# Anarchy, Neutrinoless double beta decay and Leptogenesis

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# Outline

- What is Anarchy Approach

- Anarchy approach applied to neutrino physics

    Neutrinoless double beta decay

    Leptogenesis

# What is Anarchy Approach?

*A method to study the parameter space*

known unknown

$$a = (\lambda, \theta, \alpha, \delta, m)$$

explain ↑ ↑ predict

the next-layer model ?

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$$\rho(a) da$$

- Dimensional analysis
- Symmetries
- Cuts

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Anarchy

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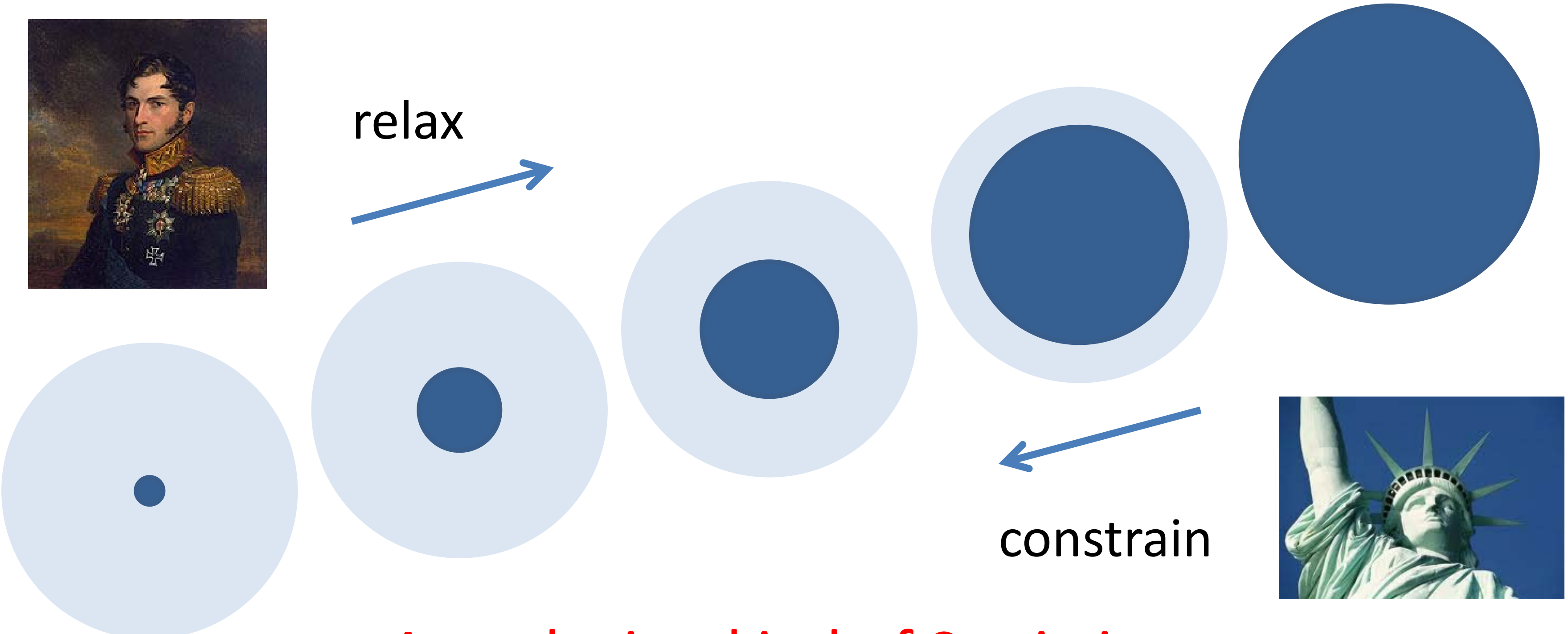
# How to understand anarchy

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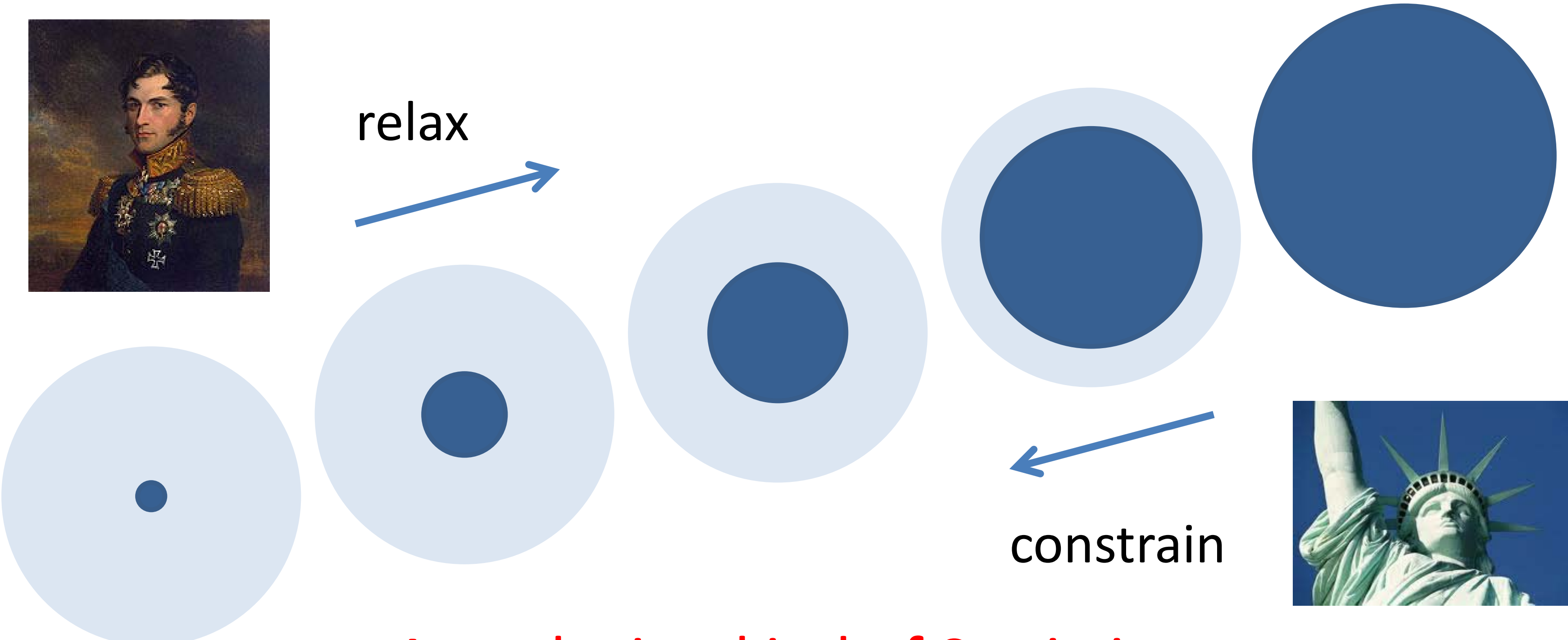


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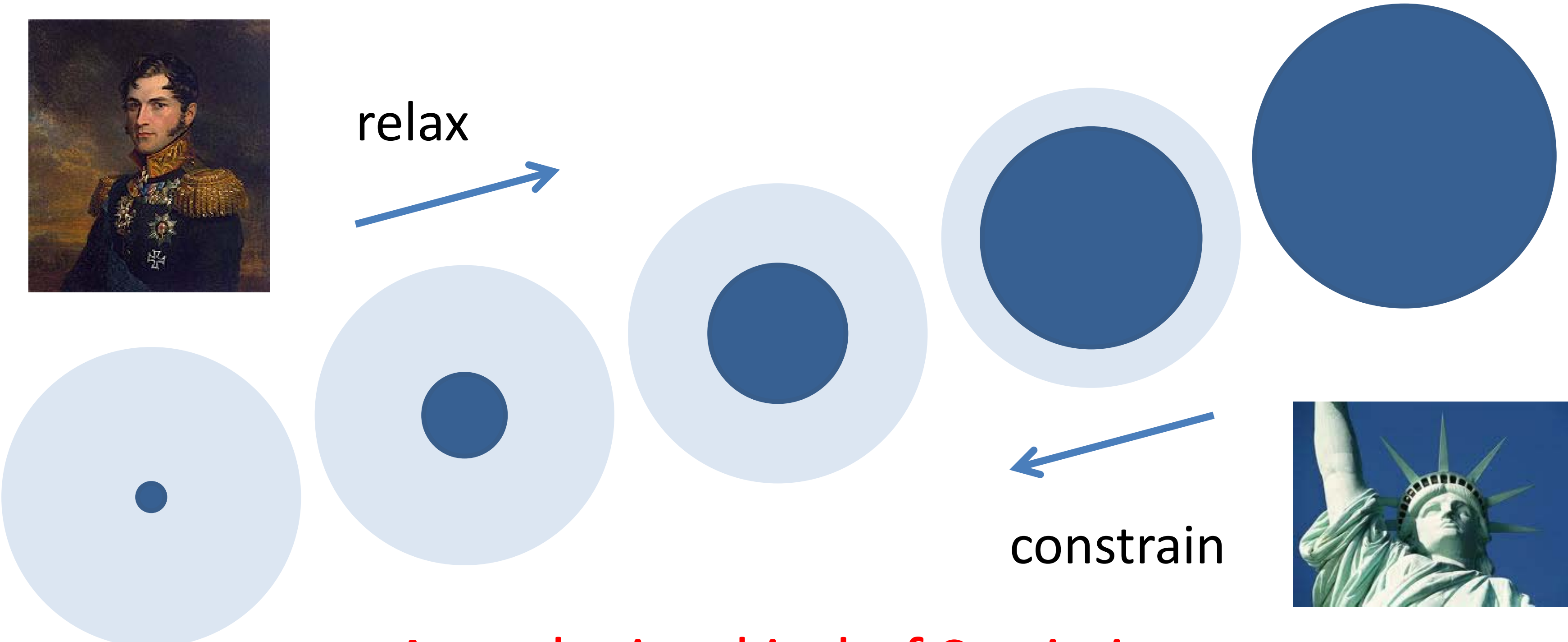


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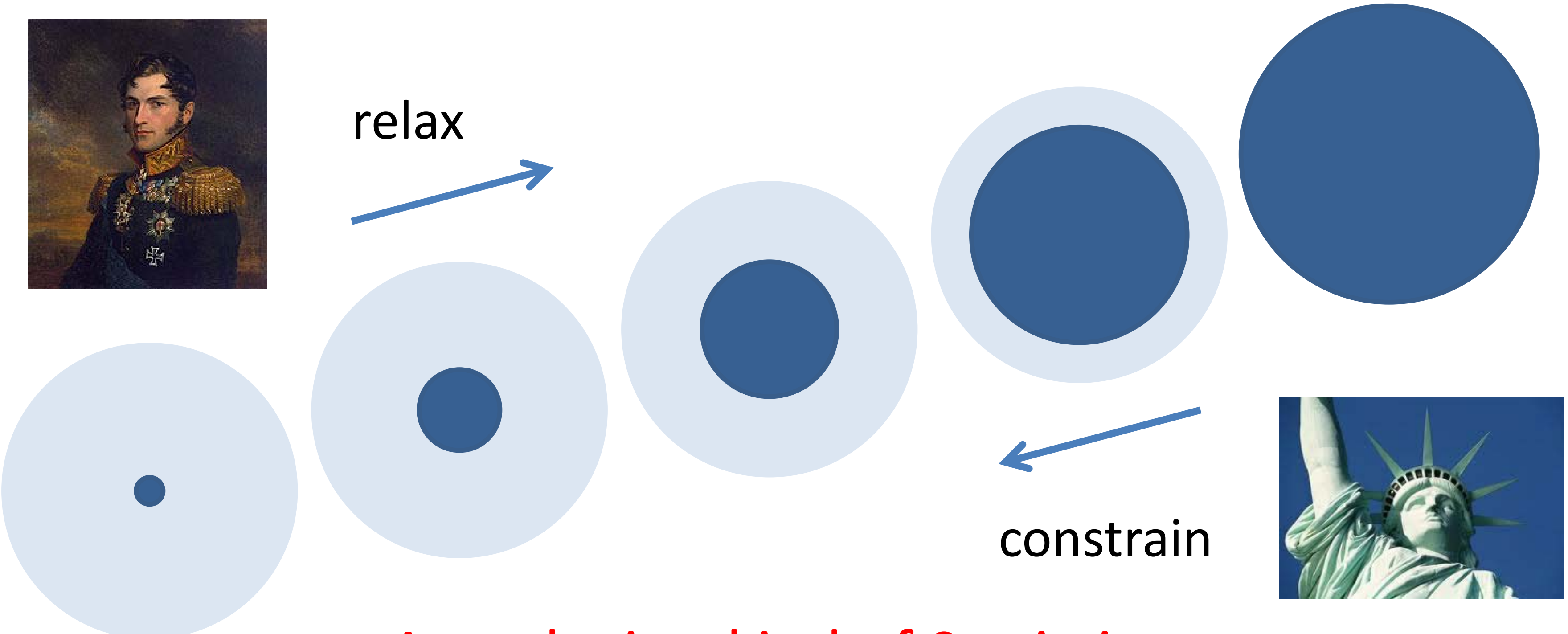


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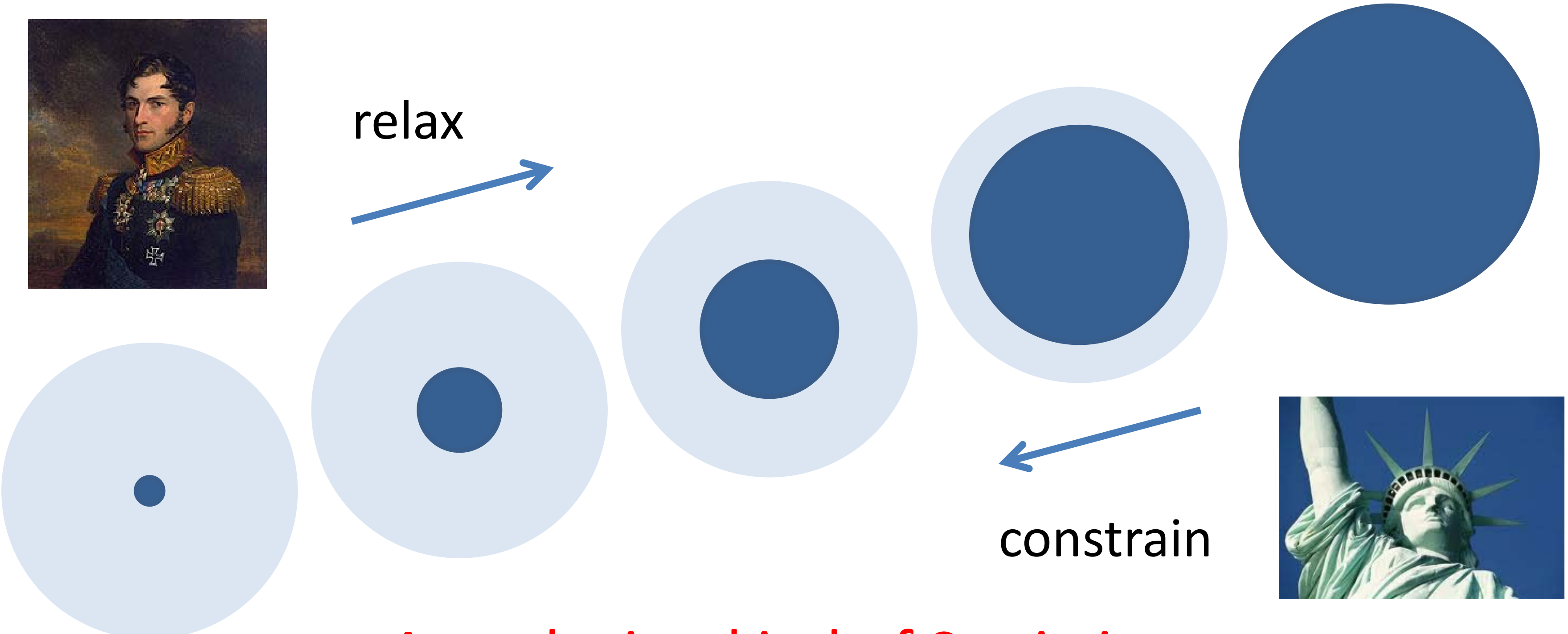


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Anarchical model

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correlation

# Neutrino Anarchy: model

**Seesaw Mechanism, 3 generations**

$$\mathcal{L} \supset -\frac{1}{2}\begin{pmatrix}\overline{\nu}_L & \overline{\nu}_R^c\end{pmatrix}\begin{pmatrix}0 & m_D \\ m_D{}^T & m_R\end{pmatrix}\begin{pmatrix}\nu_L^c \\ \nu_R\end{pmatrix} + h.c.$$

$$m_\nu = m_D m_R{}^{-1} m_D{}^T$$

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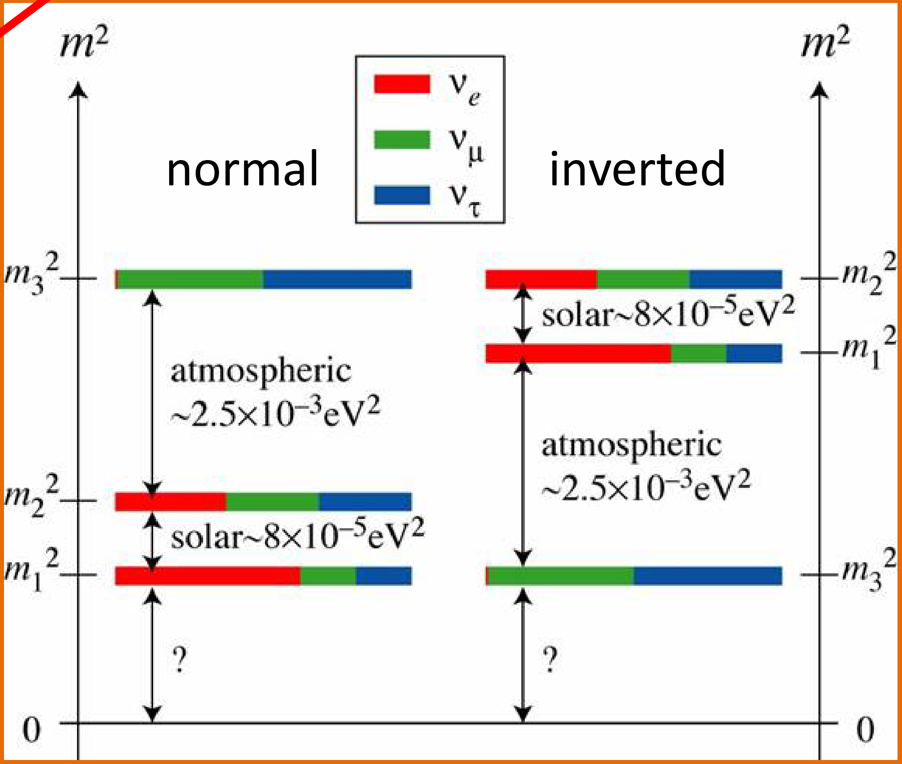
## Masses

$$\Delta m_{21}^2 \equiv m_2^2 - m_1^2$$
$$\Delta m_{32}^2 \equiv m_3^2 - m_2^2$$



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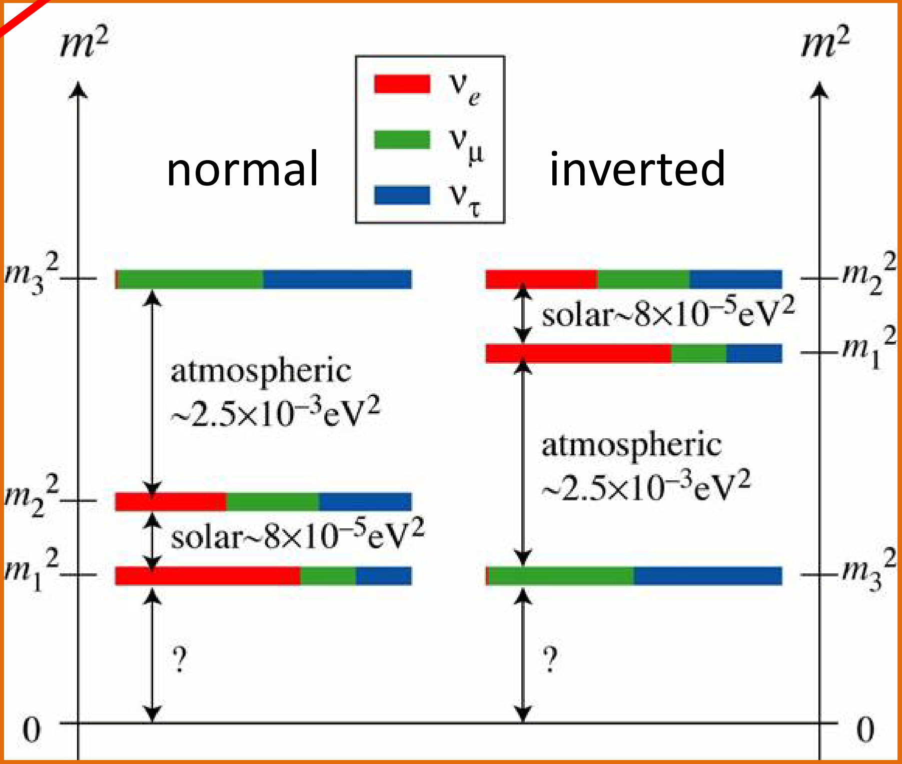


**Mixings**

$$\begin{pmatrix}\nu_e \\ \nu_\mu \\ \nu_\tau\end{pmatrix} = U_\nu \begin{pmatrix}\nu_1 \\ \nu_2 \\ \nu_3\end{pmatrix}$$

# Neutrino Anarchy: parameters

**known**

$$\Delta m_{21}^2 = (7.50 \pm 0.20) \times 10^{-5} eV^2$$

$$\left|\Delta m_{32}^2\right| = 2.32^{+0.12}_{-0.08} \times 10^{-3} eV^2$$

$$\sin^2 2\theta_{12} = 0.857 \pm 0.024$$

$$\sin^2 2\theta_{23} > 0.95 \text{ (90\% C.L.)}$$

$$\sin^2 2\theta_{13} = 0.095 \pm 0.010$$

**unknown**

Neutrino mass scale $m_1$

Sign of $\Delta m_{32}^2$ (mass hierarchy)

CP phase $\delta$

Other physical phases $\chi_1, \chi_2$

Neutrinoless double beta decay $m_{\text{eff}}$

Leptogenesis $\eta_{B0}$

# Neutrino Anarchy: parameters

distribution (measure)

$dm_D \quad dm_R$

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# What measure $dm_D, dm_R$ to choose?

A tempting choice: Entry Independence

$$m_D = \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \qquad m_R = \begin{pmatrix} * & * & * \\ & * & * \\ & & * \end{pmatrix}$$

Each free entry $*$ is generated independently

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$$\nu'_R = U_{R0}\nu_R \qquad m_R \to m'_R = U^{*}_{R0} m_R U_{R0}^{\dagger} \qquad dm'_R = dm_R$$

$$\forall U_{L0}, U_{R0}$$

# Basis Independence

Parameterization

$$m_D = \text{unitD} \cdot U_1 D_0 U_2^{\dagger}$$

$$m_R = \text{unitM} \cdot U_R D_R U_R^{T}$$

Factorize

$$dm_D = dU_1 dU_2 dD_0$$

$$dm_R = dU_R dD_R$$

Haar measure $dU_1, dU_2, dU_R$

$dU_R$ invariant $U_R \to U U_R$

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$U_1$ Haar measure ⇒ $U_\nu$ Haar measure

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$dU_R$ **invariant** $U_R \to UU_R$

$U_1$ Haar measure ⇒ $U_\nu$ Haar measure

$$U_\nu = e^{i\eta} e^{i\phi_1 \lambda_3 + i\phi_2 \lambda_8} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} e^{i\chi_1 \lambda_3 + i\chi_2 \lambda_8}$$

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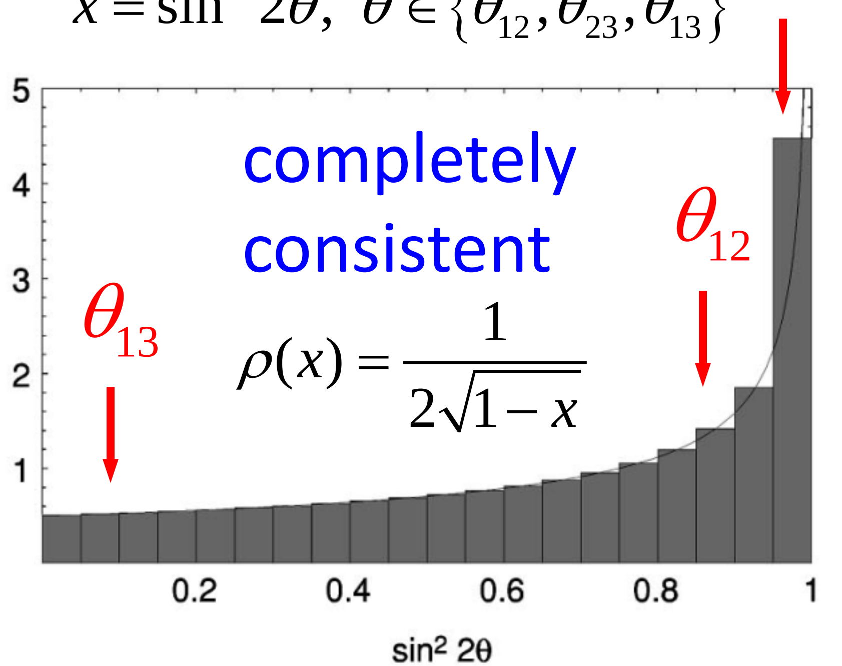
**Haar measure** $dU_\nu = ds_{23}^2 dc_{13}^4 ds_{12}^2 \cdot d\delta d\chi_1 d\chi_2 \cdot d\eta d\phi_1 d\phi_2$

# Distribution of mixing angles

$$dU_\nu = ds_{23}^2 dc_{13}^4 ds_{12}^2 \cdot d\delta d\chi_1 d\chi_2 \cdot d\eta d\phi_1 d\phi_2$$

$$x = \sin^2 2\theta, \ \ \theta \in \{\theta_{12}, \theta_{23}, \theta_{13}\}$$



# Distribution of phases

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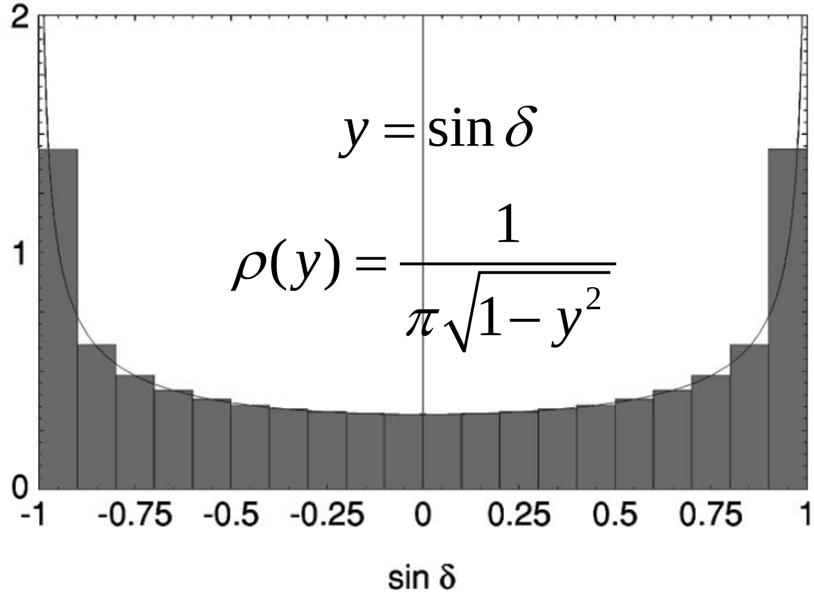

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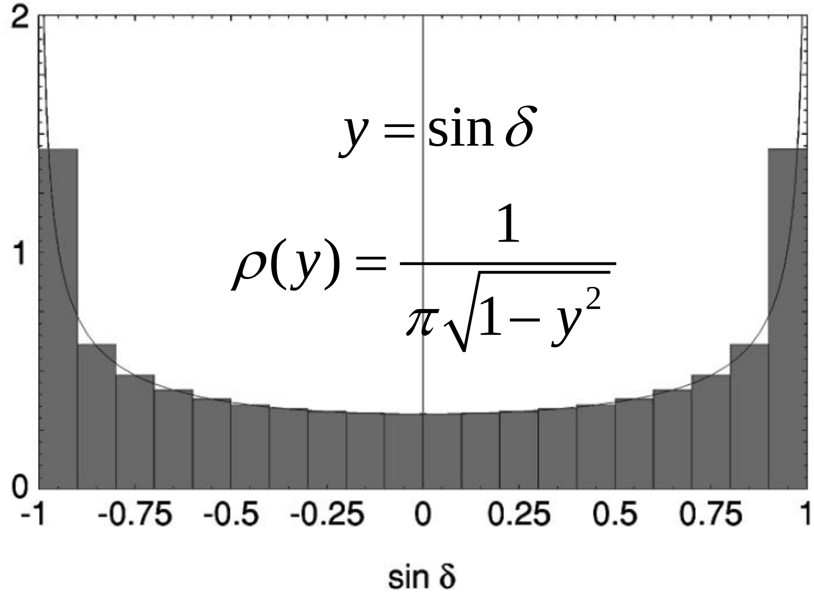


$$P_{e\mu} - P_{\bar{e}\bar{\mu}} = -16 c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta \sin\frac{\Delta m_{12}^2 L}{4E} \sin\frac{\Delta m_{13}^2 L}{4E} \sin\frac{\Delta m_{23}^2 L}{4E}$$

# Entry Independence

Parameterization

$$m_D = \text{unitD} \cdot U_1 D_0 U_2^{\dagger}$$

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Basis Independence ⇒

$U_1, U_2, D_0, U_R, D_R$ independent

$dU_1, dU_2, dU_R$ Haar measure

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Entry Independence

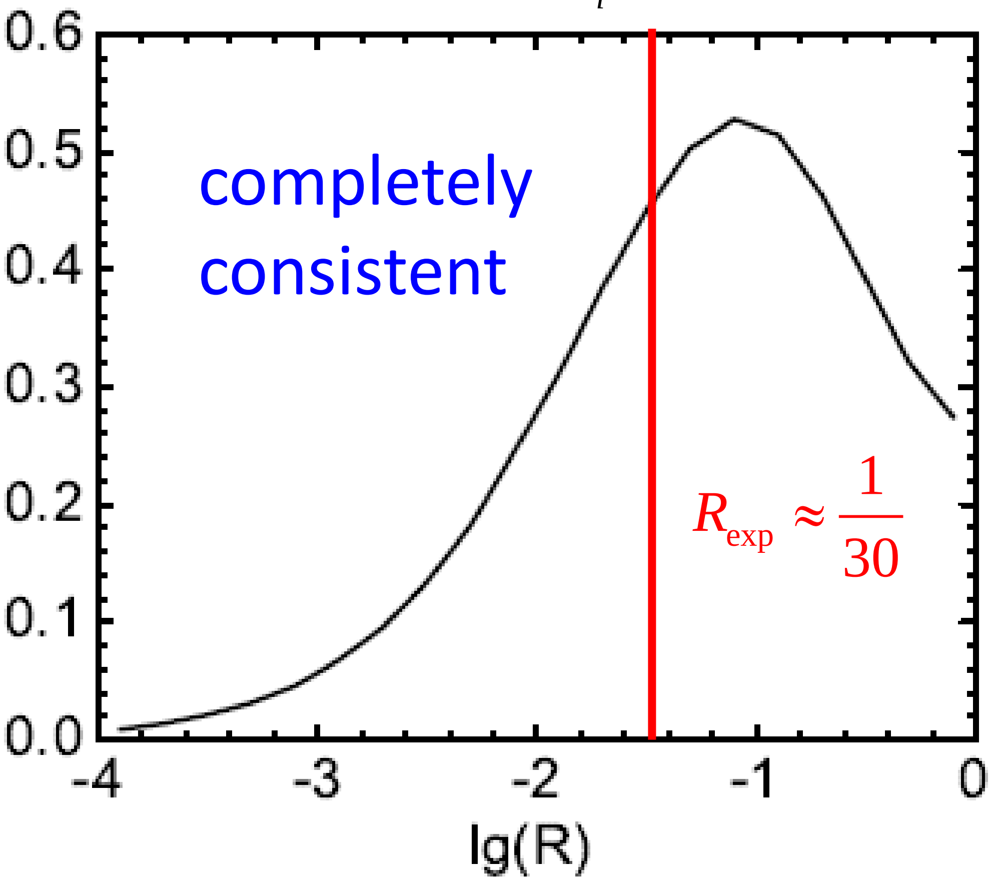
$\Rightarrow$ $U_1, U_2, D_0, U_R, D_R$ independent

$dU_1, dU_2, dU_R$ Haar measure

$\Rightarrow$ Each free entry Gaussian measure

# Mass splits and mass hierarchy

$$R \equiv \frac{\Delta m_s^2}{\Delta m_l^2}$$



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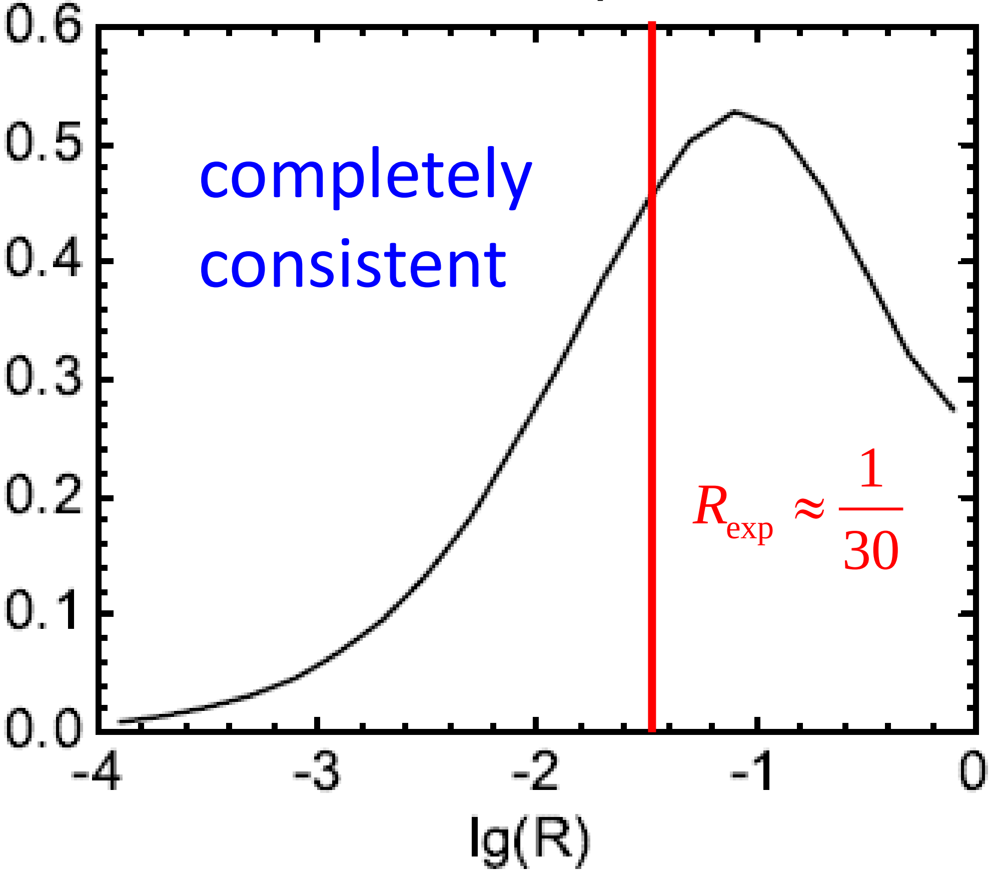


## The Cuts

$$R \in \frac{7.59\times10^{-5}}{2.32\times10^{-3}}\times(1\pm0.05)$$

$$\sin^2 2\theta_{23} = 1.0$$

$$\sin^2 2\theta_{12} = 0.861$$

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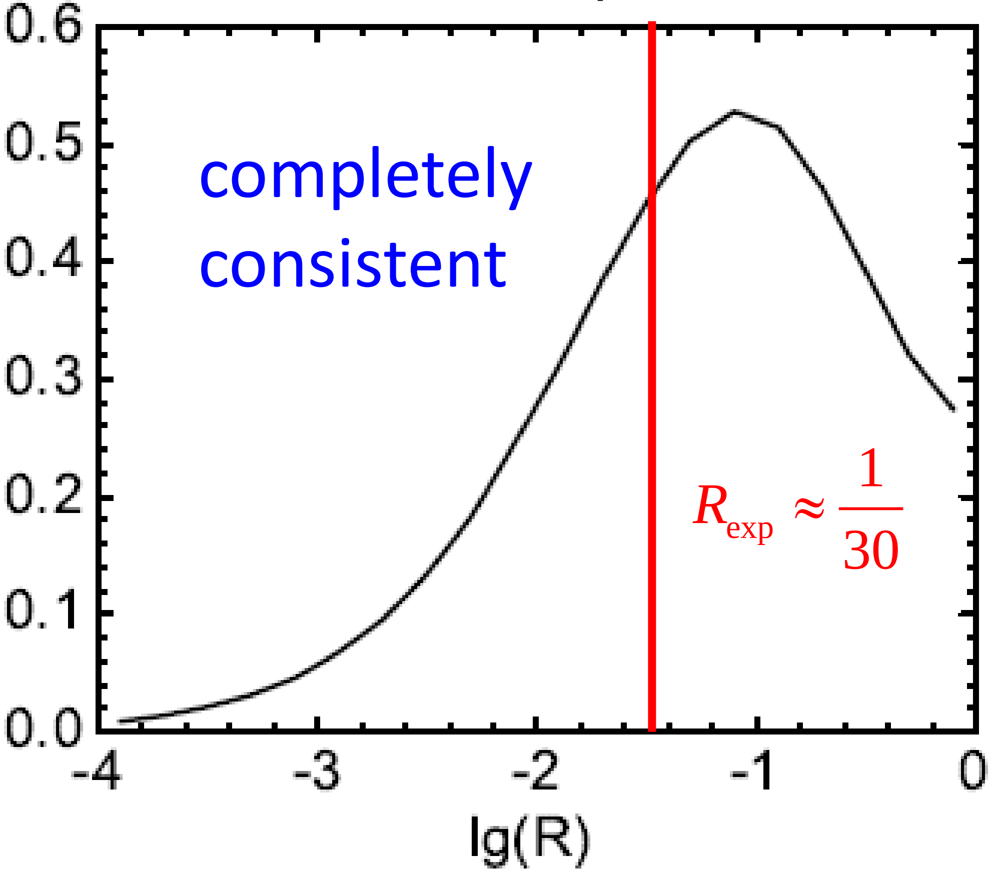


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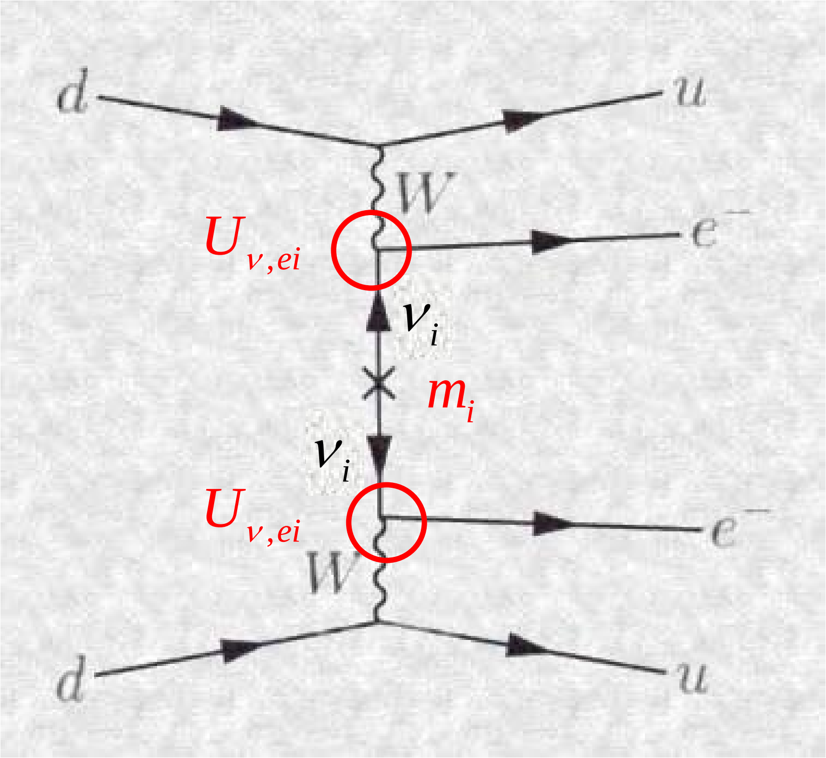
Normal Hierarchy Scenario

without cut 95.9%

with cut 99.9%

# Neutrinoless double beta decay




lepton number violation

$$\Delta L = 2$$

light-neutrino mass matrix $m_\nu$ is Majorana

Anarchy prediction:

$$m_\nu = m_D m_R^{-1} m_D^T$$

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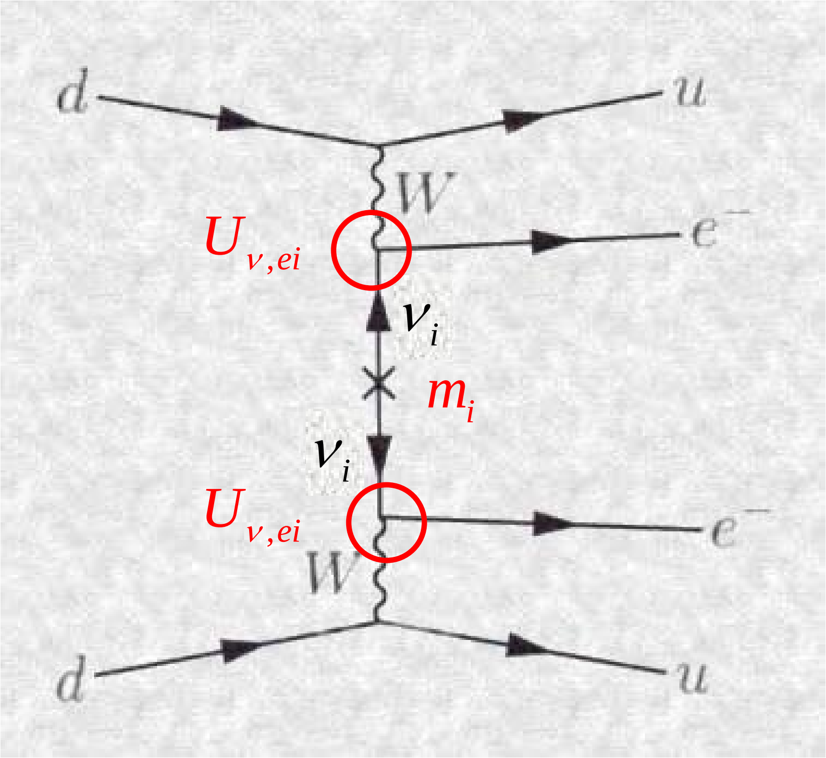


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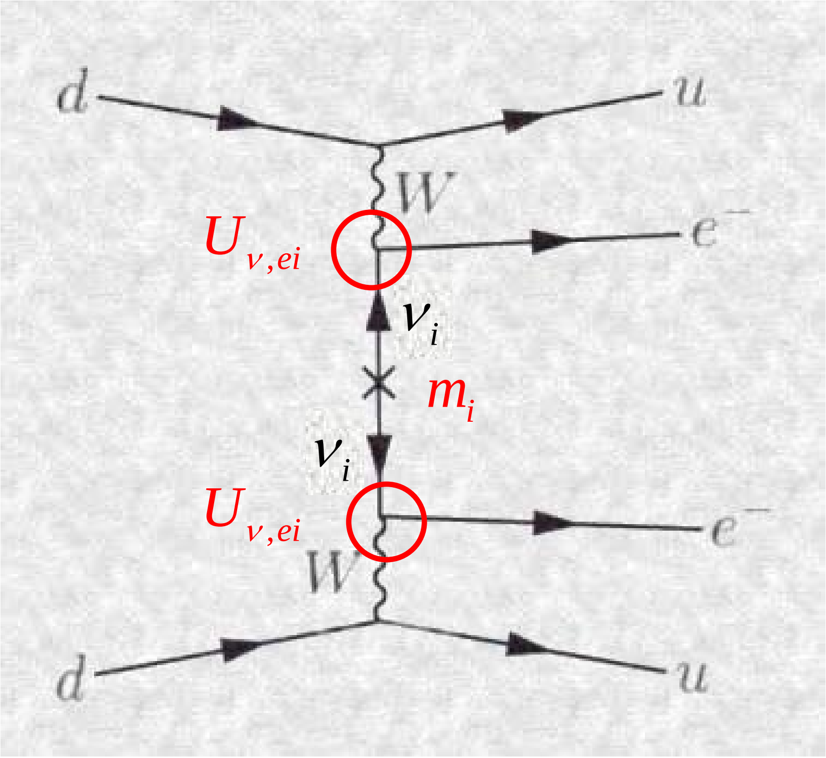


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**Anarchy prediction:**

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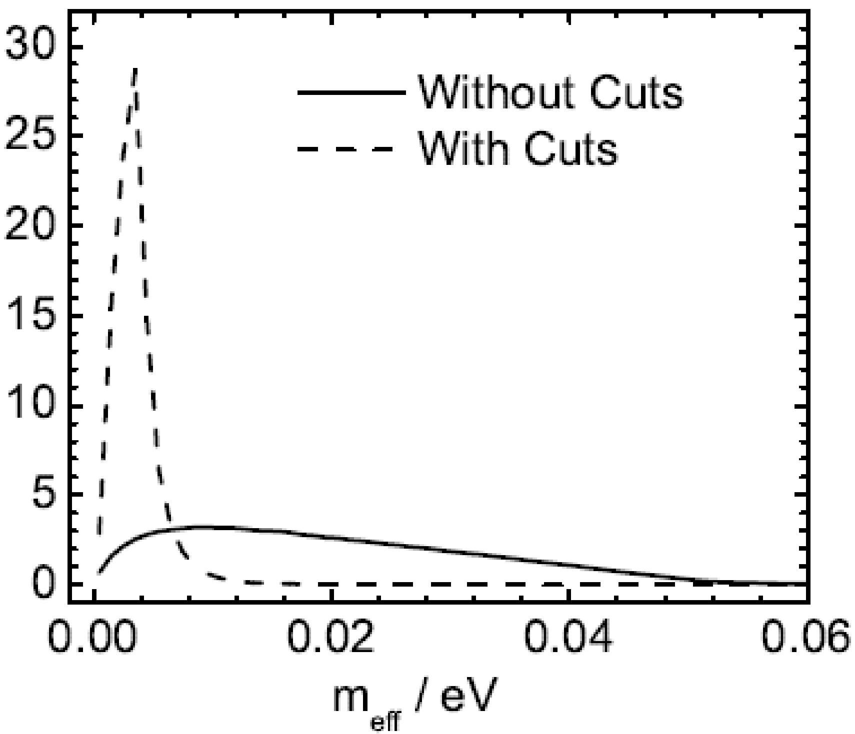
$$\Gamma_{0\nu\beta\beta} \propto m_{\text{eff}}^2, \quad m_{\text{eff}} = \left| \sum_i U_{\nu,ei}{}^2 m_i \right|$$

# Neutrinoless double beta decay

$$\text{unitD} = 30GeV$$

$$\text{unitM} \rightarrow \left|\Delta m_{32}^2\right| = 2.5\times10^{-3}eV^2$$



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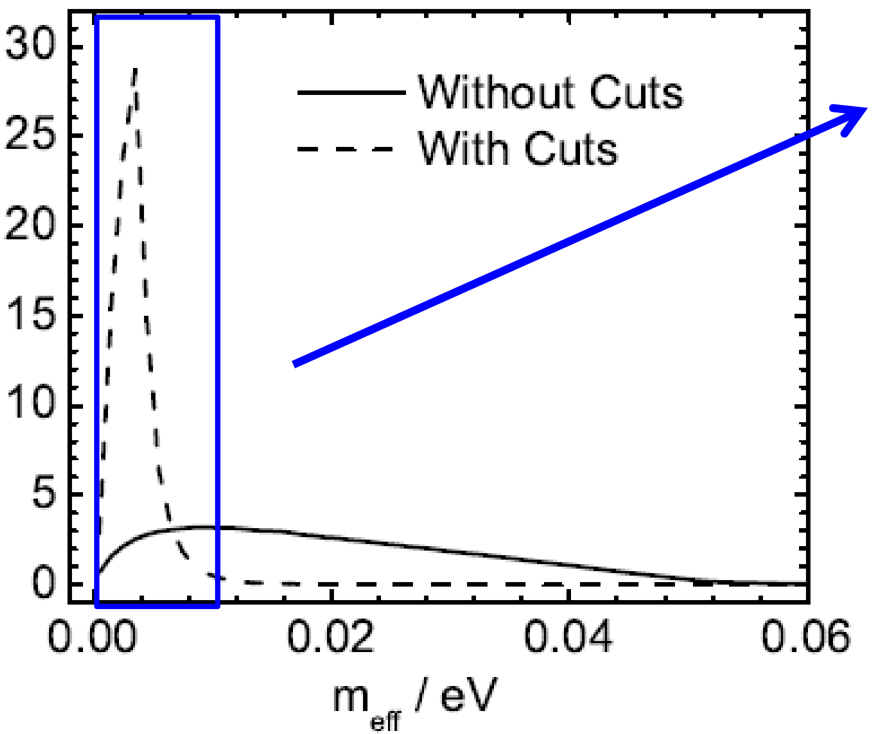


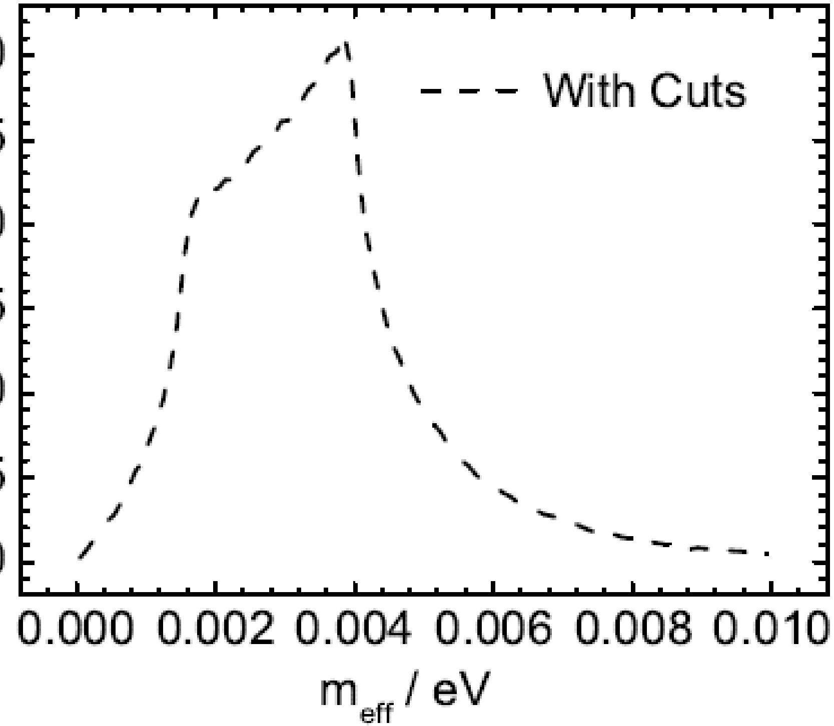

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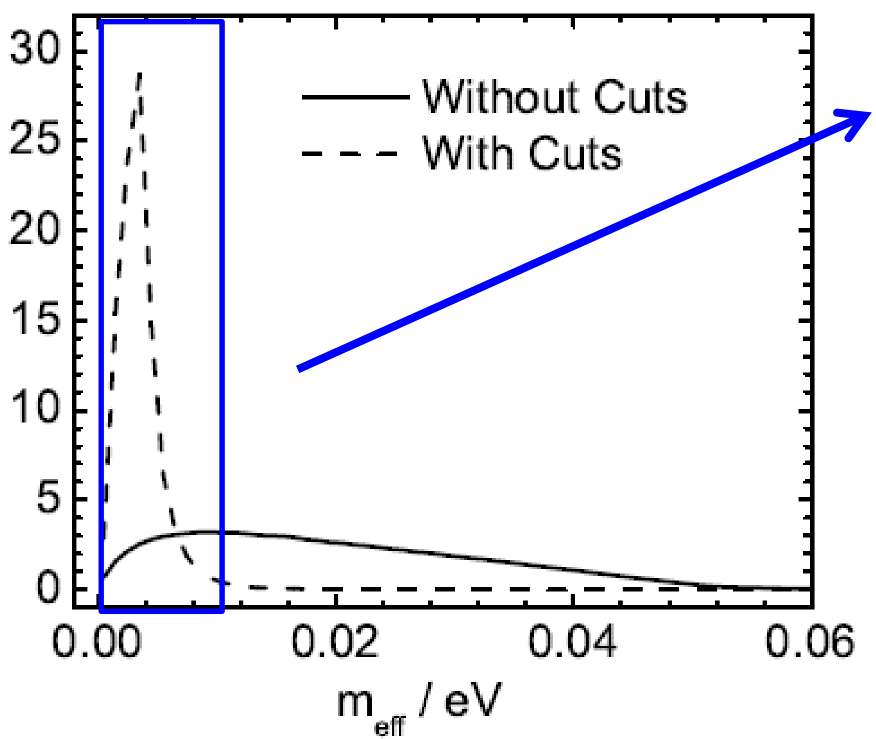


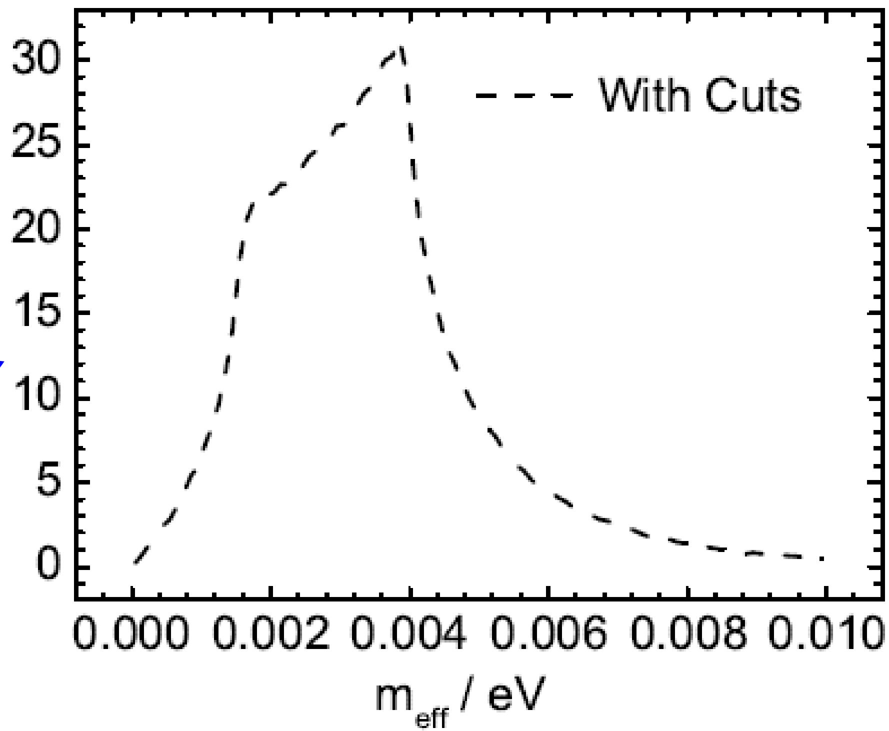


$m_{\text{eff}} \lesssim 0.05eV$ without cuts

$m_{\text{eff}} \lesssim 0.01eV$ with cuts

experimentally challenging

# Leptogenesis

Baryon asymmetry today

$$\eta_{B0} = \frac{n_{B0}}{n_\gamma} \sim 6 \times 10^{-10}$$

$m_R, m_D$ complex   CP violation

$$N_1 \to l\bar{\phi} \neq N_1 \to \bar{l}\,\phi$$

$$\downarrow$$

$$L \neq 0$$

$$\downarrow$$

$$B - L \neq 0$$

$$\downarrow$$

$$B = 0.35(B - L)$$

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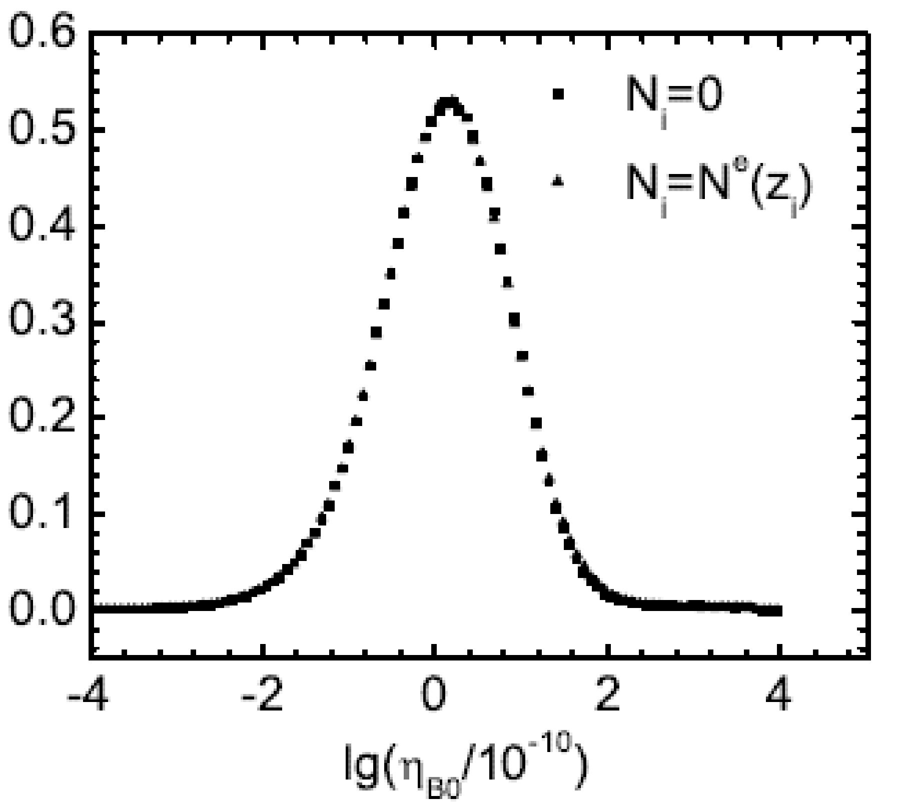
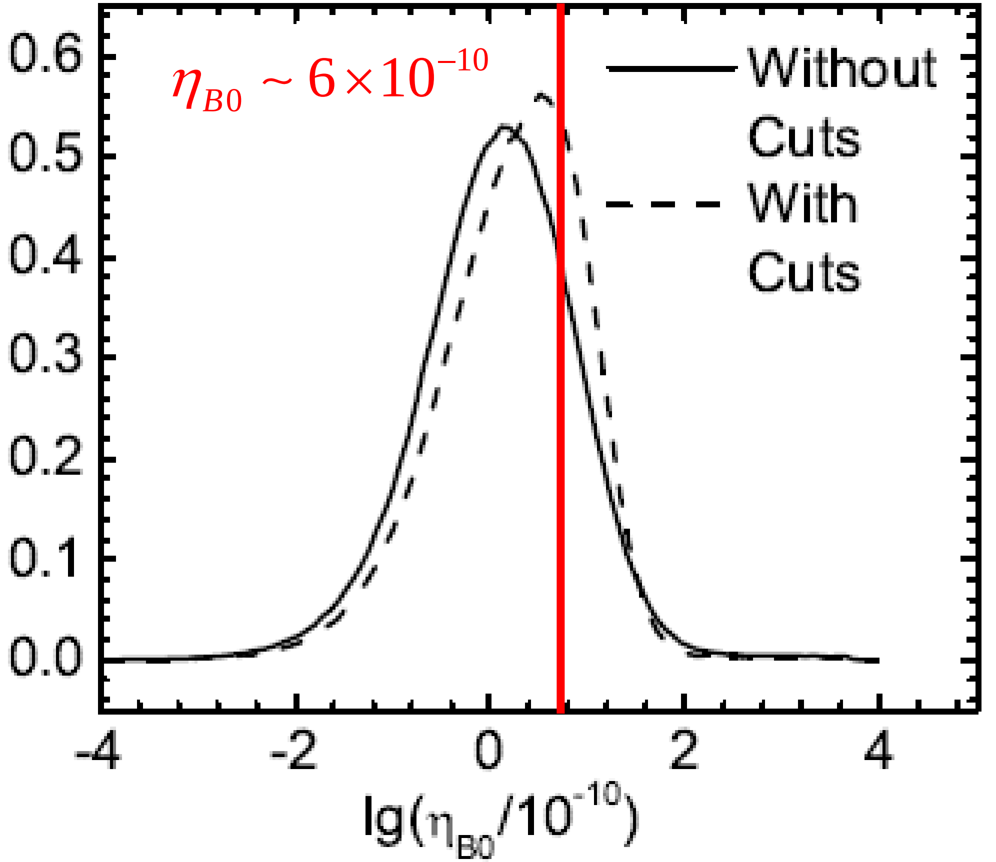
$$\downarrow$$

$$B = 0.35(B-L)$$



# Correlations

Apply the Cuts




light-neutrino masses and mixings favor leptogenesis

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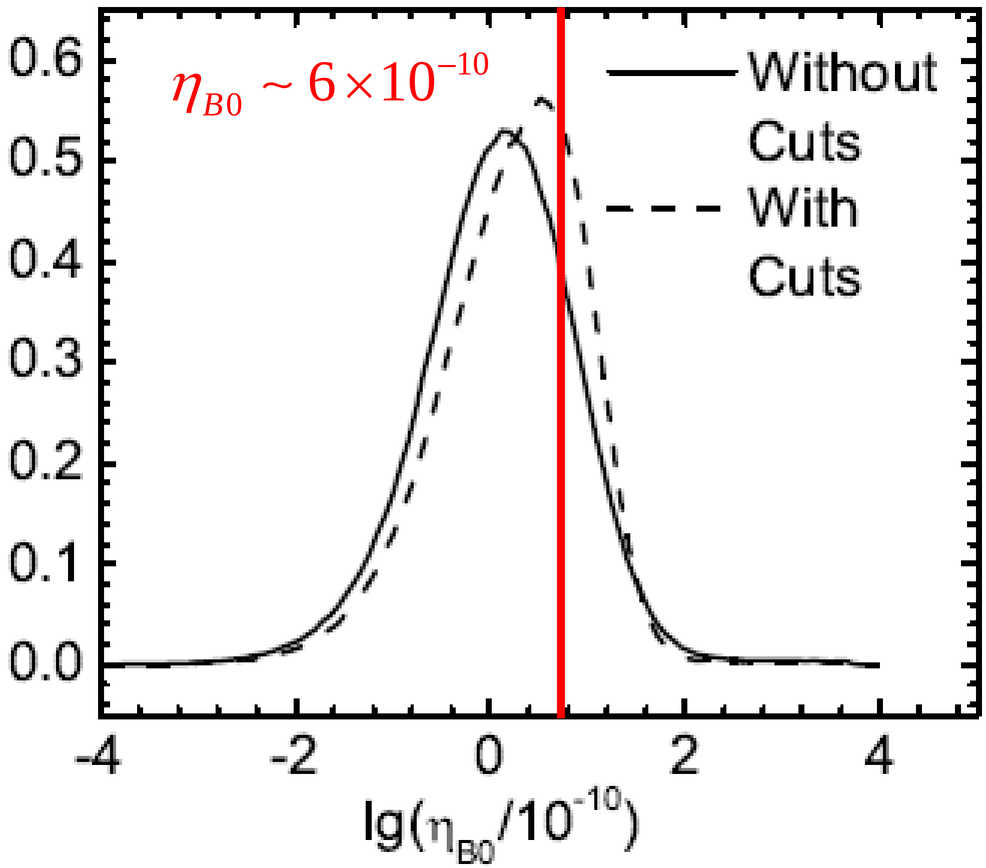
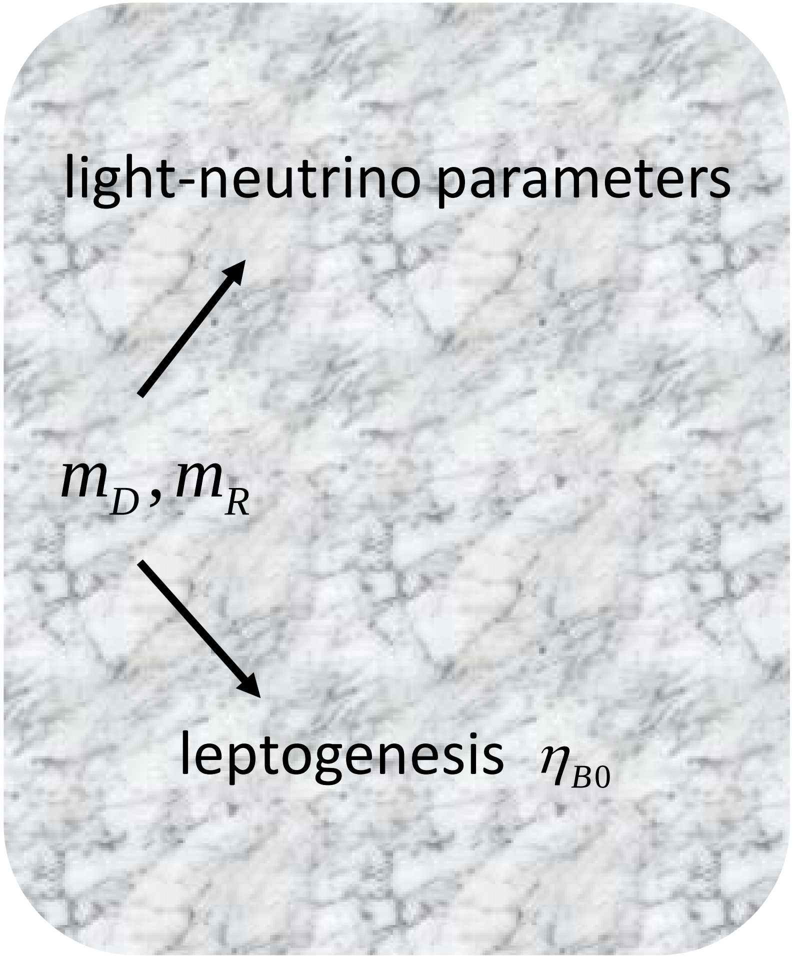
Apply the Cuts




light-neutrino masses and mixings favor leptogenesis

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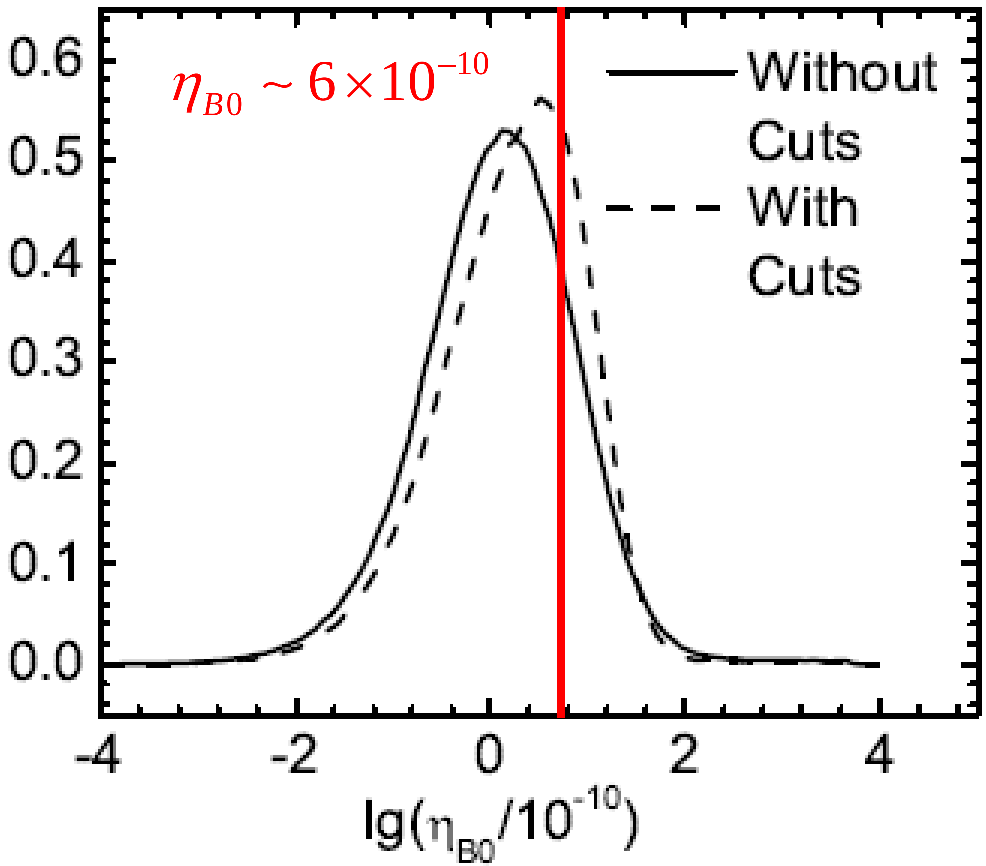
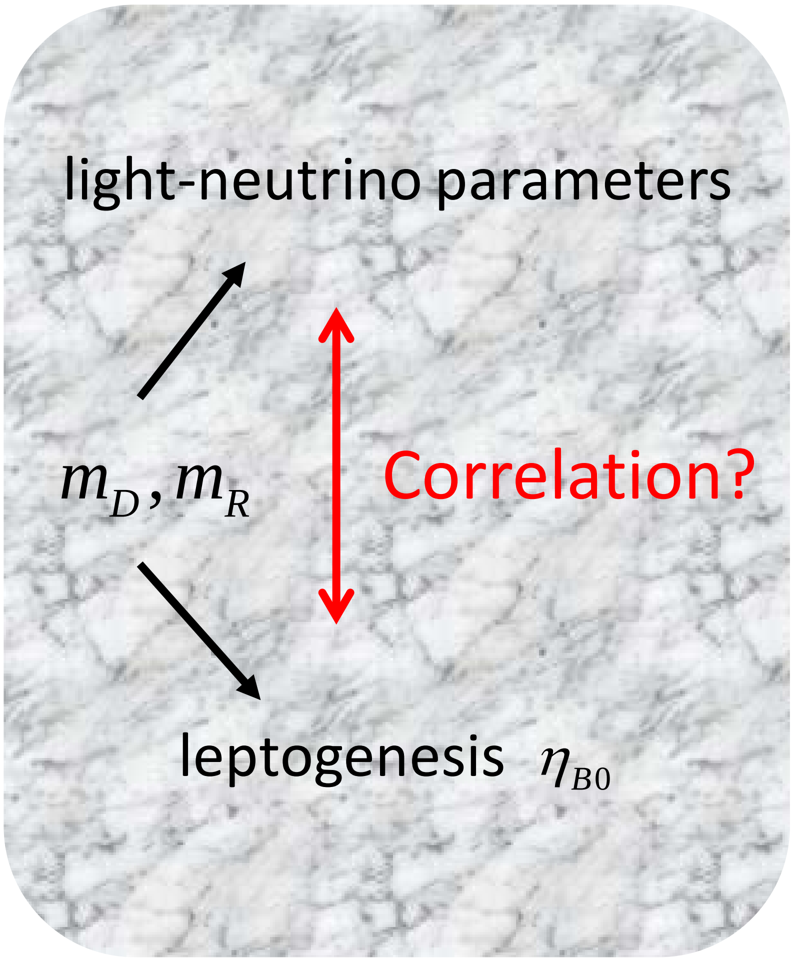



light-neutrino masses and mixings favor leptogenesis

# Correlations

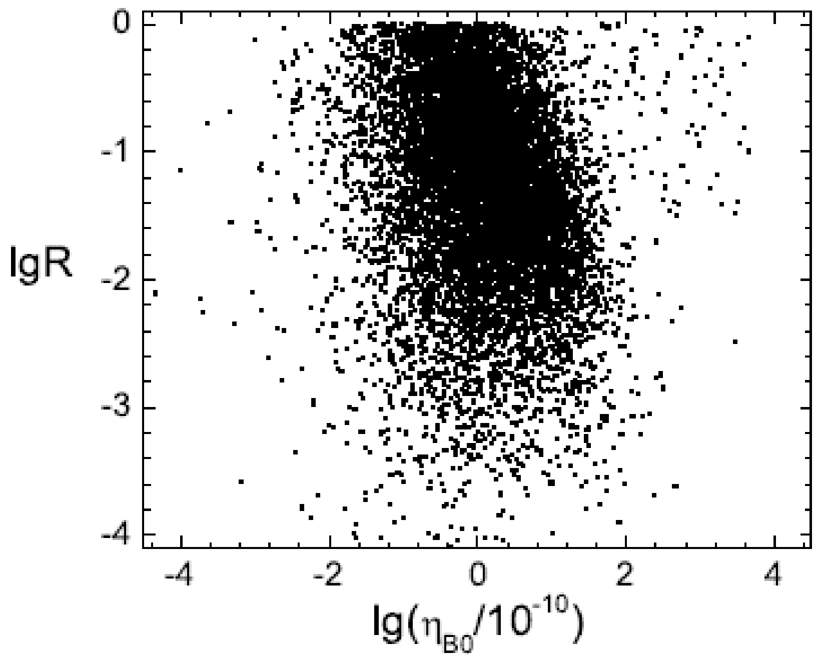
$\eta_{B0}$ has no correlation with light-neutrino mixings $U_\nu$

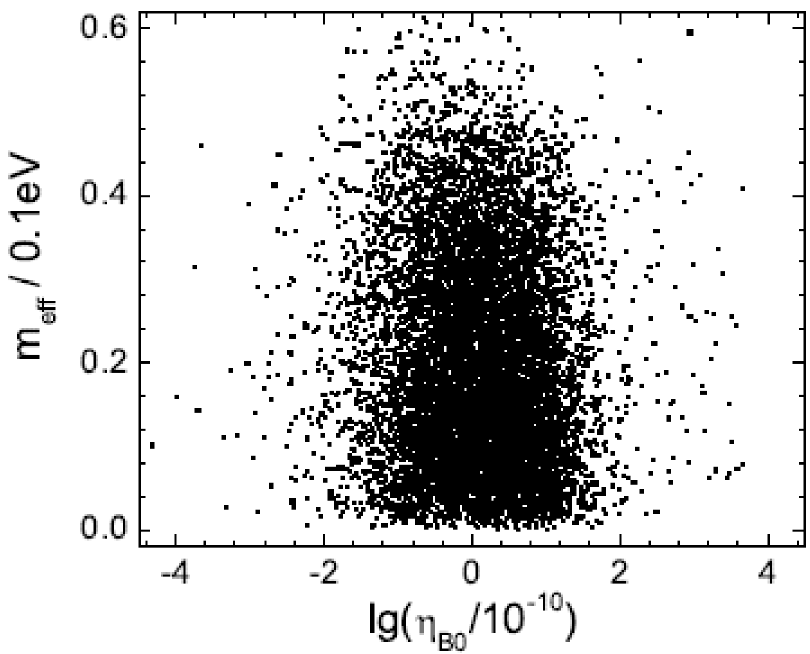
$\eta_{B0}$ has no correlations with $\theta_{12}, \theta_{13}, \theta_{23}, \chi_1, \chi_2$ or $\delta$

$\eta_{B0}$ could have correlations with $D_\nu$ and thus $R, m_{\rm eff}$

# Correlations

scatter plots:



# Correlations

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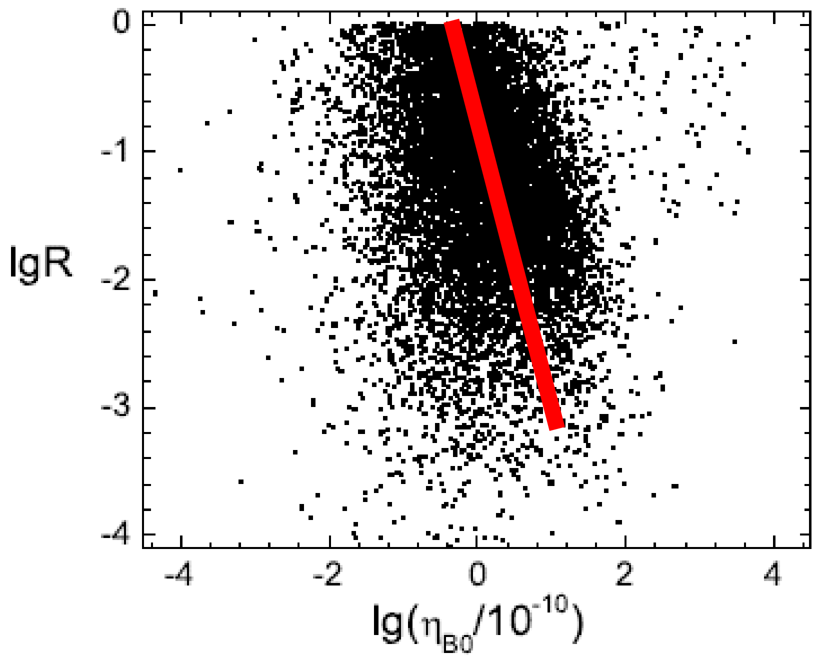
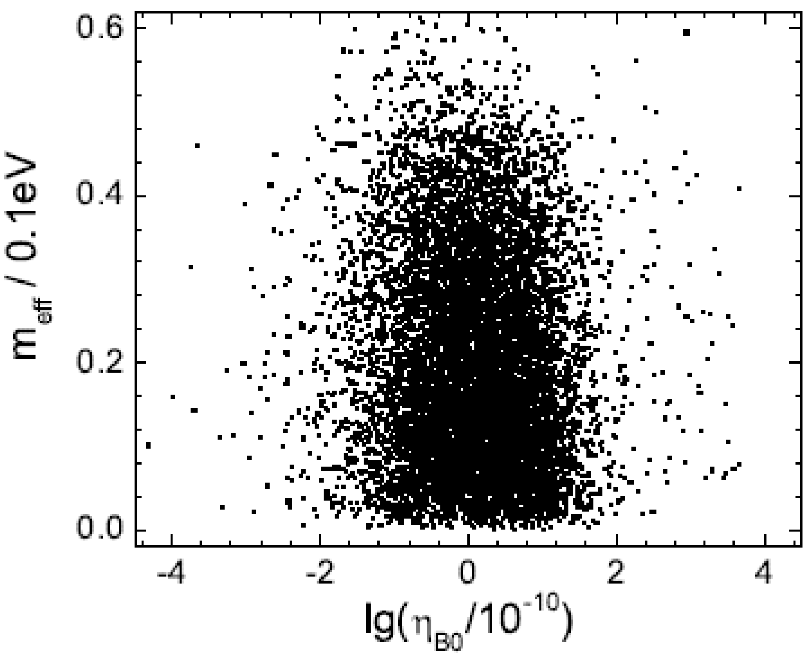
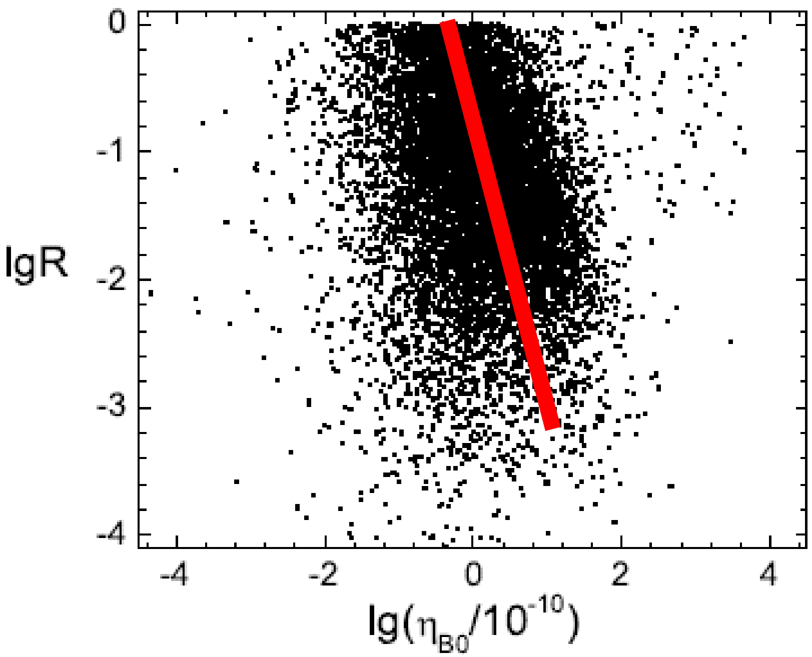
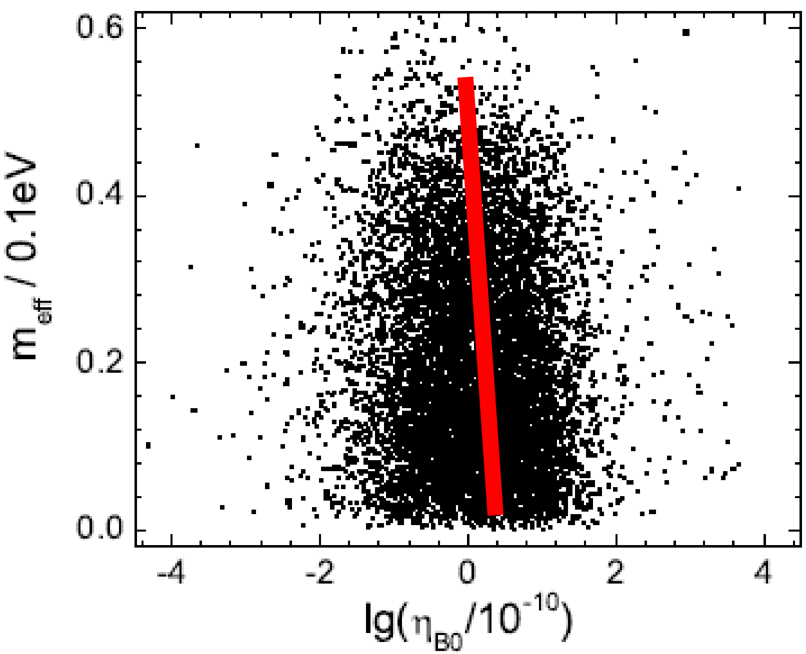


# Correlations

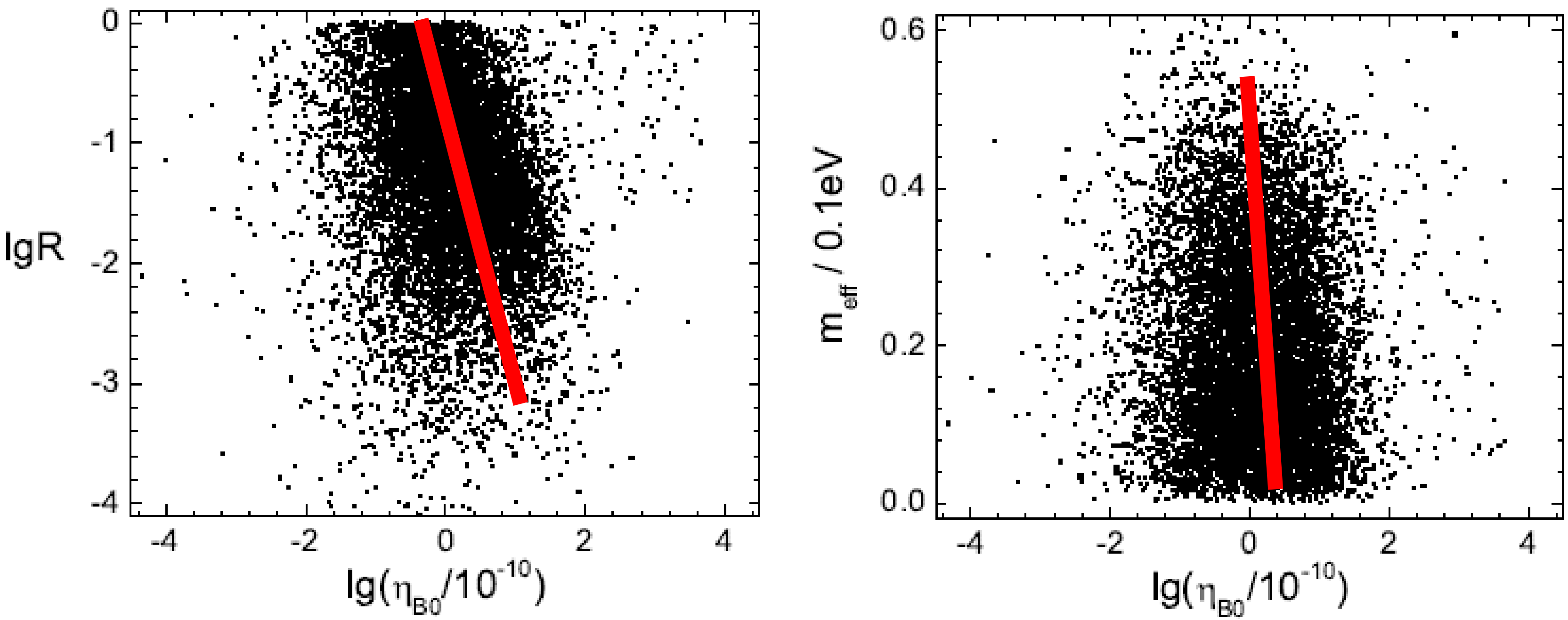
scatter plots:



# Correlations

scatter plots:




$\eta_{B0}$ has a weak negative correlation with $R$ and $m_{\text{eff}}$

# Summary

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Completely Consistent

$$\sin^2 2\theta_{12} = 0.857 \pm 0.024$$

$$\Delta m^2_{21} = (7.50 \pm 0.20) \times 10^{-5} eV^2$$

$$\sin^2 2\theta_{23} > 0.95 \text{ (90\% C.L.)}$$

$$\left|\Delta m^2_{32}\right| = 2.32^{+0.12}_{-0.08} \times 10^{-3} eV^2$$

$$\sin^2 2\theta_{13} = 0.095 \pm 0.010$$

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$\sin^2 2\theta_{12} = 0.857 \pm 0.024$

$\Delta m_{21}^2 = (7.50 \pm 0.20) \times 10^{-5} eV^2$

$\sin^2 2\theta_{23} > 0.95$ (90% C.L.)

$\left|\Delta m_{32}^2\right| = 2.32_{-0.08}^{+0.12} \times 10^{-3} eV^2$

$\sin^2 2\theta_{13} = 0.095 \pm 0.010$

Sign of $\Delta m_{32}^2$ — 99.9% normal hierarchy

CP phase $\delta$ and $\chi_1, \chi_2$ — uniform, maximal CP violation

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Sign of $\Delta m^2_{32}$ — 99.9% normal hierarchy

CP phase $\delta$ and $\chi_1, \chi_2$ — uniform, maximal CP violation

$m_{\mathrm{eff}}$ — very challenging to experimental sensitivity

$\eta_{B0}$:
- on the correct order of magnitude
- no correlations with $\theta_{12}, \theta_{13}, \theta_{23}, \chi_1, \chi_2$ or $\delta$
- weak negative correlations with $R$ and $m_{\mathrm{eff}}$

Thank you!