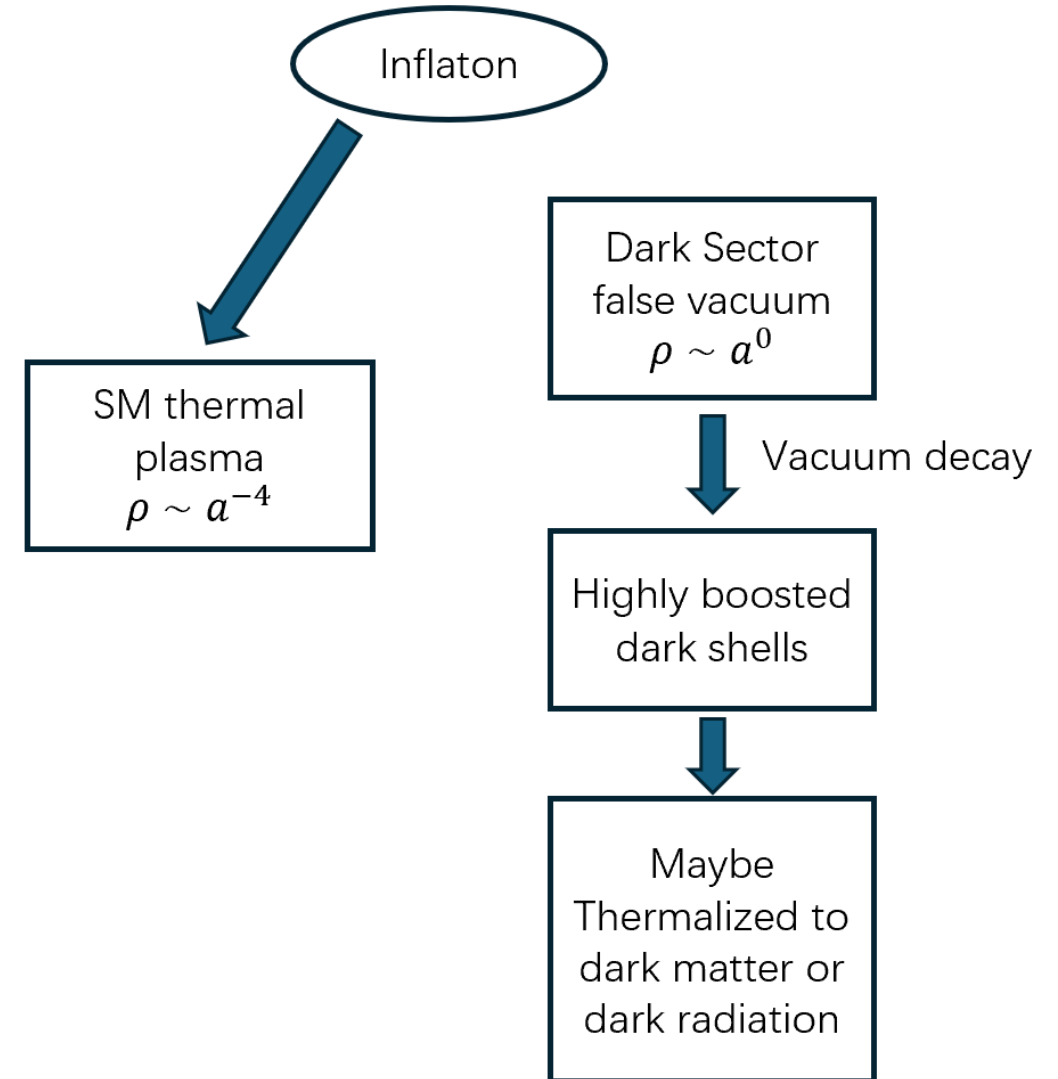
- Numerical simulation using the bulk flow model



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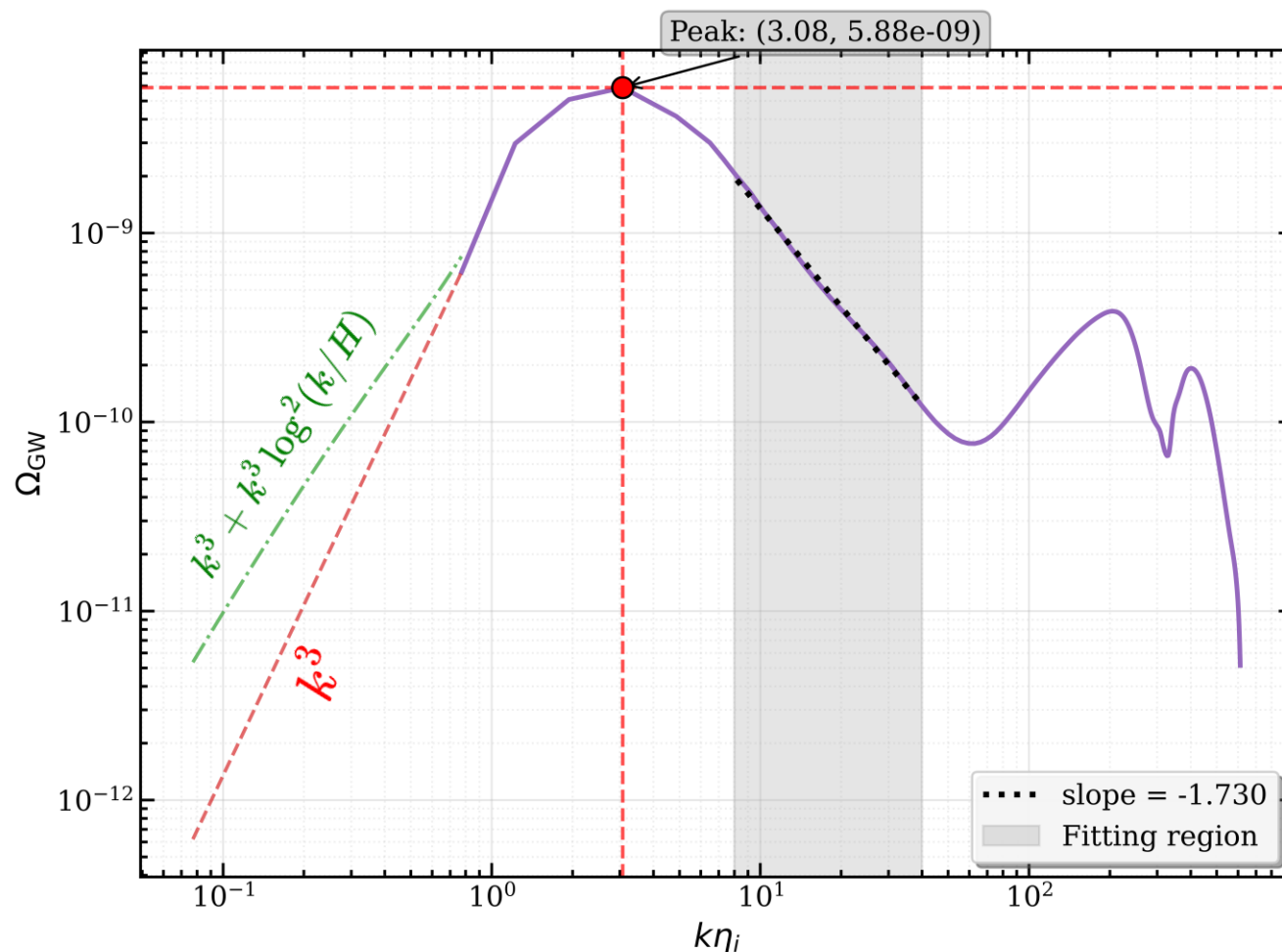
- Results:

- IR: $\frac{d\rho_{\text{GW}}^{\text{IR}}}{d\ln k} \propto k^3 \log^2 \frac{k}{H_0} + O(k^3)$

- UV: $\Omega_{\text{GW}} \propto k^{-1.73}$

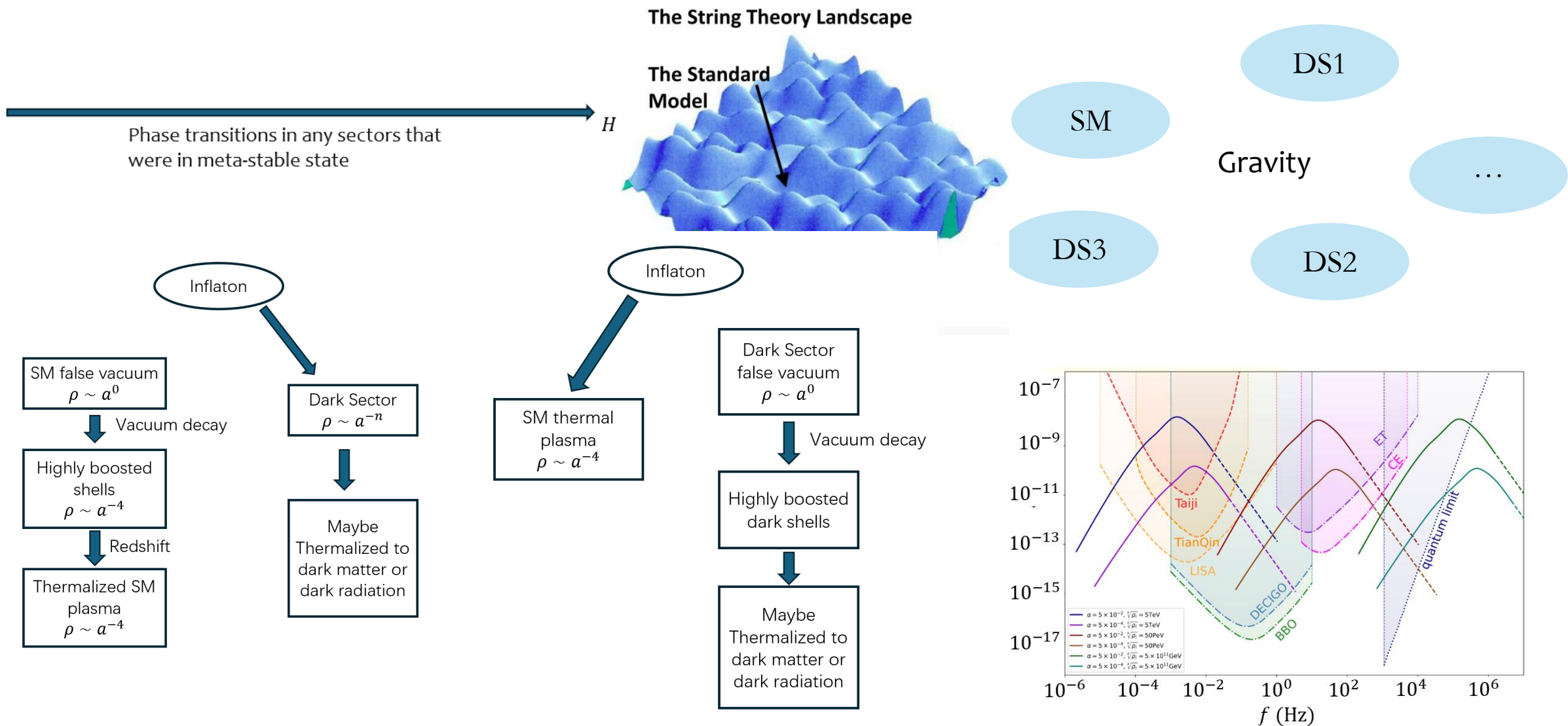
- Peak: $k_{\text{peak}}/H = 3.08$

- $\Omega_{\text{GW}}^{\text{peak}} = 1.5\Omega_R \left(\frac{L}{\rho}\right)^2$



GW power spectrum from 2048^3 simulation

Summary



Backups

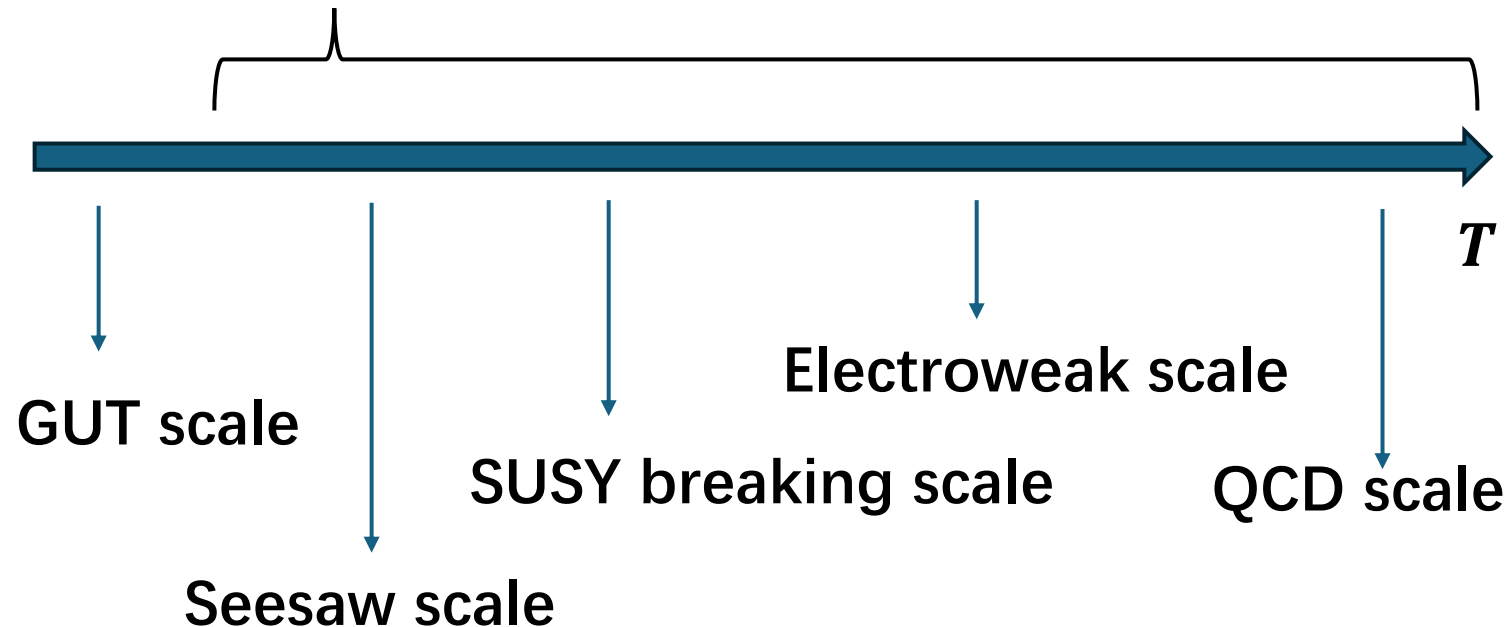
Motivations (Cosmological phase transitions)

- The temperature evolves.
- The Hubble expansion rate

$$\dot{a}/a = H \sim T^2/M_{pl}$$

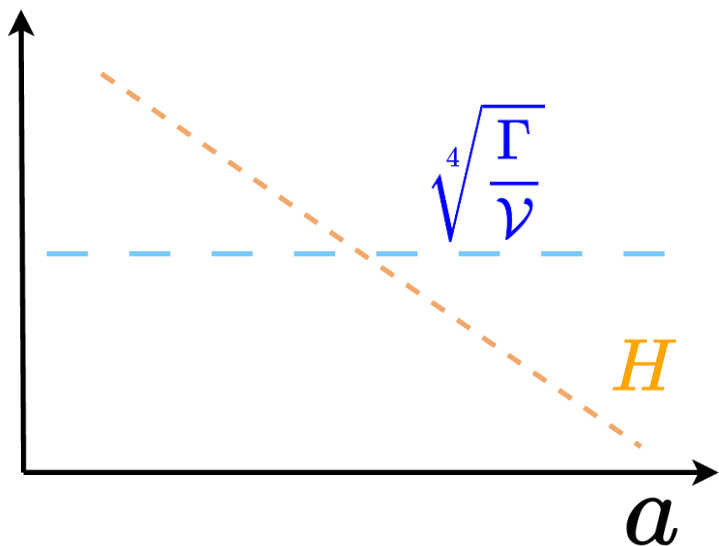
is small!

- The evolution is mostly quasi-static.
- Phase transition happens when T goes across some typical energy scales.



Motivations

- No concept of β
- Bubble radius $\sim H^{-1}$



Vacuum decay PT:

$$\frac{\Gamma}{\mathcal{V}} = \text{constant}$$

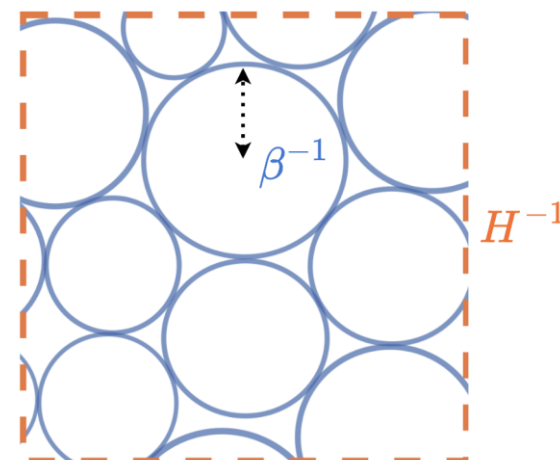
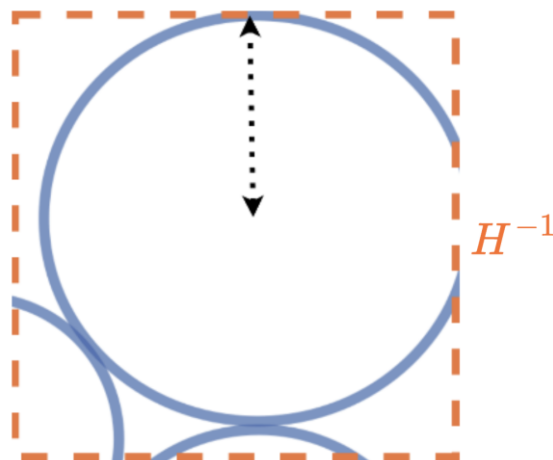
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Thermal PT:

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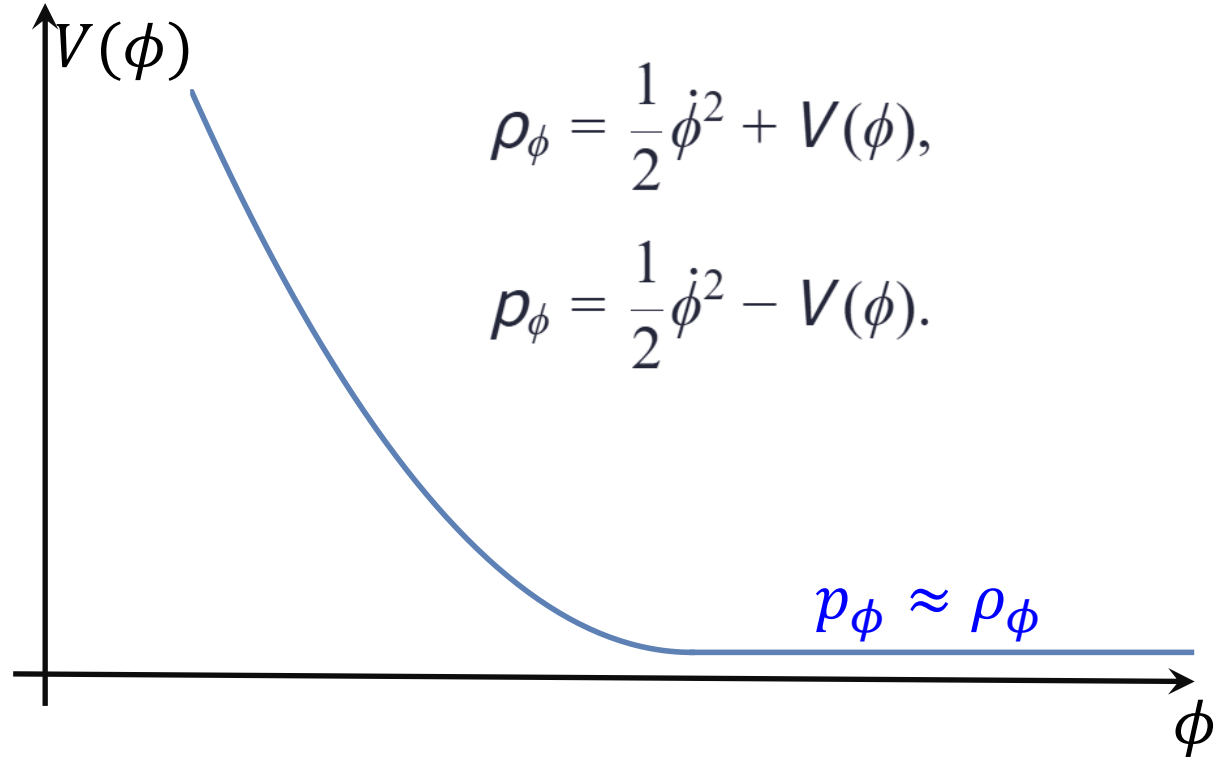
Phase transition condition:

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Simplest realization of kination domination

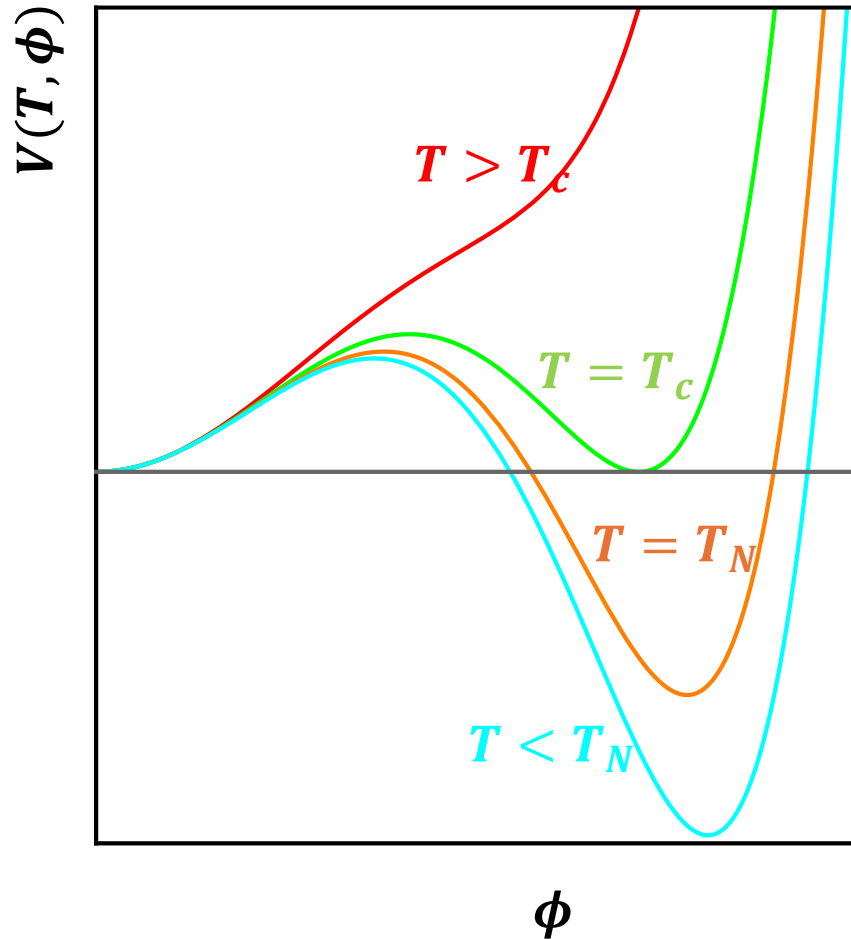
- The inflaton field potential becomes flat right after the end of inflation.
- Inflaton field transits all its energy into its kinetic energy.
- $p_\phi \approx \rho_\phi$
- $n = 6$ (Friedmann equations)



GWs from a first-order phase transition

- Phase transition starts to complete when $\Gamma/V = H^4$ at $T = T_N$.

Coleman 1977, Callan and Coleman 1977, Linde 1981



$$m^4 e^{-S_3/T}$$

$$m \sim T_N$$

S_3 : 3D bounce action

$$\frac{S_3}{T} = 4 \ln \left(\frac{m}{H} \right)$$

$$\sim 4 \ln \left(\frac{M_{pl}}{m} \right) \sim 150$$

for electroweak
phase transitions

GW frequency

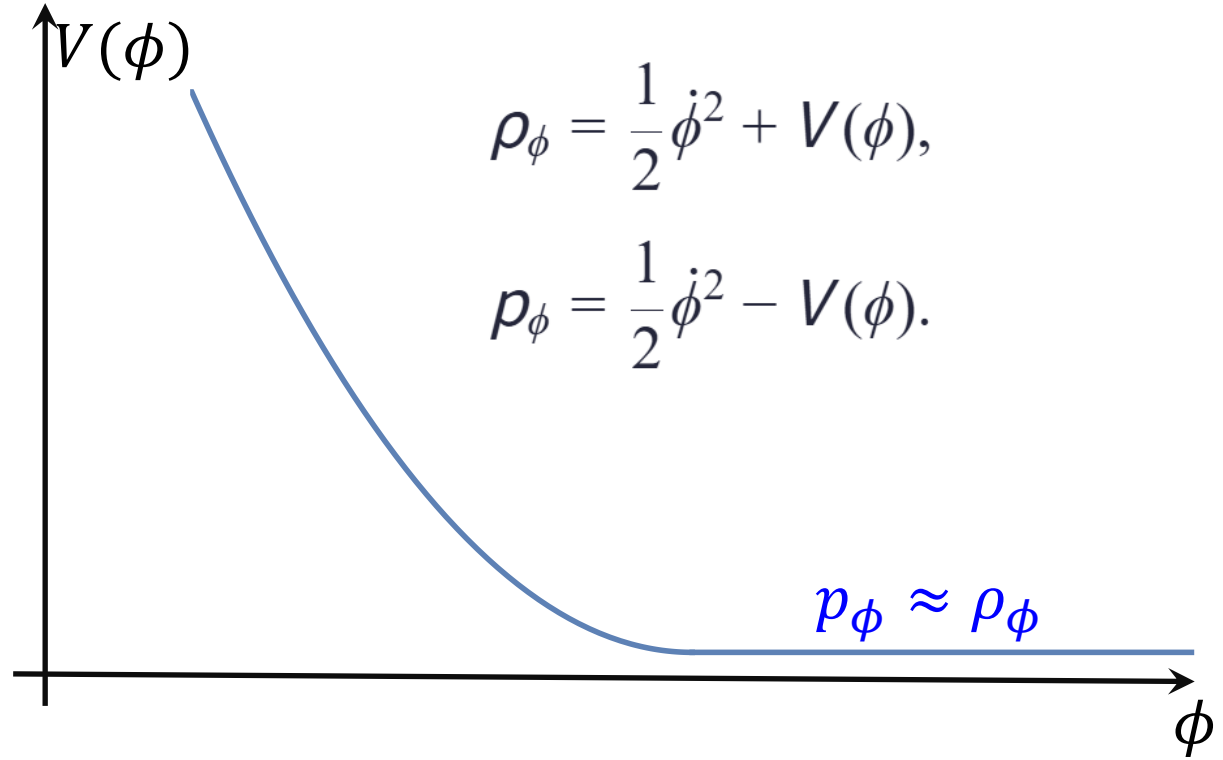
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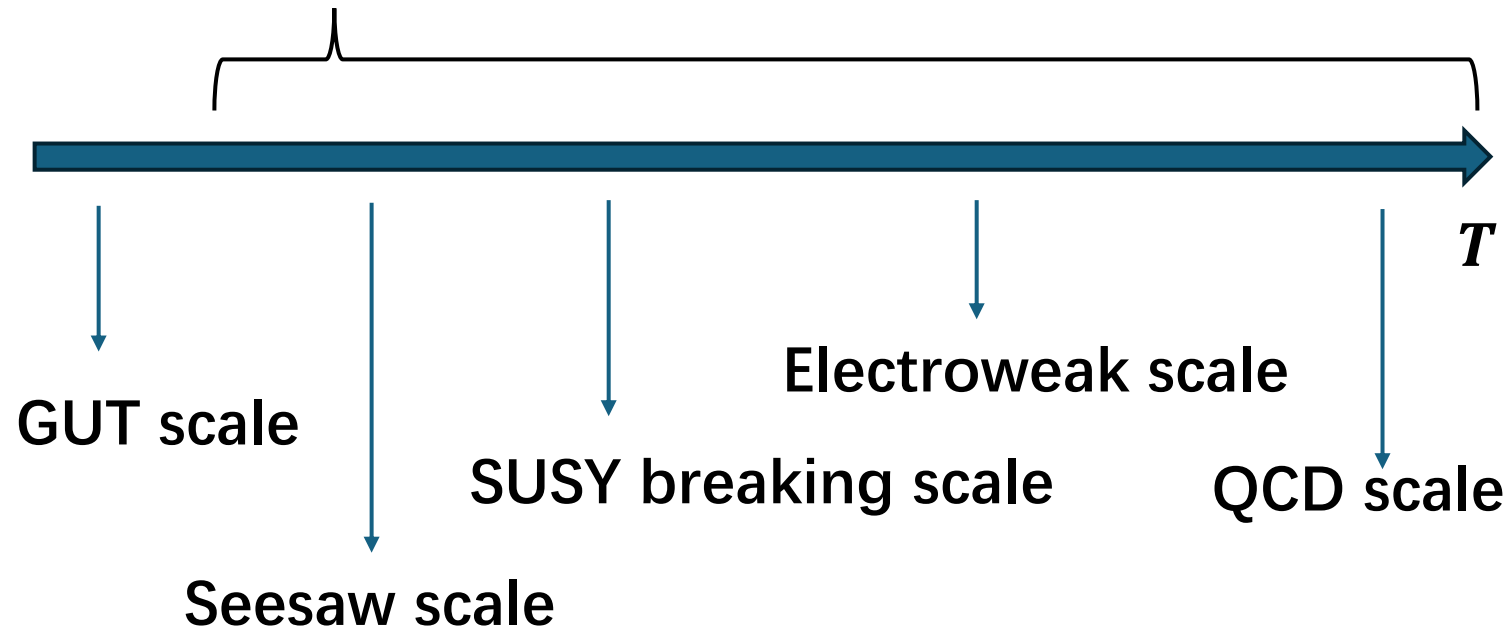
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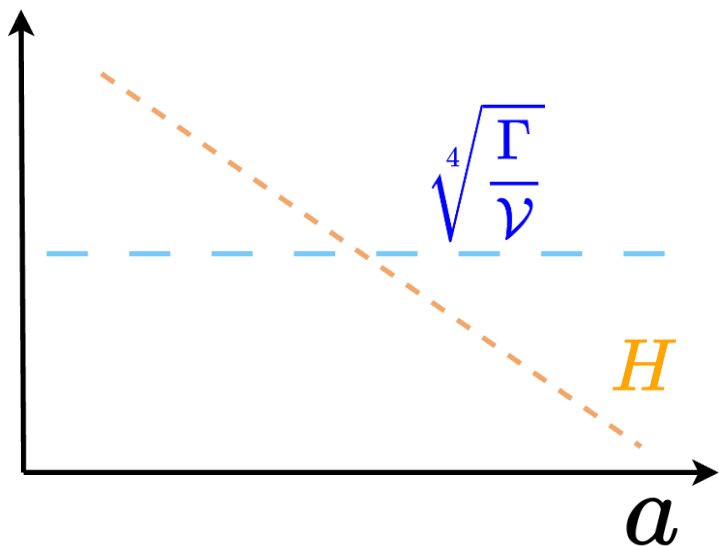
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Vacuum decay PT:

$$\frac{\Gamma}{\mathcal{V}} = \text{constant}$$

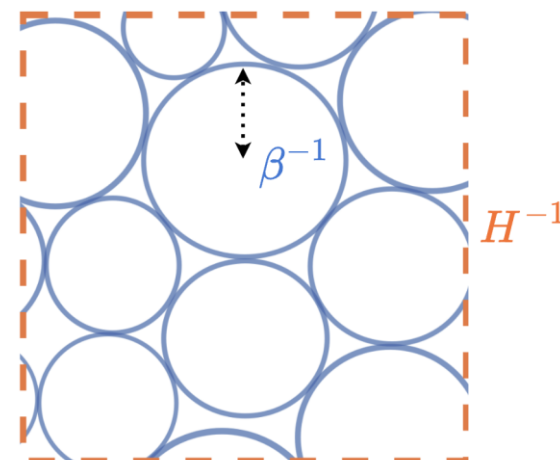
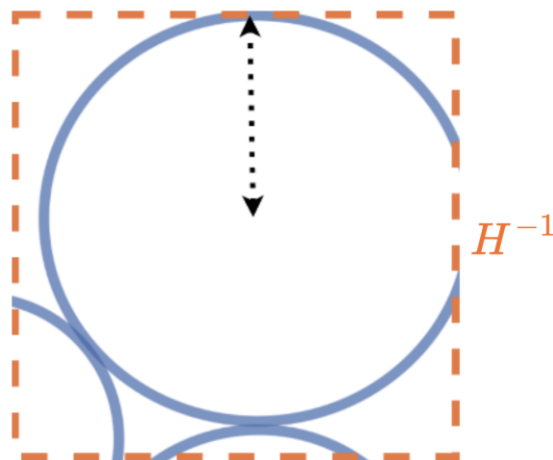
(No fluid, no plasma)

Thermal PT:

$$\frac{\Gamma}{\mathcal{V}} \sim \exp(\beta t)$$

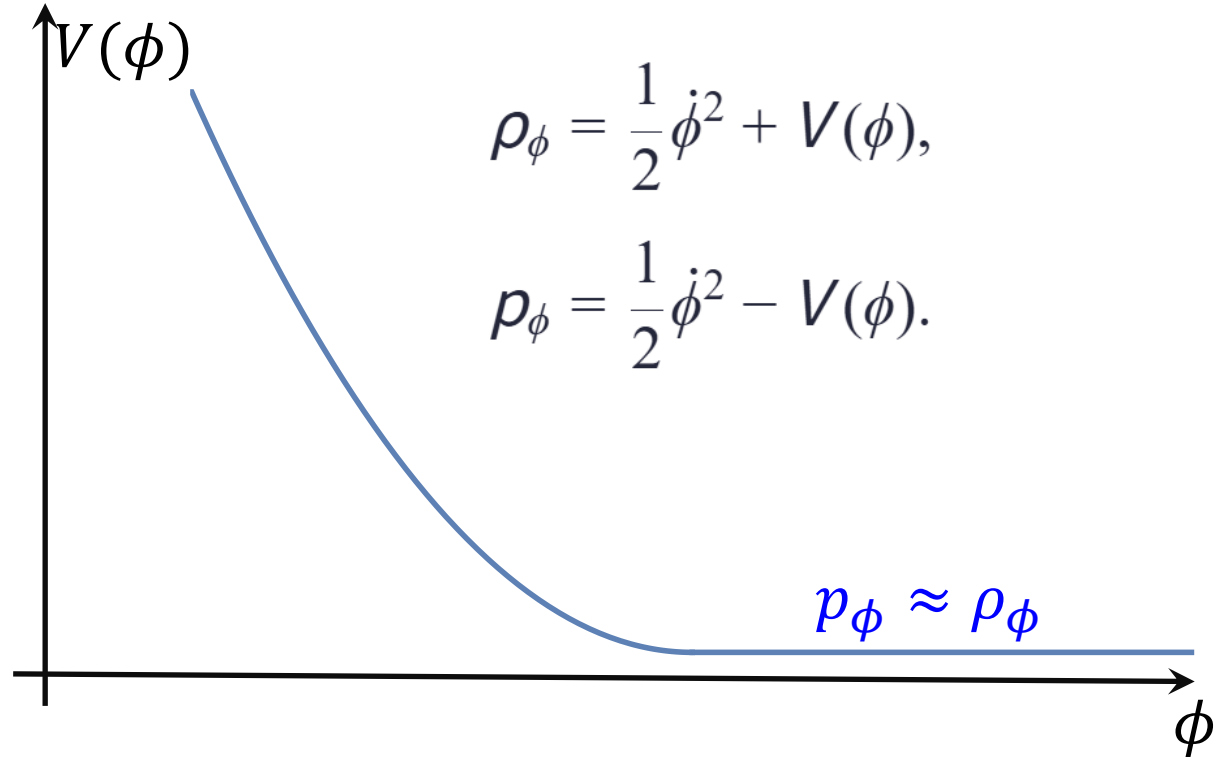
Phase transition condition:

$$\frac{\Gamma}{\mathcal{V}} \sim H^4$$



Simplest realization of kination domination

- The inflaton field potential becomes flat right after the end of inflation.
- Inflaton field transits all its energy into its kinetic energy.
- $p_\phi \approx \rho_\phi$
- $n = 6$ (Friedmann equations)



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